Chapter 12
Static Equilibrium and Elasticity

Conceptual Problems

1 • **[SSM]** True or false:
(*a*) $\sum_i \vec{F}_i = 0$ is sufficient for static equilibrium to exist.
(*b*) $\sum_i \vec{F}_i = 0$ is necessary for static equilibrium to exist.
(*c*) In static equilibrium, the net torque about any point is zero.
(*d*) An object in equilibrium cannot be moving.

(*a*) False. The conditions $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ must be satisfied.

(*b*) True. The necessary and sufficient conditions for static equilibrium are $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$.

(*c*) True. The conditions $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ must be satisfied.

(*d*) False. An object can be moving with constant speed (translational or rotational) when the conditions $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ are satisfied.

2 • True or false:
(*a*) The center of gravity is always at the geometric center of a body.
(*b*) The center of gravity must be located inside an object.
(*c*) The center of gravity of a baton is located between the two ends.
(*d*) The torque produced by the force of gravity about the center of gravity is always zero.

(*a*) False. The location of the center of gravity depends on how an object's mass is distributed.

(*b*) False. An example of an object for which the center of gravity is outside the object is a donut.

(*c*) True. The structure of a baton and the definition of the center of gravity guarantee that the center of gravity of a baton is located between the two ends.

(*d*) True. Because the force of gravity acting on an object acts through the center of gravity of the object, its lever (or moment) arm is always zero.

3 • The horizontal bar in Figure 12-27 will remain horizontal if (*a*) $L_1 = L_2$ and $R_1 = R_2$, (*b*) $M_1R_1 = M_2R_2$, (*c*) $M_2\mathrm{R}_1 = R_2M_1$, (*d*) $L_1M_1 = L_2M_2$, (*e*) $R_1L_1 = R_2L_2$.

Determine the Concept The condition that the bar is in rotational equilibrium is that the net torque acting on it equal zero; i.e., $R_1M_1 - R_2M_2 = 0$. $\boxed{(b)}$ is correct.

4 • Sit in a chair with your back straight. Now try to stand up without leaning forward. Explain why you cannot do it.

Determine the Concept You cannot stand up because, if you are to stand up, your body's center of gravity must be above your feet.

5 • You have a job digging holes for posts to support signs for a Louisiana restaurant (called Mosca's). Explain why the higher above the ground a sign is mounted, the farther the posts should extend into the ground.

Determine the Concept Flat signs of any kind experience substantial forces when the wind blows against them – the larger the surface area, the larger the force. In order to be stable, the posts which support such signs must be buried deeply enough so that the ground can exert sufficient force against the posts to keep the sign in equilibrium under the strongest winds. The pivot point around which the sign might rotate is at ground level – thus the more moment arm available below ground level, the more torque may be generated by the force of the ground on the posts. Thus the larger the surface area of the billboard, the greater will be the force applied above the surface, and hence the torque applied to the posts will be greater. As surface area increases, the preferred depth of the posts increases as well so that with the increased moment arm, the ground can exert more torque to balance the torque due to the wind.

6 • A father (mass M) and his son, (mass m) begin walking out towards opposite ends of a balanced see-saw. As they walk, the see-saw stays exactly horizontal. What can be said about the relationship between the father's speed V and the son's speed v?

Determine the Concept The question is about a situation in which an object is in static equilibrium. Both the father and son are walking outward from the center of the see-saw, which always remains in equilibrium. In order for this to happen, at any time, the net torque about any point (let's say, the pivot point at the center of the see-saw) must be zero. We can denote the father's position as X, and the son's position as x, and choose the origin of coordinates to be at the pivot point. At each moment, the see-saw exerts normal forces on the son and his father equal to their respective weights, mg and Mg. By Newton's third law, the father exerts a downward force equal in magnitude to the normal force, and the son exerts a downward force equal in magnitude to the normal force acting on him.

Apply $\sum \tau_{\text{pivot point}} = 0$ to the see-saw (assume that the father walks to the left and that counterclockwise torques are positive):

$$MgX - mgx = 0 \qquad (1)$$

Express the distance both the father and his son walk as a function of time:

$X = V\Delta t$ and $x = v\Delta t$

Substitute for X and x in equation (1) to obtain:

$$MgV\Delta t - mgv\Delta t = 0 \Rightarrow V = \boxed{\frac{m}{M}v}$$

Remarks: The father's speed is less than the son's speed by a factor of *m*/*M*.

7 • Travel mugs that people might set on the dashboards of their cars are often made with broad bases and relatively narrow mouths. Why would travel mugs be designed with this shape, rather than have the roughly cylindrical shape that mugs normally have?

Determine the Concept The main reason this is done is to lower the center of gravity of the mug as a whole. For a given volume, it is possible to make a mug with gently sloping sides that has a significantly lower center of gravity than the traditional cylinder. This is important, because as the center of gravity of an object gets lower (and as its base broadens) the object is harder to tip. When cars are traveling at constant velocity, the design of the mug is not important – but when cars are stopping and going – accelerating and decelerating – the higher center of gravity of the usual design makes it much more prone to tipping.

8 •• The sailors in the photo are using a technique called ″hiking out″. What purpose does positioning themselves in this way serve? If the wind were stronger, what would they need to do in order to keep their craft stable?

Determine the Concept Dynamically the boats are in equilibrium along their line of motion, but in the plane of their sail and the sailors, they are in static equilibrium. The torque on the boat, applied by the wind acting on the sail, has a tendency to tip the boat. The rudder counteracts that tendency to some degree, but in particularly strong winds, when the boat is sailing at particular angles with respect to the wind, the sailors need to ″hike out″ to apply some torque (due to the gravitational force of the Earth on the sailors) by leaning outward on the beam of the boat. If the wind strengthens, they need to extend their bodies further over the side and may need to get into a contraption called a ″trapeze″ that enables the sailor to have his or her entire body outside the boat.

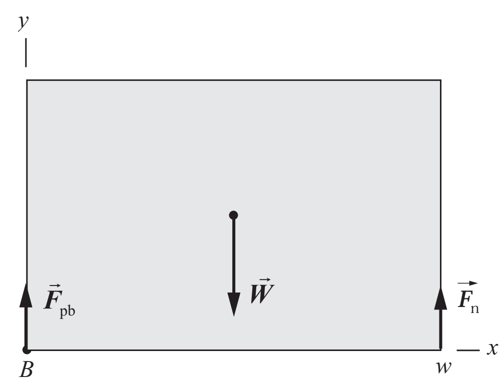
9 •• **[SSM]** An aluminum wire and a steel wire of the same length L and diameter D are joined end-to-end to form a wire of length $2L$. One end of the wire is then fastened to the ceiling and an object of mass M is attached to the other end. Neglecting the mass of the wires, which of the following statements is true? (*a*) The aluminum portion will stretch by the same amount as the steel portion. (*b*) The tensions in the aluminum portion and the steel portion are equal. (*c*) The tension in the aluminum portion is greater than that in the steel portion. (*d*) None of the above

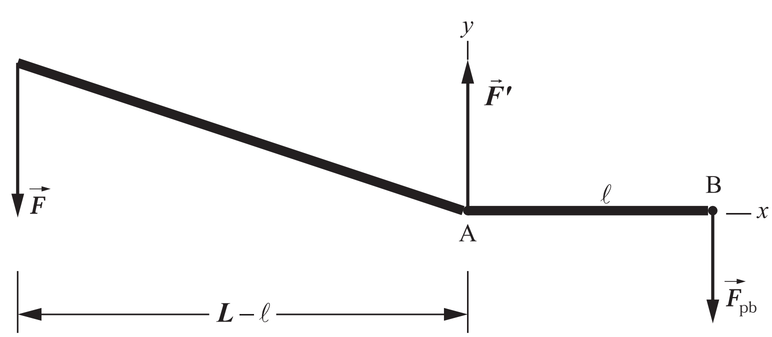
Determine the Concept We know that equal lengths of aluminum and steel wire of the same diameter will stretch different amounts when subjected to the same tension. Also, because we are neglecting the mass of the wires, the tension in them is independent of which one is closer to the roof and depends only on Mg. $\boxed{(b)}$ is correct.

Estimation and Approximation

10 •• A large crate weighing 4500 N rests on four 12-cm-high blocks on a horizontal surface (Figure 12-28). The crate is 2.0 m long, 1.2 m high and 1.2 m deep. You are asked to lift one end of the crate using a long steel pry bar. The fulcrum on the pry bar is 10 cm from the end that lifts the crate. Estimate the length of the bar you will need to lift the end of the crate.

Picture the Problem The diagram to the right shows the forces acting on the crate as it is being lifted at its left end. Note that when the crowbar lifts the crate, only half the weight of the crate is supported by the bar. Choose the coordinate system shown and let the subscript "pb" refer to the pry bar. The diagram below shows the forces acting on the pry bar as it is being used to lift the end of the crate.





Assume that the maximum force F you can apply is 500 N (about 110 lb). Let ℓ be the distance between the points of contact of the steel bar with the floor and the crate, and let L be the total length of the bar. Lacking information regarding the bend in pry bar at the fulcrum, we'll assume that it is small enough to be negligible. We can apply the condition for rotational equilibrium to the pry bar and a condition for translational equilibrium to the crate when its left end is on the verge of lifting.

Apply $\sum F_y = 0$ to the crate:

$$F_{\text{pb}} - W + F_{\text{n}} = 0 \qquad (1)$$

Apply $\sum \vec{\tau} = 0$ to the crate about an axis through point B and perpendicular to the plane of the page to obtain:

$$wF_{\text{n}} - \tfrac{1}{2}wW = 0 \Rightarrow F_{\text{n}} = \tfrac{1}{2}W$$

as noted in **Picture the Problem**.

Solve equation (1) for F_{pb} and substitute for F_{n} to obtain:

$$F_{\text{pb}} = W - \tfrac{1}{2}W = \tfrac{1}{2}W$$

Apply $\sum \vec{\tau} = 0$ to the pry bar about an axis through point A and perpendicular to the plane of the page to obtain:

$$F(L-\ell) - \ell F_{\text{pb}} = 0 \Rightarrow L = \ell\left(1 + \frac{F_{\text{pb}}}{F}\right)$$

Substitute for F_{pb} to obtain:

$$L = \ell\left(1 + \frac{W}{2F}\right)$$

Substitute numerical values and evaluate L:

$$\boldsymbol{L} = (0.10\,\text{m})\left(1 + \frac{4500\,\text{N}}{2(500\,\text{N})}\right) = \boxed{55\,\text{cm}}$$

11 •• **[SSM]** Consider an atomic model for Young's modulus. Assume that a large number of atoms are arranged in a cubic array, with each atom at a corner of a cube and each atom at a distance a from its six nearest neighbors. Imagine that each atom is attached to its 6 nearest neighbors by little springs each with force constant k. (*a*) Show that this material, if stretched, will have a Young's modulus $Y = k/a$. (*b*) Using Table 12-1 and assuming that $a \approx 1.0$ nm, estimate a typical value for the "atomic force constant" k in a metal.

Picture the Problem We can derive this expression by imagining that we pull on an area A of the given material, expressing the force each spring will experience, finding the fractional change in length of the springs, and substituting in the definition of Young's modulus.

(*a*) The definition of Young's modulus is:

$$Y = \frac{F/A}{\Delta L/L} \qquad (1)$$

Express the elongation ΔL of each spring:

$$\Delta L = \frac{F_s}{k} \qquad (2)$$

The force F_s each spring will experience as a result of a force F acting on the area A is:

$$F_s = \frac{F}{N}$$

Express the number of springs N in the area A:

$$N = \frac{A}{a^2}$$

Substituting for N yields:

$$F_s = \frac{Fa^2}{A}$$

Substitute F_s in equation (2) to obtain, for the extension of one spring:

$$\Delta L = \frac{Fa^2}{kA}$$

Assuming that the springs extend/compress linearly, the fractional extension of the springs is:

$$\frac{\Delta L_{tot}}{L} = \frac{\Delta L}{a} = \frac{1}{a}\frac{Fa^2}{kA} = \frac{Fa}{kA}$$

Substitute in equation (1) and simplify to obtain:

$$Y = \frac{\frac{F}{A}}{\frac{Fa}{kA}} = \boxed{\frac{k}{a}}$$

(*b*) From our result in Part (*a*):

$$k = Ya$$

From Table 12-1:

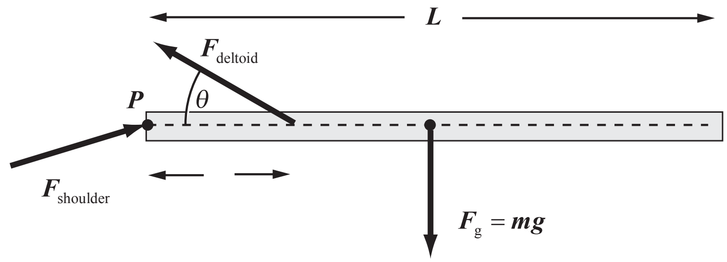
$$\boldsymbol{Y} = 200\text{GN/m}^2 = 2.00\times10^{11}\ \text{N/m}^2$$

Substitute numerical values and evaluate k:

$$\boldsymbol{k} = \left(2.00\times10^{11}\ \text{N/m}^2\right)\left(1.0\times10^{-9}\ \text{m}\right) = \boxed{2.0\ \text{N/cm}}$$

12 •• By considering the torques about the centers of the ball joints in your shoulders, estimate the force your deltoid muscles (those muscles on top of your shoulder) must exert on your upper arm, in order to keep your arm held out and extended at shoulder level. Then, estimate the force they must exert when you hold a 10-lb weight out to the side at arm's length.

Picture the Problem A model of your arm is shown in the pictorial representation. Your shoulder joint is at point P and the force the deltoid muscles exert on your extended arm $\vec{F}_{\text{deltoid}}$ is shown acting at an angle θ with the horizontal. The weight of your arm is the gravitational force $\vec{F}_{\text{g}} = m\vec{g}$ exerted by Earth through the center of gravity of your arm. We can use the condition for rotational equilibrium to estimate the forces exerted by your deltoid muscles. Note that, because its moment arm is zero, the torque due to $\vec{F}_{\text{shoulder}}$ about an axis through point P and perpendicular to the page is zero.



Apply $\sum \tau_P = 0$ to your extended arm:

$$F_{\text{deltoid}}\ell \sin\theta - \tfrac{1}{2}mgL = 0 \qquad (1)$$

Solving for F_{deltoid} yields:

$$F_{\text{deltoid}} = \frac{mgL}{2\ell\sin\theta}$$

Assuming that $\ell \approx 20$ cm, $L \approx 60$ cm, $mg \approx 10$ lb, and $\theta \approx 10°$, substitute numerical values and evaluate F_{deltoid}:

$$F_{\text{deltoid}} = \frac{(10\text{ lb})(60\text{ cm})}{2(20\text{ cm})\sin 10°} \approx \boxed{86\text{ lb}}$$

If you hold a 10-lb weight at the end of your arm, equation (1) becomes:

$$F'_{\text{deltoid}}\ell \sin\theta - \tfrac{1}{2}mgL - m'gL = 0$$

where m' is the mass of the 10-lb weight.

Solving for F_{deltoid} yields:

$$F'_{\text{deltoid}} = \frac{mgL + 2m'gL}{2\ell\sin\theta}$$

Substitute numerical values and evaluate F'_{deltoid}:

$$F_{\text{deltoid}} = \frac{(10\text{ lb})(60\text{ cm}) + 2(10\text{ lb})(60\text{ cm})}{2(20\text{ cm})\sin 10°} \approx \boxed{260\text{ lb}}$$

Conditions for Equilibrium

13 • Your crutch is pressed against the sidewalk with a force $\vec{F}_{\text{c}}$ along its own direction, as shown in Figure 12-29. This force is balanced by the normal force $\vec{F}_n$ and a frictional force $\vec{f}_s$. (*a*) Show that when the force of friction is at its maximum value, the coefficient of friction is related to the angle θ by $\mu_s = \tan\theta$. (*b*) Explain how this result applies to the forces on your foot when you are not using a crutch. (*c*) Why is it advantageous to take short steps when walking on slippery surfaces?

Picture the Problem Choose a coordinate system in which upward is the positive y direction and to the right is the positive x direction and use the conditions for translational equilibrium.

(*a*) Apply $\sum\vec{F} = 0$ to the forces acting on the tip of the crutch:

$$\sum F_x = -f_s + F_c\sin\theta = 0 \quad (1)$$

and

$$\sum F_y = F_n - F_c\cos\theta = 0 \quad (2)$$

Solve equation (2) for F_n and assuming that $f_s = f_{s,max}$, obtain:

$$f_s = f_{s,max} = \mu_s F_n = \mu_s F_c\cos\theta$$

Substitute in equation (1) and solve for μ_s:

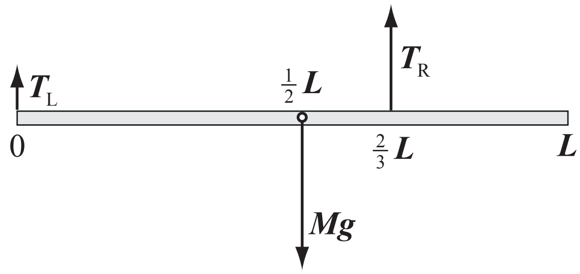
$$\mu_s = \boxed{\tan\theta}$$

(*b*) Taking long strides requires a large coefficient of static friction because θ is large for long strides.

(*c*) If μ_s is small (the surface is slippery), θ must be small to avoid slipping.

14 •• A thin rod of mass M is suspended horizontally by two vertical wires. One wire is at the left end of the rod, and the other wire is 2/3 of the way from the left end. (*a*) Determine the tension in each wire. (*b*) An object is now hung by a string attached to the right end of the rod. When this happens, it is noticed that the wire remains horizontal but the tension in the wire on the left vanishes. Determine the mass of the object.

Picture the Problem The pictorial representation shows the thin rod with the forces described in Part (*a*) acting on it. We can apply $\sum\vec{\tau}_0 = 0$ to the rod to find the forces T_L and T_R. The simplest way to determine the mass m of the object suspended from the rod in (*b*) is to apply the condition for rotational equilibrium a second time, but this time with respect to an axis perpendicular to the page and through the point at which $\vec{T}_R$ acts.



(*a*) Apply $\sum \vec{\tau}_0 = 0$ to the rod:

$$\tfrac{2}{3}LT_{\mathrm{R}} - \tfrac{1}{2}LMg = 0 \Rightarrow T_{\mathrm{R}} = \boxed{\tfrac{3}{4}Mg}$$

Apply $\sum \vec{F}_{\text{vertical}} = 0$ to the rod:

$$T_{\mathrm{L}} - Mg + T_{\mathrm{R}} = 0$$

Substitute for T_{R} to obtain:

$$T_{\mathrm{L}} - Mg + \tfrac{3}{4}Mg = 0 \Rightarrow T_{\mathrm{L}} = \boxed{\tfrac{1}{4}Mg}$$

(*b*) With an object of mass m suspended from the right end of the rod and $T_{\mathrm{L}} = 0$, applying $\sum \vec{\tau} = 0$ about an axis perpendicular to the page and through the point at which $\vec{T}_{\mathrm{R}}$ acts yields:

$$\left(\tfrac{2}{3} - \tfrac{1}{2}\right)LMg - \left(1 - \tfrac{2}{3}\right)Lmg = 0$$

Solving for m yields:

$$m = \boxed{\tfrac{1}{2}M}$$

The Center of Gravity

15 • An automobile has 58 percent of its weight on the front wheels. The front and back wheels on each side are separated by 2.0 m. Where is the center of gravity located?

Picture the Problem Let the weight of the automobile be w. Choose a coordinate system in which the origin is at the point of contact of the front wheels with the ground and the positive x axis includes the point of contact of the rear wheels with the ground. Apply the definition of the center of gravity to find its location.

Use the definition of the center of gravity to obtain:

$$\begin{aligned} x_{\mathrm{cg}}W &= \sum_i w_i x_i \\ &= 0.58w(0) + 0.42w(2.0\,\mathrm{m}) \\ &= (0.84\,\mathrm{m})w \end{aligned}$$

Because $W = w$:

$$x_{\mathrm{cg}}w = (0.84\,\mathrm{m})w \Rightarrow \boldsymbol{x}_{\mathrm{cg}} = \boxed{84\,\mathrm{cm}}$$

Static Equilibrium

16 • Figure 12-30 shows a lever of negligible mass with a vertical force F_{app} being applied to lift a load F. The *mechanical advantage* of the lever is defined as $M = F/F_{\text{app, min}}$, where $F_{\text{app, min}}$ is the smallest force necessary to lift the load F. Show that for this simple lever system, $M = x/X$, where x is the moment arm (distance to the pivot) for the applied force and X is the moment arm for the load.

Picture the Problem We can use the given definition of the mechanical advantage of a lever and the condition for rotational equilibrium to show that $M = x/X$.

Express the definition of mechanical advantage for a lever:

$$M = \frac{F}{F_{\text{app, min}}} \qquad (1)$$

Apply the condition for rotational equilibrium to the lever:

$$\sum \tau_{\text{fulcrum}} = xF_{\text{app, min}} - XF = 0$$

Solve for the ratio of F to $F_{\text{app, min}}$ to obtain:

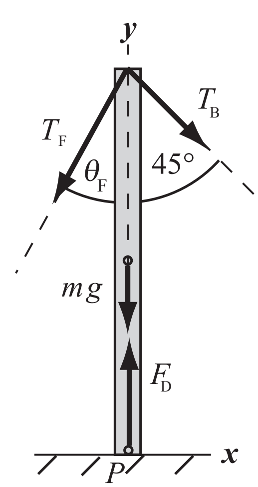
$$\frac{F}{F_{\text{app, min}}} = \frac{x}{X}$$

Substitute for $F/F_{\text{app, min}}$ in equation (1) to obtain:

$$M = \boxed{\frac{x}{X}}$$

17 • **[SSM]** Figure 12-31 shows a 25-foot sailboat. The mast is a uniform 120-kg pole that is supported on the deck and held fore and aft by wires as shown. The tension in the *forestay* (wire leading to the bow) is 1000 N. Determine the tension in the *backstay* (wire leading aft) and the normal force that the deck exerts on the mast. (Assume that the frictional force the deck exerts on the mast to be negligible.)

Picture the Problem The force diagram shows the forces acting on the mast. Let the origin of the coordinate system be at the foot of the mast with the $+x$ direction to the right and the $+y$ direction upward. Because the mast is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find the tension in the backstay, T_B, and the normal force, F_D, that the deck exerts on the mast.



Apply $\sum\vec{\tau}=0$ to the mast about an axis through point P:

$$(4.88\,\text{m})(1000\,\text{N})\sin\theta_{\text{F}} - (4.88\,\text{m})T_{\text{B}}\sin 45.0^\circ = 0$$

Solve for T_{B} to obtain:

$$T_{\text{B}} = \frac{(1000\,\text{N})\sin\theta_{\text{F}}}{\sin 45.0^\circ} \qquad (1)$$

Find θ_{F}, the angle of the forestay with the vertical:

$$\theta_{\text{F}} = \tan^{-1}\left(\frac{2.74\,\text{m}}{4.88\,\text{m}}\right) = 29.3^\circ$$

Substitute numerical values in equation (1) and evaluate T_{B}:

$$T_{\text{B}} = \frac{(1000\,\text{N})\sin 29.3^\circ}{\sin 45.0^\circ} = \boxed{692\,\text{N}}$$

Apply the condition for translational equilibrium in the y direction to the mast:

$$\sum F_y = F_{\text{D}} - T_{\text{F}}\cos\theta_{\text{F}} - T_{\text{B}}\cos 45^\circ - mg = 0$$

Solving for F_{D} yields:

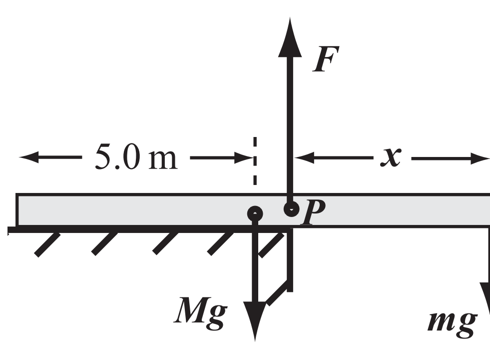
$$F_{\text{D}} = T_{\text{F}}\cos\theta_{\text{F}} + T_{\text{B}}\cos 45^\circ + mg$$

Substitute numerical values and evaluate F_{D}:

$$F_{\text{D}} = (1000\text{ N})\cos 29.3^\circ + (692\text{ N})\cos 45^\circ + (120\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{2.54\,\text{kN}}$$

18 •• A uniform 10.0-m beam of mass 300 kg extends over a ledge as in Figure 12-32. The beam is not attached, but simply rests on the surface. A 60.0-kg student intends to position the beam so that he can walk to the end of it. What is the maximum distance the beam can extend past end of the ledge and still allow him to perform this feat?

Picture the Problem The diagram shows $M\vec{g}$, the weight of the beam, $m\vec{g}$, the weight of the student, and the force the ledge exerts $\vec{F}$, acting on the beam. Because the beam is in equilibrium, we can apply the condition for rotational equilibrium to the beam to find the location of the pivot point P that will allow the student to walk to the end of the beam.



Apply $\sum\vec{\tau}=0$ about an axis through the pivot point P:

$$Mg(5.0\,\text{m} - x) - mgx = 0$$

Solving for x yields:

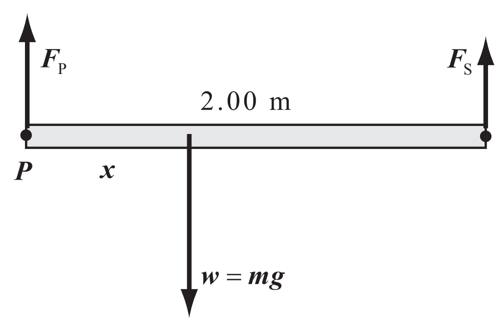
$$x = \frac{5.0M}{M + m}$$

Substitute numerical values and evaluate x:

$$\boldsymbol{x} = \frac{(5.0\,\text{m})(300\,\text{kg})}{300\,\text{kg} + 60.0\,\text{kg}} = \boxed{4.2\,\text{m}}$$

19 •• **[SSM]** A *gravity board* is a convenient and quick way to determine the location of the center of gravity of a person. It consists of a horizontal board supported by a fulcrum at one end and a scale at the other end. To demonstrate this in class, your physics professor calls on you to lie horizontally on the board with the top of your head directly above the fulcrum point as shown in Figure 12-33. The scale is 2.00 m from the fulcrum. In preparation for this experiment, you had accurately weighed yourself and determined your mass to be 70.0 kg. When you are at rest on the gravity board, the scale advances 250 N beyond its reading when the board is there by itself. Use this data to determine the location of your center of gravity relative to your feet.

Picture the Problem The diagram shows $\vec{w}$, the weight of the student, $\vec{F}_\text{P}$, the force exerted by the board at the pivot, and $\vec{F}_\text{s}$, the force exerted by the scale, acting on the student. Because the student is in equilibrium, we can apply the condition for rotational equilibrium to the student to find the location of his center of gravity.



Apply $\sum \vec{\tau} = 0$ about an axis through the pivot point P:

$$F_\text{s}(2.00\,\text{m}) - wx = 0$$

Solving for x yields:

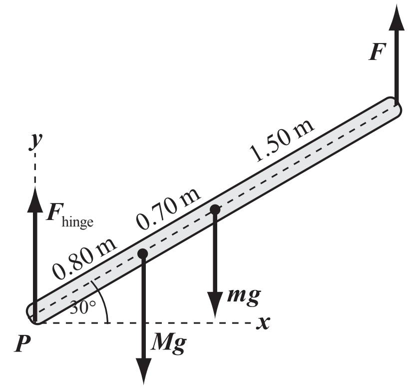
$$\boldsymbol{x} = \frac{(2.00\,\text{m})F_\text{s}}{w}$$

Substitute numerical values and evaluate x:

$$\boldsymbol{x} = \frac{(2.00\,\text{m})(250\,\text{N})}{(70.0\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.728\,\text{m}}$$

20 •• A stationary 3.0-m board of mass 5.0 kg is hinged at one end. A force $\vec{F}$ is applied vertically at the other end, and the board makes at 30° angle with the horizontal. A 60-kg block rests on the board 80 cm from the hinge as shown in Figure 12-34. (*a*) Find the magnitude of the force $\vec{F}$. (*b*) Find the force exerted by the hinge. (*c*) Find the magnitude of the force $\vec{F}$, as well as the force exerted by the hinge, if $\vec{F}$ is exerted, instead, at right angles to the board.

Picture the Problem The diagram shows $m\vec{g}$, the weight of the board, $\vec{F}_{\text{hinge}}$, the force exerted by the hinge, $M\vec{g}$, the weight of the block, and $\vec{F}$, the force acting vertically at the right end of the board. Because the board is in equilibrium, we can apply the condition for rotational equilibrium to it to find the magnitude of $\vec{F}$.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the hinge to obtain:

$$F[(3.0\,\text{m})\cos 30°] - mg[(1.50\,\text{m})\cos 30°] - Mg[(0.80\,\text{m})\cos 30°] = 0$$

Solving for F yields:

$$F = \frac{m(1.50\,\text{m}) + M(0.80\,\text{m})}{3.0\,\text{m}}g$$

Substitute numerical values and evaluate F:

$$F = \frac{(5.0\,\text{kg})(1.50\,\text{m}) + (60\,\text{kg})(0.80\,\text{m})}{3.0\,\text{m}}(9.81\,\text{m/s}^2) = 181\,\text{N} = \boxed{0.81\,\text{kN}}$$

Apply $\sum F_y = 0$ to the board to obtain:

$$F_{\text{hinge}} - Mg - mg + F = 0$$

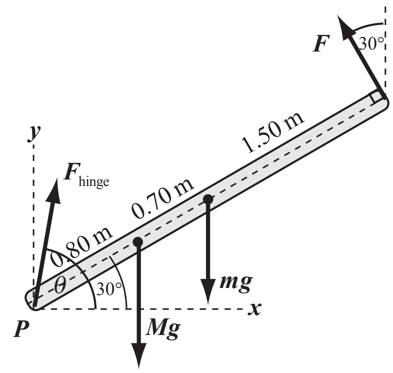
Solving for F_{hinge} yields:

$$F_{\text{hinge}} = Mg + mg - F = (M + m)g - F$$

Substitute numerical values and evaluate F_{hinge}:

$$F_{\text{hinge}} = (60\,\text{kg} + 5.0\,\text{kg})(9.81\,\text{m/s}^2) - 181\,\text{N}$$
$$= \boxed{0.46\,\text{kN}}$$

(c) The force diagram showing the force $\vec{F}$ acting at right angles to the board is shown to the right:



Apply $\sum \vec{\tau} = 0$ about the hinge:

$$F(3.0\,\text{m}) - mg[(1.5\,\text{m})\cos 30°] - Mg[(0.80\,\text{m})\cos 30°] = 0$$

Solving for F yields:

$$F = \frac{m(1.5\,\text{m}) + M(0.80\,\text{m})}{3.0\,\text{m}} g \cos 30°$$

Substitute numerical values and evaluate F:

$$F = \frac{(5.0\,\text{kg})(1.5\,\text{m}) + (60\,\text{kg})(0.80\,\text{m})}{3.0\,\text{m}}(9.81\,\text{m/s}^2)\cos 30° = 157\,\text{N} = \boxed{0.16\,\text{kN}}$$

Apply $\sum F_y = 0$ to the board:

$$F_{\text{hinge}} \sin\theta - Mg - mg + F\cos 30° = 0$$

or

$$F_{\text{hinge}} \sin\theta = (M + m)g - F\cos 30° \quad (1)$$

Apply $\sum F_x = 0$ to the board:

$$F_{\text{hinge}} \cos\theta - F\sin 30° = 0$$

or

$$F_{\text{hinge}} \cos\theta = F\sin 30° \quad (2)$$

Divide the first of these equations by the second to obtain:

$$\frac{F_{\text{hinge}} \sin\theta}{F_{\text{hinge}} \cos\theta} = \frac{(M + m)g - F\cos 30°}{F\sin 30°}$$

Solving for θ yields:

$$\theta = \tan^{-1}\left[\frac{(M + m)g - F\cos 30°}{F\sin 30°}\right]$$

Substitute numerical values and evaluate θ:

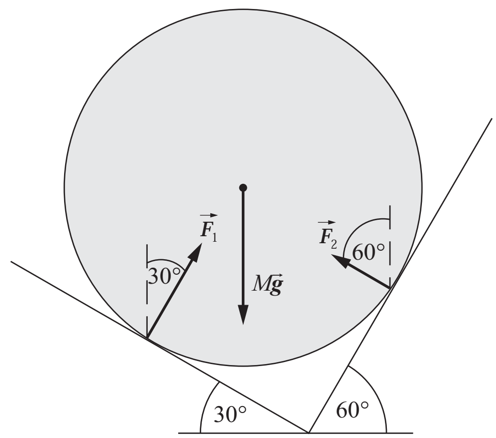
$$\theta = \tan^{-1}\left[\frac{(65\,\text{kg})(9.81\,\text{m/s}^2) - (157\,\text{N})\cos 30°}{(157\,\text{N})\sin 30°}\right] = 81.1°$$

Substitute numerical values in equation (2) and evaluate F_{hinge}:

$$\boldsymbol{F}_{\text{hinge}} = \frac{(157\ \text{N})\sin 30°}{\cos 81.1°} = \boxed{0.51\,\text{kN}}$$

21 •• A cylinder of mass M is supported by a frictionless trough formed by a plane inclined at 30º to the horizontal on the left and one inclined at 60º on the right as shown in Figure 12-35. Find the force exerted by each plane on the cylinder.

Picture the Problem The planes are frictionless; therefore, the force exerted by each plane must be perpendicular to that plane. Let $\vec{F}_1$ be the force exerted by the 30° plane, and let $\vec{F}_2$ be the force exerted by the 60° plane. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can use the conditions for translational equilibrium to find the magnitudes of $\vec{F}_1$ and $\vec{F}_2$.



Apply $\sum F_x = 0$ to the cylinder: $$F_1 \sin 30° - F_2 \sin 60° = 0 \quad (1)$$

Apply $\sum F_y = 0$ to the cylinder: $$F_1 \cos 30° + F_2 \cos 60° - Mg = 0 \quad (2)$$

Solve equation (1) for F_1: $$F_1 = \sqrt{3}F_2 \quad (3)$$

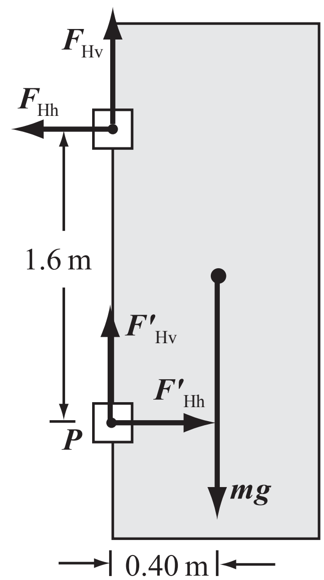
Substitute for F_1 in equation (2) to obtain: $$\sqrt{3}F_2 \cos 30° + F_2 \cos 60° - Mg = 0$$

Solve for F_2 to obtain: $$F_2 = \frac{Mg}{\sqrt{3}\cos 30° + \cos 60°} = \boxed{\tfrac{1}{2}Mg}$$

Substitute for F_2 in equation (3) to obtain: $$F_1 = \sqrt{3}\left(\tfrac{1}{2}Mg\right) = \boxed{\tfrac{\sqrt{3}}{2}Mg}$$

22 •• A uniform 18-kg door that is 2.0 m high by 0.80 m wide is hung from two hinges that are 20 cm from the top and 20 cm from the bottom. If each hinge supports half the weight of the door, find the magnitude and direction of the horizontal components of the forces exerted by the two hinges on the door.

Picture the Problem The drawing shows the door and its two supports. The center of gravity of the door is 0.80 m above (and below) the hinge, and 0.40 m from the hinges horizontally. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Denote the horizontal and vertical components of the hinge force by F_{Hh} and F_{Hv}. Because the door is in equilibrium, we can use the conditions for translational and rotational equilibrium to determine the horizontal forces exerted by the hinges.



Apply $\sum\vec{\tau} = 0$ about an axis through the lower hinge:

$$\boldsymbol{F}_{Hh}(1.6\,\text{m}) - \boldsymbol{mg}(0.40\,\text{m}) = 0$$

Solve for F_{Hh}:

$$\boldsymbol{F}_{Hh} = \frac{\boldsymbol{mg}(0.40\,\text{m})}{1.6\,\text{m}}$$

Substitute numerical values and evaluate F_{Hh}:

$$\boldsymbol{F}_{Hh} = \frac{(18\,\text{kg})(9.81\,\text{m/s}^2)(0.40\,\text{m})}{1.6\,\text{m}} = \boxed{44\,\text{N}}$$

Apply $\sum F_x = 0$ to the door and solve for F'_{Hh} :

$$F'_{Hh} - F_{Hh} = 0$$

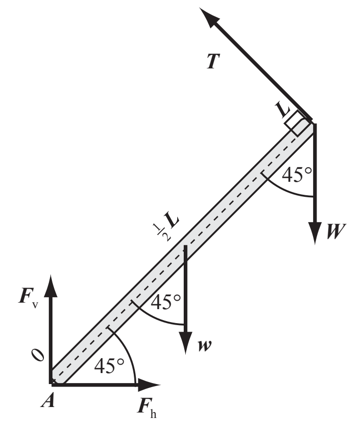
and

$$\boldsymbol{F'}_{Hh} = \boxed{44\,\text{N}}$$

Remarks: Note that the upper hinge pulls on the door and the lower hinge pushes on it.

23 •• Find the force exerted on the strut by the hinge at A for the arrangement in Figure 12-36 if (a) the strut is weightless, and (b) the strut weighs 20 N.

Picture the Problem Let T be the tension in the line attached to the wall and L be the length of the strut. The figure includes w, the weight of the strut, for part (b). Because the strut is in equilibrium, we can use the conditions for both rotational and translational equilibrium to find the force exerted on the strut by the hinge.



(a) Express the force exerted on the strut at the hinge:

$$\vec{F} = F_h\hat{i} + F_v\hat{j} \quad (1)$$

Ignoring the weight of the strut, apply $\sum\vec{\tau} = 0$ at the hinge:

$$LT - (L\cos 45°)W = 0$$

Solve for the tension in the line:

$$T = W\cos 45° = (60\,\text{N})\cos 45° = 42.4\,\text{N}$$

Apply $\sum\vec{F} = 0$ to the strut:

$$\sum F_x = F_h - T\cos 45° = 0$$

and

$$\sum F_y = F_v + T\cos 45° - Mg = 0$$

Solve for and evaluate F_h:

$$F_h = T\cos 45° = (42.4\,\text{N})\cos 45° = 30\,\text{N}$$

Solve for and evaluate F_v:

$$F_v = Mg - T\cos 45° = 60\,\text{N} - (42.4\,\text{N})\cos 45° = 30\,\text{N}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \boxed{(30\,\text{N})\hat{i} + (30\,\text{N})\hat{j}}$$

(b) Including the weight of the strut, apply $\sum\vec{\tau} = 0$ at the hinge:

$$LT - (L\cos 45°)W - \left(\frac{L}{2}\cos 45\right)w = 0$$

Solve for the tension in the line:

$$T = (\cos 45°)W + \left(\frac{1}{2}\cos 45°\right)w$$

Substitute numerical values and evaluate T:

$$T = (\cos 45°)(60\,\text{N}) + \left(\frac{1}{2}\cos 45°\right)(20\,\text{N})$$
$$= 49.5\,\text{N}$$

Apply $\sum \vec{F} = 0$ to the strut:

$$\sum F_x = F_\text{h} - T\cos 45° = 0$$
and
$$\sum F_y = F_\text{v} + T\cos 45° - W - w = 0$$

Solve for and evaluate F_h:

$$\boldsymbol{F}_\text{h} = \boldsymbol{T}\cos 45° = (49.5\,\text{N})\cos 45°$$
$$= 35\,\text{N}$$

Solve for and evaluate F_v:

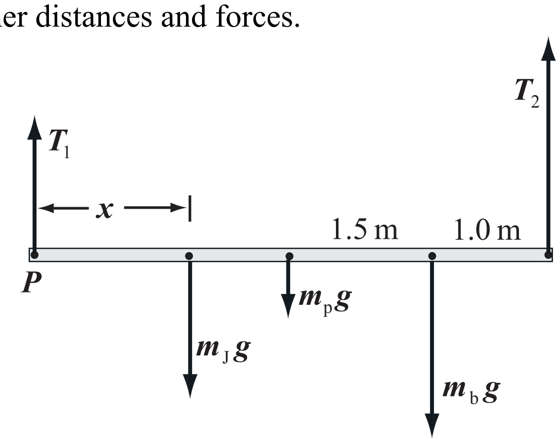
$$\boldsymbol{F}_\text{v} = \boldsymbol{W} + \boldsymbol{w} - \boldsymbol{T}\cos 45°$$
$$= 60\,\text{N} + 20\,\text{N} - (49.5\,\text{N})\cos 45°$$
$$= 45\,\text{N}$$

Substitute for F_h and F_v to obtain:

$$\vec{F} = \boxed{(35\,\text{N})\hat{\boldsymbol{i}} + (45\,\text{N})\hat{\boldsymbol{j}}}$$

24 •• Julie has been hired to help paint the trim of a building, but she is not convinced of the safety of the apparatus. A 5.0-m plank is suspended horizontally from the top of the building by ropes attached at each end. Julie knows from previous experience that the ropes being used will break if the tension exceeds 1.0 kN. Her 80-kg boss dismisses Julie's worries and begins painting while standing 1.0 m from the end of the plank. If Julie's mass is 60 kg and the plank has a mass of 20 kg, then over what range of positions can Julie stand to join her boss without causing the ropes to break?

Picture the Problem Note that if Julie is at the far left end of the plank, T_1 and T_2 are less than 1.0 kN. Let x be the distance of Julie from T_1. Because the plank is in equilibrium, we can apply the condition for rotational equilibrium to relate the distance x to the other distances and forces.



Apply $\sum \vec{\tau} = 0$ about an axis through the left end of the plank:

$$(5.0\,\text{m})\boldsymbol{T}_2 - (4.0\,\text{m})\boldsymbol{m}_\text{b}\boldsymbol{g} - (2.5\,\text{m})\boldsymbol{m}_\text{p}\boldsymbol{g} - \boldsymbol{m}_\text{J}\boldsymbol{g}\boldsymbol{x} = 0$$

Solving for x yields:

$$\boldsymbol{x} = \frac{(5.0\,\text{m})\boldsymbol{T}_2}{\boldsymbol{m}_\text{J}\boldsymbol{g}} - \frac{(4.0\,\text{m})\boldsymbol{m}_\text{b}}{\boldsymbol{m}_\text{J}} - \frac{(2.5\,\text{m})\boldsymbol{m}_\text{p}}{\boldsymbol{m}_\text{J}}$$

Substitute numerical values and simplify to obtain:

$$\boldsymbol{x} = \left(8.495\times10^{-3}\,\frac{\text{m}}{\text{N}}\right)\boldsymbol{T}_2 - 6.17\,\text{m}$$

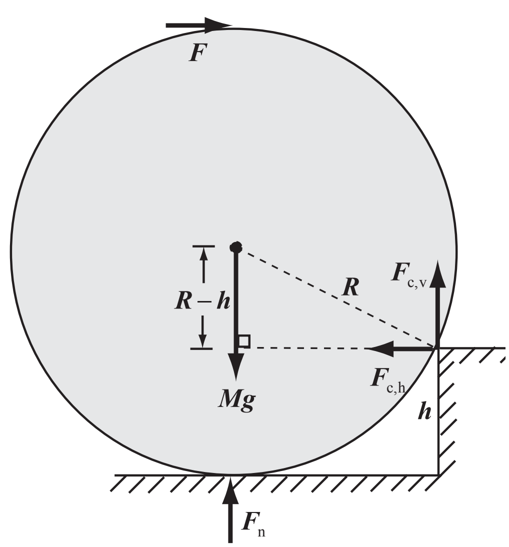
Set $T_2 = 1.0$ kN and evaluate x:

$$\boldsymbol{x} = \left(8.495\times10^{-3}\,\frac{\text{m}}{\text{N}}\right)(1.0\,\text{kN}) - 6.17\,\text{m} = 2.3\,\text{m}$$

and Julie is safe provided $\boxed{\boldsymbol{x} < 2.3\,\text{m}.}$

25 •• **[SSM]** A cylinder of mass M and radius R rolls against a step of height h as shown in Figure 12-37. When a horizontal force of magnitude F is applied to the top of the cylinder, the cylinder remains at rest. (a) Find an expression for the normal force exerted by the floor on the cylinder. (b) Find an expression for the horizontal force exerted by the edge of the step on the cylinder. (c) Find an expression for the vertical component of the force exerted by the edge of the step on the cylinder.

Picture the Problem The figure to the right shows the forces acting on the cylinder. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find F_n and the horizontal and vertical components of the force the corner of the step exerts on the cylinder.



(a) Apply $\sum \vec{\tau} = 0$ to the cylinder about the step's corner:

$$Mg\ell - F_\text{n}\ell - F(2R - h) = 0$$

Solving for F_n yields:

$$F_n = Mg - \frac{F(2R-h)}{\ell}$$

Express ℓ as a function of R and h:

$$\ell = \sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2}$$

Substitute for ℓ in the expression for F_n and simplify to obtain:

$$F_n = Mg - \frac{F(2R-h)}{\sqrt{2Rh-h^2}} = \boxed{Mg - F\sqrt{\frac{2R-h}{h}}}$$

(*b*) Apply $\sum F_x = 0$ to the cylinder:

$$-F_{c,h} + F = 0$$

Solve for $F_{c,h}$:

$$F_{c,h} = \boxed{F}$$

(*c*) Apply $\sum F_y = 0$ to the cylinder:

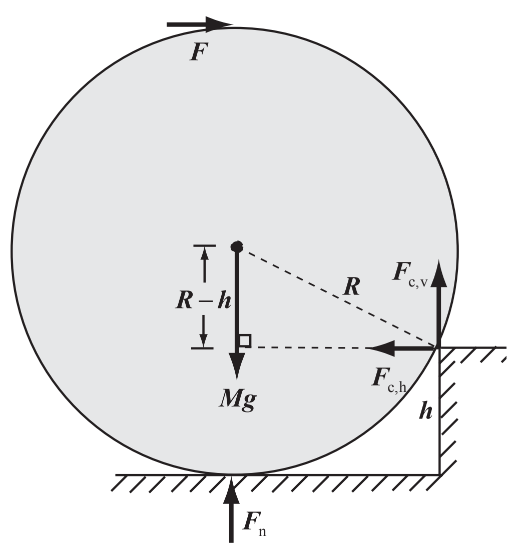
$$F_n - Mg + F_{c,v} = 0 \Rightarrow F_{c,v} = Mg - F_n$$

Substitute the result from Part (*a*) and simplify to obtain:

$$F_{c,v} = Mg - \left\{Mg - F\sqrt{\frac{2R-h}{h}}\right\} = \boxed{F\sqrt{\frac{2R-h}{h}}}$$

26 •• For the cylinder in Problem 25, find an expression for the minimum magnitude of the horizontal force $\vec{F}$ that will roll the cylinder over the step if the cylinder does not slide on the edge.

Picture the Problem The figure to the right shows the forces acting on the cylinder. Because the cylinder is in equilibrium, we can use the condition for rotational equilibrium to express F_n in terms of F. Because, to roll over the step, the cylinder must lift off the floor, we can set $F_n = 0$ in our expression relating F_n and F and solve for F.



Apply $\sum \vec{\tau} = 0$ about the step's corner:

$$Mg\ell - F_{\mathrm{n}}\ell - F(2R - h) = 0$$

Solve for F_{n}:

$$F_{\mathrm{n}} = Mg - \frac{F(2R - h)}{\ell}$$

Express ℓ as a function of R and h:

$$\ell = \sqrt{R^2 - (R - h)^2} = \sqrt{2Rh - h^2}$$

Substitute for ℓ in the expression for F_{n} and simplify to obtain:

$$F_{\mathrm{n}} = Mg - \frac{F(2R - h)}{\sqrt{2Rh - h^2}} = Mg - F\sqrt{\frac{2R - h}{h}}$$

To roll over the step, the cylinder must lift off the floor. That is, $F_{\mathrm{n}} = 0$:

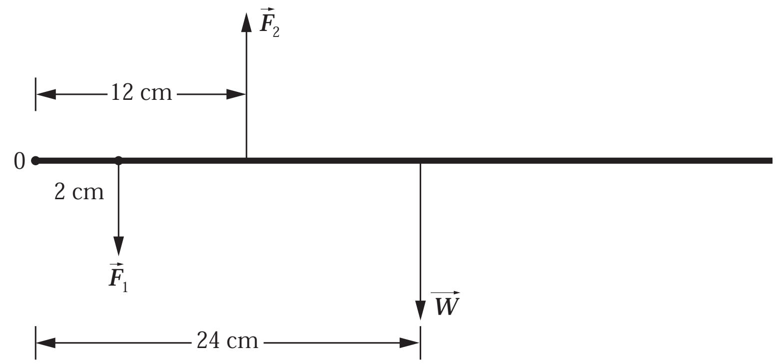
$$0 = Mg - F\sqrt{\frac{2R - h}{h}}$$

Solving for F yields:

$$F = \boxed{Mg\sqrt{\frac{h}{2R - h}}}$$

27 •• Figure 12-38 shows a hand holding an epee, a weapon used in the sport of fencing which you are taking as a physical education elective. The center of mass of your epee is 24 cm from the pommel (the end of the epee at the grip). You have weighed it so you know that the epee's mass is 0.700 kg and its full length is 110 cm. (*a*) At the beginning of a match you hold it straight out in static equilibrium. Find the total force exerted by your hand on the epee. (*b*) Find the torque exerted by your hand on the epee. (*c*) Your hand, being an extended object, actually exerts its force along the length of the epee grip. Model the total force exerted by your hand as two oppositely directed forces whose lines of action are separated by the width of your hand (taken to be 10.0 cm). Find the magnitudes and directions of these two forces.

Picture the Problem The diagram shows the forces F_1 and F_2 that the fencer's hand exerts on the epee. We can use a condition for translational equilibrium to find the upward force the fencer must exert on the epee when it is in equilibrium and the definition of torque to determine the total torque exerted. In Part (*c*) we can use the conditions for translational and rotational equilibrium to obtain two equations in F_1 and F_2 that we can solve simultaneously. In Part (*d*) we can apply Newton's 2^{nd} law in rotational form and the condition for translational equilibrium to obtain two equations in F_1 and F_2 that, again, we can solve simultaneously.



(a) Letting the upward force exerted by the fencer's hand be F, apply $\sum F_y = 0$ to the epee to obtain:

$$F - W = 0$$
and
$$F = W = mg$$

Substitute numerical values and evaluate F:

$$\boldsymbol{F} = (0.700\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{6.87\,\text{N}}$$

(b) The torque due to the weight about the left end of the epee is equal in magnitude but opposite in direction to the torque exerted by your hand on the epee:

$$\tau = \ell w$$

Substitute numerical values and evaluate τ:

$$\boldsymbol{\tau} = (0.24\,\text{m})(6.87\,\text{N}) = 1.65\,\text{N}\cdot\text{m}$$
$$= \boxed{1.7\,\text{N}\cdot\text{m}}$$

(c) Apply $\sum F_y = 0$ to the epee to obtain:

$$-F_1 + F_2 - 6.87\,\text{N} = 0 \qquad (1)$$

Apply $\sum \vec{\tau}_0 = 0$ to obtain:

$$-(0.020\,\text{m})F_1 + (0.12\,\text{m})F_2 - 1.65\,\text{N}\cdot\text{m} = 0 \qquad (2)$$

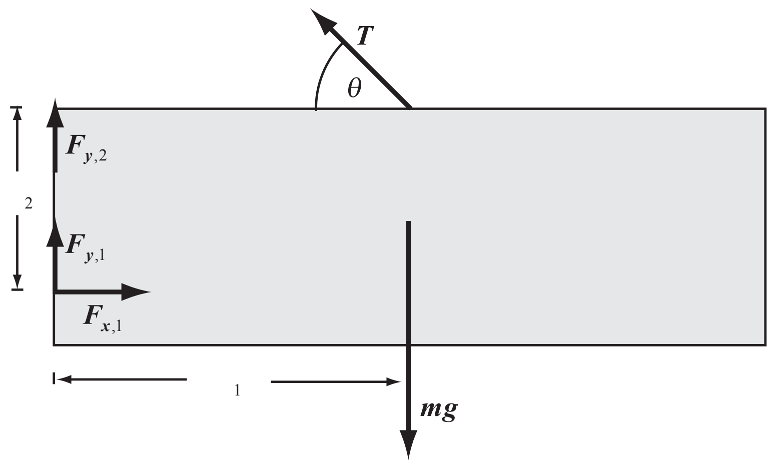
Solve equations (1) and (2) simultaneously to obtain:

$$F_1 = \boxed{8.3\,\text{N}} \text{ and } F_2 = \boxed{15\,\text{N}}.$$

Remarks: Note that the force nearest the butt of the epee is directed downward and the force nearest the hand guard is directed upward.

28 •• A large gate weighing 200 N is supported by hinges at the top and bottom and is further supported by a wire as shown in Figure 12-39. (a) What must be the tension in the wire for the force on the upper hinge to have no horizontal component? (b) What is the horizontal force on the lower hinge? (c) What are the vertical forces on the hinges?

Picture the Problem In the force diagram, the forces exerted by the hinges are $\vec{F}_{y,2}$, $\vec{F}_{y,1}$, and $\vec{F}_{x,1}$ where the subscript 1 refers to the lower hinge. Because the gate is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find the tension in the wire and the forces at the hinges.



(a) Apply $\sum \vec{\tau} = 0$ about an axis through the lower hinge and perpendicular to the plane of the page:

$$\ell_1 T \sin\theta + \ell_2 T \cos\theta - \ell_1 mg = 0$$

Solving for T yields:

$$T = \frac{\ell_1 mg}{\ell_1 \sin\theta + \ell_2 \cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(1.5\,\text{m})(200\,\text{N})}{(1.5\,\text{m})\sin 45° + (1.5\,\text{m})\cos 45°} = 141\,\text{N} = \boxed{0.14\,\text{kN}}$$

(b) Apply $\sum F_x = 0$ to the gate:

$$F_{x,1} - T\cos 45° = 0$$

Solve for and evaluate $F_{x,1}$:

$$F_{x,1} = T\cos 45° = (141\,\text{N})\cos 45° = 99.7\,\text{N} = \boxed{1.0\times10^2\,\text{N}}$$

(c) Apply $\sum F_y = 0$ to the gate:

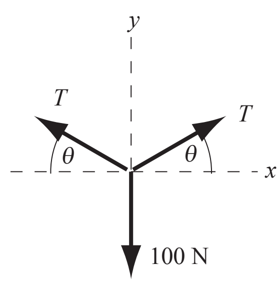
$$F_{y,1} + F_{y,2} + T\sin 45° - mg = 0$$

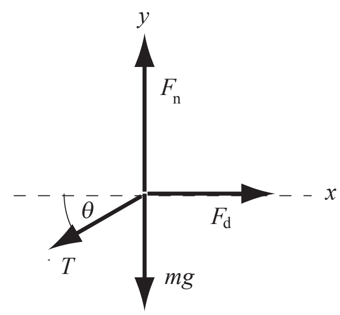
Because $F_{y,1}$ and $F_{y,2}$ cannot be determined independently, solve for and evaluate their sum:

$$F_{y,1} + F_{y,2} = mg - T\sin 45° = 200\,\text{N} - 99.7\,\text{N} = \boxed{1.0\times10^2\,\text{N}}$$

29 •• On a camping trip, you moor your boat at the end of a dock in a rapidly flowing river. It is anchored to the dock by a chain 5.0 m long, as shown in Figure 12-46. A 100-N weight is suspended from the center in the chain. This will allow the tension in the chain to change as the force of the current which pulls the boat away from the dock and to the right varies. The drag force by the water on the boat depends on the speed of the water. You decide to apply the principles of statics you learned in physics class. (Ignore the weight of the chain.) The drag force on the boat is 50 N. (*a*) What is the tension in the chain? (*b*) How far is the boat from the dock? (*c*) The maximum tension the chain can sustain is 500 N. What minimum water drag force on the boat would snap the chain?

Picture the Problem The free-body diagram shown to the left below is for the weight and the diagram to the right is for the boat. Because both are in equilibrium under the influences of the forces acting on them, we can apply a condition for translational equilibrium to find the tension in the chain.





(*a*) Apply $\sum F_x = 0$ to the boat:

$$F_d - T\cos\theta = 0 \Rightarrow T = \frac{F_d}{\cos\theta}$$

Apply $\sum F_y = 0$ to the weight:

$$2T\sin\theta - 100\text{ N} = 0 \qquad (1)$$

Substitute for T to obtain:

$$2F_d\tan\theta - 100\text{ N} = 0$$

Solve for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{100\text{ N}}{2F_d}\right]$$

Substitute for F_d and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{100\text{ N}}{2(50\text{ N})}\right] = 45°$$

Solve equation (1) for T:

$$T = \frac{100\text{ N}}{2\sin\theta}$$

Substitute for θ and evaluate T:

$$\boldsymbol{T} = \frac{100\text{ N}}{2\sin 45°} = 70.7\text{ N} = \boxed{71\text{ N}}$$

(*b*) Relate the distance *d* of the boat from the dock to the angle θ the chain makes with the horizontal:

$$\cos\theta = \frac{\frac{1}{2}d}{\frac{1}{2}L} = \frac{d}{L} \Rightarrow d = L\cos\theta$$

Substitute numerical values and evaluate *d*:

$$d = (5.0\,\text{m})\cos 45° = \boxed{3.5\,\text{m}}$$

(*c*) Relate the resultant tension in the chain to the vertical component of the tension F_v and the maximum drag force exerted on the boat by the water $F_\text{d,max}$:

$$F_\text{v}^2 + F_\text{d,max}^2 = (500\,\text{N})^2$$

Solve for $F_\text{d,max}$:

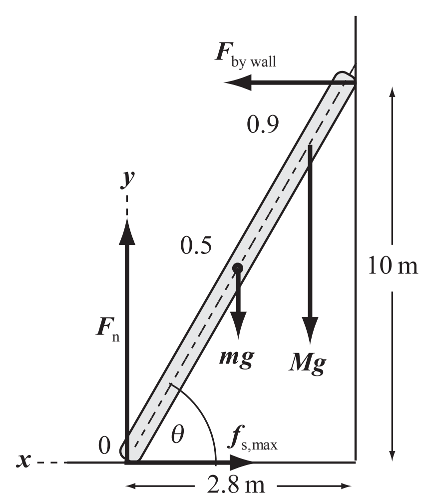
$$F_\text{d,max} = \sqrt{(500\,\text{N})^2 - F_\text{v}^2}$$

Because the vertical component of the tension is 50 N:

$$\boldsymbol{F}_\text{d,max} = \sqrt{(500\,\text{N})^2 - (50\,\text{N})^2} = \boxed{0.50\,\text{kN}}$$

30 •• Romeo takes a uniform 10-m ladder and leans it against the smooth (frictionless) wall of the Capulet residence. The ladder's mass is 22 kg and the bottom rests on the ground 2.8 m from the wall. When Romeo, whose mass is 70 kg, gets 90 percent of the way to the top, the ladder begins to slip. What is the coefficient of static friction between the ground and the ladder?

Picture the Problem The ladder and the forces acting on it at the critical moment of slipping are shown in the diagram. Use the coordinate system shown. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Using its definition, express μ_s:

$$\mu_\text{s} = \frac{f_\text{s,max}}{F_\text{n}} \qquad (1)$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder:

$$[0.9\ell\cos\theta]Mg + [0.5\ell\cos\theta]mg - [\ell\sin\theta]F_W = 0$$

Solving for F_W yields:

$$F_W = \frac{(0.9M + 0.5m)g\cos\theta}{\sin\theta}$$

Find the angle θ:

$$\theta = \cos^{-1}\left(\frac{2.8\,\text{m}}{10\,\text{m}}\right) = 73.74°$$

Substitute numerical values and evaluate F_W:

$$\boldsymbol{F}_W = \frac{[0.9(70\,\text{kg}) + 0.5(22\,\text{kg})](9.81\,\text{m/s}^2)\cos 73.74°}{\sin 73.74°} = 211.7\,\text{N}$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0$$

and

$$f_{s,max} = F_W = 211.7\,\text{N}$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - Mg - mg = 0 \Rightarrow F_n = (M + m)g$$

Substitute numerical values and evaluate F_n:

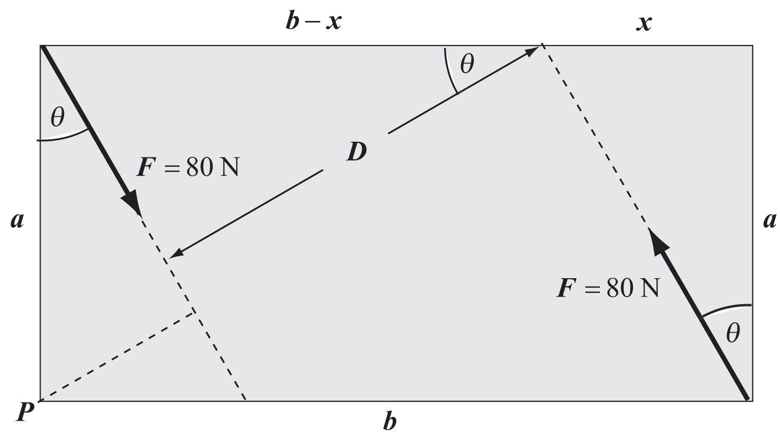
$$F_n = (70\,\text{kg} + 22\,\text{kg})(9.81\,\text{m/s}^2) = 902.5\,\text{N}$$

Substitute numerical values in equation (1) and evaluate μ_s:

$$\boldsymbol{\mu}_s = \frac{211.7\,\text{N}}{902.5\,\text{N}} = \boxed{0.23}$$

31 •• **[SSM]** Two 80-N forces are applied to opposite corners of a rectangular plate as shown in Figure 12-41. (*a*) Find the torque produced by this couple using Equation 12-6. (*b*) Show that the result is the same as if you determine the torque about the lower left-hand corner.

Picture the Problem The forces shown in the figure constitute a couple and will cause the plate to experience a counterclockwise angular acceleration. The couple equation is $\boldsymbol{\tau} = \boldsymbol{FD}$. The following diagram shows the geometric relationships between the variables in terms of a generalized angle θ.



(*a*) The couple equation is: $\tau = FD$ (1)

From the diagram, *D* is given by: $D = (b - x)\cos\theta$ (2)

Again, referring to the diagram: $x = a\tan\theta$

Substituting for *x* in equation (2) and simplifying yields:

$$\begin{aligned} D &= (b - a\tan\theta)\cos\theta \\ &= b\cos\theta - a\sin\theta \end{aligned}$$

Substituting for *D* in equation (1) yields: $\tau = F(b\cos\theta - a\sin\theta)$ (3)

Substitute numerical values and evaluate τ:

$$\begin{aligned} \tau &= (80\text{ N})(b\cos 30° - a\sin 30°) \\ &= \boxed{(69\text{ N})b - (40\text{ N})a} \end{aligned}$$

(*b*) Letting the counterclockwise direction be the positive direction, apply $\sum\vec{\tau} = 0$ about an axis normal to the plane of the rectangle and passing through point *P*:

$$F(\ell + D) - F\ell = 0$$

Substituting for *D* yields: $F(\ell + b\cos\theta - a\sin\theta) - F\ell = 0$

Solve for τ to obtain: $\tau = \boxed{F(b\cos\theta - a\sin\theta)}$, in agreement with equation (3).

32 •• A uniform cube of side *a* and mass *M* rests on a horizontal surface. A horizontal force $\vec{F}$ is applied to the top of the cube as in Figure 12-42. This force is not sufficient to move or tip the cube. (*a*) Show that the force of static friction exerted by the surface and the applied force constitute a couple, and find the torque exerted by the couple. (*b*) The torque exerted by the couple is balanced by the torque exerted by the couple consisting of the normal force on the cube and the gravitational force on the cube. Use this fact to find the effective point of application of the normal force when $F = Mg/3$. (*c*) Find the greatest magnitude of $\vec{F}$ for which the cube will not tip (Assuming the cube does not slip.).

Picture the Problem We can use the condition for translational equilibrium and the definition of a couple to show that the force of static friction exerted by the surface and the applied force constitute a couple. We can use the definition of torque to find the torque exerted by the couple. We can use our result from (*b*) to find the effective point of application of the normal force when $F = Mg/3$ and the condition for rotational equilibrium to find the greatest magnitude of $\vec{F}$ for which the cube will not tip.

(*a*) Apply $\sum \vec{F}_x = 0$ to the stationary cube:

$$\vec{F} + \vec{f}_s = 0 \Rightarrow \vec{F} = -\vec{f}_s$$

Because $\vec{F} = -\vec{f}_s$, this pair of equal, parallel, and oppositely directed forces constitute a couple.

The torque of the couple is:

$$\tau_{\text{couple}} = \boxed{Fa}$$

(*b*) Let x equal the distance from the point of application of F_n to the center of the cube. Now, $F_n = Mg$, so applying $\sum \vec{\tau} = 0$ to the cube yields:

$$Mgx - Fa = 0 \Rightarrow x = \frac{Fa}{Mg} \qquad (1)$$

Substituting for F and simplifying yields:

$$x = \frac{\frac{Mg}{3}a}{Mg} = \boxed{\frac{a}{3}}$$

(*c*) Solve equation (1) for F:

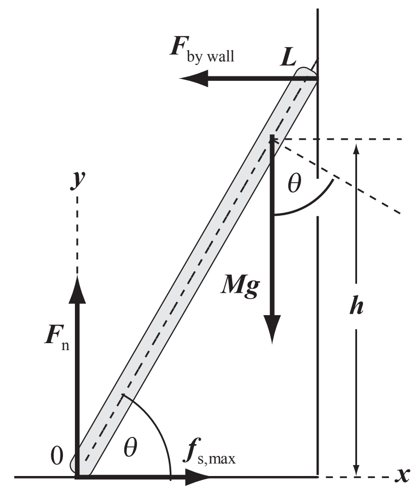
$$F = \frac{Mgx}{a}$$

Noting that $x_{\text{max}} = a/2$, express the condition that the cube will tip:

$$F > \frac{Mgx_{\text{max}}}{a} = \frac{Mg\frac{a}{2}}{a} = \boxed{\frac{Mg}{2}}$$

33 •• **[SSM]** A ladder of negligible mass and of length L leans against a slick wall making an angle of θ with the horizontal floor. The coefficient of friction between the ladder and the floor is μ_s. A man climbs the ladder. What height h can he reach before the ladder slips?

Picture the Problem Let the mass of the man be M. The ladder and the forces acting on it are shown in the diagram. Because the wall is slick, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium to it.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - Mg = 0 \Rightarrow F_n = Mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$\boldsymbol{f}_{s,max} - \boldsymbol{F}_W = 0 \Rightarrow f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder to obtain:

$$Mg\ell\cos\theta - F_W L\sin\theta = 0$$

Solving for ℓ and simplifying yields:

$$\ell = \frac{F_W L\sin\theta}{Mg\cos\theta} = \frac{f_{s,max}L}{Mg}\tan\theta = \frac{\mu_s F_n L}{Mg}\tan\theta = \mu_s L\tan\theta$$

Referring to the figure, relate ℓ to h:

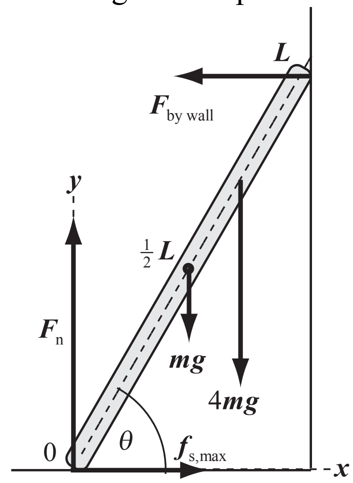
$$h = \ell\sin\theta$$

Substituting for ℓ yields:

$$h = \boxed{\mu_s L\tan\theta\sin\theta}$$

34 •• A uniform ladder of length L and mass m leans against a frictionless vertical wall, making an angle of 60º with the horizontal. The coefficient of static friction between the ladder and the ground is 0.45. If your mass is four times that of the ladder, how high can you climb before the ladder begins to slip?

Picture the Problem The ladder and the forces acting on it are shown in the drawing. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the wall is smooth, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - mg - 4mg = 0 \Rightarrow F_n = 5mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0 \Rightarrow f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta + 4mg\ell\cos\theta - F_W L\sin\theta = 0$$

Substitute for F_W and $f_{s,max}$ and solve for ℓ:

$$\ell = \frac{5\mu_s mgL\sin\theta - \frac{1}{2}mgL\cos\theta}{4mg\cos\theta}$$

Simplify to obtain:

$$\ell = \left(\frac{5\mu_s}{4}\tan\theta - \frac{1}{8}\right)L$$

Substitute numerical values to obtain:

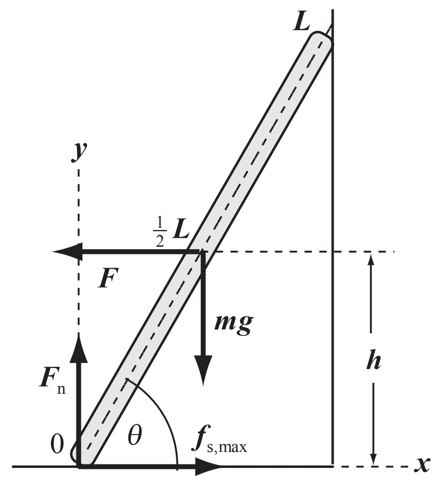
$$\ell = \left(\frac{5(0.45)}{4}\tan 60° - \frac{1}{8}\right)L = \boxed{0.85L}$$

That is, you can climb about 85% of the way to the top of the ladder before it begins to slip.

35 •• A ladder of mass m and length L leans against a frictionless vertical wall, so that it makes an angle θ with the horizontal. The center of mass of the ladder is a height h above the floor. A force F directed directly away from the wall pulls on the ladder at its midpoint. Find the minimum coefficient of static friction μ_s

for which the top end of the ladder will separate from the wall before the lower end begins to slip.

Picture the Problem The ladder and the forces acting on it are shown in the figure. Because the ladder is separating from the wall, the force the wall exerts on the ladder is zero. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



To find the force required to pull the ladder away from the wall, apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$F\left(\tfrac{1}{2}L\sin\theta\right) - mg\left(\tfrac{1}{2}L\cos\theta\right) = 0$$

or, because $\tfrac{1}{2}L\cos\theta = \dfrac{h}{\tan\theta}$,

$$\tfrac{1}{2}LF\sin\theta - \frac{mgh}{\tan\theta} = 0$$

Solving for F yields:

$$F = \frac{2mgh}{L\tan\theta\sin\theta} \quad (1)$$

Apply $\sum F_x = 0$ to the ladder:

$$f_{\text{s,max}} - F = 0 \Rightarrow F = f_{\text{s,max}} = \mu_{\text{s}} F_{\text{n}} \quad (2)$$

Apply $\sum F_y = 0$ to the ladder:

$$F_{\text{n}} - mg = 0 \Rightarrow F_{\text{n}} = mg$$

Equate equations (1) and (2) and substitute for F_{n} to obtain:

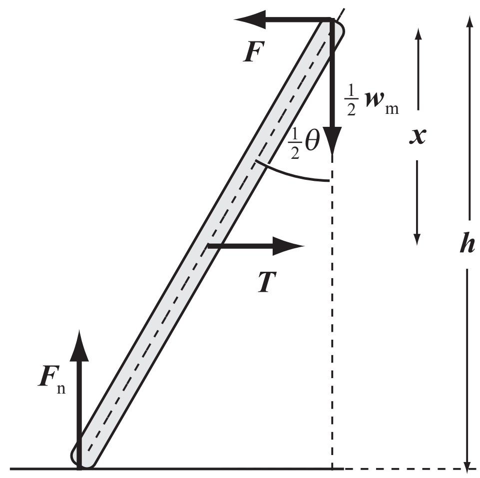
$$\mu_{\text{s}} mg = \frac{2mgh}{L\tan\theta\sin\theta}$$

Solving for μ_{s} yields:

$$\mu_{\text{s}} = \boxed{\frac{2h}{L\tan\theta\sin\theta}}$$

36 •• A 900-N man sits on top of a stepladder of negligible mass that rests on a frictionless floor as in Figure 12-43. There is a cross brace halfway up the ladder. The angle at the apex is $\theta = 30º$. (*a*) What is the force exerted by the floor on each leg of the ladder? (*b*) Find the tension in the cross brace. (*c*) If the cross brace is moved down toward the bottom of the ladder (maintaining the same angle θ), will its tension be the same, greater, or less than when it was in its higher position? Explain your answer.

Picture the Problem Assume that half the man's weight acts on each side of the ladder. The force exerted by the frictionless floor must be vertical. D is the separation between the legs at the bottom and x is the distance of the cross brace from the apex. Because each leg of the ladder is in equilibrium, we can apply the condition for rotational equilibrium to the right leg to relate the tension in the cross brace to its distance from the apex.



(a) By symmetry, each leg carries half the total weight, and the force on each leg is $\boxed{450\,\text{N.}}$

(b) Consider one of the ladder's legs and apply $\sum \vec{\tau} = 0$ about the apex:

$$F_n \frac{D}{2} - Tx = 0 \Rightarrow T = \frac{F_n D}{2x}$$

Using trigonometry, relate h and θ through the tangent function:

$$\tan\tfrac{1}{2}\boldsymbol{\theta} = \frac{\tfrac{1}{2}\boldsymbol{D}}{\boldsymbol{h}} \Rightarrow D = 2h\tan\tfrac{1}{2}\theta$$

Substitute for D in the expression for T and simplify to obtain:

$$T = \frac{2F_n h\tan\frac{1}{2}\theta}{2x} = \frac{F_n h\tan\frac{1}{2}\theta}{x}$$

Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - \tfrac{1}{2}w = 0 \Rightarrow F_n = \tfrac{1}{2}w$$

Substitute for F_n to obtain:

$$T = \frac{wh\tan\frac{1}{2}\theta}{2x} \qquad (1)$$

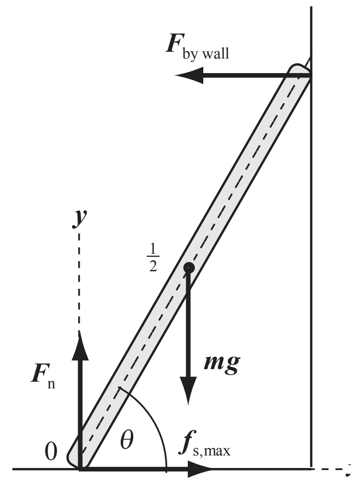
Substitute numerical values and evaluate T:

$$\boldsymbol{T} = \frac{(900\,\text{N})(4.0\,\text{m})\tan 15°}{2(2.0\,\text{m})} = \boxed{0.24\,\text{kN}}$$

(c) From equation (1) we can see that T is inversely proportional to x. Hence, if the brace is moved lower, T will decrease.

37 •• A uniform ladder rests against a frictionless vertical wall. The coefficient of static friction between the ladder and the floor is 0.30. What is the smallest angle between the ladder and the horizontal such that the ladder will not slip?

Picture the Problem The figure shows the forces acting on the ladder. Because the wall is frictionless, the force the wall exerts on the ladder is perpendicular to the wall. Because the ladder is on the verge of slipping, the static friction force is $f_{s,max}$. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the ladder:

$$\boldsymbol{f}_{s,max} - \boldsymbol{F}_W = 0 \Rightarrow \boldsymbol{F}_W = \boldsymbol{f}_{s,max} = \boldsymbol{\mu}_s \boldsymbol{F}_n$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$\boldsymbol{mg}\left(\tfrac{1}{2}\ell\cos\boldsymbol{\theta}\right) - \boldsymbol{F}_W\left(\ell\sin\boldsymbol{\theta}\right) = 0$$

Substitute for F_W and F_n and simplify to obtain:

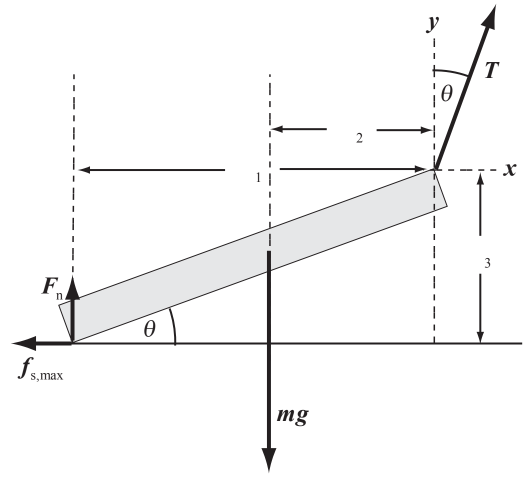
$$\tfrac{1}{2}\cos\theta - \mu_s\sin\theta = 0 \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\mu_s}\right)$$

Substitute the numerical value of μ_s and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{1}{2(0.30)}\right] = \boxed{59^\circ}$$

38 ••• A uniform log with a mass of 100 kg, a length of 4.0 m, and a radius of 12 cm is held in an inclined position, as shown in Figure 12-44. The coefficient of static friction between the log and the horizontal surface is 0.60. The log is on the verge of slipping to the right. Find the tension in the support wire and the angle the wire makes with the vertical wall.

Picture the Problem Let T = the tension in the wire; F_n = the normal force of the surface; and $f_{s,max} = \mu_s F_n$ the maximum force of static friction. Letting the point at which the wire is attached to the log be the origin, the center of mass of the log is at (−1.838 m, −0.797 m) and the point of contact with the floor is at (−3.676 m, −1.594 m). Because the log is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the log:

$$T\sin\theta - f_{s,max} = 0$$

or

$$T\sin\theta = f_{s,max} = \mu_s F_n \qquad (1)$$

Apply $\sum F_y = 0$ to the log:

$$T\cos\theta + F_n - mg = 0$$

or

$$T\cos\theta = mg - F_n \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_s F_n}{mg - F_n}$$

Solving for θ yields:

$$\theta = \tan^{-1}\left(\frac{\mu_s}{\frac{mg}{F_n} - 1}\right) \qquad (3)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the origin:

$$\ell_2 mg - \ell_1 F_n - \ell_3 \mu_s F_n = 0$$

Solve for F_n to obtain:

$$F_n = \frac{\ell_2 mg}{\ell_1 + \ell_3 \mu_s}$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_n = \frac{1.838(100\,\text{kg})(9.81\,\text{m/s}^2)}{3.676 + 1.594(0.60)} = 389\,\text{N}$$

Substitute numerical values in equation (3) and evaluate θ:

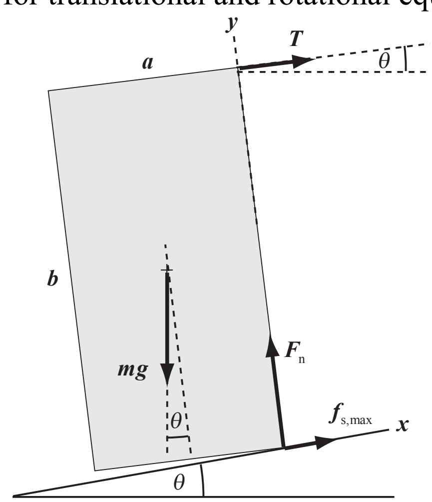
$$\theta = \tan^{-1}\left[\frac{0.60}{\frac{(100\,\text{kg})(9.81\,\text{m/s}^2)}{389\,\text{N}} - 1}\right] = 21.5° = \boxed{22°}$$

Substitute numerical values in equation (1) and evaluate T:

$$\boldsymbol{T} = \frac{(0.60)(389\,\text{N})}{\sin 21.5°} = \boxed{0.64\,\text{kN}}$$

39 ••• **[SSM]** A tall, uniform, rectangular block sits on an inclined plane as shown in Figure 12-45. A cord is attached to the top of the block to prevent it from falling down the incline. What is the maximum angle θ for which the block will not slide on the incline? Assume the block has a height-to-width ratio, b/a, of 4.0 and the coefficient of static friction between it and the incline is $\mu_s = 0.80$.

Picture the Problem Consider what happens just as θ increases beyond θ_{max}. Because the top of the block is fixed by the cord, the block will in fact rotate with only the lower right edge of the block remaining in contact with the plane. It follows that just prior to this slipping, F_n and $f_s = \mu_s F_n$ act at the lower right edge of the block. Choose a coordinate system in which up the incline is the $+x$ direction and the direction of $\vec{F}_n$ is the $+y$ direction. Because the block is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the block: $T + \mu_s F_n - mg\sin\theta = 0$ (1)

Apply $\sum F_y = 0$ to the block: $F_n - mg\cos\theta = 0$ (2)

Apply $\sum \vec{\tau} = 0$ about an axis through the lower right edge of the block: $\frac{1}{2}a(mg\cos\theta) + \frac{1}{2}b(mg\sin\theta) - bT = 0$ (3)

Eliminate F_n between equations (1) and (2) and solve for T:

$$T = mg(\sin\theta - \mu_s \cos\theta)$$

Substitute for T in equation (3):

$$\tfrac{1}{2}a(mg\cos\theta) + \tfrac{1}{2}b(mg\sin\theta) - b[mg(\sin\theta - \mu_s\cos\theta)] = 0$$

Substitute $4a$ for b:

$$\tfrac{1}{2}a(mg\cos\theta) + \tfrac{1}{2}(4.0a)(mg\sin\theta) - (4.0a)[mg(\sin\theta - \mu_s\cos\theta)] = 0$$

Simplify to obtain:

$$(1 + 8.0\boldsymbol{\mu}_s)\cos\boldsymbol{\theta} - 4.0\sin\boldsymbol{\theta} = 0$$

Solving for θ yields:

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{1 + 8.0\boldsymbol{\mu}_s}{4.0}\right]$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{1 + (8.0)(0.80)}{4.0}\right] = \boxed{62°}$$

Stress and Strain

40 • A 50-kg ball is suspended from a steel wire of length 5.0 m and radius 2.0 mm. By how much does the wire stretch?

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Find the amount the wire is stretched from Young's modulus:

$$Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta \boldsymbol{L} = \frac{(50\,\text{kg})(9.81\,\text{m/s}^2)(5.0\,\text{m})}{(200\,\text{GN/m}^2)(\boldsymbol{\pi})(2.0\times10^{-3}\,\text{m})^2} = \boxed{0.98\,\text{mm}}$$

41 • **[SSM]** Copper has a tensile strength of about 3.0×10^8 N/m^2. (*a*) What is the maximum load that can be hung from a copper wire of diameter 0.42 mm?

(*b*) If half this maximum load is hung from the copper wire, by what percentage of its length will it stretch?

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = \text{stress/strain} = (F/A)/(\Delta L/L)$.

(*a*) Express the maximum load in terms of the wire's tensile strength:

$$F_{\text{max}} = \text{tensile strength} \times A = \text{tensile strength} \times \pi r^2$$

Substitute numerical values and evaluate F_{max}:

$$F_{\text{max}} = (3.0\times10^{8}\ \text{N/m}^2)\pi(0.21\times10^{-3}\ \text{m})^2 = 41.6\ \text{N} = \boxed{42\ \text{N}}$$

(*b*) Using the definition of Young's modulus, express the fractional change in length of the copper wire:

$$\frac{\Delta L}{L} = \frac{F}{AY} = \frac{\frac{1}{2}F_{\text{max}}}{AY}$$

Substitute numerical values and evaluate $\frac{\Delta L}{L}$:

$$\frac{\Delta L}{L} = \frac{\frac{1}{2}(41.6\ \text{N})}{\pi(0.21\ \text{mm})^2(1.10\times10^{11}\ \text{N/m}^2)} = \boxed{0.14\%}$$

42 • A 4.0-kg mass is supported by a steel wire of diameter 0.60 mm and length 1.2 m. How much will the wire stretch under this load?

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Relate the amount the wire is stretched to Young's modulus:

$$Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(4.0\ \text{kg})(9.81\ \text{m/s}^2)(1.2\ \text{m})}{2\pi\times10^{11}\ \text{N/m}^2\,(0.30\times10^{-3}\ \text{m})^2} = \boxed{0.83\ \text{mm}}$$

43 • **[SSM]** As a runner's foot pushes off on the ground, the shearing force acting on an 8.0-mm-thick sole is shown in Figure 12-46. If the force of 25 N is distributed over an area of 15 cm^2, find the angle of shear θ, given that the shear modulus of the sole is 1.9×10^5 N/m^2.

Picture the Problem The shear stress, defined as the ratio of the shearing force to the area over which it is applied, is related to the shear strain through the definition of the shear modulus; $M_s = \dfrac{\text{shear stress}}{\text{shear strain}} = \dfrac{F_s/A}{\tan\theta}$.

Using the definition of shear modulus, relate the angle of shear, θ to the shear force and shear modulus:

$$\tan\theta = \frac{F_s}{M_s A} \Rightarrow \theta = \tan^{-1}\left(\frac{F_s}{M_s A}\right)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{25\,\text{N}}{\left(1.9\times10^5\ \text{N/m}^2\right)\left(15\times10^{-4}\ \text{m}^2\right)}\right] = \boxed{5.0°}$$

44 •• A steel wire of length 1.50 m and diameter 1.00 mm is joined to an aluminum wire of identical dimensions to make a composite wire of length 3.00 m. Find the resulting change in the length of this composite wire if an object with a mass of 5.00 kg is hung vertically from one of its ends. (Neglect any effects the masses of the two wires have on the changes in their lengths.)

Picture the Problem The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where L is the unstretched length of the wire, F is the force acting on it, and A is the cross-sectional area of the wire. The change in length of the composite wire is the sum of the changes in length of the steel and aluminum wires.

The change in length of the composite wire ΔL is the sum of the changes in length of the two wires:

$$\Delta \boldsymbol{L} = \Delta \boldsymbol{L}_{\text{steel}} + \Delta \boldsymbol{L}_{\text{Al}}$$

Using the defining equation for Young's modulus, substitute for ΔL_{steel} and ΔL_{Al} in equation (1) and simplify to obtain:

$$\Delta L = \frac{F}{A}\frac{L_{\text{steel}}}{Y_{\text{steel}}} + \frac{F}{A}\frac{L_{\text{Al}}}{Y_{\text{Al}}} = \frac{F}{A}\left(\frac{L_{\text{steel}}}{Y_{\text{steel}}} + \frac{L_{\text{Al}}}{Y_{\text{Al}}}\right)$$

Substitute numerical values and evaluate ΔL:

$$\Delta \boldsymbol{L} = \frac{(5.00\,\text{kg})(9.81\,\text{m/s}^2)}{\boldsymbol{\pi}(0.500\times10^{-3}\,\text{m})^2}\left(\frac{1.50\,\text{m}}{2.00\times10^{11}\,\text{N/m}^2} + \frac{1.50\,\text{m}}{0.700\times10^{11}\,\text{N/m}^2}\right) = 1.81\times10^{-3}\,\text{m}$$
$$= \boxed{1.81\,\text{mm}}$$

45 •• **[SSM]** Equal but opposite forces of magnitude F are applied to both ends of a thin wire of length L and cross-sectional area A. Show that if the wire is modeled as a spring, the force constant k is given by $k = AY/L$ and the potential energy stored in the wire is $U = \frac{1}{2}F\Delta L$, where Y is Young's modulus and ΔL is the amount the wire has stretched.

Picture the Problem We can use Hooke's law and Young's modulus to show that, if the wire is considered to be a spring, the force constant k is given by $k = AY/L$. By treating the wire as a spring we can show the energy stored in the wire is $U = \frac{1}{2}F\Delta L$.

Express the relationship between the stretching force, the force constant, and the elongation of a spring:

$$F = k\Delta L \Rightarrow k = \frac{F}{\Delta L}$$

Using the definition of Young's modulus, express the ratio of the stretching force to the elongation of the wire:

$$\frac{F}{\Delta L} = \frac{AY}{L} \qquad (1)$$

Equate these two expressions for $F/\Delta L$ to obtain:

$$k = \boxed{\frac{AY}{L}}$$

Treating the wire as a spring, express its stored energy:

$$U = \tfrac{1}{2}k(\Delta L)^2 = \tfrac{1}{2}\frac{AY}{L}(\Delta L)^2 = \tfrac{1}{2}\left(\frac{AY\Delta L}{L}\right)\Delta L$$

Solving equation (1) for F yields:

$$F = \frac{AY\Delta L}{L}$$

Substitute for F in the expression for U to obtain:

$$U = \boxed{\tfrac{1}{2}F\Delta L}$$

46 •• The steel E string of a violin is under a tension of 53.0 N. The diameter of the string is 0.200 mm and the length under tension is 35.0 cm. Find (*a*) the unstretched length of this string and (*b*) the work needed to stretch the string.

Picture the Problem Let L' represent the stretched and L the unstretched length of the wire. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where F is the force acting on it, and A is its cross-sectional area. In Problem 45 we showed that the energy stored in the wire is $U = \frac{1}{2}F\Delta L$, where Y is Young's modulus and ΔL is the amount the wire has stretched.

(*a*) Express the stretched length L' of the wire:

$$L' = L + \Delta L$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L = \frac{LF}{AY}$$

Substitute and simplify:

$$L' = L + \frac{LF}{AY} = L\left(1 + \frac{F}{AY}\right)$$

Solving for L yields:

$$L = \frac{L'}{1 + \dfrac{F}{AY}}$$

Substitute numerical values and evaluate L:

$$\boldsymbol{L} = \frac{0.350\,\text{m}}{1 + \dfrac{53.0\,\text{N}}{\boldsymbol{\pi}(0.100\times10^{-3}\,\text{m})^2(2.00\times10^{11}\,\text{N/m}^2)}} = \boxed{34.7\,\text{cm}}$$

(*b*) From Problem 45, the work done in stretching the wire is:

$$W = \Delta U = \tfrac{1}{2}F\Delta L$$

Substitute numerical values and evaluate W:

$$W = \tfrac{1}{2}(53.0\,\text{N})(0.350\,\text{m} - 0.347\,\text{m}) \approx \boxed{0.08\,\text{J}}$$

47 •• During a materials science experiment on the Young's modulus of rubber, your teaching assistant supplies you and your team with a rubber strip that is rectangular in cross section. She tells you to first measure the cross section dimensions and their values are 3.0 mm × 1.5 mm. The lab write-up calls for the rubber strip to be suspended vertically and various (known) masses to attached to it. Your team obtains the following data for the length of the strip as a function of the load (mass) on the end of the strip:

Load, kg	0.0	0.10	0.20	0.30	0.40	0.50
Length, cm	5.0	5.6	6.2	6.9	7.8	8.8

(*a*) Use a **spreadsheet** or **graphing calculator** to find Young's modulus for the rubber strip over this range of loads. *Hint: It is probably best to plot F/A versus ΔL/L. Why?*
(*b*) Find the energy stored in the strip when the load is 0.15 kg. (See Problem 45.)
(*c*) Find the energy stored in the strip when the load is 0.30 kg. Is it twice as much as your answer to Part (*b*)? Explain.

Picture the Problem We can use the definition of Young's modulus and your team's data to plot a graph whose slope is Young's modulus for the rubber strip over the given range of loads. Because the rubber strip stretches linearly for loads less than or equal to 0.20 kg, we can use linear interpolation in Part (*b*) to find the length of the rubber strip for a load of 0.15 kg. We can then use the result of Problem 45 to find the energy stored in the when the load is 0.15 kg. In Part (*c*) we can use the result of Problem 45 and the given length of the strip when its load is 0.30 kg to find the energy stored in the rubber strip.

(*a*) The equation for Young's modulus can be written as:

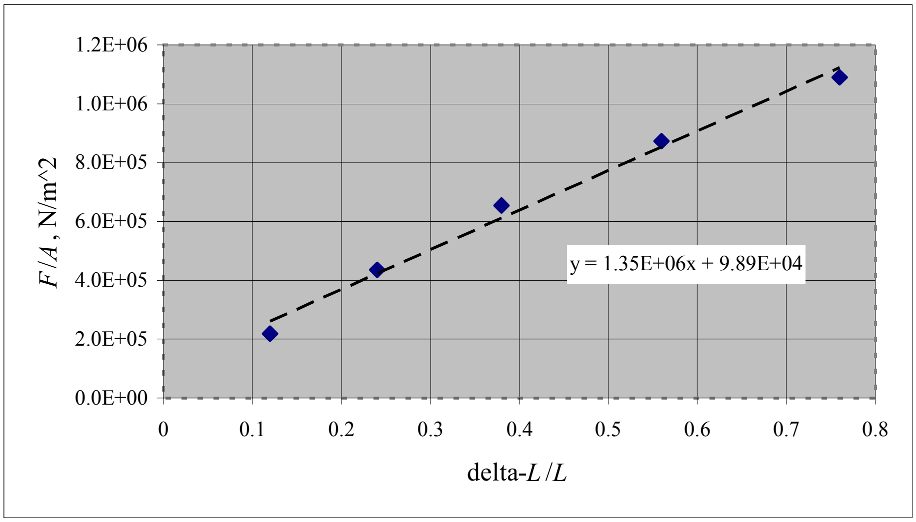
$$\frac{F}{A}=Y\frac{\Delta L}{L}$$

where Y is the slope of a graph of F/A as a function of $\Delta L/L$.

The following table summarizes the quantities, calculated using your team's data, used to plot the graph suggested in the problem statement.

Load	F	F/A	ΔL	$\Delta L/L$	U
(kg)	(N)	(N/m^2)	(m)		(J)
0.10	0.981	21.8×10^5	0.006	0.12	2.9×10^{-3}
0.20	1.962	4.36×10^5	0.012	0.24	12×10^{-3}
0.30	2.943	6.54×10^5	0.019	0.38	28×10^{-3}
0.40	3.924	8.72×10^5	0.028	0.56	55×10^{-3}
0.50	4.905	10.9×10^5	0.038	0.76	93×10^{-3}

A spreadsheet-generated graph of F/A as a function of $\Delta L/L$ follows. The spreadsheet program also plotted the regression line on the graph and added its equation to the graph.



From the regression function shown on the graph:

$$Y = \boxed{1.4\times10^{6}\ \text{N/m}^{2}}$$

(b) From Problem 45:

$$U = \tfrac{1}{2}\Delta LF$$

or, because $F = mg$,

$$U(m) = \tfrac{1}{2}\Delta Lmg$$

Interpolating from the data table we see that the length of the strip when the load on it is 0.15 kg is 5.9 cm. Substitute numerical values and evaluate U(0.15 kg):

$$U(0.30\ \text{kg}) = \tfrac{1}{2}(5.9\ \text{cm} - 5.0\ \text{cm})(0.15\ \text{kg})(9.81\ \text{m/s}^{2}) = \boxed{7\ \text{mJ}}$$

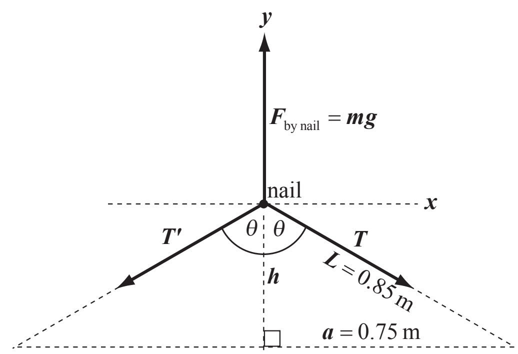
(c) Evaluate U(0.30 kg) to obtain:

$$U(0.30\ \text{kg}) = \tfrac{1}{2}(6.9\ \text{cm} - 5.0\ \text{cm})(0.30\ \text{kg})(9.81\ \text{m/s}^{2}) = \boxed{28\ \text{mJ}}$$

The energy stored in the strip when the load is 0.30 kg is four times as much as the energy stored when the load is 0.15 kg. Although the rubber strip does not stretch linearly (a conclusion you can confirm either graphically or by examining the data table), its stretch is sufficiently linear that, to a good approximation, the energy stored is quadrupled when the load is doubled.

48 •• A large mirror is hung from a nail as shown in Figure 12-47. The supporting steel wire has a diameter of 0.20 mm and an unstretched length of 1.7 m. The distance between the points of support at the top of the mirror's frame is 1.5 m. The mass of the mirror is 2.4 kg. How much will the distance between the nail and the mirror increase due to the stretching of the wire as the mirror is hung?

Picture the Problem The figure shows the forces acting on the wire where it passes over the nail. m represents the mass of the mirror and T is the tension in the supporting wires. The figure also shows the geometry of the right triangle defined by the support wires and the top of the mirror frame. The distance a is fixed by the geometry while h and L will change as the mirror is suspended from the nail.



Using the Pythagorean theorem, express the relationship between the sides of the right triangle in the diagram:

$$a^2 + h^2 = L^2$$

Express the differential of this equation and approximate differential changes with small changes:

$$2a\Delta a + 2h\Delta h = 2L\Delta L$$

or, because $\Delta a = 0$,

$$h\Delta h = L\Delta L \Rightarrow \Delta h = \frac{L\Delta L}{h}$$

Multiplying the numerator and denominator by L yields:

$$\Delta h = \frac{L^2}{h}\frac{\Delta L}{L} \qquad (1)$$

Solve the equation defining Young's modulus for $\Delta L/L$ to obtain:

$$\frac{\Delta L}{L} = \frac{T}{AY}$$

Substitute for $\Delta L/L$ in equation (1) to obtain:

$$\Delta h = \frac{L^2}{h}\frac{T}{AY} = \frac{L^2}{\sqrt{L^2 - a^2}}\frac{T}{\pi r^2 Y} \qquad (2)$$

where r is the radius of the wire.

Noting that $\boldsymbol{T} = \boldsymbol{T'}$, apply $\sum F_y = 0$ to the wire where it passes over the supporting nail:

$$mg - 2T\cos\theta = 0 \Rightarrow T = \frac{mg}{2\cos\theta}$$

Substituting for T in equation (2) yields:

$$\Delta h = \frac{L^2}{\sqrt{L^2 - a^2}}\frac{mg}{2\pi r^2 Y\cos\theta}$$

Because $\cos\theta = \frac{h}{L} = \frac{\sqrt{L^2 - a^2}}{L}$:

$$\Delta h = \frac{L^2}{\sqrt{L^2 - a^2}}\frac{mgL}{2\pi r^2 Y\sqrt{L^2 - a^2}}$$
$$= \frac{mgL^3}{2\pi r^2 Y\left(L^2 - a^2\right)}$$

Substitute numerical values and evaluate Δh:

$$\Delta \boldsymbol{h} = \frac{(2.4\,\text{kg})(9.81\,\text{m/s}^2)(0.85\,\text{m})^3}{2\boldsymbol{\pi}(0.10\times10^{-3}\,\text{m})^2(2.00\times10^{11}\,\text{N/m}^2)\left[(0.85\,\text{m})^2 - (0.75\,\text{m})^2\right]} = \boxed{7.2\,\text{mm}}$$

49 •• Two masses, M_1 and M_2, are supported by wires that have equal lengths when unstretched. The wire supporting M_1 is an aluminum wire 0.70 mm in diameter, and the one supporting M_2 is a steel wire 0.50 mm in diameter. What is the ratio M_1/M_2 if the two wires stretch by the same amount?

Picture the Problem Let the numeral 1 denote the aluminum wire and the numeral 2 the steel wire. Because their initial lengths and amount they stretch are the same, we can use the definition of Young's modulus to express the change in the lengths of each wire and then equate these expressions to obtain an equation solvable for the ratio M_1/M_2.

Using the definition of Young's modulus, express the change in length of the aluminum wire:

$$\Delta L_1 = \frac{M_1 g L_1}{A_1 Y_{\text{Al}}}$$

Using the definition of Young's modulus, express the change in length of the steel wire:

$$\Delta L_2 = \frac{M_2 g L_2}{A_2 Y_{\text{steel}}}$$

Because the two wires stretch by the same amount, equate ΔL_1 and ΔL_2 and simplify:

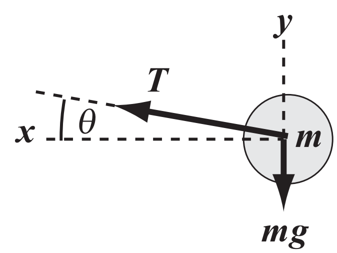
$$\frac{M_1}{A_1 Y_{\text{Al}}} = \frac{M_2}{A_2 Y_{\text{steel}}} \Rightarrow \frac{M_1}{M_2} = \frac{A_1 Y_{\text{Al}}}{A_2 Y_{\text{steel}}}$$

Substitute numerical values and evaluate M_1/M_2:

$$\frac{M_1}{M_2}=\frac{\frac{\pi}{4}(0.70\,\text{mm})^2(0.70\times10^{11}\ \text{N/m}^2)}{\frac{\pi}{4}(0.50\,\text{mm})^2(2.00\times10^{11}\ \text{N/m}^2)}=\boxed{0.69}$$

50 •• A 0.50-kg ball is attached to one end of an aluminum wire having a diameter of 1.6 mm and an unstretched length of 0.70 m. The other end of the wire is fixed to the top of a post. The ball rotates about the post in a horizontal plane at a rotational speed such that the angle between the wire and the horizontal is 5.0º. Find the tension in the wire and the increase in its length due to the tension in the wire.

Picture the Problem The free-body diagram shows the forces acting on the ball as it rotates around the post in a horizontal plane. We can apply Newton's 2nd law to find the tension in the wire and use the definition of Young's modulus to find the amount by which the aluminum wire stretches.



Apply $\sum F_y=0$ to the ball:

$$T\sin\theta-mg=0\Rightarrow T=\frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T:

$$\boldsymbol{T}=\frac{(0.50\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 5.0°}=56.3\ \text{N}=\boxed{56\,\text{N}}$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L=\frac{FL}{AY}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L=\frac{(56.3\,\text{N})(0.70\,\text{m})}{\frac{\pi}{4}(1.6\times10^{-3}\ \text{m})^2(0.70\times10^{11}\ \text{N/m}^2)}=\boxed{0.28\,\text{mm}}$$

51 •• **[SSM]** An elevator cable is to be made of a new type of composite developed by Acme Laboratories. In the lab, a sample of the cable that is 2.00 m long and has a cross-sectional area of 0.200 mm^2 fails under a load of 1000 N. The actual cable used to support the elevator will be 20.0 m long and have a cross-sectional area of 1.20 mm^2. It will need to support a load of 20,000 N safely. Will it?

Picture the Problem We can use the definition of stress to calculate the failing stress of the cable and the stress on the elevator cable. Note that the failing stress of the composite cable is the same as the failing stress of the test sample.

The stress on the elevator cable is:

$$\text{Stress}_{\text{cable}} = \frac{\boldsymbol{F}}{\boldsymbol{A}} = \frac{20.0\,\text{kN}}{1.20\times10^{-6}\ \text{m}^2} = 1.67\times10^{10}\ \text{N/m}^2$$

The failing stress of the sample is:

$$\text{Stress}_{\text{failing}} = \frac{\boldsymbol{F}}{\boldsymbol{A}} = \frac{1000\,\text{N}}{0.2\times10^{-6}\ \text{m}^2} = 0.500\times10^{10}\ \text{N/m}^2$$

Because $\text{Stress}_{\text{failing}} < \text{Stress}_{\text{cable}}$, the cable will not support the elevator.

52 •• If a material's density remains constant when it is stretched in one direction, then (because its total volume remains constant), its length must decrease in one or both of the other directions. Take a rectangular block of length *x*, width *y*, and depth z, and pull on it so that its new length $x' = x + \Delta x$. If $\Delta x << x$ and $\Delta \boldsymbol{y}/\boldsymbol{y} = \Delta \boldsymbol{z}/\boldsymbol{z}$, show that $\Delta y/y = -\frac{1}{2}\Delta x/x$.

Picture the Problem Let the length of the sides of the rectangle be *x*, *y* and *z*. Then the volume of the rectangle will be $V = xyz$ and we can express the new volume V' resulting from the pulling in the *x* direction and the change in volume ΔV in terms of Δx, Δy, and Δz. Discarding the higher order terms in ΔV and dividing our equation by V and using the given condition that $\Delta y/y = \Delta z/z$ will lead us to the given expression for $\Delta y/y$.

Express the new volume of the rectangular box when its sides change in length by Δx, Δy, and Δz:

$$V' = (x+\Delta x)(y+\Delta y)(z+\Delta z) = xyz + \Delta x(yz) + \Delta y(xz) + \Delta z(xy) + \{z\Delta x\Delta y + y\Delta x\Delta z + x\Delta y\Delta z + \Delta x\Delta y\Delta z\}$$

where the terms in brackets are very small (i.e., second order or higher).

Discard the second order and higher terms to obtain:

$$V' = V + \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

or

$$\Delta V = V' - V = \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

Because $\Delta V = 0$:

$$\Delta x(yz) = -[\Delta y(xz) + \Delta z(xy)]$$

Divide both sides of this equation by $V = xyz$ to obtain:

$$\frac{\Delta x}{x} = -\left[\frac{\Delta y}{y} + \frac{\Delta z}{z}\right]$$

Because $\Delta y/y = \Delta z/z$, our equation becomes:

$$\frac{\Delta x}{x} = -2\frac{\Delta y}{y} \Rightarrow \frac{\Delta y}{y} = \boxed{-\frac{1}{2}\frac{\Delta x}{x}}$$

53 •• **[SSM]** You are given a wire with a circular cross-section of radius r and a length L. If the wire is made from a material whose density remains constant when it is stretched in one direction, then show that $\Delta r/r = -\frac{1}{2}\Delta L/L$, assuming that $\Delta L << L$. (See Problem 52.)

Picture the Problem We can evaluate the differential of the volume of the wire and, using the assumptions that the volume of the wire does not change under stretching and that the change in its length is small compared to its length, show that $\Delta r/r = -\frac{1}{2}\Delta L/L\,L$.

Express the volume of the wire:

$$V = \pi r^2 L$$

Evaluate the differential of V to obtain:

$$dV = \pi r^2 dL + 2\pi rLdr$$

Because $dV = 0$:

$$0 = rdL + 2Ldr \Rightarrow \frac{dr}{r} = -\frac{1}{2}\frac{dL}{L}$$

Because $\Delta L << L$, we can approximate the differential changes dr and dL with small changes Δr and ΔL to obtain:

$$\frac{\Delta r}{r} = \boxed{-\frac{1}{2}\frac{\Delta L}{L}}$$

54 ••• For most materials listed in Table 12-1, the tensile strength is two to three orders of magnitude lower than Young's modulus. Consequently, most of these materials will break before their strain exceeds 1 percent. Of man-made materials, nylon has about the greatest extensibility—it can take strains of about 0.2 before breaking. But spider silk beats anything man-made. Certain forms of spider silk can take strains on the order of 10 before breaking! (*a*) If such a thread has a circular cross-section of radius r_0 and unstretched length L_0, find its new radius r when stretched to a length $L = 10L_0$. (Assume that the density of the thread remains constant as it stretches.) (*b*) If the Young's modulus of the spider thread is Y, calculate the tension needed to break the thread in terms of Y and r_0.

Picture the Problem Because the density of the thread remains constant during the stretching process, we can equate the initial and final volumes to express r_0 in terms of r. We can also use Young's modulus to express the tension needed to break the thread in terms of Y and r_0.

(*a*) Because the volume of the thread is constant during the stretching of the spider's silk:

$$\pi r^2 L = \pi r_0^2 L_0 \Rightarrow r = r_0\sqrt{\frac{L_0}{L}}$$

Substitute for *L* and simplify to obtain:

$$r = r_0\sqrt{\frac{L_0}{10L_0}} = \boxed{0.316r_0}$$

(*b*) Express Young's modulus in terms of the breaking tension *T*:

$$Y = \frac{T/A}{\Delta L/L} = \frac{T/\pi r^2}{\Delta L/L} = \frac{10T/\pi r_0^2}{\Delta L/L}$$

Solving for *T* yields:

$$T = \frac{1}{10}\pi r_0^2 Y \frac{\Delta L}{L}$$

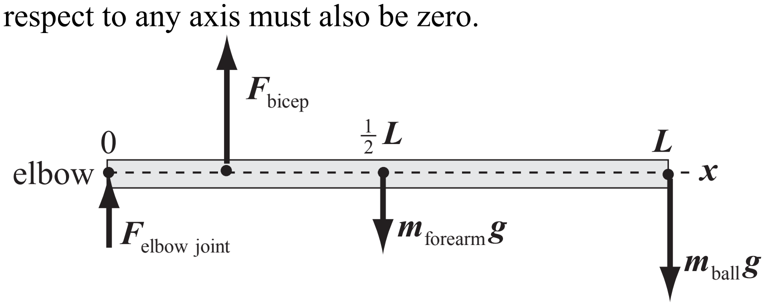
Because $\Delta L/L = 9$:

$$T = \boxed{\frac{9\pi r_0^2 Y}{10}}$$

General Problems

55 • **[SSM]** A standard bowling ball weighs 16 pounds. You wish to hold a bowling ball in front of you, with the elbow bent at a right angle. Assume that your biceps attaches to your forearm at 2.5 cm out from the elbow joint, and that your biceps muscle pulls vertically upward, that is, it acts at right angles to the forearm. Also assume that the ball is held 38 cm out from the elbow joint. Let the mass of your forearm be 5.0 kg and assume its center of gravity is located 19 cm out from the elbow joint. How much force must your biceps muscle apply to forearm in order to hold out the bowling ball at the desired angle?

Picture the Problem We can model the forearm as a cylinder of length L = 38 cm with the forces shown in the pictorial representation acting on it. Because the forearm is in both translational and rotational equilibrium under the influence of these forces, the forces in the diagram must add (vectorially) to zero and the net torque with respect to any axis must also be zero.



Apply $\sum \vec{\tau} = 0$ about an axis through the elbow and perpendicular to the plane of the diagram:

$$\ell \boldsymbol{F}_{\text{bicep}} - \tfrac{1}{2}\boldsymbol{L}\boldsymbol{m}_{\text{forearm}}\boldsymbol{g} - \boldsymbol{L}\boldsymbol{m}_{\text{ball}}\boldsymbol{g} = 0$$

Solving for F_{bicep} and simplifying yields:

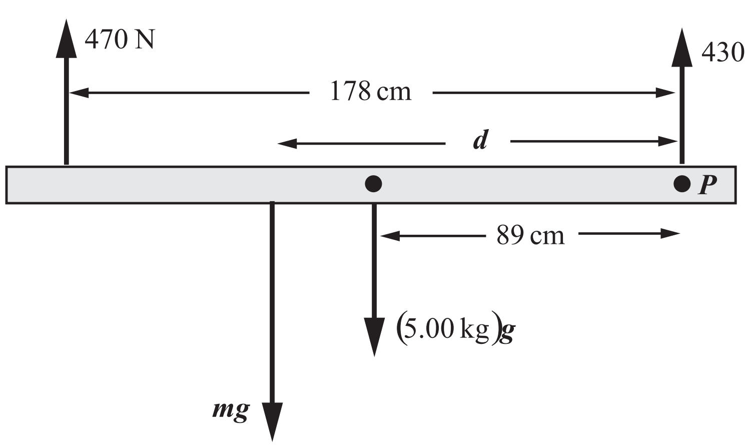
$$F_{\text{bicep}} = \frac{\tfrac{1}{2}Lm_{\text{forearm}}g + Lm_{\text{ball}}g}{\ell} = \frac{\left(\tfrac{1}{2}m_{\text{forearm}} + m_{\text{ball}}\right)Lg}{\ell}$$

Substitute numerical values and evaluate F_{bicep}:

$$\boldsymbol{F}_{\text{bicep}} = \frac{\left(\tfrac{1}{2}(5.0\text{ kg}) + 16\text{ lb} \times \dfrac{1\text{ kg}}{2.205\text{ lb}}\right)(38\text{ cm})\left(9.81\text{ m/s}^2\right)}{2.5\text{ cm}} = \boxed{1.5\text{ kN}}$$

56 •• A biology laboratory at your university is studying the location of a person's center of gravity as a function of their body weight. They pay well, and you decide to volunteer. The location of your center of gravity when standing erect is to be determined by having you lie on a uniform board (mass of 5.00 kg, length 2.00 m) supported by two scales as shown in Figure 12-54. If your height is 188 cm and the left scale reads 470 N while the right scale reads 430 N, where is your center of gravity relative to your feet? Assume the scales are both exactly the same distance from the two ends of the board, are separated by 178 cm, and are set to each read zero before you get on the platform.

Picture the Problem Because the you-board system is in equilibrium, we can apply the conditions for translational and rotational equilibrium to relate the forces exerted by the scales to the distance *d*, measured from your feet, to your center of mass and the distance to the center of gravity of the board. The following pictorial representation shows the forces acting on the board. *mg* represents your weight.



The forces responsible for a counterclockwise torque about an axis through your feet (point P) and perpendicular to the page are your weight and the weight of the board. The only force causing a clockwise torque about this axis is the 470 N force exerted by the scale under your head. Apply $\sum\vec{\tau}=0$ about an axis through your feet and perpendicular to the page:

$$\boldsymbol{m}\left(9.81\ \text{m/s}^2\right)\boldsymbol{d}-(5.0\ \text{kg})(0.89\ \text{m})-(1.78\ \text{m})(470\ \text{N})=0$$

where m is your mass.

Solve for d to obtain:

$$\boldsymbol{d}=\frac{(5.00\ \text{kg})(0.89\ \text{m})+(1.78\ \text{m})(470\ \text{N})}{\boldsymbol{m}\left(9.81\ \text{m/s}^2\right)} \quad (1)$$

Let upward be the positive y direction and apply $\sum \boldsymbol{F}_y=0$ to the plank to obtain:

$$470\ \text{N}+430\ \text{N}-\boldsymbol{m}\left(9.81\ \text{m/s}^2\right)-(5.00\ \text{kg})\left(9.81\ \text{m/s}^2\right)=0$$

Solving for m yields: $\boldsymbol{m}=86.74\ \text{kg}$

Substitute numerical values in equation (1) and evaluate d:

$$\boldsymbol{d}=\frac{(5.0\ \text{kg})(0.89\ \text{m})+(1.78\ \text{m})(470\ \text{N})}{(86.74\ \text{kg})\left(9.81\ \text{m/s}^2\right)}=\boxed{99\ \text{cm}}$$

57 •• Figure 12-49 shows a mobile consisting of four objects hanging on three rods of negligible mass. Find the values of the unknown masses of the objects if the mobile is to balance. *Hint: Find the mass m_1 first.*

Picture the Problem We can apply the balance condition $\sum\vec{\tau}=0$ successively, starting with the lowest part of the mobile, to find the value of each of the unknown weights.

Apply $\sum\vec{\tau}=0$ about an axis through the point of suspension of the lowest part of the mobile:

$$(3.0\ \text{cm})(2.0\ \text{N})-(4.0\ \text{cm})m_1 g=0$$

Solving for m_1 yields:

$$m_1=0.1529\ \text{kg}=\boxed{0.15\ \text{kg}}$$

Apply $\sum\vec{\tau} = 0$ about an axis through the point of suspension of the middle part of the mobile:

$$(2.0\,\text{cm})m_2 g - (4.0\,\text{cm})\left(\frac{2.0\,\text{N}}{g} + 0.1529\,\text{kg}\right)g = 0$$

Solving for m_2 yields: $m_2 = 0.7136\,\text{kg} = \boxed{0.71\,\text{kg}}$

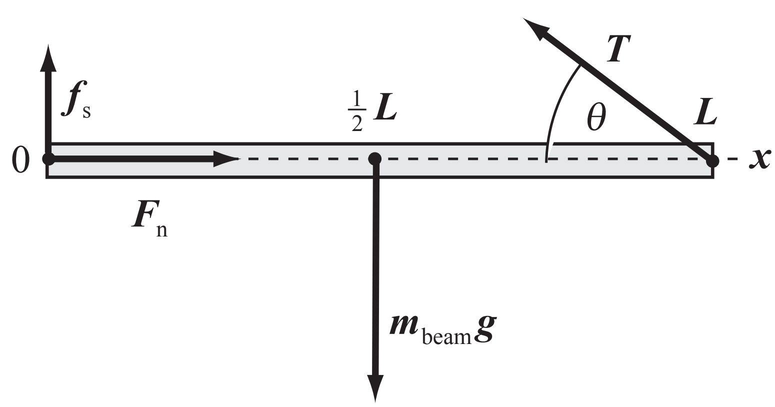
Apply $\sum\vec{\tau} = 0$ about an axis through the point of suspension of the top part of the mobile:

$$(2.0\,\text{cm})(2.0\,\text{N} + (0.7136\,\text{kg})g + (0.1529)g) - (6.0\,\text{cm})m_3 g = 0$$

Solving for m_3 yields: $m_3 = 0.3568\,\text{kg} = \boxed{0.36\,\text{kg}}$

58 •• Steel construction beams, with an industry designation of ″W12 × 22,″ have a weight of 22 pounds per foot. A new business in town has hired you to place its sign on a 4.0 m long steel beam of this type. The design calls for the beam to extend outward horizontally from the front brick wall (Figure 12-50). It is to be held in place by a 5.0 m-long steel cable. The cable is attached to one end of the beam and to the wall above the point at which the beam is in contact with the wall. During the initial stage of construction, the beam is *not* to be bolted to the wall, but to be held in place solely by friction. (*a*) What is the minimum coefficient of friction between the beam and the wall for the beam to remain in static equilibrium? (*b*)What is the tension in the cable in this case?

Picture the Problem Because the beam is in both translational and rotational equilibrium under the influence of the forces shown below in the pictorial representation, we can apply $\sum\vec{F} = 0$ and $\sum\vec{\tau} = 0$ to it to find the coefficient of static friction and the tension in the supporting cable.



(*a*) Apply $\sum \vec{F} = 0$ to the beam to obtain:

$$\sum F_x = F_n - T\cos\theta = 0 \quad (1)$$

and

$$\sum F_y = f_s - m_{beam}g + T\sin\theta = 0 \quad (2)$$

The mass of the beam m_{beam} is the product of its linear density λ and length L:

$$m_{beam} = \lambda L$$

Substituting for m_{beam} in equation (2) yields:

$$f_s - \lambda Lg + T\sin\theta = 0 \quad (3)$$

Relate the force of static friction to the normal force exerted by the wall:

$$f_s = \mu_s F_n$$

Substituting for f_s in equation (3) yields:

$$\mu_s F_n - \lambda Lg + T\sin\theta = 0$$

Solve for μ_s to obtain:

$$\mu_s = \frac{\lambda Lg - T\sin\theta}{F_n} \quad (4)$$

Solving equation (1) for F_n yields:

$$F_n = T\cos\theta$$

Substitute for F_n in equation (4) and simplify to obtain:

$$\mu_s = \frac{\lambda Lg - T\sin\theta}{T\cos\theta} = \frac{\lambda Lg}{T\cos\theta} - \tan\theta \quad (5)$$

Apply $\sum \vec{\tau} = 0$ to the beam about an axis through the origin and normal to the page to obtain:

$$(T\sin\theta)L - m_{beam}g\left(\tfrac{1}{2}L\right) = 0$$

or

$$(T\sin\theta)L - \lambda Lg\left(\tfrac{1}{2}L\right) = 0$$

Solving for T yields:

$$T = \frac{\lambda Lg}{2\sin\theta} \quad (6)$$

Substitute for T in equation (5) and simplify to obtain:

$$\mu_s = \frac{\lambda Lg}{\dfrac{\lambda Lg}{2\sin\theta}\cos\theta} - \tan\theta = \tan\theta \quad (7)$$

From Figure 12-50 we see that:

$$\tan\theta = \frac{3.0\text{ m}}{4.0\text{ m}} = \frac{3}{4}$$

Substituting for $\tan\theta$ in equation (7) yields: $\boldsymbol{\mu}_{\mathrm{s}} = \boxed{0.75}$

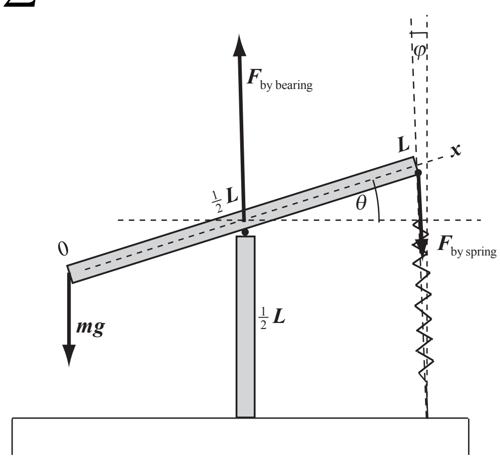
(*b*) Substitute numerical values in equation (6) and evaluate *T*:

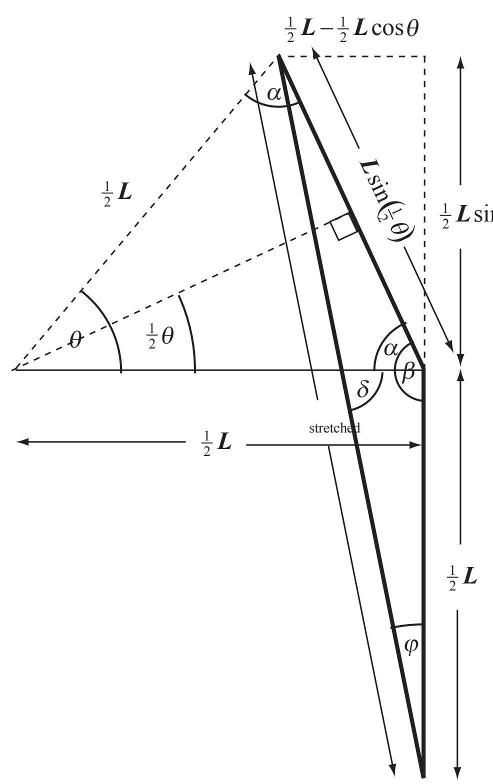
$$\boldsymbol{T} = \frac{\left(22\,\frac{\text{lb}}{\text{ft}} \times \frac{1\,\text{kg}}{2.205\,\text{lb}} \times \frac{3.281\,\text{ft}}{\text{m}}\right)(4.0\,\text{m})\left(9.81\,\text{m/s}^2\right)}{2\sin\left(\tan^{-1}\left(\frac{3}{4}\right)\right)} = \boxed{1.1\,\text{kN}}$$

Remarks: 1.1 kN is approximately 240 lb.

59 •• **[SSM]** Consider a rigid 2.5-m-long beam (Figure 12-51) that is supported by a fixed 1.25-m-high post through its center and pivots on a frictionless bearing at its center atop the vertical 1.25-m-high post. One end of the beam is connected to the floor by a spring that has a force constant $k = 1250$ N/m. When the beam is horizontal, the spring is vertical and unstressed. If an object is hung from the opposite end of the beam, the beam settles into an equilibrium position where it makes an angle of 17.5^{o} with the horizontal. What is the mass of the object?

Picture the Problem Because the beam is in rotational equilibrium, we can apply $\sum \vec{\tau} = 0$ to it to determine the mass of the object suspended from its left end.





The pictorial representation directly above shows the forces acting on the beam when it is in static equilibrium. The pictorial representation to the right is an enlarged view of the right end of the beam. We'll use this diagram to determine the length of the stretched spring.

Apply $\sum \vec{\tau} = 0$ to the beam about an axis through the bearing point to obtain:

$$\boldsymbol{mg}\left(\tfrac{1}{2}\boldsymbol{L}\cos\boldsymbol{\theta}\right) - \boldsymbol{F}_{\text{by spring}}\left(\tfrac{1}{2}\boldsymbol{L}\sin\boldsymbol{\varphi}\right) = 0$$

or, because $\boldsymbol{F}_{\text{by spring}} = \boldsymbol{k}\Delta\ell_{\text{spring}}$,

$$\boldsymbol{mg}\cos\boldsymbol{\theta} - \boldsymbol{k}\Delta\ell_{\text{spring}}\sin\boldsymbol{\varphi} = 0 \qquad (1)$$

Use the right-hand diagram above to relate the angles φ and θ:

$$\boldsymbol{\varphi} = \tan^{-1}\left(\frac{\frac{1}{2}\boldsymbol{L} - \frac{1}{2}\boldsymbol{L}\cos\boldsymbol{\theta}}{\frac{1}{2}\boldsymbol{L} + \frac{1}{2}\boldsymbol{L}\sin\boldsymbol{\theta}}\right) = \tan^{-1}\left(\frac{1-\cos\boldsymbol{\theta}}{1+\sin\boldsymbol{\theta}}\right)$$

Substitute for φ in equation (1) to obtain:

$$mg\cos\theta - k\Delta\ell_{\text{spring}}\sin\left(\tan^{-1}\left(\frac{1-\cos\theta}{1+\sin\theta}\right)\right) = 0$$

Solve for m to obtain:

$$m = \frac{k\Delta\ell_{\text{spring}}\sin\left(\tan^{-1}\left(\frac{1-\cos\theta}{1+\sin\theta}\right)\right)}{g\cos\theta} \qquad (2)$$

$\Delta\ell_{\text{spring}}$ is given by:

$$\Delta\ell_{\text{spring}} = \ell_{\text{stretched}} - \ell_{\text{unstretched}}$$

or, because $\ell_{\text{unstretched}} = \frac{1}{2}\boldsymbol{L}$,

$$\Delta\ell_{\text{spring}} = \ell_{\text{stretched}} - \tfrac{1}{2}\boldsymbol{L} \qquad (3)$$

To find the value of $\ell_{\text{stretched}}$, refer to the right-hand diagram and note that:

$$2\boldsymbol{\alpha} + \boldsymbol{\theta} = \boldsymbol{\pi} \Rightarrow \boldsymbol{\alpha} = \frac{\boldsymbol{\pi}}{2} - \frac{1}{2}\boldsymbol{\theta}$$

Again, referring to the diagram, relate β to α:

$$\boldsymbol{\beta} = \boldsymbol{\alpha} + \frac{\boldsymbol{\pi}}{2}$$

Substituting for α yields:

$$\boldsymbol{\beta} = \frac{\boldsymbol{\pi}}{2} - \frac{1}{2}\boldsymbol{\theta} + \frac{\boldsymbol{\pi}}{2} = \boldsymbol{\pi} - \frac{1}{2}\boldsymbol{\theta}$$

Apply the law of cosines to the triangle defined with bold sides:

$$\ell^2_{\text{stretched}} = \left(\tfrac{1}{2}L\right)^2 + \left(L\sin\tfrac{1}{2}\theta\right)^2 - 2\left(\tfrac{1}{2}L\right)\left(L\sin\tfrac{1}{2}\theta\right)\cos\left(\pi - \tfrac{1}{2}\theta\right)$$

Use the formula for the cosine of the difference of two angles to obtain:

$$\cos\left(\boldsymbol{\pi} - \tfrac{1}{2}\boldsymbol{\theta}\right) = -\cos\tfrac{1}{2}\boldsymbol{\theta}$$

Substituting for $\cos\left(\boldsymbol{\pi}-\frac{1}{2}\boldsymbol{\theta}\right)$ yields:

$$\begin{aligned}\ell^2_{\text{stretched}} &= \left(\tfrac{1}{2}L\right)^2+\left(L\sin\tfrac{1}{2}\theta\right)^2+2\left(\tfrac{1}{2}L\right)\left(L\sin\tfrac{1}{2}\theta\right)\cos\tfrac{1}{2}\theta\\ &= \tfrac{1}{4}L^2+L^2\sin^2\tfrac{1}{2}\theta+\tfrac{1}{2}L^2\left(2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta\right)\end{aligned}$$

Use the trigonometric identities $\sin\boldsymbol{\theta}=2\sin\frac{1}{2}\boldsymbol{\theta}\cos\frac{1}{2}\boldsymbol{\theta}$ and $\sin^2\frac{1}{2}\boldsymbol{\theta}=\dfrac{1-\cos\boldsymbol{\theta}}{2}$ to obtain:

$$\ell^2_{\text{stretched}}=\tfrac{1}{4}\boldsymbol{L}^2+\boldsymbol{L}^2\left(\frac{1-\cos\boldsymbol{\theta}}{2}\right)+\tfrac{1}{2}\boldsymbol{L}^2\left(\sin\boldsymbol{\theta}\right)$$

Simplifying yields:

$$\ell^2_{\text{stretched}}=\tfrac{1}{4}\boldsymbol{L}^2\left[3-2\left(\cos\boldsymbol{\theta}-\sin\boldsymbol{\theta}\right)\right]$$

or

$$\ell_{\text{stretched}}=\tfrac{1}{2}\boldsymbol{L}\sqrt{3-2\left(\cos\boldsymbol{\theta}-\sin\boldsymbol{\theta}\right)}$$

Substituting for $\ell_{\text{stretched}}$ in equation (3) yields:

$$\Delta\ell_{\text{spring}}=\tfrac{1}{2}L\sqrt{3-2(\cos\theta-\sin\theta)}-\tfrac{1}{2}L=\tfrac{1}{2}L\left(\sqrt{3-2(\cos\theta-\sin\theta)}-1\right)$$

Substitute for $\Delta\ell_{\text{spring}}$ in equation (2) to obtain:

$$m=\frac{\tfrac{1}{2}kL\left(\sqrt{3-2(\cos\theta-\sin\theta)}-1\right)\sin\left(\tan^{-1}\left(\dfrac{1-\cos\theta}{1+\sin\theta}\right)\right)}{g\cos\theta}$$

Substitute numerical values and evaluate m:

$$\begin{aligned}m &= \frac{\tfrac{1}{2}(1250\text{ N/m})(2.5\text{ m})\left(\sqrt{3-2(\cos17.5°-\sin17.5°)}-1\right)\sin\left(\tan^{-1}\left(\dfrac{1-\cos17.5°}{1+\sin17.5°}\right)\right)}{\left(9.81\text{ m/s}^2\right)\cos17.5°}\\ &= \boxed{1.8\text{ kg}}\end{aligned}$$

60 •• A rope and pulley system, called a *block and tackle*, is used to raise an object of mass M (Figure 12-52) at constant speed. When the end of the rope moves downward through a distance L, the height of the lower pulley is increased by h.

(*a*) What is the ratio L/h? (*b*) Assume that the mass of the block and tackle is negligible and that the pulley bearings are frictionless. Show that $FL = mgh$ by applying the work–energy principle to the block–tackle object.

Picture the Problem We can determine the ratio of L to h by noting the number of ropes supporting the load whose mass is M.

(*a*) Noting that three ropes support the pulley to which the object whose mass is M is fastened we can conclude that:

$$\frac{L}{h} = \boxed{3}$$

(*b*) Apply the work-energy principle to the block-tackle object to obtain:

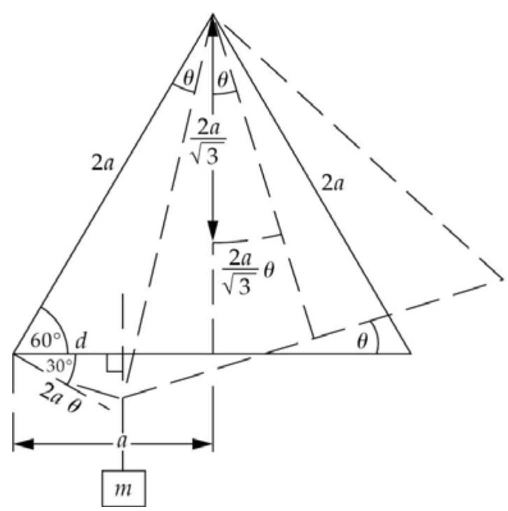
$$W_{\text{ext}} = \Delta E_{\text{system}} = \Delta U_{\text{block-tackle}}$$

or

$$FL = \boxed{mgh}$$

61 •• A plate of mass M in the shape of an equilateral triangle is suspended from one corner and a mass m is suspended from another of its corners. If the base of the triangle makes an angle of 6.0º with the horizontal, what is the ratio m/M?

Picture the Problem The figure shows the equilateral triangle without the mass m, and then the same triangle with the mass m and rotated through an angle θ. Let the side length of the triangle to be $2a$. Then the center of mass of the triangle is at a distance of $\frac{2a}{\sqrt{3}}$ from each vertex. As the triangle rotates, its center of mass shifts by $\frac{2a}{\sqrt{3}}\theta$, for $\theta \ll 1$. Also, the vertex to which m is attached moves toward the plumb line by the distance $d = 2a\theta \cos 30° = \sqrt{3}a\theta$ (see the drawing).



Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension:

$$mg\left(a - \sqrt{3}a\theta\right) - Mg\frac{2a}{\sqrt{3}}\theta = 0$$

Solving for m/M yields:

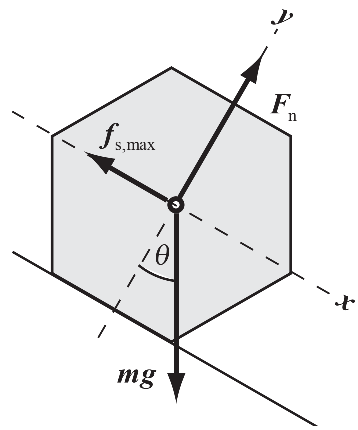
$$\frac{m}{M} = \frac{2\theta}{\sqrt{3}\left(1 - \sqrt{3}\theta\right)}$$

Substitute numerical values and evaluate m/M:

$$\frac{m}{M}=\frac{2(6.0°)\left(\frac{\pi\text{ rad}}{180°}\right)}{\sqrt{3}\left[1-\sqrt{3}(6.0°)\left(\frac{\pi\text{ rad}}{180°}\right)\right]}=\boxed{0.15}$$

62 •• A standard six-sided pencil is placed on a pad of paper (Figure 12-53) Find the minimum coefficient of static friction μ_s such that, if the pad is inclined, the pencil rolls down the incline rather than sliding.

Picture the Problem If the hexagon is to roll rather than slide, the incline's angle must be such that the center of mass falls just beyond the support base. From the geometry of the hexagon, the critical angle is 30°. The free-body diagram shows the forces acting on the hexagonal pencil when it is on the verge of sliding. We can use Newton's 2nd law to relate the coefficient of static friction to the angle of the incline for which rolling rather than sliding occurs.



Apply $\sum\vec{F}=0$ to the pencil:

$$\sum F_x = mg\sin\theta - f_{s,\max}=0 \qquad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

Substitute $f_{s,\max}=\mu_s F_n$ in equation (1):

$$mg\sin\theta-\mu_s F_n=0 \qquad (3)$$

Divide equation (3) by equation (2) to obtain:

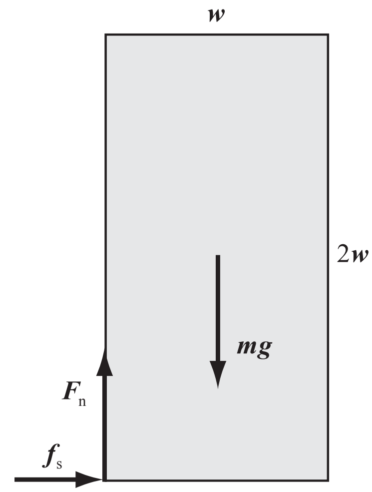
$$\tan\theta=\mu_s$$

Thus, if the pencil is to roll rather than slide when the pad is inclined:

$$\mu_s \geq \tan 30° = \boxed{0.58}$$

63 •• An 8.0-kg box that has a uniform density and is twice as tall as it is wide rests on the floor of a truck. What is the maximum coefficient of static friction between the box and floor so that the box will slide toward the rear of the truck rather than tip when the truck accelerates forward on a level road?

Picture the Problem The box and the forces acting on it are shown in the figure. The force accelerating the box is the static friction force. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s:

$$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass:

$$wf_s - \tfrac{1}{2} wF_n = 0 \Rightarrow \frac{f_s}{F_n} = \frac{1}{2}$$

Substitute for $\frac{f_s}{F_n}$ to obtain the condition for tipping:

$$\mu_s \geq 0.50$$

Therefore, if the box is to slide:

$$\mu_s < \boxed{0.50}$$

64 •• A balance scale has unequal arms. The scale is balanced with a 1.50-kg block on the left pan and a 1.95 kg block on the right pan (Figure 12-54). If the 1.95-kg block is removed from the right pan and the 1.50-kg block is then moved to the right pan, what mass on the left pan will balance the scale?

Picture the Problem Because the balance is in equilibrium, we can use the condition for rotational equilibrium to relate the masses of the blocks to the lever arms of the balance in the two configurations described in the problem statement.

Apply $\sum \vec{\tau} = 0$ about an axis through the fulcrum:

$$(1.50\,\text{kg})L_1 - (1.95\,\text{kg})L_2 = 0$$

Solving for L_1/L_2 yields:

$$\frac{L_1}{L_2}=1.30$$

Apply $\sum\vec{\tau}=0$ about an axis through the fulcrum with 1.50 kg at L_2:

$$ML_1-(1.50\text{ kg})L_2=0$$

Solving for M yields:

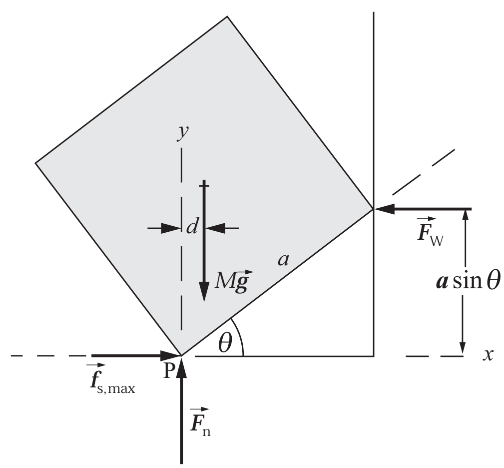
$$M=\frac{(1.50\text{ kg})L_2}{L_1}=\frac{1.50\text{ kg}}{L_1/L_2}$$

Substitute for L_1/L_2 and evaluate M:

$$M=\frac{1.50\text{ kg}}{1.30}=\boxed{1.15\text{ kg}}$$

65 •• **[SSM]** A cube leans against a frictionless wall making an angle of θ with the floor as shown in Figure 12-55. Find the minimum coefficient of static friction μ_s between the cube and the floor that is needed to keep the cube from slipping.

Picture the Problem Let the mass of the cube be M. The figure shows the location of the cube's center of mass and the forces acting on the cube. The opposing couple is formed by the friction force $f_{s,max}$ and the force exerted by the wall. Because the cube is in equilibrium, we can use the condition for translational equilibrium to establish that $f_{s,max}=F_W$ and $F_n=Mg$ and the condition for rotational equilibrium to relate the opposing couples.



Apply $\sum\vec{F}=0$ to the cube:

$$\sum F_y=F_n-Mg=0\Rightarrow F_n=Mg$$

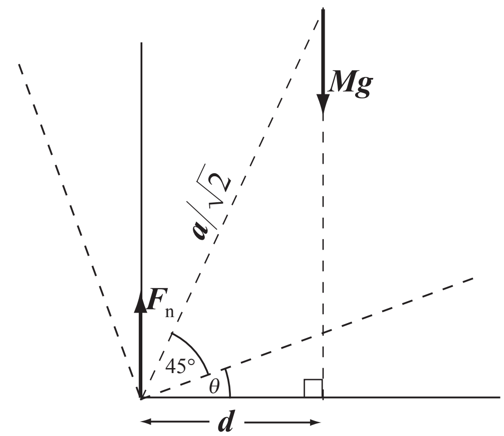
and

$$\sum F_x=f_s-F_W=0\Rightarrow F_W=f_s$$

Noting that $\vec{f}_{s,max}$ and $\vec{F}_W$ form a couple (their magnitudes are equal), as do $\vec{F}_n$ and $M\vec{g}$, apply $\sum\vec{\tau}=0$ about an axis though point P to obtain:

$$f_{s,max}a\sin\theta-Mgd=0$$

Referring to the diagram to the right, note that $d = \frac{a}{\sqrt{2}}\sin(45° + \theta)$.



Substitute for d and $f_{s,max}$ to obtain:

$$\mu_s Mga\sin\theta - Mg\frac{a}{\sqrt{2}}\sin(45° + \theta) = 0$$

or

$$\mu_s \sin\theta - \frac{1}{\sqrt{2}}\sin(45° + \theta) = 0$$

Solve for μ_s and simplify to obtain:

$$\mu_s = \frac{1}{\sqrt{2}\sin\theta}\sin(45° + \theta) = \frac{1}{\sqrt{2}\sin\theta}(\sin 45°\cos\theta + \cos 45°\sin\theta)$$
$$= \frac{1}{\sqrt{2}\sin\theta}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right) = \boxed{\frac{1}{2}(\cot\theta + 1)}$$

66 •• Figure 12-56 shows a 5.00-kg rod hinged to a vertical wall and supported by a thin wire. The wire and rod each make angles of 45º with the vertical. When a 10.0-kg block is suspended from the midpoint of the rod, the tension T in the supporting wire is 52.0 N. If the wire will break when the tension exceeds 75 N, what is the maximum distance from the hinge at which the block can be suspended?

Picture the Problem Because the rod is in equilibrium, we can apply the condition for rotational equilibrium to find the maximum distance from the hinge at which the block can be suspended.

Apply $\sum \vec{\tau} = 0$ about an axis through the hinge to obtain:

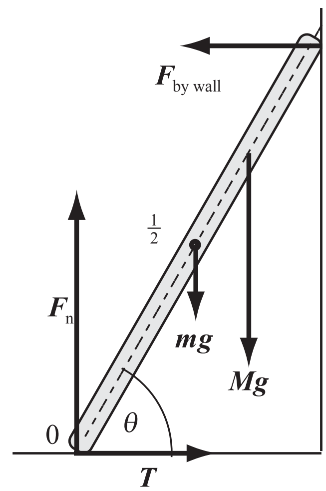
$$(1.00\,\text{m})(75\,\text{N}) - (0.50\,\text{m})(5.00\,\text{kg})(9.81\,\text{m/s}^2)\cos 45°$$
$$- \boldsymbol{d}(10.0\,\text{kg})(9.81\,\text{m/s}^2)\cos 45° = 0$$

Solving for d yields:

$$\boldsymbol{d} = \boxed{83\,\text{cm}}$$

67 •• **[SSM]** Figure 12-57 shows a 20.0-kg ladder leaning against a frictionless wall and resting on a frictionless horizontal surface. To keep the ladder from slipping, the bottom of the ladder is tied to the wall with a thin wire. When no one is on the ladder, the tension in the wire is 29.4 N. (The wire will break if the tension exceeds 200 N.) (*a*) If an 80.0-kg person climbs halfway up the ladder, what force will be exerted by the ladder against the wall? (*b*) How far from the bottom end of the ladder can an 80.0-kg person climb?

Picture the Problem Let *m* represent the mass of the ladder and *M* the mass of the person. The force diagram shows the forces acting on the ladder for Part (*b*). From the condition for translational equilibrium, we can conclude that $T = F_{\text{by wall}}$, a result we'll need in Part (*b*). Because the ladder is also in rotational equilibrium, summing the torques about the bottom of the ladder will eliminate both F_n and T.



(*a*) Apply $\sum\vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$F_{\text{by wall}}(\ell\sin\theta) - mg\left(\tfrac{1}{2}\ell\cos\theta\right) - Mg\left(\tfrac{1}{2}\ell\cos\theta\right) = 0$$

Solve for $F_{\text{by wall}}$ and simplify to obtain:

$$F_{\text{by wall}} = \frac{(m+M)g\cos\theta}{2\sin\theta} = \frac{(m+M)g}{2\tan\theta}$$

Refer to Figure 12-57 to determine θ:

$$\theta = \tan^{-1}\left(\frac{5.0\text{ m}}{1.5\text{ m}}\right) = 73.3°$$

Substitute numerical values and evaluate $F_{\text{by wall}}$:

$$F_{\text{by wall}} = \frac{(20\text{ kg} + 80\text{ kg})(9.81\text{ m/s}^2)}{2\tan 73.3°} = \boxed{0.15\text{ kN}}$$

(*b*) Apply $\sum F_x = 0$ to the ladder to obtain:

$$T - F_{\text{by wall}} = 0 \Rightarrow T = F_{\text{by wall}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder subject to the condition that $\boldsymbol{F}_{\text{by wall}} = \boldsymbol{T}_{\max}$:

$$T_{\max}(\ell \sin\theta) - mg\left(\tfrac{1}{2}\ell\cos\theta\right) - Mg(L\cos\theta) = 0$$

where L is the maximum distance along the ladder that the person can climb without exceeding the maximum tension in the wire.

Solving for L and simplifying yields:

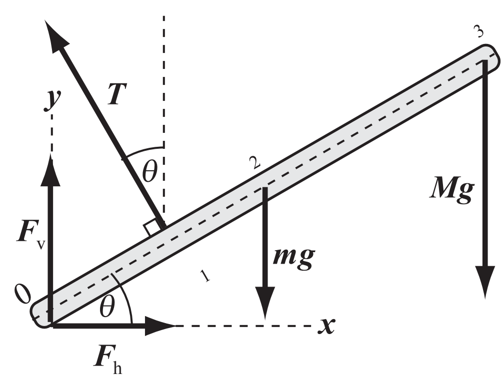
$$L = \frac{T_{\max}\ell\sin\theta - \frac{1}{2}mg\ell\cos\theta}{Mg\cos\theta}$$

Substitute numerical values and evaluate L:

$$\boldsymbol{L} = \frac{(200\text{ N})(5.0\text{ m}) - \frac{1}{2}(20\text{ kg})(9.81\text{ m/s}^2)(1.5\text{ m})}{(80\text{ kg})(9.81\text{ m/s}^2)\cos 73.3°} = \boxed{3.8\text{ m}}$$

68 •• A 360-kg object is supported on a wire attached to a 15-m-long steel bar that is pivoted at a vertical wall and supported by a cable as shown in Figure 12-58. The mass of the bar is 85 kg. With the cable attached to the bar 5.0 m from the lower end as shown, find the tension in the cable and the force exerted by the wall on the steel bar.

Picture the Problem Let m represent the mass of the bar, M the mass of the suspended object, F_v the vertical component of the force the wall exerts on the bar, F_h the horizontal component of the force the wall exerts on the bar, and T the tension in the cable. The force diagram shows these forces and their points of application on the bar. Because the bar is in equilibrium, we can apply the conditions for translational and rotational equilibrium to relate the various forces and distances.



Apply $\sum \vec{\tau} = 0$ to the bar about an axis through the hinge:

$$T\ell_1 - mg\ell_2\cos\theta - Mg\ell_3\cos\theta = 0$$

Solving for T yields:

$$T = \frac{(m\ell_2 + M\ell_3)g\cos\theta}{\ell_1}$$

Substitute numerical values and evaluate T:

$$\boldsymbol{T}=\frac{\left((85\text{ kg})(7.5\text{ m})+(360\text{ kg})(15\text{ m})\right)\left(9.81\text{ m/s}^2\right)\cos 30°}{5.0\text{ m}}=10.3\text{ kN}=\boxed{10\text{ kN}}$$

The magnitude and direction of the force exerted by the wall are given by:

$$\boldsymbol{F}_{\text{by wall}}=\sqrt{\boldsymbol{F}_{\text{v}}^2+\boldsymbol{F}_{\text{h}}^2} \qquad (1)$$

and

$$\boldsymbol{\phi}=\tan^{-1}\left(\frac{\boldsymbol{F}_{\text{v}}}{\boldsymbol{F}_{\text{h}}}\right) \qquad (2)$$

Apply $\sum\vec{F}=0$ to the bar to obtain:

$$\sum \boldsymbol{F}_{\boldsymbol{y}}=\boldsymbol{F}_{\text{v}}+\boldsymbol{T}\cos 30°-\boldsymbol{mg}-\boldsymbol{Mg}=0$$

and

$$\sum \boldsymbol{F}_{\boldsymbol{x}}=\boldsymbol{F}_{\text{h}}-\boldsymbol{T}\sin 30°=0$$

Solving the y equation for F_{v} yields:

$$\boldsymbol{F}_{\text{v}}=(\boldsymbol{m}+\boldsymbol{M})\boldsymbol{g}-\boldsymbol{T}\cos 30°$$

Solve the x equation for F_{h} to obtain:

$$\boldsymbol{F}_{\text{h}}=\boldsymbol{T}\sin 30°$$

Substituting for F_{v} and F_{h} in equation (1) yields:

$$F_{\text{by wall}}=\sqrt{\left((m+M)g-T\cos 30°\right)^2+\left(T\sin 30°\right)^2}$$

Substitute numerical values and evaluate $F_{\text{by wall}}$:

$$\boldsymbol{F}_{\text{by wall}}=\sqrt{\left((85\text{ kg}+360\text{ kg})\left(9.81\text{ m/s}^2\right)-(10.3\text{ kN})\cos 30°\right)^2+\left((10.3\text{ kN})\sin 30°\right)^2}$$
$$=\boxed{6.9\text{ kN}}$$

Substituting for F_{v} and F_{h} in equation (2) yields:

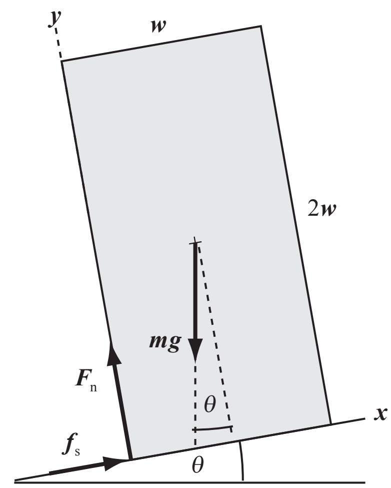
$$\phi=\tan^{-1}\left(\frac{(m+M)g-T\cos 30°}{T\sin 30°}\right)$$

Substitute numerical values and evaluate ϕ:

$$\phi=\tan^{-1}\left(\frac{(85\text{ kg}+360\text{ kg})\left(9.81\text{ m/s}^2\right)-(10.3\text{ kN})\cos 30°}{(10.3\text{ kN})\sin 30°}\right)=-41.49°$$
$$=\boxed{-41°}\ \text{That is, }41°\text{ below the horizontal.}$$

69 •• Repeat Problem 63 if the truck accelerates up a hill that makes an angle of 9.0º with the horizontal.

Picture the Problem The box and the forces acting on it are shown in the figure. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s: $$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass: $$wf_s - \tfrac{1}{2} wF_n = 0 \Rightarrow \frac{f_s}{F_n} = \frac{1}{2}$$

Substitute for $\frac{f_s}{F_n}$ to obtain the condition for tipping: $$\mu_s \geq 0.50$$

Therefore, if the box is to slide: $\mu_s < \boxed{0.50}$, as in Problem 63.

Remarks: The difference between problems 63 and 69 is that in 63 the maximum acceleration before slipping is 0.5*g*, whereas in 69 it is (0.5 cos9.0°– sin9.0°)*g* = 0.337*g*.

70 •• A thin uniform rod 60 cm long is balanced 20 cm from one end when an object whose mass is (2*m* + 2.0 grams) is at the end nearest the pivot and an object of mass *m* is at the opposite end (Figure 12-59*a*). Balance is again achieved if the object whose mass is (2*m* + 2.0 grams) is replaced by the object of mass *m* and no object is placed at the other end (Figure 12-59*b*). Determine the mass of the rod.

Picture the Problem Let the mass of the rod be represented by *M*. Because the rod is in equilibrium, we can apply the condition for rotational equilibrium to relate the masses of the objects placed on the rod to its mass.

Apply $\sum \vec{\tau} = 0$ about an axis through the pivot for the initial condition:

$$(20\,\text{cm})(2m + 2.0\,\text{g}) - (40\,\text{cm})m - (10\,\text{cm})M = 0$$

Simplifying yields:

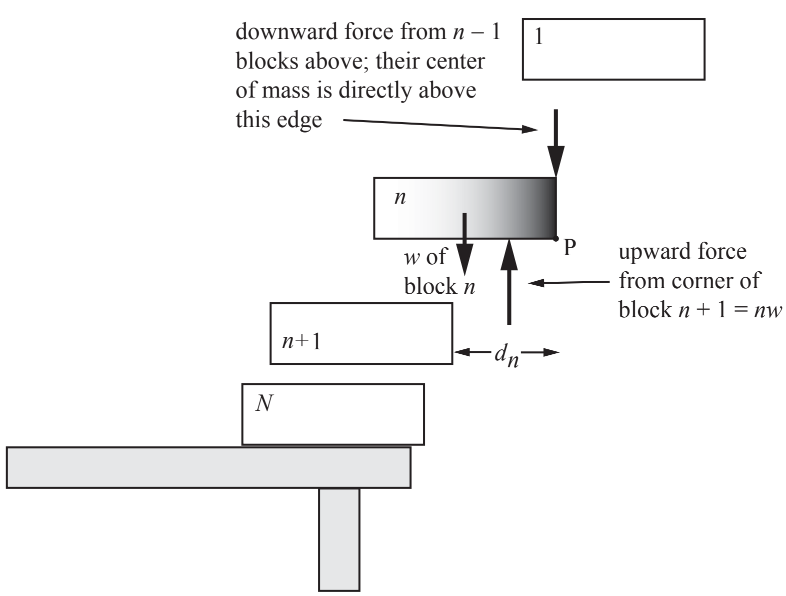
$$2(2m + 2.0\,\text{g}) - 4m - M = 0$$

Solve for M to obtain:

$$M = \boxed{4.0\,\text{g}}$$

71 ••• **[SSM]** There are a large number of identical uniform bricks, each of length L. If they are stacked one on top of another lengthwise (see Figure 12-60), the maximum offset that will allow the top brick to rest on the bottom brick is $L/2$. (a) Show that if this two-brick stack is placed on top of a third brick, the maximum offset of the second brick on the third brick is $L/4$. (b) Show that, in general, if you build a stack of N bricks, the maximum overhang of the ($n - 1$)th brick (counting down from the top) on the nth brick is $L/2n$. (c) Write a **spreadsheet** program to calculate total offset (the sum of the individual offsets) for a stack of N bricks, and calculate this for $L = 20$ cm and $N = 5$, 10, and 100. (d) Does the sum of the individual offsets approach a finite limit as $N \to \infty$? If so, what is that limit?

Picture the Problem Let the weight of each uniform brick be w. The downward force of all the bricks above the nth brick *must act at its corner,* because the upward reaction force points through the center of mass of all the bricks above the nth one. Because there is no vertical acceleration, the upward force exerted by the (n + 1)th brick on the nth brick must equal the total weight of the bricks above it. Thus this force is just nw. Note that it is convenient to develop the general relationship of Part (b) initially and then extract the answer for Part (a) from this general result.



(*a*) and (*b*) Noting that the line of action of the downward forces exerted by the blocks above the *n*th block passes through the point P, resulting in a lever arm of zero, apply $\sum \tau_P = 0$ to the *n*th brick to obtain:

$$w\left(\tfrac{1}{2}L\right) - nwd_n = 0$$

where d_n is the overhang of the *n*th brick beyond the edge of the (n + 1)th brick.

Solving for d_n yields:

$$d_n = \boxed{\frac{L}{2n}} \text{ where } n = 1,2,3,\ldots$$

For block number 2:

$$d_2 = \boxed{\frac{L}{4}}$$

(*c*) A spreadsheet program to calculate the sum of the offsets as a function of *n* is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B5	B4+1	$n + 1$
C5	C4+B1/(2*B5)	$d_n + \frac{L}{2n}$

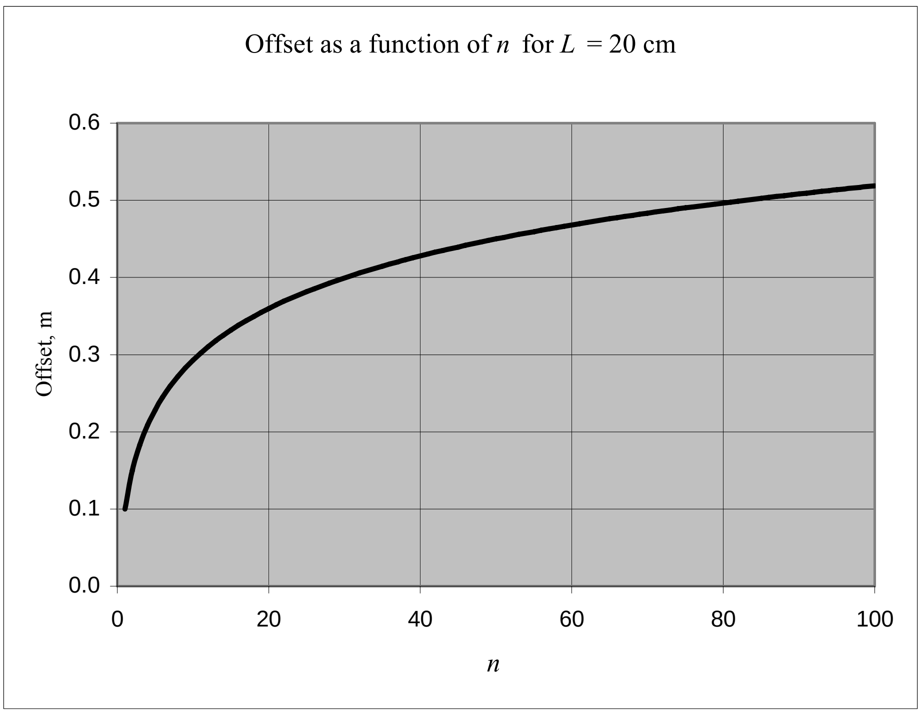
	A	B	C	D
1	L=	0.20	m	
2				
3		n	offset	
4		1	0.100	
5		2	0.150	
6		3	0.183	
7		4	0.208	
8		5	0.228	
9		6	0.245	
10		7	0.259	
11		8	0.272	
12		9	0.283	
13		10	0.293	
98		95	0.514	
99		96	0.515	
100		97	0.516	
101		98	0.517	
102		99	0.518	
103		100	0.519	

From the table we see that $d_5 = \boxed{15\,\text{cm},}$ $d_{10} = \boxed{26\,\text{cm},}$ and $d_{100} = \boxed{0.52\,\text{cm.}}$

(*d*) The sum of the individual offsets S is given by:

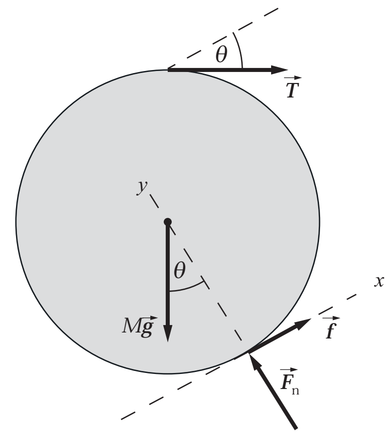
$$S = \sum_{n=1}^{N} d_n = \frac{L}{2}\sum_{n=1}^{N}\frac{1}{n}$$

Because this series is a harmonic series, S approaches infinity as the number of blocks N grows without bound. The following graph, plotted using a spreadsheet program, suggests that S has no limit.



72 •• A uniform sphere of radius R and mass M is held at rest on an inclined plane of angle θ by a horizontal string, as shown in Figure 12-61. Let R = 20 cm, M = 3.0 kg, and θ = 30º. (*a*) Find the tension in the string. (*b*) What is the normal force exerted on the sphere by the inclined plane? (*c*) What is the frictional force acting on the sphere?

Picture the Problem The four forces acting on the sphere: its weight, mg; the normal force of the plane, F_n; the frictional force, f, acting parallel to the plane; and the tension in the string, T, are shown in the figure. Because the sphere is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find f, F_n, and T.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the center of the sphere:

$$fR - TR = 0 \Rightarrow T = f$$

Apply $\sum F_x = 0$ to the sphere:

$$f + T\cos\theta - Mg\sin\theta = 0$$

Substituting for f and solving for T yields:

$$T = \frac{Mg\sin\theta}{1+\cos\theta}$$

Substitute numerical values and evaluate T:

$$\boldsymbol{T} = \frac{(3.0\,\text{kg})(9.81\,\text{m/s}^2)\sin 30°}{1+\cos 30°} = 7.89\,\text{N}$$
$$= \boxed{7.9\,\text{N}}$$

(*b*) Apply $\sum F_y = 0$ to the sphere:

$$F_n - T\sin\theta - Mg\cos\theta = 0$$

Solve for F_n:

$$F_n = T\sin\theta + Mg\cos\theta$$

Substitute numerical values and evaluate F_n:

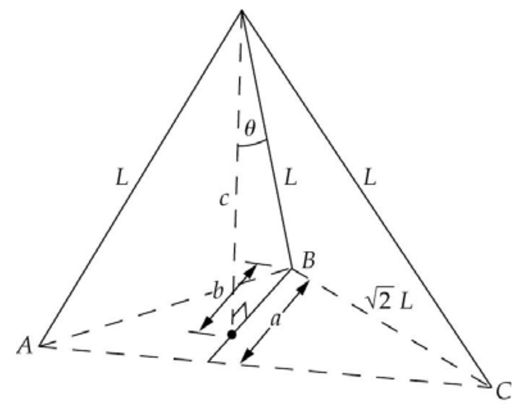
$$\boldsymbol{F}_n = (7.89\,\text{N})\sin 30° + (3.0\,\text{kg})(9.81\,\text{m/s}^2)\cos 30°$$
$$= \boxed{29\,\text{N}}$$

(*c*) In Part (*a*) we showed that $f = T$:

$$\boldsymbol{f} = \boxed{7.9\,\text{N}}$$

73 ••• The legs of a tripod make equal angles of 90º with each other at the apex, where they join together. A 100-kg block hangs from the apex. What are the compressional forces in the three legs?

Picture the Problem Let L be the length of each leg of the tripod. Applying the Pythagorean theorem leads us to conclude that the distance a shown in the figure is $\sqrt{3/2}\boldsymbol{L}$ and the distance b, the distance to the centroid of the triangle ABC is $\frac{2}{3}\sqrt{3/2}\boldsymbol{L}$, and the distance c is $\boldsymbol{L}/\sqrt{3}$. These results allow us to conclude that $\cos\boldsymbol{\theta} = \boldsymbol{L}/\sqrt{3}$. Because the tripod is in equilibrium, we can apply the condition for translational equilibrium to find the compressional forces in each leg.



Letting F_C represent the compressional force in a leg of the tripod, apply $\sum\vec{F} = 0$ to the apex of the tripod:

$$3F_C\cos\theta - mg = 0 \Rightarrow F_C = \frac{mg}{3\cos\theta}$$

Substitute for $\cos\theta$ and simplify to obtain:

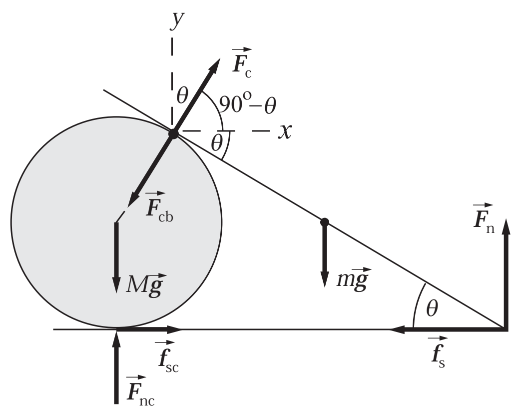
$$F_C = \frac{mg}{3\left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{3}mg$$

Substitute numerical values and evaluate F_C:

$$\boldsymbol{F}_C = \frac{\sqrt{3}}{3}(100\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{566\,\text{N}}$$

74 ••• Figure 12-63 shows a 20-cm-long uniform beam resting on a cylinder that has a radius of 4.0-cm. The mass of the beam is 5.0 kg and that of the cylinder is 8.0 kg. The coefficient of static friction between beam and cylinder is zero, whereas the coefficients of static friction between the cylinder and the floor, and between the beam and the floor, are *not* zero. Are there any values for these coefficients of static friction such that the system is in static equilibrium? If so, what are these values? If not, explain why none exist.

Picture the Problem The forces that act on the beam are its weight, mg; the force of the cylinder, F_c, acting along the radius of the cylinder; the normal force of the ground, F_n; and the friction force $f_s = \mu_s F_n$. The forces acting on the cylinder are its weight, Mg; the force of the beam on the cylinder, $F_{cb} = F_c$ in magnitude, acting radially inward; the normal force of the ground on the cylinder, F_{nc}; and the force of friction, $f_{sc} = \mu_{sc}F_{nc}$. Choose the coordinate system shown in the figure and apply the conditions for rotational and translational equilibrium.



Express $\mu_{s,\text{beam-floor}}$ in terms of f_s and F_n:

$$\mu_{s,\text{beam-floor}} = \frac{f_s}{F_n} \qquad (1)$$

Express $\mu_{s,\text{cylinder-floor}}$ in terms of f_{sc} and F_{nc}:

$$\mu_{s,\text{cylinder-floor}} = \frac{f_{sc}}{F_{nc}} \qquad (2)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the right end of the beam:

$$[(10\,\text{cm})\cos\theta]mg - (15\,\text{cm})F_c = 0$$

Solve for F_c to obtain:

$$F_c = \frac{[(10\,\text{cm})\cos\theta]mg}{15\,\text{cm}}$$

Substitute numerical values and evaluate F_c:

$$\boldsymbol{F}_c = \frac{[10\cos 30°](5.0\,\text{kg})(9.81\,\text{m/s}^2)}{15} = 28.3\,\text{N}$$

Apply $\sum F_y = 0$ to the beam:

$$\boldsymbol{F}_n + \boldsymbol{F}_c \cos\boldsymbol{\theta} - \boldsymbol{mg} = 0$$

Solving for F_n yields:

$$F_n = mg - F_c\cos\theta$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_n = (5.0\,\text{kg})(9.81\,\text{m/s}^2) - (28.3\,\text{N})\cos 30° = 24.5\,\text{N}$$

Apply $\sum F_x = 0$ to the beam:

$$-f_s + F_c\cos(90° - \theta) = 0$$

Solve for f_s to obtain:

$$\boldsymbol{f}_s = \boldsymbol{F}_c \cos(90° - \boldsymbol{\theta})$$

Substitute numerical values and evaluate f_s:

$$f_s = F_c\cos(90° - \theta) = (28.3\,\text{N})\cos 60°$$
$$= 14.2\,\text{N}$$

$\vec{F}_{cb}$ is the reaction force to $\vec{F}_c$:

$F_{cb} = F_c = 28.3\,\text{N}$ radially inward.

Apply $\sum F_y = 0$ to the cylinder:

$$F_{nc} - F_{cb}\cos\theta - Mg = 0$$

Solve for F_{nc} to obtain:

$$F_{nc} = F_{cb}\cos\theta + Mg$$

Substitute numerical values and evaluate F_{nc}:

$$\boldsymbol{F}_{nc} = (28.3\,\text{N})\cos 30° + (8.0\,\text{kg})(9.81\,\text{m/s}^2)$$
$$= 103\,\text{N}$$

Apply $\sum F_x = 0$ to the cylinder:

$$f_{sc} - F_{cb}\cos(90° - \theta) = 0$$

Solve for and evaluate f_{sc}:

$$f_{sc} = F_{cb}\cos(90° - \theta) = (28.3\,\text{N})\cos 60°$$
$$= 14.2\,\text{N}$$

Substitute numerical values in equations (1) and (2) and evaluate $\mu_{s,\text{beam-floor}}$ and $\mu_{s,\text{cylinder-floor}}$:

$$\boldsymbol{\mu}_{s,\text{beam–floor}} = \frac{14.2\,\text{N}}{24.5\,\text{N}} = \boxed{0.58}$$

and

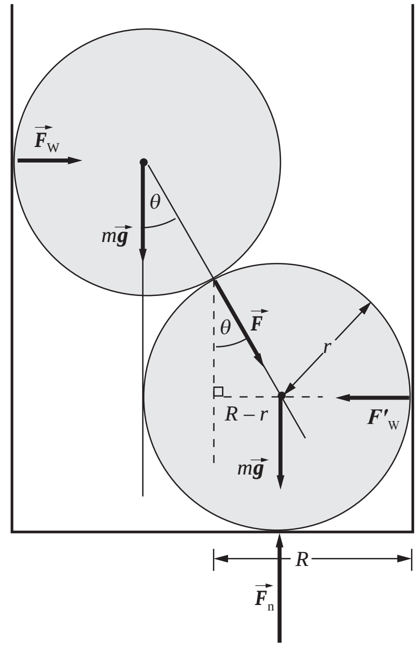
$$\boldsymbol{\mu}_{s,\text{cylinder–floor}} = \frac{14.2\,\text{N}}{103\,\text{N}} = \boxed{0.14}$$

75 ••• **[SSM]** Two solid smooth (frictionless) spheres of radius r are placed inside a cylinder of radius R, as in Figure 12-62. The mass of each sphere is m. Find the force exerted by the bottom of the cylinder on the bottom sphere, the force exerted by the wall of the cylinder on each sphere, and the force exerted by one sphere on the other. All forces should be expressed in terms of m, R, and r.

Picture the Problem The geometry of the system is shown in the drawing. Let upward be the positive y direction and to the right be the positive x direction. Let the angle between the vertical center line and the line joining the two centers be θ. Then

$$\sin\theta = \frac{R-r}{r} \text{ and } \tan\theta = \frac{R-r}{\sqrt{R(2r-R)}}.$$

The force exerted by the bottom of the cylinder is just $2mg$. Let F be the force that the top sphere exerts on the lower sphere. Because the spheres are in equilibrium, we can apply the condition for translational equilibrium.



Apply $\sum F_y = 0$ to the spheres:

$$F_n - mg - mg = 0 \Rightarrow F_n = \boxed{2mg}$$

Because the cylinder wall is smooth, $F\cos\theta = mg$, and:

$$F = \frac{mg}{\cos\theta} = \boxed{mg\frac{r}{R(2r-R)}}$$

Express the x component of F:

$$F_x = F\sin\theta = mg\tan\theta$$

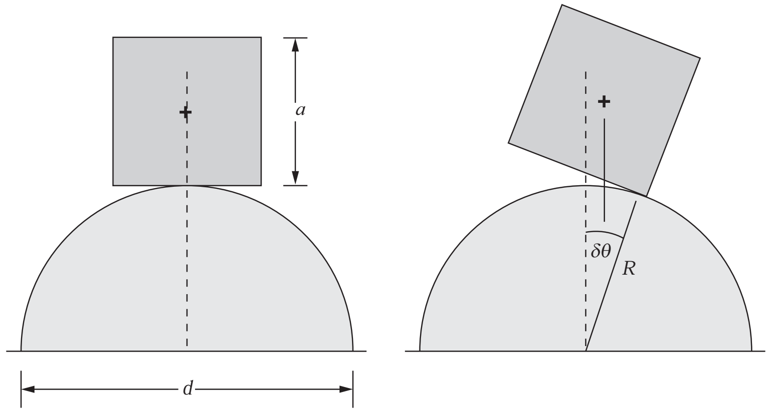
Express the force that the wall of the cylinder exerts:

$$F_W = \boxed{mg\frac{R-r}{\sqrt{R(2r-R)}}}$$

Remarks: Note that as *r* approaches *R*/2, $F_W \to \infty$.

76 ••• A solid cube of side length a balanced atop a cylinder of diameter d is in unstable equilibrium if $d << a$ (Figure 12-64) and is in stable equilibrium if $d >> a$. Determine the minimum value of the ratio d/a for which the cube is in stable equilibrium.

Picture the Problem Consider a small rotational displacement, $\delta\theta$ of the cube from equilibrium. This shifts the point of contact between cube and cylinder by $R\delta\theta$, where $R = d/2$. As a result of that motion, the cube itself is rotated through the same angle $\delta\theta$, and so its center is shifted in the same direction by the amount $(a/2)\,\delta\theta$, neglecting higher order terms in $\delta\theta$.



If the displacement of the cube's center of mass is less than that of the point of contact, the torque about the point of contact is a restoring torque, and the cube will return to its equilibrium position. If, on the other hand, $(a/2)\,\delta\theta > (d/2)\,\delta\theta,$ then the torque about the point of contact due to mg is in the direction of $\delta\theta,$ and will cause the displacement from equilibrium to increase. We see that the minimum value of d/a for stable equilibrium is

$d/a = 1.$