SUM/DIFFERENCE IDENTITIES NAME Key

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^o \cos 43^o + \cos 97^o \sin 43^o$

$\sin(97+43) = \sin 140^o$

2. $\cos 72^o \cos 130^o + \sin 72^o \sin 130^o$

$\cos(72-130) = \cos(-58)$

3. $\dfrac{\tan 140^o - \tan 60^\circ}{1 + \tan 140^o \tan 60^\circ}$

$\tan(140-60) = \tan 80^o$

4. $\sin\dfrac{\pi}{5}\cos\dfrac{2\pi}{3} - \cos\dfrac{\pi}{5}\sin\dfrac{2\pi}{3}$

$\sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(-\frac{7\pi}{15}\right)$

$\frac{3\pi}{15} - \frac{10\pi}{15}$

5. $\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{7} - \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{7}$

$\cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \cos\left(\frac{13\pi}{42}\right)$

$\frac{7\pi}{42} + \frac{6\pi}{42}$

6. $\dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \tan\left(\frac{7\pi}{12}\right)$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. $\tan(-105^o)$

$\tan(-60 + -45) = \dfrac{\tan(-60) + \tan(-45)}{1 - \tan(-60)\tan(-45)}$

$= \dfrac{-\sqrt{3} - 1}{1 - (-\sqrt{3})(-1)} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}} = \dfrac{-\sqrt{3} - 3 - 1 - \sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3}$

$= \dfrac{-4 - 2\sqrt{3}}{-2} = 2 + \sqrt{3}$

8. $\sin 345^o$

$\sin(300 + 45) = \sin 300 \cos 45 + \cos 300 \sin 45$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \dfrac{-\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

#9-11. Given: $\csc\alpha = \dfrac{13}{5}$, $\dfrac{\pi}{2} \le \alpha \le \pi$, and $\tan\beta = -\dfrac{3}{4}$, $\dfrac{3\pi}{2} \le \beta \le 2\pi$, find the following:

9. $\sin(\alpha - \beta)$

$\sin\alpha = \frac{5}{13}$

CW 10. $\cos(\beta + \alpha)$

11. $\tan(\alpha - \beta)$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right)$

$\frac{20}{65} - \frac{36}{65}$

$\boxed{-\frac{16}{65}}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{5}{13}\right)^2 + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{144}{169}$

$\cos\alpha = -\frac{12}{13}$

$\tan^2\beta + 1 = \sec^2\beta$

$\left(-\frac{3}{4}\right)^2 + 1 = \sec^2\beta$

$\frac{9}{16} + \frac{16}{16} = \sec^2\beta$

$\frac{25}{16} = \sec^2\beta$

$\frac{5}{4} = \sec\beta$

$\frac{4}{5} = \cos\beta \Leftrightarrow \sin\beta = -\frac{3}{5}$

#12-13. If $\sin\theta = -\frac{3}{5}$ and θ is in the third quadrant, find the following:

12. $\cos(\theta + \frac{\pi}{3})$

$\cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}$

$\cos\theta\left(\frac{1}{2}\right) + \frac{3}{5}\cdot\frac{\sqrt{3}}{2}$

$\left(-\frac{4}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{-4+3\sqrt{3}}{10}$

13. $\tan 2\theta$

#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\frac{3\pi}{2} + x) = -\cos x$

16. $\cos(30^0 - x) + \cos(30^o + x) = \sqrt{3}\cos x$

$\cos 30 \cos x + \cancel{\sin 30 \sin x} + \cos 30 \cos x - \cancel{\sin 30 \sin x}$

$2\left(\frac{\sqrt{3}}{2}\right)\cos x$

$\sqrt{3}\cos x$

17. $\frac{\sin(\beta - \alpha)}{\sin\alpha \sin\beta} = \cot\alpha - \cot\beta$

CW 18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

19. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

$\cancel{\sin x \cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} - \left[\cancel{\sin x \cos\frac{\pi}{6}} - \cos x \sin\frac{\pi}{6}\right] = \frac{1}{2}$

$2\cos x\left(\frac{1}{2}\right)$

$\cos x = \frac{1}{2}$

$\frac{\pi}{3}, \frac{5\pi}{3}$

20. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

21. $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$

$\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} - \left(\cos x \cos\frac{3\pi}{2} - \sin x \sin\frac{3\pi}{2}\right) = 0$

$\sin x \cdot 0 + \cos x \cdot 1 - \left(\cos x \cdot 0 - \sin x(-1)\right)$

$\cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$