MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. The figure shows the graph of a function. At the given value of \( x \), does the function appear to be differentiable, continuous but not differentiable, or neither continuous nor differentiable?

1. \( x=0 \)
   - A. Differentiable
   - B. Continuous but not differentiable
   - C. Neither continuous nor differentiable

2. \( x=0 \)
   - A. Differentiable
   - B. Continuous but not differentiable
   - C. Neither continuous nor differentiable

3. \( x=-1 \)
   - A. Differentiable
   - B. Continuous but not differentiable
   - C. Neither continuous nor differentiable

4. \( x=1 \)
   - A. Differentiable
   - B. Continuous but not differentiable
   - C. Neither continuous nor differentiable
If the function is not differentiable at the given value of $x$, tell whether the problem is a corner, cusp, vertical tangent, or a discontinuity.

5. $y = -3|x| - 9$, at $x = 0$
   A. vertical tangent
   B. cusp
   C. corner
   D. function is differentiable at $x=0$

6. $y = \frac{8}{x+2}$, at $x = -2$
   A. vertical tangent
   B. cusp
   C. corner
   D. function is differentiable at $x=2$

7. $y = 3 - \sqrt[3]{x}$, at $x = 0$
   A. vertical tangent
   B. cusp
   C. corner
   D. function is differentiable at $x=0$

8. $y = \sqrt[3]{|x|+12}$, at $x = -12$
   A. vertical tangent
   B. cusp
   C. corner
   D. function is differentiable at $x=-12$

Determine the values of $x$ for which the function is differentiable.

9) $y = 6x - 1$
   A) All reals except 0.16666667
   B) All reals
   C) All reals except -1
   D) All reals except 6

10) $y = \frac{1}{x-7}$
    A) All reals except 1
    B) All reals
    C) All reals except 7
    D) All reals except -7

11) $y = \frac{1}{x^2-121}$
    A) All reals
    B) All reals except 11
    C) All reals except 121
    D) All reals except -11 and 11

12) $y = x^2 - 49$
    A) All reals except 7
    B) All reals except 49
    C) All reals
    D) All reals except -7 and 7

13) $y = \frac{1}{x^2+64}$
    A) All reals except 64
    B) All reals
    C) All reals except -8 and 8
    D) All reals except 8

14) $y = \sqrt{x-5}$
    A) All reals greater than -5
    B) All reals greater than or equal to 5
    C) All reals except 5
    D) All reals greater than 5

15) $y = \sqrt{x^2+9}$
    A) All reals except 3
    B) All reals except 9
    C) All reals
    D) All reals except -3 and 3
Solve the problem.
16) The graph shows the yearly average interest rates for 30-year mortgages for years since 1988 (Year 0 corresponds to 1988). Sketch a graph of the rate of change of interest rates with respect to time.

A)  

B)  

C)  

D)
Sketch a graph of the derivative of the given function.

17. 

18. 

19. 

20. (7, -12) (-2, -6) (-7, 12) (2, 6)
21.

22.

23.

-2\pi \leq x \leq 2\pi \quad x\text{-scale} = \frac{\pi}{2}
-4 \leq y \leq 4 \quad y\text{-scale} = 1
1. The graph of $f$ is given below. State the numbers at which $f$ is not differentiable and why. Your reason should be based on the definition of differentiability at a number.

![Graph of f]

- $f$ is not differentiable at $x = -2$ because $f'(-2)$ does not exist.
- $f$ is not differentiable at $x = 2$ because $f'(2) = f'(2)$ does not exist.
- $f$ is not differentiable at $x = 5$ because $f'(5) = f'(5)$ does not exist.
- $f$ is not differentiable at $x = 7$ because $f'(7) = f'(7)$ does not exist.

2. The graph of $f$ is given below. State the numbers at which $f$ is not differentiable and why. Your reason should be based on the definition of differentiability at a number. Also state where $f$ is not continuous and tell why. Your reason should be based on the definition of continuity at a number.

![Graph of f]

- $f$ is not differentiable at $x = 3$ because $f$ is not continuous at $x = 3$.
- $f$ is not differentiable at $x = 4$ because $f'(4) = f'(4)$ does not exist.
- $f$ is not differentiable at $x = 6$ because $f'(6) = f'(6)$ does not exist.
3. Show that \( f(x) = |x - 6| \) is not differentiable at \( x = 6 \).

\[
f(x) = \begin{cases} 
  x - 6 & \text{if } x \geq 6 \\
  6 - x & \text{if } x < 6
\end{cases}
\]

\[
f'(x) = \begin{cases} 
  1 & \text{if } x > 6 \\
  -1 & \text{if } x < 6
\end{cases}
\]

Now, since \( f'(6) = 1 \) and \( f'(6) = -1 \), then \( f'(x) \neq f'(6) \). \( f'(6) \) does not exist and \( f \) is not differentiable at \( x = 6 \).

4. Where is the greatest integer function \( f(x) = \lceil x \rceil \) not differentiable?

\( f \) is not differentiable at any integer because \( f \) is not continuous at any integer.

5. Where and why is the following function not continuous? Where and why is it not differentiable?

\[
f(x) = \begin{cases} 
  \frac{x^3 - x}{x^2 + x} & \text{if } x < 1 \text{ but } x \neq 0 \\
  0 & \text{if } x = 0 \\
  1 - x & \text{if } x \geq 1
\end{cases}
\]

Continuity test at \( x = 0 \)

i) \( f(0) = 0 \)

ii) \( \lim_{x \to 0} f(x) = \lim_{x \to 0} (x - 1) = -1 \)

Since \( f(0) \neq \lim_{x \to 0} f(x) \), \( f \) is not continuous at \( x = 0 \) and therefore \( f \) is not differentiable at \( x = 0 \).

Continuity test at \( x = 1 \)

i) \( f(1) = 0 \)

ii) \( \lim_{x \to 1} f(x) = 0 \)

\( \lim_{x \to 1^+} f(x) = 0 \)

\( \lim_{x \to 1^-} f(x) = 1 \)

Since \( \lim_{x \to 1} f(x) \), \( f \) is continuous at \( x = 1 \).

Differentiability test at \( x = 1 \)

Since \( f'(1) = -1 \) and \( f'(1) = 1 \), \( f'(1) \) and \( f'(1) \) does not exist. \( f \) is not differentiable at \( x = 1 \).

6. If \( f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 0 \\
  x - 4 & \text{if } x > 0
\end{cases} \), find \( f'(x) \) and tell where (if anywhere) the derivative does not exist.

\[
f'(x) = \begin{cases} 
  2x & \text{if } x < 0 \\
  1 & \text{if } x > 0
\end{cases}
\]

Since \( f'(0) = 1 \) and \( f'(0) = 0 \), then \( f'(0) \neq f'(0) \). \( f'(0) \) does not exist and \( f \) is not differentiable at \( x = 0 \).