In 1-4, explain why the function has a zero in the given interval.

1	$f(x)=\frac{1}{16}x^4-x^3+3$; $[1,2]$	Continuous $f(1)=2.0625$. $f(2)=-4$ Sign Change
2	$f(x)=x^3+3x-2$; $[0,1]$	Continuous $f(0)=-2$ $f(1)=2$ Sign Change
3	$f(x)=x^2-x-\cos x$; $[0,\pi]$	Continuous $f(0)=-1$ $f(\pi)=7.728$ Sign Change
4	$f(x)=-\frac{4}{x}+\tan\left(\frac{\pi x}{8}\right)$; $[1,3]$	Continuous $f(1)=-.9289322$ $f(3)=15.737734$ Sign Change.

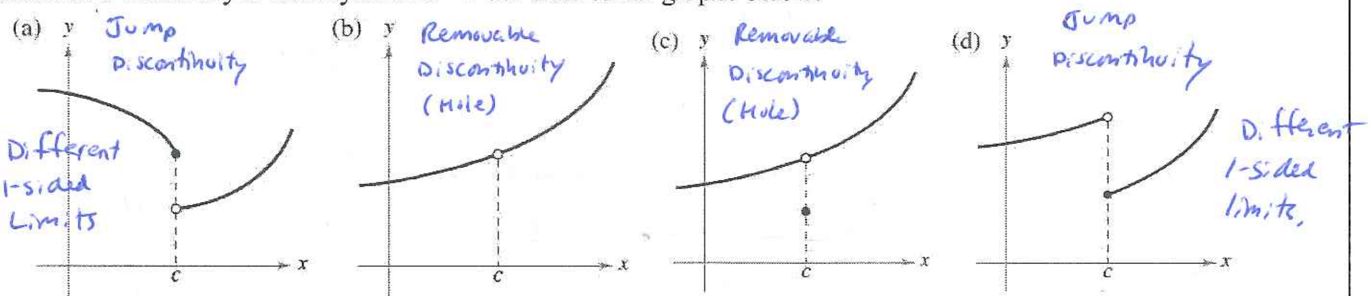
In 5-8, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval $[0,1]$ for the given function. Use a graphing calculator to find the zero.

5	$f(x)=x^3+x-1$	Continuous $f(0)=-1$ $f(1)=1$ $x=.682$
6	$f(x)=x^3+3x-2$	Continuous $f(0)=-2$ $f(1)=2$ $x=.596$
7	$g(t)=2\cos t-3t$	Continuous $f(0)=2$ $f(1)=-1.919$ $x=.564$
8	$h(\theta)=1+\theta-3\tan\theta$	Continuous $f(0)=1$ $f(1)=-2.672$ $x=.450$

In 9-12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem. No calculator is permitted on these problems.

9	$f(x)=x^2+x-1,\quad [0,5],\quad f(c)=11$	Continuous $f(0)=-1$ $f(5)=29$ $c=3$
10	$f(x)=x^2-6x+8,\quad [0,3],\quad f(c)=0$	Continuous $f(0)=8$ $f(3)=-1$ $c=4$
11	$f(x)=x^3-x^2+x-2,\quad [0,3],\quad f(c)=4$	Continuous $f(0)=-2$ $f(3)=19$ $c=2$
12	$f(x)=\frac{x^2+x}{x-1},\quad \left[\frac{5}{2},4\right],\quad f(c)=6$	Continuous $f\left(\frac{5}{2}\right)=\frac{35}{6}$ $f(4)=\frac{20}{3}$ $c=3$

13 State how continuity is destroyed at $x=c$ for each of the graphs below:



14 Use the Intermediate Value Theorem to show that for all spheres with radii in the interval $[1,5]$, there is one with a volume of 275 cubic centimeters.

15 Show that $f(x)$ is continuous at $x=2$ for $f(x)=\begin{cases}5-x, & -1\le x\le 2\\ x^2-1, & 2<x\le 3\end{cases}$.

$5-2=3$

$2^2-1=3$

$\therefore$ Continuous.

16 Determine whether $f(x)$ is continuous at $x=-1$ for $f(x)=\begin{cases}\frac{1}{x}, & x\le -1\\ \frac{x-1}{2}, & -1<x<1\\ \sqrt{x}, & x\ge 1\end{cases}$

$\frac{1}{-1}=-1$

$\frac{-1-1}{2}=-1$

yes continuous at $x=-1$

#9

$$x^2 + x - 1 = 11$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4 \quad \boxed{x = 3}$$

#10

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x+2) = 0$$

$$\boxed{x = 4} \quad x = -2$$

#11

$$x^3 - x^2 + x - 2 = 4$$

$$x^3 - x^2 + x - 6 = 0$$

$$(x-2)(x^2 + x + 3) = 0$$

$$\boxed{x = 2}$$

#12

$$\frac{x^2 + x}{x-1} = 6$$

$$x^2 + x = 6(x-1)$$

$$x^2 + x = 6x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$\boxed{x = 3} \quad x = 2$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$$x^2 + x + 3$$

#14 $V = \frac{4}{3}\pi r^3$

$$V(1) = \frac{4}{3}\pi \approx 4.189$$

$$V(5) = \frac{4}{3}\pi(5)^3 \approx 523.599$$

$$V(1) < 275 < V(5)$$

Calculus 1 Name *Key*

Worksheet – Extreme Value Theorem

Extreme Value Theorem:

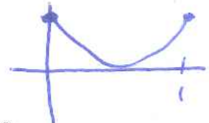
If a function is continuous on a closed interval [a, b], then the function will have both a maximum value and a minimum value.

Find the maximum and minimum values for each function on the given interval. You can use your calculator.

1) $f(x) = 4x^2 - 4x + 1$ $[0,1]$

$f(0) = 1$

$f(1) = 1$



Continuous on $[0,1]$

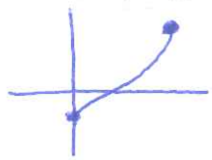
max: $(0,1)$ $(1,1)$

min: $(\frac{1}{2}, 0)$

2) $f(x) = (x-1)^3$ $[0,4]$

$f(0) = -1$

$f(4) = 27$



Continuous on $[0,4]$

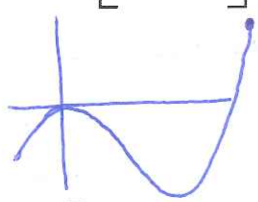
max: $(4,27)$

min: $(0,-1)$

3) $f(x) = 2x^3 - 9x^2$ $[-1,5]$

$f(-1) = -11$

$f(5) = 25$



Continuous on $[-1,5]$

max: $(5,25)$

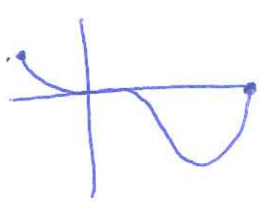
min: $(3,-27)$

Local max $(0,0)$ Local min: $(-1,-11)$

4) $f(x) = x^4 - 4x^3$ $[-1,4]$

$f(-1) = 5$

$f(4) = 0$



Continuous on $[-1,4]$

max: $(-1,5)$

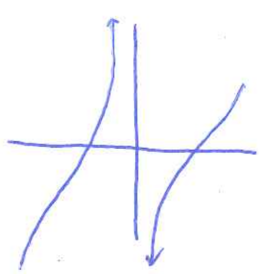
min: $(3,-27)$

Local max: $(4,0)$

5) $f(x) = 2x^3 - \frac{1}{x}$ $[-3,1.5]$

$f(-3) = -53\frac{2}{3}$

$f(1.5) = 6\frac{1}{12}$



NOT Continuous!

Local max: $(1.5, 6\frac{1}{12})$

Local min: $(-3, -53\frac{2}{3})$