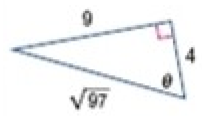
4-1 Right Triangle Trigonometry

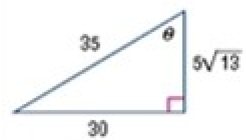
Find the exact values of the six trigonometric functions of θ.

3. 

SOLUTION:

The length of the side opposite θ is 9, the length of the side adjacent to θ is 4, and the length of the hypotenuse is $\sqrt{97}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{\sqrt{97}} \text{ or } \frac{9\sqrt{97}}{97}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{\sqrt{97}} \text{ or } \frac{4\sqrt{97}}{97}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{4}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{97}}{9}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{97}}{4}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{9}$$

6. 

SOLUTION:

The length of the side opposite θ is 30, the length of the side adjacent to θ is $5\sqrt{13}$, and the length of the hypotenuse is 35.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{30}{35} \text{ or } \frac{6}{7}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5\sqrt{13}}{35} \text{ or } \frac{\sqrt{13}}{7}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{30}{5\sqrt{13}} \text{ or } \frac{6\sqrt{13}}{13}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{7}{6}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{7}{\sqrt{13}} \text{ or } \frac{7\sqrt{13}}{13}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5\sqrt{13}}{30} \text{ or } \frac{\sqrt{13}}{6}$$

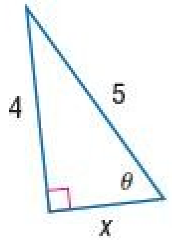
4-1 Right Triangle Trigonometry

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ.

9. $\sin \theta = \frac{4}{5}$

SOLUTION:

Draw a right triangle and label one acute angle θ. Because $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$, label the opposite side 4 and the hypotenuse 5.



By the Pythagorean Theorem, the length of the side adjacent to θ is 3.

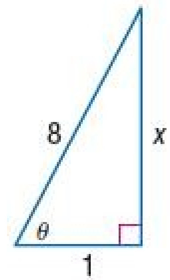
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

12. $\sec \theta = 8$

SOLUTION:

Draw a right triangle and label one acute angle θ.

Because $\sec \theta = \frac{\text{hyp}}{\text{adj}} = 8 = \frac{8}{1}$, label the hypotenuse 8 and the adjacent side 1.



By the Pythagorean Theorem, the length of the side adjacent to θ is $\sqrt{8^2 - 1^2} = \sqrt{63} = 3\sqrt{7}$.

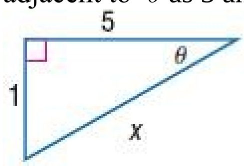
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3\sqrt{7}}{8}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{8}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3\sqrt{7}}{1} \text{ or } 3\sqrt{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{8}{3\sqrt{7}} \text{ or } \frac{8\sqrt{7}}{21}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{3\sqrt{7}} \text{ or } \frac{\sqrt{7}}{21}$$

15. $\cot\theta = 5$

SOLUTION:

Draw a right triangle and label one acute angle θ.

Because $\cot\theta = \frac{\text{adj}}{\text{opp}} = 5 = \frac{5}{1}$, label the side adjacent to θ as 5 and the opposite side 1.



By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{5^2+1^2} = \sqrt{26}$.

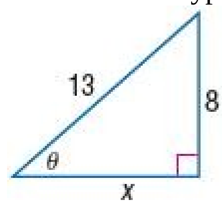
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{26}} \text{ or } \frac{\sqrt{26}}{26}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{26}} \text{ or } \frac{5\sqrt{26}}{26}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{26}}{1} \text{ or } \sqrt{26}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{26}}{5}$$

18. $\sin\theta = \frac{8}{13}$

SOLUTION:

Draw a right triangle and label one acute angle θ.

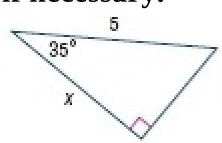
Because $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{13}$, label the side opposite θ as 8 and the hypotenuse 13.



By the Pythagorean Theorem, the length of the adjacent side is $\sqrt{13^2-8^2} = \sqrt{105}$.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{105}}{13}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{\sqrt{105}} \text{ or } \frac{8\sqrt{105}}{105}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{8}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{\sqrt{105}} \text{ or } \frac{13\sqrt{105}}{105}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{105}}{8}$$

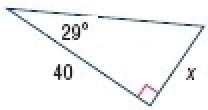
Find the value of *x*. Round to the nearest tenth, if necessary.

21. 

SOLUTION:

An acute angle measure and the length of the hypotenuse are given, so the cosine function can be used to find the length of the side adjacent to the angle.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 35° = \frac{x}{5}$$
$$5\cos 35° = x$$
$$4.1 \approx x$$

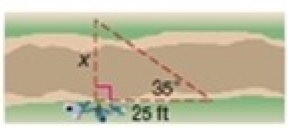
24. 

SOLUTION:

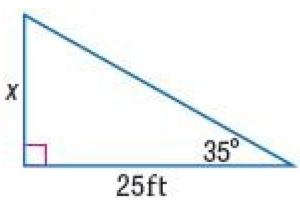
An acute angle measure and the length of the side adjacent to it are given, so the tangent function can be used to find the length of the side opposite the angle .

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 29° = \frac{x}{40}$$
$$40\tan 29° = x$$
$$22.2 \approx x$$

27. **MOUNTAIN CLIMBING** A team of climbers must determine the width of a ravine in order to set up equipment to cross it. If the climbers walk 25 feet along the ravine from their crossing point, and sight the crossing point on the far side of the ravine to be at a 35º angle, how wide is the ravine?



SOLUTION:

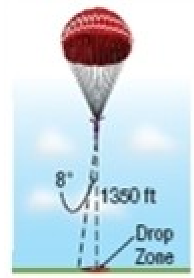


An acute angle measure and the adjacent side length are given, so the tangent function can be used to find the length of the opposite side.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 35° = \frac{x}{25}$$
$$25\tan 35° = x$$
$$17.5 \approx x$$

Therefore, the ravine is about 17.5 feet wide.

30. **PARACHUTING** A paratrooper encounters stronger winds than anticipated while parachuting from 1350 feet, causing him to drift at an 8º angle. How far from the drop zone will the paratrooper land?



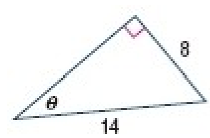
SOLUTION:

Because an acute angle and the length of the side that is adjacent to the angle are given, the tangent function can be used to find the length of the opposite side.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 8° = \frac{x}{1350}$$
$$1350\tan 8° = x$$
$$189.7 \approx x$$

So, the paratrooper will land about 190 feet away from the drop zone.

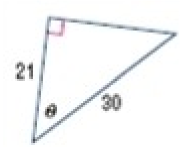
Find the measure of angle θ. Round to the nearest degree, if necessary.

33. 

SOLUTION:

Because the length of the hypotenuse and side opposite θ are given, the sine function can be used to find θ .

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin\theta = \frac{8}{14}$$
$$\theta = \sin^{-1}\frac{8}{14} \text{ or about } 35°$$

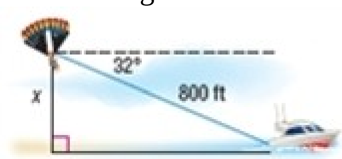
36. 

SOLUTION:

Because the length of the hypotenuse and side adjacent to θ are given, the cosine function can be used to find θ.

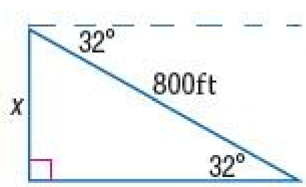
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\cos\theta = \frac{21}{30}$$
$$\theta = \cos^{-1}\frac{21}{30} \text{ or about } 46°$$

39. **PARASAILING** Kayla decided to try parasailing. She was strapped into a parachute towed by a boat. An 800-foot line connected her parachute to the boat, which was at a 32º angle of depression below her. How high above the water was Kayla?



SOLUTION:

The angle of elevation from the boat to the parachute is equivalent to the angle of depression from the parachute to the boat because the two angles are alternate interior angles, as shown below.



Because an acute angle and the hypotenuse are given, the sine function can be used to find *x*.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 32° = \frac{x}{800}$$
$$800 \sin 32° = x$$
$$423.9 \approx x$$

Therefore, Kayla was about 424 feet above the water.

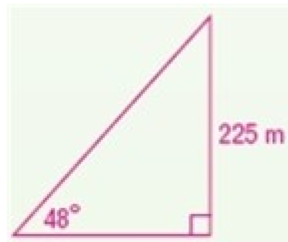
42. **SKI LIFT** A company is installing a new ski lift on a 225-meter-high mountain that will ascend at a 48º angle of elevation.

a. Draw a diagram to represent the situation.

b. Determine the length of cable the lift requires to extend from the base to the peak of the mountain.

SOLUTION:

a. Draw a diagram of a right triangle. The height of the mountain is give. Label the vertical side 225. Then label the acute angle opposite the 225 m side as 48º.

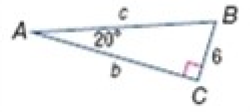


b. Because an acute angle and the length of the side opposite the angle are given, you can use the sine function to find the length of the hypotenuse.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 48° = \frac{225}{x}$$
$$x \sin 48° = 225$$
$$x = \frac{225}{\sin 48°}$$
$$x \approx 302.8$$

So, the company will need about 303 meters of cable.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

47. 

SOLUTION:

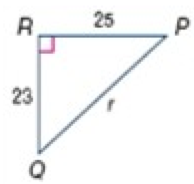
Use trigonometric functions to find b and c.

$$\begin{aligned} \tan 20^\circ &= \frac{6}{b} \\ b\tan 20^\circ &= 6 \\ b &= \frac{6}{\tan 20^\circ} \\ b &\approx 16.5 \end{aligned} \qquad \begin{aligned} \sin 20^\circ &= \frac{6}{c} \\ c\sin 20^\circ &= 6 \\ c &= \frac{6}{\sin 20^\circ} \\ c &\approx 17.5 \end{aligned}$$

Because the measures of two angles are given, B can be found by subtracting A from 90°.

$$\begin{aligned} 20^\circ + B &= 90^\circ \\ B &= 70^\circ \end{aligned}$$

Therefore, $B = 70^\circ$, $b \approx 16.5$, $c \approx 17.5$.

49. 

SOLUTION:

Use the Pythagorean Theorem to find r.

$$\begin{aligned} r &= \sqrt{23^2 + 25^2} \\ &\approx 34.0 \end{aligned}$$

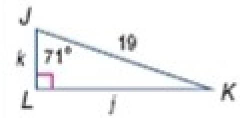
Use the tangent function to find P.

$$\begin{aligned} \tan P &= \frac{23}{25} \\ P &= \tan^{-1}\frac{23}{25} \\ P &\approx 42.61^\circ \end{aligned}$$

Because the measures of two angles are now known, you can find Q by subtracting P from 90°.

$$\begin{aligned} 42.61^\circ + Q &\approx 90^\circ \\ Q &\approx 47.39^\circ \end{aligned}$$

Therefore, $P \approx 43^\circ$, $Q \approx 47^\circ$, and $r \approx 34.0$.

51. 

SOLUTION:

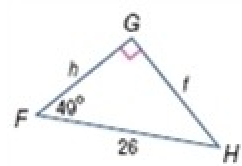
Use trigonometric functions to find j and k.

$$\begin{aligned} \sin 71^\circ &= \frac{j}{19} \\ 19\sin 71^\circ &= j \\ 18.0 &\approx j \end{aligned} \qquad \begin{aligned} \cos 71^\circ &= \frac{k}{19} \\ 19\cos 71^\circ &= k \\ 6.2 &\approx k \end{aligned}$$

Because the measures of two angles are given, K can be found by subtracting J from 90°.

$$\begin{aligned} 71^\circ + K &\approx 90^\circ \\ K &\approx 19^\circ \end{aligned}$$

Therefore, $K \approx 19^\circ$, $j \approx 18.0$, $k \approx 6.2$.

53. 

SOLUTION:

Use trigonometric functions to find f and h.

$$\begin{aligned} \sin 49^\circ &= \frac{f}{26} \\ 26\sin 49^\circ &= f \\ 19.6 &\approx f \end{aligned} \qquad \begin{aligned} \cos 49^\circ &= \frac{h}{26} \\ 26\cos 49^\circ &= h \\ 17.1 &\approx h \end{aligned}$$

Because the measures of two angles are given, H can be found by subtracting F from 90°.

$$\begin{aligned} 49^\circ + H &= 90^\circ \\ H &= 41^\circ \end{aligned}$$

Therefore, $H = 41^\circ$, $f \approx 19.6$, $h \approx 17.1$.