Chapter 23
Electrical Potential

Conceptual Problems

1 • **[SSM]** A proton is moved to the left in a uniform electric field that points to the right. Is the proton moving in the direction of increasing or decreasing electric potential? Is the electrostatic potential energy of the proton increasing or decreasing?

Determine the Concept The proton is moving to a region of higher potential. The proton's electrostatic potential energy is increasing.

2 • An electron is moved to the left in a uniform electric field that points to the right. Is the electron moving in the direction of increasing or decreasing electric potential? Is the electrostatic potential energy of the electron increasing or decreasing?

Determine the Concept The electron is moving to a region of higher electric potential. The electron's electrostatic potential energy is decreasing.

3 • If the electric potential is uniform throughout a region of space, what can be said about the electric field in that region?

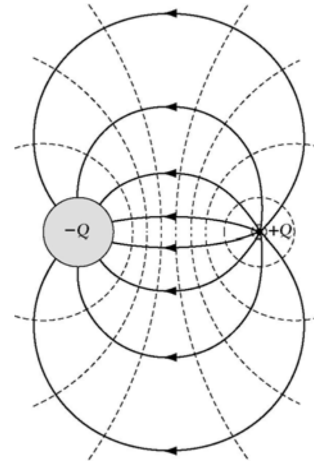
Determine the Concept If V is constant, its gradient is zero; consequently the electric field is zero throughout the region.

4 • If V is known at only a single point in space, can $\vec{E}$ be found at that point? Explain your answer.

Determine the Concept No. $\vec{E}$ can be determined without knowing V at a continuum of points.

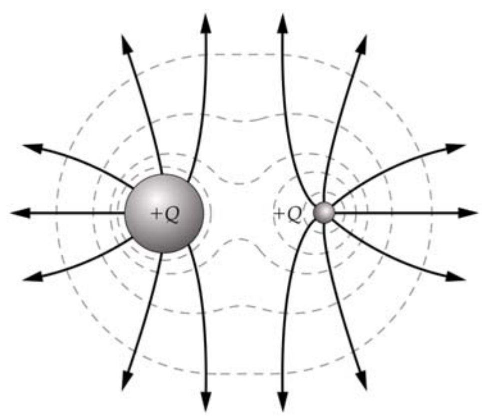
5 •• **[SSM]** Figure 23-29 shows a point particle that has a positive charge $+Q$ and a metal sphere that has a charge $-Q$. Sketch the electric field lines and equipotential surfaces for this system of charges.

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $-Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface.



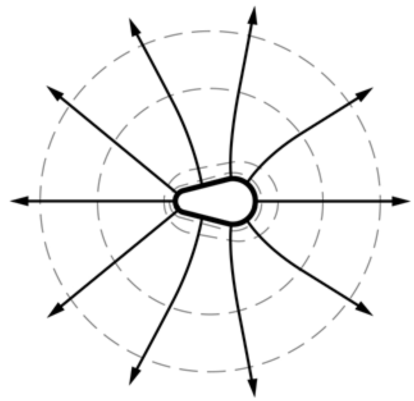
6 •• Figure 23-30 shows a point particle that has a negative charge $-Q$ and a metal sphere that has a charge $+Q$. Sketch the electric field lines and equipotential surfaces for this system of charges.

Picture the Problem The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $+Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere's surface. Very far from both charges, the equipotential surfaces and field lines approach those of a point charge $2Q$ located at the midpoint.



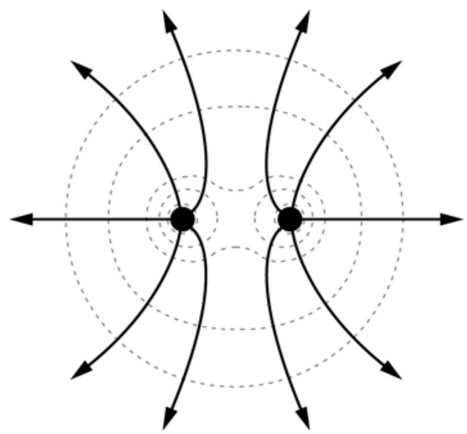
7 •• Sketch the electric field lines and equipotential surfaces for the region surrounding the charged conductor shown in Figure 23-31, assuming that the conductor has a net positive charge.

Picture the Problem The equipotential surfaces are shown with dashed lines, the field lines are shown in solid lines. It is assumed that the conductor carries a positive charge. Near the conductor the equipotential surfaces follow the conductor's contours; far from the conductor, the equipotential surfaces are spheres centered on the conductor. The electric field lines are perpendicular to the equipotential surfaces.



8 •• Two equal positive point charges are separated by a finite distance. Sketch the electric field lines and the equipotential surfaces for this system.

Picture the Problem The equipotential surfaces are shown with dashed lines, the electric field lines are shown with solid lines. Near each charge, the equipotential surfaces are spheres centered on each charge; far from the charges, the equipotential surface is a sphere centered at the midpoint between the charges. The electric field lines are perpendicular to the equipotential surfaces.



9 •• Two point charges are fixed on the *x*-axis. (*a*) Each has a positive charge *q*. One is at $x = -a$ and the other is at $x = +a$. At the origin, which of the following is true?

(1) $\vec{E} = 0$ and $V = 0$,
(2) $\vec{E} = 0$ and $V = 2kq/a$,
(3) $\vec{E} = \left(2kq/a^2\right)\hat{i}$ and $V = 0$,
(4) $\vec{E} = \left(2kq/a^2\right)\hat{i}$ and $V = 2kq/a$,
(5) None of the above.

(*b*) One has a positive charge $+q$ and the other has a negative charge $-q$. The positive point charge is at $x = -a$ and the negative point charge is at $x = +a$. At the origin, which of the following is true?

(1) $\vec{E} = 0$ and $V = 0$,

(2) $\vec{E} = 0$ and $V = 2kq/a$,
(3) $\vec{E} = (2kq/a^2)\hat{i}$ and $V = 0$,
(4) $\vec{E} = (2kq/a^2)\hat{i}$ and $V = 2kq/a$,
(5) None of the above.

Picture the Problem We can use Coulomb's law and the superposition of fields to find E at the origin and the definition of the electric potential due to a point charge to find V at the origin.

(*a*) Apply Coulomb's law and the superposition of fields to find the electric field E at the origin:

$$\vec{E} = \vec{E}_{+q\text{ at }-a} + \vec{E}_{+q\text{ at }+a} = \frac{kq}{a^2}\hat{i} - \frac{kq}{a^2}\hat{i} = 0$$

The potential V at the origin is given by:

$$V = V_{+q\text{ at }-a} + V_{+q\text{ at }+a} = \frac{kq}{a} + \frac{kq}{a} = \frac{2kq}{a}$$

and $\boxed{(2)}$ is correct.

(*b*) Apply Coulomb's law and the superposition of fields to find the electric field E at the origin:

$$\vec{E} = \vec{E}_{+q\text{ at }-a} + \vec{E}_{-q\text{ at }+a} = \left(\frac{kq}{a^2}\right)\hat{i} + \left(\frac{kq}{a^2}\right)\hat{i} = \left(\frac{2kq}{a^2}\right)\hat{i}$$

The potential V at the origin is given by:

$$V = V_{+q\text{ at }-a} + V_{-q\text{ at }+a} = \frac{kq}{a} + \frac{k(-q)}{a} = 0$$

and $\boxed{(3)}$ is correct.

10 •• The electrostatic potential (in volts) is given by $V(x, y, z) = 4.00|x| + V_0$, where V_0 is a constant, and x is in meters. (*a*) Sketch the electric field for this potential. (*b*) Which of the following charge distributions is most likely responsible for this potential: (1) A negatively charged flat sheet in the $z = 0$ plane, (2) a point charge at the origin, (3) a positively charged flat sheet in the $x = 0$ plane, or (4) a uniformly charged sphere centered at the origin. Explain your answer.

Picture the Problem We can use $\vec{E} = -\frac{\partial V}{\partial x}\hat{i}$ to find the electric field corresponding to the given potential and then compare its form to those produced by the four alternatives listed.

(*a*) Find the electric field corresponding to this potential function:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -\frac{\partial}{\partial x}\left[4.00|x| + V_0\right]\hat{i} = -4.00\frac{\partial}{\partial x}\left[|x|\right]\hat{i}$$

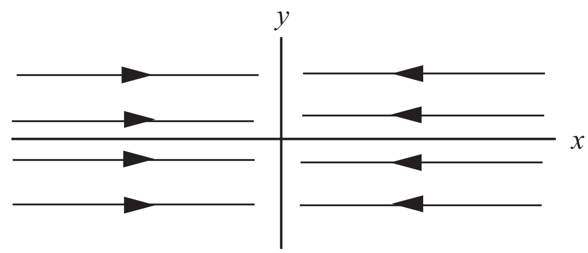
If $x > 0$, then $\frac{\partial}{\partial x}\left[|x|\right] = 1$ and:

$$\vec{E}_{x>0} = \left(-4.00\frac{\text{V}}{\text{m}}\right)\hat{i}$$

If $x < 0$, then $\frac{\partial}{\partial x}\left[|x|\right] = -1$ and:

$$\vec{E}_{x<0} = \left(4.00\frac{\text{V}}{\text{m}}\right)\hat{i}$$

A sketch of the electric field in this region follows:



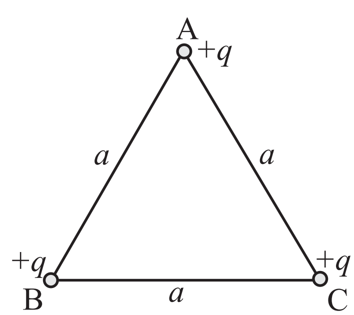
(*b*) $\boxed{(1)}$ is correct because field lines end on negative charges.

11 •• [SSM] The electric potential is the same everywhere on the surface of a conductor. Does this mean that the surface charge density is also the same everywhere on the surface? Explain your answer.

Determine the Concept No. The local surface charge density is proportional to the normal component of the electric field, not the potential on the surface.

12 •• Three identical positive point charges are located at the vertices of an equilateral triangle. If the length of each side of the triangle shrinks to one-fourth of its original length, by what factor does the electrostatic potential energy of this system change? (The electrostatic potential energy approaches zero if the length of each side of the triangle approaches infinity.)

Picture the Problem Points A, B, and C are at the vertices of an equilateral triangle of side *a*. The electrostatic potential energy of the system of the three equal positive point charges is the total work that must be done on the charges to bring them from infinity to this configuration.



The electrostatic potential energy is the work required to assemble the three charges at the vertices of the equilateral triangle:

$$U = W_A + W_B + W_C \qquad (1)$$

Place the first charge at point A. To accomplish this step, the work W_A that is needed is zero:

$$W_A = 0$$

Bring the second charge to point B. The work required is $\boldsymbol{W}_B = \boldsymbol{q}\boldsymbol{V}_A$, where V_A is the potential at point B due to the first charge at point A a distance a away:

$$W_B = qV_A = q\left(\frac{kq}{a}\right) = \frac{kq^2}{a}$$

Similarly, W_C is given by:

$$W_C = qV_C = q\left(\frac{kq}{a} + \frac{kq}{a}\right) = \frac{2kq^2}{a}$$

Substituting for W_A, W_B, and W_C in equation (1) yields:

$$U = 0 + \frac{kq^2}{a} + \frac{2kq^2}{a} = \frac{3kq^2}{a}$$

If the triangle is expanded to four times its original size, its electrostatic potential energy U' becomes:

$$U' = \frac{3kq^2}{4a} = \frac{1}{4}\left(\frac{3kq^2}{a}\right) = \tfrac{1}{4}U$$

Hence the electrostatic potential energy of this system changes by a factor of $\boxed{4}$.

Estimation and Approximation Problems

13 • [SSM] Estimate maximum the potential difference between a thundercloud and Earth, given that the electrical breakdown of air occurs at fields of roughly 3.0×10^6 V/m.

Picture the Problem The field of a thundercloud must be of order 3.0×10^6 V/m just before a lightning strike.

Express the potential difference between the cloud and the earth as a function of their separation d and electric field E between them:

$$V = Ed$$

Assuming that the thundercloud is at a distance of about 1 km above the surface of the earth, the potential difference is approximately:

$$V = (3.0\times10^6\ \text{V/m})(10^3\ \text{m}) = \boxed{3.0\times10^9\ \text{V}}$$

Note that this is an upper bound, as there will be localized charge distributions on the thundercloud which raise the local electric field above the average value.

14 • The specifications for the gap width of typical automotive spark plug is approximately equal to the thickness of the cardboard used for matchbook covers. Because of the high compression of the air-gas mixture in the cylinder, the dielectric strength of the mixture is roughly 2.0×10^7 V/m. Estimate the maximum potential difference across the spark gap during operating conditions.

Picture the Problem The potential difference between the electrodes of the spark plug is the product of the electric field in the gap and the separation of the electrodes. We'll assume that the separation of the electrodes is 1.0 mm.

Express the potential difference between the electrodes of the spark plug as a function of their separation d and electric field E between them:

$$V = Ed$$

Substitute numerical values and evaluate V:

$$V = (2.0\times10^7\ \text{V/m})(1.0\times10^{-3}\ \text{m}) = \boxed{2.0\,\text{kV}}$$

15 •• The radius of a proton is approximately 1.0×10^{-15} m. Suppose two protons having equal and opposite momenta undergo a head-on collision. Estimate the minimum kinetic energy (in MeV) required by each proton to allow the protons to overcome electrostatic repulsion and collide. *Hint: The rest energy of a proton is 938 MeV. If the kinetic energies of the protons are much less than this rest energy, then a non-relativistic calculation is justified.*

Picture the Problem We can use conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential energy when they have approached each other to the given "radius."

Apply conservation of energy to relate the initial kinetic energy of the protons to their electrostatic potential when they are separated by a distance r:

$$K_i + U_i = K_f + U_f$$

or, because $U_i = K_f = 0$,

$$K_i = U_f$$

Because each proton has kinetic energy K:

$$2K = \frac{e^2}{4\pi \in_0 r} \Rightarrow K = \frac{e^2}{8\pi \in_0 r}$$

Substitute numerical values and evaluate K:

$$K = \frac{\left(1.602\times10^{-19}\text{ C}\right)^2}{8\pi\left(8.854\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(1.0\times10^{-15}\text{ m}\right)} = 1.153\times10^{-13}\text{ J}\times\frac{1\text{eV}}{1.602\times10^{-19}\text{ J}}$$
$$= \boxed{0.72\,\text{MeV}}$$

Remarks: Because the kinetic energies of the protons is approximately 0.08% of their rest energy ($K/E_{\text{rest}} = 0.72\text{ MeV}/938\text{ MeV} \approx 0.08\%$,) the non-relativistic calculation was justified.

16 • When you touch a friend after walking across a rug on a dry day, you typically draw a spark of about 2.0 mm. Estimate the potential difference between you and your friend just before the spark.

Picture the Problem The magnitude of the electric field for which dielectric breakdown occurs in air is about 3.0 MV/m. We can estimate the potential difference between you and your friend from the product of the length of the spark and the dielectric constant of air.

Express the product of the length of the spark and the dielectric constant of air:

$$V = (3.0\,\text{MV/m})(2.0\,\text{mm}) = \boxed{6.0\,\text{kV}}$$

17 • Estimate the maximum surface charge density that can exist at the end of a sharp lightning rod so that no dielectric breakdown of air occurs.

Picture the Problem The maximum electric field E_{max} just outside the end of the lightning rod is related to the maximum surface charge density σ_{max}.

Express the maximum electric field just outside the end of the lightning rod as a function of the maximum surface charge density σ_{max}:

$$E_{\text{max}} = \frac{\sigma_{\text{max}}}{\in_0} \Rightarrow \sigma_{\text{max}} = \in_0 E_{\text{max}}$$

Substitute numerical values and evaluate σ_{max}:

$$\sigma_{\text{max}} = \left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(3.0\times10^{6}\,\frac{\text{V}}{\text{m}}\right) \approx \boxed{27\ \mu\text{C/m}^2}$$

18 •• The electric-field strength near the surface of Earth is about 300 V/m. (*a*) Estimate the magnitude of the charge density on the surface of Earth. (*b*) Estimate the total charge on Earth. (*c*) What is value of the electric potential at Earth′s surface? (Assume the potential is zero at infinity.) (*d*) If all Earth′s electrostatic potential energy could be harnessed and converted to electric energy at reasonable efficiency, how long could it be used to run the consumer households in the United States? Assume the average American household consumes about 500 kW·h of electric energy per month.

Picture the Problem (*a*) The charge density is the product of ϵ_0 and the magnitude of the electric field at the surface of Earth. (*b*) The total charge on Earth is the product of its surface charge density and its area. (*c*) The electric potential at Earth's surface is given by $V = kQ/R$ where Q is the charge on Earth and R is its radius. (*d*) We can estimate how long Earth's electrostatic energy could run households in the United States by dividing the energy available by the rate of consumption of electrical energy.

(*a*) The magnitude of the charge density σ on the surface of Earth is proportional to the electric field strength E at the surface of Earth:

$$\sigma = \epsilon_0\, \boldsymbol{E}$$

Substitute numerical values and evaluate σ:

$$\sigma = \left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(300\,\frac{\text{V}}{\text{m}}\right) = \boxed{2.66\ \text{nC/m}^2}$$

(*b*) The total charge on Earth is given by:

$$Q = \sigma A = 4\pi\sigma R^2$$

Substitute numerical values and evaluate Q:

$$\boldsymbol{Q} = 4\boldsymbol{\pi}\left(2.656\,\frac{\text{nC}}{\text{m}^2}\right)(6370\ \text{km})^2 = \boxed{1.35\ \text{MC}}$$

(*c*) The electric potential V at the surface of Earth is given by:

$$V = \frac{kQ}{R}$$

where R is Earth's radius.

Multiplying and dividing by R and substituting for kQ/R^2 yields:

$$V=\frac{kQ}{R}=\left(\frac{kQ}{R^2}\right)R=ER$$

Substitute numerical values and evaluate V:

$$V=\left(300\frac{\text{V}}{\text{m}}\right)(6370\text{ km})=\boxed{1.91\text{ GV}}$$

(*d*) Express the available energy:

$$E_{\text{avail}}=eU=e\left(\tfrac{1}{2}QV\right)$$

where e is the efficiency of energy conversion.

Assuming an efficiency of 1/3 yields:

$$E_{\text{avail}}=\tfrac{1}{3}\left(\tfrac{1}{2}QV\right)=\tfrac{1}{6}QV$$

Express the rate at which energy must be supplied to households in the United States:

$$P_{\text{req}}=P_{\substack{\text{req per}\\ \text{household}}}N_{\text{households}}$$

Assuming 80 million households in the United States, substitute numerical values and evaluate the lifetime of the electrical energy derived from Earth's electrostatic energy:

$$\begin{aligned}\frac{E_{\text{avail}}}{P_{\text{req}}}&=\frac{\tfrac{1}{6}QV}{P_{\substack{\text{req per}\\ \text{household}}}N_{\text{households}}}\\&=\frac{(1.35\text{ MC})(1.91\text{ GV})}{6\left(500\dfrac{\text{kWh}\times\dfrac{3600\text{ s}}{\text{h}}}{\text{household}\cdot\text{month}}\times\dfrac{1\text{ month}}{30.4\text{ d}}\times\dfrac{1\text{ d}}{24\text{ h}}\right)\left(80\times10^6\text{ households}\right)}\\&\approx\boxed{2.2\text{ h}}\end{aligned}$$

Electrostatic Potential Difference, Electrostatic Energy and Electric Field

19 • A point particle has a charge equal to +2.00 μC and is fixed at the origin. (*a*) What is the electric potential V at a point 4.00 m from the origin assuming that $V = 0$ at infinity? (*b*) How much work must be done to bring a second point particle that has a charge of +3.00 μC from infinity to a distance of 4.00 m from the +2.00-μC charge?

Picture the Problem The Coulomb potential at a distance r from the origin relative to $V = 0$ at infinity is given by $V = kq/r$ where q is the charge at the origin. The work that must be done by an outside agent to bring a charge from infinity to a position a distance r from the origin is the product of the magnitude of the charge and the potential difference due to the charge at the origin.

(*a*) The Coulomb potential of the charge is given by:

$$V = \frac{kq}{r}$$

Substitute numerical values and evaluate V:

$$V = \frac{\left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(2.00\ \mu\mathrm{C}\right)}{4.00\ \mathrm{m}} = 4.494\ \mathrm{kV} = \boxed{4.49\ \mathrm{kV}}$$

(*b*) The work that must be done is given by:

$$W = q\Delta V$$

Substitute numerical values and evaluate W:

$$W = \left(3.00\ \mu\mathrm{C}\right)\left(4.494\ \mathrm{kV}\right) = \boxed{13.5\ \mathrm{mJ}}$$

20 •• The facing surfaces of two large parallel conducting plates separated by 10.0 cm have uniform surface charge densities that are equal in magnitude but opposite in sign. The difference in potential between the plates is 500 V. (*a*) Is the positive or the negative plate at the higher potential? (*b*) What is the magnitude of the electric field between the plates? (*c*) An electron is released from rest next to the negatively charged surface. Find the work done by the electric field on the electron as the electron moves from the release point to the positive plate. Express your answer in both electron volts and joules. (*d*) What is the change in potential energy of the electron when it moves from the release point plate to the positive plate? (*e*) What is its kinetic energy when it reaches the positive plate?

Picture the Problem Because the electric field is uniform, we can find its magnitude from $E = \Delta V/\Delta x$. We can find the work done by the electric field on the electron from the difference in potential between the plates and the charge of the electron and find the change in potential energy of the electron from the work done on it by the electric field. We can use conservation of energy to find the kinetic energy of the electron when it reaches the positive plate.

(*a*) Because the electric force on a test charge is away from the positive plate and toward the negative plate, the positive plate is at the higher potential.

(*b*) Express the magnitude of the electric field between the plates in terms of their separation and the potential difference between them:

$$E = \frac{\Delta V}{\Delta x} = \frac{500\ \mathrm{V}}{0.100\ \mathrm{m}} = \boxed{5.00\ \mathrm{kV/m}}$$

(*c*) Relate the work done by the electric field on the electron to the difference in potential between the plates and the charge of the electron:

$$W = q\Delta V = \left(1.602\times10^{-19}\ \text{C}\right)\left(500\text{V}\right) = \boxed{8.01\times10^{-17}\ \text{J}}$$

Converting 8.01×10^{-17} J to eV yields:

$$W = \left(8.01\times10^{-17}\ \text{J}\right)\left(\frac{1\,\text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = \boxed{500\,\text{eV}}$$

(*d*) Relate the change in potential energy of the electron to the work done on it as it moves from the negative plate to the positive plate:

$$\Delta U = -W = \boxed{-500\,\text{eV}}$$

(*e*) Apply conservation of energy to obtain:

$$\Delta K = -\Delta U = \boxed{500\,\text{eV}}$$

21 •• A uniform electric field that has a magnitude 2.00 kV/m points in the $+x$ direction. (*a*) What is the electric potential difference between the $x = 0.00$ m plane and the $x = 4.00$ m plane? A point particle that has a charge of $+3.00\ \mu$C is released from rest at the origin. (*b*) What is the change in the electric potential energy of the particle as it travels from the $x = 0.00$ m plane to the $x = 4.00$ m plane? (*c*) What is the kinetic energy of the particle when it arrives at the $x = 4.00$ m plane? (*d*) Find the expression for the electric potential $V(x)$ if its value is chosen to be zero at $x = 0$.

Picture the Problem (*a*) and (*b*) We can use the definition of potential difference to find the potential difference $V(4.00\text{ m}) - V(0)$ and (*c*) conservation of energy to find the kinetic energy of the charge when it is at $x = 4.00$ m. (*d*) We can find $V(x)$ if $V(x)$ is assigned various values at various positions from the definition of potential difference.

(*a*) Apply the definition of finite potential difference to obtain:

$$V(4.00\,\text{m}) - V(0) = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_0^{4.00\,\text{m}} E\,d\ell = -\left(2.00\,\text{kN/C}\right)\left(4.00\,\text{m}\right) = \boxed{-8.00\,\text{kV}}$$

(*b*) By definition, ΔU is given by:

$$\Delta U = q\Delta V = \left(3.00\,\mu\text{C}\right)\left(-8.00\,\text{kV}\right) = \boxed{-24.0\,\text{mJ}}$$

(*c*) Use conservation of energy to relate ΔU and ΔK:

$$\Delta K + \Delta U = 0$$
or
$$K_{4\,\text{m}} - K_0 + \Delta U = 0$$

Because $K_0 = 0$:

$$K_{4\,\text{m}} = -\Delta U = \boxed{24.0\,\text{mJ}}$$

Use the definition of finite potential difference to obtain:

$$V(x) - V(x_0) = -E_x(x - x_0) = -(2.00\,\text{kV/m})(x - x_0)$$

(*d*) For $V(0) = 0$:

$$V(x) - 0 = -(2.00\,\text{kV/m})(x - 0)$$
or
$$V(x) = \boxed{-(2.00\,\text{kV/m})x}$$

22 •• In a potassium chloride molecule the distance between the potassium ion (K^+) and the chlorine ion (Cl^-) in a potassium chloride molecule is 2.80×10^{-10} m. (*a*) Calculate the energy (in eV) required to separate the two ions to an infinite distance apart. (Model the two ions as two point particles initially at rest.) (*b*) If twice the energy determined in Part (*a*) is actually supplied, what is the total amount of kinetic energy that the two ions have when they were an infinite distance apart?

Picture the Problem In general, the work done by an external agent in separating the two ions changes both their kinetic and potential energies. Here we're assuming that they are at rest initially and that they will be at rest when they are infinitely far apart. Because their potential energy is also zero when they are infinitely far apart, the energy W_{ext} required to separate the ions to an infinite distance apart is the negative of their potential energy when they are a distance r apart.

(*a*) Express the energy required to separate the ions in terms of the work required by an external agent to bring about this separation:

$$W_{\text{ext}} = \Delta K + \Delta U = 0 - U_{\text{i}} = -\frac{kq_-q_+}{r} = -\frac{k(-e)e}{r} = \frac{ke^2}{r}$$

Substitute numerical values and evaluate W_{ext}:

$$W_{\text{ext}} = \frac{(8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19}\ \text{C})^2}{2.80 \times 10^{-10}\ \text{m}} = 8.238 \times 10^{-19}\ \text{J}$$

Convert this energy to eV:

$$W_{ext} = \left(8.238\times10^{-19}\ \text{J}\right)\left(\frac{1\,\text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = \boxed{5.14\,\text{eV}}$$

(*b*) Apply the work-energy theorem to the system of ions to obtain:

$$2W_{ext} = \Delta K + \Delta U = K_f - U_i$$

where K_f is the kinetic energy of the ions when they are an infinite distance apart.

Solving for K_f yields:

$$K_f = 2W_{ext} + U_i$$

From Part (*a*), $U_i = -W_{ext}$:

$$K_f = 2W_{ext} - W_{ext} = W_{ext} = \boxed{5.14\,\text{eV}}$$

23 •• [SSM] Protons are released from rest in a Van de Graaff accelerator system. The protons initially are located where the electrical potential has a value of 5.00 MV and then they travel through a vacuum to a region where the potential is zero. (*a*) Find the final speed of these protons. (*b*) Find the accelerating electric-field strength if the potential changed *uniformly* over a distance of 2.00 m.

Picture the Problem We can find the final speeds of the protons from the potential difference through which they are accelerated and use $E = \Delta V/\Delta x$ to find the accelerating electric field.

(*a*) Apply the work-kinetic energy theorem to the accelerated protons:

$$W = \Delta K = K_f \Rightarrow e\Delta V = \tfrac{1}{2}mv^2$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2\left(1.602\times10^{-19}\ \text{C}\right)\left(5.00\,\text{MV}\right)}{1.673\times10^{-27}\ \text{kg}}} = \boxed{3.09\times10^{7}\ \text{m/s}}$$

(*b*) Assuming the same potential change occurred *uniformly* over the distance of 2.00 m, we can use the relationship between E, ΔV, and Δx express and evaluate E:

$$E = \frac{\Delta V}{\Delta x} = \frac{5.00\,\text{MV}}{2.00\,\text{m}} = \boxed{2.50\,\text{MV/m}}$$

24 •• The picture tube of a television set was, until recently, invariably a cathode-ray tube. In a typical cathode-ray tube, an electron "gun" arrangement is used to accelerate electrons from rest to the screen. The electrons are accelerated through a potential difference of 30.0 kV. (*a*) Which region is at a higher electric potential, the screen or the electron's starting location? Explain your answer. (*b*) What is the kinetic energy (in both eV and joules) of an electron as it reaches the screen?

Picture the Problem The work done on the electrons by the electric field changes their kinetic energy. Hence we can use the work-kinetic energy theorem to find the kinetic energy and the speed of impact of the electrons.

(*a*) Because positively charged objects are accelerated from higher potential to lower potential regions, the screen must be at the higher electric potential to accelerate electrons toward it.

(*b*) Use the work-kinetic energy theorem to relate the work done by the electric field to the change in the kinetic energy of the electrons:

$$W = \Delta K = K_f$$

or

$$K_f = e\Delta V \qquad (1)$$

Substitute numerical values and evaluate K_f:

$$\boldsymbol{K}_f = (1\boldsymbol{e})(30.0\,\text{kV}) = \boxed{3.00\times10^4\ \text{eV}}$$

Convert this energy to eV:

$$\boldsymbol{K}_f = (3.00\times10^4\ \text{eV})\left(\frac{1.602\times10^{-19}\ \text{J}}{\text{eV}}\right) = \boxed{4.81\times10^{-15}\ \text{J}}$$

25 ••• (*a*) A positively charged particle is on a trajectory to collide head-on with a massive positively charge nucleus that is initially at rest The particle initially has kinetic energy K_i. In addition, the particle is initially far from the nucleus. Derive an expression for the distance of closest approach. Your expression should be in terms of the initial kinetic energy *K* of the particle, the charge *ze* on the particle, and the charge *Ze* on the nucleus, where both *z* and *Z* are integers. (*b*) Find the numerical value for the distance of closest approach between a 5.00 MeV α-particle and between a 9.00 MeV α-particle and a stationary gold nucleus. (The values 5.00 MeV and 9.00 MeV are the initial kinetic energies of the alpha particles. Neglect the motion of the gold nucleus following the collisions.) (*c*) The radius of the gold nucleus is about 7×10^{-15} m. If α-particles approach the nucleus closer than 7×10^{-15} m, they experience the strong nuclear force in addition to the electric force of repulsion. In the early 20th century, before the strong nuclear force was known, Ernest Rutherford bombarded gold nuclei with α-particles that had kinetic energies of about 5 MeV. Would you expect this experiment to reveal the existence of this strong nuclear force? Explain your answer.

Picture the Problem We know that energy is conserved in the interaction between the α particle and the massive nucleus. Under the assumption that the recoil of the massive nucleus is negligible, we know that the initial kinetic energy of the α particle will be transformed into potential energy of the two-body system when the particles are at their distance of closest approach.

(*a*) Apply conservation of energy to the system consisting of the positively charged particle and the massive nucleus:

$$\Delta K + \Delta U = 0$$
or
$$K_f - K_i + U_f - U_i = 0$$

Because $K_f = U_i = 0$:

$$-K_i + U_f = 0$$

Letting r be the separation of the particles at closest approach, express U_f:

$$U_f = \frac{kq_{\text{nucleus}}q_{\text{particle}}}{r} = \frac{k(Ze)(ze)}{r} = \frac{kzZe^2}{r}$$

Substitute for U_f to obtain:

$$-K_i + \frac{kzZe^2}{r} = 0 \Rightarrow r = \boxed{\frac{kzZe^2}{K_i}}$$

(*b*) For a 5.00-MeV α particle and a stationary gold nucleus:

$$r_5 = \frac{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.602\times10^{-19}\ \text{C}\right)^2}{(5.00\,\text{MeV})\left(1.602\times10^{-19}\ \text{J/eV}\right)} = \boxed{46\ \text{fm}}$$

For a 9.00-MeV α particle and a stationary gold nucleus:

$$r_9 = \frac{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.602\times10^{-19}\ \text{C}\right)^2}{(9.00\,\text{MeV})\left(1.602\times10^{-19}\ \text{J/eV}\right)} = \boxed{25\ \text{fm}}$$

(*c*) No. The distance of closest approach for a 5-MeV alpha particle found above (46 fm) is much larger than the 7 fm radius of a gold nucleus. Hence the scattering was solely the result of the inverse-square Coulomb force.

Potential Due to a System of Point Charges

26 • Four point charges, each having a magnitude of 2.00 μC, are fixed at the corners of a square whose edges are 4.00-mlong. Find the electric potential at the center of the square if (*a*) all the charges are positive, (*b*) three of the charges are positive and one charge is negative, and (*c*) two charges are positive and two charges are negative. (Assume the potential is zero very far from all charges.)

Picture the Problem Let the numerals 1, 2, 3, and 4 denote the charges at the four corners of square and r the distance from each charge to the center of the square. The potential at the center of square is the algebraic sum of the potentials due to the four charges.

Express the potential at the center of the square:

$$V = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r} + \frac{kq_4}{r} = \frac{k}{r}(q_1 + q_2 + q_3 + q_4) = \frac{k}{r}\sum_{i=1}^{4} q_i$$

(*a*) If the charges are positive:

$$V = \frac{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{2.00\sqrt{2}\ \text{m}}(4)(2.00\ \mu\text{C}) = \boxed{25.4\,\text{kV}}$$

(*b*) If three of the charges are positive and one is negative:

$$V = \frac{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{2.00\sqrt{2}\ \text{m}}(2)(2.00\ \mu\text{C}) = \boxed{12.7\,\text{kV}}$$

(*c*) If two are positive and two are negative:

$$V = \boxed{0}$$

27 • [SSM] Three point charges are fixed at locations on the x-axis: q_1 is at $x = 0.00$ m, q_2 is at $x = 3.00$ m, and q_3 is at $x = 6.00$ m. Find the electric potential at the point on the y axis at $y = 3.00$ m if (*a*) $q_1 = q_2 = q_3 = +2.00\ \mu$C, (*b*) $q_1 = q_2 = +2.00\ \mu$C and $q_3 = -2.00\ \mu$C, and (*c*) $q_1 = q_3 = +2.00\ \mu$C and $q_2 = -2.00\ \mu$C. (Assume the potential is zero very far from all charges.)

Picture the Problem The potential at the point whose coordinates are (0, 3.00 m) is the algebraic sum of the potentials due to the charges at the three locations given.

Express the potential at the point whose coordinates are (0, 3.00 m):

$$V = k\sum_{i=1}^{3}\frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

(*a*) For $q_1 = q_2 = q_3 = 2.00\ \mu\text{C}$:

$$V = \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.00\ \mu\text{C})\left(\frac{1}{3.00\,\text{m}}+\frac{1}{3.00\sqrt{2}\ \text{m}}+\frac{1}{3.00\sqrt{5}\ \text{m}}\right)$$

$$= \boxed{12.9\,\text{kV}}$$

(*b*) For $q_1 = q_2 = 2.00\ \mu\text{C}$ and $q_3 = -2.00\ \mu\text{C}$:

$$\mathbf{V} = \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.00\ \mu\text{C})\left(\frac{1}{3.00\,\text{m}}+\frac{1}{3.00\sqrt{2}\ \text{m}}-\frac{1}{3.00\sqrt{5}\ \text{m}}\right)$$

$$= \boxed{7.55\,\text{kV}}$$

(*c*) For $q_1 = q_3 = 2.00\ \mu\text{C}$ and $q_2 = -2.00\ \mu\text{C}$:

$$\mathbf{V} = \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.00\ \mu\text{C})\left(\frac{1}{3.00\,\text{m}}-\frac{1}{3.00\sqrt{2}\ \text{m}}+\frac{1}{3.00\sqrt{5}\ \text{m}}\right)$$

$$= \boxed{4.43\,\text{kV}}$$

28 • Points *A*, *B*, and *C* are fixed at the vertices of an equilateral triangle whose edges are 3.00-m long. A point particle with a charge of +2.00 μC is fixed at each of vertices *A* and *B*. (*a*) What is the electric potential at point *C*? (Assume the potential is zero very far from all charges.) (*b*) How much work is required to move a point particle having a charge of +5.00 μC from a distance of infinity to point *C*? (*c*) How much additional work is required to move the +5.00-μC point particle from point *C* to the midpoint of side *AB*?

Picture the Problem (*a*) The potential at vertex *C* is the algebraic sum of the potentials due to the point charges at vertices *A* and *B*. (*b*) The work required to bring a charge from infinity to vertex *C* equals the change in potential energy of the system during this process. (*c*) The additional work required to move the +5.00-μC point particle from point *C* to the midpoint of side *AB* is the product of +5.00-μC and the difference in potential between point *C* and the midpoint of side *AB*.

(*a*) Express the potential at vertex *C* as the sum of the potentials due to the point charges at vertices *A* and *B*:

$$V_C = k\left(\frac{q_A}{r_A}+\frac{q_B}{r_B}\right)$$

Because $q_A = q_B = q_2$:

$$V_C = kq_2\left(\frac{1}{r_A} + \frac{1}{r_B}\right)$$

Substitute numerical values and evaluate V_C:

$$V_C = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(2.00\,\mu\mathrm{C}\right)\left(\frac{1}{3.00\,\mathrm{m}} + \frac{1}{3.00\,\mathrm{m}}\right)$$
$$= 11.98\ \mathrm{kV} = \boxed{12.0\,\mathrm{kV}}$$

(*b*) Express the required work in terms of the change in the potential energy of the system:

$$W_{\infty\to C} = \Delta U = U_C - U_\infty = U_C$$

Substituting for U_C yields:

$$W_{\infty\to C} = q_5 V_C$$

Substitute numerical values and evaluate $\boldsymbol{W_{\infty\to C}}$:

$$W_{\infty\to C} = (5.00\,\mu\mathrm{C})(11.98\,\mathrm{kV})$$
$$= \boxed{59.9\,\mathrm{mJ}}$$

(*c*) Express the required work in terms of the change in the potential energy of the system:

$$W_{C\to\mathrm{midpoint}} = \Delta U = U_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ AB}} - U_C$$

Substituting for $\boldsymbol{U}_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ \boldsymbol{AB}}}$ and U_C yields:

$$W_{C\to\mathrm{midpoint}} = q_5 V_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ AB}} - q_5 V_C$$
$$= q_5\left(V_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ AB}} - V_C\right) \qquad (1)$$

The potential at the midpoint of AB is the sum of the potentials due to the point charges at vertices A and B:

$$V_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ AB}} = k\left(\frac{q_A}{r_A} + \frac{q_B}{r_B}\right)$$

Because $q_A = q_B = q_2$ and $r_A = r_B = r$:

$$V_{\substack{\mathrm{midpoint}\\ \mathrm{of}\ AB}} = \frac{2kq_2}{r}$$

where r is the distance from vertex A (and vertex B) to the midpoint of side AB of the triangle.

Substituting in equation (1) and simplifying yields:

$$W_{C\to\mathrm{midpoint}} = q_5\left(\frac{2kq_2}{r} - V_C\right)$$

Substitute numerical values and evaluate $W_{C\to\text{midpoint}}$:

$$W_{C\to\text{midpoint}} = (+5.00\ \mu\text{C})\left(\frac{2(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(+2.00\ \mu\text{C})}{1.50\ \text{m}} - 11.98\ \text{kV}\right)$$
$$= \boxed{59.9\ \text{mJ}}$$

29 •• Three identical point particles with charge q are at the vertices of an equilateral triangle that is circumscribed by a circle of radius a that lies in the $z = 0$ plane and is centered at the origin. The values of q and a are +3.00 μC and 60.0 cm, respectively. (Assume the potential is zero very far from all charges.) (*a*) What is the electric potential at the origin? (*b*) What is the electric potential at the point on the z axis at $z = a$? (*c*) How would your answers to Parts (*a*) and (*b*) change if the charges were still on the circle but one is no longer at a vertex of the triangle? Explain your answer.

Picture the Problem The electric potential at the origin and at $z = a$ is the algebraic sum of the potentials at those points due to the individual charges distributed along the equator.

(*a*) Express the potential at the origin as the sum of the potentials due to the charges placed at 120° intervals along the equator of the sphere:

$$V_{\text{origin}} = k\sum_{i=1}^{3}\frac{q_i}{r_i} = 3\frac{kq}{a}$$

Substitute numerical values and evaluate V_{origin}:

$$V_{\text{origin}} = \frac{3(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\ \mu\text{C})}{0.600\ \text{m}}$$
$$= \boxed{135\ \text{kV}}$$

(*b*) Using geometry, find the distance from each charge to $z = a$:

$$\boldsymbol{a} = 0.600\sqrt{2}\ \text{m}$$

Proceed as in (*a*) with $a = 0.600\sqrt{2}\ \text{m}$:

$$V = k\sum_{i=1}^{3}\frac{q_i}{r_i'} = 3\frac{kq}{a'}$$
$$= \frac{3(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\ \mu\text{C})}{0.600\sqrt{2}\ \text{m}}$$
$$= \boxed{95.3\ \text{kV}}$$

(c) Because the two field points are equidistant from all points on the circle, the answers for Parts (a) and (b) would not change.

30 •• Two point charges q and q' are separated by a distance a. At a point $a/3$ from q and along the line joining the two charges the potential is zero. (Assume the potential is zero very far from all charges.) (a) Which of the following statements is true?

(1) The charges have the same sign.
(2) The charges have opposite signs.
(3) The relative signs of the charges can not be determined by using data given.

(b) Which of the following statements is true?

(1) $|\boldsymbol{q}| > |\boldsymbol{q}'|$.
(2) $|\boldsymbol{q}| < |\boldsymbol{q}'|$.
(3) $|\boldsymbol{q}| = |\boldsymbol{q}'|$.
(4) The relative magnitudes of the charges cannot be determined by using the data given.

(c) Find the ratio q/q'.

Picture the Problem We can use the fact that the electric potential at the point of interest is the algebraic sum of the potentials at that point due to the charges q and q' to find the ratio q/q'.

(a) The only way that, in the absence of other point charges, the potential can be zero at $a/3$ is if q and q' have opposite signs. $\boxed{2}$ is correct.

(b) Because the point of interest is closer to q, the magnitude of q must be less than the magnitude of q'. $\boxed{2}$ is correct.

(c) Express the potential at the point of interest as the sum of the potentials due to the two charges:

$$\frac{kq}{\frac{1}{3}a}+\frac{kq'}{\frac{2}{3}a}=0$$

Simplify to obtain:

$$q+\frac{q'}{2}=0 \Rightarrow \frac{q}{q'}=\boxed{-\frac{1}{2}}$$

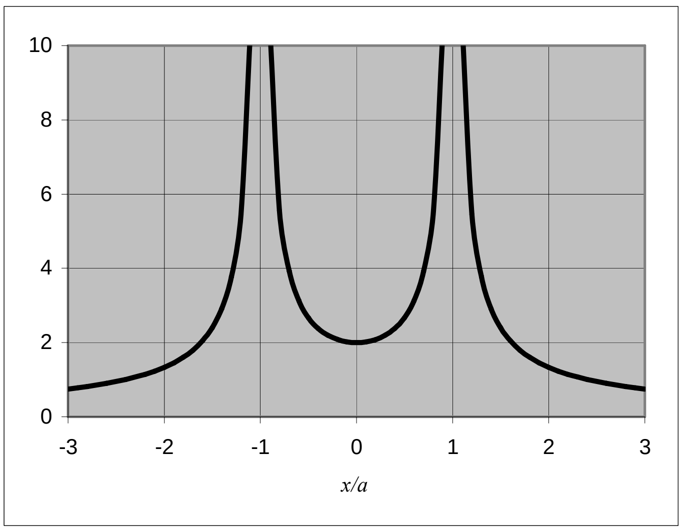
31 •• [SSM] Two identical positively charged point particles are fixed on the x-axis at $x = +a$ and $x = -a$. (a) Write an expression for the electric potential $V(x)$ as a function of x for all points on the x-axis. (b) Sketch $V(x)$ versus x for all points on the x axis.

Picture the Problem For the two charges, $r = |x - a|$ and $|x + a|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = +a$ and $x = -a$.

(a) Express $V(x)$ as the sum of the potentials due to the charges at $x = +a$ and $x = -a$:

$$V = \boxed{kq\left(\frac{1}{|x-a|} + \frac{1}{|x+a|}\right)}$$

(b) The following graph of V as a function of x/a was plotted using a spreadsheet program:



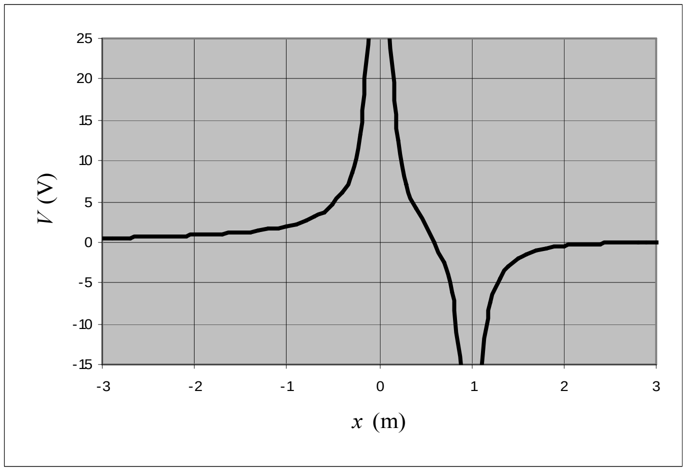
32 •• A point charge of $+3e$ is at the origin and a second point charge of $-2e$ is on the x-axis at $x = a$. (a) Sketch the potential function $V(x)$ versus x for all points on the x axis. (b) At what point or points, if any, is V= zero on the x axis? (c) What point or points, if any, on the x-axis is the electric field zero? Are these locations the same locations found in Part (b)? Explain your answer. (d) How much work is needed to bring a third charge $+e$ to the point $x = \frac{1}{2}a$ on the x-axis?

Picture the Problem For the two charges, $r = |x - a|$ and $|x|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = a$ and $x = 0$. We can use the graph and the function found in Part (a) to identify the points at which $V(x) = 0$. We can find the work needed to bring a third charge $+e$ to the point $x = \frac{1}{2}a$ on the x axis from the change in the potential energy of this third charge.

(a) The potential at x is the sum of the potentials due to the point charges $+3e$ and $-2e$:

$$V(x)=\frac{k(3e)}{|x|}+\frac{k(-2e)}{|x-a|}$$

The following graph of $V(x)$ for $ke = 1$ and $a = 1$ was plotted using a spreadsheet program.



(b) From the graph we can see that $V(x) = 0$ when:

$$x=\boxed{\pm\infty}$$

Examining the function, we see that $V(x)$ is also zero provided:

$$\frac{3}{|x|}-\frac{2}{|x-a|}=0$$

For $x > 0$, $V(x) = 0$ when:

$$x=\boxed{3a}$$

For $0 < x < a$, $V(x) = 0$ when:

$$x=\boxed{0.6a}$$

(c) The electric field at x is the sum of the electric fields due to the point charges $+3e$ and $-2e$:

$$E(x)=\frac{k(3e)}{x^2}+\frac{k(-2e)}{(x-r)^2}$$

Setting $E(x) = 0$ and simplifying yields:

$$x^2-6ax+3a^2=0$$

Solve this equation to find the points on the x-axis where the electric field is zero:

$\boldsymbol{x}=\boxed{5.4\boldsymbol{a}}$ and $\boldsymbol{x}=\boxed{0.55\boldsymbol{a}}$

Note that the zeros of the electric field are different from the zeros of the electric potential. This is generally the case although, in special cases, they can be the same.

(*d*) Express the work that must be done in terms of the change in potential energy of the charge:

$$W = \Delta U = qV\left(\tfrac{1}{2}a\right)$$

Evaluate the potential at $x = \frac{1}{2}a$:

$$V\left(\tfrac{1}{2}a\right) = \frac{k(3e)}{\left|\frac{1}{2}a\right|} + \frac{k(-2e)}{\left|\frac{1}{2}a - a\right|} = \frac{6ke}{a} - \frac{4ke}{a} = \frac{2ke}{a}$$

Substitute to obtain:

$$W = e\left(\frac{2ke}{a}\right) = \boxed{\frac{2ke^2}{a}}$$

33 ••• [SSM] A dipole consists of equal but opposite point charges $+q$ and $-q$. It is located so that its center is at the origin, and its axis is aligned with the z-axis (Figure 23-32) The distance between the charges is L. Let $\vec{r}$ be the vector from the origin to an arbitrary field point and θ be the angle that $\vec{r}$ makes with the $+z$ direction. (*a*) Show that at large distances from the dipole (that is for $r >> L$), the dipole's electric potential is given by $V(r,\theta) \approx k\vec{p}\cdot\hat{r}/r^2 = kp\cos\theta/r^2$, where $\vec{p}$ is the dipole moment of the dipole and θ is the angle between $\vec{r}$ and $\vec{p}$. (*b*) At what points in the region $r >> L$, other than at infinity, is the electric potential zero?

Picture the Problem The potential at the arbitrary field point is the sum of the potentials due to the equal but opposite point charges.

(*a*) Express the potential at the arbitrary field point at a large distance from the dipole:

$$V = V_+ + V_- = \frac{kq}{r_+} + \frac{k(-q)}{r_-} = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = kq\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

Referring to the figure, note that, for the far field ($r >> L$):

$$r_- - r_+ \approx L\cos\theta \text{ and } r_+ \approx r_- \approx r$$

Substituting and simplifying yields:

$$V(r,\theta) = kq\left(\frac{L\cos\theta}{r^2}\right) = \frac{kqL\cos\theta}{r^2}$$

or, because $p = qL$,

$$V(r,\theta)=\frac{kp\cos\theta}{r^2}$$

Finally, because $\vec{\boldsymbol{p}}\cdot\hat{\boldsymbol{r}}=\boldsymbol{p}\cos\boldsymbol{\theta}$:

$$\boldsymbol{V}(\boldsymbol{r},\boldsymbol{\theta})=\boxed{\frac{\boldsymbol{k}\vec{\boldsymbol{p}}\cdot\hat{\boldsymbol{r}}}{\boldsymbol{r}^2}}$$

(*b*) $\boldsymbol{V}(\boldsymbol{r},\boldsymbol{\theta})=0$ where $\cos\boldsymbol{\theta}=0$:

$\boldsymbol{\theta}=\cos^{-1}0=90°\Rightarrow V=0$ at points on the *z* axis. Note that these locations are equidistant from the two oppositely-charged ends of the dipole.

34 ••• A charge configuration consists of three point charges located on the *z* axis (Figure 23-33). One has a charge equal to $-2q$, and is located at the origin. The other two each have a charge equal to $+q$, one is located at $z=+L$ and the other is located at $z=-L$. This charge configuration can be modeled as two dipoles: one centered at $z=+L/2$ and with a dipole moment in the $+z$ direction, the other centered at $z=-L/2$ and with a dipole moment in the $-z$ direction. Each of these dipoles has a dipole moment that has a magnitude equal to qL. Two dipoles arranged in this fashion form a *linear electric quadrupole*. (There are other geometrical arrangements of dipoles that create quadrupoles but they are not linear.) (*a*) Using the result from Problem 33, show that at large distances from the quadrupole (that is for $r \gg L$), the electric potential is given by $V_{\text{quad}}(r,\theta)=2kB\cos^2\theta/r^3$, where $B=qL^2$. (*B* is the magnitude of the quadrupole moment of the charge configuration.) (*b*) Show that on the positive *z* axis, this potential gives an electric field (for $z \gg L$) of $\vec{E}=\left(6kB/z^4\right)\hat{k}$. (*c*) Show that you get the result of Part (*b*) by adding the electric fields from the three point charges.

Picture the Problem (*a*) The electric potential due to the linear electric quadrupole is the sum of the potentials of the two dipoles. (*b*) The electric field on the *y* axis can be obtained from the electric potential on the *y* axis using $\vec{\boldsymbol{E}}_{z\,\text{axis}}=-\frac{\partial\boldsymbol{V}_{z\,\text{axis}}}{\partial\boldsymbol{z}}\hat{\boldsymbol{k}}$.

(*a*) The electric potential due to the linear electric quadrupole is given by:

$$V_{\text{quad}}=V_{\uparrow}+V_{\downarrow}$$

From Problem 23-33:

$$V_{\uparrow}=\frac{kqL\cos\theta}{r_{\uparrow}^2}\text{ and }V_{\downarrow}=-\frac{kqL\cos\theta}{r_{\downarrow}^2}$$

Substituting for $V_\uparrow$ and $V_\downarrow$ yields:

$$V_{\text{quad}} = \frac{kqL\cos\theta}{r_\uparrow^2} - \frac{kqL\cos\theta}{r_\downarrow^2}$$

Simplify to obtain:

$$V_{\text{quad}} = kqL\cos\theta\left(\frac{r_\downarrow^2 - r_\uparrow^2}{r_\uparrow^2 r_\downarrow^2}\right) = kqL\cos\theta\left(\frac{(r_\downarrow - r_\uparrow)(r_\downarrow + r_\uparrow)}{r_\uparrow^2 r_\downarrow^2}\right)$$

Referring to the figure, note that, for the far field ($r >> L$):

$\boldsymbol{r}_\downarrow - \boldsymbol{r}_\uparrow \approx \boldsymbol{L}\cos\boldsymbol{\theta}$, $\boldsymbol{r}_\uparrow + \boldsymbol{r}_\downarrow \approx 2\boldsymbol{r}$, and $r_\uparrow \approx r_\downarrow \approx 2r$

Substitute and simplify to obtain:

$$V_{\text{quad}}(r,\theta) \approx kqL\cos\theta\left(\frac{2rL\cos\theta}{r^4}\right) = \frac{2kqL^2\cos^2\theta}{r^3}$$

Because $B = qL^2$:

$$V_{\text{quad}}(r,\theta) \approx \boxed{\frac{2kB\cos^2\theta}{r^3}}$$

(*b*) The electric field on the *z* axis is related to the electric potential on the *z* axis:

$$\vec{\boldsymbol{E}}_{z\text{ axis}} = -\frac{\partial \boldsymbol{V}_{z\text{ axis}}}{\partial \boldsymbol{z}}\hat{\boldsymbol{k}}$$

On the *y* axis, $\theta = 0$ and $\cos\boldsymbol{\theta} = 1$. Hence:

$$V_{y\text{ axis}} \approx \frac{2kB}{y^3}$$

Substituting for $V_{z\text{ axis}}$ yields:

$$\vec{\boldsymbol{E}}_{z\text{ axis}} = -\frac{\partial}{\partial \boldsymbol{z}}\left(\frac{2\boldsymbol{kB}}{\boldsymbol{z}^3}\right)\hat{\boldsymbol{k}} = \boxed{\frac{6\boldsymbol{kB}}{\boldsymbol{z}^4}\hat{\boldsymbol{k}}}$$

(*c*) The *E* field on the *z* axis is given by:

$$\vec{\boldsymbol{E}}_{z\text{ axis}} = \boldsymbol{kq}\left(\frac{1}{(\boldsymbol{z}-\boldsymbol{L})^2} - \frac{2}{\boldsymbol{z}^2} + \frac{1}{(\boldsymbol{z}+\boldsymbol{L})^2}\right)\hat{\boldsymbol{k}}$$

Letting $\boldsymbol{w} = \boldsymbol{L}^2/\boldsymbol{z}^2$ yields:

$$\vec{\boldsymbol{E}}_{z\text{ axis}} = \frac{\boldsymbol{kq}}{\boldsymbol{z}^2}\left(\frac{1}{(1-\boldsymbol{w})^2} - 2 + \frac{1}{(1+\boldsymbol{w})^2}\right)\hat{\boldsymbol{k}}$$

Expand $(1-w)^{-2}$ and $(1+w)^{-2}$ binomially to obtain:

$$(1-w)^{-2} = 1 + 2w + 3w^2 + \text{higher - order terms}$$

and

$$(1+w)^{-2} = 1 - 2w + 3w^2 + \text{higher - order terms}$$

For $w << 1$:

$$(1-w)^{-2} \approx 1 + 2w + 3w^2$$

and

$$(1+w)^{-2} \approx 1 - 2w + 3w^2$$

Substituting in the expression for $\vec{\boldsymbol{E}}_{z\,\text{axis}}$ and simplifying yields:

$$\vec{\boldsymbol{E}}_{z\,\text{axis}} \approx \frac{\boldsymbol{kq}}{\boldsymbol{z}^2}\left(1 + 2\boldsymbol{w} + 3\boldsymbol{w}^2 - 2 + 1 - 2\boldsymbol{w} + 3\boldsymbol{w}^2\right)\hat{\boldsymbol{k}} = \frac{\boldsymbol{kq}}{\boldsymbol{z}^2}\left(6\boldsymbol{w}^2\right)\hat{\boldsymbol{k}}$$

Finally, substitute for w to obtain:

$$\vec{\boldsymbol{E}}_{z\,\text{axis}} \approx \frac{\boldsymbol{kq}}{\boldsymbol{z}^2}\left(\frac{6\boldsymbol{L}^2}{\boldsymbol{z}^2}\right)\hat{\boldsymbol{k}} = \frac{6\boldsymbol{kqL}^2}{\boldsymbol{z}^4}\hat{\boldsymbol{k}} = \boxed{\frac{6\boldsymbol{kB}}{\boldsymbol{z}^4}\hat{\boldsymbol{k}}}$$

Computing the Electric Field from the Potential

35 • A uniform electric field is in the $-x$ direction. Points a and b are on the x-axis, with a at $x = 2.00$ m and b at $x = 6.00$ m. (*a*) Is the potential difference $V_b - V_a$ positive or negative? (*b*) If $|V_b - V_a|$ is 100 kV, what is the magnitude of the electric field?

Picture the Problem We can use the relationship $E_x = -(dV/dx)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta x$ to find E.

(*a*) Because $E_x = -(dV/dx)$, V is greater for larger values of x. So:

$$\boxed{V_b - V_a \text{ is positive.}}$$

(*b*) Express E in terms of $V_b - V_a$ and the separation of points a and b:

$$E_x = \frac{\Delta V}{\Delta x} = \frac{V_b - V_a}{\Delta x}$$

Substitute numerical values and evaluate E_x:

$$\boldsymbol{E}_x = \frac{100\,\text{kV}}{6.00\,\text{m} - 2.00\,\text{m}} = \boxed{25.0\text{ kV/m}}$$

36 • An electric field is given by the expression $\vec{E} = bx^3\hat{i}$, where $b = 2.00$ kV/m^4. Find the potential difference between the point at $x = 1.00$ m and the point at $x = 2.00$ m. Which of these points is at the higher potential?

Picture the Problem Because $V(x)$ and E_x are related through $E_x = -\,dV/dx$, we can find V from E by integration.

Separate variables in $E_x = -\,dV/dx$ and substitute for b to obtain:

$$dV = -E_x dx = -\left(2.00x^3\,\frac{\text{kV}}{\text{m}^4}\right)dx$$

Integrate V from V_1 to V_2 and x from $x = 1.00$ m to $x = 2.00$ m:

$$\int_{V_1}^{V_2} dV = V_2 - V_1$$

$$= -\left(2.00\,\frac{\text{kV}}{\text{m}^4}\right)\int_{1.00\,\text{m}}^{2.00\,\text{m}} x^3 dx$$

$$= -\left(2.00\,\frac{\text{kV}}{\text{m}^4}\right)\left[\tfrac{1}{4}x^4\right]_{1.00\,\text{m}}^{2.00\,\text{m}}$$

Simplify to obtain:

$$V_2 - V_1 = \boxed{-7.50\,\text{kV}}$$

Because $V_2 = V_1 + 7.50\,\text{kV}$, the point at $x = 2.00$ m is at the higher potential.

37 •• The electric field on the x axis due to a point charge fixed at the origin is given by $\vec{E} = \left(b/x^2\right)\hat{i}$, where $b = 6.00$ kV·m and $x \neq 0$. (*a*) Find the magnitude and sign of the point charge. (*b*) Find the potential difference between the points on the x-axis at $x = 1.00$ m and $x = 2.00$ m. Which of these points is at the higher potential.

Picture the Problem We can integrate $E_x = -dV_x/dx$ to obtain $V(x)$ and then use this function to find the electric potential difference between the given points.

(*a*) We know that this field is due to a point charge because it varies inversely with the square of the distance from the point charge. Because $\vec{E}_x$ is positive, the sign of the charge must be positive.

Because the given electric field is that due to a point charge, it follows that:

$$kq = 6.00\,\frac{\text{kV}}{\text{m}} \Rightarrow q = \frac{6.00\,\dfrac{\text{kV}}{\text{m}}}{k}$$

Substitute the numerical value of k and evaluate q:

$$q = \frac{6.00\,\dfrac{\text{kV}}{\text{m}}}{8.988\times10^{9}\,\dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = \boxed{668\,\text{nC}}$$

(b) The potential difference between the points on the x-axis at x = 1.00 m and x = 2.00 m is given by:

$$\Delta V = V_1 - V_2 \qquad (1)$$

From $E_x = -\dfrac{dV_x}{dx}$ we have:

$$\int_{\infty}^{x} dV = -\int_{\infty}^{x} E_x dx = -kq\int_{\infty}^{x} x^{-2} dx$$

or

$$V(x) - V(\infty) = \frac{kq}{x}$$

Letting $\boldsymbol{V}(\infty) = 0$ yields:

$$V(x) = \frac{kq}{x} = \frac{6.00\,\text{kV}\cdot\text{m}}{x}$$

Substituting in equation (1) yields:

$$\boldsymbol{V}_1 - \boldsymbol{V}_2 = \frac{6.00\,\text{kV}\cdot\text{m}}{1.00\,\text{m}} - \frac{6.00\,\text{kV}\cdot\text{m}}{2.00\,\text{m}}$$
$$= \boxed{3.00\,\text{kV}}$$

Because $\boldsymbol{V}_1 = \boldsymbol{V}_2 + 3.00\,\text{kV}$, the point at x = 2.00 m is at the higher potential.

38 •• The electric potential due to a particular charge distribution is measured at many points along the x-axis. A plot of this data is shown in Figure 23-34. At what location (or locations) is the x component of the electric field equal to zero? At this location (or these locations) is the potential also equal to zero? Explain your answer.

Picture the Problem Because $\boldsymbol{E_x = -dV/dx}$, we can find the point(s) at which $E_x = 0$ by identifying the values for x for which $dV/dx = 0$.

Examination of the graph indicates that $dV/dx = 0$ at approximately 4.5 m. Thus $E_x = 0$ at $\boldsymbol{x} \approx \boxed{4.5\,\text{m}}$. At this location, the potential is not zero. The electric field is zero when the slope of the potential function is zero.

Use the graph of $V(x)$ to estimate the negative of the slope at the given points:

$$E_x(1\,\mathrm{m}) = -\left.\frac{dV}{dx}\right|_{x=1\,\mathrm{m}} \approx \boxed{2\,\frac{\mathrm{V}}{\mathrm{m}}},$$

$$E_x(3\,\mathrm{m}) = -\left.\frac{dV}{dx}\right|_{x=3\,\mathrm{m}} \approx \boxed{0.3\,\frac{\mathrm{V}}{\mathrm{m}}},$$

and

$$E_x(7\,\mathrm{m}) = -\left.\frac{dV}{dx}\right|_{x=7\,\mathrm{m}} \approx \boxed{-0.5\,\frac{\mathrm{V}}{\mathrm{m}}}$$

39 •• Three identical point charges, each with a charge equal to q, lie in the xy plane. Two of the charges are on the y-axis at $y = -a$ and $y = +a$, and the third charge is on the x-axis at $x = a$. (*a*) Find the potential as a function of position along the x axis. (*b*) Use the Part (*a*) result to obtain an expression for $E_x(x)$, the x component of the electric field as a function of x, on. Check your answers to Parts (*a*) and (*b*) at the origin and as x approaches ∞ to see if they yield the expected results.

Picture the Problem Let r_1 be the distance from $(0, a)$ to $(x, 0)$, r_2 the distance from $(0, -a)$, and r_3 the distance from $(a, 0)$ to $(x, 0)$. We can express $V(x)$ as the sum of the potentials due to the charges at $(0, a)$, $(0, -a)$, and $(a, 0)$ and then find E_x from $-dV/dx$.

(*a*) Express $V(x)$ as the sum of the potentials due to the charges at (0, a), (0, $-a$), and (a, 0):

$$V(x) = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$$

where $q_1 = q_2 = q_3 = q$

At $x = 0$, the fields due to q_1 and q_2 cancel, so $E_x(0) = -kq/a^2$; this is also obtained from (*b*) if $x = 0$.

As $x\to\infty$, i.e., for $x >> a$, the three charges appear as a point charge $3q$, so $E_x = 3kq/x^2$; this is also the result one obtains from (*b*) for $x >> a$.

Substitute for the r_i to obtain:

$$V(x) = kq\left(\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right) = \boxed{kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{|x-a|}\right) \qquad x \neq a}$$

(*b*) For $x > a$, $x - a > 0$ and:

$$|x-a| = x - a$$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{x-a}\right)\right] = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}} + \frac{kq}{(x-a)^2} \quad x > a}$$

For $x < a$, $x - a < 0$ and: $|x-a| = -(x-a) = a - x$

Use $E_x = -dV/dx$ to find E_x:

$$E_x(x) = -\frac{d}{dx}\left[kq\left(\frac{2}{\sqrt{x^2+a^2}} + \frac{1}{a-x}\right)\right] = \boxed{\frac{2kqx}{\left(x^2+a^2\right)^{3/2}} - \frac{kq}{(a-x)^2} \quad x < a}$$

Calculations of *V* for Continuous Charge Distributions

40 • A charge of +10.0 μC is uniformly distributed on a thin spherical shell of radius 12.0 cm. (Assume the potential is zero very far from all charges.) (*a*) What is the magnitude of the electric field just outside and just inside the shell? (*b*) What is the magnitude of the electric potential just outside and just inside the shell? (*c*) What is the electric potential at the center of the shell? (*d*) What is the magnitude of the electric field at the center of the shell?

Picture the Problem We can construct Gaussian surfaces just inside and just outside the spherical shell and apply Gauss's law to find the electric field at these locations. We can use the expressions for the electric potential inside and outside a spherical shell to find the potential at these locations.

(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r < 12.0$ cm:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

because the charge resides on the outer surface of the spherical surface. Hence $\vec{E}(r < 12.0\,\text{cm}) = \boxed{0}$

Apply Gauss's law to a spherical Gaussian surface of radius $r > 12.0$ cm:

$$E\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

and

$$E(r > 12.0\,\text{cm}) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{kq}{r^2}$$

Substitute numerical values and evaluate $\boldsymbol{E}(\boldsymbol{r} > 12.0\,\text{cm})$:

$$\boldsymbol{E}(\boldsymbol{r} > 12.0\,\text{cm}) = \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10.0\,\mu\text{C})}{(0.120\,\text{m})^2} = \boxed{6.24\,\text{MV/m}}$$

(*b*) Express and evaluate the potential just inside the spherical shell:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10.0\,\mu\text{C})}{0.120\,\text{m}} = \boxed{749\,\text{kV}}$$

Express and evaluate the potential just outside the spherical shell:

$$V(r \ge R) = \frac{kq}{r} = \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10.0\,\mu\text{C})}{0.120\,\text{m}} = \boxed{749\,\text{kV}}$$

(*c*) The electric potential inside a uniformly charged spherical shell is constant and given by:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10.0\,\mu\text{C})}{0.120\,\text{m}} = \boxed{749\,\text{kV}}$$

(*d*) In Part (*a*) we showed that: $\vec{\boldsymbol{E}}(\boldsymbol{r} < 12.0\,\text{cm}) = \boxed{0}$

41 • [SSM] An infinite line charge of linear charge density +1.50 μC/m lies on the *z*-axis. Find the electric potential at distances from the line charge of (*a*) 2.00 m, (*b*) 4.00 m, and (*c*) 12.0 m. Assume that we choose $V = 0$ at a distance of 2.50 m from the line of charge.

Picture the Problem We can use the expression for the potential due to a line charge $V(r) = -2k\lambda\ln\left(\frac{r}{a}\right)$, where $V = 0$ at some distance $r = a$, to find the potential at these distances from the line.

Express the potential due to a line charge as a function of the distance from the line:

$$V(r) = -2k\lambda\ln\left(\frac{r}{a}\right)$$

Because $V = 0$ at $r = 2.50$ m:

$$0 = -2\boldsymbol{k\lambda}\ln\left(\frac{2.50\,\text{m}}{\boldsymbol{a}}\right)$$

$$\Rightarrow 0 = \ln\left(\frac{2.50\,\text{m}}{\boldsymbol{a}}\right)$$

and

$$\frac{2.50\,\text{m}}{\boldsymbol{a}} = \ln^{-1}(0) = 1 \Rightarrow a = 2.50\text{ m}$$

Thus we have $a = 2.50$ m and:

$$V(r) = -2\left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.5\,\frac{\mu\text{C}}{\text{m}}\right)\ln\left(\frac{r}{2.50\,\text{m}}\right)$$
$$= -\left(2.696\times10^{4}\text{ N}\cdot\text{m/C}\right)\ln\left(\frac{r}{2.50\,\text{m}}\right)$$

(*a*) Evaluate $V(2.00\text{ m})$:

$$V(2.00\,\text{m}) = -\left(2.696\times10^{4}\,\frac{\text{N}\cdot\text{m}}{\text{C}}\right)$$
$$\times\ln\left(\frac{2.00\,\text{m}}{2.50\,\text{m}}\right)$$
$$= \boxed{6.02\,\text{kV}}$$

(*b*) Evaluate $V(4.00\text{ m})$:

$$V(4.00\,\text{m}) = -\left(2.696\times10^{4}\,\frac{\text{N}\cdot\text{m}}{\text{C}}\right)$$
$$\times\ln\left(\frac{4.00\,\text{m}}{2.50\,\text{m}}\right)$$
$$= \boxed{-12.7\,\text{kV}}$$

(*c*) Evaluate $V(12.0\text{ m})$:

$$V(12.0\,\text{m}) = -\left(2.696\times10^{4}\,\frac{\text{N}\cdot\text{m}}{\text{C}}\right)$$
$$\times\ln\left(\frac{12.0\,\text{m}}{2.50\,\text{m}}\right)$$
$$= \boxed{-42.3\,\text{kV}}$$

42 • (*a*) Find the maximum net charge that can be placed on a spherical conductor of radius 16 cm before dielectric breakdown of the air occurs. (*b*) What is the electric potential of the sphere when it has this maximum charge? (Assume the potential is zero very far from all charges.)

Picture the Problem We can relate the dielectric strength of air (about 3 MV/m) to the maximum net charge that can be placed on a spherical conductor using the expression for the electric field at its surface. We can find the potential of the sphere when it carries its maximum charge using $V = kQ_{\text{max}}/R$.

(*a*) Express the dielectric strength of a spherical conductor in terms of the charge on the sphere:

$$E_{\text{breakdown}} = \frac{kQ_{\text{max}}}{R^2} \Rightarrow Q_{\text{max}} = \frac{E_{\text{breakdown}}R^2}{k}$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{\text{max}} = \frac{(3\,\text{MV/m})(0.16\,\text{m})^2}{8.988\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 8.545\,\mu\text{C}$$
$$\approx \boxed{9\,\mu\text{C}}$$

(*b*) Because the charge carried by the sphere could be either positive or negative:

$$V_{\text{max}} = \pm\frac{kQ_{\text{max}}}{R}$$
$$= \pm\frac{\left(8.988\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(8.545\,\mu\text{C})}{0.16\,\text{m}}$$
$$= \boxed{\pm5\times10^5\ \text{V}}$$

43 • Find the maximum surface charge density σ_{max} that can exist on the surface of any conductor before dielectric breakdown of the air occurs.

Picture the Problem We can solve the equation giving the electric field at the surface of a conductor for the greatest surface charge density that can exist before dielectric breakdown of the air occurs.

Relate the electric field at the surface of a conductor to the surface charge density:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ under dielectric breakdown of the air conditions:

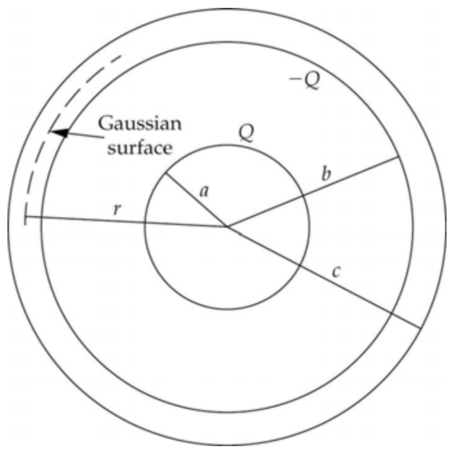
$$\sigma_{\text{max}} = \epsilon_0\, E_{\text{breaddown}}$$

Substitute numerical values and evaluate σ_{max}:

$$\sigma_{\text{max}} = \left(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(3\,\text{MV/m})$$
$$\approx \boxed{3\times10^{-5}\,\text{C/m}^2}$$

44 •• A conducting spherical shell of inner radius b and outer radius c is concentric with a small metal sphere of radius $a < b$. The metal sphere has a positive charge Q. The total charge on the conducting spherical shell is $-Q$. (Assume the potential is zero very far from all charges.) (*a*) What is the electric potential of the spherical shell? (*b*) What is the electric potential of the metal sphere?

Picture the Problem The diagram is a cross-sectional view showing the charges on the sphere and the spherical conducting shell. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $r > b$ is also shown. For $a < r < b$, the sphere acts like point charge Q and the potential of the metal sphere is the sum of the potential due to a point charge at its center and the potential at its surface due to the charge on the inner surface of the spherical shell.



(*a*) Express $V_{r>b}$:

$$V_{r>b} = -\int E_{r>b}\,dr$$

Apply Gauss's law for $r > b$:

$$\oint_S \vec{E}_r \cdot \hat{n}\,dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r>b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r > b$.

Substitute for $E_{r>b}$ to obtain:

$$V_{r>b} = -\int (0)\,dr = \boxed{0}$$

(*b*) Express the potential of the metal sphere:

$$V_a = V_{Q\text{ at its center}} + V_{\text{surface}}$$

Express the potential at the surface of the metal sphere:

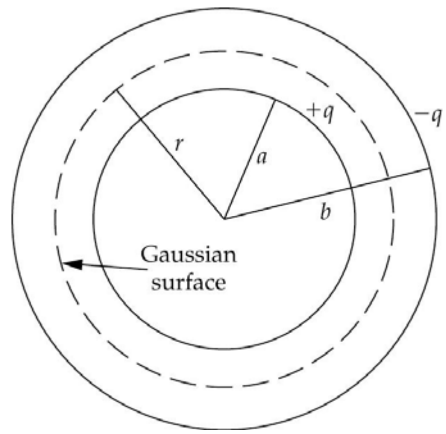
$$V_{\text{surface}} = \frac{k(-Q)}{b} = -\frac{kQ}{b}$$

Substitute and simplify to obtain:

$$V_a = \frac{kQ}{a} - \frac{kQ}{b} = \boxed{kQ\left(\frac{1}{a} - \frac{1}{b}\right)}$$

45 •• **[SSM]** Two coaxial conducting cylindrical shells have equal and opposite charges. The inner shell has charge $+q$ and an outer radius a, and the outer shell has charge $-q$ and an inner radius b. The length of each cylindrical shell is L, and L is very long compared with b. Find the potential difference, $V_a - V_b$ between the shells.

Picture the Problem The diagram is a cross-sectional view showing the charges on the inner and outer conducting shells. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $a < r < b$ is also shown. Once we've determined how E varies with r, we can find $V_a - V_b$ from $V_b - V_a = -\int E_r dr$.



Express the potential difference $V_b - V_a$:

$$V_b - V_a = -\int E_r dr \Rightarrow \boldsymbol{V_a - V_b = \int E_r dr}$$

Apply Gauss's law to cylindrical Gaussian surface of radius r and length L:

$$\oint_S \vec{E} \cdot \hat{n} dA = E_r(2\pi rL) = \frac{q}{\epsilon_0}$$

Solving for E_r yields:

$$E_r = \frac{q}{2\pi \epsilon_0 rL}$$

Substitute for E_r and integrate from $r = a$ to b:

$$V_a - V_b = \frac{q}{2\pi \epsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \boxed{\frac{2kq}{L}\ln\left(\frac{b}{a}\right)}$$

46 •• Positive charge is placed on two conducting spheres that are very far apart and connected by a long very-thin conducting wire. The radius of the smaller sphere is 5.00 cm and the radius of the larger sphere is 12.0 cm. The electric field strength at the surface of the larger sphere is 200 kV/m. Estimate the surface charge density on each sphere.

Picture the Problem Let L and S refer to the larger and smaller spheres, respectively. We can use the fact that both spheres are at the same potential to estimate the electric fields near their surfaces. Knowing the electric fields, we can use $\sigma = \epsilon_0 E$ to estimate the surface charge density of each sphere.

Express the electric fields at the surfaces of the two spheres:

$$E_S = \frac{kQ_S}{R_S^2} \text{ and } E_L = \frac{kQ_L}{R_L^2}$$

Divide the first of these equations by the second to obtain:

$$\frac{E_S}{E_L} = \frac{\frac{kQ_S}{R_S^2}}{\frac{kQ_L}{R_L^2}} = \frac{Q_S R_L^2}{Q_L R_S^2}$$

Because the potentials are equal at the surfaces of the spheres:

$$\frac{kQ_L}{R_L} = \frac{kQ_S}{R_S} \text{ and } \frac{Q_S}{Q_L} = \frac{R_S}{R_L}$$

Substitute for $\frac{Q_S}{Q_L}$ to obtain:

$$\frac{E_S}{E_L} = \frac{R_S R_L^2}{R_L R_S^2} = \frac{R_L}{R_S} \Rightarrow E_S = \frac{R_L}{R_S} E_L$$

Substitute numerical values and evaluate E_S:

$$E_S = \frac{12.0\,\text{cm}}{5.00\,\text{cm}}(200\,\text{kV/m}) = 480\,\text{kV/m}$$

Use $\sigma = \epsilon_0 E$ to estimate the surface charge density of each sphere:

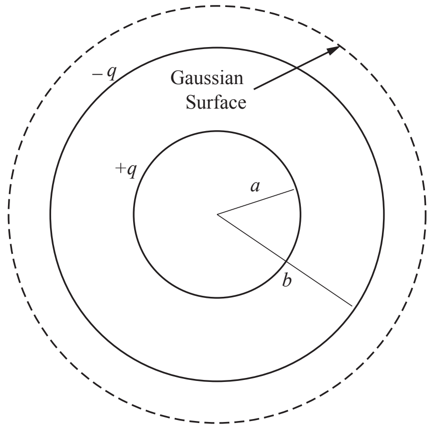
$$\boldsymbol{\sigma}_{12\,\text{cm}} = \epsilon_0 \boldsymbol{E}_{12\,\text{cm}} = \left(8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(200\,\text{kV/m}) = \boxed{1.77\,\mu\text{C/m}^2}$$

and

$$\boldsymbol{\sigma}_{5\,\text{cm}} = \epsilon_0 \boldsymbol{E}_{5\,\text{cm}} = \left(8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(480\,\text{kV/m}) = \boxed{4.25\,\mu\text{C/m}^2}$$

47 •• Two concentric conducting spherical shells have equal and opposite charges. The inner shell has outer radius a and charge $+q$; the outer shell has inner radius b and charge $-q$. Find the potential difference, $V_a - V_b$ between the shells.

Picture the Problem The diagram is a cross-sectional view showing the charges on the concentric spherical shells. The Gaussian surface over which we'll integrate E in order to find V in the region $r \geq b$ is also shown. We'll also find E in the region for which $a < r < b$. We can then use the relationship $V = -\int E dr$ to find V_a and V_b and their difference.



Express V_b:

$$V_b = -\int_{\infty}^{b} E_{r\geq b}\,dr$$

Apply Gauss's law for $r \geq b$:

$$\oint_S \vec{\boldsymbol{E}}_r \cdot \hat{\boldsymbol{n}}\,dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r\geq b} = 0$ because $\boldsymbol{Q}_{\text{enclosed}} = 0$ for $r \geq b$.

Substitute for $E_{r\geq b}$ to obtain:

$$V_b = -\int_{\infty}^{b} (0)\,dr = 0$$

Express V_a:

$$V_a = -\int_{b}^{a} E_{r\geq a}\,dr$$

Apply Gauss's law for $r \geq a$:

$$E_{r\geq a}\left(4\pi\, r^2\right) = \frac{q}{\epsilon_0}$$

and

$$E_{r\geq a} = \frac{q}{4\pi \epsilon_0\, r^2} = \frac{kq}{r^2}$$

Substitute for $E_{r\geq a}$ to obtain:

$$V_a = -kq\int_{b}^{a} \frac{dr}{r^2} = \frac{kq}{a} - \frac{kq}{b}$$

The potential difference between the shells is given by:

$$V_a - V_b = V_a = \boxed{kq\left(\frac{1}{a} - \frac{1}{b}\right)}$$

48 •• The electric potential at the surface of a uniformly charged sphere is 450 V. At a point outside the sphere at a (radial) distance of 20.0 cm from its surface, the electric potential is 150 V. (The potential is zero very far from the sphere.) What is the radius of the sphere, and what is the charge of the sphere?

Picture the Problem Let R be the radius of the sphere and Q its charge. We can express the potential at the two locations given and solve the resulting equations simultaneously for R and Q.

Relate the potential of the sphere at its surface to its radius:

$$\frac{kQ}{R} = 450\,\text{V} \qquad (1)$$

Express the potential at a distance of 20.0 cm from its surface:

$$\frac{kQ}{R + 0.200\,\text{m}} = 150\,\text{V} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\dfrac{kQ}{R}}{\dfrac{kQ}{R + 0.200\,\text{m}}} = \frac{450\,\text{V}}{150\,\text{V}}$$

or

$$\frac{R + 0.200\,\text{m}}{R} = 3 \Rightarrow R = \boxed{10.0\,\text{cm}}$$

Solving equation (1) for Q yields:

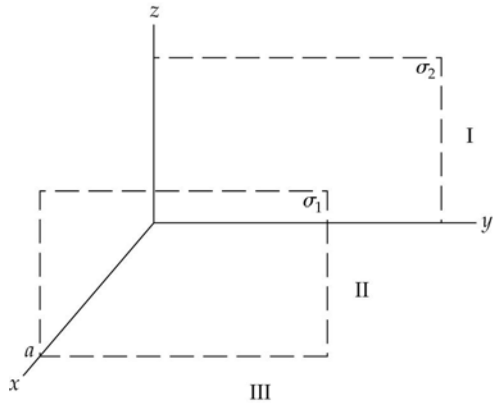
$$Q = (450\,\text{V})\frac{R}{k}$$

Substitute numerical values and evaluate Q:

$$\boldsymbol{Q} = (450\,\text{V})\frac{(0.100\,\text{m})}{\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{5.01\,\text{nC}}$$

49 •• Consider two infinite parallel thin sheets of charge, one in the $x = 0$ plane and the other in the $x = a$ plane. The potential is zero at the origin. (*a*) Find the electric potential everywhere in space if the planes have equal positive charge densities $+\sigma$. (*b*) Find the electric potential everywhere in space if the sheet in the $x = 0$ plane has a charge density $+\sigma$ and the sheet in the $x = a$ plane has a charge density $-\sigma$.

Picture the Problem Let the charge density on the infinite plane at $x = a$ be σ_1 and that on the infinite plane at $x = 0$ be σ_2. Call that region in space for which $x < 0$, region I, the region for which $0 < x < a$, region II, and the region for which $a < x$, region III. We can integrate E due to the planes of charge to find the electric potential in each of these regions.



(*a*) Express the potential in region I in terms of the electric field in that region:

$$V_{\mathrm{I}} = -\int_0^x \vec{\boldsymbol{E}}_{\mathrm{I}} \cdot d\vec{\boldsymbol{x}}$$

Express the electric field in region I as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{\boldsymbol{E}}_{\mathrm{I}} = -\frac{\sigma_1}{2\in_0}\hat{\boldsymbol{i}} - \frac{\sigma_2}{2\in_0}\hat{\boldsymbol{i}}$$
$$= -\frac{\sigma}{2\in_0}\hat{\boldsymbol{i}} - \frac{\sigma}{2\in_0}\hat{\boldsymbol{i}} = -\frac{\sigma}{\in_0}\hat{\boldsymbol{i}}$$

Substitute for $\vec{\boldsymbol{E}}_{\mathrm{I}}$ and evaluate V_{I}:

$$V_{\mathrm{I}} = -\int_0^x \left(-\frac{\sigma}{\in_0}\right)dx = \frac{\sigma}{\in_0}x + V(0)$$
$$= \frac{\sigma}{\in_0}x + 0 = \boxed{\frac{\sigma}{\in_0}x}$$

Express the potential in region II in terms of the electric field in that region:

$$V_{\mathrm{II}} = -\int \vec{\boldsymbol{E}}_{\mathrm{II}} \cdot d\vec{\boldsymbol{x}} + V(0)$$

Express the electric field in region II as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{\boldsymbol{E}}_{\mathrm{II}} = -\frac{\sigma_1}{2\in_0}\hat{\boldsymbol{i}} + \frac{\sigma_2}{2\in_0}\hat{\boldsymbol{i}}$$
$$= -\frac{\sigma}{2\in_0}\hat{\boldsymbol{i}} + \frac{\sigma}{2\in_0}\hat{\boldsymbol{i}} = 0$$

Substitute for $\vec{\boldsymbol{E}}_{\mathrm{II}}$ and evaluate V_{II}:

$$V_{\mathrm{II}} = -\int_0^x (0)dx = 0 + V(0) = \boxed{0}$$

Express the potential in region III in terms of the electric field in that region:

$$V_{\mathrm{III}} = -\int_a^x \vec{\boldsymbol{E}}_{\mathrm{III}} \cdot d\vec{\boldsymbol{x}}$$

Express the electric field in region III as the sum of the fields due to the charge densities σ_1 and σ_2:

$$\vec{E}_{\mathrm{III}} = \frac{\sigma_1}{2\in_0}\hat{i} + \frac{\sigma_2}{2\in_0}\hat{i} = \frac{\sigma}{2\in_0}\hat{i} + \frac{\sigma}{2\in_0}\hat{i} = \frac{\sigma}{\in_0}\hat{i}$$

Substitute for $\vec{E}_{\mathrm{III}}$ and evaluate V_{III}:

$$V_{\mathrm{III}} = -\int_a^x \left(\frac{\sigma}{\in_0}\right)dx = -\frac{\sigma}{\in_0}x + \frac{\sigma}{\in_0}a = \boxed{-\frac{\sigma}{\in_0}(x-a)}$$

(*b*) Proceed as in (*a*) with $\sigma_1 = -\sigma$ and $\sigma_2 = \sigma$ to obtain:

$V_{\mathrm{I}} = \boxed{0}$, $V_{\mathrm{II}} = \boxed{-\frac{\sigma}{\in_0}x}$, and

$V_{\mathrm{III}} = \boxed{0}$

These results are summarized in the following table:

Region	$x \le 0$	$0 \le x \le a$	$x \ge a$
Part (*a*)	$\frac{\sigma}{\in_0}x$	0	$-\frac{\sigma}{\in_0}(x-a)$
Part (*b*)	0	$-\frac{\sigma}{\in_0}x$	0

50 ••• The expression for the potential along the axis of a thin uniformly charge disk is given by $V = 2\pi k\sigma|z|\left(\sqrt{1+\frac{R^2}{z^2}}-1\right)$ (Equation 23-20), where R and σ are the radius and the charge per unit area of the disk, respectively. Show that this expression reduces to $V = kQ/|z|$ for $|\boldsymbol{z}| >> \boldsymbol{R}$, where $Q = \sigma\pi R^2$ is the total charge on the disk. Explain why this result is expected. *Hint: Use the binomial theorem to expand the radical.*

Picture the Problem

Expand the radical expression binomially to obtain:

$$\sqrt{1+\frac{R^2}{z^2}} = 1+\left(\frac{1}{2}\right)\left(\frac{R^2}{z^2}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{R^2}{z^2}\right)^2 + \text{higher order terms}$$

For $|z| >> R$:

$$\sqrt{1+\frac{R^2}{z^2}} \approx 1+\left(\frac{1}{2}\right)\left(\frac{R^2}{z^2}\right)$$

and

$$\sqrt{1+\frac{R^2}{z^2}} - 1 \approx \frac{R^2}{2z^2}$$

Substituting in Equation 23-20 yields:

$$V = 2\pi k\sigma|z|\left(\frac{R^2}{2z^2}\right) = \pi k\sigma|z|\left(\frac{R^2}{z^2}\right)$$

The total charge on the disk is given by:

$$Q = \sigma\pi R^2 \Rightarrow \sigma = \frac{Q}{\pi R^2}$$

Substitute for σ and simplify to obtain:

$$V = \pi k\left(\frac{Q}{\pi R^2}\right)|z|\left(\frac{R^2}{z^2}\right) = \frac{k|z|Q}{z^2}$$

If $\mathbf{z} \geq 0$, then $|\mathbf{z}| = \mathbf{z}$ and:

$$V = \frac{kzQ}{z^2} = \frac{kQ}{z} = \frac{kQ}{|z|}$$

If $\mathbf{z} < 0$, then $|\mathbf{z}| = -\mathbf{z}$ and:

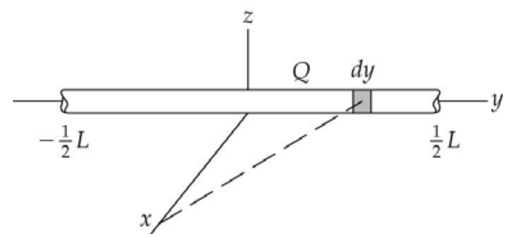
$$V = \frac{k(-z)Q}{z^2} = \frac{kQ}{-z} = \frac{kQ}{|z|}$$

Thus, for $|z| >> R$, Equation 23-20 reduces to:

$$V = \boxed{\frac{kQ}{|z|}}$$

51 •• **[SSM]** A rod of length L has a total charge Q uniformly distributed along its length. The rod lies along the y-axis with its center at the origin. (*a*) Find an expression for the electric potential as a function of position along the x-axis. (*b*) Show that the result obtained in Part (*a*) reduces to $V = kQ/|x|$ for $|x| >> L$. Explain why this result is expected.

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to the charge element λdy and integrate to find $V(x, 0)$.



(*a*) Express the element of potential dV due to the line element dy:

$$dV = \frac{k\lambda}{r}dy$$

where $r = \sqrt{x^2 + y^2}$ and $\lambda = \frac{Q}{L}$.

Substituting for r and λ yields:

$$dV = \frac{kQ}{L}\frac{dy}{\sqrt{x^2 + y^2}}$$

Use a table of integrals to integrate dV from $y = -L/2$ to $y = L/2$:

$$V(x,0) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{\sqrt{x^2 + y^2}}$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{\sqrt{x^2 + \frac{1}{4}L^2} + \frac{1}{2}L}{\sqrt{x^2 + \frac{1}{4}L^2} - \frac{1}{4}L^2}\right)}$$

(*b*) Factor x from the numerator and denominator within the parentheses to obtain:

$$V(x,0) = \frac{kQ}{L}\ln\left(\frac{\sqrt{1 + \frac{L^2}{4x^2}} + \frac{L}{2x}}{\sqrt{1 + \frac{L^2}{4x^2}} - \frac{L}{2x}}\right)$$

Use $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ to obtain:

$$V(x,0) = \frac{kQ}{L}\left\{\ln\left(\sqrt{1 + \frac{L^2}{4x^2}} + \frac{L}{2x}\right) - \ln\left(\sqrt{1 + \frac{L^2}{4x^2}} - \frac{L}{2x}\right)\right\}$$

Let $\varepsilon = \frac{L^2}{4x^2}$ and use $(1 + \varepsilon)^{1/2} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \ldots$ to expand $\sqrt{1 + \frac{L^2}{4x^2}}$:

$$\left(1 + \frac{L^2}{4x^2}\right)^{1/2} = 1 + \frac{1}{2}\frac{L^2}{4x^2} - \frac{1}{8}\left(\frac{L^2}{4x^2}\right)^2 + \ldots \approx 1 \text{ for } |\boldsymbol{x}| >> \boldsymbol{L}.$$

Substitute for $\left(1+\frac{L^2}{4x^2}\right)^{1/2}$ to obtain:

$$V(x,0)=\frac{kQ}{L}\left\{\ln\left(1+\frac{L}{2x}\right)-\ln\left(1-\frac{L}{2x}\right)\right\}$$

Let $\delta=\frac{L}{2x}$ and use $\ln(1+\delta)=\delta-\frac{1}{2}\delta^2+\ldots$ to expand $\ln\left(1\pm\frac{L}{2x}\right)$:

$$\ln\left(1+\frac{L}{2x}\right)\approx\frac{L}{2x}-\frac{L^2}{4x^2}\text{ and }\ln\left(1-\frac{L}{2x}\right)\approx-\frac{L}{2x}-\frac{L^2}{4x^2}\text{ for }x>>L.$$

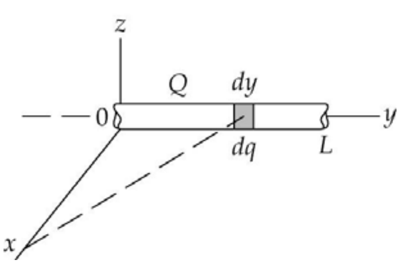
Substitute for $\ln\left(1+\frac{L}{2x}\right)$ and $\ln\left(1-\frac{L}{2x}\right)$ and simplify to obtain:

$$V(x,0)=\frac{kQ}{L}\left\{\frac{L}{2x}-\frac{L^2}{4x^2}-\left(-\frac{L}{2x}-\frac{L^2}{4x^2}\right)\right\}=\boxed{\frac{kQ}{x}}$$

Because, for $|x|>>\boldsymbol{L}$, the charge carried by the rod is far enough away from the point of interest to look like a point charge, this result is what we would expect.

52 •• A rod of length L has a charge Q uniformly distributed along its length. The rod lies along the y-axis with one end at the origin. (*a*) Find an expression for the electric potential as a function of position along the x-axis. (*b*) Show that the result obtained in Part (*a*) reduces to $V=kQ/|x|$ for $|x|>>\boldsymbol{L}$. Explain why this result is expected.

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy. We can express the potential dV at any point on the x axis due to λdy and integrate to find $V(x, 0)$.



(*a*) Express the element of potential dV due to the line element dy:

$$dV=\frac{k\lambda}{r}dy$$

where $r=\sqrt{x^2+y^2}$ and $\lambda=\frac{Q}{L}$.

Substituting for r and λ yields:

$$dV = \frac{kQ}{L}\frac{dy}{\sqrt{x^2+y^2}}$$

Use a table of integrals to integrate dV from $y = 0$ to $y = L$:

$$\begin{aligned} V(x,0) &= \frac{kQ}{L}\int_0^L \frac{dy}{\sqrt{x^2+y^2}} \\ &= \frac{kQ}{L}\ln\left(y+\sqrt{x^2+y^2}\right)\Big|_0^L \\ &= \frac{kQ}{L}\left[\ln\left(L+\sqrt{x^2+L^2}\right)-\ln(x)\right] \end{aligned}$$

Because $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$:

$$V(x,0) = \boxed{\frac{kQ}{L}\left[\ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right)\right]} \quad (1)$$

(b) Factor x^2 under the radical to obtain:

$$\ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right) = \ln\left(\frac{L+\sqrt{x^2\left(1+\left(\frac{L}{x}\right)^2\right)}}{x}\right) = \ln\left(\frac{L+\sqrt{x^2}\sqrt{\left(1+\left(\frac{L}{x}\right)^2\right)}}{x}\right)$$

Because $|x| >> L$:

$$\begin{aligned} \ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right) &\approx \ln\left(\frac{L+\sqrt{x^2}}{x}\right) \\ &= \ln\left(1+\frac{L}{x}\right) \end{aligned}$$

Expanding $\ln\left(1+\frac{L}{x}\right)$ binomially yields:

$$\ln\left(1+\frac{L}{x}\right) = \frac{L}{x} - \frac{1}{2}\left(\frac{L}{x}\right)^2 + \text{higher order terms}$$

Again, because $|\boldsymbol{x}| >> \boldsymbol{L}$:

$$\ln\left(1+\frac{L}{x}\right) \approx \frac{L}{x}$$

Substitute in equation (2) to obtain:

$$\ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right) \approx \frac{L}{x}$$

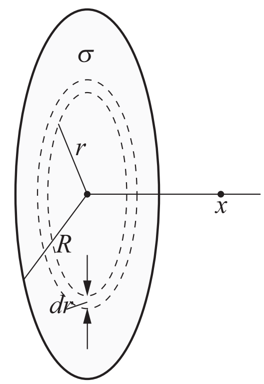
Finally, substituting in equation (1) yields:

$$V(x,0)=\frac{kQ}{L}\left[\frac{L}{x}\right]=\boxed{\frac{kQ}{x}}$$

Because, for $|\boldsymbol{x}| >> \boldsymbol{L}$, the charge carried by the rod is far enough away from the point of interest to look like a point charge, this result is what we would expect.

53 •• [SSM] A disk of radius R has a surface charge distribution given by $\sigma=\sigma_0 r^2/R^2$ where σ_0 is a constant and r is the distance from the center of the disk. (a) Find the total charge on the disk. (b) Find an expression for the electric potential at a distance z from the center of the disk on the axis that passes through the disk′s center and is perpendicular to its plane.

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(a) Express the charge dq on a ring of radius r and thickness dr:

$$dq=2\pi r\sigma dr=2\pi r\left(\sigma_0\frac{r^2}{R^2}\right)dr$$

$$=\frac{2\pi\sigma_0}{R^2}r^3dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q=\frac{2\pi\sigma_0}{R^2}\int_0^R r^3dr=\boxed{\tfrac{1}{2}\pi\sigma_0 R^2}$$

(b)Express the potential on the axis of the disk due to a circular element of charge $dq=\dfrac{2\pi\sigma_0}{R^2}r^3dr$:

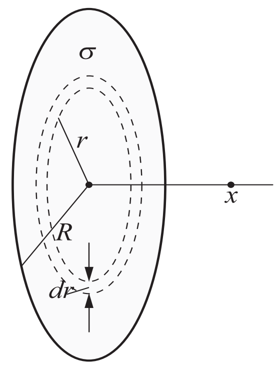
$$dV=\frac{kdq}{r'}=\frac{2\pi k\sigma_0}{R^2}\frac{r^3}{\sqrt{x^2+r^2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$V=\frac{2\pi k\sigma_0}{R^2}\int_0^R\frac{r^3dr}{\sqrt{x^2+r^2}}=\boxed{\frac{2\pi k\sigma_0}{R^2}\left(\frac{R^2-2x^2}{3}\sqrt{x^2+R^2}+\frac{2x^3}{3}\right)}$$

54 ••• A disk of radius R has a surface charge distribution given by $\sigma = \sigma_0 R/r$ where σ_0 is a constant and r is the distance from the center of the disk. (*a*) Find the total charge on the disk. (*b*) Find an expression for the electric potential at a distance x from the center of the disk on the axis that passes through the disk′s center and is perpendicular to its plane.

Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(*a*) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{R}{r}\right)dr$$
$$= 2\pi\sigma_0 R dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = 2\pi\sigma_0 R\int_0^R dr = \boxed{2\pi\sigma_0 R^2}$$

(*b*) Express the potential on the axis of the disk due to a circular element of charge $dq = 2\pi r\sigma dr$:

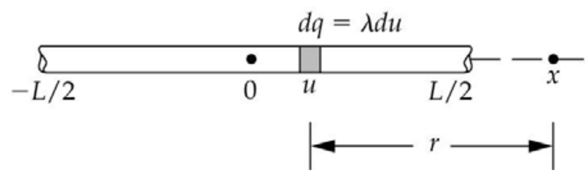
$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0 R dr}{\sqrt{x^2 + r^2}}$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$V = 2\pi k\sigma_0 R\int_0^R \frac{dr}{\sqrt{x^2 + r^2}}$$
$$= \boxed{2\pi k\sigma_0 R\ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)}$$

55 •• A rod of length L has a total charge Q uniformly distributed along its length. The rod lies along the x-axis with its center at the origin. (*a*) What is the electric potential as a function of position along the x-axis for $x > L/2$? (*b*) Show that for $x >> L/2$, your result reduces to that due to a point charge Q.

Picture the Problem We can express the electric potential dV at x due to an elemental charge dq on the rod and then integrate over the length of the rod to find $V(x)$. In the second part of the problem we use a binomial expansion to show that, for $x >> L/2$, our result reduces to that due to a point charge Q.



(*a*) Express the potential at *x* due to the element of charge *dq* located at *u*:

$$dV = \frac{kdq}{r} = \frac{k\lambda du}{x-u}$$

or, because $\lambda = Q/L$,

$$dV = \frac{kQ}{L}\frac{du}{x-u}$$

Integrate *V* from $u = -L/2$ to $u = L/2$ and simplify to obtain:

$$V(x) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{du}{x-u}$$

$$= \frac{kQ}{L}\ln(x-u)\Big|_{-L/2}^{L/2}$$

$$= \frac{kQ}{L}\left[-\ln\left(x-\tfrac{1}{2}L\right)+\ln\left(x+\tfrac{1}{2}L\right)\right]$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{x+\frac{1}{2}L}{x-\frac{1}{2}L}\right)}$$

(*b*) Divide the numerator and denominator of the argument of the logarithm by *x* to obtain:

$$\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right) = \ln\left(\frac{1+\frac{L}{2x}}{1-\frac{L}{2x}}\right) = \ln\left(\frac{1+a}{1-a}\right)$$

where $a = L/2x$.

Divide $1 + a$ by $1 - a$ and simplify to obtain:

$$\ln\left(\frac{1+a}{1-a}\right) = \ln\left(1+2a+\frac{2a^2}{1-a}\right)$$

$$= \ln\left(1+\frac{L}{x}+\frac{\frac{L^2}{x^2}}{2-\frac{L}{x}}\right)$$

$$\approx \ln\left(1+\frac{L}{x}\right)$$

provided $x >> L$.

Expand $\ln\left(1+\frac{L}{x}\right)$ binomially to obtain:

$\ln\left(1+\frac{L}{x}\right)\approx\frac{L}{x}$

provided $x >> L$.

Substitute to express $V(x)$ for $x >> L/2$:

$V(x)=\frac{kQ}{L}\frac{L}{x}=\boxed{\frac{kQ}{x}}$, the potential due to a point charge Q.

56 ••• A circle of radius a is removed from the center of a uniformly charged thin circular disk of radius b and charge per unit area σ. (*a*) Find an expression for the potential on the x axis a distance x from the center of the disk. (*b*) Show that for $x >> b$ the electric potential on the axis of the uniformly charged disk with cutout approaches kQ/x, where $Q = \sigma\pi(b^2 - a^2)$ is the total charge on the disk.

Picture the Problem The potential on the axis of the uniformly charged disk is the sum of the potential V_b due to the disk of radius b and the potential V_a due to the disk of radius a that has been removed. We can think of the charged disk that has been removed as having a negative charge density $-\sigma$. Note that if $x >> b$, then it is also true that $x >> a$.

(*a*)The potential on the axis of the circular disk is:

$V(x)=V_b(x)+V_a(x)$

where

$V_b(x)=2\pi k\sigma\left[\left(x^2+b^2\right)^{1/2}-x\right]$

and

$V_a(x)=2\pi k(-\sigma)\left[\left(x^2+a^2\right)^{1/2}-x\right]$

Substitute for $V_b(x)$ and $V_a(x)$ and simplify to obtain:

$$\begin{aligned}V(x)&=2\pi k\sigma\left[\left(x^2+b^2\right)^{1/2}-x\right]+2\pi k(-\sigma)\left[\left(x^2+a^2\right)^{1/2}-x\right]\\&=2\pi k\sigma\left[\left(x^2+b^2\right)^{1/2}-\left(x^2+a^2\right)^{1/2}\right]\\&=\boxed{2\pi k\sigma\left(\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right)}\end{aligned}$$

(*b*) Expanding $\left(1+\frac{\boldsymbol{b}^2}{\boldsymbol{x}^2}\right)^{1/2}$ binomially yields:

$$\begin{aligned}\left(1+\frac{\boldsymbol{b}^2}{\boldsymbol{x}^2}\right)^{1/2}&=1+\frac{\boldsymbol{b}^2}{2\boldsymbol{x}^2}\\&\quad+\text{higher order terms}\\&\approx1+\frac{\boldsymbol{b}^2}{2\boldsymbol{x}^2}\end{aligned}$$

Expanding $\left(1+\frac{\boldsymbol{a}^2}{\boldsymbol{x}^2}\right)^{1/2}$ binomially yields:

$$\left(1+\frac{\boldsymbol{a}^2}{\boldsymbol{x}^2}\right)^{1/2} \approx 1+\frac{\boldsymbol{a}^2}{2\boldsymbol{x}^2}$$

Substituting in the expression for $V(x)$ and simplifying yields:

$$V(x) \approx 2\pi k\sigma x\left[1+\frac{b^2}{2x^2}-\left(1+\frac{a^2}{2x^2}\right)\right] = 2\pi k\sigma x\left[\frac{b^2}{2x^2}-\frac{a^2}{2x^2}\right] = \frac{\pi k\sigma\left(b^2-a^2\right)}{x}$$

The total charge on the disk is:

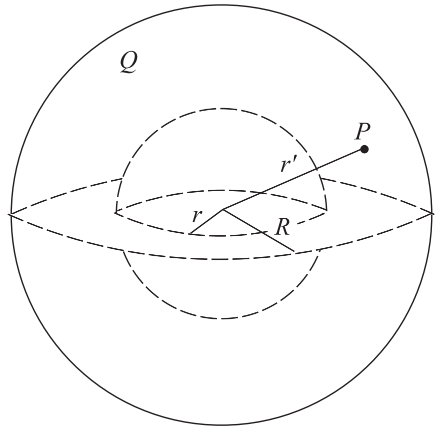
$$Q = \sigma\pi\left(b^2-a^2\right) \Rightarrow \sigma = \frac{Q}{\pi\left(b^2-a^2\right)}$$

Substituting for σ yields:

$$V(x) \approx \frac{\pi k\left(\frac{Q}{\pi\left(b^2-a^2\right)}\right)\left(b^2-a^2\right)}{x} = \boxed{\frac{kQ}{x}}$$

57 ••• The expression for the electric potential inside a uniformly charged solid sphere is given by $V(r) = \frac{kQ}{2R}\left(3-\frac{r^2}{R^2}\right)$, where R is the radius of the sphere and r is the distance from the center. This expression was obtained in Example 23-12 by first finding the electric field. In this problem, you derive the same expression by modeling the sphere as a nested collection of thin spherical shells, and then adding the potentials of these shells at a field point inside the sphere. The potential dV that is a distance r' from the center of a uniformly charged thin spherical shell that has a radius r' and a charge dQ is given by $dV = kdQ/r'$ for $r' \geq r$ and $dV = kdQ/r$ for $r' \leq r$ (Equation 23-22). Consider a sphere of radius R containing a charge Q that is uniformly distributed and you want to find V at some point inside the sphere (that is for $r < R$). (*a*) Find an expression for the charge dQ on a spherical shell of radius r' and thickness dr'. (*b*) Find an expression for the potential dV at r due to the charge in a shell of radius r' and thickness dr', where $r \leq r' \leq R$. (*c*) Integrate your expression in Part (*b*) from $r' = r$ to $r' = R$ to find the potential at r due to all the charge in the region farther from r than the center of the sphere. (*d*) Find an expression for the potential dV at r due to the charge in a shell of radius r' and thickness dr', where $r' \leq r$. (*e*) Integrate your expression in Part (*d*) from $r' = 0$ to $r' = r$ to find the potential at r due to all the charge in the region closer than r to the center of the sphere. (*f*) Find the total potential V at r by adding your Part (*c*) and Part (*e*) results.

Picture the Problem The diagram shows a uniformly charged sphere of radius R and the field point P at which we wish to find the total potential. We can use the definition of charge density to find the charge inside a sphere of radius r and the potential at r due to this charge. We can express the potential at r due to the charge in a shell of radius r' and thickness dr' at $r' \geq r$ using $\boldsymbol{dV} = \boldsymbol{kdq'}/\boldsymbol{r}$ and then integrate this expression from $r' = r$ to $r' = R$ to find V.



(*a*) The charge dQ in a shell of radius r' and thickness dr' at $r' > r$ is given by:

$$dQ = \rho dV' = \rho A' dr'$$

Because $A' = 4\pi r'^2$:

$$dQ = 4\pi r'^2 \rho dr'$$

Because the sphere is uniformly charged:

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

Substituting for ρ and simplifying yields:

$$dQ = 4\pi r'^2\left(\frac{3Q}{4\pi R^3}\right)dr' = \boxed{\frac{3Q}{R^3}r'^2 dr'}$$

(*b*) Express the potential dV in the interval $r \leq r' \leq R$ due to dQ:

$$dV = \frac{kdQ}{r'}$$

Substituting for dQ and simplifying yields:

$$dV = \frac{k}{r'}\left(\frac{3Q}{R^3}r'^2 dr'\right) = \boxed{\frac{3kQ}{R^3}r' dr'}$$

(*c*) Integrate dV from $r' = r$ to $r' = R$ to find V:

$$V = \frac{3kQ}{R^3}\int_r^R r' dr' = \boxed{\frac{3kQ}{2R^3}\left(R^2 - r^2\right)}$$

(*d*) The potential dV at r due to the charge in a shell of radius r' and thickness dr', where $r' \leq r$, is given by:

$$dV = \frac{kdQ}{r} = \frac{k\rho dV'}{r} = \frac{k\rho A' dr'}{r}$$
$$= \frac{k\rho\left(4\pi r'^2\right)dr'}{r} = \frac{4\pi k\rho}{r}r'^2\, dr'$$

Substituting for ρ and simplifying yields:

$$dV = \frac{4\pi k}{r}\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)r'^2\,dr' = \boxed{\left(\frac{3kQ}{R^3 r}\right)r'^2\,dr'}$$

(*e*) Integrate from $r' = 0$ to $r' = r$ to find the potential at r due to all the charge in the region closer than r to the center of the sphere:

$$V = \left(\frac{3kQ}{R^3 r}\right)\int_0^r r'^2\,dr' = \boxed{\frac{kQ}{R^3}r^2}$$

(*f*) The sum of our results from Part (*c*) and Part (*e*) is:

$$V = \frac{kQ}{R^3}r^2 + \frac{3kQ}{2R^3}\left(R^2 - r^2\right) = \boxed{\frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$

58 •• Calculate the electric potential at the point a distance $R/2$ from the center of a uniformly charged thin spherical shell of radius R and charge Q. (Assume the potential is zero far from the shell.)

Picture the Problem We can find the potential relative to infinity at a distance $R/2$ from the center of the spherical shell by integrating the electric field for 0 to ∞. We can apply Gauss's law to find the electric field both inside and outside the spherical shell.

The potential relative to infinity at a distance $R/2$ from the center of the spherical shell is given by:

$$V = -\int_{\infty}^{R/2}\vec{E}\cdot d\vec{r} = -\int_{\infty}^{R}E_{r>R}dr - \int_{R}^{R/2}E_{r<R}dr$$

Because there is no charge inside the spherical shell, $E_{r<R} = 0$ and:

$$\int_{R}^{R/2}E_{r<R}dr = 0 \text{ and } V = -\int_{\infty}^{R}E_{r>R}dr \quad (1)$$

Apply Gauss's law to a spherical surface of radius $r > R$ to obtain:

$$\int_S E_n dA = E_{r>R}\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Solving for $E_{r>R}$ yields:

$$E_{r>R} = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

Substitute for $E_{r>R}$ in equation (1) to obtain:

$$V = -kQ\int_{\infty}^{R}\frac{dr}{r^2}$$

Evaluating this integral yields:

$$V=-kQ\left[-\frac{1}{r}\right]_{\infty}^{R}=\boxed{\frac{kQ}{R}}$$

59 •• **[SSM]** A circle of radius a is removed from the center of a uniformly charged thin circular disk of radius b. Show that the potential at a point on the central axis of the disk a distance z from its geometrical center is given by $V(z)=2\pi k\sigma\left(\sqrt{z^2+b^2}-\sqrt{z^2+a^2}\right)$, where σ is the charge density of the disk.

Picture the Problem We can find the electrostatic potential of the conducting washer by treating it as two disks with equal but opposite charge densities.

The electric potential due to a charged disk of radius R is given by:

$$V(x)=2\pi k\sigma|x|\left(\sqrt{1+\frac{R^2}{x^2}}-1\right)$$

Superimpose the electrostatic potentials of the two disks with opposite charge densities and simplify to obtain:

$$\begin{aligned}V(x)&=2\pi k\sigma|x|\left(\sqrt{1+\frac{b^2}{x^2}}-1\right)-2\pi k\sigma|x|\left(\sqrt{1+\frac{a^2}{x^2}}-1\right)\\&=2\pi k\sigma|x|\left(\left(\sqrt{1+\frac{b^2}{x^2}}-1\right)-\left(\sqrt{1+\frac{a^2}{x^2}}-1\right)\right)\\&=2\pi k\sigma|x|\left(\sqrt{1+\frac{b^2}{x^2}}-\sqrt{1+\frac{a^2}{x^2}}\right)=2\pi k\sigma|x|\left(\sqrt{\frac{x^2+b^2}{x^2}}-\sqrt{\frac{x^2+a^2}{x^2}}\right)\\&=2\pi k\sigma|x|\left(\frac{\sqrt{x^2+b^2}}{\sqrt{x^2}}-\frac{\sqrt{x^2+a^2}}{\sqrt{x^2}}\right)=2\pi k\sigma|x|\left(\frac{\sqrt{x^2+b^2}}{|x|}-\frac{\sqrt{x^2+a^2}}{|x|}\right)\\&=2\pi k\sigma\left(\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right)\end{aligned}$$

The charge density σ is given by:

$$\sigma=\frac{Q}{\pi\left(b^2-a^2\right)}$$

Substituting for σ yields:

$$V(x)=\boxed{\frac{2kQ}{\left(b^2-a^2\right)}\left(\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right)}$$

Equipotential Surfaces

60 • An infinite flat sheet of charge has a uniform surface charge density equal to 3.50 μC/m^2. How far apart are the equipotential surfaces whose potentials differ by 100 V?

Picture the Problem We can equate the expression for the electric field due to an infinite plane of charge and $-\Delta V/\Delta x$ and solve the resulting equation for the separation of the equipotential surfaces.

Express the electric field due to the infinite plane of charge:

$$E = \frac{\sigma}{2\in_0}$$

Relate the electric field to the potential:

$$E = -\frac{\Delta V}{\Delta x}$$

Equate these expressions and solve for Δx to obtain:

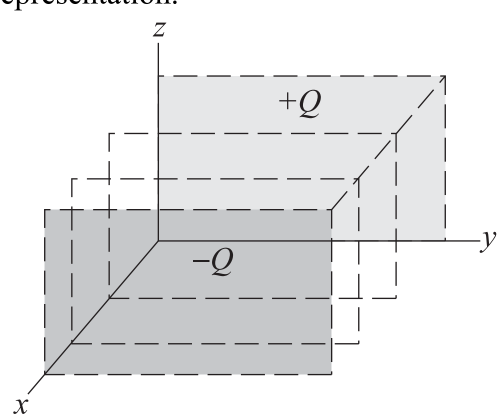
$$\Delta x = \frac{2\in_0 \Delta V}{\sigma}$$

Substitute numerical values and evaluate $|\Delta x|$:

$$|\Delta \boldsymbol{x}| = \frac{2\left(8.854\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)(100\ \mathrm{V})}{3.50\ \mu\mathrm{C/m}^2} = \boxed{0.506\ \mathrm{mm}}$$

61 •• [SSM] Consider two parallel uniformly charged infinite planes that are equal but oppositely charged. (*a*) What is (are) the shape(s) of the equipotentials in the region between them? Explain your answer. (*b*) What is (are) the shape(s) of the equipotentials in the regions not between them? Explain your answer.

Picture the Problem The two parallel planes, with their opposite charges, are shown in the pictorial representation.



(*a*) Because the electric field between the charged plates is uniform and perpendicular to the plates, the equipotential surfaces are planes parallel to the charged planes.

(*b*) The regions to either side of the two charged planes are equipotential regions, so any surface in either of these regions is an equipotential surface.

62 •• A Geiger tube consists of two elements, a long metal cylindrical shell and a long straight metal wire running down its central axis. Model the tube as if both the wire and cylinder are infinitely long. The central wire is positively charged and the outer cylinder is negatively charge. The potential difference between the wire and the cylinder is 1.00 kV. (*a*) What is the direction of the electric field inside the tube? (*b*) Which element is at a higher electric potential? (*c*) What is (are) the shape(s) of the equipotentials inside the tube? (*d*) Consider two equipotentials described in Part (*c*). Suppose they differ in electric potential by 10 V. Do two such equipotentials near the central wire have the same spacing as they would near the outer cylinder? If not, where in the tube are the equipotentials that are more widely spaced? Explain your answer.

Determine the Concept

(*a*) The direction of the electric field inside the tube is the direction of the force the electric field exerts on a positively charged object. Because the wire is positively charged and the tube is negatively charged, and because of the cylindrical geometry of the Geiger tube, the electric field is directed radially away from the central wire.

(*b*) Because it would require more work to bring a positively charged object from infinity to the surface of the wire than it would to bring this test object to the surface of the cylindrical tube, the central wire is at the higher electric potential.

(*c*) Because of the cylindrical geometry of the Geiger tube, the equipotential surfaces are cylinders concentric with the central wire.

(*d*) No. Because the magnitude of the electric field, which is the rate of change with distance (also known as the gradient) of the potential decreases with distance from the wire, the spacing between adjacent equipotential surfaces having the same potential difference between them decreases as you get farther from the central wire.

63 •• **[SSM]** Suppose the cylinder in the Geiger tube in Problem 62 has an inside diameter of 4.00 cm and the wire has a diameter of 0.500 mm. The cylinder is grounded so its potential is equal to zero. (*a*) What is the radius of the equipotential surface that has a potential equal to 500 V? Is this surface closer to the wire or to the cylinder? (*b*) How far apart are the equipotential surfaces that have potentials of 200 and 225 V? (*c*) Compare your result in Part (*b*) to the distance between the two surfaces that have potentials of 700 and 725 V respectively. What does this comparison tell you about the electric field strength as a function of the distance from the central wire?

Picture the Problem If we let the electric potential of the cylinder be zero, then the surface of the central wire is at +1000 V and we can use Equation 23-23 to find the electric potential at any point between the outer cylinder and the central wire.

(*a*) From Equation 23-23 we have:

$$V(r) = 2k\lambda \ln\frac{R_{\text{ref}}}{r} \qquad (1)$$

where R_{ref} is the radius of the outer cylinder and r is the distance from the center of the central wire and $r < R_{\text{ref}}$.

Solving for $2k\lambda$ yields:

$$2k\lambda = \frac{V(r)}{\ln\frac{R_{\text{ref}}}{r}}$$

At the surface of the wire, $V = 1000$ V and $r = 0.250$ mm. Hence:

$$2k\lambda = \frac{1000\ \text{V}}{\ln\frac{2.00\ \text{cm}}{0.250\ \text{mm}}} = 228.2\ \text{V}$$

and

$$V(r) = (228.2\ \text{V})\ln\frac{2.00\ \text{cm}}{r}$$

Setting $V = 500$ V yields:

$$500\ \text{V} = (228.2\ \text{V})\ln\frac{2.00\ \text{cm}}{\boldsymbol{r}}$$

or

$$\ln\frac{2.00\ \text{cm}}{\boldsymbol{r}} = 2.191 \Rightarrow \frac{2.00\ \text{cm}}{\boldsymbol{r}} = \boldsymbol{e}^{2.191}$$

Solve for r to obtain:

$$r = \frac{2.00\ \text{cm}}{e^{2.191}} = \boxed{0.224\ \text{cm}}$$, closer to the wire.

(*b*) The separation of the equipotential surfaces that have potential values of 200 and 225 V is:

$$\Delta\boldsymbol{r} = \left|\boldsymbol{r}_{225\ \text{V}} - \boldsymbol{r}_{200\ \text{V}}\right| \qquad (2)$$

Solving equation (1) for r yields:

$$r = R_{\text{ref}}e^{-\frac{V}{2k\lambda}} = (2.00\ \text{cm})e^{-\frac{V}{228.2\ \text{V}}}$$

Substitute for the radii in equation (2), simplify, and evaluate Δr to obtain:

$$\Delta r = \left|(2.00\text{ cm})e^{-\frac{225\text{ V}}{228.2\text{ V}}} - (2.00\text{ cm})e^{-\frac{200\text{ V}}{228.2\text{ V}}}\right| = (2.00\text{ cm})\left|e^{-\frac{225\text{ V}}{228.2\text{ V}}} - e^{-\frac{200\text{ V}}{228.2\text{ V}}}\right|$$

$$= \boxed{0.864\text{ mm}}$$

(c) The distance between the 700 V and the 725 V equipotentials is:

$$\Delta r = (2.00\text{ cm})\left|e^{-\frac{725\text{ V}}{228.2\text{ V}}} - e^{-\frac{700\text{ V}}{228.2\text{ V}}}\right|$$

$$= \boxed{0.0966\text{ mm}}$$

This closer spacing of these two equipotential surfaces was to be expected. Close to the central wire, two equipotential surfaces with the same difference in potential should be closer together to reflect the fact that the higher electric field strength is greater closer to the wire.

64 •• A point particle that has a charge of +11.1 nC is at the origin. (a) What is (are) the shapes of the equipotential surfaces in the region around this charge? (b) Assuming the potential to be zero at $r = \infty$, calculate the radii of the five surfaces that have potentials equal to 20.0 V, 40.0 V, 60.0 V, 80.0 V and 100.0 V, and sketch them to scale centered on the charge. (c) Are these surfaces equally spaced? Explain your answer. (d) Estimate the electric field strength between the 40.0-V and 60.0-V equipotential surfaces by dividing the difference between the two potentials by the difference between the two radii. Compare this estimate to the exact value at the location midway between these two surfaces.

Picture the Problem We can integrate the expression for the electric field due to a point charge to find an expression for the electric potential of the point particle.

(a) The equipotential surfaces are spheres centered on the charge.

(b) From the relationship between the electric potential due to the point charge and the electric field of the point charge we have:

$$\int_a^b dV = -\int_{r_a}^{r_b} \vec{\boldsymbol{E}} \cdot d\vec{\mathbf{r}} = -kQ\int_{r_a}^{r_b} r^{-2}dr$$

or

$$V_b - V_a = kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Taking the potential to be zero at $r_a = \infty$ yields:

$$V_b - 0 = kQ\left(\frac{1}{r_b}\right) \Rightarrow V = \frac{kQ}{r} \Rightarrow r = \frac{kQ}{V}$$

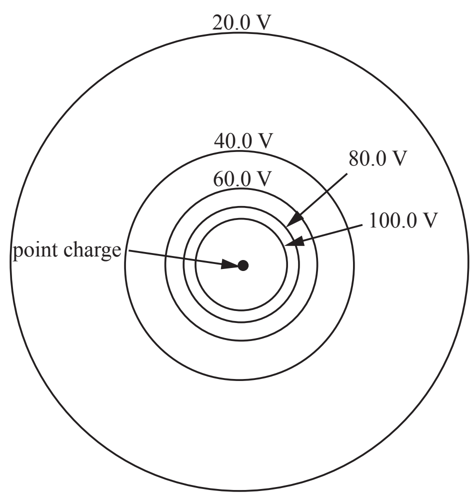
Because $Q = +1.11 \times 10^{-8}$ C:

$$r = \frac{\left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.11\times10^{-8}\,\text{C}\right)}{V} = \frac{99.77\,\frac{\text{N}\cdot\text{m}^2}{\text{C}}}{V} \quad (1)$$

Use equation (1) to complete the following table:

V (V)	20.0	40.0	60.0	80.0	100.0
r (m)	4.99	2.49	1.66	1.25	1.00

The equipotential surfaces are shown in cross-section to the right:



(c) No. The equipotential surfaces are closest together where the electric field strength is greatest.

(d) The average value of the magnitude of the electric field between the 40.0-V and 60.0-V equipotential surfaces is given by:

$$E = -\frac{\Delta V}{\Delta r} = -\frac{40\text{ V} - 60\text{ V}}{\Delta r}$$

Drop perpendiculars to the r axis from 40.0 V and 60.0 V to approximate the radii corresponding to each of these potential surfaces:

$$\boldsymbol{E}_{\text{est}} \approx -\frac{40\text{ V} - 60\text{ V}}{2.4\text{ m} - 1.7\text{ m}} = 29\,\frac{\text{V}}{\text{m}}$$

The exact value of the electric field at the location midway between these two surfaces is given by $E = kQ/r^2$, where r is the average of the radii of the 40.0-V and 60.0-V equipotential surfaces. Substitute numerical values and evaluate E_{exact}.

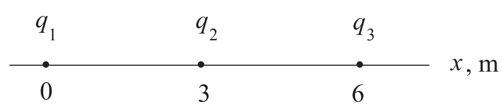
$$E_{\text{exact}} = \frac{\left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.11\times10^{-8}\ \text{C}\right)}{\left(\frac{1.66\ \text{m}+2.49\ \text{m}}{2}\right)^2} = 23\,\frac{\text{V}}{\text{m}}$$

The estimated value for E differs by about 21% from the exact value.

Electrostatic Potential Energy

65 • Three point charges are on the x-axis: q_1 is at the origin, q_2 is at $x = +3.00$ m, and q_3 is at $x = +6.00$ m. Find the electrostatic potential energy of this system of charges for the following charge values: (a) $q_1 = q_2 = q_3 = +2.00\ \mu$C; ($b$) $q_1 = q_2 = +2.00\ \mu$C and $q_3 = -2.00\ \mu$C; and (c) $q_1 = q_3 = +2.00\ \mu$C and $q_2 = -2.00\ \mu$C. (Assume the potential energy is zero when the charges are very far from each other.)

The electrostatic potential energy of this system of three point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram.



Express the work required to assemble this system of charges:

$$U = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_1q_3}{r_{1,3}} + \frac{kq_2q_3}{r_{2,3}}$$
$$= k\left(\frac{q_1q_2}{r_{1,2}} + \frac{q_1q_3}{r_{1,3}} + \frac{q_2q_3}{r_{2,3}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, and $r_{2,3}$:

$$r_{1,2} = 3\,\text{m}, r_{2,3} = 3\,\text{m}, \text{and}\ r_{1,3} = 6\,\text{m}$$

(a) Evaluate U for $q_1 = q_2 = q_3 = 2.00\ \mu$C:

$$U = \left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{(2.00\ \mu\text{C})(2.00\ \mu\text{C})}{3.00\,\text{m}} + \frac{(2.00\ \mu\text{C})(2.00\ \mu\text{C})}{6.00\,\text{m}} + \frac{(2.00\ \mu\text{C})(2.00\ \mu\text{C})}{3.00\,\text{m}}\right)$$

$$= \boxed{30.0\,\text{mJ}}$$

(*b*) Evaluate U for $q_1 = q_2 = 2.00\ \mu C$ and $q_3 = -2.00\ \mu C$:

$$U = \left(8.988\times10^{9}\ \frac{N\cdot m^2}{C^2}\right)\left(\frac{(2.00\,\mu C)(2.00\,\mu C)}{3.00\,m}+\frac{(2.00\,\mu C)(-2.00\,\mu C)}{6.00\,m}+\frac{(2.00\,\mu C)(-2.00\,\mu C)}{3.00\,m}\right)$$

$$= \boxed{-5.99\,mJ}$$

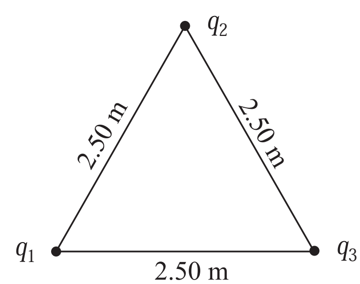
(*c*) Evaluate U for $q_1 = q_3 = 2.00\ \mu C$ and $q_2 = -2.00\ \mu C$:

$$U = \left(8.988\times10^{9}\ \frac{N\cdot m^2}{C^2}\right)\left(\frac{(2.00\,\mu C)(-2.00\,\mu C)}{3.00\,m}+\frac{(2.00\,\mu C)(2.00\,\mu C)}{6.00\,m}+\frac{(-2.00\,\mu C)(2.00\,\mu C)}{3.00\,m}\right)$$

$$= \boxed{-18.0\,mJ}$$

66 • Point charges q_1, q_2, and q_3 are fixed at the vertices of an equilateral triangle whose sides are 2.50 m-long. Find the electrostatic potential energy of this system of charges for the following charge values:
(*a*) $q_1 = q_2 = q_3 = +4.20\ \mu C$, (*b*) $q_1 = q_2 = +4.20\ \mu C$ and $q_3 = -4.20\ \mu C$; and
(*c*) $q_1 = q_2 = -4.20\ \mu C$ and $q_3 = +4.20\ \mu C$. (Assume the potential energy is zero when the charges are very far from each other.)

Picture the Problem The electrostatic potential energy of this system of three point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram.



Express the work required to assemble this system of charges:

$$U = \frac{kq_1q_2}{r_{1,2}}+\frac{kq_1q_3}{r_{1,3}}+\frac{kq_2q_3}{r_{2,3}} = k\left(\frac{q_1q_2}{r_{1,2}}+\frac{q_1q_3}{r_{1,3}}+\frac{q_2q_3}{r_{2,3}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, and $r_{2,3}$:

$$\boldsymbol{r}_{1,2} = \boldsymbol{r}_{2,3} = \boldsymbol{r}_{1,3} = 2.50\,m$$

(*a*) Evaluate U for $q_1 = q_2 = q_3 = 4.20\ \mu$C:

$$\boldsymbol{U} = \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}}\right]$$

$$= \boxed{190\,\text{mJ}}$$

(*b*) Evaluate U for $q_1 = q_2 = 4.20\ \mu$C and $q_3 = -4.20\ \mu$C:

$$U = \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(4.20\,\mu\text{C})(-4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(4.20\,\mu\text{C})(-4.20\,\mu\text{C})}{2.50\,\text{m}}\right]$$

$$= \boxed{-63.4\,\text{mJ}}$$

(*c*) Evaluate U for $q_1 = q_2 = -4.20\ \mu$C and $q_3 = +4.20\ \mu$C:

$$U = \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(-4.20\,\mu\text{C})(-4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(-4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}} + \frac{(-4.20\,\mu\text{C})(4.20\,\mu\text{C})}{2.50\,\text{m}}\right]$$

$$= \boxed{-63.4\,\text{mJ}}$$

67 •• [SSM] (*a*) How much charge is on the surface of an isolated spherical conductor that has a 10.0-cm radius and is charged to 2.00 kV? (*b*)What is the electrostatic potential energy of this conductor? (Assume the potential is zero far from the sphere.)

Picture the Problem The potential of an isolated spherical conductor is given by $V = kQ/r$, where Q is its charge and r its radius, and its electrostatic potential energy by $U = \frac{1}{2}QV$. We can combine these relationships to find the sphere's electrostatic potential energy.

(*a*) The potential of the isolated spherical conductor at its surface is related to its radius:

$$V = \frac{kQ}{R} \Rightarrow Q = \frac{RV}{k}$$

where R is the radius of the spherical conductor.

Substitute numerical values and evaluate Q:

$$Q = \frac{(10.0\text{ cm})(2.00\text{ kV})}{8.988\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 22.25\text{ nC} = \boxed{22.3\text{ nC}}$$

(*b*) Express the electrostatic potential energy of the isolated spherical conductor as a function of its charge Q and potential V:

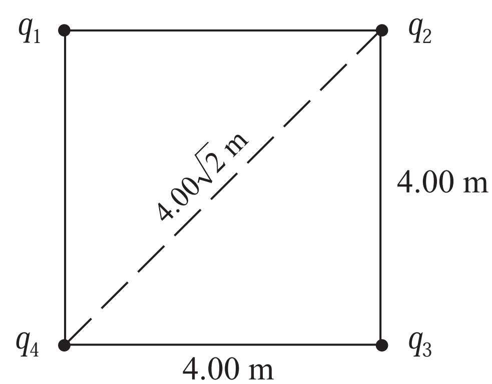
$$U = \tfrac{1}{2}QV$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(22.25\text{ nC})(2.00\text{ kV}) = \boxed{22.3\ \mu\text{J}}$$

68 ••• Four point charges, each having a charge with a magnitude of 2.00 μC, are at the corners of a square whose sides are 4.00 m-long. Find the electrostatic potential energy of this system under the following conditions: (*a*) all of the charges are negative, (*b*) three of the charges are positive and one of the charges is negative, and (*c*) the charges at two adjacent corners are positive and the other two charges are negative. (*d*) the charges at two opposite corners are positive and the other two charges are negative. (Assume the potential energy is zero when the point charges are very far from each other.)

Picture the Problem The electrostatic potential energy of this system of four point charges is the work needed to bring the charges from an infinite separation to the final positions shown in the diagram.



The work required to assemble this system of charges equals the potential energy of the assembled system:

$$U = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_1q_3}{r_{1,3}} + \frac{kq_1q_4}{r_{1,4}} + \frac{kq_2q_3}{r_{2,3}} + \frac{kq_2q_4}{r_{2,4}} + \frac{kq_3q_4}{r_{3,4}}$$
$$= k\left(\frac{q_1q_2}{r_{1,2}} + \frac{q_1q_3}{r_{1,3}} + \frac{q_1q_4}{r_{1,4}} + \frac{q_2q_3}{r_{2,3}} + \frac{q_2q_4}{r_{2,4}} + \frac{q_3q_4}{r_{3,4}}\right)$$

Find the distances $r_{1,2}$, $r_{1,3}$, $r_{1,4}$, $r_{2,3}$, $r_{2,4}$, and $r_{3,4}$,:

$$r_{1,2} = r_{2,3} = r_{3,4} = r_{1,4} = 4.00\,\text{m}$$

and

$$\boldsymbol{r}_{1,3} = \boldsymbol{r}_{2,4} = 4.00\sqrt{2}\,\text{m}$$

(*a*) Evaluate U for $q_1 = q_2 = q_3 = q_4 = -2.00\ \mu\text{C}$:

$$\begin{aligned}U &= \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}}\right.\\ &\quad + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}}\\ &\quad \left. + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}} + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}}\right]\\ &= \boxed{48.7\,\text{mJ}}\end{aligned}$$

(*b*) Evaluate U for $q_1 = q_2 = q_3 = 2\ \mu\text{C}$ and $q_4 = -2.00\ \mu\text{C}$:

$$\begin{aligned}U &= \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(2.00\,\mu\text{C})(2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(2.00\,\mu\text{C})(2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}}\right.\\ &\quad + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(2.00\,\mu\text{C})(2.00\,\mu\text{C})}{4.00\,\text{m}}\\ &\quad \left. + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}} + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}}\right]\\ &= \boxed{0}\end{aligned}$$

(*c*) Let $q_1 = q_2 = 2.00\ \mu\text{C}$ and $q_3 = q_4 = -2.00\ \mu\text{C}$:

$$\begin{aligned}U &= \left(8.988\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(2.00\,\mu\text{C})(2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}}\right.\\ &\quad + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}} + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}}\\ &\quad \left. + \frac{(2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\sqrt{2}\,\text{m}} + \frac{(-2.00\,\mu\text{C})(-2.00\,\mu\text{C})}{4.00\,\text{m}}\right]\\ &= \boxed{-12.7\,\text{mJ}}\end{aligned}$$

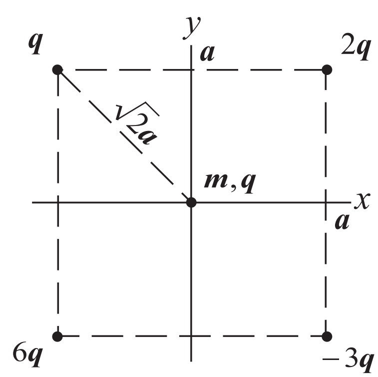
(*d*) Let $q_1 = q_3 = 2.00\ \mu$C and $q_2 = q_4 = -2.00\ \mu$C:

$$U = \left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(2.00\ \mu\text{C})(-2.00\ \mu\text{C})}{4.00\,\text{m}} + \frac{(2.00\ \mu\text{C})(2.00\ \mu\text{C})}{4.00\sqrt{2}\ \text{m}} + \frac{(2.00\ \mu\text{C})(-2.00\ \mu\text{C})}{4.00\,\text{m}} + \frac{(-2.00\ \mu\text{C})(2.00\ \mu\text{C})}{4.00\,\text{m}} + \frac{(-2.00\ \mu\text{C})(-2.00\ \mu\text{C})}{4.00\sqrt{2}\ \text{m}} + \frac{(2.00\ \mu\text{C})(-2.00\ \mu\text{C})}{4.00\,\text{m}}\right]$$

$$= \boxed{-23.2\,\text{mJ}}$$

69 •• [SSM] Four point charges are fixed at the corners of a square centered at the origin. The length of each side of the square is *2a.* The charges are located as follows: +*q* is at (–*a*, +*a*), +2*q* is at (+*a*, +*a*), –3*q* is at (+*a*, –*a*), and +6*q* is at (–*a*, –*a*). A fifth particle that has a mass *m* and a charge +*q* is placed at the origin and released from rest. Find its speed when it is a very far from the origin.

Picture the Problem The diagram shows the four point charges fixed at the corners of the square and the fifth charged particle that is released from rest at the origin. We can use conservation of energy to relate the initial potential energy of the particle to its kinetic energy when it is at a great distance from the origin and the electrostatic potential at the origin to express U_i.



Use conservation of energy to relate the initial potential energy of the particle to its kinetic energy when it is at a great distance from the origin:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

Express the initial potential energy of the particle to its charge and the electrostatic potential at the origin:

$$U_i = qV(0)$$

Substitute for K_f and U_i to obtain:

$$\tfrac{1}{2}mv^2 - qV(0) = 0 \Rightarrow v = \sqrt{\frac{2qV(0)}{m}}$$

Express the electrostatic potential at the origin:

$$V(0)=\frac{kq}{\sqrt{2}a}+\frac{2kq}{\sqrt{2}a}+\frac{-3kq}{\sqrt{2}a}+\frac{6kq}{\sqrt{2}a}=\frac{6kq}{\sqrt{2}a}$$

Substitute for $V(0)$ and simplify to obtain:

$$v=\sqrt{\frac{2q}{m}\left(\frac{6kq}{\sqrt{2}a}\right)}=\boxed{q\sqrt{\frac{6\sqrt{2}k}{ma}}}$$

70 •• Consider two point particles that each have charge $+e$, are at rest, and are separated by 1.50×10^{-15} m. (*a*) How much work was required to bring them together from a very large separation distance? (*b*) If they are released, how much kinetic energy will each have when they are separated by twice their separation at release? (*c*) The mass of each particle is 1.00 u (1.00 AMU). What speed will each have when they are very far from each other?

Picture the Problem (*a*) In the absence of other charged bodies, no work is required to bring the first proton from infinity to its initial position. We can use the work- energy theorem to find the work required to bring the second proton to a position 1.50×10^{-15} m away from the first proton. (*b*) and (*c*) We can apply conservation of mechanical energy to the two-proton system to find the kinetic energy of each proton when they are separated by twice their separation at release and when they are separated by a large distance.

(*a*) Apply the work-energy theorem to the second proton to obtain:

$$\boldsymbol{W}_{\text{ext}}=\Delta\boldsymbol{K}+\Delta\boldsymbol{U}=0+\frac{\boldsymbol{ke}^2}{\boldsymbol{r}}=\frac{\boldsymbol{ke}^2}{\boldsymbol{r}}$$

Substitute numerical values and evaluate W_{ext}:

$$W_{\text{ext}}=\frac{\left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.602\times10^{-19}\ \text{C}\right)^2}{1.50\times10^{-15}\ \text{m}}=1.538\times10^{-13}\ \text{J}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\ \text{J}}=\boxed{960\,\text{keV}}$$

(*b*) Apply conservation of mechanical energy to the separating protons to obtain:

$$\Delta K+\Delta U=0\Rightarrow K_{\text{f}}-K_{\text{i}}+U_{\text{f}}-U_{\text{i}}=0$$

or, because $K_{\text{i}} = 0$,

$$K_{\text{f}}+U_{\text{f}}-U_{\text{i}}=0$$

Substituting for U_{i} and U_{f} and simplifying yields:

$$K_{\text{f}}=U_{\text{i}}-U_{\text{f}}=\frac{ke^2}{r_{\text{i}}}-\frac{ke^2}{r_{\text{f}}}=\frac{ke^2}{r}-\frac{ke^2}{2r}=\frac{ke^2}{2r}$$

Remembering that K_f is the kinetic energy of both protons, substitute numerical values and evaluate $K_{\text{f, each proton}}$:

$$K_{\text{f, each proton}} = \frac{1}{2}\frac{\left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.602\times10^{-19}\ \text{C}\right)^2}{2\left(1.50\times10^{-15}\ \text{m}\right)}$$
$$= 3.844\times10^{-14}\ \text{J}\times\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}$$
$$= \boxed{240\ \text{keV}}$$

(*c*) Apply conservation of mechanical energy to the separating protons to obtain:

$\Delta K + \Delta U = 0 \Rightarrow K_f - K_i + U_f - U_i = 0$

or, because $K_i = U_f = 0$,

$K_f - U_i = 0 \Rightarrow \frac{1}{2}m_p v_\infty^2 - U_i = 0$

Solving for v_∞ yields:

$v_\infty = \sqrt{\frac{2U_i}{m_p}}$ where U_i is half the initial potential energy of the two-proton system.

Substitute numerical values and evaluate v_∞:

$$\boldsymbol{v}_\infty = \sqrt{\frac{2\left(480\ \text{keV}\times\frac{1.602\times10^{-19}\ \text{J}}{\text{eV}}\right)}{1.673\times10^{-27}\ \text{kg}}}$$
$$= \boxed{9.59\times10^{6}\ \text{m/s}}$$

71 ••• Consider an electron and a proton that are initially at rest and are separated by 2.00 nm. Neglecting any motion of the much more massive proton, what is the minimum (*a*) kinetic energy and (*b*) speed that the electron must be projected at so it reaches a point a distance of 12.0 nm from the proton? Assume the electron's velocity is directed radially away from the proton. (*c*) How far will the electron travel away from the proton if it has twice that initial kinetic energy?

Picture the Problem We can apply the conservation of mechanical energy to the electron-proton system to find the minimum initial kinetic energy (that is, the initial kinetic energy that corresponds to a final kinetic energy of zero) required in Part (*a*) and, in Part (*c*), to find how far the electron will travel away from the proton if it has twice the initial kinetic energy found in Part (*a*).

(a) Apply conservation of mechanical energy to the electron-stationary proton system with $K_f = 0$ to obtain:

$$\Delta K + \Delta U = 0 \Rightarrow -K_{i,\,min} + U_f - U_i = 0$$

Solving for $\boldsymbol{K}_{i,\,min}$ and substituting for $\boldsymbol{U}_f$ and $\boldsymbol{U}_i$ yields:

$$K_{i,\,min} = U_f - U_i = -\frac{ke^2}{r_f} - \left(-\frac{ke^2}{r_i}\right)$$

or, simplifying further,

$$K_{i,\,min} = ke^2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

Substitute numerical values and evaluate $K_{i,\,min}$:

$$K_{i,\,min} = \left(8.988\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.602\times10^{-19}\ \text{C}\right)^2\left(\frac{1}{2.00\ \text{nm}} - \frac{1}{12.0\ \text{nm}}\right)$$

$$= 9.611\times10^{-20}\ \text{J} = \boxed{9.61\times10^{-20}\ \text{J}}$$

(b) Relate the minimum initial kinetic energy of the electron to its initial speed:

$$K_{i,\,min} = \tfrac{1}{2}m_e v_i^2 \Rightarrow v_i = \sqrt{\frac{2K_{i,\,min}}{m_e}}$$

Substitute numerical values and evaluate v_i:

$$v_i = \sqrt{\frac{2\left(9.611\times10^{-20}\ \text{J}\right)}{9.109\times10^{-31}\ \text{kg}}} = \boxed{4.59\times10^5\ \text{m/s}}$$

(c) Apply conservation of mechanical energy to the electron- proton system to obtain:

$$\Delta K + \Delta U = 0 \Rightarrow K_f - K_i + U_f - U_i = 0$$

Assuming, as we did in Part (a), that $K_f = 0$ yields:

$$-2K_{i,\,min} + U_f - U_i = 0$$

Substituting for $\boldsymbol{U}_f$ and $\boldsymbol{U}_i$ yields:

$$-2K_{i,\,min} - \frac{ke^2}{r_f} + \frac{ke^2}{r_i} = 0$$

Solve for $\boldsymbol{r}_f$ to obtain:

$$r_f = \frac{r_i}{1 - \dfrac{2K_{i,\,min}}{ke^2}}$$

Substituting numerical values and evaluating $\boldsymbol{r}_\mathrm{f}$ yields a negative value; a result that is not physical and suggests that, contrary to our assumption, $K_\mathrm{f} \neq 0$. To confirm that this is the case, assume that the electron escapes from the proton (it's final electrostatic potential will then be equal to zero) and find the initial kinetic energy required for this to occur.

If the electron is to escape the influence of the proton, its final electrostatic potential energy will be zero and:

$$K_{\mathrm{i,\,escape}} = -U_\mathrm{i} = -\left(-\frac{ke^2}{r_\mathrm{i}}\right) = \frac{ke^2}{r_\mathrm{i}}$$

Substitute numerical values and evaluate $K_{\mathrm{i,\,escape}}$:

$$K_{\mathrm{i,\,escape}} = \frac{\left(8.988\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(1.602\times10^{-19}\ \mathrm{C}\right)^2}{2.00\ \mathrm{nm}} = 1.15\times10^{-19}\ \mathrm{J}$$

Because $2K_{\mathrm{i,\,min}} > K_{\mathrm{i,\,escape}}$, the electron escapes from the proton with residual kinetic energy.

General Problems

72 • A positive point charge equal to 4.80×10^{-19} C is separated from a negative point charge of the same magnitude by 6.40×10^{-10} m. What is the electric potential at a point 9.20×10^{-10} m from each of the two charges?

Picture the Problem Because the charges are point charges, we can use the expression for the Coulomb potential to find the field at any distance from them.

Using the expression for the potential due to a system of point charges, express the potential at the point 9.20×10^{-10} m from each of the two charges:

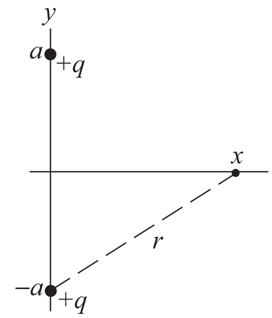
$$V = \frac{kq_+}{r} + \frac{kq_-}{r} = \frac{k}{r}\left(q_+ + q_-\right)$$

Because $q_+ = -q_-$:

$$\boldsymbol{V} = \boxed{0}$$

73 • **[SSM]** Two positive point charges, each have a charge of $+q$, and are fixed on the y-axis at $y = +a$ and $y = -a$. (a) Find the electric potential at any point on the x-axis. (b) Use your result in Part (a) to find the electric field at any point on the x-axis.

Picture the Problem The potential V at any point on the x axis is the sum of the Coulomb potentials due to the two point charges. Once we have found V, we can use $\vec{E} = -(\partial V_x/\partial x)\hat{i}$ to find the electric field at any point on the x axis.



(*a*) Express the potential due to a system of point charges:

$$V = \sum_i \frac{kq_i}{r_i}$$

Substitute to obtain:

$$V(x) = V_{\text{charge at }+a} + V_{\text{charge at }-a} = \frac{kq}{\sqrt{x^2+a^2}} + \frac{kq}{\sqrt{x^2+a^2}} = \boxed{\frac{2kq}{\sqrt{x^2+a^2}}}$$

(*b*) The electric field at any point on the x axis is given by:

$$\vec{E}(x) = -\frac{\partial V_x}{\partial x}\hat{i} = -\frac{d}{dx}\left[\frac{2kq}{\sqrt{x^2+a^2}}\right]\hat{i} = \boxed{\frac{2kqx}{(x^2+a^2)^{3/2}}\hat{i}}$$

74 • If a conducting sphere is to be charged to a potential of 10.0 kV, what is the smallest possible radius of the sphere so that the electric field near the surface of the sphere will not exceed the dielectric strength of air?

Picture the Problem The radius of the sphere is related to the electric field and the potential at its surface. The dielectric strength of air is about 3 MV/m.

Relate the electric field at the surface of a conducting sphere to the potential at the surface of the sphere:

$$E_r = \frac{V(r)}{r} \Rightarrow r = \frac{V(r)}{E_r}$$

When E is a maximum, r is a minimum:

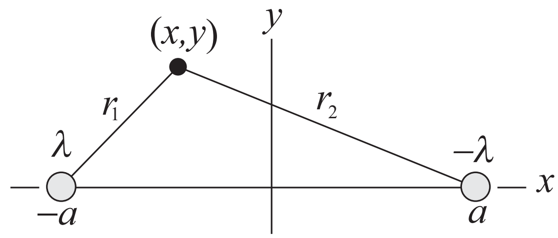
$$r_{\text{min}} = \frac{V(r)}{E_{\text{max}}}$$

Substitute numerical values and evaluate r_{min}:

$$r_{\text{min}} = \frac{10.0\,\text{kV}}{3\,\text{MV/m}} \approx \boxed{3\,\text{mm}}$$

75 •• [SSM] Two infinitely long parallel wires have a uniform charge per unit length λ and $-\lambda$ respectively. The wires are parallel with the z-axis. The positively charged wire intersects the x-axis at $x = -a$, and the negatively charged wire intersects the x-axis at $x = +a$. (a) Choose the origin as the reference point where the potential is zero, and express the potential at an arbitrary point (x, y) in the xy plane in terms of x, y, λ, and a. Use this expression to solve for the potential everywhere on the y axis. (b) Using $a = 5.00$ cm and $\lambda = 5.00$ nC/m, obtain the equation for the equipotential in the xy plane that passes through the point $x = \frac{1}{4}a$, $y = 0$. (c) Use a **spreadsheet** program to plot the equipotential found in part (b).

Picture the Problem The geometry of the wires is shown below. The potential at the point whose coordinates are (x, y) is the sum of the potentials due to the charge distributions on the wires.



(a) Express the potential at the point whose coordinates are (x, y):

$$V(x,y) = V_{\text{wire at } -a} + V_{\text{wire at } a}$$
$$= 2k\lambda \ln\left(\frac{r_{\text{ref}}}{r_1}\right) + 2k(-\lambda)\ln\left(\frac{r_{\text{ref}}}{r_2}\right)$$
$$= 2k\lambda\left[\ln\left(\frac{r_{\text{ref}}}{r_1}\right) - \ln\left(\frac{r_{\text{ref}}}{r_2}\right)\right]$$
$$= \frac{\lambda}{2\pi \in_0}\ln\left(\frac{r_2}{r_1}\right)$$

where $V(0) = 0$.

Because $r_1 = \sqrt{(x+a)^2 + y^2}$ and $r_2 = \sqrt{(x-a)^2 + y^2}$:

$$V(x,y) = \boxed{\frac{\lambda}{2\pi \in_0}\ln\left(\frac{\sqrt{(x-a)^2 + y^2}}{\sqrt{(x+a)^2 + y^2}}\right)}$$

On the y-axis, $x = 0$ and:

$$V(0,y) = \frac{\lambda}{2\pi \in_0}\ln\left(\frac{\sqrt{a^2 + y^2}}{\sqrt{a^2 + y^2}}\right)$$
$$= \frac{\lambda}{2\pi \in_0}\ln(1) = \boxed{0}$$

(*b*) Evaluate the potential at $\left(\frac{1}{4}a,0\right)=(1.25\,\text{cm},0)$:

$$V\left(\tfrac{1}{4}a,0\right)=\frac{\lambda}{2\pi\in_0}\ln\left(\frac{\sqrt{\left(\frac{1}{4}a-a\right)^2}}{\sqrt{\left(\frac{1}{4}a+a\right)^2}}\right)=\frac{\lambda}{2\pi\in_0}\ln\left(\frac{3}{5}\right)$$

Equate $V(x,y)$ and $V\left(\frac{1}{4}a,0\right)$:

$$\frac{3}{5}=\frac{\sqrt{(x-5)^2+y^2}}{\sqrt{(x+5)^2+y^2}}$$

Solve for y to obtain:

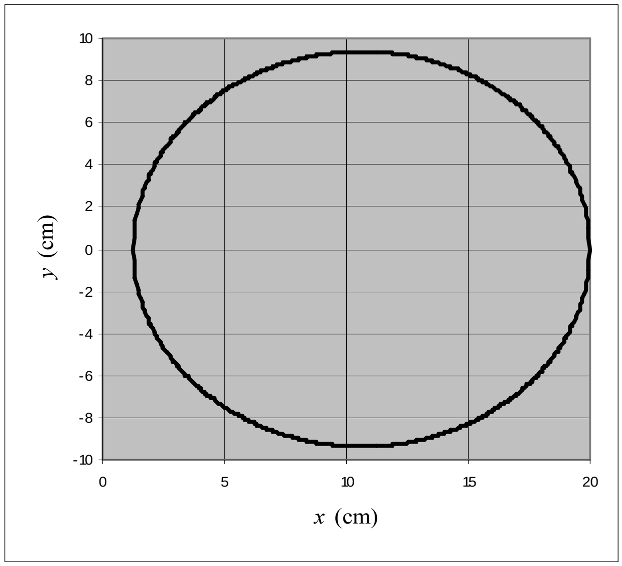
$$\boldsymbol{y}=\boxed{\pm\sqrt{21.25\boldsymbol{x}-\boldsymbol{x}^2-25}}$$

(*c*) A spreadsheet program to plot $y=\pm\sqrt{21.25x-x^2-25}$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A2	1.25	$\frac{1}{4}a$
A3	A2 + 0.05	$x+\Delta x$
B2	SQRT(21.25*A2 – A2^2 – 25)	$y=\sqrt{21.25x-x^2-25}$
B4	–B2	$y=-\sqrt{21.25x-x^2-25}$

	A	B	C
1	x	y_{pos}	y_{neg}
2	1.25	0.00	0.00
3	1.30	0.97	–0.97
4	1.35	1.37	–1.37
5	1.40	1.67	–1.67
6	1.45	1.93	–1.93
7	1.50	2.15	–2.15
370	19.65	2.54	–2.54
371	19.70	2.35	–2.35
372	19.75	2.15	–2.15
373	19.80	1.93	–1.93
374	19.85	1.67	–1.67
375	19.90	1.37	–1.37
376	19.95	0.97	–0.97

The following graph shows the equipotential curve in the *xy* plane for
$V\left(\tfrac{1}{4}a,0\right)=\dfrac{\lambda}{2\pi\in_0}\ln\left(\dfrac{3}{5}\right)$.



76 •• The equipotential curve graphed in Problem 75 should be a circle. (*a*) Show mathematically that it is a circle. (*b*) The equipotential circle in the *xy* plane is the intersection of a three-dimensional equipotential surface and the *xy* plane. Describe the three-dimensional surface using one or two sentences.

Picture the Problem We can use the expression for the potential at any point in the *xy* plane to show that the equipotential curve is a circle.

(*a*) Equipotential surfaces must satisfy the condition:

$$V=\frac{\lambda}{2\pi\in_0}\ln\left(\frac{r_2}{r_1}\right)$$

Solving for r_2/r_1 yields:

$$\frac{r_2}{r_1}=e^{\frac{2\pi\in_0 V}{\lambda}}=C \text{ or } r_2=Cr_1$$

where C is a constant.

Substitute for r_1 and r_2 to obtain:

$$(x-a)^2+y^2=C^2\left[(x+a)^2+y^2\right]$$

Expand this expression, combine like terms, and simplify to obtain:

$$x^2+2a\frac{C^2+1}{C^2-1}x+y^2=-a^2$$

Complete the square by adding $\left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right]$ to both sides of the equation:

$$x^2+2a\frac{C^2+1}{C^2-1}x+\left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right]+y^2=\left[a^2\left(\frac{C^2+1}{C^2-1}\right)^2\right]-a^2=\frac{4a^2C^2}{\left(C^2-1\right)^2}$$

Letting $\alpha=2a\frac{C^2+1}{C^2-1}$ and $\beta=2a\frac{C}{C^2-1}$ yields:

$\boxed{(x+\alpha)^2+y^2=\beta^2,}$ the equation of circle in the xy plane with its center at $(-\alpha,0)$.

(b) The three-dimensional surfaces are cylinders whose axes are parallel to the wires and are in the $y=0$ plane.

77 ••• The hydrogen atom in its ground state can be modeled as a positive point charge of magnitude $+e$ (the proton) surrounded by a negative charge distribution that has a charge density (the electron) that varies with the distance from the center of the proton r as: $\rho(r)=-\rho_0e^{-2r/a}$ (a result obtained from quantum mechanics), where a = 0.523 nm is the most probable distance of the electron from the proton. (a) Calculate the value of ρ_0 needed for the hydrogen atom to be neutral. (b) Calculate the electrostatic potential (relative to infinity) of this system as a function of the distance r from the proton.

Picture the Problem (a) Expressing the charge dq in a spherical shell of volume $4\pi r^2dr$ within a distance r of the proton and setting the integral of this expression equal to the charge of the electron will allow us to solve for the value of ρ_0 needed for charge neutrality. (b) The electrostatic potential of this system is the sum of the electrostatic potentials due to the proton and electron's charge density. The potential due to the proton is ke/r. We can use the given charge density to express the potential function due to the electron's charge distribution and then integrate this function to find the potential due to the electron.

(a) Express the charge dq in a spherical shell of volume $dV=4\pi r^2dr$ at a distance r from the proton:

$$dq=\rho dV=\left(-\rho_0e^{-2r/a}\right)\left(4\pi r^2dr\right)$$
$$=-4\pi\rho_0r^2e^{-2r/a}dr$$

Express the condition for charge neutrality:

$$e=-4\pi\rho_0\int_0^\infty r^2e^{-2r/a}dr$$

From a table of integrals we have:

$$\int x^2e^{bx}dx=\frac{e^{bx}}{b^3}\left(b^2x^2-2bx+2\right)$$

Using this result yields:

$$\int_0^\infty r^2 e^{-2r/a}\,dr = \frac{a^3}{4}$$

Substitute in the expression for e to obtain:

$$e = -4\pi\rho_0 \frac{a^3}{4} = -\pi\rho_0 a^3 \Rightarrow \rho_0 = -\frac{e}{\pi a^3}$$

Substitute numerical values and evaluate ρ_0:

$$\rho_0 = -\frac{1.602\times10^{-19}\ \text{C}}{\pi(0.523\ \text{nm})^3} = \boxed{-3.56\times10^{8}\ \text{C/m}^3}$$

(b) The electrostatic potential of this proton-electron system is the sum of the electrostatic potentials due to the proton and the electron's charge density:

$$V = V_1 + V_2 \qquad (1)$$

where

$$V_1 = \frac{ke}{r} + \frac{kQ_1}{r},$$

$$V_2 = \int_r^\infty \frac{k\rho(r')4\pi r'^2\,dr'}{r'} \qquad (2)$$

and

$$Q_1 = \int_0^r \rho(r')4\pi r'^2\,dr'$$

Substituting for $\rho(r')$ in the expression for Q_1 yields:

$$Q_1 = 4\pi\rho_0 \int_0^r r'^2 e^{-2r'/a}\,dr'$$

From a table of integrals we have:

$$\int x^2 e^{bx}\,dx = \frac{e^{bx}}{b^3}\left(b^2x^2 - 2bx + 2\right)$$

Using this result to evaluate $\int_0^r r'^2 e^{-2r'/a}\,dr'$ yields:

$$\begin{aligned}
\int_0^r x^2 e^{-2x/a}\,dr' &= -\frac{a^3 e^{-2x/a}}{8}\left(\frac{4}{a^2}x^2 + 2\left(\frac{2}{a}\right)x + 2\right)\Bigg|_0^r \\
&= -\frac{a^3 e^{-2r/a}}{8}\left(\frac{4}{a^2}r^2 + \frac{4}{a}r + 2\right) - \left(-\frac{a^3}{8}(2)\right) \\
&= -\frac{a^3 e^{-2r/a}}{8}\left(\frac{4}{a^2}r^2 + \frac{4}{a}r + 2\right) + \frac{a^3}{4}
\end{aligned}$$

and

$$Q_1 = 4\pi\rho_0\left[-\frac{a^3e^{-2r/a}}{8}\left(\frac{4}{a^2}r^2+\frac{4}{a}r+2\right)+\frac{a^3}{4}\right]$$

Substituting for Q_1 in the expression for V_1 yields:

$$V_1 = \frac{ke}{r}+\frac{4\pi k\rho_0}{r}\left[-\frac{a^3e^{-2r/a}}{8}\left(\frac{4}{a^2}r^2+\frac{4}{a}r+2\right)+\frac{a^3}{4}\right]$$

Substitute for ρ_0 from Part (*a*) and simplify to obtain:

$$\begin{aligned}V_1 &= \frac{ke}{r}+\frac{4\pi k\left(-\frac{e}{\pi a^3}\right)}{r}\left[-\frac{a^3e^{-2r/a}}{8}\left(\frac{4}{a^2}r^2+\frac{4}{a}r+2\right)+\frac{a^3}{4}\right]\\ &= \frac{ke}{r}e^{-2r/a}\left(\frac{2}{a^2}r^2+\frac{2}{a}r+1\right)\end{aligned}$$

Substituting for $\rho(r')$ in equation (2) and simplifying yields:

$$\begin{aligned}V_2 &= \int_r^\infty \frac{k\rho_0 e^{-2r/a}4\pi r'^2\,dr'}{r'}\\ &= 4\pi k\rho_0\int_r^\infty e^{-2r/a}r'\,dr'\end{aligned}$$

From a table of integrals we have:

$$\int xe^{bx}dx = \frac{e^{bx}}{b^2}(bx-1)$$

Using this result to evaluate $\int_r^\infty e^{-2r/a}r'\,dr'$ yields:

$$\begin{aligned}\int_r^\infty e^{-2x/a}x\,dx &= \frac{a^2}{4}e^{-2x/a}\left(\frac{2}{a}x+1\right)\Big|_r^\infty\\ &= -\frac{a^2}{4}e^{-2x/a}\left(\frac{2}{a}r+1\right)\end{aligned}$$

Substitute for $\int_r^\infty e^{-2r/a}r'\,dr'$ and ρ_0 in the expression for V_2 to obtain:

$$\begin{aligned}V_2 &= 4\pi k\left(-\frac{e}{\pi a^3}\right)e^{-2x/a}\left[-\frac{a^2}{4}\left(\frac{2}{a}r+1\right)\right]\\ &= ke\left(\frac{1}{a}\right)e^{-2r/a}\left(\frac{2}{a}r+1\right)\end{aligned}$$

Substituting for V_1 and V_2 in equation (1) and simplifying yields:

$$V = \frac{ke}{r}e^{-2r/a}\left(\frac{2}{a^2}r^2+\frac{2}{a}r+1\right)+\frac{ke}{a}e^{-2r/a}\left(\frac{2}{a}r+1\right)=\boxed{ke\left(\frac{1}{a}+\frac{1}{r}\right)e^{-2r/a}}$$

78 •• Charge is supplied to the metal dome of a Van de Graaff generator by the belt at the rate of 200 μC/s when the potential difference between the belt and the dome is 1.25 MV. The dome transfers charge to the atmosphere at the same rate, so the 1.25 MV potential difference is maintained. What minimum power is needed to drive the moving belt and maintain the 1.25 MV potential difference?

Picture the Problem We can use the definition of power and the expression for the work done in moving a charge through a potential difference to find the minimum power needed to drive the moving belt.

Relate the power needed to drive the moving belt to the rate at which the generator is doing work:

$$P = \frac{dW}{dt}$$

Express the work done in moving a charge q through a potential difference ΔV:

$$W = q\Delta V$$

Substitute for W to obtain:

$$P = \frac{d}{dt}\left[q\Delta V\right] = \Delta V\frac{dq}{dt}$$

Substitute numerical values and evaluate P:

$$P = (1.25\,\text{MV})(200\,\mu\text{C/s}) = \boxed{250\,\text{W}}$$

79 •• A positive point charge $+Q$ is located on the x axis at $x = -a$. (a) How much work is required to bring an identical point charge from infinity to the point on the x axis at $x = +a$? (b) With the two identical point charges in place at $x = -a$ and $x = +a$, how much work is required to bring a third point charge $-Q$ from infinity to the origin? (c) How much work is required to move the charge $-Q$ from the origin to the point on the x axis at $x = 2a$ along the semicircular path shown (Figure 23-35)?

Picture the Problem We can use $W_{q\to\text{final position}} = Q\Delta V_{i\to f}$ to find the work required to move these charges between the given points.

(a) Express the required work in terms of the charge being moved and the potential due to the charge at $x = +a$ and simplify to obtain:

$$W_{+Q\to+a} = Q\Delta V_{\infty\to+a} = Q\left[V(a) - V(\infty)\right]$$
$$= QV(a) = Q\left(\frac{kQ}{2a}\right) = \boxed{\frac{kQ^2}{2a}}$$

(*b*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$ and simplify to obtain:

$$W_{-Q\to 0} = -Q\Delta V_{\infty\to 0} = -Q\left[V(0) - V(\infty)\right]$$
$$= -QV(0) = -Q\left[V_{\substack{\text{charge}\\ \text{at } -a}} + V_{\substack{\text{charge}\\ \text{at } +a}}\right]$$
$$= -Q\left(\frac{kQ}{a} + \frac{kQ}{a}\right) = \boxed{\frac{-2kQ^2}{a}}$$

(*c*) Express the required work in terms of the charge being moved and the potentials due to the charges at $x = +a$ and $x = -a$ and simplify to obtain:

$$W_{-Q\to 2a} = -Q\Delta V_{0\to 2a}$$
$$= -Q\left[V(2a) - V(0)\right]$$
$$= -Q\left[V_{\substack{\text{charge}\\ \text{at } -a}} + V_{\substack{\text{charge}\\ \text{at } +a}} - V(0)\right]$$
$$= -Q\left(\frac{kQ}{3a} + \frac{kQ}{a} - \frac{2kQ}{a}\right)$$
$$= \boxed{\frac{2kQ^2}{3a}}$$

80 •• A charge of +2.00 nC is uniformly distributed on a ring of radius 10.0 cm that lies in the $x = 0$ plane and is centered at the origin. A point charge of +1.00 nC is initially located on the x axis at $x = 50.0$ cm. Find the work required to move the point charge to the origin.

Picture the Problem Let q represent the charge being moved from $x = 50.0$ cm to the origin, Q the ring charge, and a the radius of the ring. We can use $W_{q\to\text{final position}} = q\Delta V_{i\to f}$, where V is the expression for the axial field due to a ring charge, to find the work required to move q from $x = 50.0$ cm to the origin.

Express the required work in terms of the charge being moved and the potential due to the ring charge at $x = 50.0$ cm and $x = 0$:

$$W = q\Delta V = q\left[V(0) - V(0.500\,\text{m})\right]$$

The potential on the axis of a uniformly charged ring is given by:

$$V(x) = \frac{kQ}{\sqrt{x^2 + a^2}}$$

At $x = 50.0$ cm:

$$V(0.500\,\text{m}) = \frac{kQ}{\sqrt{(0.500\,\text{m})^2 + a^2}}$$

At $x = 0$:

$$V(0) = \frac{kQ}{\sqrt{a^2}} = \frac{kQ}{a}$$

Substituting for $V(0)$ and $V(0.500\text{ m})$ yields:

$$W = q\left[\frac{kQ}{a} - \frac{kQ}{\sqrt{(0.500\text{ m})^2 + a^2}}\right] = kQq\left[\frac{1}{a} - \frac{1}{\sqrt{(0.500\text{ m})^2 + a^2}}\right]$$

Substitute numerical values and evaluate W:

$$W = \left(8.988\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.00\,\text{nC})(1.00\,\text{nC})$$

$$\times\left[\frac{1}{0.100\text{ m}} - \frac{1}{\sqrt{(0.500\text{ m})^2 + (0.100\text{ m})^2}}\right]$$

$$= 1.445\times10^{-7}\text{ J} = \boxed{1.4\times10^{-7}\text{ J}} = 1.445\times10^{-7}\text{ J}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\text{ J}}$$

$$= \boxed{9.0\times10^{11}\text{ eV}}$$

81 •• Two metal spheres each have a radius of 10.0 cm. The centers of the two spheres are 50.0 cm apart. The spheres are initially neutral, but a charge Q is transferred from one sphere to the other, creating a potential difference between the spheres of 100 V. A proton is released from rest at the surface of the positively charged sphere and travels to the negatively charged sphere. (*a*) What is the proton′s kinetic energy just as it strikes the negatively charged sphere? (*b*) At what speed does it strike the sphere?

Picture the Problem The proton′s kinetic energy just as it strikes the negatively charged sphere is the product of its charge and the potential difference through which it has been accelerated. We can find the speed of the proton as it strikes the negatively charged sphere from its kinetic energy and, in turn, its kinetic energy from the potential difference through which it is accelerated.

(*a*) Apply the work-kinetic energy theorem to the proton to obtain:

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}}$$

or, because $K_{\text{i}} = 0$,

$$W_{\text{net}} = K_{\text{f}} = K_{\text{p}}$$

The net work done on the proton is given by:

$$W_{\text{net}} = q_{\text{p}}V$$

Equating W_{net} and K_{p} yields:

$$K_{\text{p}} = q_{\text{p}}V = e(100\text{ V}) = \boxed{100\text{ eV}}$$

(*b*) Use the definition of kinetic energy to express the speed of the proton when it strikes the negatively charged sphere:

$$K_p = \tfrac{1}{2}m_p v^2 \Rightarrow v = \sqrt{\frac{2K_p}{m_p}} = \sqrt{\frac{2\Delta V}{m_p}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(100\ \text{V})}{1.673\times10^{-27}\ \text{kg}}} = \boxed{1.38\times10^{5}\ \text{m/s}}$$

82 •• (*a*) Using a **spreadsheet** program, graph $V(z)$ versus z for a uniformly charged ring in the $z = 0$ plane and centered at the origin. The potential on the z axis is given by $V(z) = kQ/\sqrt{a^2+z^2}$ (Equation 23-19). (*b*) Use your graph to estimate the points on the z axis where the electric field strength is greatest.

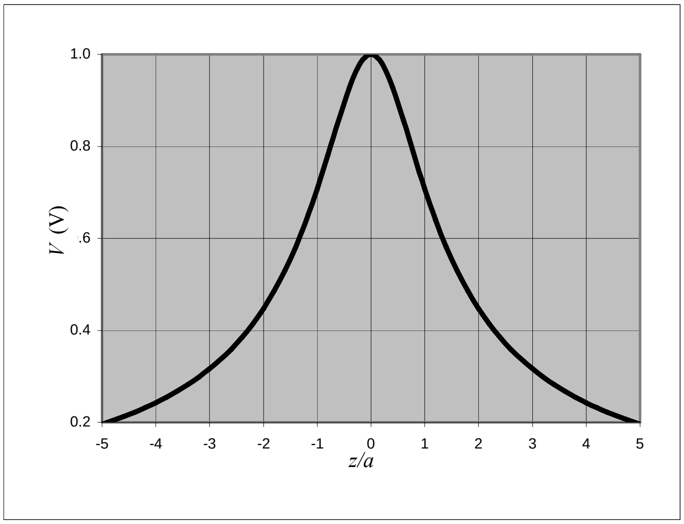
Picture the Problem (*b*) The electric field strength is greatest where the magnitude of the slope of the graph of electric potential is greatest.

(*a*) A spreadsheet solution is shown below for $kQ = a = 1$. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
A4	A3 + 0.1	$z + \Delta z$
B3	1/(1+A3^2)^(1/2)	$\dfrac{kQ}{\sqrt{a^2+z^2}}$

	A	B
1		
2	z/a	$V(z/a)$
3	−5.0	0.196
4	−4.8	0.204
5	−4.6	0.212
6	−4.4	0.222
7	−4.2	0.232
8	−4.0	0.243
9	−3.8	0.254
49	4.2	0.232
50	4.4	0.222
51	4.6	0.212
52	4.8	0.204
53	5.0	0.196

The following graph, plotted using a spreadsheet program, shows V as a function of z/a:



Examining the graph we see that the magnitude of the slope is maximum at $z/a \approx \boxed{0.7}$ and at $z/a \approx \boxed{-0.7}$.

83 •• A spherical conductor of radius R_1 is charged to 20 kV. When it is connected by a long very-thin conducting wire to a second conducting sphere far away, its potential drops to 12 kV. What is the radius of the second sphere?

Picture the Problem Let R_2 be the radius of the second sphere and Q_1 and Q_2 the charges on the spheres when they have been connected by the wire. When the spheres are connected, the charge initially on the sphere of radius R_1 will redistribute until the spheres are at the same potential.

Express the common potential of the spheres when they are connected:

$$12\,\text{kV} = \frac{kQ_1}{R_1} \quad (1)$$

and

$$12\,\text{kV} = \frac{kQ_2}{R_2} \quad (2)$$

Express the potential of the first sphere before it is connected to the second sphere:

$$20\,\text{kV} = \frac{k(Q_1 + Q_2)}{R_1} \quad (3)$$

Solve equation (1) for Q_1:

$$Q_1 = \frac{(12\,\text{kV})R_1}{k}$$

Solve equation (2) for Q_2:

$$Q_2 = \frac{(12\,\text{kV})R_2}{k}$$

Substitute for Q_1 and Q_2 in equation (3) and simplify to obtain:

$$20\,\text{kV} = \frac{k\left(\frac{(12\,\text{kV})R_1}{k} + \frac{(12\,\text{kV})R_2}{k}\right)}{R_1} = 12\,\text{kV} + 12\,\text{kV}\left(\frac{R_2}{R_1}\right)$$

or

$$8 = 12\left(\frac{R_2}{R_1}\right) \Rightarrow R_2 = \boxed{\frac{2}{3}R_1}$$

84 •• A metal sphere centered at the origin has a surface charge density that has a magnitude of 24.6 nC/m^2 and a radius less than 2.00 m. A distance of 2.00 m from the origin, the electric potential is 500 V and the electric field strength is 250 V/m. (Assume the potential is zero very far from the sphere.) (*a*) What is the radius of the metal sphere? (*b*) What is the sign of the charge on the sphere? Explain your answer.

Picture the Problem We can use the definition of surface charge density to relate the radius R of the sphere to its charge Q and the potential function $V(r) = kQ/r$ to relate Q to the potential at r = 2.00 m.

(*a*) Use its definition, relate the surface charge density σ to the charge Q on the sphere and the radius R of the sphere:

$$\sigma = \frac{Q}{4\pi R^2} \Rightarrow R = \sqrt{\frac{Q}{4\pi\sigma}}$$

Relate the potential to the charge on the sphere:

$$V(r) = \frac{kQ}{r} \Rightarrow Q = \frac{rV(r)}{k}$$

Substitute for Q in the expression for R and simplify to obtain:

$$R = \sqrt{\frac{rV(r)}{4\pi k\sigma}} = \sqrt{\frac{4\pi\in_0 rV(r)}{4\pi\sigma}} = \sqrt{\frac{\in_0 rV(r)}{\sigma}}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{\frac{(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(2.00\ \mathrm{m})(500\ \mathrm{V})}{24.6\ \mathrm{nC/m^2}}} = \boxed{60.0\ \mathrm{cm}}$$

(b) The charge on the sphere is positive. The formula for the electric potential outside a uniformly charged spherical shell is $V = kQ/r$. If V is positive, then so is Q.

85 •• Along the central axis of a uniformly charged disk, at a point 0.60 m from the center of the disk, the potential is 80 V and the magnitude of the electric field is 80 V/m. At a distance of 1.5 m, the potential is 40 V and the magnitude of the electric field is 23.5 V/m. (Assume the potential is zero very far from the sphere.) Find the total charge on the disk.

Picture the Problem We can use the definition of surface charge density to relate the radius R of the disk to its charge Q and the potential function $V(r) = kQ/r$ to relate Q to the potential at r = 1.5 m.

Use its definition, relate the surface charge density σ to the charge Q on the disk and the radius R of the disk:

$$\sigma = \frac{Q}{\pi R^2} \Rightarrow Q = \pi\sigma R^2 \qquad (1)$$

Relate the potential at r to the charge on the disk:

$$V(r) = 2\pi k\sigma\left(\sqrt{x^2 + R^2} - x\right)$$

Substitute V(0.60 m) = 80 V:

$$80\ \mathrm{V} = 2\pi k\sigma\left(\sqrt{(0.60\ \mathrm{m})^2 + R^2} - 0.60\ \mathrm{m}\right)$$

Substitute V(1.5 m) = 40 V:

$$40\ \mathrm{V} = 2\pi k\sigma\left(\sqrt{(1.5\ \mathrm{m})^2 + R^2} - 1.5\ \mathrm{m}\right)$$

Divide the first of these equations by the second to obtain:

$$2 = \frac{\sqrt{(0.60\ \mathrm{m})^2 + R^2} - 0.60\ \mathrm{m}}{\sqrt{(1.5\ \mathrm{m})^2 + R^2} - 1.5\ \mathrm{m}}$$

Solving for R yields:

$$R = 0.80\ \mathrm{m}$$

Express the electric field on the axis of a disk charge:

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

Solving for σ yields:

$$\sigma = \frac{E_x}{2\pi k\left(1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right)} = \frac{2\in_0 E_x}{1 - \dfrac{x}{\sqrt{x^2 + R^2}}}$$

Substitute for σ in equation (1) to obtain:

$$Q = \frac{2\pi \in_0 R^2 E_x}{1 - \dfrac{x}{\sqrt{x^2 + R^2}}}$$

Substitute numerical values and evaluate Q:

$$Q = \frac{2\pi\left(8.854\times10^{-12}\ \dfrac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(0.80\ \mathrm{m})^2(23.5\ \mathrm{V/m})}{1 - \dfrac{1.5\ \mathrm{m}}{\sqrt{(1.5\ \mathrm{m})^2 + (0.80\ \mathrm{m})^2}}} = \boxed{7.1\ \mathrm{nC}}$$

86 •• A radioactive ^{210}Po nucleus emits an α-particle that has a charge $+2e$. When the α-particle is a large distance from the nucleus it has a kinetic energy of 5.30 MeV. Assume that the α-particle had negligible kinetic energy as it left at the surface of the nucleus. The "daughter" (or residual) nucleus ^{206}Pb has a charge $+82e$. Determine the radius of the ^{206}Pb nucleus. (Neglect the radius of the α particle and assume the ^{206}Pb nucleus remains at rest.)

Picture the Problem We can use $U = kq_1q_2/r$ to relate the electrostatic potential energy of the particles to their separation.

Express the electrostatic potential energy of the two particles in terms of their charge and separation:

$$U = \frac{kq_1q_2}{r} \Rightarrow r = \frac{kq_1q_2}{U}$$

Substitute numerical values and evaluate r:

$$r = \frac{(8.988\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(82)(1.602\times10^{-19}\ \mathrm{C})^2}{5.30\ \mathrm{MeV}\times\dfrac{1.602\times10^{-19}\ \mathrm{C}}{\mathrm{eV}}} = \boxed{44.6\ \mathrm{fm}}$$

87 ••• **[SSM]** Configuration A consists of two point particles, one particle has a charge of $+q$ and is on the x axis at $x = +d$ and the other particle has a charge of $-q$ and is at $x = -d$ (Figure 23-36a). (a) Assuming the potential is zero at large distances from these charged particles, show that the potential is also zero everywhere on the $x = 0$ plane. (*b*) Configuration *B* consists of a flat metal plate of infinite extent and a point particle located a distance *d* from the plate (Figure 23-36*b*) The point particle has a charge equal to $+q$ and the plate is grounded. (Grounding the plate forces its potential to equal zero.) Choose the line perpendicular to the plate and through the point charge as the *x* axis, and choose the origin at the surface of the plate nearest the particle. (These choices put the particle on the *x* axis at $x = +d$.) For configuration *B*, the electric potential is zero both at all points in the half-space $x \geq 0$ that are very far from the particle and at all points on the $x = 0$ plane—just as was the case for configuration *A*. (*c*) A theorem, called the *uniqueness theorem*, implies that throughout the half-space $x \geq 0$ the potential function *V*—and thus the electric field $\vec{E}$—for the two configurations are identical. Using this result, obtain the electric field $\vec{E}$ at every point in the $x = 0$ plane in the configuration *B*. (The uniqueness theorem tells us that in configuration *B* the electric field at each point in the $x = 0$ plane is the same as it is in configuration *A*.) Use this result to find the surface charge density σ at each point in the conducting plane (in configuration *B*).

Picture the Problem We can use the relationship between the potential and the electric field to show that this arrangement is equivalent to replacing the plane by a point charge of magnitude $-q$ located a distance *d* beneath the plane. In (*b*) we can first find the field at the plane surface and then use $\sigma = \epsilon_0 E$ to find the surface charge density. In (*c*) the work needed to move the charge to a point 2*d* away from the plane is the product of the potential difference between the points at distances 2*d* and 3*d* from $-q$ multiplied by the separation Δx of these points.

(*a*) The potential anywhere on the plane is 0 and the electric field is perpendicular to the plane in both configurations, so they must give the same potential everywhere in the *xy* plane. Also, because the net charge is zero, the potential at infinity is zero.

(*b*) The surface charge density is given by:

$$\sigma = \epsilon_0 E \quad (1)$$

At any point on the plane, the electric field points in the negative *x* direction and has magnitude:

$$E = \frac{kq}{d^2 + r^2}\cos\theta$$

where θ is the angle between the horizontal and a vector pointing from the positive charge to the point of interest on the *xz* plane and *r* is the distance along the plane from the origin (that is, directly to the left of the charge).

Because $\cos\theta = \dfrac{d}{\sqrt{d^2 + r^2}}$:

$$E = \frac{kq}{d^2 + r^2}\frac{d}{\sqrt{d^2 + r^2}} = \frac{kqd}{\left(d^2 + r^2\right)^{3/2}}$$

$$= \frac{qd}{4\pi \in_0 \left(d^2 + r^2\right)^{3/2}}$$

Substitute for E in equation (1) and simplify to obtain:

$$\sigma = \boxed{\frac{qd}{4\pi\left(d^2 + r^2\right)^{3/2}}}$$

88 ••• A particle that has a mass m and a positive charge q is constrained to move along the x-axis. At $x = -L$ and $x = L$ are two ring charges of radius L (Figure 23-38). Each ring is centered on the x-axis and lies in a plane perpendicular to it. Each ring has a total positive charge Q uniformly distributed on it. (*a*) Obtain an expression for the potential $V(x)$ on the x axis due to the charge on the rings. (*b*) Show that $V(x)$ has a minimum at $x = 0$. (*c*) Show that for $|x| << L$, the potential approaches the form $V(x) = V(0) + \alpha x^2$. (*d*) Use the result of Part (*c*) to derive an expression for the angular frequency of oscillation of the mass m if it is displaced slightly from the origin and released. (Assume the potential equals zero at points far from the rings.)

Picture the Problem We can express the potential due to the ring charges as the sum of the potentials due to each of the ring charges. To show that $V(x)$ is a minimum at $x = 0$, we must show that the first derivative of $V(x) = 0$ at $x = 0$ and that the second derivative is positive. In Part (*c*) we can use a Taylor expansion to show that, for $|\boldsymbol{x}| << \boldsymbol{L}$, the potential approaches the form $V(x) = V(0) + \alpha x^2$. In Part (*d*) we can obtain the potential energy function from the potential function and, noting that it is quadratic in x, find the "spring" constant and the angular frequency of oscillation of the particle provided its displacement from its equilibrium position is small.

(*a*) Express the potential due to the ring charges as the sum of the potentials due to each of their charges:

$$V(x) = V_{\substack{\text{ring to}\\\text{the left}}} + V_{\substack{\text{ring to}\\\text{the right}}}$$

The potential for a ring of charge is:

$$V(x) = \frac{kQ}{\sqrt{x^2 + a^2}}$$

where a is the radius of the ring and Q is its charge.

For the ring to the left we have:

$$V_{\substack{\text{ring to}\\\text{the left}}} = \frac{kQ}{\sqrt{(x+L)^2+L^2}}$$

For the ring to the right we have:

$$V_{\substack{\text{ring to}\\\text{the right}}} = \frac{kQ}{\sqrt{(x-L)^2+L^2}}$$

Substitute for $V_{\substack{\text{ring to}\\\text{the left}}}$ and $V_{\substack{\text{ring to}\\\text{the right}}}$ to obtain:

$$V(x) = \boxed{\frac{kQ}{\sqrt{(x+L)^2+L^2}} + \frac{kQ}{\sqrt{(x-L)^2+L^2}}}$$

(*b*) Evaluate dV/dx to obtain:

$$\frac{dV}{dx} = kQ\left\{\frac{L-x}{\left[(L-x)^2+L^2\right]^{3/2}} - \frac{L+x}{\left[(L+x)^2+L^2\right]^{3/2}}\right\} = 0 \text{ for extrema}$$

Solving for x yields:

$$x = 0$$

Evaluate d^2V/dx^2 to obtain:

$$\frac{d^2V}{dx^2} = kQ\left\{\frac{3(L-x)^2}{\left[(L-x)^2+L^2\right]^{5/2}} - \frac{1}{\left[(L-x)^2+L^2\right]^{3/2}} + \frac{3(L+x)^2}{\left[(L+x)^2+L^2\right]^{5/2}} - \frac{1}{\left[(L-x)^2+L^2\right]^{3/2}}\right\}$$

Evaluating this expression for $x = 0$ yields:

$$\frac{d^2V(0)}{dx^2} = \frac{kQ}{2\sqrt{2}L^3} > 0$$

$$\Rightarrow \boxed{V(x) \text{ is a maximum at } x = 0.}$$

(*c*) The Taylor expansion of $V(x)$ is:

$$V(x) = V(0) + V'(0)x + \tfrac{1}{2}V''(0)x^2 + \text{higher order terms}$$

For $x << L$:

$$V(x) \approx V(0) + V'(0)x + \tfrac{1}{2}V''(0)x^2$$

Substitute our results from Parts (*a*) and (*b*) to obtain:

$$V(x)=\frac{\sqrt{2}kQ}{L}+(0)x+\tfrac{1}{2}\left(\frac{kQ}{2\sqrt{2}L^3}\right)x^2$$
$$=\frac{\sqrt{2}kQ}{L}+\frac{kQ}{4\sqrt{2}L^3}x^2$$

or

$$V(x)=\boxed{V(0)+\alpha x^2}$$

where

$$V(0)=\boxed{\frac{\sqrt{2}kQ}{L}} \text{ and } \alpha=\boxed{\frac{kQ}{4\sqrt{2}L^3}}$$

(*d*) Express the angular frequency of oscillation of a simple harmonic oscillator:

$$\omega=\sqrt{\frac{k'}{m}}$$

where k' is the restoring constant.

From our result for Part (*c*) and the definition of electric potential:

$$U(x)=qV(0)+\frac{1}{2}\left(\frac{kqQ}{2\sqrt{2}L^3}\right)x^2$$
$$=qV(0)+\tfrac{1}{2}k'x^2$$

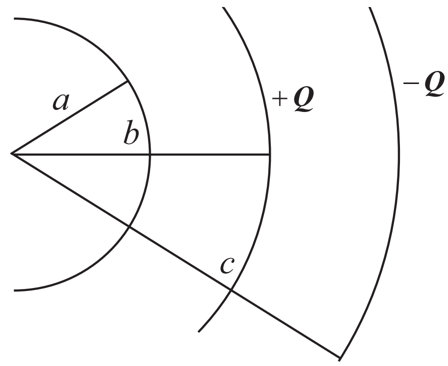
where $k'=\dfrac{kqQ}{2\sqrt{2}L^3}$

Substituting for k' in the expression for ω yields:

$$\omega=\boxed{\sqrt{\frac{kqQ}{2m\sqrt{2}L^3}}}$$

89 ••• Three concentric conducting thin spherical shells have radii *a*, *b*, and *c* so that $a < b < c$. Initially, the inner shell is uncharged, the middle shell has a positive charge $+Q$, and the outer shell has a charge $-Q$. (Assume the potential equals zero at points far from the shells.) (*a*) Find the electric potential of each of the three shells. (*b*) If the inner and outer shells are now connected by a conducting wire that is insulated as it passes through a small hole in the middle shell, what is the electric potential of each of the three shells, and what is the final charge on each shell?

Picture the Problem The diagram shows part of the shells in a cross-sectional view under the conditions of Part (*a*) of the problem. We can use Gauss's law to find the electric field in the regions defined by the three surfaces and then find the electric potentials from the electric fields. In Part (*b*) we can use the redistributed charges to find the charge on and potentials of the three surfaces.



(*a*) Apply Gauss's law to a spherical Gaussian surface of radius $r \geq c$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_r = 0$ because the net charge enclosed by the Gaussian surface is zero.

Because $E_r(c) = 0$:

$$V(c) = \boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius $b < r < c$ to obtain:

$$E_r\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

and

$$E_r(b < r < c) = \frac{kQ}{r^2}$$

Use $E_r(b < r < c)$ to find the potential difference between c and b:

$$V(b) - V(c) = -kQ\int_c^b \frac{dr}{r^2}$$

$$= kQ\left(\frac{1}{b} - \frac{1}{c}\right)$$

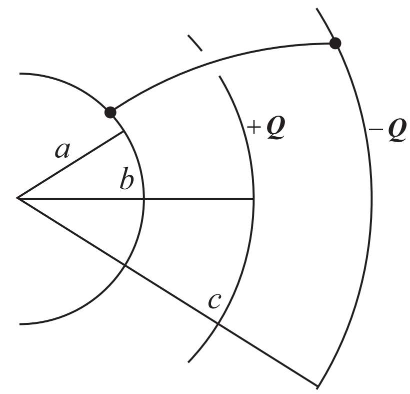
Because $V(c) = 0$:

$$V(b) = \boxed{kQ\left(\frac{1}{b} - \frac{1}{c}\right)}$$

The inner shell carries no charge, so the field between $r = a$ and $r = b$ is zero and:

$$V(a) = V(b) = \boxed{kQ\left(\frac{1}{b} - \frac{1}{c}\right)}$$

(*b*) When the inner and outer shells are connected their potentials become equal as a consequence of the redistribution of charge.



The charges on surfaces *a* and *c* are related according to:

$$Q_a + Q_c = -Q \qquad (1)$$

Q_b does not change with the connection of the inner and outer shells:

$$Q_b = \boxed{Q}$$

Express the potentials of shells *a* and *c*:

$$V(a) = V(c) = \boxed{0}$$

In the region between the $r = a$ and $r = b$, the field is kQ_a/r^2 and the potential at $r = b$ is then:

$$V(b) = kQ_a\left(\frac{1}{b} - \frac{1}{a}\right) \qquad (2)$$

The enclosed charge for $b < r < c$ is $Q_a + Q$, and by Gauss's law the field in this region is:

$$E_{b<r<c} = \frac{k(Q_a + Q)}{r^2}$$

Express the potential difference between *b* and *c*:

$$V(c) - V(b) = k(Q_a + Q)\left(\frac{1}{c} - \frac{1}{b}\right)$$
$$= -V(b)$$

because $V(c) = 0$.

Solve for $V(b)$ to obtain:

$$V(b) = k(Q_a + Q)\left(\frac{1}{b} - \frac{1}{c}\right) \qquad (3)$$

Equate equations (2) and (3) and solve for Q_a to obtain:

$$Q_a = \boxed{-Q\frac{a(c-b)}{b(c-a)}} \qquad (4)$$

Substitute equation (4) in equation (1) and solve for Q_c to obtain:

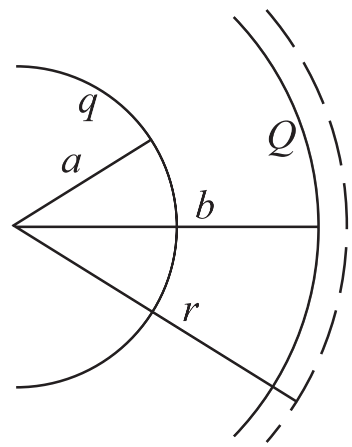
$$Q_c = \boxed{-Q\frac{c(b-a)}{b(c-a)}} \qquad (5)$$

Substitute equations (4) and (5) in equation (3) to obtain:

$$V(b)=\boxed{kQ\frac{(c-b)(b-a)}{b^2(c-a)}}$$

90 ••• Consider two concentric spherical thin metal shells of radii a and b, where $b > a$. The outer shell has a charge Q, but the inner shell is grounded. This means that the potential on the inner shell is the same as the potential at points far from the shells. Find the charge on the inner shell.

Picture the Problem The diagram shows a cross-sectional view of a portion of the concentric spherical shells. Let the charge on the inner shell be q. The dashed line represents a spherical Gaussian surface over which we can integrate $\vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}dA$ in order to find E_r for $r \geq b$. We can find $V(b)$ from the integral of E_r between $r = \infty$ and $r = b$. We can obtain a second expression for $V(b)$ by considering the potential difference between a and b and solving the two equations simultaneously for the charge q on the inner shell.



Apply Gauss's law to a spherical surface of radius $r \geq b$:

$$E_r\left(4\pi r^2\right)=\frac{Q+q}{\epsilon_0}\Rightarrow E_r=\frac{k(Q+q)}{r^2}$$

Use E_r to find $V(b)$:

$$V(b)=-k(Q+q)\int_{\infty}^{b}\frac{dr}{r^2}=\frac{k(Q+q)}{b}$$

We can also determine $V(b)$ by considering the potential difference between a, i.e., 0 and b:

$$V(b)=kq\left(\frac{1}{b}-\frac{1}{a}\right)$$

Equate these expressions for $V(b)$ to obtain:

$$\frac{k(Q+q)}{b}=ka\left(\frac{1}{b}-\frac{1}{a}\right)\Rightarrow q=\boxed{-\frac{a}{b}Q}$$

91 ••• **[SSM]** Show that the total work needed to assemble a uniformly charged sphere that has a total charge of Q and radius R is given by $3Q^2/(20\pi\epsilon_0 R)$. Energy conservation tells us that this result is the same as the resulting electrostatic potential energy of the sphere. *Hint: Let ρ be the charge density of the sphere that has charge Q and radius R. Calculate the work dW to bring in charge dq from infinity to the surface of a uniformly charged sphere that has radius r (r < R) and charge density ρ. (No additional work is required to smear dq throughout a spherical shell of radius r, thickness dr, and charge density ρ. Why?)*

Picture the Problem We can use the hint to derive an expression for the electrostatic potential energy dU required to bring in a layer of charge of thickness dr and then integrate this expression from $r = 0$ to R to obtain an expression for the required work.

If we build up the sphere in layers, then at a given radius r the net charge on the sphere will be given by:

$$Q(r) = Q\left(\frac{r}{R}\right)^3$$

When the radius of the sphere is r, the potential relative to infinity is:

$$V(r) = \frac{Q(r)}{4\pi \in_0 r} = \frac{Q}{4\pi \in_0}\frac{r^2}{R^3}$$

Express the work dW required to bring in charge dQ from infinity to the surface of a uniformly charged sphere of radius r:

$$\begin{aligned} dW &= dU = V(r)dQ \\ &= \frac{Q}{4\pi \in_0}\frac{r^2}{R^3}\left(4\pi r^2 \frac{3Q}{4\pi R^3}dr\right) \\ &= \frac{3Q^2}{4\pi \in_0 R^6}r^4 dr \end{aligned}$$

Integrate dW from 0 to R to obtain:

$$\begin{aligned} W = U &= \frac{3Q^2}{4\pi \in_0 R^6}\int_0^R r^4 dr \\ &= \frac{3Q^2}{4\pi \in_0 R^6}\left[\frac{r^5}{5}\right]_0^R = \frac{3Q^2}{20\pi \in_0 R} \\ &= \boxed{\frac{3Q^2}{20\pi \in_0 R}} \end{aligned}$$

92 ••• (*a*) Use the result of Problem 91 to calculate the *classical electron radius*, the radius of a uniform sphere that has a charge $-e$ has and an electrostatic potential energy equal to the rest energy of the electron (5.11×10^5 eV). Comment on the shortcomings of this model for the electron. (*b*) Repeat the calculation in Part (*a*) for a proton using its rest energy of 938 MeV. Experiments indicate the proton has an approximate radius of about 1.2×10^{-15} m. Is your result close to this value?

Picture the Problem We can equate the rest energy of an electron and the result of Problem 91 in order to obtain an expression that we can solve for the classical electron radius.

(*a*) From Problem 91 we have:

$$U = \frac{3e^2}{20\pi \in_0 R}$$

The rest energy of the electron is given by:

$$E_0 = m_0 c^2$$

Equate these energies to obtain:

$$\frac{3e^2}{20\pi \in_0 R} = m_0 c^2$$

Solving for *R* yields:

$$R = \frac{3e^2}{20\pi \in_0 m_0 c^2} \qquad (1)$$

Substitute numerical values and evaluate *R*:

$$R = \frac{3\left(1.602\times10^{-19}\ \text{C}\right)^2}{20\pi\left(8.854\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(5.11\times10^{5}\ \text{eV}\times\frac{1.602\times10^{-19}\ \text{J}}{\text{eV}}\right)} = \boxed{1.69\times10^{-15}\ \text{m}}$$

This model does not explain how the electron holds together against its own mutual repulsion.

(*b*) For a proton, equation (1) yields:

$$R = \frac{3\left(1.602\times10^{-19}\ \text{C}\right)^2}{20\pi\left(8.854\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(938\ \text{MeV}\times\frac{1.602\times10^{-19}\ \text{J}}{\text{eV}}\right)} = \boxed{9.21\times10^{-19}\ \text{m}}$$

This result is way too small to agree with the experimental value of 1.2×10^{-15} m.

93 ••• [SSM] (*a*) Consider a uniformly charged sphere that has radius *R* and charge *Q* and is composed of an incompressible fluid, such as water. If the sphere fissions (splits) into two halves of equal volume and equal charge, and if these halves stabilize into uniformly charged spheres, what is the radius *R*′ of each? (*b*) Using the expression for potential energy shown in Problem 90, calculate the change in the total electrostatic potential energy of the charged fluid. Assume that the spheres are separated by a large distance.

Picture the Problem Because the post-fission volumes of the fission products are equal, we can express the post-fission radii in terms of the radius of the pre-fission sphere.

(*a*) Relate the initial volume V of the uniformly charged sphere to the volumes V' of the fission products:

$$V = 2V'$$

Substitute for V and V':

$$\tfrac{4}{3}\pi R^3 = 2\left(\tfrac{4}{3}\pi R'^3\right)$$

Solving for R' yields:

$$R' = \frac{1}{\sqrt[3]{2}}R = \boxed{0.794R}$$

(*b*) Express the difference ΔE in the total electrostatic energy as a result of fissioning:

$$\Delta E = E' - E$$

From Problem 91 we have:

$$E = \frac{3Q^2}{20\pi\in_0 R}$$

After fissioning:

$$E' = 2\left(\frac{3Q'^2}{20\pi\in_0 R'}\right) = 2\left[\frac{3\left(\frac{1}{2}Q\right)^2}{20\pi\in_0 \frac{1}{\sqrt[3]{2}}R}\right]$$

$$= \frac{\sqrt[3]{2}}{2}\left(\frac{3Q^2}{20\pi\in_0 R}\right) = 0.630E$$

Substitute for E and E' to obtain:

$$\Delta E = 0.630E - E = \boxed{-0.370E}$$

94 ••• Problem 93 can be modified to be used as a very simple model for nuclear fission. When a ^{235}U nucleus absorbs a neutron, it can fission into the fragments ^{140}Xe, ^{94}Sr, and 2 neutrons. The ^{235}U has 92 protons, while ^{140}Xe has 54 protons and ^{94}Sr has 38 protons. Estimate the energy released during this fission process (in MeV), assuming that the mass density of the nucleus is constant and has a value of 4×10^{17} kg/m^3.

Picture the Problem We can use the definition of density to express the radius R of a nucleus as a function of its atomic mass N. We can then use the result derived in Problem 93 to express the electrostatic energies of the ^{235}U nucleus and the nuclei of the fission fragments ^{140}Xe and ^{94}Sr.

The energy released by this fission process is:

$$\Delta E = U_{^{235}\text{U}} - \left(U_{^{140}\text{Xe}} + U_{^{94}\text{Sr}}\right) \quad (1)$$

Express the mass of a nucleus in terms of its density and volume:

$$Nm = \tfrac{4}{3}\rho\pi R^3 \Rightarrow R = \sqrt[3]{\frac{3Nm}{4\pi\rho}}$$

where N is the nuclear number.

Substitute numerical values and evaluate R as a function of N:

$$R = \sqrt[3]{\frac{3(1.660\times10^{-27}\ \text{kg})}{4\pi(4\times10^{17}\ \text{kg/m}^3)}}N^{1/3} = (9.97\times10^{-16}\ \text{m})N^{1/3}$$

The 'radius' of the ^{235}U nucleus is therefore:

$$R_U = (9.97\times10^{-16}\ \text{m})(235)^{1/3} = 6.15\times10^{-15}\ \text{m}$$

From Problem 91 we have:

$$U = \frac{3Q^2}{20\pi\in_0 R}$$

Substitute numerical values and evaluate the electrostatic energy of the ^{235}U nucleus:

$$\boldsymbol{U}_{^{235}\text{U}} = \frac{3(92\times1.602\times10^{-19}\ \text{C})^2}{20\boldsymbol{\pi}\left(8.854\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(6.15\times10^{-15}\ \text{m})}$$

$$= 1.90\times10^{-10}\ \text{J}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\ \text{J/eV}} = 1189\,\text{MeV}$$

The radii of ^{140}Xe and ^{94}Sr are:

$$R_{^{149}\text{Xe}} = (9.97\times10^{-16}\ \text{m})(140)^{1/3} = 5.177\times10^{-15}\ \text{m}$$

and

$$R_{^{94}\text{Sr}} = (9.97\times10^{-16}\ \text{m})(94)^{1/3} = 4.533\times10^{-15}\ \text{m}$$

Proceed as above to find the electrostatic energy of the fission fragments ^{140}Xe and ^{94}Sr:

$$U_{^{140}\mathrm{Xe}} = \frac{3\left(54\times1.602\times10^{-19}\ \mathrm{C}\right)^2}{20\pi\left(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(5.18\times10^{-15}\ \mathrm{m}\right)}$$
$$= 7.791\times10^{-11}\ \mathrm{J}\times\frac{1\,\mathrm{eV}}{1.602\times10^{-19}\ \mathrm{J/eV}} = 486\,\mathrm{MeV}$$

and

$$U_{^{94}\mathrm{Sr}} = \frac{3\left(38\times1.602\times10^{-19}\ \mathrm{C}\right)^2}{20\pi\left(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.53\times10^{-15}\ \mathrm{m}\right)}$$
$$= 4.412\times10^{-11}\ \mathrm{J}\times\frac{1\,\mathrm{eV}}{1.602\times10^{-19}\ \mathrm{J/eV}} = 275\,\mathrm{MeV}$$

Substitute for $U_{^{235}\mathrm{U}}$, $U_{^{140}\mathrm{Xe}}$, and $U_{^{94}\mathrm{Sr}}$ in equation (1) and evaluate ΔE:

$$\Delta E = 1189\,\mathrm{MeV} - \left(486\,\mathrm{MeV} + 275\,\mathrm{MeV}\right) \approx \boxed{428\,\mathrm{MeV}}$$