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4-4 Study Guide and Intervention

Graphing Sine and Cosine Functions

Transformations of Sine and Cosine Functions A sinusoid is a transformation of the graph of the sine function. The general form of the sinusoidal functions sine and cosine are $y = a \sin (bx + c) + d$ or $y = a \cos (bx + c) + d$. The graphs of $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$ have the following characteristics.

Amplitude $= |a|$ Period $= \frac{2\pi}{|b|}$ Frequency $= \frac{|b|}{2\pi}$ or $\frac{1}{\text{period}}$

Phase Shift $= -\frac{c}{|b|}$ Vertical Shift $= d$ Midline $y = d$

Example **State the amplitude, period, frequency, phase shift, and vertical shift of $y = -2 \cos \left(\frac{x}{4} - \frac{\pi}{3}\right) + 2$. Then graph two periods of the function.**

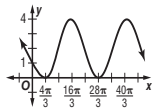
Amplitude $= |a| = |-2| = 2$

Period $= \frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{1}{4}\right|}$ or 8π

Frequency $= \frac{|b|}{2\pi} = \frac{\left|\frac{1}{4}\right|}{2\pi}$ or $\frac{1}{8\pi}$

Phase Shift $= -\frac{c}{|b|} = \frac{-\left(-\frac{\pi}{3}\right)}{\left|\frac{1}{4}\right|}$ or $\frac{4\pi}{3}$

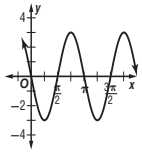
Vertical Shift $= d$ or 2



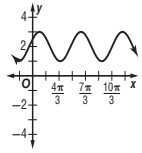
Exercises

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

1. $y = 3 \sin (2x + \pi)$ **$3, \pi, \frac{1}{\pi}, -\frac{\pi}{2}, 0$**



2. $y = \cos \left(x - \frac{\pi}{3}\right) + 2$ **$1, 2\pi, \frac{1}{2\pi}, \frac{\pi}{3}, 2$**



State the frequency and midline of each function.

3. $y = 3 \sin \left(\frac{1}{2}x + \frac{4\pi}{3}\right) + 1$ **$\frac{1}{4\pi}$; $y = 1$**

4. $y = \cos (3x) - 2$ **$\frac{3}{2\pi}$; $y = -2$**

Write a sine function with the given characteristics.

5. amplitude = 2, period = 4, phase shift = $\frac{1}{2}$, vertical shift = 4

Sample answer: $y = 2 \sin \left(\frac{\pi}{2}x + \frac{\pi}{4}\right) + 4$

6. amplitude = 1.2, phase shift = $\frac{3\pi}{2}$, vertical shift = 1

Sample answer: $y = 1.2 \sin \left(x - \frac{3\pi}{2}\right) + 1$

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4-4 Study Guide and Intervention *(continued)*

Graphing Sine and Cosine Functions

Applications of Sinusoidal Functions You can use sinusoidal functions to solve certain application problems.

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan, in degrees Fahrenheit (°F). Write a sinusoidal function that models the average monthly temperatures as a function of time x, where $x = 1$ represents January.**

Month	Temperature
Jan	30°
Feb	34°
Mar	45°
Apr	59°
May	71°
June	80°
July	84°
Aug	81°
Sept	74°
Oct	62°
Nov	48°
Dec	35°

The data can be modeled by a sinusoidal function of the form $y = a \sin (bx + c) + d$. Find the maximum M and minimum m values of the data, and use these values to find a, b, c, and d.

$a = \frac{1}{2}(M - m)$ Amplitude formula

$= \frac{1}{2}(84 - 30)$ or 27 $M = 84$ and $m = 30$

$d = \frac{1}{2}(M + m)$ Vertical shift formula

$= \frac{1}{2}(84 + 30)$ or 57 $M = 84$ and $m = 30$

Period $= 2(x_{max} - x_{min})$ x_{max} = July or month 7 and

$= 2(7 - 1)$ or 12 x_{min} = January or month 1

$|b| = \frac{2\pi}{12}$ or $\frac{\pi}{6}$ $|b| = \frac{2\pi}{\text{period}}$

Phase shift $= -\frac{c}{|b|}$ Phase shift formula

$4 = -\frac{c}{\frac{\pi}{6}}$ Phase shift = 4 and $|b| = \frac{\pi}{6}$

$c = -\frac{2\pi}{3}$ Solve for c.

Therefore, $y = 27 \sin \left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.

Exercise

1. MUSEUM ATTENDANCE The table gives the number of visitors in thousands to a museum for each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Visitors	10	8	11	15	24	30	32	29

a. Write a trigonometric function that models the monthly attendance at the museum using $x = 1$ to represent January.

$y = 12 \sin \left(\frac{\pi}{5}x - \frac{9\pi}{10}\right) + 20$

b. According to your model, how many people should the museum expect to visit during October? **16,292 people**

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4-4 Practice

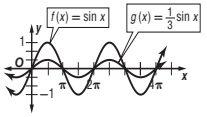
Graphing Sine and Cosine Functions

Describe how the graphs of $f(x)$ and $g(x)$ are related. Then find the amplitude of $g(x)$ and sketch two periods of both functions on the same coordinate axes.

1. $f(x) = \sin x$

$g(x) = \frac{1}{3}\sin x$

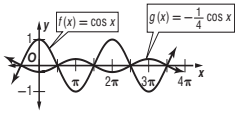
The graph of $g(x)$ is the graph of $f(x)$ compressed vertically. The amplitude of $g(x)$ is $\frac{1}{3}$.



2. $f(x) = \cos x$

$g(x) = -\frac{1}{4}\cos x$

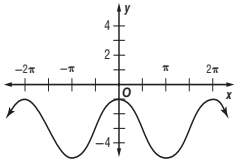
The graph of $g(x)$ is the graph of $f(x)$ compressed vertically and reflected in the x-axis. The amplitude of $g(x)$ is $\frac{1}{4}$.



State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

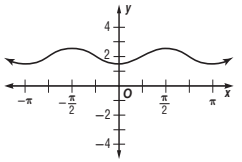
3. $y = 2\sin\left(x + \frac{\pi}{2}\right) - 3$

amplitude = 2; period = 2π; frequency = $\frac{1}{2\pi}$; phase shift = $-\frac{\pi}{2}$; vertical shift = -3



4. $y = \frac{1}{2}\cos(2x - \pi) + 2$

amplitude = $\frac{1}{2}$; period = π; frequency = $\frac{1}{\pi}$; phase shift = $\frac{\pi}{2}$; vertical shift = 2



Write a sinusoidal function with the given amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = -10

$y = \pm 15 \sin\left(\frac{x}{2} - \frac{\pi}{4}\right) - 10$

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

$y = \pm\frac{2}{3}\cos(6x + 2\pi) + 5$

7. MUSIC A piano tuner strikes a tuning fork note A above middle C and sets in motion vibrations that can be modeled by $y = 0.001 \sin 880t\pi$. Find the amplitude and period of the function. **0.001; $\frac{1}{440}$**

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4-4 Word Problem Practice

Graphing Sine and Cosine Functions

1. METEOROLOGY The average monthly temperatures for Baltimore, Maryland, are shown below.

Month	Temperature (°F)	Month	Temperature (°F)
Jan	32	July	77
Feb	35	Aug	76
Mar	44	Sept	69
Apr	53	Oct	57
May	63	Nov	47
June	73	Dec	37

a. Determine the amplitude, period, phase shift, and vertical shift of a sinusoidal function that models the monthly temperatures using $x = 1$ to represent January.

amplitude = 22.5; period = 12 months; phase shift: −5 months; vertical shift = 54.5°

b. Write an equation of a sinusoidal function that models the monthly temperatures.

Sample answer:
$y = 22.5\cos\left(\frac{\pi}{6}x + \frac{5\pi}{6}\right) + 54.5$

c. According to your model, what is Baltimore's average temperature in July? December?

Sample answer: 77°; 35°

2. BOATING A buoy, bobbing up and down in the water as waves pass it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. Determine the amplitude and period of a sinusoidal function that models the bobbing buoy.

1.5; 10 s

b. Write an equation of a sinusoidal function that models the bobbing buoy, using $x = 0$ as its highest point.

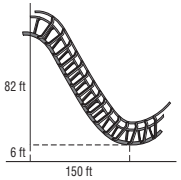
Sample answer:
$y = 1.5\cos\left(\frac{\pi}{5}x\right)$

3. A student graphed a periodic function with a period of n. The student then translated the graph c units to the right and obtained the original graph. Describe the relationship between c and n.

Sample answer: c is a positive multiple of n.

4. SWING Marsha is pushing her brother Bobby on a rope swing over a creek. When she starts the swing, he is 7 feet over land away from the edge of the creek. After 2 seconds, Bobby is 11 feet over the water past the edge of the creek. Assume that the distance from the edge of the creek varies sinusoidally with time and that the distance y is positive when Bobby is over the water and negative when he is over land. Write a trigonometric function that models the distance Bobby is from the edge of the creek at time t seconds. $y = -9\cos\frac{\pi}{2}t + 2$

5. ROLLER COASTER Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically as shown. The low point is 6 feet above the ground.



a. Write a sinusoidal function that models the distance the roller coaster track is above the ground at a given horizontal distance x.

$y = 41\cos\frac{\pi}{150}x + 47$

b. Point A is 40 feet to the right of the y-axis. How far above the ground is the track at point A? **74.4 ft**