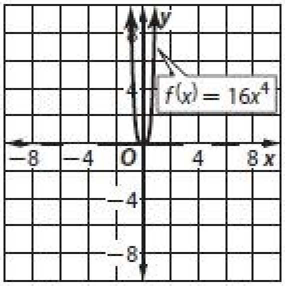
2-2 Polynomial Functions

Graph each function.

5. $f(x) = 16x^4$

SOLUTION:

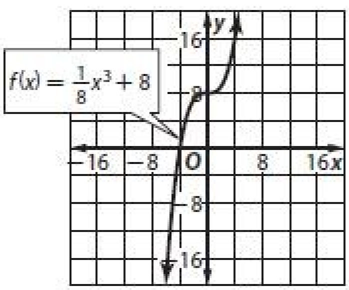
The graph of $f(x) = 16x^4$ is the graph of $y = x^4$ stretched vertically by a factor of 16.



10. $f(x) = \frac{1}{8}x^3 + 8$

SOLUTION:

The graph of $f(x) = \frac{1}{8}x^3 + 8$ is the graph of $y = x^3$ compressed vertically by a factor of $\frac{1}{8}$ and translated 8 units up.



Describe the end behavior of the graph of each polynomial function using limits. Explain your reasoning using the leading term test.

15. $g(x) = -7x^3 + 8x^4 - 6x^6$

SOLUTION:

$g(x) = -7x^3 + 8x^4 - 6x^6 = -6x^6 + 8x^4 - 7x^3$

The degree is 6, and the leading coefficient is −6. Because the degree is even and the leading coefficient is negative, $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$.

2-2 Polynomial Functions

20. $f(x) = -x(x - 4)(x + 5)$

SOLUTION:

$$\begin{aligned} -x(x-4)(x+5) &= \left(-x^2+4x\right)(x+5) \\ &= -x^3-5x^2+4x^2+20x \\ &= -x^3-x^2+20x \end{aligned}$$

The degree is 3, and the leading coefficient is −1. Because the degree is odd and the leading coefficient is negative, $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$.

State the number of possible real zeros and turning points of each function. Then determine all of the real zeros by factoring.

25. $f(x) = x^4 + 4x^2 - 21$

SOLUTION:

The degree of $f(x)$ is 4, so it will have at most four real zeros and three turning points.

Let $u = x^2$.

$$0 = \left(x^2\right)^2 + 4(x^2) - 21$$

$$0 = u^2 + 4u - 21$$

$$0 = (u + 7)(u - 3)$$

$$0 = (x^2 + 7)(x^2 - 3)$$

$$x^2 + 7 = 0 \quad \text{or} \quad x^2 - 3 = 0$$

$$x^2 = -7 \qquad x^2 = 3$$

$$x = \pm\sqrt{-7} \qquad x = \pm\sqrt{3}$$

Because $\pm\sqrt{-7}$ are not real zeros, f has two distinct real zeros, $\pm\sqrt{3}$.

30. $f(x) = 6x^5 - 150x^3$

SOLUTION:

The degree of $f(x)$ is 5, so it will have at most five real zeros and four turning points.

$$0 = 6x^5 - 150x^3$$

$$0 = 6x^3(x^2 - 25)$$

$$0 = 6x^3(x - 5)(x + 5)$$

So, the zeros are 0, 5, and −5.

For each function, (a) apply the leading-term test, (b) determine the zeros and state the multiplicity of any repeated zeros, (c) find a few additional points, and then (d) graph the function.

35. $f(x) = -x(x + 3)^2(x - 5)$

SOLUTION:

a.

$$-x(x+3)^2(x-5)$$
$$= (x^2+6x+9)(-x^2+5x)$$
$$= -x^4+5x^3-6x^3+30x^2-9x^2+45x$$
$$= -x^4-x^3+21x^2+45x$$

The degree is 4, and the leading coefficient is −1. Because the degree is even and the leading coefficient is negative, $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$.

b. The zeros are 0, −3, and 5. The zero −3 has multiplicity 2 since (x + 3) is a factor of the polynomial twice.

c. Sample answer: Evaluate the function for a few x-values in its domain.

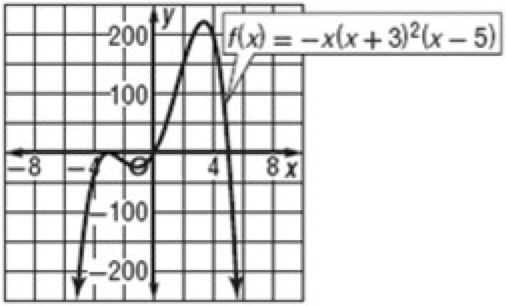
Choose x-values that fall in the intervals determined by the zeros of the function.

Interval	x	f(x)
(–∞, –3)	−4	−36
(–3, 0)	−1	−24
(0, 5)	2	150
(5, ∞)	6	−486

d. Evaluate the function for several x-values in its domain.

x	f(x)
−5	−200
−3	0
−2	−14
0	0
1	64
3	216
4	196

Use these points to construct a graph.



2-2 Polynomial Functions

40. $f(x) = -2x^3 - 4x^2 + 6x$

SOLUTION:

a. The degree is 3, and the leading coefficient is −2. Because the degree is odd and the leading coefficient is negative, $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$.

b.

$$0 = -2x^3 - 4x^2 + 6x$$
$$0 = -2x(x^2 + 2x - 3)$$
$$0 = -2x(x + 3)(x - 1)$$

The zeros are 0, −3, and 1.

c. Sample answer: Evaluate the function for a few *x*-values in its domain.

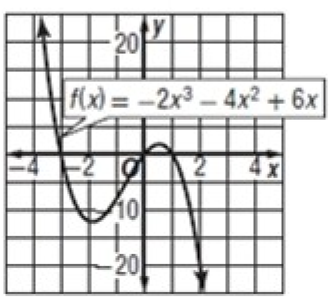
Choose *x*-values that fall in the intervals determined by the zeros of the function.

Interval	*x*	*f(x)*
(–∞, –3)	−4	40
(–3, 0)	−2	−12
(0, 1)	0.5	1.75
(0, ∞)	2	−20

d. Evaluate the function for several *x*-values in its domain.

x	*f(x)*
−3	0
−1	−8
0	0
1	0
3	−72
4	−168

Use these points to construct a graph.



2-2 Polynomial Functions

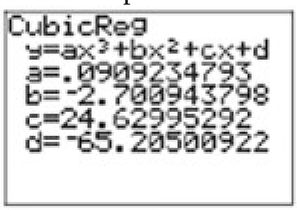
Use a graphing calculator to write a polynomial function to model each set of data.

45.

x	f(x)
5	2
7	5
8	6
10	4
11	−1
12	−3
15	5
16	9

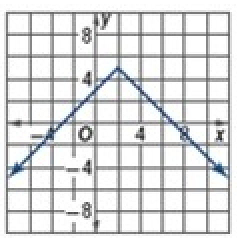
SOLUTION:

Sample answer: Since the data shows two possible turning points, a third-degree polynomial may be the best model to represent the data. Use the cubic regression function on the graphing calculator.



$f(x) = 0.09x^3 – 2.70x^2 + 24.63x – 65.21$

Determine whether each graph could show a polynomial function. Write *yes* or *no*. If not, explain why not.

50. 

SOLUTION:

The graph is not a polynomial function. There is a sharp turn at $x = 2$. Polynomials have smooth turns. This is most likely an absolute value function.

Find a polynomial function of degree *n* with only the following real zeros. More than one answer is possible.

55. 3; $n = 3$

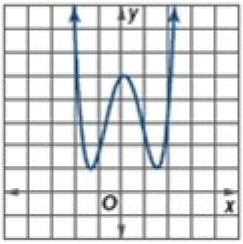
SOLUTION:

Sample answer: If 3 is a zero of the polynomial function, then $(x − 3)$ is a factor. For the function to have a degree of 3, raise $(x − 3)$ to the third power.

$$\begin{aligned} f(x) &= (x-3)^3 \\ &= x^3 + 3(-3)x^2 + 3(-3)^2x + (-3)^3 \\ &= x^3 - 9x^2 + 27x - 27 \end{aligned}$$

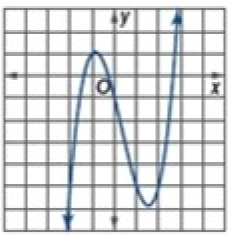
2-2 Polynomial Functions

Determine whether the degree *n* of the polynomial for each graph is *even* or *odd* and whether its leading coefficient a_n is *positive* or *negative*.

64. 

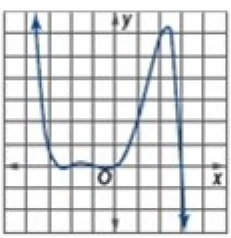
SOLUTION:

Since $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$, *n* is even and a_n is positive.

65. 

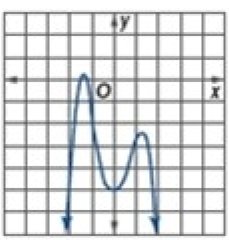
SOLUTION:

Since $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$, *n* is odd and a_n is positive.

66. 

SOLUTION:

Since $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$, *n* is odd and a_n is negative.

67. 

SOLUTION:

Since $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$, *n* is even and a_n is negative.