Chapter 9
Rotation

Conceptual Problems

1 • Two points are on a disk that is turning about a fixed-axis through its center, perpendicular to the disk and through its center, at increasing angular velocity. One point on the rim and the other point is halfway between the rim and the center. (*a*) Which point moves the greater distance in a given time? (*b*) Which point turns through the greater angle? (*c*) Which point has the greater speed? (*d*) Which point has the greater angular speed? (*e*) Which point has the greater tangential acceleration? (*f*) Which point has the greater angular acceleration? (*g*) Which point has the greater centripetal acceleration?

Determine the Concept (*a*) Because r is greater for the point on the rim, it moves the greater distance. (*b*) Both points turn through the same angle. (*c*) Because r is greater for the point on the rim, it has the greater speed. (*d*) Both points have the same angular speed. (*e*) Both points have zero tangential acceleration. (*f*) Both have zero angular acceleration. (*g*) Because r is greater for the point on the rim, it has the greater centripetal acceleration.

2 • True or false:

(*a*) Angular speed and linear velocity have the same dimensions.
(*b*) All parts of a wheel rotating about a fixed axis must have the same angular speed.
(*c*) All parts of a wheel rotating about a fixed axis must have the same angular acceleration.
(*d*) All parts of a wheel rotating about a fixed axis must have the same centripetal acceleration.

(*a*) False. Angular speed has the dimensions $[1/\boldsymbol{T}]$ whereas linear velocity has dimensions $[\boldsymbol{L}/\boldsymbol{T}]$.

(*b*) True. The angular speed of all points on a wheel is $d\theta/dt$.

(*c*) True. The angular acceleration of all points on the wheel is $d\omega/dt$.

(*d*) False. The centripetal acceleration at a point on a rotating wheel is directly proportional to its distance from the center of the wheel

3 • Starting from rest and rotating at constant angular acceleration, a disk takes 10 revolutions to reach an angular speed ω. How many additional revolutions at the same angular acceleration are required for it to reach an angular speed of 2ω? (*a*) 10 rev, (*b*) 20 rev, (*c*) 30 rev, (*d*) 40 rev, (*e*) 50 rev?

Picture the Problem The constant-acceleration equation that relates the given variables is $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. We can set up a proportion to determine the number of revolutions required to double ω and then subtract to find the number of additional revolutions to accelerate the disk to an angular speed of 2ω.

Using a constant-acceleration equation, relate the initial and final angular velocities to the angular acceleration:

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

or, because $\omega_0^2 = 0$,

$$\omega^2 = 2\alpha\Delta\theta$$

Let $\Delta\theta_{10}$ represent the number of revolutions required to reach an angular speed ω:

$$\omega^2 = 2\alpha\Delta\theta_{10} \quad (1)$$

Let $\Delta\theta_{2\omega}$ represent the number of revolutions required to reach an angular speed ω:

$$(2\omega)^2 = 2\alpha\Delta\theta_{2\omega} \quad (2)$$

Divide equation (2) by equation (1) and solve for $\Delta\theta_{2\omega}$:

$$\Delta\theta_{2\omega} = \frac{(2\omega)^2}{\omega^2}\Delta\theta_{10} = 4\Delta\theta_{10}$$

The number of *additional* revolutions is:

$$4\Delta\boldsymbol{\theta}_{10} - \Delta\boldsymbol{\theta}_{10} = 3\Delta\boldsymbol{\theta}_{10} = 3(10\,\text{rev}) = 30\,\text{rev}$$

and $\boxed{(\boldsymbol{c})}$ is correct.

4 • You are looking down from above at a merry-go-round, and observe that is rotating counterclockwise and its rotation rate is slowing. If we designate counterclockwise as positive, what is the sign of the angular acceleration?

Determine the Concept Because the merry-go-round is slowing, the sign of its angular acceleration is $\boxed{\text{negative.}}$

5 • Chad and Tara go for a ride on a merry-go-round. Chad sits on a pony that is 2.0 m from the rotation axis, and Tara sits on a pony 4.0 m from the axis. The merry-go-round is traveling counterclockwise and is speeding up. Does Chad or Tara have (*a*) the larger linear speed? (*b*) the larger centripetal acceleration? (*c*) the larger tangential acceleration?

Determine the Concept

(*a*) The linear speed of all points on the merry-go-round is given by $v = r\omega$. Because she is farther from the rotation axis, Tara has the larger linear speed.

(*b*) The centripetal acceleration of all points on the merry-go-round is given by $a_c = r\omega^2$. Because she is farther from the rotation axis, Tara has the larger centripetal acceleration.

(*c*) The tangential acceleration of all points on the merry-go-round is given by $a_t = r\alpha$. Because the angular acceleration is the same for all points on the merry-go-round and she is farther from the rotation axis, Tara has the larger tangential acceleration.

6 • Disk B was identical to disk A before a hole was drilled though the center of disk B. Which disk has the largest moment of inertia about its symmetry axis center? Explain your answer.

Determine the Concept Because its mass is now greater than that of disk B, disk $\boxed{\text{A}}$ has the larger moment of inertia about its axis of symmetry.

7 • **[SSM]** During a baseball game, the pitcher has a blazing fastball. You have not been able to swing the bat in time to hit the ball. You are now just trying to make the bat contact the ball, hit the ball foul, and avoid a strikeout. So, you decide to take your coach's advice and grip the bat high rather than at the very end. This change should increase bat speed; thus you will be able to swing the bat quicker and increase your chances of hitting the ball. Explain how this theory works in terms of the moment of inertia, angular acceleration, and torque of the bat.

Determine the Concept The closer the rotation axis to the center of mass, the smaller the moment of inertia of the bat. By choking up, you are rotating the bat about an axis closer to the center of mass, thus reducing the bat's moment of inertia. The smaller the moment of inertia the larger the angular acceleration (a quicker bat).

8 • (*a*) Is the direction of an object's angular velocity necessarily the same as the direction of the net torque on it? Explain. (*b*) If the net torque and angular velocity are in opposite directions, what does that tell you about the angular speed? (*c*) Can the angular velocity be zero even if the net torque is not zero? If your answer is yes, give an example.

(*a*) No. If the object is slowing down, they are oppositely directed.

(*b*) The angular speed of the object will decrease.

(*c*) Yes. At the instant the object is stopping and turning around (angularly), such as a pendulum at its turnaround (highest) point, the net torque is not zero and the angular velocity is zero.

9 • A disk is free to rotate about a fixed axis. A tangential force applied a distance *d* from the axis causes an angular acceleration α. What angular acceleration is produced if the same force is applied a distance 2*d* from the axis? (*a*) α, (*b*) 2α, (*c*) $\alpha/2$, (*d*) 4α, (*e*) $\alpha/4$?

Determine the Concept The angular acceleration of a rotating object is proportional to the *net* torque acting on it. The net torque is the product of the tangential force and its lever arm.

Express the angular acceleration of the disk as a function of the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Fd}{I} = \frac{F}{I}d \Rightarrow \alpha \propto d$$

Because $\alpha \propto d$, doubling *d* will double the angular acceleration. $\boxed{(\boldsymbol{b})}$ is correct.

10 • The moment of inertia of an object about an axis that does not pass through its center of mass is ______ the moment of inertia about a parallel axis through its center of mass. (*a*) always less than, (*b*) sometimes less than, (*c*) sometimes equal to, (*d*) always greater than.

Determine the Concept From the parallel-axis theorem we know that $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, *M* is the mass of the object, and *h* is the distance between the parallel axes. Therefore, *I* is always greater than I_{cm} by Mh^2. $\boxed{(\boldsymbol{d})}$ is correct.

11 • **[SSM]** The motor of a merry-go-round exerts a constant torque on it. As it speeds up from rest, the power output of the motor (*a*) is constant, (*b*) increases linearly with the angular speed of the merry-go-round, (*c*) is zero. (*d*) None of the above.

Determine the Concept The power delivered by the constant torque is the product of the torque and the angular speed of the merry-go-round. Because the constant torque causes the merry-go-round to accelerate, the power output increases linearly with the angular speed of the merry-go-round. $\boxed{(\boldsymbol{b})}$ is correct.

12 • A constant net torque acts on a merry-go-round from startup until it reaches its operating speed. During this time, the merry-go-round's rotational kinetic energy (*a*) is constant, (*b*) increases linearly with angular speed, (*c*) increases quadratically as the square of the angular speed, (*d*) none of the above.

Determine the Concept The work done by the net torque increases the rotational kinetic energy of the merry-go-round. Because $\boldsymbol{K}_{\text{rot}} = \frac{1}{2}\boldsymbol{I\omega}^2$, $\boxed{(\boldsymbol{c})}$ is correct.

13 • Most doors knobs are designed so the knob is located on the side opposite the hinges (rather than in the center of the door, for example). Explain why this practice makes doors easier to open.

Determine the Concept The moment arm of the force (pull) on the knob increases with the radial distance of the knob from the rotation axis. The larger the moment arm the greater the torque for the same pull. Thus, with the knob farthest from the axis the pull is most effective in rotating the door.

14 • A wheel of radius R and angular speed ω is rolling without slipping toward the north on a flat, stationary surface. The velocity of the point on the rim that is (momentarily) in contact with the surface is (*a*) equal in magnitude to $R\omega$ and directed toward the north, (*b*) equal to in magnitude $R\omega$ and directed toward the south, (*c*) zero, (*d*) equal to the speed of the center of mass and directed toward the north, (*e*) equal to the speed of the center of mass and directed toward the south.

Determine the Concept If the wheel is rolling without slipping, a point at the top of the wheel moves with a speed twice that of the center of mass of the wheel, but the bottom of the wheel is momentarily at rest. $\boxed{(\boldsymbol{c})}$ is correct.

15 • A uniform solid cylinder and a uniform solid sphere have equal masses. Both roll without slipping on a horizontal surface without slipping. If their total kinetic energies are the same, then (*a*) the translational speed of the cylinder is greater than the translational speed of the sphere, (*b*) the translational speed of the cylinder is less than the translational speed of the sphere, (*c*) the translational speeds of the two objects are the same, (*d*) (*a*), (*b*), or (*c*) could be correct depending on the radii of the objects.

Picture the Problem The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to decide which of their translational speeds is greater.

Express the kinetic energy of the cylinder:

$$K_{\text{cyl}} = \tfrac{1}{2}I_{\text{cyl}}\omega_{\text{cyl}}^2 + \tfrac{1}{2}mv_{\text{cyl}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v_{\text{cyl}}^2}{r^2} + \tfrac{1}{2}mv_{\text{cyl}}^2 = \tfrac{3}{4}mv_{\text{cyl}}^2$$

Express the kinetic energy of the sphere:

$$K_{\text{sph}} = \tfrac{1}{2}I_{\text{sph}}\omega_{\text{sphl}}^2 + \tfrac{1}{2}mv_{\text{sph}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v_{\text{sph}}^2}{r^2} + \tfrac{1}{2}mv_{\text{sph}}^2 = \tfrac{7}{10}mv_{\text{sph}}^2$$

Equate the kinetic energies and simplify to obtain:

$v_{\text{cyl}} = \sqrt{\tfrac{14}{15}}v_{\text{sph}} < v_{\text{sph}}$ and $\boxed{(\boldsymbol{b})}$ is correct.

16 • Two identical-looking 1.0-m-long pipes are each plugged with 10 kg of lead. In the first pipe, the lead is concentrated at the middle of the pipe, while in the second the lead is divided into two 5-kg masses placed at opposite ends of the pipe. The ends of the pipes are then sealed using four identical caps. Without opening either pipe, how could you determine which pipe has the lead at both ends?

Determine the Concept You could spin the pipes about their center. The one which is easier to spin has its mass concentrated closer to the center of mass and, hence, has a smaller moment of inertia.

17 •• Starting simultaneously from rest, a coin and a hoop roll without slipping down an incline. Which of the following statements is true? (*a*) The hoop reaches the bottom first. (*b*) The coin reaches the bottom first. (*c*) The coin and hoop arrive at the bottom simultaneously. (*d*) The race to the bottom depends on their relative masses. (*e*) The race to the bottom depends on their relative diameters.

Picture the Problem The object moving the fastest when it reaches the bottom of the incline will arrive there first. Because the coin and the hoop begin from the same elevation, they will have the same kinetic energy at the bottom of the incline. The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to their common potential energy loss to decide which of their translational speeds is greater at the bottom of the incline.

Express the kinetic energy of the coin at the bottom of the incline:

$$K_{\text{coin}} = \tfrac{1}{2} I_{\text{cyl}} \omega_{\text{coin}}^2 + \tfrac{1}{2} m_{\text{coin}} v_{\text{coin}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_{\text{coin}} r^2\right)\frac{v_{\text{coin}}^2}{r^2} + \tfrac{1}{2} m_{\text{coin}} v_{\text{coin}}^2 = \tfrac{3}{4} m_{\text{coin}} v_{\text{coin}}^2$$

Express the kinetic energy of the hoop at the bottom of the incline:

$$\boldsymbol{K}_{\text{hoop}} = \tfrac{1}{2} \boldsymbol{I}_{\text{hoop}} \boldsymbol{\omega}_{\text{hoop}}^2 + \tfrac{1}{2} \boldsymbol{m}_{\text{hoop}} \boldsymbol{v}_{\text{hoop}}^2 = \tfrac{1}{2}\left(\boldsymbol{m}_{\text{hoop}} \boldsymbol{r}^2\right)\frac{\boldsymbol{v}_{\text{hoop}}^2}{\boldsymbol{r}^2} + \tfrac{1}{2} \boldsymbol{m}_{\text{hoop}} \boldsymbol{v}_{\text{hoop}}^2 = \boldsymbol{m}_{\text{hoop}} \boldsymbol{v}_{\text{hoop}}^2$$

Equate the kinetic energy of the coin to its change in potential energy as it rolled down the incline and solve for v_{coin}:

$$\tfrac{3}{4} \boldsymbol{m}_{\text{coin}} \boldsymbol{v}_{\text{coin}}^2 = \boldsymbol{m}_{\text{coin}} \boldsymbol{gh} \Rightarrow \boldsymbol{v}_{\text{coin}} = \sqrt{\tfrac{4}{3} \boldsymbol{gh}}$$

Equate the kinetic energy of the hoop to its change in potential energy as it rolled down the incline and solve for v_{hoop}:

$$\boldsymbol{m}_{\text{hoop}} \boldsymbol{v}_{\text{hoop}}^2 = \boldsymbol{m}_{\text{hoop}} \boldsymbol{gh} \Rightarrow \boldsymbol{v}_{\text{hoop}} = \sqrt{\boldsymbol{gh}}$$

Express the ratio of these speeds to obtain:

$$\frac{\boldsymbol{v}_{\text{coin}}}{\boldsymbol{v}_{\text{hoop}}} = \frac{\sqrt{\tfrac{4}{3} \boldsymbol{gh}}}{\sqrt{\boldsymbol{gh}}} = \sqrt{\tfrac{4}{3}} \Rightarrow \boldsymbol{v}_{\text{coin}} > \boldsymbol{v}_{\text{hoop}}$$

and $\boxed{(\boldsymbol{b})}$ is correct.

18 •• For a hoop of mass *M* and radius *R* that is rolling without slipping, which is larger, its translational kinetic energy or its rotational kinetic energy? (*a*) Its translational kinetic energy is larger. (*b*) Its rotational kinetic energy is larger. (*c*) Both energies have the same magnitude. (*d*) The answer depends on the radius of the hoop. (*e*) The answer depends on the mass of the hoop.

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the hoop and the moment of inertia of a hoop (ring) to express and compare the kinetic energies.

Express the ratio of the translational kinetic energy of the hoop to its rotational kinetic energy and simplify to obtain:

$$\frac{K_{\text{trans}}}{K_{\text{rot}}} = \frac{\tfrac{1}{2} m v^2}{\tfrac{1}{2} I_{\text{hoop}} \omega^2} = \frac{m v^2}{\left(m r^2\right)\dfrac{v^2}{r^2}} = 1$$

Therefore, the translational and rotational kinetic energies are the same and $\boxed{(c)}$ is correct.

19 •• For a disk of mass M and radius R that is rolling without slipping, which is larger, its translational kinetic energy or its rotational kinetic energy? (*a*) Its translational kinetic energy is larger. (*b*) Its rotational kinetic energy is larger. (*c*) Both energies have the same magnitude. (*d*) The answer depends on the radius of the disk. (*e*) The answer depends on the mass of the disk.

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the disk and the moment of inertia of a disk (cylinder) to express and compare the kinetic energies.

Express the ratio of the translational kinetic energy of the disk to its rotational kinetic energy:

$$\frac{K_{\text{trans}}}{K_{\text{rot}}}=\frac{\frac{1}{2}mv^2}{\frac{1}{2}I_{\text{disk}}\omega^2}=\frac{mv^2}{\left(\frac{1}{2}mr^2\right)\frac{v^2}{r^2}}=2$$

Therefore, the translational kinetic energy is greater by a factor of two and $\boxed{(a)}$ is correct.

20 •• A perfectly rigid ball rolls without slipping along a perfectly rigid horizontal plane. Show that the frictional force acting on the ball must be zero. *Hint: Consider a possible direction for the action of the frictional force and what effects such a force would have on the velocity of the center of mass and on the angular velocity.*

Picture the Problem Let us assume that $f \neq 0$ and acts along the direction of motion. Now consider the acceleration of the center of mass and the angular acceleration about the point of contact with the plane. Because $F_{\text{net}} \neq 0$, $a_{\text{cm}} \neq 0$. However, $\tau = 0$ because $\ell = 0$, and so $\alpha = 0$. But $\alpha = 0$ is not consistent with $a_{\text{cm}} \neq 0$. Consequently, $f = 0$.

21 •• **[SSM]** A spool is free to rotate about a fixed axis (see Figure 9-42*a*), and a string wrapped around the axle causes the spool to rotate in a counterclockwise direction. However, if the spool is set on a horizontal tabletop (Figure 9-42*b*), the spool instead (given sufficient frictional force between the table and the spool) rotates in a clockwise direction, and rolls to the right. By considering torque about the appropriate axes, show that these conclusions are consistent with Newton's second law for rotations.

Determine the Concept First, visualize the situation. The string pulling to the right exerts a torque on the spool with a moment arm equal in length to the radius of the inner portion of the spool. When the spool is freely rotating about that axis, then the torque due to the pulling string causes a counter clockwise rotation. Second, in the situation in which the spool is resting on the horizontal tabletop, one should (for ease of understanding) consider torques not about the central axle of the spool, but about the point of contact with the tabletop. In this situation, there is only one force that can produce a torque – the applied force. The motion of the spool can then be understood in terms of the force applied by the string and the moment arm equal to the difference between the outer radius and the inner radius. This torque will cause a clockwise rotation about the point of contact between spool and table – and thus the spool rolls to the right (whereas we might have thought the spool would rotate in a counter-clockwise sense, and thus move left).

22 • You want to locate the center of gravity of an arbitrarily shaped flat object. You are told to suspend the object from a point, and to suspend a plumb line from the same point. Then draw a vertical line on the object to represent the plumb line. Next, you repeat the process using a different suspension point. The center of gravity will be at the intersection of the drawn lines. Explain the principle(s) behind this process.

Determine the Concept You are finding positions at which gravity exerts no torque on the object, so the gravitational force (weight) passes through the center of mass. Thus you are triangulating, and in theory, two such lines should intersect at the center of mass. In practice, several lines do a better and more accurate job.

Estimation and Approximation

23 •• A baseball is thrown at 88 mi/h, and with a spin rate of 1500 rev/min. If the distance between the pitcher's point of release and the catcher's glove is about 61 feet, estimate how many revolutions the ball makes between release and catch. Neglect any effects of gravity or air resistance on the ball's flight.

Picture the Problem The number of revolutions made by the ball is the ratio of the angle through which it rotates to 2π rad/rev.

The number of revolutions N the ball makes between release and catch is given by:

$$\boldsymbol{N} = \frac{\Delta\boldsymbol{\theta}}{2\boldsymbol{\pi}\ \text{rad/rev}} \qquad (1)$$

where $\Delta\theta$ is the angular displacement of the ball as it travels from the pitcher to the catcher.

Because $\boldsymbol{\omega} = 2\boldsymbol{\pi f}$, $\Delta\theta$ is given by:

$$\Delta\boldsymbol{\theta} = \boldsymbol{\omega}\Delta\boldsymbol{t} = 2\boldsymbol{\pi f}\Delta\boldsymbol{t}$$

Substituting for $\Delta\theta$ in equation (1) yields:

$$N = \frac{2\pi f\Delta t}{2\pi\ \text{rad/rev}} = \frac{f\Delta t}{\text{rad/rev}} \qquad (2)$$

Express the time-of-flight of the ball:

$$\Delta t = \frac{d}{v}$$

where d is the distance from the release point to the catcher's glove and v is the speed of the ball.

Substituting for Δt in equation (2) yields:

$$N = \frac{fd}{v\ \text{rad/rev}}$$

Substitute numerical values and evaluate N:

$$N = \frac{\left(1500\,\frac{\text{rev}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)(61\,\text{ft})}{\left(88\,\frac{\text{mi}}{\text{h}}\times\frac{5280\,\text{ft}}{\text{mi}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)\frac{\text{rad}}{\text{rev}}} \approx \boxed{12\ \text{rev}}$$

24 •• Consider the Crab Pulsar, discussed on page 293. Justify the statement that the loss in rotational energy is equivalent to the power output of 100 000 Suns. The total power radiated by the Sun is about 4×10^{26} W. Assume that the pulsar has a mass that is 2×10^{30} kg, has a radius of 20 km, is rotating at about 30 rev/s, and has a rotational period that is increasing at 10^{-5} s/y.

Picture the Problem The power dissipated in the loss of rotational kinetic energy is the rate at which the rotational kinetic energy of the Crab Pulsar is decreasing.

Express the rate at which the rotational kinetic energy of the Crab Pulsar is changing:

$$P_{\text{Crab Pulsar}} = \frac{\Delta K}{\Delta t} = \frac{K_f - K_i}{\Delta t}$$

Substitute for K_f and K_i and simplify to obtain:

$$P_{\text{Crab Pulsar}} = \frac{\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2}{\Delta t} = \frac{\frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)}{\Delta t}$$

Letting $\omega_f = \omega_i - \delta\omega$ yields:

$$P_{\text{Crab Pulsar}} = \frac{\frac{1}{2}I\left[(\omega_i - \delta\omega)^2 - \omega_i^2\right]}{\Delta t}$$

Expand the binomial expression and simplify to obtain:

$$P_{\text{Crab Pulsar}} = \frac{\frac{1}{2}I\left[(\delta\omega)^2 - 2\omega_i\delta\omega\right]}{\Delta t} \quad (1)$$

Express $\delta\omega$ in terms of ω_f and ω_i:

$$\delta\omega = \omega_f - \omega_i = \frac{2\pi}{T_f} - \omega_i$$

Substituting for T_f and ω_i yields:

$$\delta\omega = \frac{2\pi}{T_i + 10^{-5}\text{ s}} - 60\pi\text{ rad/s}$$

Substitute for T_i and simplify to obtain:

$$\delta\omega = \frac{2\pi}{\frac{2\pi}{60\pi}\text{s} + 10^{-5}\text{ s}} - 60\pi\text{ rad/s}$$
$$= -0.0565\text{ s}^{-1}$$

Because $\delta\omega << \omega_i$:

$$(\delta\omega)^2 - 2\omega_i\delta\omega \approx -2\omega_i\delta\omega$$

With this substitution, equation (1) becomes:

$$P_{\text{Crab Pulsar}} = \frac{\frac{1}{2}I[-2\omega_i\delta\omega]}{\Delta t} = \frac{-I\omega_i\delta\omega}{\Delta t} \quad (2)$$

The moment of inertia of a sphere of mass M and radius R is:

$$I = \tfrac{2}{5}MR^2$$

Substitute for I in equation (2) to obtain:

$$P_{\text{Crab Pulsar}} = \frac{-\frac{2}{5}MR^2\omega_i\delta\omega}{\Delta t} \quad (3)$$

Dividing both sides of equation (3) by the power radiated by the Sun yields:

$$\frac{P_{\text{Crab Pulsar}}}{P_{\text{sun}}} = \frac{-\frac{2}{5}MR^2\omega_i\delta\omega}{P_{\text{sun}}}$$

Substitute numerical values and evaluate $P_{\text{Crab Pulsar}}/P_{\text{Sun}}$:

$$\frac{P_{\text{Crab Pulsar}}}{P_{\text{Sun}}} = \frac{-\frac{2}{5}\left(2\times10^{30}\text{ kg}\right)\left(20\text{ km}\right)^2\left(60\pi\,\frac{\text{rad}}{\text{s}}\right)\left(-0.0565\text{ s}^{-1}\right)}{\left(4\times10^{26}\text{ W}\right)\left(1\text{ y}\times\frac{3.156\times10^{7}\text{ s}}{\text{y}}\right)} \approx \boxed{10^5}$$

25 •• A 14-kg bicycle has 1.2-m-diameter wheels, each with a mass of 3.0 kg. The mass of the rider is 38 kg. Estimate the fraction of the total kinetic energy of the rider-bicycle system is associated with rotation of the wheels.

Picture the Problem Assume the wheels are hoops. That is, neglect the mass of the spokes, and express the total kinetic energy of the bicycle and rider. Let M represent the mass of the rider, m the mass of the bicycle, m_w the mass of each bicycle wheel, and r the radius of the wheels.

Express the ratio of the kinetic energy associated with the rotation of the wheels to that associated with the total kinetic energy of the bicycle and rider:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{K_{\text{rot}}}{K_{\text{trans}} + K_{\text{rot}}} \qquad (1)$$

Express the translational kinetic energy of the bicycle and rider:

$$K_{\text{trans}} = K_{\text{bicycle}} + K_{\text{rider}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2$$

Express the rotational kinetic energy of the bicycle wheels:

$$K_{\text{rot}} = 2K_{\text{rot, 1 wheel}} = 2\left(\tfrac{1}{2}I_{\text{w}}\omega^2\right) = \left(m_{\text{w}}r^2\right)\frac{v^2}{r^2} = m_{\text{w}}v^2$$

Substitute for K_{rot} and K_{trans} in equation (1) and simplify to obtain:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{m_{\text{w}}v^2}{\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 + m_{\text{w}}v^2} = \frac{m_{\text{w}}}{\frac{1}{2}m + \frac{1}{2}M + m_{\text{w}}} = \frac{2}{2 + \dfrac{m+M}{m_{\text{w}}}}$$

Substitute numerical values and evaluate $K_{\text{rot}}/K_{\text{trans}}$:

$$\frac{\boldsymbol{K}_{\text{rot}}}{\boldsymbol{K}_{\text{tot}}} = \frac{2}{2 + \dfrac{14\,\text{kg} + 38\,\text{kg}}{3.0\,\text{kg}}} = \boxed{10\%}$$

26 •• Why does toast falling off a table always land jelly-side down? The question may sound silly, but it has been a subject of serious scientific enquiry. The analysis is too complicated to reproduce here, but R. D. Edge and Darryl Steinert showed that a piece of toast, pushed gently over the edge of a table until it tilts off, typically falls off the table when it makes an angle of about 30° with the horizontal (Figure 9-43) and at that instant has an angular speed of $\boldsymbol{\omega} = 0.956\sqrt{\boldsymbol{g}/\ell}$, where ℓ is the length of one edge of the piece of toast (assumed to be square). Assuming that a piece of toast is jelly-side up, what side will it land on if it falls from a 0.500-m-high table? If it falls from a 1.00-m-high table? Assume that ℓ = 10.0 cm. Ignore any forces due to air resistance.

Picture the Problem We can apply the definition of angular speed to find the angular orientation of the slice of toast when it has fallen a distance of 0.500 m (or 1.00 m) from the edge of the table. We can then interpret the orientation of the toast to decide whether it lands jelly-side up or down.

Relate the angular orientation θ of the toast to its initial angular orientation, its angular speed ω, and time of fall Δt:

$$\theta = \theta_0 + \omega\Delta t$$

Substituting the expression given for ω in the problem statement to obtain:

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + 0.956\sqrt{\frac{\boldsymbol{g}}{\ell}}\Delta \boldsymbol{t} \qquad (1)$$

Using a constant-acceleration equation, relate the distance the toast falls Δy to its time of fall Δt:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Substitute for Δt in equation (1) and simplify to obtain:

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + 0.956\sqrt{\frac{\boldsymbol{g}}{\ell}}\sqrt{\frac{2\Delta \boldsymbol{y}}{\boldsymbol{g}}} = \boldsymbol{\theta}_0 + 0.956\sqrt{\frac{2\Delta \boldsymbol{y}}{\ell}}$$

Substitute numerical values and evaluate θ for $\Delta y = 0.500$ m:

$$\boldsymbol{\theta}_{0.50\,\mathrm{m}} = \frac{\boldsymbol{\pi}}{6} + 0.956\sqrt{\frac{2(0.500\,\mathrm{m})}{0.100\,\mathrm{m}}} = 3.547\,\mathrm{rad}\times\frac{180°}{\boldsymbol{\pi}\,\mathrm{rad}} = \boxed{203°}$$

Substitute numerical values and evaluate θ for $\Delta y = 1.00$ m:

$$\boldsymbol{\theta}_{1.0\,\mathrm{m}} = \frac{\boldsymbol{\pi}}{6} + 0.956\sqrt{\frac{2(1.00\,\mathrm{m})}{0.100\,\mathrm{m}}} = 4.799\,\mathrm{rad}\times\frac{180°}{\boldsymbol{\pi}\,\mathrm{rad}} = \boxed{275°}$$

The orientation of the slice of toast will therefore be at angles of 203° and 275° with respect to ground; that is, with the jelly-side down.

27 •• Consider your moment of inertia about a vertical axis through the center of your body, both when you are standing straight up with your arms flat against your sides, and when you are standing straight up holding yours arms straight out to the side. Estimate the ratio of the moment of inertia with your arms straight out to the moment of inertia with your arms flat against your sides.

Picture the Problem Assume that the mass of an average adult male is about 80 kg, and that we can model his body when he is standing straight up with his arms at his sides as a cylinder. From experience in men's clothing stores, a man's average waist circumference seems to be about 34 inches, and the average chest circumference about 42 inches. We'll also assume that about 20% of your body's mass is in your two arms, and that each has a length L = 1 m, so that each arm has a mass of about m = 8 kg.

Letting I_{out} represent his moment of inertia with his arms straight out and I_{in} his moment of inertia with his arms at his side, the ratio of these two moments of inertia is:

$$\frac{I_{out}}{I_{in}} = \frac{I_{body} + I_{arms}}{I_{in}} \quad (1)$$

Express the moment of inertia of the ″man as a cylinder″:

$$I_{in} = \tfrac{1}{2}MR^2$$

Express the moment of inertia of his arms:

$$I_{arms} = 2\left(\tfrac{1}{3}\right)mL^2$$

Express the moment of inertia of his body-less-arms:

$$I_{body} = \tfrac{1}{2}(M-m)R^2$$

Substitute in equation (1) to obtain:

$$\frac{I_{out}}{I_{in}} = \frac{\tfrac{1}{2}(M-m)R^2 + 2\left(\tfrac{1}{3}\right)mL^2}{\tfrac{1}{2}MR^2}$$

Assume the circumference of the cylinder to be the average of the average waist circumference and the average chest circumference:

$$c_{av} = \frac{34\,\text{in} + 42\,\text{in}}{2} = 38\,\text{in}$$

Find the radius of a circle whose circumference is 38 in:

$$R = \frac{c_{av}}{2\pi} = \frac{38\,\text{in}\times\dfrac{2.54\,\text{cm}}{\text{in}}\times\dfrac{1\,\text{m}}{100\,\text{cm}}}{2\pi} = 0.154\,\text{m}$$

Substitute numerical values and evaluate I_{out}/I_{in}:

$$\frac{\boldsymbol{I}_{out}}{\boldsymbol{I}_{in}} = \frac{\tfrac{1}{2}(80\,\text{kg}-16\,\text{kg})(0.154\,\text{m})^2 + \tfrac{2}{3}(8\,\text{kg})(1\,\text{m})^2}{\tfrac{1}{2}(80\,\text{kg})(0.154\,\text{m})^2} \approx \boxed{6}$$

Angular Velocity, Angular Speed and Angular Acceleration

28 • A particle moves with a constant speed of 25 m/s in a 90-m-radius circle. (*a*) What is its angular speed, in radians per second, about the center of the circle? (*b*) How many revolutions does it make in 30 s?

Picture the Problem The tangential and angular velocities of a particle moving in a circle are directly proportional. The number of revolutions made by the particle in a given time interval is proportional to both the time interval and its angular speed.

(*a*) Relate the angular speed of the particle to its speed along the circumference of the circle:

$$\boldsymbol{\omega} = \frac{\boldsymbol{v}}{\boldsymbol{r}}$$

Substitute numerical values and evaluate ω:

$$\boldsymbol{\omega} = \frac{25\,\text{m/s}}{90\,\text{m}} = 0.278\,\text{rad/s} = \boxed{0.28\,\text{rad/s}}$$

(*b*) Using a constant-acceleration equation, relate the number of revolutions made by the particle in a given time interval to its angular speed:

$$\Delta\boldsymbol{\theta} = \boldsymbol{\omega}\,\Delta\boldsymbol{t} = \left(0.278\,\frac{\text{rad}}{\text{s}}\right)(30\,\text{s})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$
$$= \boxed{1.3\,\text{rev}}$$

29 • **[SSM]** A wheel released from rest is rotating with constant angular acceleration of 2.6 rad/s^2. At 6.0 s after its release: (*a*) What is its angular speed? (*b*) Through what angle has the wheel turned? (*c*) How many revolutions has it completed? (*d*) What is the linear speed, and what is the magnitude of the linear acceleration, of a point 0.30 m from the axis of rotation?

Picture the Problem Because the angular acceleration is constant, we can find the various physical quantities called for in this problem by using constant-acceleration equations.

(*a*) Using a constant-acceleration equation, relate the angular speed of the wheel to its angular acceleration:

$$\omega = \omega_0 + \alpha\Delta t$$
or, when $\omega_0 = 0$,
$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 6.0$ s:

$$\boldsymbol{\omega} = \left(2.6\,\text{rad/s}^2\right)(6.0\,\text{s}) = 15.6\,\text{rad/s}$$
$$= \boxed{16\,\text{rad/s}}$$

(*b*) Using another constant-acceleration equation, relate the angular displacement to the wheel's angular acceleration and the time it has been accelerating:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$
or, when $\omega_0 = 0$,
$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Evaluate $\Delta\theta$ when $\Delta t = 6.0$ s:

$$\Delta\boldsymbol{\theta}(6\,\text{s}) = \tfrac{1}{2}\left(2.6\,\text{rad/s}^2\right)(6.0\,\text{s})^2 = 46.8\,\text{rad}$$
$$= \boxed{47\,\text{rad}}$$

(*c*) Convert $\Delta\boldsymbol{\theta}(6.0\,\text{s})$ from radians to revolutions:

$$\Delta\boldsymbol{\theta}(6.0\,\text{s}) = 46.8\,\text{rad} \times \frac{1\,\text{rev}}{2\boldsymbol{\pi}\,\text{rad}} = \boxed{7.4\,\text{rev}}$$

(*d*) Relate the angular speed of the particle to its tangential speed and evaluate the latter when $\Delta t = 6.0$ s:

$$\boldsymbol{v} = \boldsymbol{r\omega} = (0.30\,\text{m})(15.6\,\text{rad/s})$$
$$= \boxed{4.7\,\text{m/s}}$$

Relate the resultant acceleration of the point to its tangential and centripetal accelerations when $\Delta t = 6.0$ s:

$$a = \sqrt{a_\text{t}^2 + a_\text{c}^2} = \sqrt{(r\alpha)^2 + \left(r\omega^2\right)^2}$$
$$= r\sqrt{\alpha^2 + \omega^4}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = (0.30\,\text{m})\sqrt{\left(2.6\,\text{rad/s}^2\right)^2 + (15.6\,\text{rad/s})^4} = \boxed{73\,\text{m/s}^2}$$

30 • When a turntable rotating at 33 rev/min is shut off, it comes to rest in 26 s. Assuming constant angular acceleration, find (*a*) the angular acceleration. During the 26 s, find (*b*) the average angular speed, and (*c*) the angular displacement, in revolutions.

Picture the Problem Because we're assuming constant angular acceleration; we can find the various physical quantities called for in this problem by using constant-acceleration equations for rotational motion.

(*a*) The angular acceleration of the turntable is given by:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{0 - 33\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{26\,\text{s}}$$
$$= \boxed{0.13\,\text{rad/s}^2}$$

(b) Because the angular acceleration is constant, the average angular speed is given by:

$$\omega_{av} = \frac{\omega_0 + \omega}{2}$$

Substitute numerical values and evaluate ω_{av}:

$$\omega_{av} = \frac{33\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{2} = 1.73\,\text{rad/s} = \boxed{1.7\,\text{rad/s}}$$

(c) Using the definition of ω_{av}, find the angular displacement of the turntable as it slows to a stop:

$$\Delta\theta = \omega_{av}\Delta t = (1.73\,\text{rad/s})(26\,\text{s}) = 44.9\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.2\,\text{rev}}$$

31 • A 12-cm-radius disk that is begins to rotate about its axis at $t = 0$, rotates with a constant angular acceleration of 8.0 rad/s^2. At $t = 5.0$ s, (a) what is the angular speed of the disk, and (b) what are the tangential and centripetal components of the acceleration of a point on the edge of the disk?

Picture the Problem Because the angular acceleration of the disk is constant, we can use a constant-acceleration equation to relate its angular speed to its acceleration and the time it has been accelerating. We can find the tangential and centripetal accelerations from their relationships to the angular speed and angular acceleration of the disk.

(a) Using a constant-acceleration equation, relate the angular speed of the disk to its angular acceleration and time during which it has been accelerating:

$$\omega = \omega_0 + \alpha\,\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\,\Delta t$$

Evaluate ω when $t = 5.0$ s:

$$\boldsymbol{\omega}(5.0\,\text{s}) = (8.0\,\text{rad/s}^2)(5.0\,\text{s}) = \boxed{40\,\text{rad/s}}$$

(b) Express a_t in terms of α:

$$a_t = r\alpha$$

Evaluate a_t when $t = 5.0$ s:

$$\boldsymbol{a}_t(5.0\,\text{s}) = (0.12\,\text{m})(8.0\,\text{rad/s}^2) = \boxed{0.96\,\text{m/s}^2}$$

Express a_c in terms of ω:

$$a_c = r\omega^2$$

Evaluate a_c when $t = 5.0$ s:

$$\boldsymbol{a}_c(5.0\,\text{s}) = (0.12\,\text{m})(40\,\text{rad/s})^2 = \boxed{0.19\,\text{km/s}^2}$$

32 • A 12-m-radius Ferris wheel rotates once each 27 s. (*a*) What is its angular speed (in radians per second)? (*b*) What is the linear speed of a passenger? (*c*) What is the acceleration of a passenger?

Picture the Problem We can find the angular speed of the Ferris wheel from its definition and the linear speed and centripetal acceleration of the passenger from the relationships between those quantities and the angular speed of the Ferris wheel.

(*a*) Find ω from its definition:

$$\boldsymbol{\omega} = \frac{\Delta\boldsymbol{\theta}}{\Delta\boldsymbol{t}} = \frac{2\pi\,\text{rad}}{27\,\text{s}} = 0.233\,\text{rad/s} = \boxed{0.23\,\text{rad/s}}$$

(*b*) Find the linear speed of the passenger from his/her angular speed:

$$\boldsymbol{v} = \boldsymbol{r\omega} = (12\,\text{m})(0.233\,\text{rad/s}) = \boxed{2.8\,\text{m/s}}$$

Find the passenger's centripetal acceleration from his/her angular speed:

$$\boldsymbol{a}_c = \boldsymbol{r\omega}^2 = (12\,\text{m})(0.233\,\text{rad/s})^2 = \boxed{0.65\,\text{m/s}^2}$$

33 • A cyclist accelerates uniformly from rest. After 8.0 s, the wheels have rotated 3.0 rev. (*a*) What is the angular acceleration of the wheels? (*b*) What is the angular speed of the wheels at the end of the 8.0 s?

Picture the Problem Because the angular acceleration of the wheels is constant, we can use constant-acceleration equations in rotational form to find their angular acceleration and their angular speed at any given time.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its angular acceleration and the time it has been accelerating:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2 \Rightarrow \alpha = \frac{2\Delta\theta}{(\Delta t)^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(3.0\,\text{rev})\left(\dfrac{2\pi\,\text{rad}}{\text{rev}}\right)}{(8.0\,\text{s})^2} = 0.589\,\text{rad/s}^2 = \boxed{0.59\,\text{rad/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the angular speed of the wheel to its angular acceleration and the time it has been accelerating:

$$\omega = \omega_0 + \alpha\Delta t$$

or, when $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 8.0$ s:

$$\omega(8.0\,\text{s}) = (0.589\,\text{rad/s}^2)(8.0\,\text{s}) = \boxed{4.7\,\text{rad/s}}$$

34 • What is the angular speed of Earth, in radians per second, as it rotates about its axis?

Picture the Problem Earth rotates through 2π radians every 24 hours.

Apply the definition of angular speed to obtain:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{24\,\text{h}\times\dfrac{3600\,\text{s}}{\text{h}}} = \boxed{73\,\mu\text{rad/s}}$$

35 • A wheel rotates through 5.0 rad in 2.8 s as it is brought to rest with constant angular acceleration. Determine the wheel's initial angular speed before braking began.

Picture the Problem When the angular acceleration of a wheel is constant, its average angular speed is the average of its initial and final angular velocities. We can combine this relationship with the always applicable definition of angular speed to find the initial angular velocity of the wheel.

Express the average angular speed of the wheel in terms of its initial and final angular speeds:

$$\omega_{av} = \frac{\omega_0 + \omega}{2}$$

or, because $\omega = 0$,

$$\omega_{av} = \tfrac{1}{2}\omega_0$$

The average angular speed of the wheel is also given by:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

Equate these two expressions for ω_{av} and solve for ω_0 to obtain:

$$\omega_0 = \frac{2\Delta\theta}{\Delta t}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{2(5.0\,\text{rad})}{2.8\,\text{s}} = \boxed{3.6\,\text{rad/s}}$$

36 • A bicycle has 0.750-m-diameter wheels. The bicyclist accelerates from rest with constant acceleration to 24.0 km/h in 14.0 s. What is the angular acceleration of the wheels?

Picture the Problem The tangential and angular accelerations of the wheel are directly proportional to each other with the radius of the wheel as the proportionality constant. Provided there is no slippage, the acceleration of a point on the rim of the wheel is the same as the acceleration of the bicycle. We can use its defining equation to determine the acceleration of the bicycle.

Relate the tangential acceleration of a point on the wheel (equal to the acceleration of the bicycle) to the wheel's angular acceleration and solve for its angular acceleration:

$$a = a_t = r\alpha \Rightarrow \alpha = \frac{a}{r}$$

Use its definition to express the acceleration of the wheel:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t}$$

or, because $v_0 = 0$,

$$a = \frac{v}{\Delta t}$$

Substitute in the expression for α to obtain:

$$\alpha = \frac{v}{r\Delta t} = \frac{v}{\frac{1}{2}d\Delta t} = \frac{2v}{d\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2\left(24.0\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right)}{(0.750\,\text{m})(14.0\,\text{s})} = \boxed{1.3\,\text{rad/s}^2}$$

37 •• **[SSM]** The tape in a standard VHS videotape cassette has a total length of 246 m, which is enough for the tape to play for 2.0 h (Figure 9-44). As the tape starts, the full reel has a 45-mm outer radius and a 12-mm inner radius. At some point during the play, both reels have the same angular speed. Calculate this angular speed in radians per second and in revolutions per minute. (*Hint: Between the two reels the tape moves at constant speed.*)

Picture the Problem The two tapes will have the same tangential and angular velocities when the two reels are the same size, i.e., have the same area. We can calculate the tangential speed of the tape from its length and running time and relate the angular speed to the constant tangential speed and the radius of the reels when they are turning with the same angular speed.

Relate the angular speed of the tape to its tangential speed:

$$\omega = \frac{v}{r} \quad (1)$$

Letting R_f represent the outer radius of the reel when the reels have the same area, express the condition that they have the same speed:

$$\pi R_f^2 - \pi r^2 = \tfrac{1}{2}\left(\pi R^2 - \pi r^2\right)$$

Solving for R_f yields:

$$R_f = \sqrt{\frac{R^2 + r^2}{2}}$$

Substitute numerical values and evaluate R_f:

$$R_f = \sqrt{\frac{(45\,\text{mm})^2 + (12\,\text{mm})^2}{2}} = 32.9\,\text{mm}$$

Find the tangential speed of the tape from its length and running time:

$$\boldsymbol{v} = \frac{\boldsymbol{L}}{\Delta \boldsymbol{t}} = \frac{246\,\text{m} \times \dfrac{100\,\text{cm}}{\text{m}}}{2.0\,\text{h} \times \dfrac{3600\,\text{s}}{\text{h}}} = 3.42\,\text{cm/s}$$

Substitute in equation (1) and evaluate ω:

$$\boldsymbol{\omega} = \frac{3.42\,\text{cm/s}}{32.9\,\text{mm} \times \dfrac{1\,\text{cm}}{10\,\text{mm}}} = 1.04\,\text{rad/s}$$

$$= \boxed{1.0\,\text{rad/s}}$$

Convert 1.04 rad/s to rev/min:

$$1.04\,\text{rad/s} = 1.04\frac{\text{rad}}{\text{s}} \times \frac{1\,\text{rev}}{2\boldsymbol{\pi}\,\text{rad}} \times \frac{60\,\text{s}}{\text{min}}$$

$$= \boxed{9.9\,\text{rev/min}}$$

38 •• To start a lawn mower, you must pull on a rope wound around the perimeter of a flywheel. After you pull the rope for 0.95 s, the flywheel is rotating at 4.5 revolutions per second, at which point the rope disengages. This attempt at starting the mower does not work, however, and the flywheel slows, coming to rest 0.24 s after the disengagement. Assume constant acceleration during both spin up and spin down. (*a*) Determine the average angular acceleration during the 4.5-s spin-up and again during the 0.24-s spin-down. (*b*) What is the maximum angular speed reached by the flywheel? (*c*) Determine the ratio of the number of revolutions made during spin-up to the number made during spin-down.

Picture the Problem The average angular acceleration of the starter is the ratio of the change in angular speed to the time during which the starter either speeds up slows down. The number of revolutions through which the starter turns is the product of its average angular speed and the elapsed time.

(*a*) The average angular acceleration of the starter is given by:

$$\boldsymbol{\alpha}_{\text{av}} = \frac{\Delta\boldsymbol{\omega}}{\Delta\boldsymbol{t}} = \frac{\boldsymbol{\omega}_{\text{f}} - \boldsymbol{\omega}_{\text{i}}}{\Delta\boldsymbol{t}}$$

Evaluate α_{av} for $\omega_{\text{i}} = 0$ and $\omega_{\text{f}} = 4.5$ rev/s:

$$\boldsymbol{\alpha}_{\text{av}} = \frac{4.5\,\frac{\text{rev}}{\text{s}}\times\frac{2\boldsymbol{\pi}\text{ rad}}{\text{rev}} - 0}{0.95\text{ s}} = 29.8\text{ rad/s}^2$$

$$= \boxed{30\text{ rad/s}^2}$$

Evaluate α_{av} for $\omega_{\text{i}} = 4.5$ rev/s and $\omega_{\text{f}} = 0$:

$$\alpha_{\text{av}} = \frac{0 - 4.5\,\frac{\text{rev}}{\text{s}}\times\frac{2\pi\text{ rad}}{\text{rev}}}{0.24\text{ s}}$$

$$= -118\text{ rad/s}^2 = \boxed{-1.2\times10^2\text{ rad/s}^2}$$

(*b*) The maximum angular speed reached by the starter is the angular speed it had when the rope comes off:

$$\boldsymbol{\omega}_{\text{max}} = 4.5\,\frac{\text{rev}}{\text{s}}\times\frac{2\boldsymbol{\pi}\text{ rad}}{\text{rev}} = \boxed{28\text{ rad/s}}$$

(*c*) Use a constant-acceleration equation to express the ratio of the number of revolutions during startup to the number of revolutions during slowdown:

$$\frac{\Delta\boldsymbol{\theta}_{\text{spin up}}}{\Delta\boldsymbol{\theta}_{\text{spin down}}} = \frac{\boldsymbol{\omega}_{\text{av,spin up}}(\Delta\boldsymbol{t})_{\text{spin up}}}{\boldsymbol{\omega}_{\text{av,spin down}}(\Delta\boldsymbol{t})_{\text{spin down}}}$$

or, because $\boldsymbol{\omega}_{\text{av, spin up}} = \boldsymbol{\omega}_{\text{av, spin down}}$,

$$\frac{\Delta\boldsymbol{\theta}_{\text{spin up}}}{\Delta\boldsymbol{\theta}_{\text{spin down}}} = \frac{(\Delta\boldsymbol{t})_{\text{spin up}}}{(\Delta\boldsymbol{t})_{\text{spin down}}}$$

Substitute numerical values and evaluate $\Delta\theta_{\text{spin up}}/\Delta\theta_{\text{spin down}}$:

$$\frac{\Delta\boldsymbol{\theta}_{\text{spin up}}}{\Delta\boldsymbol{\theta}_{\text{spin down}}} = \frac{0.95\text{ s}}{0.24\text{ s}} = \boxed{4.0}$$

39 ••• During a period of 687 days Mars orbits the Sun at a mean orbital radius of 228 Gm (1 Gm = 10^9 m) and has an orbital period of 687 d. Earth orbits the Sun at a mean orbital radius of 149.6 Gm. (*a*) The Earth-Sun line sweeps out an angle of 360° during one Earth year. Approximately what angle is swept out by the Mars-Sun line during one Earth-year? (*b*) How frequently are Mars and the Sun in opposition (on diametrically opposite sides of Earth)?

Picture the Problem The angle between the Earth-Sun line and the Mars-Sun line is $\Delta\theta = (1-f)\theta_{\text{Earth}}$ where *f* is the ratio of the periods of Earth and Mars.

(*a*) The angle between the Earth-Sun line and the Mars-Sun line is given by:

$$\Delta\theta = \theta_{\text{Earth}} - \theta_{\text{Mars}} = \theta_{\text{Earth}} - f\theta_{\text{Earth}} = (1-f)\theta_{\text{Earth}}$$

where *f* is the ratio of the periods of Earth and Mars.

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\theta = \left(1 - \frac{365.24\ \text{d}}{687\ \text{d}}\right)(2\pi\ \text{rad}) = \boxed{2.94\ \text{rad}}$$

(*b*) The second alignment of Earth and Mars will occur when both planets have the same angular displacement from their initial alignment-with Earth having made one full revolution more than Mars:

$$\theta_{\text{Earth}} = \omega_{\text{Earth}}\Delta t = 2\pi + \theta_{\text{Mars}} \qquad (1)$$

The angular position of Mars at this time is:

$$\theta_{\text{Mars}} = \omega_{\text{Mars}}\Delta t \qquad (2)$$

Substituting for θ_{Mars} in equation (1) yields:

$$\omega_{\text{Earth}}\Delta t = 2\pi + \omega_{\text{Mars}}\Delta t$$

Solve for Δt to obtain:

$$\Delta t = \frac{2\pi}{\omega_{\text{Earth}} - \omega_{\text{Mars}}} \qquad (3)$$

The angular speeds of Earth and Mars are related to their periods:

$$\omega_{\text{Earth}} = \frac{2\pi}{T_{\text{Earth}}} \text{ and } \omega_{\text{Mars}} = \frac{2\pi}{T_{\text{Mars}}}$$

Substituting for ω_{Earth} and ω_{Mars} in equation (3) and simplifying yields:

$$\Delta t = \frac{2\pi}{\dfrac{2\pi}{T_{\text{Earth}}} - \dfrac{2\pi}{T_{\text{Mars}}}} = \frac{T_{\text{Earth}}T_{\text{Mars}}}{T_{\text{Mars}} - T_{\text{Earth}}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(365.24\ \text{d})(687\ \text{d})}{687\ \text{d} - 365.24\ \text{d}} = \boxed{780\ \text{d}}$$

Calculating the Moment of Inertia

40 • A tennis ball has a mass of 57 g and a diameter of 7.0 cm. Find the moment of inertia about its diameter. Model the ball as a thin spherical shell.

Picture the Problem One can find the formula for the moment of inertia of a thin spherical shell in Table 9-1.

The moment of inertia of a thin spherical shell about its diameter is:

$$I = \tfrac{2}{3}MR^2$$

Substitute numerical values and evaluate I:

$$I = \tfrac{2}{3}(0.057\ \text{kg})(0.035\ \text{m})^2 = \boxed{4.7\times10^{-5}\ \text{kg}\cdot\text{m}^2}$$

41 • **[SSM]** Four particles, one at each of the four corners of a square with 2.0-m long edges, are connected by massless rods (Figure 9-45). The masses of the particles are $m_1 = m_3 = 3.0$ kg and $m_2 = m_4 = 4.0$ kg. Find the moment of inertia of the system about the z axis.

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

Use the definition of the moment of inertia of a system of four particles to obtain:

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate $I_{z\ \text{axis}}$:

$$I_{z\ \text{axis}} = (3.0\ \text{kg})(2.0\ \text{m})^2 + (4.0\ \text{kg})\left(2\sqrt{2}\ \text{m}\right)^2 + (4.0\ \text{kg})(2.0\ \text{m})^2 + (3.0\ \text{kg})(0)^2 = \boxed{60\ \text{kg}\cdot\text{m}^2}$$

42 •• Use the parallel-axis theorem and the result for Problem 41 to find the moment of inertia of the four-particle system in Figure 9-45 about an axis that passes through the center of mass and is parallel with the z axis. Check your result by direct computation.

Picture the Problem According to the parallel-axis theorem, $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes. Note that the center of mass of the system is not at the intersection of the diagonals connecting the four masses. Hence we'll need to determine its position as a part of our confirmation of our results using the parallel-axis theorem.

Express the parallel axis theorem:

$$\boldsymbol{I} = \boldsymbol{I}_{\text{cm}} + \boldsymbol{M}\boldsymbol{h}^2$$

Solve for I_{cm}:

$$I_{\text{cm}} = I_{z\text{ axis}} - Mh^2 \qquad (1)$$

Use the definition of the moment of inertia of a system of four particles to express I_{cm}:

$$\begin{aligned} I_{\text{cm}} &= \sum_{\text{i}} m_{\text{i}} r_{\text{i}}^2 \\ &= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 \end{aligned} \qquad (2)$$

Express the x coordinate of the center of mass of the four-particle system:

$$\boldsymbol{x}_{\text{cm}} = \frac{\boldsymbol{m}_1\boldsymbol{x}_1 + \boldsymbol{m}_2\boldsymbol{x}_2 + \boldsymbol{m}_3\boldsymbol{x}_3 + \boldsymbol{m}_4\boldsymbol{x}_4}{\boldsymbol{m}_1 + \boldsymbol{m}_2 + \boldsymbol{m}_3 + \boldsymbol{m}_4}$$

Substitute numerical values and evaluate x_{cm}:

$$\boldsymbol{x}_{\text{cm}} = \frac{(3.0\text{ kg})(0) + (4.0\text{ kg})(2.0\text{ m}) + (3.0\text{ kg})(0) + (4.0\text{ kg})(2.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 3.0\text{ kg} + 4.0\text{ kg}} = 1.143\text{ m}$$

Express the y coordinate of the center of mass of the four-particle system:

$$\boldsymbol{y}_{\text{cm}} = \frac{\boldsymbol{m}_1\boldsymbol{y}_1 + \boldsymbol{m}_2\boldsymbol{y}_2 + \boldsymbol{m}_3\boldsymbol{y}_3 + \boldsymbol{m}_4\boldsymbol{y}_4}{\boldsymbol{m}_1 + \boldsymbol{m}_2 + \boldsymbol{m}_3 + \boldsymbol{m}_4}$$

Substitute numerical values and evaluate y_{cm}:

$$\boldsymbol{y}_{\text{cm}} = \frac{(3.0\text{ kg})(2.0\text{ m}) + (4.0\text{ kg})(2.0\text{ m}) + (3.0\text{ kg})(0) + (4.0\text{ kg})(0)}{3.0\text{ kg} + 4.0\text{ kg} + 3.0\text{ kg} + 4.0\text{ kg}} = 1.000\text{ m}$$

Find the square of the distance from the center of mass to the z axis (note that this distance is also the distance from the center of mass to the center of the object whose mass is m_1):

$$\begin{aligned} \boldsymbol{h}^2 &= \boldsymbol{r}_1^2 \\ &= (0 - 1.143\text{ m})^2 + (2.0\text{ m} - 1.00\text{ m})^2 \\ &= 2.306\text{ m}^2 \end{aligned}$$

Substitute numerical values in equation (1) and evaluate I_{cm}:

$$I_{cm} = 60\,\text{kg}\cdot\text{m}^2 - (14\,\text{kg})(2.306\,\text{m}^2) = \boxed{28\,\text{kg}\cdot\text{m}^2}$$

Find the square of the distance from the center of mass to the center of the object whose mass is m_2:

$$r_2^2 = (2.0 - 1.143\,\text{m})^2 + (2.0\,\text{m} - 1.00\,\text{m})^2 = 1.734\,\text{m}^2$$

Because $m_1 = m_3$, $m_2 = m_4$, $r_1^2 = r_3^2$, and $r_2^2 = r_4^2$, equation (2) becomes:

$$\begin{aligned} I_{cm} &= m_1r_1^2 + m_2r_2^2 + m_1r_1^2 + m_2r_2^2 \\ &= 2m_1r_1^2 + 2m_2r_2^2 \\ &= 2(m_1r_1^2 + m_2r_2^2) \end{aligned}$$

Substitute numerical values and evaluate I_{cm}:

$$I_{cm} = 2[(3.0\,\text{kg})(2.306\,\text{m}^2) + (4.0\,\text{kg})(1.734\,\text{m}^2)] = \boxed{28\,\text{kg}\cdot\text{m}^2}$$

43 • For the four-particle system of Figure 9-45, (*a*) find the moment of inertia I_x about the x axis, which passes through m_2 and m_3, and (*b*) find the moment of inertia I_y about the y axis, which passes through $m_1$and m_2.

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

(*a*) Apply the definition of the moment of inertia of a system of particles to express I_x:

$$I_x = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$$

Substitute numerical values and evaluate I_x:

$$\begin{aligned} I_x &= (3.0\,\text{kg})(2.0\,\text{m})^2 + (4.0\,\text{kg})(0)^2 \\ &\quad + (4.0\,\text{kg})(0)^2 + (3.0\,\text{kg})(2.0\,\text{m})^2 \\ &= \boxed{28\,\text{kg}\cdot\text{m}^2} \end{aligned}$$

(*b*) Apply the definition of the moment of inertia of a system of particles to express I_y:

$$I_y = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$$

Substitute numerical values and evaluate I_y:

$$\begin{aligned} I_y &= (3.0\,\text{kg})(0) + (4.0\,\text{kg})(0)^2 \\ &\quad + (3.0\,\text{kg})(2.0\,\text{m})^2 + (4.0\,\text{kg})(2.0\,\text{m})^2 \\ &= \boxed{32\,\text{kg}\cdot\text{m}^2} \end{aligned}$$

44 • Determine the moment of inertia of a uniform solid sphere of mass M and radius R about an axis that is tangent to the surface of the sphere (Figure 9-46).

Picture the Problem According to the parallel-axis theorem, $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes.

The moment of inertia of a solid sphere of mass M and radius R about an axis that is tangent to the sphere is given by:

$$I = I_{\text{cm}} + Mh^2 \qquad (1)$$

Use Table 9-1 to find the moment of inertia of a sphere with respect to an axis through its center of mass:

$$I_{\text{cm}} = \tfrac{2}{5}MR^2$$

Substitute for I_{cm} and h in equation (1) and simplify to obtain:

$$\boldsymbol{I} = \tfrac{2}{5}\boldsymbol{MR}^2 + \boldsymbol{MR}^2 = \boxed{\tfrac{7}{5}\boldsymbol{MR}^2}$$

45 •• A 1.00-m-diameter wagon wheel consists of a thin rim having a mass of 8.00 kg and 6 spokes, each with a mass of 1.20 kg. Determine the moment of inertia of the wagon wheel about its axis.

Picture the Problem The moment of inertia of the wagon wheel is the sum of the moments of inertia of the rim and the six spokes.

Express the moment of inertia of the wagon wheel as the sum of the moments of inertia of the rim and the spokes:

$$I_{\text{wheel}} = I_{\text{rim}} + I_{\text{spokes}} \qquad (1)$$

Using Table 9-1, find formulas for the moments of inertia of the rim and spokes:

$$I_{\text{rim}} = M_{\text{rim}}R^2$$

and

$$I_{\text{spoke}} = \tfrac{1}{3}M_{\text{spoke}}L^2$$

Substitute for I_{rim} and I_{spoke} in equation (1) to obtain:

$$\begin{aligned} I_{\text{wheel}} &= M_{\text{rim}}R^2 + 6\left(\tfrac{1}{3}M_{\text{spoke}}L^2\right) \\ &= M_{\text{rim}}R^2 + 2M_{\text{spoke}}L^2 \end{aligned}$$

Substitute numerical values and evaluate I_{wheel}:

$$I_{\text{wheel}} = (8.00\,\text{kg})(0.50\,\text{m})^2 + 2(1.20\,\text{kg})(0.50\,\text{m})^2 = \boxed{2.6\,\text{kg}\cdot\text{m}^2}$$

46 •• Two point masses $m_1$and m_2 are separated by a massless rod of length L. (*a*) Write an expression for the moment of inertia I about an axis perpendicular to the rod and passing through it a distance x from mass m_1. (*b*) Calculate dI/dx and show that I is at a minimum when the axis passes through the center of mass of the system.

Picture the Problem The moment of inertia of a system of particles depends on the axis with respect to which it is calculated. Once this choice is made, the moment of inertia is the sum of the products of the mass of each particle and the square of its distance from the chosen axis.

(*a*) Apply the definition of the moment of inertia of a system of particles:

$$I = \sum_{\text{i}} m_{\text{i}} r_{\text{i}}^2 = \boxed{m_1 x^2 + m_2 (L - x)^2}$$

(*b*) Set the derivative of I with respect to x equal to zero in order to identify values for x that correspond to either maxima or minima:

$$\begin{aligned}\frac{dI}{dx} &= 2m_1 x + 2m_2 (L - x)(-1) \\ &= 2(m_1 x + m_2 x - m_2 L) \\ &= 0 \text{ for extrema}\end{aligned}$$

If $\dfrac{dI}{dx} = 0$, then:

$$m_1 x + m_2 x - m_2 L = 0$$

Solving for x yields:

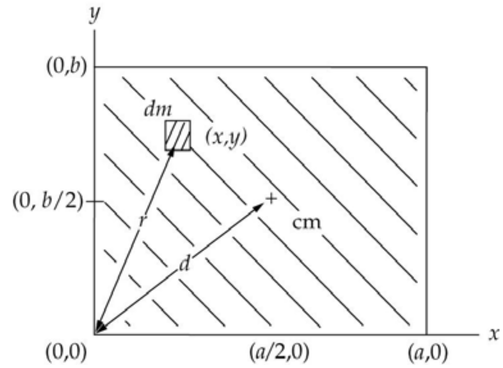
$$x = \frac{m_2 L}{m_1 + m_2}$$

Convince yourself that you've found a minimum by showing that $\boldsymbol{d^2 I/dx^2}$ is positive at this point.

$$\boxed{x = \frac{m_2 L}{m_1 + m_2} \text{ is, by definition, the distance of the center of mass from } m.}$$

47 •• A uniform rectangular plate has mass m and edges of lengths a and b. (*a*) Show by integration that the moment of inertia of the plate about an axis that is perpendicular to the plate and passes through one corner is $m(a^2 + b^2)/3$.
(*b*) What is the moment of inertia about an axis that is perpendicular to the plate and passes through its center of mass?

Picture the Problem Let σ be the mass per unit area of the uniform rectangular plate. Then the elemental unit has mass $dm = \sigma\, dxdy$. Let the corner of the plate through which the axis runs be the origin. The distance of the element whose mass is dm from the corner r is related to the coordinates of dm through the Pythagorean relationship $r^2 = x^2 + y^2$.



(*a*) Express the moment of inertia of the element whose mass is dm with respect to an axis perpendicular to it and passing through one of the corners of the uniform rectangular plate:

$$dI = \sigma\left(x^2 + y^2\right)dxdy$$

Integrate this expression to find I:

$$\boldsymbol{I = \sigma\int_0^a\int_0^b\left(x^2 + y^2\right)dxdy = \tfrac{1}{3}\sigma\left(a^3b + ab^3\right)}$$

$$= \boxed{\boldsymbol{\tfrac{1}{3}m\left(a^2 + b^3\right)}}$$

(*b*) Letting d represent the distance from the origin to the center of mass of the plate, use the parallel axis theorem to relate the moment of inertia found in (*a*) to the moment of inertia with respect to an axis through the center of mass:

$$I = I_{\text{cm}} + md^2$$

or

$$I_{\text{cm}} = I - md^2 = \tfrac{1}{3}m\left(a^2 + b^2\right) - md^2$$

Using the Pythagorean theorem, relate the distance d to the center of mass to the lengths of the sides of the plate:

$$d^2 = \left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}b\right)^2 = \tfrac{1}{4}\left(a^2 + b^2\right)$$

Substitute for d^2 in the expression for I_{cm} and simplify to obtain:

$$I_{\text{cm}} = \tfrac{1}{3}m\left(a^2 + b^2\right) - \tfrac{1}{4}m\left(a^2 + b^2\right)^2$$

$$= \boxed{\tfrac{1}{12}m\left(a^2 + b^2\right)}$$

48 •• In attempting to ensure a spot on the pep squad, you and your friend Corey research baton-twirling. Each of you is using "The Beast" as a model baton: two uniform spheres, each of mass 500 g and radius 5.00 cm, mounted at the ends of a 30.0-cm uniform rod of mass 60.0 g (Figure 9-47). You want to determine the moment of inertia *I* of ″The Beast″ about an axis perpendicular to the rod and passing through its center. Corey uses the approximation that the two spheres can be treated as point particles that are 20.0 cm from the axis of rotation, and that the mass of the rod is negligible. You, however, decide to do an exact calculation. (*a*) Compare the two results. (Give the percentage difference between them). (*b*) Suppose the spheres were replaced by two thin spherical shells, each of the same mass as the original solid spheres. Give a conceptual argument explaining how this replacement would, or would not, change the value of *I*.

Picture the Problem Corey will use the point-particle relationship $I_{app} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$ for his calculation whereas your calculation will take into account not only the rod but also the fact that the spheres are not point particles.

(*a*) Using the point-mass approximation and the definition of the moment of inertia of a system of particles, express I_{app}:

$$I_{app} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$$

Substitute numerical values and evaluate I_{app}:

$$\boldsymbol{I}_{app} = (0.500\,\text{kg})(0.200\,\text{m})^2 + (0.500\,\text{kg})(0.200\,\text{m})^2 = 0.0400\,\text{kg}\cdot\text{m}^2$$

Express the moment of inertia of the two spheres and connecting rod system:

$$I = I_{spheres} + I_{rod}$$

Use Table 9-1 to find the moments of inertia of a sphere (with respect to its center of mass) and a rod (with respect to an axis through its center of mass):

$$I_{sphere} = \tfrac{2}{5} M_{sphere} R^2$$

and

$$I_{rod} = \tfrac{1}{12} M_{rod} L^2$$

Because the spheres are not on the axis of rotation, use the parallel axis theorem to express their moment of inertia with respect to the axis of rotation:

$$\boldsymbol{I}_{sphere} = \tfrac{2}{5} \boldsymbol{M}_{sphere} \boldsymbol{R}^2 + \boldsymbol{M}_{sphere} \boldsymbol{h}^2$$

where *h* is the distance from the center of mass of a sphere to the axis of rotation.

Substitute to obtain:

$$I = 2\left[\tfrac{2}{5} M_{\text{sphere}} R^2 + M_{\text{sphere}} h^2\right] + \tfrac{1}{12} M_{\text{rod}} L^2$$

Substitute numerical values and evaluate I:

$$I = 2\left[\tfrac{2}{5}(0.500\,\text{kg})(0.0500\,\text{m})^2 + (0.500\,\text{kg})(0.200\,\text{m})^2\right] + \tfrac{1}{12}(0.0600\,\text{kg})(0.300\,\text{m})^2$$
$$= 0.0415\,\text{kg}\cdot\text{m}^2$$

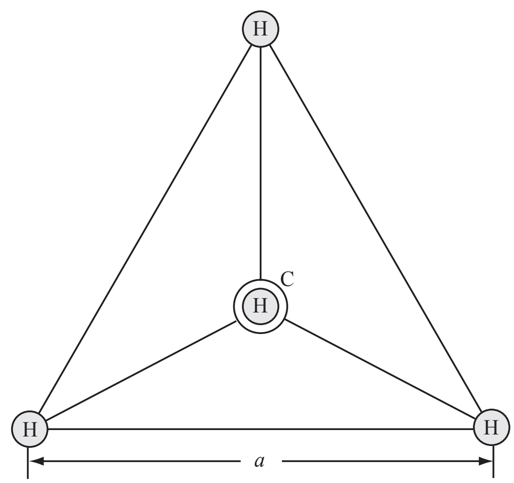
The percent difference between I and I_{app} is:

$$\frac{I - I_{\text{app}}}{I} = \frac{0.0415\,\text{kg}\cdot\text{m}^2 - 0.0400\,\text{kg}\cdot\text{m}^2}{0.0415\,\text{kg}\cdot\text{m}^2} = \boxed{3.6\ \%}$$

(*b*) The rotational inertia would increase because I_{cm} of a hollow sphere is greater than I_{cm} of a solid sphere.

49 •• The methane molecule (CH_4) has four hydrogen atoms located at the vertices of a regular tetrahedron of edge length 0.18 nm, with the carbon atom at the center of the tetrahedron (Figure 9-48). Find the moment of inertia of this molecule for rotation about an axis that passes through the centers of the carbon atom and one of the hydrogen atoms.

Picture the Problem The axis of rotation passes through the center of the base of the tetrahedron. The carbon atom and the hydrogen atom at the apex of the tetrahedron do not contribute to I because the distance of their nuclei from the axis of rotation is zero. From the geometry, the distance of the three H nuclei from the rotation axis is $a/\sqrt{3}$, where a is the length of a side of the tetrahedron.



Apply the definition of the moment of inertia for a system of particles to obtain:

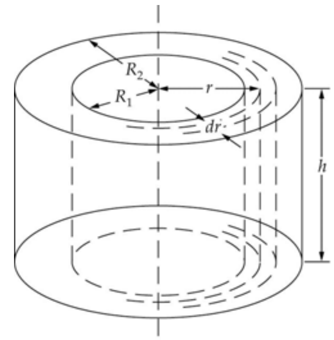
$$I = \sum_i m_i r_i^2 = m_H r_1^2 + m_H r_2^2 + m_H r_3^2$$
$$= 3m_H\left(\frac{a}{\sqrt{3}}\right)^2 = m_H a^2$$

Substitute numerical values and evaluate I:

$$I = \left(1.67\times10^{-27}\,\text{kg}\right)\left(0.18\times10^{-9}\,\text{m}\right)^2$$
$$= \boxed{5.4\times10^{-47}\,\text{kg}\cdot\text{m}^2}$$

50 •• A hollow cylinder has mass m, an outside radius R_2, and an inside radius R_1. Use integration to show that the moment of inertia about its axis is given by $I = \frac{1}{2}m\left(R_2^2 + R_1^2\right)$. *Hint: Review Section 9-3, where the moment of inertia is calculated for a solid cylinder by direct integration.*

Picture the Problem Let the mass of the element of volume dV be $dm = \rho dV = 2\pi\rho hrdr$ where h is the height of the cylinder. We'll begin by expressing the moment of inertia dI for the element of volume and then integrating it between R_1 and R_2.



Express the moment of inertia of an element of mass dm:

$$dI = r^2dm = 2\pi\rho\, hr^3dr$$

Integrate dI from R_1 to R_2 to obtain:

$$I = 2\pi\rho\, h\int_{R_1}^{R_2} r^3dr = \tfrac{1}{2}\pi\rho\, h\left(R_2^4 - R_1^4\right)$$

$$= \tfrac{1}{2}\pi\rho\, h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)$$

The mass of the hollow cylinder is $m = \pi\,\rho\, h\left(R_2^2 - R_1^2\right)$, so:

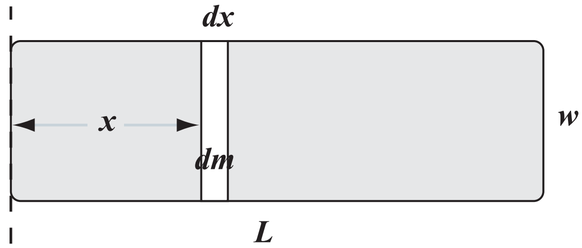
$$\rho = \frac{m}{\pi\, h\left(R_2^2 - R_1^2\right)}$$

Substitute for ρ and simplify to obtain:

$$I = \tfrac{1}{2}\pi\left(\frac{m}{\pi\, h\left(R_2^2 - R_1^2\right)}\right)h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right) = \boxed{\tfrac{1}{2}m\left(R_2^2 + R_1^2\right)}$$

51 •• While slapping the water's surface with his tail to communicate danger, a beaver must rotate it about one of its narrow ends. Let us model the tail as a rectangle of uniform thickness and density (Figure 9-49). Estimate its moment of inertia about the line passing through its narrow end (dashed line). Assume the tail measures 15 by 30 cm with a thickness of 1.0 cm and that the flesh has the density of water.

Picture the Problem The pictorial representation shows our model of the beaver tail pivoted about the dashed line shown to the left. We can apply $\boldsymbol{I} = \int \boldsymbol{x^2dm}$ to this configuration to derive an expression for the moment of inertia of the beaver tail.



The moment of inertia, about an axis through the short side of the rectangular object, is:

$$I = \int x^2 dm$$

Letting the density of the beaver's tail be represented by ρ:

$$dm = \rho dV = \rho wt dx$$

Substituting for dm yields:

$$I = \int \rho wt x^2 dx = \rho wt \int x^2 dx$$

Integrating over the length L of the beaver's tail yields:

$$I = \rho wt \int_0^L x^2 dx = \tfrac{1}{3}\rho wt L^3$$

Substitute numerical values and evaluate I:

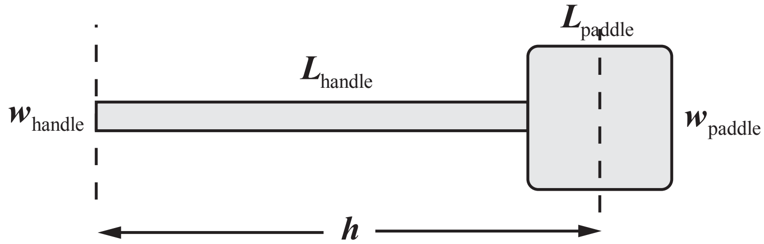
$$I = \tfrac{1}{3}\left(1.0\,\frac{\text{g}}{\text{cm}^3}\right)(15\text{ cm})(1.0\text{ cm})(30\text{ cm})^3$$

$$= \boxed{1.4\times10^{-2}\text{ kg}\cdot\text{m}^2}$$

Remarks: Had we substituted $m = \rho wtL$ in the expression for I, we would have obtained $I = \tfrac{1}{3}mL^2$ as the expression for the moment of inertia, about an axis through its short side, of a rectangular plate of uniform thickness.

52 •• To prevent damage to her shoulders, your elderly grandmother wants to purchase the rug beater (Figure 9-50) with the lowest moment of inertia about its grip end. Knowing you are taking physics, she asks your advice. There are two models to choose from. Model A has a 1.0-m-long handle and a 40-cm-edge-length square, where the masses of the handle and square are 1.0 kg and 0.5 kg, respectively. Model B has a 0.75-m-long handle and a 30-cm-edge-length square, where the masses of the handle and square are 1.0 kg and 0.5 kg, respectively. Which model should you recommend? Determine which beater is easiest to swing from the very end by computing the moment of inertia for both beaters.

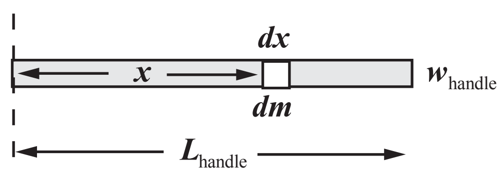
Picture the Problem The pictorial representation models the rug beater as two rectangles of different dimensions. The moment of inertia of the rug beater, about the axis shown to the left, is the sum of the moments of inertia of the handle and the paddle. The rug beater that is easiest for your grandmother to use is the one with the smaller moment of inertia about an axis through the grip end of the handle.



The moment of inertia of the rug beater, with respect to an axis through the end of its handle, is the sum of the moments of inertia of its handle and paddle:

$$I = I_{\text{handle}} + I_{\text{paddle}} \quad (1)$$

The handle of the rug beater is shown to the right. We can apply $I = \int x^2 dm$ to this configuration to derive an expression for I_{handle}.



The moment of inertia of the handle, about an axis through the grip end of the handle, is:

$$I = \int x^2 dm$$

Let t_{handle} represent the thickness of the handle and ρ its density yields:

$$dm = \rho dV = \rho w_{\text{handle}} t_{\text{handle}} dx$$

Substituting for dm yields:

$$I_{\text{handle}} = \int \rho w_{\text{handle}} t_{\text{handle}} x^2 dx = \rho w_{\text{handle}} t_{\text{handle}} \int x^2 dx$$

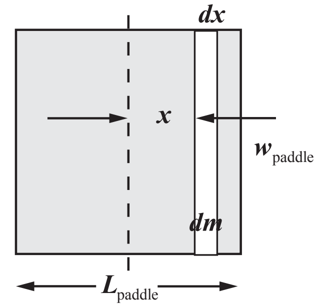
Integrating over the length of the handle yields:

$$I = \rho w_{\text{handle}} t_{\text{handle}} \int_0^{L_{\text{handle}}} x^2 dx = \tfrac{1}{3} \rho w_{\text{handle}} t_{\text{handle}} L_{\text{handle}}^3$$

Because the mass of the handle is given by $m_{\text{handle}} = \rho w_{\text{handle}} t_{\text{handle}} L_{\text{handle}}$:

$$I_{\text{handle}} = \tfrac{1}{3} m_{\text{handle}} L_{\text{handle}}^2$$

We can find the moment of inertia of the paddle, relative to an axis through the grip end of the handle, by first finding its moment of inertia with respect to an axis through its center of mass and then applying the parallel-axis theorem.



The moment of inertia, about an axis through the center of mass of the paddle, is:

$$I_{\text{paddle}} = \int x^2 dm$$

The mass of the infinitesimal element of the paddle is given by:

$$dm = \rho dV = \rho w_{\text{paddle}} t_{\text{paddle}} dx$$

Substituting for *dm* yields:

$$\begin{aligned} I &= \int \rho w_{\text{paddle}} t_{\text{paddle}} x^2 dx \\ &= \rho w_{\text{paddle}} t_{\text{paddle}} \int x^2 dx \end{aligned}$$

Integrating over the length L_{paddle} of the rectangular object yields:

$$\begin{aligned} I_{\text{paddle}} &= \rho w_{\text{paddle}} t_{\text{paddle}} \int_{-\frac{1}{2}L_{\text{paddle}}}^{\frac{1}{2}L_{\text{paddle}}} x^2 dx \\ &= \tfrac{1}{12} \rho w_{\text{paddle}} t_{\text{paddle}} L_{\text{paddle}}^3 \end{aligned}$$

Because the mass of the paddle is given by $m_{\text{paddle}} = \rho w_{\text{paddle}} t_{\text{paddle}} L_{\text{paddle}}$:

$$I_{\text{paddle, cm}} = \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2$$

Apply the parallel-axis theorem to express the moment of inertia of the paddle with respect to an axis through the grip end of the handle:

$$I_{\text{paddle}} = I_{\text{cm}} + m_{\text{paddle}} h^2$$

or, because $h = L_{\text{handle}} + \frac{1}{2} L_{\text{paddle}}$,

$$\begin{aligned} I_{\text{paddle}} &= I_{\text{paddle, cm}} \\ &\quad + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2 \end{aligned}$$

Substituting for $I_{\text{paddle, cm}}$ yields:

$$I_{\text{paddle}} = \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2 + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2$$

Substitute for I_{handle} and I_{paddle} in equation (1) to obtain:

$$I = \tfrac{1}{3} m_{\text{handle}} L_{\text{handle}}^2 + \tfrac{1}{12} m_{\text{paddle}} L_{\text{paddle}}^2 + m_{\text{paddle}} \left(L_{\text{handle}} + \tfrac{1}{2} L_{\text{paddle}} \right)^2$$

Substitute numerical values and evaluate I_1, the moment of inertia of the rug beater with the shorter handle:

$$\begin{aligned} \boldsymbol{I}_1 &= \tfrac{1}{3}(1.0\,\text{kg})(1.0\,\text{m})^2 + \tfrac{1}{12}(0.50\,\text{kg})(0.40\,\text{m})^2 + (0.50\,\text{kg})\left(1.0\,\text{m} + \tfrac{1}{2}(0.4\,\text{m})\right)^2 \\ &= 1.06\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Substitute numerical values and evaluate I_2, the moment of inertia of the rug beater with the longer handle:

$$\begin{aligned} \boldsymbol{I}_2 &= \tfrac{1}{3}(1.5\,\text{kg})(0.75\,\text{m})^2 + \tfrac{1}{12}(0.60\,\text{kg})(0.30\,\text{m})^2 + (0.60\,\text{kg})\left(0.75\,\text{m} + \tfrac{1}{2}(0.3\,\text{m})\right)^2 \\ &= 0.77\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Because $I_2 < I_1$, the more massive rug beater will be easier to swing.

53 ••• **[SSM]** Use integration to show that the moment of inertia of a thin spherical shell of radius R and mass m about an axis through its center is $2mR^2/3$.

Picture the Problem We can derive the given expression for the moment of inertia of a spherical shell by following the procedure outlined in the problem statement.

Find the moment of inertia of a sphere, with respect to an axis through a diameter, in Table 9-1:

$$I = \tfrac{2}{5}mR^2$$

Express the mass of the sphere as a function of its density and radius:

$$m = \tfrac{4}{3}\pi\rho R^3$$

Substitute for m to obtain:

$$I = \tfrac{8}{15}\pi\rho R^5$$

Express the differential of this expression:

$$dI = \tfrac{8}{3}\pi\rho R^4 dR \qquad (1)$$

Express the increase in mass dm as the radius of the sphere increases by dR:

$$dm = 4\pi\rho R^2 dR \qquad (2)$$

Eliminate dR between equations (1) and (2) to obtain:

$$dI = \tfrac{2}{3}R^2 dm$$

Integrate over the mass of the spherical shell to obtain:

$$I_{\text{spherical shell}} = \boxed{\tfrac{2}{3}\boldsymbol{mR^2}}$$

54 ••• According to one model, the density of Earth varies with the distance r from the center of Earth as $\rho = C\,[1.22 - (r/R)]$, where R is the radius of Earth and C is a constant. (*a*) Find C in terms of the total mass M and the radius R. (*b*) According to this model, what is the moment of inertia of Earth about and axis through its center. (See Problem 53.)

Picture the Problem We can find C in terms of M and R by integrating a spherical shell of mass dm with the given density function to find the mass of Earth as a function of M and then solving for C. In Part (*b*), we'll start with the moment of inertia of the same spherical shell, substitute the Earth's density function, and integrate from 0 to R. Let the axis of rotation be the Earth's axis.

(*a*) Express the mass of Earth using the given density function:

$$M = \int dm = \int_0^R 4\pi\,\rho\,r^2 dr$$
$$= 4\pi C\int_0^R 1.22r^2 dr - \frac{4\pi C}{R}\int_0^R r^3 dr$$
$$= \frac{4\pi}{3}1.22CR^3 - \pi CR^3$$

Solve for C as a function of M and R to obtain:

$$C = \boxed{0.508\frac{M}{R^3}}$$

(*b*) From Problem 9-53 we have:

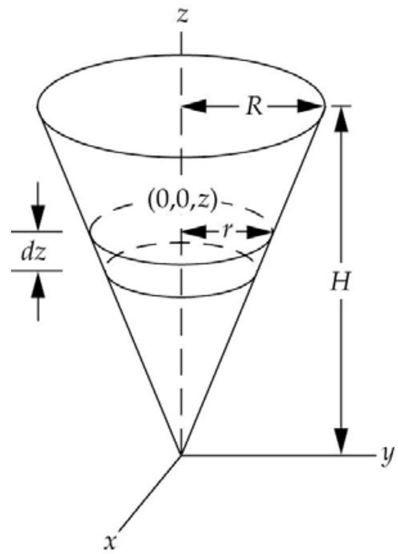
$$dI = \tfrac{8}{3}\pi\,\rho\,r^4 dr$$

Integrate and simplify to obtain:

$$\boldsymbol{I} = \tfrac{8}{3}\boldsymbol{\pi}\int_0^{\boldsymbol{R}}\boldsymbol{\rho\, r^4 dr} = \frac{8\boldsymbol{\pi}(0.508)\boldsymbol{M}}{3\boldsymbol{R}^3}\left[\int_0^{\boldsymbol{R}}1.22\boldsymbol{r}^4\boldsymbol{dr} - \frac{1}{\boldsymbol{R}}\int_0^{\boldsymbol{R}}\boldsymbol{r}^5\boldsymbol{dr}\right] = \frac{4.26\boldsymbol{M}}{\boldsymbol{R}^3}\left[\frac{1.22}{5}\boldsymbol{R}^5 - \frac{1}{6}\boldsymbol{R}^5\right]$$
$$= \boxed{0.329\boldsymbol{MR}^2}$$

55 ••• Use integration to determine the moment of inertia about its axis of a uniform right circular cone of height H, base radius R, and mass M.

Picture the Problem Let the origin be at the apex of the cone, with the z axis along the cone's symmetry axis. Then the radius of the elemental ring, at a distance z from the apex, can be obtained from the proportion $\frac{r}{z} = \frac{R}{H}$. The mass dm of the elemental disk is $\rho dV = \rho\pi r^2 dz$. We'll integrate $r^2 dm$ to find the moment of inertia of the disk in terms of R and H and then integrate dm to obtain a second equation in R and H that we can use to eliminate H in our expression for I.



Express the moment of inertia of the cone in terms of the moment of inertia of the elemental disk:

$$I = \tfrac{1}{2}\int r^2 dm = \tfrac{1}{2}\int_0^H \frac{R^2}{H^2}z^2\left(\rho\pi\frac{R^2}{H^2}z^2\right)dz = \frac{\pi\rho R^4}{2H^4}\int_0^H z^4 dz = \frac{\pi\rho R^4 H}{10}$$

Express the total mass of the cone in terms of the mass of the elemental disk:

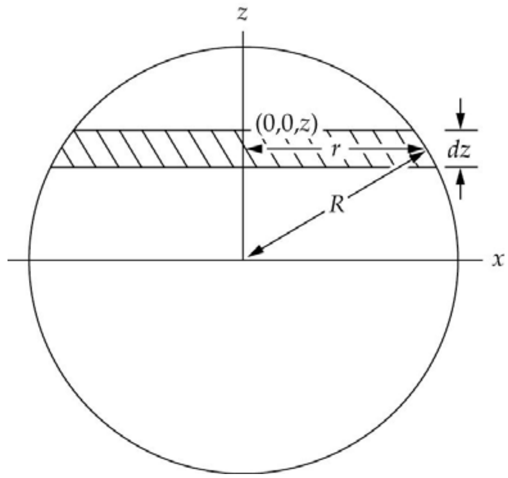
$$M = \pi\rho\int_0^H r^2 dz = \pi\rho\int_0^H \frac{R^2}{H^2}z^2 dz = \tfrac{1}{3}\pi\rho R^2 H$$

Divide I by M, simplify, and solve for I to obtain:

$$I = \boxed{\tfrac{3}{10}MR^2}$$

56 ••• Use integration to determine the moment of inertia about its axis of a thin uniform disk of mass M and radius R. Check your answer by referring to Table 9-1.

Picture the Problem Let the axis of rotation be the x axis. The radius r of the elemental area is $\sqrt{R^2 - z^2}$ and its mass, dm, is $\sigma\, dA = 2\sigma\sqrt{R^2 - z^2}\,dz$. We'll integrate $z^2\, dm$ to determine I in terms of σ and then divide this result by M in order to eliminate σ and express I in terms of M and R.



Express the moment of inertia about the x axis:

$$\boldsymbol{I = \int z^2 dm = \int z^2 \sigma\, dA}$$

Substitute for dm to obtain:

$$\boldsymbol{I = \int_{-R}^{R} z^2 \left(2\sigma\sqrt{R^2 - z^2}\, dz\right)}$$

Carrying out the integration yields:

$$\boldsymbol{I = \tfrac{1}{4}\sigma\pi R^4}$$

The mass of the thin uniform disk is:

$$M = \sigma\pi R^2$$

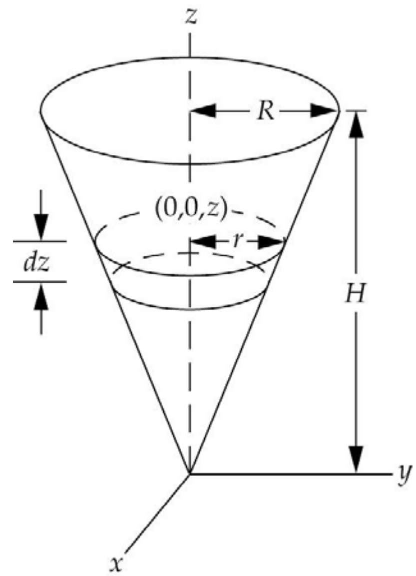
Divide I by M, simplify, and solve for I to obtain:

$I = \boxed{\tfrac{1}{4}MR^2}$, a result in agreement with the expression given in Table 9-1 for a cylinder of length $L = 0$.

57 ••• An advertising firm has contacted your engineering firm to create a new advertisement for a local ice-cream stand. The owner of this stand wants to add rotating solid cones (painted to look like ice-cream cones, of course) to catch the eye of travelers. Each cone will rotate about an axis parallel to its base and passing through its apex. The actual size of the cones is to be decided upon, and the owner wonders if it would be more energy-efficient to rotate smaller cones than larger ones. He asks your firm to write a report showing the determination of the moment of inertia of a homogeneous right circular cone of height H, base radius R, and mass M. What is the result of your report?

Picture the Problem Choose the coordinate system shown in the diagram to the right. Then the radius of the elemental disk, at a distance z from the apex, can be obtained from the proportion $\frac{r}{z} = \frac{R}{H}$. The mass dm of the elemental disk is $\rho dV = \rho\pi r^2 dz$. Each elemental disk rotates about an axis that is parallel to its diameter but removed from it by a distance z. We can use the result from Problem

9-56 for the moment of inertia of the elemental disk with respect to a diameter and then use the parallel axis theorem to express the moment of inertia of the cone with respect to the x axis.



Using the parallel axis theorem, express the moment of inertia of the elemental disk with respect to the x axis:

$$dI_x = dI_{\text{disk}} + dm\, z^2 \qquad (1)$$

where

$$dm = \rho\, dV = \rho\pi r^2 dz$$

In Problem 9-56 it was established that the moment of inertia of a thin uniform disk of mass M and radius R rotating about a diameter is $\frac{1}{4}\boldsymbol{MR}^2$. Express this result in terms of our elemental disk:

$$\boldsymbol{dI}_{\text{disk}} = \tfrac{1}{4}\left(\boldsymbol{\rho\pi r^2 dz}\right)\boldsymbol{r}^2 = \tfrac{1}{4}\boldsymbol{\rho\pi}\left(\frac{\boldsymbol{R}^2}{\boldsymbol{H}^2}\boldsymbol{z}^2\right)^2 \boldsymbol{dz}$$

Substituting for dI_{disk} in equation (1) yields:

$$\boldsymbol{dI_x} = \boldsymbol{\pi\rho}\left[\frac{1}{4}\left(\frac{\boldsymbol{R}^2}{\boldsymbol{H}^2}\boldsymbol{z}^2\right)^2\right]\boldsymbol{dz} + \left(\boldsymbol{\pi\rho}\left(\frac{\boldsymbol{R}}{\boldsymbol{H}}\boldsymbol{z}\right)^2 \boldsymbol{dz}\right)\boldsymbol{z}^2$$

Integrate from 0 to H to obtain:

$$I_x = \pi\rho\int_0^H \left[\frac{1}{4}\left(\frac{R^2}{H^2}z^2\right)^2 + \frac{R^2}{H^2}z^4\right]dz$$

$$= \pi\rho\left(\frac{R^4 H}{20} + \frac{R^2 H^3}{5}\right)$$

Express the total mass of the cone in terms of the mass of the elemental disk:

$$M = \pi\rho\int_0^H r^2 dz = \pi\rho\int_0^H \frac{R^2}{H^2}z^2 dz$$
$$= \tfrac{1}{3}\pi\rho R^2 H$$

Divide I_x by M, simplify, and solve for I_x to obtain:

$$I_x = \boxed{3M\left(\frac{H^2}{5} + \frac{R^2}{20}\right)}$$

Remarks: Because both *H* and *R* appear in the numerator, the larger the cones are, the greater their moment of inertia and the greater the energy consumption required to set them into motion.

Torque, Moment of Inertia, and Newton's Second Law for Rotation

58 • A firm wants to determine the amount of frictional torque in their current line of grindstones, so they can redesign them to be more energy efficient. To do this, they ask you to test the best-selling model, which is basically a disk-shaped grindstone of mass 1.70 kg and radius 8.00 cm which operates at 730 rev/min. When the power is shut off, you time the grindstone and find it takes 31.2 s for it to stop rotating. (*a*) What is the angular acceleration of the grindstone? (Assume constant angular acceleration.) (*b*) What is the frictional torque exerted on the grindstone?

Picture the Problem (*a*) We can use the definition of angular acceleration to find the angular acceleration of the grindstone. (*b*) Apply Newton's 2nd law in rotational form will allow us to find the torque exerted by the friction force acting on the grindstone.

(*a*) From the definition of angular acceleration we have:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

or, because $\omega = 0$,

$$\alpha = \frac{-\omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = -\frac{730\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}}{31.2\,\text{s}}$$
$$= \boxed{-2.45\,\text{rad/s}^2}$$

where the minus sign means that the grindstone is slowing down.

(*b*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the grindstone to the frictional torque slowing it:

$$\boldsymbol{\tau}_{\text{frictional}} = -\boldsymbol{I\alpha}$$

Express the moment of inertia of disk with respect to its axis of rotation:

$$I = \tfrac{1}{2}MR^2$$

Substitute for I to obtain:

$$\boldsymbol{\tau}_{\text{frictional}} = -\tfrac{1}{2}\boldsymbol{MR^2\alpha}$$

Substitute numerical values and evaluate $\tau_{\text{frictional}}$:

$$\boldsymbol{\tau}_{\text{frictional}} = -\tfrac{1}{2}(1.70\,\text{kg})(0.0800\,\text{m})^2\left(-2.45\,\text{rad/s}^2\right) = \boxed{0.0133\,\text{N}\cdot\text{m}}$$

59 • [SSM] A 2.5-kg 11-cm-radius cylinder, initially at rest, is free to rotate about the axis of the cylinder. A rope of negligible mass is wrapped around it and pulled with a force of 17 N. Assuming that the rope does not slip, find (*a*) the torque exerted on the cylinder by the rope, (*b*) the angular acceleration of the cylinder, and (*c*) the angular speed of the cylinder after 0.50 s.

Picture the Problem We can find the torque exerted by the 17-N force from the definition of torque. The angular acceleration resulting from this torque is related to the torque through Newton's 2nd law in rotational form. Once we know the angular acceleration, we can find the angular speed of the cylinder as a function of time.

(*a*) The torque exerted by the rope is:

$$\boldsymbol{\tau} = \boldsymbol{F}\ell = (17\,\text{N})(0.11\,\text{m}) = 1.87\,\text{N}\cdot\text{m} = \boxed{1.9\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law in rotational form to relate the acceleration resulting from this torque to the torque:

$$\alpha = \frac{\tau}{I}$$

Express the moment of inertia of the cylinder with respect to its axis of rotation:

$$I = \tfrac{1}{2}MR^2$$

Substitute for I and simplify to obtain:

$$\alpha = \frac{2\tau}{MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(1.87\,\text{N}\cdot\text{m})}{(2.5\,\text{kg})(0.11\,\text{m})^2} = 124\,\text{rad/s}^2$$
$$= \boxed{1.2\times10^2\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, express the angular speed of the cylinder as a function of time:

$$\omega = \omega_0 + \alpha t$$
or, because $\omega_0 = 0$,
$$\omega = \alpha t$$

Substitute numerical values and evaluate ω (5.0 s):

$$\omega(5.0\,\text{s}) = (124\,\text{rad/s}^2)(5.0\,\text{s})$$
$$= \boxed{6.2\times10^2\,\text{rad/s}}$$

60 •• A grinding wheel is initially at rest. A constant external torque of 50.0 N·m is applied to the wheel for 20.0 s, giving the wheel an angular speed of 600 rev/min. The external torque is then removed, and the wheel comes to rest 120 s later. Find (*a*) the moment of inertia of the wheel, and (*b*) the frictional torque, which is assumed to be constant.

Picture the Problem We can apply Newton's 2nd law in rotational form to both the speeding up and slowing down motions of the wheel to obtain two equations in *I,* the moment of inertia of the wheel, and τ_{fr}, the frictional torque, that we can solve simultaneously for *I* and τ_{fr}. We'll assume that both the speeding-up and slowing-down of the wheel took place under constant-acceleration conditions.

Apply $\sum\tau = I\alpha$ to the wheel during the speeding-up portion of its motion:

$$\tau_{\text{ext}} - \tau_{\text{fr}} = I\alpha_{\text{speeding up}} \qquad (1)$$

Apply $\sum\tau = I\alpha$ to the wheel during the slowing-down portion of its motion:

$$\tau_{\text{fr}} = I\alpha_{\text{slowing down}} \qquad (2)$$

(*a*) Eliminating τ_{fr} between equations (1) and (2) and solving for *I* yields:

$$I = \frac{\tau_{\text{ext}}}{\alpha_{\text{speeding up}} + \alpha_{\text{slowing down}}}$$

Substituting for $\alpha_{\text{speeding up}}$ and $\alpha_{\text{slowing down}}$ yields:

$$I = \frac{\tau_{\text{ext}}}{\dfrac{\Delta\omega_{\text{speeding up}}}{\Delta t_{\text{speeding up}}} + \dfrac{\Delta\omega_{\text{slowing down}}}{\Delta t_{\text{slowing down}}}}$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I}=\frac{50.0\ \text{N}\cdot\text{m}}{\dfrac{600\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\ \text{rad}}{\text{rev}}\times\dfrac{1\ \text{min}}{60\ \text{s}}}{20.0\ \text{s}}+\dfrac{-600\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\ \text{rad}}{\text{rev}}\times\dfrac{1\ \text{min}}{60\ \text{s}}}{120\ \text{s}}}=19.10\ \text{kg}\cdot\text{m}^2$$

$$=\boxed{19.1\ \text{kg}\cdot\text{m}^2}$$

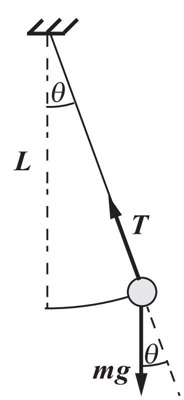
(b) Substitute numerical values in equation (2) and evaluate τ_{fr}:

$$\boldsymbol{\tau}_{\text{fr}}=\left(19.10\ \text{kg}\cdot\text{m}^2\right)\left(\frac{-600\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\ \text{rad}}{\text{rev}}\times\dfrac{1\ \text{min}}{60\ \text{s}}}{120\ \text{s}}\right)=\boxed{-10.0\ \text{N}\cdot\text{m}}$$

where the minus sign is a consequence of the fact that the frictional torque opposes the motion of the wheel.

61 •• A pendulum consisting of a string of length L attached to a bob of mass m swings in a vertical plane. When the string is at an angle θ to the vertical, (a) Using $\sum F_{\text{t}}=ma_{\text{t}}$, calculate the tangential acceleration of the bob? (b) What is the torque exerted about the pivot point? (c) Show that $\sum\boldsymbol{\tau}=\boldsymbol{I\alpha}$ with $\boldsymbol{a}_{\text{t}}=\boldsymbol{L\alpha}$ gives the same tangential acceleration as found in Part (a).

Picture the Problem The pendulum and the forces acting on it are shown in the free-body diagram. Note that the tension in the string is radial, and so exerts no tangential force on the ball. We can use Newton's 2nd law in both translational and rotational form to find the tangential component of the acceleration of the bob.



(a) Referring to the free-body diagram, express the component of $m\vec{g}$ that is tangent to the circular path of the bob:

$$F_{\text{t}}=mg\sin\theta$$

Use Newton's 2nd law to express the tangential acceleration of the bob:

$$a_{\text{t}}=\frac{F_{\text{t}}}{m}=\frac{mg\sin\theta}{m}=\boxed{g\sin\theta}$$

(*b*) Noting that, because the line-of-action of the tension passes through the pendulum's pivot point, its lever arm is zero and the net torque is due to the weight of the bob, sum the torques about the pivot point to obtain:

$$\sum \tau_{\text{pivot point}} = \boxed{mgL\sin\theta}$$

(*c*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the pendulum to the net torque acting on it:

$$\tau_{\text{net}} = mgL\sin\theta = I\alpha$$

Solve for α to obtain:

$$\alpha = \frac{mgL\sin\theta}{I}$$

Express the moment of inertia of the bob with respect to the pivot point:

$$I = mL^2$$

Substitute for I and simplify to obtain:

$$\alpha = \frac{mgL\sin\theta}{mL^2} = \frac{g\sin\theta}{L}$$

Relate α to a_t, substitute for α, and simplify to obtain:

$$\boldsymbol{a}_{\text{t}} = \boldsymbol{r\alpha} = \boldsymbol{L}\left(\frac{\boldsymbol{g}\sin\boldsymbol{\theta}}{\boldsymbol{L}}\right) = \boxed{\boldsymbol{g}\sin\boldsymbol{\theta}}$$

62 ••• A uniform rod of mass M and length L is pivoted at one end and hangs as in Figure 9-51 so that it is free to rotate without friction about its pivot. It is struck a sharp horizontal blow a distance x below the pivot, as shown. (*a*) Show that, just after the rod is struck, the speed of the center of mass of the rod is given by $v_0 = 3xF_0\Delta t/(2ML)$, where F_0 and Δt are the average force and duration, respectively, of the blow. (*b*) Find the horizontal component of the force exerted by the pivot on the rod, and show that this force component is zero if $\boldsymbol{x} = \frac{2}{3}\boldsymbol{L}$. This point (the point of impact when the horizontal component of the pivot force is zero) is called the *center of percussion* of the rod-pivot system.

Picture the Problem We can express the velocity of the center of mass of the rod in terms of its distance from the pivot point and the angular speed of the rod. We can find the angular speed of the rod by using Newton's 2nd law to find its angular acceleration and then a constant-acceleration equation that relates ω to α. We'll use the impulse-momentum relationship to derive the expression for the force delivered to the rod by the pivot. Finally, the location of the *center of percussion* of the rod will be verified by setting the force exerted by the pivot to zero.

(*a*) Relate the velocity of the center of mass to its distance from the pivot point:

$$v_{cm} = \frac{L}{2}\omega \qquad (1)$$

Express the torque due to F_0:

$$\tau = F_0 x = I_{pivot}\alpha \Rightarrow \alpha = \frac{F_0 x}{I_{pivot}}$$

Express the moment of inertia of the rod with respect to an axis through its pivot point:

$$I_{pivot} = \tfrac{1}{3}ML^2$$

Substitute for I_{pivot} and simplify to obtain:

$$\alpha = \frac{3F_0 x}{ML^2}$$

Express the angular speed of the rod in terms of its angular acceleration:

$$\omega = \alpha\,\Delta t = \frac{3F_0 x\Delta t}{ML^2}$$

Substitute in equation (1) to obtain:

$$v_{cm} = \boxed{\frac{3F_0 x\Delta t}{2ML}}$$

(*b*) Let I_P be the impulse exerted by the pivot on the rod. Then the total impulse (equal to the change in momentum of the rod) exerted on the rod is:

$$I_P + F_0\Delta t = Mv_{cm}$$

and

$$I_P = Mv_{cm} - F_0\Delta t$$

Substitute your result from (*a*) to obtain:

$$I_P = \frac{3F_0 x\Delta t}{2L} - F_0\Delta t = F_0\Delta t\left(\frac{3x}{2L} - 1\right)$$

Because $I_P = F_P\Delta t$:

$$F_P = \boxed{F_0\left(\frac{3x}{2L} - 1\right)}$$

If F_P is zero, then:

$$\frac{3x}{2L} - 1 = 0 \Rightarrow x = \boxed{\frac{2L}{3}}$$

63 ••• A uniform horizontal disk of mass M and radius R is spinning about the vertical axis through its center with an angular speed ω. When the spinning disk is dropped onto a horizontal tabletop, kinetic-frictional forces on the disk oppose its spinning motion. Let μ_k be the coefficient of kinetic friction between the disk and the tabletop. (*a*) Find the torque $d\tau$ exerted by the force of friction on a circular element of radius r and width dr. (*b*) Find the total torque exerted by friction on the disk. (*c*) Find the time required for the disk to stop rotating.

Picture the Problem We'll first express the torque exerted by the force of friction on the elemental disk and then integrate this expression to find the torque on the entire disk. We'll use Newton's 2nd law to relate this torque to the angular acceleration of the disk and then to the stopping time for the disk.

(*a*) Express the torque exerted on the elemental disk in terms of the friction force and the distance to the elemental disk:

$$d\tau_{\text{f}} = r\,df_{\text{k}} \quad (1)$$

Using the definition of the coefficient of friction, relate the force of friction to μ_{k} and the weight of the circular element:

$$df_{\text{k}} = \mu_{\text{k}}\,g\,dm \quad (2)$$

Letting σ represent the mass per unit area of the disk, express the mass of the circular element:

$$dm = 2\pi\,r\sigma\,dr \quad (3)$$

Substitute equations (2) and (3) in (1) to obtain:

$$d\tau_{\text{f}} = 2\pi\,\mu_{\text{k}}\sigma\,g\,r^2dr \quad (4)$$

Because $\sigma = \dfrac{M}{\pi R^2}$:

$$d\tau_{\text{f}} = \boxed{\frac{2\,\mu_{\text{k}}M\,g}{R^2}r^2dr}$$

(*b*) Integrate $d\tau_{\text{f}}$ to obtain the total torque on the elemental disk:

$$\tau_{\text{f}} = \frac{2\,\mu_{\text{k}}M\,g}{R^2}\int_0^R r^2dr = \boxed{\tfrac{2}{3}MR\mu_{\text{k}}g}$$

(*c*) Relate the disk's stopping time to its angular speed and acceleration:

$$\Delta t = \frac{\omega}{\alpha}$$

Using Newton's 2nd law, express α in terms of the net torque acting on the disk:

$$\alpha = \frac{\tau_{\text{f}}}{I}$$

Substituting for α yields:

$$\Delta t = \frac{\omega}{\dfrac{\tau_{\text{f}}}{I}} = \frac{I\omega}{\tau_{\text{f}}}$$

The moment of inertia of the disk, with respect to its axis of rotation, is:

$$I = \tfrac{1}{2}MR^2$$

Substitute for I and τ_f (from Part (a)) and simplify to obtain:

$$\Delta t = \frac{\frac{1}{2}MR^2\omega}{\frac{2}{3}MR\mu_k g} = \boxed{\frac{3R\omega}{4\mu_k g}}$$

Energy Methods Including Rotational Kinetic Energy

64 • The particles in Figure 9-52 are connected by a very light rod. They rotate about the y axis at 2.0 rad/s. (a) Find the speed of each particle, and use it to calculate the kinetic energy of this system directly from $\sum \frac{1}{2}\boldsymbol{m}_i\boldsymbol{v}_i^2$. (b) Find the moment of inertia about the y axis, calculate the kinetic energy from $\boldsymbol{K} = \frac{1}{2}\boldsymbol{I\omega}^2$, and compare your result with your Part-(a) result.

Picture the Problem The kinetic energy of this rotating system of particles can be calculated either by finding the tangential velocities of the particles and using these values to find the kinetic energy or by finding the moment of inertia of the system and using the expression for the rotational kinetic energy of a system.

(a) Use the relationship between v and ω to find the speed of each particle:

$$\boldsymbol{v}_3 = \boldsymbol{r}_3\boldsymbol{\omega} = (0.20\,\text{m})(2{,}0\,\text{rad/s}) = 0.40\,\text{m/s}$$

and

$$\boldsymbol{v}_1 = \boldsymbol{r}_1\boldsymbol{\omega} = (0.40\,\text{m})(2.0\,\text{rad/s}) = 0.80\,\text{m/s}$$

The kinetic energy of the system is:

$$\boldsymbol{K} = 2\boldsymbol{K}_3 + 2\boldsymbol{K}_1 = \boldsymbol{m}_3\boldsymbol{v}_3^2 + \boldsymbol{m}_1\boldsymbol{v}_1^2$$

Substitute numerical values and evaluate K:

$$\boldsymbol{K} = (3.0\,\text{kg})(0.40\,\text{m/s})^2 + (1.0\,\text{kg})(0.80\,\text{m/s})^2 = \boxed{1.1\,\text{J}}$$

(b) Use the definition of the moment of inertia of a system of particles to obtain:

$$I = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I} = (1.0\,\text{kg})(0.40\,\text{m})^2 + (3.0\,\text{kg})(0.20\,\text{m})^2 + (1.0\,\text{kg})(0.40\,\text{m})^2 + (3.0\,\text{kg})(0.20\,\text{m})^2 = 0.560\,\text{kg}\cdot\text{m}^2$$

The kinetic energy of the system of particles is given by:

$$\boldsymbol{K} = \tfrac{1}{2}\boldsymbol{I\omega}^2$$

Substitute numerical values and evaluate K:

$$\boldsymbol{K} = \tfrac{1}{2}\left(0.560\,\text{kg}\cdot\text{m}^2\right)\left(2.0\,\text{rad/s}\right)^2 = \boxed{1.1\,\text{J}}$$

in agreement with our Part (a) result.

65 • **[SSM]** A 1.4-kg 15-cm-diameter solid sphere is rotating about its diameter at 70 rev/min. (a) What is its kinetic energy? (b) If an additional 5.0 mJ of energy are added to the kinetic energy, what is the new angular speed of the sphere?

Picture the Problem We can find the kinetic energy of this rotating ball from its angular speed and its moment of inertia. In Part (b) we can use the work-kinetic energy theorem to find the angular speed of the sphere when additional kinetic energy has been added to the sphere.

(a) The initial rotational kinetic energy of the ball is:

$$\boldsymbol{K}_\text{i} = \tfrac{1}{2}\boldsymbol{I}\boldsymbol{\omega}_\text{i}^2$$

Express the moment of inertia of the ball with respect to its diameter:

$$I = \tfrac{2}{5}MR^2$$

Substitute for I to obtain:

$$\boldsymbol{K}_\text{i} = \tfrac{1}{5}\boldsymbol{M}\boldsymbol{R}^2\boldsymbol{\omega}_\text{i}^2$$

Substitute numerical values and evaluate K:

$$\boldsymbol{K}_\text{i} = \tfrac{1}{5}(1.4\,\text{kg})(0.075\,\text{m})^2\left(70\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)^2 = 84.6\,\text{mJ} = \boxed{85\,\text{mJ}}$$

(b) Apply the work-kinetic energy theorem to the sphere to obtain:

$$\boldsymbol{W} = \Delta\boldsymbol{K} = \boldsymbol{K}_\text{f} - \boldsymbol{K}_\text{i}$$

or

$$\boldsymbol{W} = \tfrac{1}{2}\boldsymbol{I}\boldsymbol{\omega}_\text{f}^2 - \boldsymbol{K}_\text{i} \Rightarrow \boldsymbol{\omega}_\text{f} = \sqrt{\frac{2(\boldsymbol{W}+\boldsymbol{K}_\text{i})}{\boldsymbol{I}}}$$

Substitute for I and simplify to obtain:

$$\boldsymbol{\omega}_\text{f} = \sqrt{\frac{2(\boldsymbol{W}+\boldsymbol{K}_\text{i})}{\tfrac{2}{5}\boldsymbol{M}\boldsymbol{R}^2}} = \sqrt{\frac{5(\boldsymbol{W}+\boldsymbol{K}_\text{i})}{\boldsymbol{M}\boldsymbol{R}^2}}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \sqrt{\frac{5(84.6\text{ mJ} + 5.0\text{ mJ})}{(1.4\text{ kg})(7.5\text{ cm})^2}}$$
$$= 7.542\frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{\text{min}}$$
$$= \boxed{72\text{ rev/min}}$$

66 •• Calculate the kinetic energy of Earth due to its spinning about its axis, and compare your answer with the kinetic energy of the orbital motion of Earth's center of mass about the Sun. Assume Earth to be a homogeneous sphere of mass 6.0×10^{24} kg and radius 6.4×10^6 m. The radius of Earth's orbit is 1.5×10^{11} m.

Picture the Problem The earth's rotational kinetic energy is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2$ where I is its moment of inertia with respect to its axis of rotation. The center of mass of the earth-sun system is so close to the center of the sun and the earth-sun distance so large that we can use the earth-sun distance as the separation of their centers of mass and assume each to be point mass.

Express the rotational kinetic energy of the earth:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad (1)$$

Find the angular speed of the earth's rotation using the definition of ω:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{24\text{ h} \times \frac{3600\text{ s}}{\text{h}}}$$
$$= 7.27 \times 10^{-5}\text{ rad/s}$$

From Table 9-1, for the moment of inertia of a homogeneous sphere, we find:

$$I = \frac{2}{5}MR^2$$
$$= \frac{2}{5}(6.0 \times 10^{24}\text{ kg})(6.4 \times 10^6\text{ m})^2$$
$$= 9.83 \times 10^{37}\text{ kg} \cdot \text{m}^2$$

Substitute numerical values in equation (1) to obtain:

$$K_{\text{rot}} = \frac{1}{2}(9.83 \times 10^{37}\text{ kg} \cdot \text{m}^2) \times (7.27 \times 10^{-5}\text{ rad/s})^2$$
$$= 2.60 \times 10^{29}\text{ J} = \boxed{2.6 \times 10^{29}\text{ J}}$$

The earth's orbital kinetic energy is:

$$K_{\text{orb}} = \frac{1}{2}I\omega_{\text{orb}}^2 \qquad (2)$$

Find the angular speed of the center of mass of the Earth-Sun system:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{365.24\,\text{days} \times 24\,\frac{\text{h}}{\text{day}} \times \frac{3600\,\text{s}}{\text{h}}} = 1.99 \times 10^{-7}\ \text{rad/s}$$

The orbital moment of inertia of Earth is:

$$I = M_E R_{orb}^2 = \left(6.0 \times 10^{24}\ \text{kg}\right)\left(1.50 \times 10^{11}\ \text{m}\right)^2 = 1.35 \times 10^{47}\,\text{kg} \cdot \text{m}^2$$

Substitute for I in equation (2) and evaluate K_{orb}:

$$K_{orb} = \tfrac{1}{2}\left(1.35 \times 10^{47}\ \text{kg} \cdot \text{m}^2\right) \times \left(1.99 \times 10^{-7}\ \text{rad/s}\right)^2 = 2.68 \times 10^{33}\ \text{J}$$

Evaluate the ratio $\frac{K_{orb}}{K_{rot}}$:

$$\frac{K_{orb}}{K_{rot}} = \frac{2.68 \times 10^{33}\ \text{J}}{2.60 \times 10^{29}\ \text{J}} \approx \boxed{10^4}$$

67 •• **[SSM]** A 2000-kg block is lifted at a constant speed of 8.0 cm/s by a steel cable that passes over a massless pulley to a motor-driven winch (Figure 9-53). The radius of the winch drum is 30 cm. (*a*) What is the tension in the cable? (*b*) What torque does the cable exert on the winch drum? (*c*) What is the angular speed of the winch drum? (*d*) What power must be developed by the motor to drive the winch drum?

Picture the Problem Because the load is not being accelerated, the tension in the cable equals the weight of the load. The role of the massless pulley is to change the direction the force (tension) in the cable acts.

(*a*) Because the block is lifted at constant speed:

$$T = mg = (2000\,\text{kg})\left(9.81\,\text{m/s}^2\right) = \boxed{19.6\,\text{kN}}$$

(*b*) Apply the definition of torque at the winch drum:

$$\tau = Tr = (19.6\,\text{kN})(0.30\,\text{m}) = \boxed{5.9\,\text{kN} \cdot \text{m}}$$

(*c*) Relate the angular speed of the winch drum to the rate at which the load is being lifted (the tangential speed of the cable on the drum):

$$\omega = \frac{v}{r} = \frac{0.080\,\text{m/s}}{0.30\,\text{m}} = \boxed{0.27\,\text{rad/s}}$$

(*d*) The power developed by the motor in terms is the product of the tension in the cable and the speed with which the load is being lifted:

$$\boldsymbol{P} = \boldsymbol{T}\boldsymbol{v} = (19.6\,\text{kN})(0.080\,\text{m/s})$$
$$= \boxed{1.6\,\text{kW}}$$

68 •• A uniform disk that has a mass *M* and a radius *R* can rotate freely about a fixed horizontal axis that passes through its center and perpendicular to the plane of the disk. A small particle that has a mass *m* is attached to the rim of the disk at the top, directly above the pivot. The system is gently nudged, and the disk begins to rotate. As the particle passes through its lowest point, (*a*) what is the angular speed of the disk, and (*b*) what force is exerted by the disk on the particle?

Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the small particle. We can use conservation of energy to find the angular speed of the disk when the particle is at its lowest point and Newton's 2nd law to find the force the disk will have to exert on the particle to keep it from falling off.

(*a*) Use conservation of energy to relate the initial potential energy of the system to its rotational kinetic energy when the small particle is at its lowest point:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}\left(I_{\text{disk}} + I_{\text{particle}}\right)\omega_f^2 - mg\Delta h = 0$$

Solving for ω_f yields:

$$\omega_f = \sqrt{\frac{2mg\Delta h}{I_{\text{disk}} + I_{\text{particle}}}}$$

Substitute for I_{disk}, I_{particle}, and Δh and simplify to obtain:

$$\omega_f = \sqrt{\frac{2mg(2R)}{\frac{1}{2}MR^2 + mR^2}}$$
$$= \boxed{\sqrt{\frac{8mg}{R(2m+M)}}}$$

(*b*) The mass is in uniform circular motion at the bottom of the disk, so the sum of the force *F* exerted by the disk and the gravitational force must be the centripetal force:

$$F - mg = mR\omega_f^2 \Rightarrow F = mg + mR\omega_f^2$$

Substituting for ω_f^2 and simplifying yields::

$$F = mg + mR\left(\frac{8mg}{R(2m+M)}\right)$$
$$= \boxed{mg\left(1+\frac{8m}{2m+M}\right)}$$

69 •• A uniform 1.5-m-diameter ring is pivoted at a point on its perimeter so that it is free to rotate about a horizontal axis that is perpendicular to the plane of the ring. The ring is released with the center of the ring at the same height as the axis (Figure 9-54). (*a*) If the ring was released from rest, what was its maximum angular speed? (*b*) What minimum angular speed must it be given at release if it is to rotate a full 360°?

Picture the Problem Let the zero of gravitational potential energy be at the center of mass of the ring when it is directly below the point of support. We'll use conservation of energy to relate the maximum angular speed and the initial angular speed required for a complete revolution to the changes in the potential energy of the ring.

(*a*) Use conservation of energy to relate the initial potential energy of the ring to its rotational kinetic energy when its center of mass is directly below the point of support:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}I_P\omega_{max}^2 - mg\Delta h = 0 \qquad (1)$$

Use the parallel axis theorem and Table 9-1 to express the moment of inertia of the ring with respect to its pivot point P:

$$I_P = I_{cm} + mR^2$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_{max}^2 - mgR = 0$$

Solving for ω_{max} yields:

$$\omega_{max} = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_{max}:

$$\omega_{max} = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.6\,\text{rad/s}}$$

(*b*) Use conservation of energy to relate the final potential energy of the ring to its initial rotational kinetic energy:

$$\Delta K + \Delta U = 0$$

or, because $U_i = K_f = 0$,

$$-\tfrac{1}{2}I_P\omega_i^2 + mg\Delta h = 0$$

Noting that the center of mass must rise a distance R if the ring is to make a complete revolution, substitute for I_P and Δh to obtain:

$$-\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_i^2 + mgR = 0$$

Solving for ω_i yields:

$$\omega_i = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_i:

$$\omega_i = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.6\,\text{rad/s}}$$

70 •• You set out to design a car that uses the energy stored in a flywheel consisting of a uniform 100-kg cylinder of radius R that has a maximum angular speed of 400 rev/s. The flywheel must deliver an average of 2.00 MJ of energy for each kilometer of distance. Find the smallest value of R for which the car can travel 300 km without the flywheel needing to be recharged.

Picture the Problem We can find the energy that must be stored in the flywheel and relate this energy to the radius of the wheel and use the definition of rotational kinetic energy to find the wheel's radius.

Relate the kinetic energy of the flywheel to the energy it must deliver:

$$\boldsymbol{K}_{\text{rot}} = \tfrac{1}{2}\boldsymbol{I}_{\text{cyl}}\boldsymbol{\omega}^2$$

Express the moment of inertia of the flywheel:

$$I_{\text{cyl}} = \tfrac{1}{2}MR^2$$

Substitute for I_{cyl} to obtain:

$$K_{\text{rot}} = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 \Rightarrow R = \frac{2}{\omega}\sqrt{\frac{K_{\text{rot}}}{M}}$$

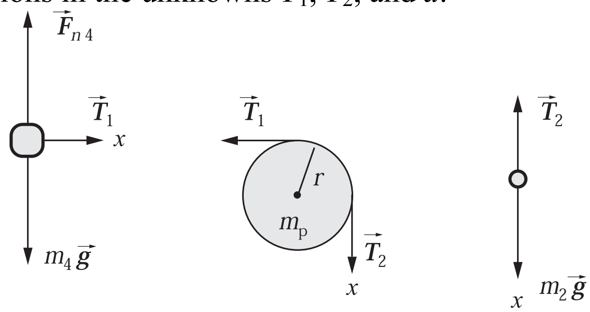
Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \frac{2}{400\,\dfrac{\text{rev}}{\text{s}} \times \dfrac{2\boldsymbol{\pi}\,\text{rad}}{\text{rev}}}\sqrt{\frac{\left(2.00\,\dfrac{\text{MJ}}{\text{km}}\right)(300\,\text{km})}{100\,\text{kg}}} = \boxed{1.95\,\text{m}}$$

Pulleys, Yo-Yos, and Hanging Things

71 •• **[SSM]** The system shown in Figure 9-55consists of a 4.0-kg block resting on a frictionless horizontal ledge. This block is attached to a string that passes over a pulley, and the other end of the string is attached to a hanging 2.0-kg block. The pulley is a uniform disk of radius 8.0 cm and mass 0.60 kg. Find the acceleration of each block and the tension in the string.

Picture the Problem The diagrams show the forces acting on each of the masses and the pulley. We can apply Newton's 2nd law to the two blocks and the pulley to obtain three equations in the unknowns T_1, T_2, and a.



Apply Newton's 2nd law to the two blocks and the pulley:

$$\sum F_x = T_1 = m_4 a\,, \qquad (1)$$

$$\sum \boldsymbol{\tau}_p = (\boldsymbol{T}_2 - \boldsymbol{T}_1)\boldsymbol{r} = \boldsymbol{I}_\mathrm{p}\boldsymbol{\alpha}\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Substitute for I_p and α in equation (2) to obtain:

$$T_2 - T_1 = \tfrac{1}{2} M_p a \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a:

$$a = \frac{m_2 g}{m_2 + m_4 + \frac{1}{2}M_p}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{(2.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{2.0\,\mathrm{kg} + 4.0\,\mathrm{kg} + \frac{1}{2}(0.60\,\mathrm{kg})} = 3.11\,\mathrm{m/s^2} = \boxed{3.1\,\mathrm{m/s^2}}$$

Using equation (1), evaluate T_1:

$$\boldsymbol{T}_1 = (4.0\,\mathrm{kg})(3.11\,\mathrm{m/s^2}) = \boxed{12\,\mathrm{N}}$$

Solve equation (3) for T_2:

$$T_2 = m_2(g - a)$$

Substitute numerical values and evaluate T_2:

$$\boldsymbol{T}_2 = (2.0\,\text{kg})(9.81\,\text{m/s}^2 - 3.11\,\text{m/s}^2) = \boxed{13\,\text{N}}$$

Remarks: Note that the only effect of the pulley is to change the direction of the force in the string.

72 •• For the system in Problem 71, the 2.0-kg block is released from rest. (*a*) Find the speed of the block after it falls a distance of 2.5 m. (*b*) What is the angular speed of the pulley at this instant?

Picture the Problem We'll solve this problem for the general case in which the mass of the block on the ledge is M, the mass of the hanging block is m, and the mass of the pulley is M_p, and R is the radius of the pulley. Let the zero of gravitational potential energy be 2.5 m below the initial position of the 2.0-kg block and R represent the radius of the pulley. Let the system include both blocks, the shelf and pulley, and the earth. The initial potential energy of the 2.0-kg block will be transformed into the translational kinetic energy of both blocks plus rotational kinetic energy of the pulley.

(*a*) Use energy conservation to relate the speed of the 2 kg block when it has fallen a distance Δh to its initial potential energy and the kinetic energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_\text{i} = U_\text{f} = 0$,

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}I_\text{pulley}\omega^2 - mgh = 0$$

Substitute for I_pulley and ω to obtain:

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2} - mgh = 0$$

Solving for v yields:

$$v = \sqrt{\frac{2mgh}{M + m + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

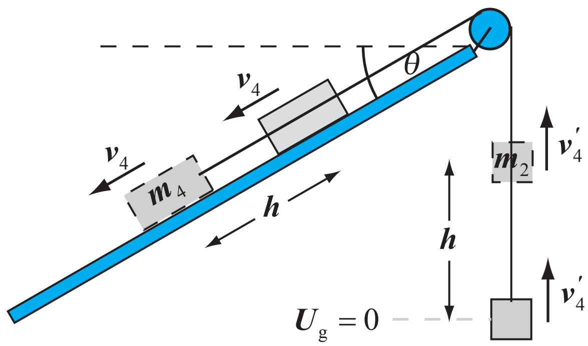
$$v = \sqrt{\frac{2(2.0\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m})}{4.0\,\text{kg} + 2.0\,\text{kg} + \frac{1}{2}(0.60\,\text{kg})}} = 3.946\,\text{m/s} = \boxed{3.9\,\text{m/s}}$$

(*b*) The angular speed of the pulley is the ratio of its tangential speed to its radius:

$$\boldsymbol{\omega} = \frac{\boldsymbol{v}}{\boldsymbol{R}} = \frac{3.946\,\text{m/s}}{0.080\,\text{m}} = \boxed{49\,\text{rad/s}}$$

73 •• For the system in Problem 71, if the (frictionless) ledge were adjustable in angle, at what angle would it have to be tilted upward so that once the system is set into motion the blocks will continue to move at constant speed?

Picture the Problem The pictorial representation shows the ledge with its left end lowered and the 4.0-kg object moving with a constant speed v_4. As this object whose mass is m_4 slides down the frictionless incline, the object whose mass is m_2 rises. Let the system include the earth, the ledge, and both objects and apply conservation of mechanical energy to determine the angle of inclination of the ledge.



Apply conservation of mechanical energy to the two moving objects to obtain:

$$\Delta K_2 + \Delta K_4 + \Delta U_2 + \Delta U_4 = 0$$

Because the objects are moving at constant speed, $\Delta K_2 = \Delta K_4 = 0$ and:

$$\Delta U_2 + \Delta U_4 = 0$$

Substituting for ΔU_2 and ΔU_4 yields:

$$m_2 gh - m_4 gh \sin\theta = 0$$

Solving for θ yields:

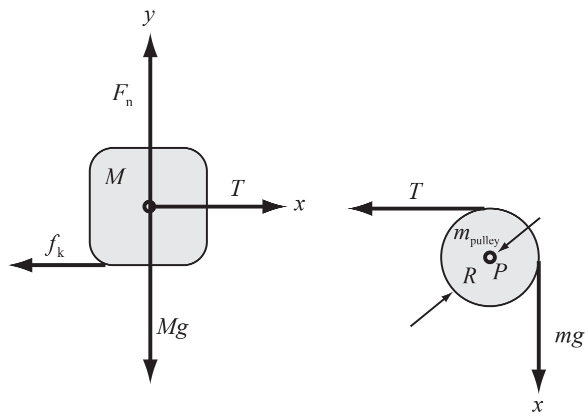
$$\theta = \sin^{-1}\left(\frac{m_2}{m_4}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{2.0\text{ kg}}{4.0\text{ kg}}\right) = \boxed{30°}$$

74 •• In the system shown in Figure 9-55, there is a 4.0-kg block resting on a horizontal ledge. The coefficient of kinetic friction between the ledge and the block is 0.25. The block is attached to a string that passes over a pulley and the other end of the string is attached to a hanging 2.0-kg block. The pulley is a uniform disk of radius 8.0 cm and mass 0.60 kg. Find the acceleration of each block and the tension in the string.

Picture the Problem Assuming that the string does not stretch or slip on the pulley, we can apply Newton's 2nd law in translational form to the 4.0-kg block and Newton's 2nd law in rotational form to the pulley to obtain simultaneous equations in the tension T in the string and the common acceleration a of the blocks.



Apply $\sum \vec{F} = m\vec{a}$ to the 4.0-kg block:

$$\sum F_x = T - f_k = Ma \quad (1)$$

and

$$\sum F_y = F_n - Mg = 0 \Rightarrow F_n = Mg$$

Because $f_k = \mu_k F_n$, equation (1) becomes:

$$T - \mu_k F_n = Ma$$

or

$$T - \mu_k Mg = Ma \quad (2)$$

Apply $\sum \tau_P = I_P \alpha$ to the pulley:

$$mgR - TR = I_P \alpha$$

Assuming the string does not slip on the pulley, $\alpha = a/R$ and:

$$mgR - TR = I_P \frac{a}{R}$$

From Table 9-1, the moment of inertia of the pulley about an axis through P and perpendicular to the plane of the pulley is:

$$I_P = \tfrac{1}{2} m_{\text{pulley}} R^2$$

Substituting for I_P and simplifying yields:

$$mgR - TR = \left(\tfrac{1}{2} m_{\text{pulley}} R^2\right)\frac{a}{R}$$

or

$$mg - T = \tfrac{1}{2} m_{\text{pulley}} a \quad (3)$$

Add equations (2) and (3) to obtain:

$$mg - \mu_k Mg = Ma + \tfrac{1}{2} m_{\text{pulley}} a$$

Solving for a yields:

$$a = \frac{(m - \mu_k M)g}{M + \frac{1}{2} m_{\text{pulley}}}$$

Substitute numerical values and evaluate a:

$$a = \frac{(2.0\text{ kg} - (0.25)(4.0\text{ kg}))(9.81\text{ m/s}^2)}{4.0\text{ kg} + \frac{1}{2}(0.60\text{ kg})}$$
$$= 2.281\text{ m/s}^2 = \boxed{2.3\text{ m/s}^2}$$

Solving equation (3) for T yields:

$$T = mg - \tfrac{1}{2} m_{\text{pulley}} a$$

Substitute numerical values and evaluate T:

$$T = (4.0\text{ kg})(9.81\text{ m/s}^2) - \tfrac{1}{2}(0.60\text{ kg})(2.281\text{ m/s}^2) = \boxed{39\text{ N}}$$

75 •• A 1200-kg car is being raised over water by a winch. At the moment the car is 5.0 m above the water (Figure 9-56), the gearbox breaks—allowing the winch drum to spin freely as the car falls. During the car's fall, there is no slipping between the (massless) rope, the pulley wheel, and the winch drum. The moment of inertia of the winch drum is 320 kg·m^2, and the moment of inertia of the pulley wheel is 4.00 kg·m^2. The radius of the winch drum is 0.800 m, and the radius of the pulley is 0.300 m. Assume the car starts to fall from rest. Find the speed of the car as it hits the water.

Picture the Problem Let the zero of gravitational potential energy be at the water's surface and let the system include the winch, the car, and the earth. We'll apply conservation of energy to relate the car's speed as it hits the water to its initial potential energy. Note that some of the car's initial potential energy will be transformed into rotational kinetic energy of the winch and pulley.

Use conservation of mechanical energy to relate the car's speed as it hits the water to its initial potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_w\omega_w^2 + \tfrac{1}{2}I_p\omega_p^2 - mg\Delta h = 0$$

Express ω_w^2 and ω_p^2 in terms of the speed v of the rope, which is the same throughout the system:

$$\omega_w^2 = \frac{v^2}{r_w^2} \text{ and } \omega_p^2 = \frac{v^2}{r_p^2}$$

Substitute for ω_w^2 and ω_p^2 to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_w\frac{v^2}{r_w^2} + \tfrac{1}{2}I_p\frac{v^2}{r_p^2} - mg\Delta h = 0$$

Solving for v yields:

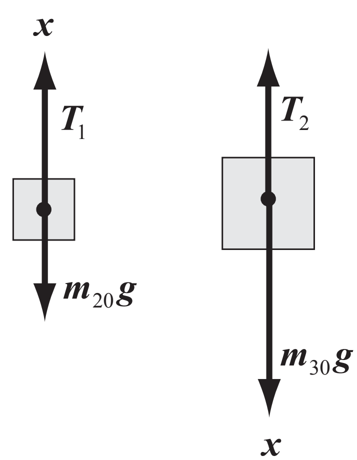
$$v=\sqrt{\frac{2mg\Delta h}{m+\frac{I_w}{r_w^2}+\frac{I_p}{r_p^2}}}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2(1200\,\text{kg})(9.81\,\text{m/s}^2)(5.00\,\text{m})}{1200\,\text{kg}+\frac{320\,\text{kg}\cdot\text{m}^2}{(0.800\,\text{m})^2}+\frac{4.00\,\text{kg}\cdot\text{m}^2}{(0.300\,\text{m})^2}}}=\boxed{8.21\,\text{m/s}}$$

76 •• The system in Figure 9-57 is released from rest when the 30-kg block is 2.0 m above the ledge. The pulley is a uniform 5.0-kg disk with a radius of 10 cm. Just before the 30-kg block hits the ledge, find (*a*) its speed, (*b*) the angular speed of the pulley, and (*c*) the tensions in the strings. (*d*) Find the time of descent for the 30-kg block. Assume that the string does not slip on the pulley.

Picture the Problem Let the system include the blocks, the pulley and the earth. Choose the zero of gravitational potential energy to be at the ledge and apply energy conservation to relate the impact speed of the 30-kg block to the initial potential energy of the system. We can use a constant-acceleration equations and Newton's 2nd law to find the tensions in the strings and the descent time.



(*a*) Use conservation of mechanical energy to relate the impact speed of the 30-kg block to the initial potential energy of the system:

$$\Delta K+\Delta U=0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}m_{30}v^2+\tfrac{1}{2}m_{20}v^2+\tfrac{1}{2}I_p\omega_p^2 + m_{20}g\Delta h-m_{30}g\Delta h=0$$

Substitute for ω_p and I_p to obtain:

$$\tfrac{1}{2}m_{30}v^2+\tfrac{1}{2}m_{20}v^2+\tfrac{1}{2}\left(\tfrac{1}{2}M_p r^2\right)\left(\frac{v^2}{r^2}\right) + m_{20}g\Delta h-m_{30}g\Delta h=0$$

Solving for v yields:

$$v=\sqrt{\frac{2g\Delta h(m_{30}-m_{20})}{m_{20}+m_{30}+\frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2\left(9.81\,\text{m/s}^2\right)(2.0\,\text{m})(30\,\text{kg}-20\,\text{kg})}{20\,\text{kg}+30\,\text{kg}+\frac{1}{2}(5.0\,\text{kg})}}$$
$$=2.73\,\text{m/s}=\boxed{2.7\,\text{m/s}}$$

(b) Find the angular speed at impact from the tangential speed at impact and the radius of the pulley:

$$\boldsymbol{\omega}=\frac{\boldsymbol{v}}{\boldsymbol{r}}=\frac{2.73\,\text{m/s}}{0.10\,\text{m}}=\boxed{27\,\text{rad/s}}$$

(c) Apply Newton's 2nd law to the blocks:

$$\sum F_x=T_1-m_{20}g=m_{20}a \qquad (1)$$
and
$$\sum F_x=m_{30}g-T_2=m_{30}a \qquad (2)$$

Using a constant-acceleration equation, relate the speed at impact to the fall distance and the acceleration:

$$v^2=v_0^2+2a\Delta h$$
or, because $v_0=0$,
$$v^2=2a\Delta h\Rightarrow \boldsymbol{a}=\frac{\boldsymbol{v}^2}{2\Delta \boldsymbol{h}}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a}=\frac{(2.73\,\text{m/s})^2}{2(2.0\,\text{m})}=1.87\,\text{m/s}^2$$

Solve equation (1) for T_1 to obtain:

$$\boldsymbol{T}_1=\boldsymbol{m}_{20}(\boldsymbol{g}+\boldsymbol{a})$$

Substitute numerical values and evaluate T_1:

$$\boldsymbol{T}_1=(20\,\text{kg})\left(9.81\,\text{m/s}^2+1.87\,\text{m/s}^2\right)$$
$$=\boxed{0.23\,\text{kN}}$$

Solve equation (2) for T_2 to obtain:

$$\boldsymbol{T}_2=\boldsymbol{m}_{30}(\boldsymbol{g}-\boldsymbol{a})$$

Substitute numerical values and evaluate T_2:

$$\boldsymbol{T}_2=(30\,\text{kg})\left(9.81\,\text{m/s}^2-1.87\,\text{m/s}^2\right)$$
$$=\boxed{0.24\,\text{kN}}$$

(d) Noting that the initial speed of the 30-kg block is zero, express the time-of-fall in terms of the fall distance and the block's average speed:

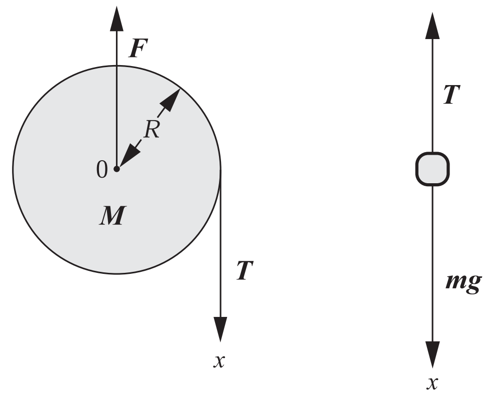
$$\Delta t=\frac{\Delta h}{v_{\text{av}}}=\frac{\Delta h}{\frac{1}{2}v}=\frac{2\Delta h}{v}$$

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t}=\frac{2(2.0\,\text{m})}{2.73\,\text{m/s}}=\boxed{1.5\,\text{s}}$$

77 •• A uniform solid sphere of mass M and radius R is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to an object of mass m (Figure 9-58). Assume that the string does not slip on the sphere. Find (*a*) the acceleration of the object, and (*b*) the tension in the string.

Picture the Problem The force diagram shows the forces acting on the sphere and the hanging object. The tension in the string is responsible for the angular acceleration of the sphere and the difference between the weight of the object and the tension is the net force acting on the hanging object. We can use Newton's 2nd law to obtain two equations in a and T that we can solve simultaneously.



(*a*) Noting that $T = T'$, apply Newton's 2nd law to the sphere and the hanging object:

$$\sum \tau_0 = TR = I_{\text{sphere}}\alpha \quad (1)$$

and

$$\sum F_x = mg - T = ma \quad (2)$$

Substitute for I_{sphere} and α in equation (1) to obtain:

$$TR = \left(\tfrac{2}{5}MR^2\right)\frac{a}{R} \quad (3)$$

Eliminate T between equations (2) and (3) and solve for a to obtain:

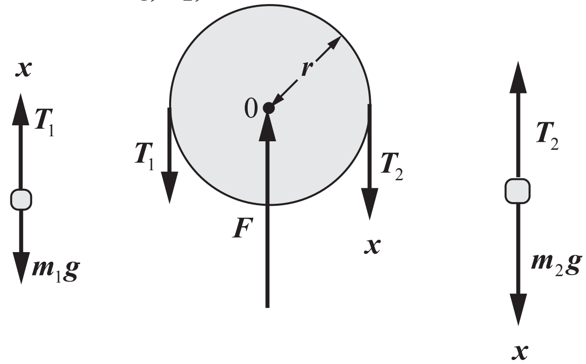
$$a = \boxed{\frac{g}{1+\dfrac{2M}{5m}}}$$

(*b*) Substitute for a in equation (2) and solve for T to obtain:

$$T = \boxed{\frac{2mMg}{5m+2M}}$$

78 •• Two objects, of masses m_1= 500 g and m_2 = 510 g, are connected by a string of negligible mass that passes over a pulley with frictionless bearings (Figure 9-59). The pulley is a uniform 50.0-g disk with a radius of 4.00 cm. The string does not slip on the pulley. (*a*) Find the accelerations of the objects. (*b*) What is the tension in the string between the 500-g block and the pulley? What is the tension in the string between the 510-g block and the pulley? By how much do these tensions differ? (*c*) What would your answers be if you neglected the mass of the pulley?

Picture the Problem The diagram shows the forces acting on both objects and the pulley. The direction of motion has been chosen to be the positive x direction. By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and a that we can solve simultaneously.



(*a*) Apply Newton's 2nd law to the pulley and the two objects:

$$\sum F_x = T_1 - m_1 g = m_1 a\,, \qquad (1)$$

$$\sum \tau_0 = (T_2 - T_1)r = I_0\alpha\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Substitute for $I_0 = I_{\text{pulley}}$ and α in equation (2) to obtain:

$$(T_2 - T_1)r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a to obtain:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{1}{2}m}$$

Substitute numerical values and evaluate a:

$$a = \frac{(80.25\,\text{g} - 75.25\,\text{g})(981\,\text{cm/s}^2)}{75.25\,\text{g} + 80.25\,\text{g} + \frac{1}{2}(50.00\,\text{g})}$$

$$= 27.175\,\text{cm/s}^2 = \boxed{27.2\,\text{cm/s}^2}$$

(*b*) Substitute for a in equation (1) and solve for T_1 to obtain:

$$\boldsymbol{T}_1 = \boldsymbol{m}_1(\boldsymbol{g} + \boldsymbol{a})$$

$$= (0.07525\,\text{kg})$$

$$\times(9.81\,\text{m/s}^2 + 0.27\,\text{m/s}^2)$$

$$= \boxed{0.76\,\text{N}}$$

Substitute for a in equation (3) and solve for T_2 to obtain:

$$\boldsymbol{T}_2 = \boldsymbol{m}_2(\boldsymbol{g}-\boldsymbol{a}) = (0.08025\,\text{kg}) \times (9.81\,\text{m/s}^2 - 0.27\,\text{m/s}^2) = \boxed{0.77\,\text{N}}$$

ΔT is the difference between T_2 and T_1:

$$\Delta \boldsymbol{T} = \boldsymbol{T}_2 - \boldsymbol{T}_1 = 0.77\,\text{N} - 0.76\,\text{N} = \boxed{0.01\,\text{N}}$$

(c) If we ignore the mass of the pulley, our acceleration equation becomes:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Substitute numerical values and evaluate a:

$$a = \frac{(80.25\,\text{g} - 75.25\,\text{g})(981\,\text{cm/s}^2)}{75.25\,\text{g} + 80.25\,\text{g}} = 31.543\,\text{cm/s}^2 = \boxed{31.5\,\text{cm/s}^2}$$

Substitute for a in equation (1) and solve for T_1 to obtain:

$$T_1 = m_1(g + a)$$

Substitute numerical values and evaluate T_1:

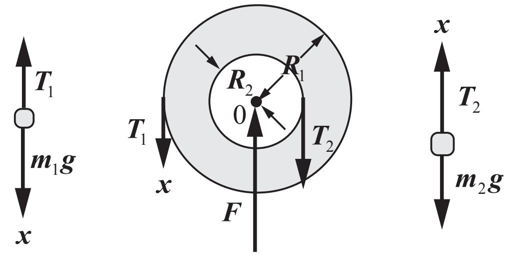
$$\boldsymbol{T}_1 = (0.08025\,\text{kg})(9.81\,\text{m/s}^2 + 0.32\,\text{m/s}^2) = \boxed{0.81\,\text{N}}$$

From equation (4), if $m = 0$:

$$\boldsymbol{T}_2 = \boxed{\boldsymbol{T}_1}$$

79 •• **[SSM]** Two objects are attached to ropes that are attached to two wheels on a common axle, as shown in Figure 9-60. The two wheels are attached together so that they form a single rigid object. The moment of inertia of the rigid object is 40 kg·m^2. The radii of the wheels are R_1 = 1.2 m and R_2 = 0.40 m. (a) If m_1= 24 kg, find m_2 such that there is no angular acceleration of the wheels. (b) If 12 kg is placed on top of m_1, find the angular acceleration of the wheels and the tensions in the ropes.

Picture the Problem The following diagram shows the forces acting on both objects and the pulley for the conditions of Part (b). By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and α that we can solve simultaneously.



(*a*) When the system does not accelerate, $\boldsymbol{T}_1 = \boldsymbol{m}_1\boldsymbol{g}$ and $\boldsymbol{T}_2 = \boldsymbol{m}_2\boldsymbol{g}$. Under these conditions:

$$\sum \tau_0 = m_1 g R_1 - m_2 g R_2 = 0$$

Solving for m_2 yields:

$$m_2 = m_1 \frac{R_1}{R_2}$$

Substitute numerical values and evaluate m_2:

$$\boldsymbol{m}_2 = (24\,\text{kg})\frac{1.2\,\text{m}}{0.40\,\text{m}} = \boxed{72\,\text{kg}}$$

(*b*) Apply Newton's 2nd law to the objects and the pulley:

$$\sum F_x = m_1 g - T_1 = m_1 a\,, \qquad (1)$$

$$\sum \tau_0 = T_1 R_1 - T_2 R_2 = I_0 \alpha\,, \qquad (2)$$

and

$$\sum F_x = T_2 - m_2 g = m_2 a \qquad (3)$$

Eliminate a in favor of α in equations (1) and (3) and solve for T_1 and T_2:

$$T_1 = m_1 (g - R_1 \alpha) \qquad (4)$$

and

$$T_2 = m_2 (g + R_2 \alpha) \qquad (5)$$

Substitute for T_1 and T_2 in equation (2) and solve for α to obtain:

$$\alpha = \frac{(m_1 R_1 - m_2 R_2) g}{m_1 R_1^2 + m_2 R_2^2 + I_0}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{[(36\,\text{kg})(1.2\,\text{m}) - (72\,\text{kg})(0.40\,\text{m})](9.81\,\text{m/s}^2)}{(36\,\text{kg})(1.2\,\text{m})^2 + (72\,\text{kg})(0.40\,\text{m})^2 + 40\,\text{kg}\cdot\text{m}^2} = 1.37\,\text{rad/s}^2 = \boxed{1.4\,\text{rad/s}^2}$$

Substitute numerical values in equation (4) to find T_1:

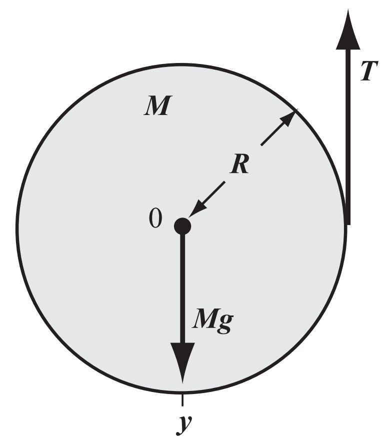
$$\boldsymbol{T}_1 = (36\,\text{kg})[9.81\,\text{m/s}^2 - (1.2\,\text{m})(1.37\,\text{rad/s}^2)] = \boxed{0.29\,\text{kN}}$$

Substitute numerical values in equation (5) to find T_2:

$$\boldsymbol{T}_2 = (72\,\text{kg})\left[9.81\,\text{m/s}^2 + (0.40\,\text{m})(1.37\,\text{rad/s}^2)\right] = \boxed{0.75\,\text{kN}}$$

80 •• The upper end of the string wrapped around the cylinder in Figure 9-61 is held by a hand that is accelerated upward so that the center of mass of the cylinder does not move as the cylinder spins up. Find (*a*) the tension in the string, (*b*) the angular acceleration of the cylinder, and (*c*) the acceleration of the hand.

Picture the Problem By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T and a. In (*b*) we can use the torque equation from (*a*) and our value for T to find α. In (*c*) we use the condition that the acceleration of a point on the rim of the cylinder is the same as the acceleration of the hand, together with the angular acceleration of the cylinder, to find the acceleration of the hand.



(*a*) Apply $\sum \tau_0 = I\alpha$ to the cylinder about an axis through its center of mass:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum \boldsymbol{F}_y = \boldsymbol{Mg} - \boldsymbol{T} = 0 \qquad (2)$$

Solving equation (2) for T yields:

$$T = \boxed{Mg}$$

(*b*) Solving equation (1) for α yields:

$$\alpha = \frac{TR}{I_0}$$

From Table 9-1, the moment of inertia of a cylinder about an axis through 0 and perpendicular to the end of the cylinder is:

$$I_0 = \tfrac{1}{2}MR^2$$

Substitute for T and I_0 and simplify to obtain:

$$\alpha = \frac{MgR}{\frac{1}{2}MR^2} = \boxed{\frac{2g}{R}}$$

(c) Relate the acceleration a of the hand to the angular acceleration of the cylinder:

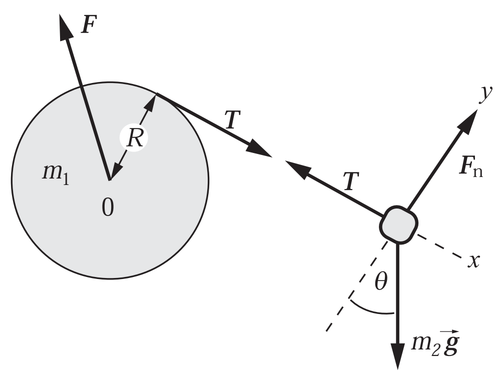
$$a = R\alpha$$

Substitute for α and simplify to obtain:

$$\boldsymbol{a} = \boldsymbol{R}\left(\frac{2\boldsymbol{g}}{\boldsymbol{R}}\right) = \boxed{2\boldsymbol{g}}$$

81 •• **[SSM]** A uniform cylinder of mass m_1 and radius R is pivoted on frictionless bearings. A massless string wrapped around the cylinder is connected to a block of mass m_2 that is on a frictionless incline of angle θ, as shown in Figure 9-62. The system is released from rest with the block a vertical distance h above the bottom of the incline. (a) What is the acceleration of the block? (b) What is the tension in the string? (c) What is the speed of the block as it reaches the bottom of the incline? (d) Evaluate your answers for the special case where θ = 90° and m_1 = 0. Are your answers what you would expect for this special case? Explain.

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. By applying Newton's 2nd law to the cylinder and the block we can obtain simultaneous equations in a, T, and α from which we can express a and T. By applying the conservation of energy, we can derive an expression for the speed of the block when it reaches the bottom of the incline.



(a) Apply Newton's 2nd law to the cylinder and the block:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum F_x = m_2 g\sin\theta - T = m_2 a \qquad (2)$$

Substitute for α and I_0 in equation (1), solve for T, and substitute in equation (2) and solve for a to obtain:

$$a = \boxed{\frac{g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(b) Substituting for a in equation (2) and solve for T yields:

$$T = \boxed{\frac{\frac{1}{2}m_1 g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(*c*) Noting that the block is released from rest, express the total energy of the system when the block is at height *h*:

$$E = U + K = \boxed{m_2 gh}$$

(*d*) Use the fact that this system is conservative to express the total energy at the bottom of the incline:

$$E_{\text{bottom}} = \boxed{m_2 gh}$$

(*e*) Express the total energy of the system when the block is at the bottom of the incline in terms of its kinetic energies:

$$\begin{aligned} E_{\text{bottom}} &= K_{\text{tran}} + K_{\text{rot}} \\ &= \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} I_0 \omega^2 \end{aligned}$$

Substitute for ω and I_0 to obtain:

$$\tfrac{1}{2} m_2 v^2 + \tfrac{1}{2}\left(\tfrac{1}{2} m_1 r^2\right)\frac{v^2}{r^2} = m_2 gh$$

Solving for *v* yields:

$$v = \boxed{\sqrt{\frac{2gh}{1 + \dfrac{m_1}{2m_2}}}}$$

For $\theta = 0$:

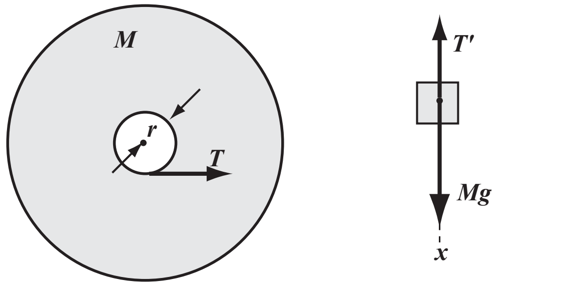
$$\boldsymbol{a} = \boxed{0} \text{ and } \boldsymbol{T} = \boxed{0}$$

(*f*) For $\theta = 90°$ and $m_1 = 0$:

$$\boldsymbol{a} = \boxed{\boldsymbol{g}},\ T = \boxed{0},\ \text{and } v = \boxed{\sqrt{2gh}}$$

82 •• A device for measuring the moment of inertia of an object is shown in Figure 9-63. The circular platform is attached to a concentric drum of radius *R*, and are free to rotate about a frictionless vertical axis. The string that is wound around the drum passes over a frictionless and massless pulley to a block of mass *M*. The block is released from rest, and the time t_1 it takes for it to drop a distance *D* is measured. The system is then rewound, the object whose moment of inertia *I* we wish to measure is placed on the platform, and the system is again released from rest. The time t_2 required for the block to drop the same distance *D* then provides the data needed to calculate *I*. Using $R = 10$ cm, $M = 2.5$ kg, $D = 1.8$ m and $t_1 = 4.2$ s and $t_2 = 6.8$ s, (*a*) find the moment of inertia of the platform-drum combination, (*b*) Using $t_2 = 6.9$ s, find the moment of inertia of the platform-drum-object combination. (*c*) Use your results for Parts (*a*) and (*b*) to find the moment of inertia of the object.

Picture the Problem Let r be the radius of the concentric drum (10 cm) and let I_0 be the moment of inertia of the drum plus platform. We can use Newton's 2nd law in both translational and rotational forms to express I_0 in terms of a and a constant-acceleration equation to express a and then find I_0. We can use the same equation to find the total moment of inertia when the object is placed on the platform and then subtract to find its moment of inertia.



Top view of platform   Side view of falling object

(*a*) Apply Newton's 2nd law to the platform and the weight:

$$\sum \tau_0 = Tr = I_0\alpha \qquad (1)$$

and

$$\sum F_x = Mg - T = Ma \qquad (2)$$

Substitute a/r for α in equation (1) and solve for T:

$$T = \frac{I_0}{r^2}a$$

Substitute for T in equation (2) and solve for I_0 to obtain:

$$I_0 = \frac{Mr^2(g-a)}{a} \qquad (3)$$

Using a constant-acceleration equation, relate the distance of fall to the acceleration of the weight and the time of fall and solve for the acceleration:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta x = D$,

$$a = \frac{2D}{(\Delta t)^2}$$

Substitute for a in equation (3) to obtain:

$$I_0 = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_0:

$$I_0 = (2.5\,\text{kg})(0.10\,\text{m})^2\left[\frac{(9.81\,\text{m/s}^2)(4.2\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.2\,\text{kg}\cdot\text{m}^2}$$

(*b*) Relate the moments of inertia of the platform, drum, shaft, and pulley (I_0) to the moment of inertia of the object and the total moment of inertia:

$$I_{\text{tot}} = I_0 + I = Mr^2\left(\frac{g}{a} - 1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D} - 1\right)$$

Substitute numerical values and evaluate I_{tot}:

$$\boldsymbol{I}_{\text{tot}} = (2.5\,\text{kg})(0.10\,\text{m})^2\left[\frac{(9.81\,\text{m/s}^2)(6.8\,\text{s})^2}{2(1.8\,\text{m})} - 1\right] = 3.125\,\text{kg}\cdot\text{m}^2 = \boxed{3.1\,\text{kg}\cdot\text{m}^2}$$

I is the difference between I_{tot} and I_0:

$$\boldsymbol{I} = \boldsymbol{I}_{\text{tot}} - \boldsymbol{I}_0$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I} = 3.125\,\text{kg}\cdot\text{m}^2 - 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.9\,\text{kg}\cdot\text{m}^2}$$

Objects Rotating and Rolling Without Slipping

83 • A homogeneous 60-kg cylinder of radius 18 cm is rolling without slipping along a horizontal floor at a speed of 15 m/s. What is the minimum amount of work that was required to give it this motion?

Picture the Problem Any work done on the cylinder by a net force will change its kinetic energy. Therefore, the work needed to give the cylinder this motion is equal to its kinetic energy.

Express the relationship between the work needed to give the cylinder this motion:

$$|W| = |\Delta K| = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

or, because $v = r\omega$ (the cylinder is rolling without slipping),

$$|\boldsymbol{W}| = \tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}^2 + \tfrac{1}{2}\boldsymbol{I}\left(\frac{\boldsymbol{v}}{\boldsymbol{r}}\right)^2$$

Substitute for I (see Table 9-1) and simplify to obtain:

$$|\boldsymbol{W}| = \tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}\boldsymbol{m}\boldsymbol{r}^2\right)\frac{\boldsymbol{v}^2}{\boldsymbol{r}^2} = \tfrac{3}{4}\boldsymbol{m}\boldsymbol{v}^2$$

Substitute numerical values and evaluate $|\boldsymbol{W}|$:

$$|\boldsymbol{W}| = \tfrac{3}{4}(60\,\text{kg})(15\,\text{m/s})^2 = \boxed{10\,\text{kJ}}$$

84 • An object is rolling without slipping. What percentage of its total kinetic energy is its translational kinetic energy if the object is (*a*) a uniform sphere, (*b*) a uniform cylinder, or (*c*) a hoop.

Picture the Problem The total kinetic energy of any object that is rolling without slipping is given by $K = K_{\text{trans}} + K_{\text{rot}}$. We can find the percentages associated with each motion by expressing the moment of inertia of the objects as kmr^2 and deriving a general expression for the ratios of rotational kinetic energy to total kinetic energy and translational kinetic energy to total kinetic energy and substituting the appropriate values of k.

Express the total kinetic energy associated with a rotating and translating object:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmr^2\right)\frac{v^2}{r^2}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 = \tfrac{1}{2}mv^2(1+k)$$

Express the ratio $\dfrac{K_{\text{trans}}}{K}$:

$$\frac{\boldsymbol{K}_{\text{trans}}}{\boldsymbol{K}} = \frac{\frac{1}{2}\boldsymbol{mv}^2}{\frac{1}{2}\boldsymbol{mv}^2(1+\boldsymbol{k})} = \frac{1}{1+\boldsymbol{k}}$$

(*a*) Substitute $\boldsymbol{k} = \frac{2}{5}$ for a uniform sphere to obtain:

$$\left.\frac{\boldsymbol{K}_{\text{trans}}}{\boldsymbol{K}}\right|_{\text{sphere}} = \frac{1}{1+0.4} = 0.714 = \boxed{71.4\%}$$

(*b*) Substitute $\boldsymbol{k} = \frac{1}{2}$ for a uniform cylinder to obtain:

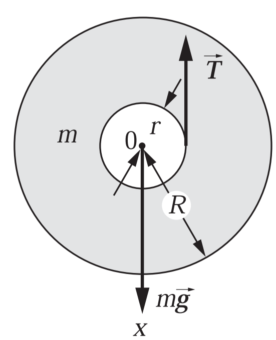
$$\left.\frac{\boldsymbol{K}_{\text{trans}}}{\boldsymbol{K}}\right|_{\text{cylinder}} = \frac{1}{1+0.5} = 0.667 = \boxed{66.7\%}$$

(*c*) Substitute $k = 1$ for a hoop to obtain:

$$\left.\frac{\boldsymbol{K}_{\text{trans}}}{\boldsymbol{K}}\right|_{\text{hoop}} = \frac{1}{1+1} = 0.500 = \boxed{50.0\%}$$

85 •• **[SSM]** In 1993 a giant 400-kg yo-yo with a radius of 1.5 m was dropped from a crane at a height of 57 m. One end of the string was tied to the top of the crane, so the yo-yo unwound as it descended. Assuming that the axle of the yo-yo had a radius of 0.10 m, estimate its linear speed at the end of the fall.

Picture the Problem The forces acting on the yo-yo are shown in the figure. We can use a constant-acceleration equation to relate the velocity of descent at the end of the fall to the yo-yo's acceleration and Newton's 2nd law in both translational and rotational form to find the yo-yo's acceleration.



Using a constant-acceleration equation, relate the yo-yo's final speed to its acceleration and fall distance:

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

$$v = \sqrt{2a\Delta h} \quad (1)$$

Use Newton's 2nd law to relate the forces that act on the yo-yo to its acceleration:

$$\sum F_x = mg - T = ma \quad (2)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \quad (3)$$

Use $a = r\alpha$ to eliminate α in equation (3)

$$Tr = I_0 \frac{a}{r} \quad (4)$$

Eliminate T between equations (2) and (4) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \quad (5)$$

Substitute $\frac{1}{2}mR^2$ for I_0 in equation (5):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma \Rightarrow a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

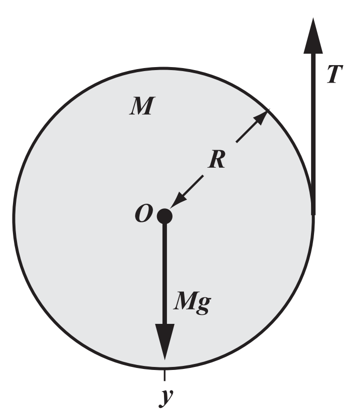
$$\boldsymbol{a} = \frac{9.81\,\text{m/s}^2}{1 + \frac{(1.5\,\text{m})^2}{2(0.10\,\text{m})^2}} = 0.0864\,\text{m/s}^2$$

Substitute in equation (1) and evaluate v:

$$\boldsymbol{v} = \sqrt{2(0.0864\,\text{m/s}^2)(57\,\text{m})} = \boxed{3.1\,\text{m/s}}$$

86 •• A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed, and the cylinder falls vertically as shown in Figure 9-64. (*a*) Show that the acceleration of the cylinder is downward with a magnitude $a = 2g/3$. (*b*) Find the tension in the string.

Picture the Problem The forces acting on the cylinder are shown in the diagram. Choose a coordinate system in which the $+y$ directions is downward. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T, a and α that we can solve simultaneously.



(*a*) Apply Newton's 2nd law to the cylinder:

$$\sum \tau_0 = TR = I_0\alpha \quad (1)$$

and

$$\sum \boldsymbol{F}_y = \boldsymbol{Mg} - \boldsymbol{T} = \boldsymbol{Ma} \quad (2)$$

Substitute for α and I_0 (see Table 9-1) in equation (1) to obtain:

$$TR = \left(\tfrac{1}{2}MR^2\right)\left(\frac{a}{R}\right) \Rightarrow T = \tfrac{1}{2}Ma \ (3)$$

Substitute for T in equation (2) to obtain:

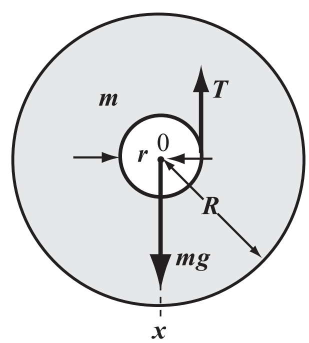
$$Mg - \tfrac{1}{2}Ma = Ma \Rightarrow a = \boxed{\tfrac{2}{3}g}$$

(*b*) Substitute for a in equation (3) and simplify to obtain:

$$\boldsymbol{T} = \tfrac{1}{2}\boldsymbol{M}\left(\tfrac{2}{3}\boldsymbol{g}\right) = \boxed{\tfrac{1}{3}\boldsymbol{Mg}}$$

87 •• A 0.10-kg yo-yo consists of two solid disks, each of radius 10 cm, is joined by a massless rod of radius 1.0 cm. A string is wrapped around the rod. One end of the string is held fixed and is under tension as the yo-yo is released. The yo-yo rotates as it descends vertically. Find (*a*) the acceleration of the yo-yo, and (*b*) the tension *T*.

Picture the Problem The forces acting on the yo-yo are shown in the figure. Choose a coordinate system in which the $+x$ direction is downward. Apply Newton's 2nd law in both translational and rotational form to obtain simultaneous equations in T, a, and α from which we can eliminate α and solve for T and a.



Apply Newton's 2nd law to the yo-yo:

$$\sum F_x = mg - T = ma \quad (1)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \quad (2)$$

Use $a = r\alpha$ to eliminate α in equation (2)

$$Tr = I_0\frac{a}{r} \quad (3)$$

Eliminate T between equations (1) and (3) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \quad (4)$$

Substitute $\frac{1}{2}mR^2$ for I_0 (see Table 9-1) in equation (4):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma \Rightarrow a = \frac{g}{1+\frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{9.81\,\text{m/s}^2}{1+\frac{(0.10\,\text{m})^2}{2(0.010\,\text{m})^2}} = 0.192\,\text{m/s}^2$$
$$= \boxed{0.19\,\text{m/s}^2}$$

Solving equation (1) for T yields:

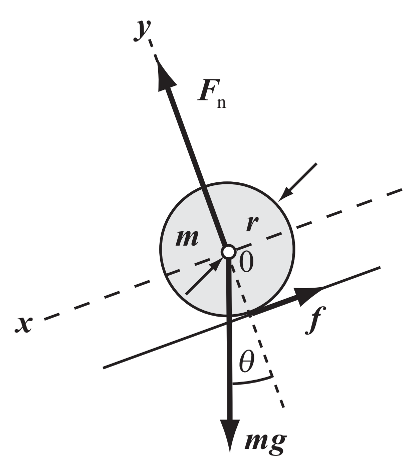
$$T = m(g-a)$$

Substitute numerical values and evaluate T:

$$T = (0.10\,\text{kg})\left(9.81\,\text{m/s}^2 - 0.192\,\text{m/s}^2\right)$$
$$= \boxed{0.96\,\text{N}}$$

88 •• A uniform solid sphere rolls down an incline without slipping. If the linear acceleration of the center of mass of the sphere is $0.2g$, then what is the angle the incline makes with the horizontal?

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass of the uniform solid sphere equals the net force acting on the sphere divided by its mass. The forces acting on the sphere are its weight $m\vec{g}$ downward, the normal force $\vec{F}_\text{n}$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the sphere accelerates down the incline, the angular speed of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_\text{n}$ act through the center of mass. Choose the $+x$ direction to be down the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the sphere: $$mg\sin\theta - f = ma_{\text{cm}} \qquad (1)$$

Apply $\sum \tau = I_{\text{cm}}\alpha$ to the sphere: $$fr = I_{\text{cm}}\alpha$$

Use the nonslip condition to eliminate α and solve for f: $$fr = I_{\text{cm}}\frac{a_{\text{cm}}}{r} \Rightarrow f = \frac{I_{\text{cm}}}{r^2}a_{\text{cm}}$$

Substitute this result for f in equation (1) to obtain: $$mg\sin\theta - \frac{I_{\text{cm}}}{r^2}a_{\text{cm}} = ma_{\text{cm}} \qquad (2)$$

From Table 9-1 we have, for a solid sphere: $$I_{\text{cm}} = \tfrac{2}{5}mr^2$$

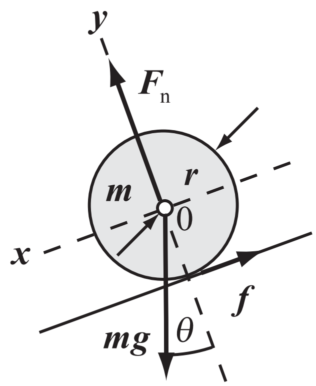
Substitute for I_{cm} in equation (1) and simplify to obtain: $$\boldsymbol{mg}\sin\boldsymbol{\theta} - \tfrac{2}{5}\boldsymbol{ma}_{\text{cm}} = \boldsymbol{ma}_{\text{cm}}$$

Solving for θ yields: $$\boldsymbol{\theta} = \sin^{-1}\left(\frac{7\boldsymbol{a}_{\text{cm}}}{5\boldsymbol{g}}\right)$$

Substitute numerical values and evaluate θ: $$\boldsymbol{\theta} = \sin^{-1}\left[\frac{7(0.2\boldsymbol{g})}{5\boldsymbol{g}}\right] = \boxed{16°}$$

89 •• A thin spherical shell rolls down an incline without slipping. If the linear acceleration of the center of mass of the shell is 0.20*g*, then what is the angle the incline makes with the horizontal?

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass of the thin spherical shell equals the net force acting on the spherical shell divided by its mass. The forces acting on the thin spherical shell are its weight $m\vec{g}$ downward, the normal force $\vec{F}_{\text{n}}$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the spherical shell accelerates down the incline, the angular speed of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the shell to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_{\text{n}}$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the thin spherical shell:

$$mg\sin\theta - f = ma_{cm} \quad (1)$$

Apply $\sum \tau = I_{cm}\alpha$ to the thin spherical shell:

$$fr = I_{cm}\alpha$$

Use the nonslip condition to eliminate α and solve for f:

$$fr = I_{cm}\frac{a_{cm}}{r} \text{ and } f = \frac{I_{cm}}{r^2}a_{cm}$$

Substitute this result for f in equation (1) to obtain:

$$mg\sin\theta - \frac{I_{cm}}{r^2}a_{cm} = ma_{cm} \quad (2)$$

From Table 9-1 we have, for a thin spherical shell:

$$I_{cm} = \tfrac{2}{3}mr^2$$

Substitute for I_{cm} in equation (1) and simplify to obtain:

$$\boldsymbol{mg}\sin\boldsymbol{\theta} - \tfrac{2}{3}\boldsymbol{ma}_{cm} = \boldsymbol{ma}_{cm}$$

Solving for θ yields:

$$\boldsymbol{\theta} = \sin^{-1}\left(\frac{5\boldsymbol{a}_{cm}}{3\boldsymbol{g}}\right)$$

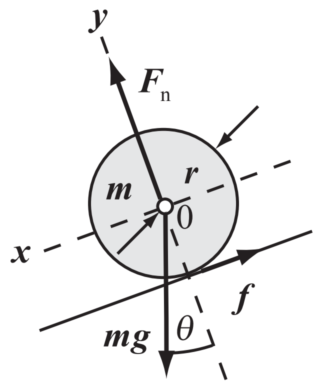
Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{5(0.2\boldsymbol{g})}{3\boldsymbol{g}}\right] = \boxed{20°}$$

Remarks: This larger angle makes sense, as the moment of inertia for a given mass is larger for a thin spherical shell than for a solid one.

90 •• A basketball of mass m rolls without slipping down an incline of angle θ. The coefficient of static friction is μ_s. Model the ball as a thin spherical shell. Find (*a*) the acceleration of the center of mass of the ball, (*b*) the frictional force acting on the ball, and (*c*) the maximum angle of the incline for which the ball will roll without slipping.

Picture the Problem The three forces acting on the basketball are the weight of the ball, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. Let the mass of the basketball be m and apply Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the ball:

$$\sum F_x = mg\sin\theta - f_s = ma, \quad (1)$$
$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$
and
$$\sum \tau_0 = f_s r = I_0\alpha \quad (3)$$

Because the basketball is rolling without slipping we know that:

$$\alpha = \frac{a}{r}$$

Substitute in equation (3) to obtain:

$$f_s r = I_0\frac{a}{r} \quad (4)$$

From Table 9-1 we have:

$$I_0 = \tfrac{2}{3}mr^2$$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{2}{3}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{2}{3}ma \quad (5)$$

Substitute for f_s in equation (1) and solve for a:

$$a = \boxed{\tfrac{3}{5}g\sin\theta}$$

(*b*) Find f_s using equation (5):

$$\boldsymbol{f}_s = \tfrac{2}{3}\boldsymbol{m}\left(\tfrac{3}{5}\boldsymbol{g}\sin\boldsymbol{\theta}\right) = \boxed{\tfrac{2}{5}\boldsymbol{mg}\sin\boldsymbol{\theta}}$$

(*c*) Solve equation (2) for F_n:

$$F_n = mg\cos\theta$$

Use the definition of $f_{s,max}$ to obtain:

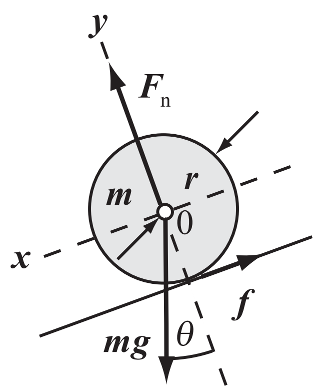
$$f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$$

Use the result of Part (*b*) to obtain: $\frac{2}{5}mg\sin\theta_{\text{max}} = \mu_s mg\cos\theta_{\text{max}}$

Solve for θ_{max}: $\theta_{\text{max}} = \boxed{\tan^{-1}\left(\frac{5}{2}\mu_s\right)}$

91 •• A uniform solid cylinder of wood rolls without slipping down an incline of angle θ. The coefficient of static friction is μ_s. Find (*a*) the acceleration of the center of mass of the cylinder, (*b*) the frictional force acting on the cylinder, and (*c*) the maximum angle of the incline for which the cylinder will roll without slipping.

Picture the Problem The three forces acting on the cylinder are the weight of the cylinder, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. Let the mass of the cylinder by *m* and use Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the cylinder:

$$\sum F_x = mg\sin\theta - f_s = ma\,, \quad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \quad (3)$$

Because the cylinder is rolling without slipping we know that:

$$\alpha = \frac{a}{r}$$

Substitute in equation (3) to obtain:

$$f_s r = I_0\frac{a}{r} \quad (4)$$

From Table 9-1 we have:

$$I_0 = \tfrac{1}{2}mr^2$$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{1}{2}ma \quad (5)$$

Substitute for f_s in equation (1) and solve for a:

$$a = \boxed{\tfrac{2}{3} g \sin\theta}$$

(*b*) Find f_s using equation (5):

$$\boldsymbol{f_s = \tfrac{1}{2} m\left(\tfrac{2}{3} g \sin\theta\right) = \boxed{\tfrac{1}{3} mg \sin\theta}}$$

(*c*) Solve equation (2) for F_n:

$$F_n = mg\cos\theta$$

Use the definition of $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$$

Use the result of part (*b*) to obtain:

$$\tfrac{1}{3} mg \sin\theta_{max} = \mu_s mg\cos\theta_{max}$$

Solve for θ_{max}:

$$\theta_{max} = \boxed{\tan^{-1}(3\mu_s)}$$

92 •• A thin spherical shell and solid sphere of the same mass *m* and radius *R* roll without slipping down an incline through the same vertical drop *H* (Figure 9-64). Each is moving horizontally as it leaves the ramp. The spherical shell hits the ground a horizontal distance *L* from the end of the ramp and the solid sphere hits the ground a distance *L*′ from the end of the ramp. Find the ratio *L*′/*L*.

Picture the Problem Let the zero of gravitational potential energy be at the elevation where the spheres leave the ramp. The distances the spheres will travel are directly proportional to their speeds when they leave the ramp.

Express the ratio of the distances traveled by the two spheres in terms of their speeds when they leave the ramp:

$$\frac{L'}{L} = \frac{v_{solid}\Delta t}{v_{shell}\Delta t} = \frac{v_{solid}}{v_{shell}} \qquad (1)$$

Use conservation of mechanical energy to find the speed of the spheres when they leave the ramp:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0 \qquad (2)$$

Express K_f for the spheres and simplify to obtain (Note that $k = 2/3$ for the spherical shell and 2/5 for the uniform sphere):

$$\begin{aligned} K_f &= K_{trans} + K_{rot} = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I_{cm}\omega^2 \\ &= \tfrac{1}{2} mv^2 + \tfrac{1}{2}\left(kmR^2\right)\frac{v^2}{R^2} \\ &= \tfrac{1}{2} mv^2 + \tfrac{1}{2} kmv^2 = (1+k)\tfrac{1}{2} mv^2 \end{aligned}$$

Substitute for K_f in equation (2) to obtain:

$$(1+k)\tfrac{1}{2} mv^2 = mgH \Rightarrow v = \sqrt{\frac{2gH}{1+k}}$$

Substitute for v_{shell} and v_{solid} in equation (1) and simplify to obtain:

$$\frac{L'}{L}=\sqrt{\frac{1+k_{\text{shell}}}{1+k_{\text{solid}}}}=\sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}}=\boxed{1.09}$$

93 •• **[SSM]** A uniform thin cylindrical shell and a solid cylinder roll horizontally without slipping. The speed of the cylindrical shell is *v*. The cylinder and the hollow cylinder encounter an incline that they climb without slipping. If the maximum height they reach is the same, find the initial speed *v*′ of the solid cylinder.

Picture the Problem Let the subscripts u and h refer to the uniform and thin-walled spheres, respectively. Because the cylinders climb to the same height, their kinetic energies at the bottom of the incline must be equal.

Express the total kinetic energy of the thin-walled cylinder at the bottom of the inclined plane:

$$K_{\text{h}}=K_{\text{trans}}+K_{\text{rot}}=\tfrac{1}{2}m_{\text{h}}v^2+\tfrac{1}{2}I_{\text{h}}\omega^2$$
$$=\tfrac{1}{2}m_{\text{h}}v^2+\tfrac{1}{2}\left(m_{\text{h}}r^2\right)\frac{v^2}{r^2}=m_{\text{h}}v^2$$

Express the total kinetic energy of the solid cylinder at the bottom of the inclined plane:

$$\boldsymbol{K}_{\text{u}}=\boldsymbol{K}_{\text{trans}}+\boldsymbol{K}_{\text{rot}}=\tfrac{1}{2}\boldsymbol{m}_{\text{u}}\boldsymbol{v}'^2+\tfrac{1}{2}\boldsymbol{I}_{\text{u}}\boldsymbol{\omega}'^2$$
$$=\tfrac{1}{2}\boldsymbol{m}_{\text{u}}\boldsymbol{v}'^2+\tfrac{1}{2}\left(\tfrac{1}{2}\boldsymbol{m}_{\text{u}}\boldsymbol{r}^2\right)\frac{\boldsymbol{v}'^2}{\boldsymbol{r}^2}=\tfrac{3}{4}\boldsymbol{m}_{\text{u}}\boldsymbol{v}'^2$$

Because the cylinders climb to the same height:

$$\tfrac{3}{4}m_{\text{u}}v'^2=m_{\text{u}}gh$$
and
$$m_{\text{h}}v^2=m_{\text{h}}gh$$

Divide the first of these equations by the second:

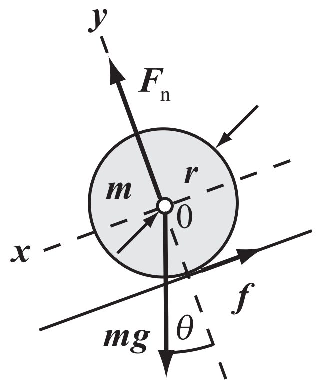
$$\frac{\tfrac{3}{4}m_{\text{u}}v'^2}{m_{\text{h}}v^2}=\frac{m_{\text{u}}gh}{m_{\text{h}}gh}$$

Simplify to obtain:

$$\frac{3v'^2}{4v^2}=1\Rightarrow v'=\boxed{\sqrt{\frac{4}{3}}v}$$

94 •• A thin cylindrical shell and a solid sphere start from rest and roll without slipping down a 3.0-m-long inclined plane. The cylinder arrives at the bottom of the incline 2.4 s after the sphere does. Determine the angle the incline makes with the horizontal.

Picture the Problem Let the subscripts s and c refer to the solid sphere and thin-walled cylinder, respectively. Because the cylinder and sphere descend from the same height, their kinetic energies at the bottom of the incline must be equal. The force diagram shows the forces acting on the solid sphere. We'll use Newton's 2nd law to relate the accelerations to the angle of the incline and use a constant acceleration to relate the accelerations to the distances traveled down the incline.



Apply Newton's 2nd law to the sphere:	$\sum F_x = mg\sin\theta - f_s = ma_s$, (1) $\sum F_y = F_n - mg\cos\theta = 0$, (2) and $\sum \tau_0 = f_s r = I_0\alpha$ (3)
Substitute for I_0 and α in equation (3) and solve for f_s:	$f_s r = \left(\frac{2}{5}mr^2\right)\frac{a}{r} \Rightarrow f_s = \frac{2}{5}ma_s$
Substitute for f_s in equation (1) and solve for a:	$a_s = \frac{5}{7}g\sin\theta$
Proceed as above for the thin cylindrical shell to obtain:	$a_c = \frac{1}{2}g\sin\theta$
Using a constant-acceleration equation, relate the distance traveled down the incline to its acceleration and the elapsed time:	$\Delta s = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$ or, because $v_0 = 0$, $\Delta s = \frac{1}{2}a(\Delta t)^2$ (4)
Because Δs is the same for both objects:	$a_s t_s^2 = a_c t_c^2$ where $\boldsymbol{t}_c^2 = (\boldsymbol{t}_s + 2.4)^2 = \boldsymbol{t}_s^2 + 4.80\boldsymbol{t}_s + 5.760$ provided t_c and t_s are in seconds.
Substitute for a_s and a_c and simplify to obtain the quadratic equation:	$0.429\boldsymbol{t}_s^2 - 4.80\boldsymbol{t}_s - 5.760 = 0$

Use the quadratic formula or your graphing calculator to solve for the positive root:

$$t_s = 12.28\,\text{s}$$

Substitute in equation (4), simplify, and solve for θ:

$$\theta = \sin^{-1}\left[\frac{14\Delta s}{5 g t_s^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{14(3.0\,\text{m})}{5(9.81\,\text{m/s}^2)(12.28\,\text{s})^2}\right] = \boxed{0.33°}$$

95 •• A wheel has a thin 3.0-kg rim and four spokes, each of mass 1.2 kg. Find the kinetic energy of the wheel when it is rolling at 6.0 m/s on a horizontal surface.

Picture the Problem The kinetic energy of the wheel is the sum of its translational and rotational kinetic energies. Because the wheel is a composite object, we can model its moment of inertia by treating the rim as a cylindrical shell and the spokes as rods.

Express the kinetic energy of the wheel:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\frac{v^2}{R^2}$$

where $M_{\text{tot}} = M_{\text{rim}} + 4M_{\text{spoke}}$.

The moment of inertia of the wheel is the sum of the moments of inertia of the rim and spokes:

$$I_{\text{cm}} = I_{\text{rim}} + I_{\text{spokes}} = M_{\text{rim}}R^2 + 4\left(\tfrac{1}{3}M_{\text{spoke}}R^2\right) = \left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2$$

Substitute for I_{cm} in the equation for K:

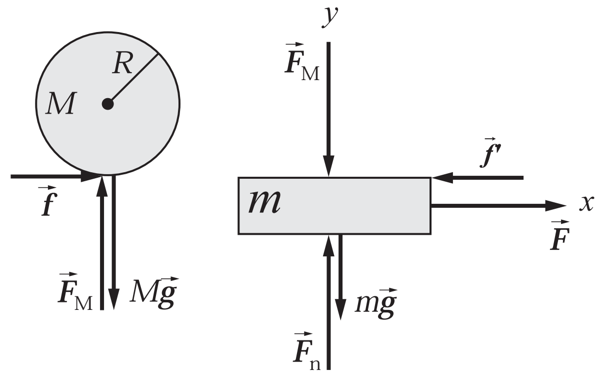
$$K = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}\left[\left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2\right]\frac{v^2}{R^2} = \left[\tfrac{1}{2}\left(M_{\text{tot}} + M_{\text{rim}}\right) + \tfrac{2}{3}M_{\text{spoke}}\right]v^2$$

Substitute numerical values and evaluate K:

$$K = \left[\tfrac{1}{2}(7.8\,\text{kg} + 3.0\,\text{kg}) + \tfrac{2}{3}(1.2\,\text{kg})\right](6.0\,\text{m/s})^2 = \boxed{0.22\,\text{kJ}}$$

96 ••• A uniform solid cylinder of mass M and radius R is at rest on a slab of mass m, which in turn rests on a horizontal, frictionless table (Figure 9-65). If a horizontal force $\vec{F}$ is applied to the slab, it accelerates and the cylinder rolls without slipping. Find the acceleration of the slab in terms of M, R, and F.

Picture the Problem Let the letter S identify the slab and the letter C the cylinder. We can find the accelerations of the slab and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



Apply $\sum F_x = ma_x$ to the slab: $F - f' = ma_S$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_C$, (2)

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder: $fR = I_{CM}\alpha$ (3)

Substitute for I_{CM} in equation (3) and solve for $f = f'$ to obtain: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the slab to the acceleration of the cylinder:

$a_C = a_S + a_{CS}$

or, because $a_{CS} = -R\alpha$ is the acceleration of the cylinder relative to the slab,

$a_C = a_S - R\alpha \Rightarrow R\alpha = a_S - a_C$ (5)

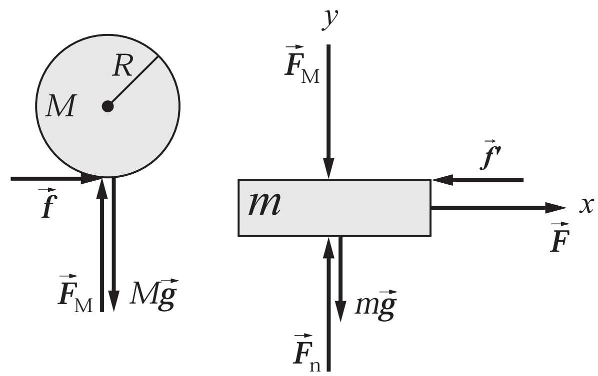
Equate equations (2) and (4) and substitute from (5) to obtain: $a_S = 3a_C$

Substitute equation (4) in equation (1) and substitute for a_C to obtain:

$$F - \tfrac{1}{3}Ma_S = ma_B \Rightarrow a_S = \boxed{\frac{3F}{M+3m}}$$

97 ••• (*a*) Find the angular acceleration of the cylinder in Problem 96. Is the cylinder rotating clockwise or counterclockwise? (*b*) What is the cylinder's linear acceleration (magnitude and direction) relative to the table? (*c*) What is the magnitude and direction of the linear acceleration of the center of mass of the cylinder relative to the slab?

Picture the Problem Let the letter S identify the slab and the letter C the cylinder. In this problem, as in Problem 96, we can find the accelerations of the slab and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



(*a*) Apply $\sum F_x = ma_x$ to the slab: $F - f' = ma_S$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_C$, (2)

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder: $fR = I_{CM}\alpha$ (3)

Substitute for I_{CM} in equation (3) and solve for $f = f'$ to obtain: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the slab to the acceleration of the cylinder:

$a_C = a_S + a_{CS}$

or, because $a_{CS} = -R\alpha$,

$$a_C = a_S - R\alpha \Rightarrow \alpha = \frac{a_S - a_C}{R} \quad (5)$$

Equate equations (2) and (4) and substitute from (5) to obtain: $a_S = 3a_C$

Substitute for a_S in equation (5) and simplify to obtain:

$$\alpha = \frac{3a_C - a_C}{R} = \frac{2a_C}{R}$$

$$= \boxed{\frac{2F}{R(M+3m)}, \text{ counterclockwise}}$$

(*b*) From equations (1) and (4) we have:

$$F - \tfrac{1}{3}Ma_S = ma_S \Rightarrow a_S = \frac{3F}{M + 3m}$$

Because $a_S = 3a_C$:

$$a_C = \tfrac{1}{3}a_S = \boxed{\frac{F}{M+3m}\text{, in the direction of } \vec{F}}$$

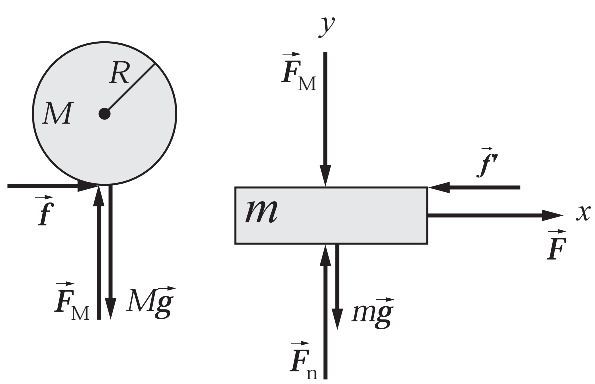
(*c*) The acceleration of the cylinder relative to the slab is the difference between the acceleration of the cylinder and the acceleration of the slab:

$$a_{CS} = a_C - a_S = a_C - 3a_C = -2a_C$$

$$= \boxed{-\frac{2F}{M+3m}\text{, opposite to the direction of } \vec{F}}$$

98 ••• If the force in Problem 96 acts over a distance *d*, in terms of the symbols given, find (*a*) the kinetic energy of the slab, and (*b*) the total kinetic energy of the cylinder. (*c*) Show that the total kinetic energy of the slab-cylinder system is equal to the work done by the force.

Picture the Problem Let the system include Earth, the cylinder (C), and the slab (S). Then $\vec{F}$ is an external force that changes the energy of the system by doing work on it. We can find the kinetic energy of the slab from its speed when it has traveled a distance *d.* We can find the kinetic energy of the cylinder from the sum of its translational and rotational kinetic energies. In Part (*c*) we can add the kinetic energies of the slab and the cylinder to show that their sum is the work done by $\vec{F}$ in displacing the system a distance *d*.



(*a*) Express the kinetic energy of the slab:

$$K_S = W_{\text{on slab}} = \tfrac{1}{2}mv_S^2$$

Using a constant-acceleration equation, relate the velocity of the slab to its acceleration and the distance traveled:

$v_S^2 = v_0^2 + 2a_S d$

or, because the slab starts from rest,

$v_S^2 = 2a_S d$

Substitute for v_S^2 to obtain:

$K_S = \frac{1}{2}m(2a_S d) = ma_S d$ (1)

Apply $\sum F_x = ma_x$ to the slab:

$F - f = ma_S$ (2)

Apply $\sum F_x = ma_x$ to the cylinder:

$f = Ma_C$, (3)

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder:

$fR = I_{CM}\alpha$ (4)

Substitute for I_{CM} in equation (4) and solve for f:

$f = \frac{1}{2}MR\alpha$ (5)

Relate the acceleration of the slab to the acceleration of the cylinder:

$a_C = a_S + a_{CS}$

or, because $a_{CS} = -R\alpha$,

$a_C = a_S - R\alpha$

Solving for $R\alpha$ yields:

$R\alpha = a_S - a_C$ (6)

Equate equations (3) and (5) and substitute in (6) to obtain:

$a_S = 3a_C$

Substitute equation (5) in equation (2) and use $a_S = 3a_C$ to obtain:

$F - Ma_C = ma_S$

or

$F - \frac{1}{3}Ma_S = ma_S$

Solving for a_S yields:

$$a_S = \frac{F}{m + \frac{1}{3}M}$$

Substitute for a_S in equation (1) to obtain:

$$K_B = \boxed{\frac{mFd}{m + \frac{1}{3}M}}$$

(*b*) Express the total kinetic energy of the cylinder:

$$K_C = K_{trans} + K_{rot} = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\omega^2 = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\frac{v_{CB}^2}{R^2} \quad (7)$$

where $v_{CS} = v_C - v_S$.

In Part (*a*) it was established that: $a_S = 3a_C$

Integrate both sides of $a_S = 3a_C$ with respect to time to obtain: $v_S = 3v_C + \text{constant}$
where the constant of integration is determined by the initial conditions that $v_C = 0$ when $v_S = 0$.

Substitute the initial conditions to obtain: constant $= 0$ and $v_S = 3v_C$

Substitute in the expression for v_{CS} to obtain: $v_{CS} = v_C - v_S = v_C - 3v_C = -2v_C$

Substitute for I_{CM} and v_{CS} in equation (7) to obtain:

$$K_C = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{(-2v_C)^2}{R^2} = \tfrac{3}{2}Mv_C^2 \quad (8)$$

Because $v_C = \tfrac{1}{3}v_S$: $v_C^2 = \tfrac{1}{9}v_S^2$

It Part (*a*) it was established that: $v_S^2 = 2a_S d$ and $a_S = \dfrac{F}{m + \frac{1}{3}M}$

Substitute to obtain:

$$v_C^2 = \tfrac{1}{9}(2a_S d) = \tfrac{2}{9}\left(\frac{F}{m + \frac{1}{3}M}\right)d = \frac{2Fd}{9\left(m + \frac{1}{3}M\right)}$$

Substituting in equation (8) and simplifying yields:

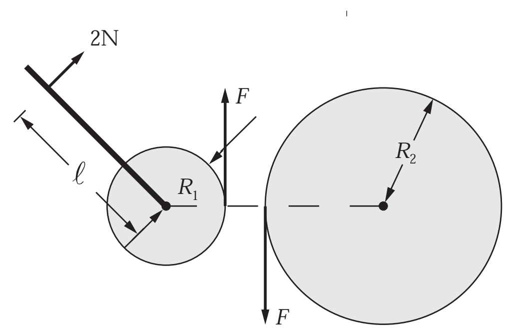
$$K_C = \tfrac{3}{2}M\left(\frac{2Fd}{9\left(m + \frac{1}{3}M\right)}\right) = \boxed{\frac{MFd}{3\left(m + \frac{1}{3}M\right)}}$$

(*c*) Express the total kinetic energy of the system and simplify to obtain:

$$K_{\text{tot}} = K_S + K_C = \frac{mFd}{m+\frac{1}{3}M} + \frac{MFd}{3\left(m+\frac{1}{3}M\right)} = \frac{(3m+M)}{3\left(m+\frac{1}{3}M\right)}Fd = \boxed{Fd}$$

99 ••• **[SSM]** Two large gears that are being designed as part of a large machine and are shown in Figure 9-66; each is free to rotate about a fixed axis through its center. The radius and moment of inertia of the smaller gear are 0.50 m and 1.0 kg·m^2, respectively, and the radius and moment of inertia of the larger gear are 1.0 m and 16 kg·m^2, respectively. The lever attached to the smaller gear is 1.0 m long and has a negligible mass. (*a*) If a worker will typically apply a force of 2.0 N to the end of the lever, as shown, what will be the angular accelerations of gears the two gears? (*b*) Another part of the machine (not shown) will apply a force tangentially to the outer edge of the larger gear to temporarily keep the gear system from rotating. What should the magnitude and direction of this force (clockwise or counterclockwise) be?

Picture the Problem The forces responsible for the rotation of the gears are shown in the diagram to the right. The forces acting through the centers of mass of the two gears have been omitted because they produce no torque. We can apply Newton's 2$^{\text{nd}}$ law in rotational form to obtain the equations of motion of the gears and the not slipping condition to relate their angular accelerations.



(*a*) Apply $\sum \tau = I\alpha$ to the gears to obtain their equations of motion:

$$2.0\ \text{N}\cdot\text{m} - \boldsymbol{FR_1} = \boldsymbol{I_1\alpha_1} \qquad (1)$$

and

$$FR_2 = I_2\alpha_2 \qquad (2)$$

where F is the force keeping the gears from slipping with respect to each other.

Because the gears do not slip relative to each other, the tangential accelerations of the points where they are in contact must be the same:

$$R_1\alpha_1 = R_2\alpha_2$$

or

$$\alpha_2 = \frac{R_1}{R_2}\alpha_1 = \tfrac{1}{2}\alpha_1 \qquad (3)$$

Divide equation (1) by R_1 to obtain:

$$\frac{2.0\ \text{N}\cdot\text{m}}{R_1} - F = \frac{I_1}{R_1}\alpha_1$$

Divide equation (2) by R_2 to obtain:

$$F = \frac{I_2}{R_2}\alpha_2$$

Adding these equations yields:

$$\frac{2.0\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{R_2}\alpha_2$$

Use equation (3) to eliminate α_2:

$$\frac{2.0\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{2R_2}\alpha_1$$

Solving for α_1 yields:

$$\alpha_1 = \frac{2.0\,\text{N}\cdot\text{m}}{I_1 + \frac{R_1}{2R_2}I_2}$$

Substitute numerical values and evaluate α_1:

$$\alpha_1 = \frac{2.0\,\text{N}\cdot\text{m}}{1.0\,\text{kg}\cdot\text{m}^2 + \frac{0.50\,\text{m}}{2(1.0\,\text{m})}\left(16\,\text{kg}\cdot\text{m}^2\right)}$$
$$= 0.400\,\text{rad/s}^2 = \boxed{0.40\,\text{rad/s}^2}$$

Use equation (3) to evaluate α_2:

$$\boldsymbol{\alpha}_2 = \tfrac{1}{2}\left(0.400\,\text{rad/s}^2\right) = \boxed{0.20\,\text{rad/s}^2}$$

(*b*) To counterbalance the 2.0-N·m torque, a counter torque of 2.0 N·m must be applied to the first gear:

$$2.0\ \text{N}\cdot\text{m} - FR_1 = 0 \Rightarrow F = \frac{2.0\,\text{N}\cdot\text{m}}{R_1}$$

Substitute numerical values and evaluate F:

$$F = \frac{2.0\,\text{N}\cdot\text{m}}{0.50\,\text{m}} = \boxed{4.0\,\text{N, clockwise}}$$

100 ••• As the chief design engineer for a major toy company, you are in charge of designing a ″loop-the-loop″ toy for youngsters. The idea, as shown in Figure 9-67, is that a ball of mass *m* and radius *r* will roll down an inclined track and around the loop without slipping. The ball starts from rest at a height *h* above the tabletop that supports the whole track. The loop radius is *R*. Determine the minimum height *h*, in terms of *R* and *r*, for which the ball will remain in contact with the track during the whole of its loop-the-loop journey. (Do not neglect the size of the ball’s radius when doing this calculation.)

Picture the Problem Choose $U_g = 0$ at the bottom of the loop and let the system include the ball, the track, and the earth. We can apply conservation of energy to this system to relate *h* to the speed of the ball *v* at the top of the ″loop-to-loop″, the radius *r* of the ball, and the radius *R* of the ″loop-to-loop.″ We can then apply Newton’s 2nd law to the ball at the top of the loop to eliminate *v*.

Apply conservation of energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U_g$$

and, because $W_{ext} = 0$,

$$\Delta K_{trans} + \Delta K_{rot} + \Delta U_g = 0$$

Substituting for ΔK_{trans}, ΔK_{rot}, and ΔU_g yields:

$$K_{trans,f} - K_{trans,i} + K_{rot,f} - K_{rot,i} + U_{g,f} - U_{g,i} = 0$$

Because $K_{trans,i} = K_{rot,i} = 0$:

$$K_{trans,f} + K_{rot,f} + U_{g,f} - U_{g,i} = 0$$

Substitute for $K_{trans,f}$, $K_{rot,f}$, $U_{g,f}$, and $U_{g,i}$ to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mg(2R - r) - mgh = 0$$

Noting that, because the ball rolls without slipping, $v = r\omega$ and that $I_{sphere} = \frac{2}{5}mr^2$, substitute to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 + mg(2R - r) - mgh = 0 \Rightarrow h = \frac{7v^2}{10g} + 2R - r$$

Noting that, for the minimum speed for which the ball remains in contact with the track at the top of the loop the centripetal force is the gravitational force, apply Newton's second law to the ball to obtain:

$$mg = m\frac{v^2}{R - r} \Rightarrow v^2 = g(R - r)$$

Substituting for v^2 and simplifying yields:

$$h = \frac{7g(R - r)}{10g} + 2R - r$$
$$= \boxed{2.7R - 1.7r}$$

Rolling With Slipping

101 •• A bowling ball of mass M and radius R is released so that at the instant it touches the floor it is moving horizontally with a speed v_0 and is not rotating. It slides for a time t_1 a distance s_1 before it begins to roll without slipping. (*a*) If μ_k is the coefficient of kinetic friction between the ball and the floor, find s_1, t_1, and the final speed v_1 of the ball. (*b*) Find the ratio of the final kinetic energy to the initial kinetic energy of the ball. (*c*) Evaluate s_1, t_1, and v_1 for $v_0 = 8.0$ m/s and $\mu_k = 0.060$.

Picture the Problem Part (*a*) of this problem is identical to Example 9-16. In Part (*b*) we can use the definitions of translational and rotational kinetic energy to find the ratio of the final and initial kinetic energies.

(*a*) From Example 9-16:

$$s_1 = \boxed{\frac{12}{49}\frac{v_0^2}{\mu_k g}}, t_1 = \boxed{\frac{2}{7}\frac{v_0}{\mu_k g}}, \text{ and}$$

$$\boldsymbol{v}_1 = \tfrac{5}{2}\mu_k \boldsymbol{g}\boldsymbol{t}_1 = \boxed{\frac{5}{7}\boldsymbol{v}_0}$$

(*b*) When the ball rolls without slipping, $v_1 = r\omega$. The final kinetic energy of the ball is given by:

$$\boldsymbol{K}_f = \boldsymbol{K}_{trans} + \boldsymbol{K}_{rot} = \tfrac{1}{2}\boldsymbol{M}\boldsymbol{v}_1^2 + \tfrac{1}{2}\boldsymbol{I}\boldsymbol{\omega}^2$$

Substituting for I and simplifying yields:

$$\boldsymbol{K}_f = \tfrac{1}{2}\boldsymbol{M}\boldsymbol{v}_1^2 + \tfrac{1}{2}\left(\tfrac{2}{5}\boldsymbol{M}\boldsymbol{r}^2\right)\frac{\boldsymbol{v}_1^2}{\boldsymbol{r}^2} = \tfrac{7}{10}\boldsymbol{M}\boldsymbol{v}_1^2 = \tfrac{5}{14}\boldsymbol{M}\boldsymbol{v}_0^2$$

Express the ratio of the final and initial kinetic energies:

$$\frac{\boldsymbol{K}_f}{\boldsymbol{K}_i} = \frac{\tfrac{5}{14}\boldsymbol{M}\boldsymbol{v}_0^2}{\tfrac{1}{2}\boldsymbol{M}\boldsymbol{v}_0^2} = \boxed{\frac{5}{7}}$$

(*c*) Substitute numerical values in the expression from (*a*) and evaluate s_1:

$$\boldsymbol{s}_1 = \frac{12}{49}\frac{(8.0\,\text{m/s})^2}{(0.060)(9.81\,\text{m/s}^2)} = \boxed{27\,\text{m}}$$

Substitute numerical values in the expression from (*a*) and evaluate t_1:

$$\boldsymbol{t}_1 = \frac{2}{7}\frac{8.0\,\text{m/s}}{(0.060)(9.81\,\text{m/s}^2)} = \boxed{3.9\,\text{s}}$$

Substitute numerical values in the expression from (*a*) and evaluate v_1:

$$\boldsymbol{v}_1 = \frac{5}{7}(8.0\,\text{m/s}) = \boxed{5.7\,\text{m/s}}$$

102 •• During a game of pool, the cue ball (a uniform sphere of radius r) is at rest on the horizontal pool table (Figure 9-68). You strike the ball horizontally with your cue stick, which delivers a large horizontal force of magnitude F_0 for a short time. The stick strikes the ball at a point a vertical height h above the tabletop. Assume the striking location is above the ball's center. Show that the ball's angular speed ω is related to the initial linear speed of its center of mass v_{cm} by $\omega = (5/2)v_{cm}(h - r)/r^2$. Estimate the ball's rotation rate just after the hit using reasonable estimates for h, r and v_{cm}.

Picture the Problem We can apply Newton's 2nd law in rotational form and the impulse-momentum theorem to obtain two equations that we can solve simultaneously for ω.

Apply Newton's 2nd law in rotational form to the cue ball:

$$F_0(h-r) = I_{\text{cm}}\alpha = \tfrac{2}{5}mr^2\frac{\Delta\omega}{\Delta t}$$
$$= \tfrac{2}{5}mr^2\frac{\omega}{\Delta t}$$

Solving for ω yields:

$$\omega = \frac{5F_0(h-r)\Delta t}{2mr^2} \qquad (1)$$

From the impulse-momentum theorem:

$$I = F_0\Delta t = \Delta p = mv_0 \Rightarrow \Delta t = \frac{mv_0}{F_0}$$

Substitute for Δt in equation (1) and simplify to obtain:

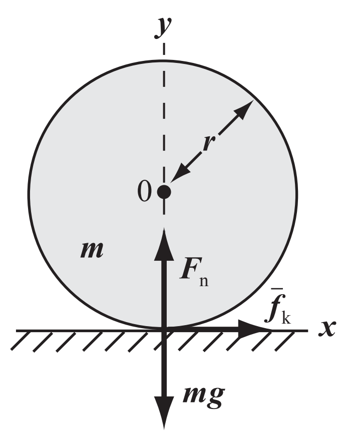
$$\omega = \frac{5F_0(h-r)\dfrac{mv_0}{F_0}}{2mr^2} = \boxed{\frac{5v_0(h-r)}{2r^2}}$$

Assume that $v_0 \approx 1.0$ m/s, $r \approx 4.0$ cm, and $h \approx 6.0$ cm, to estimate ω:

$$\omega = \frac{5(1.0\text{ m/s})(6.0\text{ cm} - 4.0\text{ cm})}{2(4.0\text{ cm})^2}$$
$$\approx \boxed{30\text{ rad/s}}$$

103 •• A uniform solid sphere is set rotating about a horizontal axis at an angular speed ω_0 and then is placed on the floor with its center of mass at rest. If the coefficient of kinetic friction between the sphere and the floor is μ_k, find the speed of the center of mass of the sphere when the sphere begins to roll without slipping.

Picture the Problem The angular speed of the rotating sphere will decrease until the condition for rolling without slipping is satisfied and then it will begin to roll. The force diagram shows the forces acting on the sphere. We can apply Newton's 2nd law to the sphere and use the condition for rolling without slipping to find the speed of the center of mass when the sphere begins to roll without slipping.

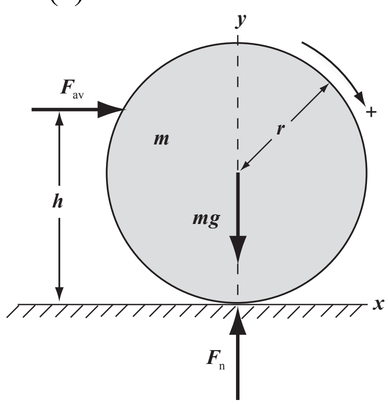


Relate the velocity of the sphere when it begins to roll to its acceleration at that moment and the elapsed time:	$v = a\Delta t$	(1)
Apply Newton's 2nd law to the sphere:	$\sum F_x = f_k = ma$,	(2)
	$\sum F_y = F_n - mg = 0$,	(3)
	and	
	$\sum \tau_0 = f_k r = I_0 \alpha$	(4)
Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:	$a = \mu_k g$	
Substitute in equation (1) to obtain:	$v = a\Delta t = \mu_k g\Delta t$	(5)
Solving for α in equation (4) yields:	$\alpha = \frac{f_k r}{I_0} = \frac{mar}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu_k g}{r}$	
Express the angular speed of the sphere when it has been moving for a time Δt:	$\omega = \omega_0 - \alpha\,\Delta t = \omega_0 - \frac{5\mu_k g}{2r}\Delta t$	(6)
Express the condition that the sphere rolls without slipping:	$v = r\omega$	
Substitute from equations (5) and (6) and solve for the elapsed time until the sphere begins to roll:	$\Delta t = \frac{2}{7}\frac{r\omega_0}{\mu_k g}$	
Use equation (5) to find v when the sphere begins to roll:	$\boldsymbol{v = \mu_k g\Delta t = \frac{2}{7}\frac{r\omega_0\mu_k g}{\mu_k g}} = \boxed{\frac{2r\boldsymbol{\omega}_0}{7}}$	

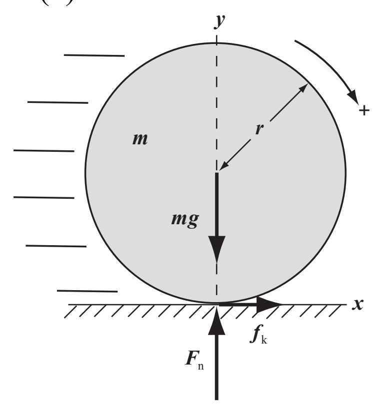
104 •• A uniform solid ball rests on a horizontal surface and has a mass that is 0.80 kg and a radius that is 5.0 cm. A sharp force is applied to the ball in a horizontal direction 9.0 cm above the horizontal surface. The force increases linearly from 0 N to 40 kN in 1.0×10^{-4} s, and then decreases linearly to 0 N in 1.0×10^{-4} s. (*a*) What is the speed of the ball just after impact? (*b*) What is the angular speed of the ball after impact? (*c*) What is the speed of the ball when it begins to roll without slipping? (*d*) How far does the ball travel along the surface before it begins to roll without slipping? Assume that $\mu_k = 0.50$.

Picture the Problem (*a*) The sharp force delivers a translational impulse to the ball that changes its linear momentum. We can use the impulse-momentum theorem to find the speed of the ball after impact. (*b*) We can find the angular speed of the ball after impact by applying Newton's 2nd law in rotational form. Because the ball has a forward spin, the friction force is in the direction of motion and will cause the ball's translational speed to increase. In Parts (*c*) and (*d*) we can apply Newton's 2nd law to the ball to obtain equations describing both the translational and rotational motion of the ball. We can then solve these equations to find the constant accelerations that allow us to apply constant-acceleration equations to find the velocity of the ball when it begins to roll and its sliding time.

(*a*) and (*b*)



(*c*) and (*d*)



(*a*) Apply the impulse-momentum theorem to the ball to obtain:

$$I = F_{av}\Delta t = \Delta p = mv_0$$

Solving for v_0 yields:

$$v_0 = \frac{F_{av}\Delta t}{m} \qquad (1)$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{(20\,\text{kN})(2.0\times10^{-4}\,\text{s})}{0.80\,\text{kg}} = \boxed{5.0\,\text{m/s}}$$

(*b*) Referring to the force diagram for (*a*) and (*b*), apply Newton's 2nd law in rotational form to the ball to obtain:

$$F_{av}(h-r) = I_{cm}\alpha = I_{cm}\frac{\omega_0}{\Delta t}$$

Solving for ω_0 yields:

$$\omega_0 = \frac{F_{av}(h-r)\Delta t}{I_{cm}} = \frac{F_{av}(h-r)\Delta t}{\frac{2}{5}mr^2}$$

Solve equation (1) for Δt:

$$\Delta t = \frac{mv_0}{F_{av}}$$

Substitute for Δt and simplify to obtain:

$$\omega_0 = \frac{F_{\text{av}}(h-r)\dfrac{mv_0}{F_{\text{av}}}}{\frac{2}{5}mr^2} = \frac{5v_0(h-r)}{2r^2}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{5(5.0\,\text{m/s})(0.090\,\text{m} - 0.050\,\text{m})}{2(.050\,\text{m})^2} = \boxed{200\ \text{rad/s}}$$

(*c*) Use a constant-acceleration equation to relate the speed of the ball to the acceleration and the time:

$$v_{\text{cm}\,x} = v_0 + a_{\text{cm}\,x}t \qquad (1)$$

Referring to the force diagram for (*c*) and (*d*), apply Newton's 2nd law to the ball to obtain:

$$\sum F_x = f_{\text{k}} = ma_{\text{cm}\,x}, \qquad (2)$$

$$\sum F_y = F_{\text{n}} - mg = 0, \qquad (3)$$

and

$$\sum \tau_{\text{cm}} = -f_{\text{k}}r = I_{\text{cm}}\alpha \qquad (4)$$

Using the definition of f_{k} and F_{n} from equation (3), substitute in equation (2) to obtain:

$$\mu_{\text{k}}mg = ma_{\text{cm}\,x} \Rightarrow a_{\text{cm}\,x} = \mu_{\text{k}}g$$

Substitute for $a_{\text{cm}\,x}$ in equation (1) to obtain:

$$v_{\text{cm}\,x} = v_0 + \mu_{\text{k}}gt \qquad (5)$$

Solving for α in equation (4) yields:

$$\alpha = -\frac{f_{\text{k}}r}{I_{\text{cm}}} = -\frac{\mu_{\text{k}}mgr}{\frac{2}{5}mr^2} = -\frac{5\mu_{\text{k}}g}{2r}$$

Use a constant-acceleration equation to relate the angular speed of the ball to the angular acceleration and the time:

$$\omega = \omega_0 + \alpha t = \omega_0 - \frac{5\mu_{\text{k}}g}{2r}t \qquad (6)$$

When the ball rolls without slipping:

$$v_{\text{cm}\,x} = r\omega = r\left(\omega_0 - \frac{5\mu_{\text{k}}g}{2r}t\right) = r\omega_0 - \frac{5\mu_{\text{k}}g}{2}t \qquad (7)$$

Equate equations (5) and (7) to obtain:

$$r\omega_0 - \frac{5\mu_{\text{k}}g}{2}t = v_0 + \mu_{\text{k}}gt$$

Solve for the time t at which $v_{\text{cm}\,x} = r\omega$:

$$t = \frac{2(r\omega_0 - v_0)}{7\mu_k g}$$

Substitute numerical values and evaluate t:

$$t = \frac{2((0.050\text{ m})(200\text{ rad/s}) - 5.0\text{ m/s})}{7(0.50)(9.81\text{ m/s}^2)} = 0.2912\text{ s}$$

Substitute numerical values in equation (5) and evaluate $v_{\text{cm}\,x}(0.2912\text{ s})$:

$$v_{\text{cm}\,x}(0.2912\text{ s}) = 5.0\text{ m/s} + (0.50)(9.81\text{ m/s}^2)(0.2912\text{ s}) = 6.429\text{ m/s} = \boxed{6.4\text{ m/s}}$$

(d) The distance traveled in time t is:

$$\Delta x = v_0 t + \tfrac{1}{2}a_{\text{cm}}t^2 = v_0 t + \tfrac{1}{2}\mu_k g t^2$$

Substitute numerical values and evaluate $\Delta x(0.2912\text{ s})$:

$$\Delta x(0.2912\text{ s}) = (5.0\text{ m/s})((0.2912\text{ s})\text{s}) + \tfrac{1}{2}(0.50)(9.81\text{ m/s}^2)((0.2912\text{ s})\text{s})^2 = \boxed{1.7\text{ m}}$$

105 •• **[SSM]** A 0.16-kg billiard ball whose radius is 3.0 cm is given a sharp blow by a cue stick. The applied force is horizontal and the line of action of the force passes through the center of the ball. The speed of the ball just after the blow is 4.0 m/s, and the coefficient of kinetic friction between the ball and the billiard table is 0.60. (a) How long does the ball slide before it begins to roll without slipping? (b) How far does it slide? (c) What is its speed once it begins rolling without slipping?

Picture the Problem Because the impulse is applied through the center of mass, $\omega_0 = 0$. We can use the results of Example 9-16 to find the rolling time without slipping, the distance traveled to rolling without slipping, and the velocity of the ball once it begins to roll without slipping.

(a) From Example 9-16 we have:

$$t_1 = \frac{2}{7}\frac{v_0}{\mu_k g}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{2}{7}\frac{4.0\text{ m/s}}{(0.60)(9.81\text{ m/s}^2)} = \boxed{0.19\text{ s}}$$

(b) From Example 9-16 we have:

$$s_1 = \frac{12}{49}\frac{v_0^2}{\mu_k g}$$

Substitute numerical values and evaluate s_1:

$$s_1 = \frac{12}{49}\frac{(4.0\,\text{m/s})^2}{(0.60)(9.81\,\text{m/s}^2)} = \boxed{0.67\,\text{m}}$$

(c) From Example 9-16 we have:

$$v_1 = \frac{5}{7}v_0$$

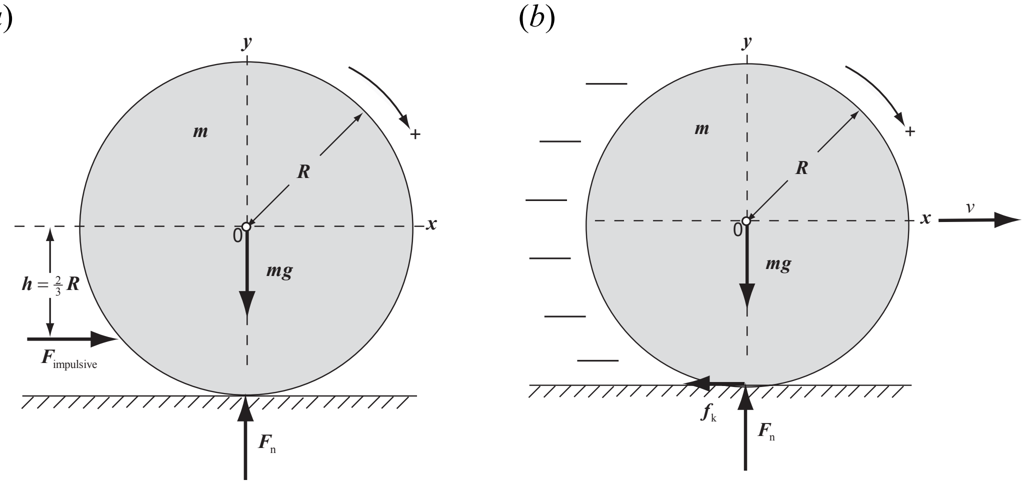
Substitute numerical values and evaluate v_1:

$$v_1 = \frac{5}{7}(4.0\,\text{m/s}) = \boxed{2.9\,\text{m/s}}$$

106 •• A billiard ball that is initially at rest is given a sharp blow by a cue stick. The force is horizontal and is applied at a distance $2R/3$ below the centerline, as shown in Figure 9-69. The speed of the ball just after the blow is v_0 and the coefficient of kinetic friction between the ball and the billiard table is μ_k. (a) What is the angular speed of the ball just after the blow? (b) What is the speed of the ball once it begins to roll without slipping? (c) What is the kinetic energy of the ball just after the hit?

Picture the Problem Because the impulsive force is applied below the center line, the ball will have a backward spin and the direction of the friction force is opposite the direction of motion. This will cause the ball's translational speed to decrease. We'll use the impulse-momentum theorem and Newton's 2nd law in rotational form to find the linear and rotational speeds and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time-to-rolling without slipping.

(a) (b)



(a) Apply Newton's 2nd law in rotational form to the ball to obtain:

$$\sum \tau_0 = F_{av}(h - R) = I_{cm}\alpha = I_{cm}\frac{\omega_0}{\Delta t}$$

Solving for ω_0 yields:

$$\omega_0 = \frac{F_{av}(h-R)\Delta t}{I_{cm}} = \frac{F_{av}(h-R)\Delta t}{\frac{2}{5}mr^2}$$

Solve equation (1) for Δt:

$$\boldsymbol{\Delta t} = \frac{\boldsymbol{mv}_0}{\boldsymbol{F}_{\text{av}}}$$

Substitute for Δt and simplify to obtain:

$$\boldsymbol{\omega}_0 = \frac{\boldsymbol{F}_{\text{av}}(\boldsymbol{h}-\boldsymbol{r})\dfrac{\boldsymbol{mv}_0}{\boldsymbol{F}_{\text{av}}}}{\frac{2}{5}\boldsymbol{mr}^2} = \boxed{\frac{5\boldsymbol{v}_0(\boldsymbol{h}-\boldsymbol{r})}{2\boldsymbol{r}^2}}$$

(*b*) Apply Newton's 2nd law to the ball when it is rolling without slipping to obtain:

$$\sum \tau_0 = f_{\text{k}} R = I_{\text{cm}}\alpha\,, \qquad (1)$$

$$\sum F_y = F_{\text{n}} - mg = 0\,, \qquad (2)$$

and

$$\sum F_x = -f_{\text{k}} = ma \qquad (3)$$

Using the definition of f_{k} and F_{n} from equation (2), solve for α:

$$\alpha = \frac{\mu_{\text{k}} mgR}{I_{\text{cm}}} = \frac{\mu_{\text{k}} mgR}{\frac{2}{5}mR^2} = \frac{5\mu_{\text{k}} g}{2R}$$

Using a constant-acceleration equation, relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 + \alpha\Delta t = \omega_0 + \frac{5\mu_{\text{k}} g}{2R}\Delta t$$

Using the definition of f_{k} and F_{n} from equation (2), solve equation (3) for a:

$$a = -\mu_{\text{k}} g$$

Using a constant-acceleration equation, relate the speed of the ball to its acceleration:

$$v = v_0 + a\Delta t = v_0 - \mu_{\text{k}} g\Delta t \qquad (4)$$

Impose the condition for rolling without slipping to obtain:

$$R\left(\omega_0 + \frac{5\mu_{\text{k}} g}{2R}\Delta t\right) = v_0 - \mu_{\text{k}} g\Delta t$$

Solving for Δt yields:

$$\Delta t = \frac{16}{21}\frac{v_0}{\mu_{\text{k}} g}$$

Substitute in equation (4) to obtain:

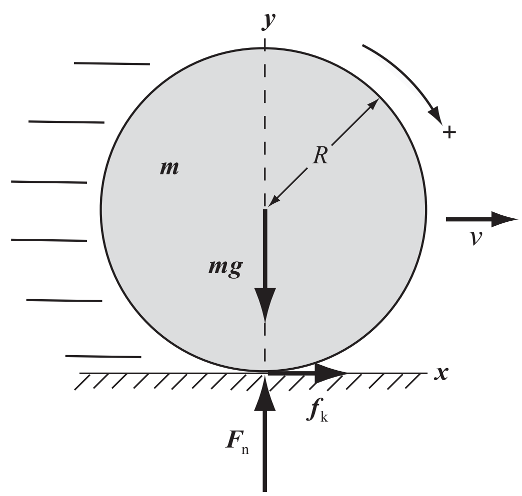
$$\boldsymbol{v} = \boldsymbol{v}_0 - \boldsymbol{\mu}_{\text{k}}\boldsymbol{g}\left(\frac{16}{21}\frac{\boldsymbol{v}_0}{\boldsymbol{\mu}_{\text{k}}\boldsymbol{g}}\right) = \boxed{\frac{5}{21}\boldsymbol{v}_0}$$

(*c*) Express the initial kinetic energy of the ball and simplify to obtain:

$$K_i = K_{trans} + K_{rot} = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2$$
$$= \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\left(\frac{5v_0}{3R}\right)^2$$
$$= \boxed{\frac{19}{18}mv_0^2}$$

107 •• A bowling ball of radius R has an initial speed v_0 down the lane and a forward spin $\omega_0 = 3v_0/R$ just after its release. The coefficient of kinetic friction is μ_k. (*a*) What is the speed of the ball just as it begins rolling without slipping? (*b*) For how long a time does the ball slide before it begins rolling without slipping? (*c*) What distance does the ball slide down the lane before it begins rolling without slipping?

Picture the Problem The figure shows the forces acting on the bowling during the sliding phase of its motion. Because the ball has a forward spin, the friction force is in the direction of motion and will cause the ball's translational speed to increase. We'll apply Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time to rolling without slipping.



(*a*) and (*b*) Relate the velocity of the ball when it begins to roll to its acceleration and the elapsed time:

$$v = v_0 + a\Delta t \qquad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = f_k = ma\,, \qquad (2)$$
$$\sum F_y = F_n - mg = 0\,, \qquad (3)$$
and
$$\sum \tau_0 = -f_k R = I_0\alpha \qquad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = v_0 + a\Delta t = v_0 + \mu_k g\Delta t \qquad (5)$$

Solve for α in equation (4):

$$\boldsymbol{\alpha} = -\frac{\boldsymbol{f}_\text{k}\boldsymbol{R}}{\boldsymbol{I}_0} = -\frac{\boldsymbol{maR}}{\frac{2}{5}\boldsymbol{mR}^2} = -\frac{5}{2}\frac{\boldsymbol{\mu}_\text{k}\boldsymbol{g}}{\boldsymbol{R}}$$

Relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 - \frac{5}{2}\frac{\mu_\text{k} g}{R}\Delta t$$

Apply the condition for rolling without slipping and simplify to obtain:

$$\begin{aligned}\boldsymbol{v} = \boldsymbol{R\omega} &= \boldsymbol{R}\left(\boldsymbol{\omega}_0 - \frac{5}{2}\frac{\boldsymbol{\mu}_\text{k}\boldsymbol{g}}{\boldsymbol{R}}\Delta\boldsymbol{t}\right)\\ &= \boldsymbol{R}\left(\frac{3\boldsymbol{v}_0}{\boldsymbol{R}} - \frac{5}{2}\frac{\boldsymbol{\mu}_\text{k}\boldsymbol{g}}{\boldsymbol{R}}\Delta\boldsymbol{t}\right) \qquad (6)\\ &= 3\boldsymbol{v}_0 - \frac{5}{2}\boldsymbol{\mu}_\text{k}\boldsymbol{g}\Delta\boldsymbol{t}\end{aligned}$$

Equate equations (5) and (6) and solve for Δt:

$$\Delta t = \boxed{\frac{4}{7}\frac{v_0}{\mu_\text{k} g}}$$

Substitute for Δt in equation (6) to obtain:

$$\boldsymbol{v} = \boxed{\frac{11}{7}\boldsymbol{v}_0}$$

(*c*) Relate Δx to the average speed of the ball and the time it moves before beginning to roll without slipping:

$$\Delta\boldsymbol{x} = \boldsymbol{v}_\text{av}\Delta\boldsymbol{t} = \tfrac{1}{2}(\boldsymbol{v}_0 + \boldsymbol{v})\Delta\boldsymbol{t}$$

Substitute for $v_0 + v$ and Δt and simplify to obtain:

$$\Delta\boldsymbol{x} = \tfrac{1}{2}\left(\boldsymbol{v}_0 + \frac{11}{7}\boldsymbol{v}_0\right)\left(\frac{4\boldsymbol{v}_0}{7\boldsymbol{\mu}_\text{k}\boldsymbol{g}}\right) = \boxed{\frac{36}{49}\frac{\boldsymbol{v}_0^2}{\boldsymbol{\mu}_\text{k}\boldsymbol{g}}}$$

General Problems

108 •• The radius of a small playground merry-go-round is 2.2 m. To start it rotating, you wrap a rope around its perimeter and pull with a force of 260 N for 12 s. During this time, the merry-go-round makes one complete rotation. Neglect any effects of friction. (*a*) Find the angular acceleration of the merry-go-round. (*b*) What torque is exerted by the rope on the merry-go-round? (*c*) What is the moment of inertia of the merry-go-round?

Picture the Problem The force you exert on the rope results in a net torque that accelerates the merry-go-round. The moment of inertia of the merry-go-round, its angular acceleration, and the torque you apply are related through Newton's 2nd law.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the merry-go-round to its angular acceleration and acceleration time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2 \Rightarrow \boldsymbol{\alpha} = \frac{2\Delta\boldsymbol{\theta}}{(\Delta \boldsymbol{t})^2}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{2(2\pi\,\text{rad})}{(12\,\text{s})^2} = 0.0873\,\text{rad/s}^2$$
$$= \boxed{0.087\,\text{rad/s}^2}$$

(*b*) Use the definition of torque to obtain:

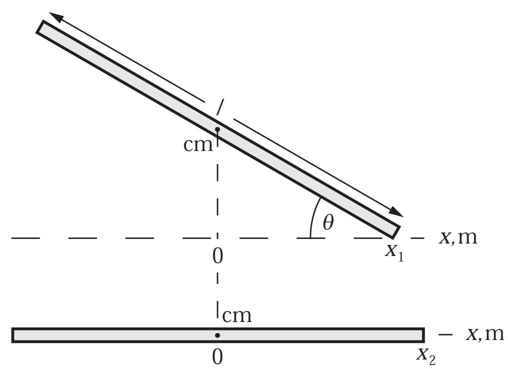
$$\boldsymbol{\tau} = \boldsymbol{Fr} = (260\,\text{N})(2.2\,\text{m}) = 572\,\text{N}\cdot\text{m}$$
$$= \boxed{0.57\,\text{kN}\cdot\text{m}}$$

(*c*) Use Newton's 2nd law to find the moment of inertia of the merry-go-round:

$$\boldsymbol{I} = \frac{\boldsymbol{\tau}_{\text{net}}}{\boldsymbol{\alpha}} = \frac{572\,\text{N}\cdot\text{m}}{0.0873\,\text{rad/s}^2}$$
$$= \boxed{6.6\times10^3\,\text{kg}\cdot\text{m}^2}$$

109 •• A uniform 2.00-m-long stick is raised at an angle of 30° to the horizontal above a sheet of ice. The bottom end of the stick rests on the ice. The stick is released from rest. The bottom of the stick remains in contact with the ice at all times. How far will the bottom end of the stick have traveled during the time the rest of the stick is falling to the ice? Assume that the ice is frictionless.

Picture the Problem Because there are no horizontal forces acting on the stick, the center of mass of the stick will not move in the horizontal direction. Choose a coordinate system in which the origin is at the horizontal position of the center of mass. The diagram shows the stick in its initial raised position and when it has fallen to the ice.



Express the displacement of the right end of the stick Δx as the difference between the position coordinates x_1 and x_2:

$$\Delta x = x_2 - x_1 \qquad (1)$$

Using trigonometry, find the initial coordinate of the right end of the stick:

$$\boldsymbol{x}_1 = \tfrac{1}{2}\ell\cos\boldsymbol{\theta}$$
$$= \tfrac{1}{2}(2.00\,\text{m})\cos30° = 0.866\,\text{m}$$

Because the center of mass has not moved horizontally:

$$x_2 = \tfrac{1}{2}\ell = 1.00\,\text{m}$$

Substitute for x_1 and x_2 in equation (1) to find the displacement of the right end of the stick:

$$\Delta x = 1.00\,\text{m} - 0.866\,\text{m} = \boxed{13\,\text{cm}}$$

110 •• A uniform 5.0-kg disk that has a 0.12-m radius is pivoted so that it rotates freely about its axis (Figure 9-70). A string wrapped around the disk is pulled with a force equal to 20 N. (*a*) What is the torque being exerted by this force about the rotation axis? (*b*) What is the angular acceleration of the disk? (*c*) If the disk starts from rest, what is its angular speed after 5.0 s? (*d*) What is its kinetic energy after the 5.0 s? (*e*) What is the angular displacement of the disk during the 5.0 s? (*f*) Show that the work done by the torque, $\tau\Delta\theta$, equals the kinetic energy.

Picture the Problem The force applied to the string results in a torque about the center of mass of the disk that accelerates it. We can relate these quantities to the moment of inertia of the disk through Newton's 2nd law and then use constant-acceleration equations to find the disks angular speed the angle through which it has rotated in a given period of time. The disk's rotational kinetic energy can be found from its definition.

(*a*) Use the definition of torque to obtain:

$$\tau = FR = (20\,\text{N})(0.12\,\text{m}) = 2.40\,\text{N}\cdot\text{m}$$
$$= \boxed{2.4\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law to express the angular acceleration of the disk in terms of the net torque acting on it and its moment of inertia:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{\tau_{\text{net}}}{\frac{1}{2}MR^2} = \frac{2\tau_{\text{net}}}{MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(2.40\,\text{N}\cdot\text{m})}{(5.0\,\text{kg})(0.12\,\text{m})^2} = 66.7\,\text{rad/s}^2$$
$$= \boxed{67\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, relate the angular speed of the disk to its angular acceleration and the elapsed time:

$$\omega = \omega_0 + \alpha\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Substitute numerical values and evaluate ω:

$$\omega = (66.7\,\text{rad/s}^2)(5.0\,\text{s}) = 333\,\text{rad/s} = \boxed{3.3\times10^2\ \text{rad/s}}$$

(*d*) Use the definition of rotational kinetic energy to obtain:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 = \tfrac{1}{4}MR^2\omega^2$$

Substitute numerical values and evaluate K_{rot}:

$$K_{\text{rot}} = \tfrac{1}{4}(5.0\,\text{kg})(0.12\,\text{m})^2(333\,\text{rad/s})^2 = \boxed{2.0\,\text{kJ}}$$

(*e*) Using a constant-acceleration equation, relate the angle through which the disk turns to its angular acceleration and the elapsed time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Substitute numerical values and evaluate $\Delta\theta$:

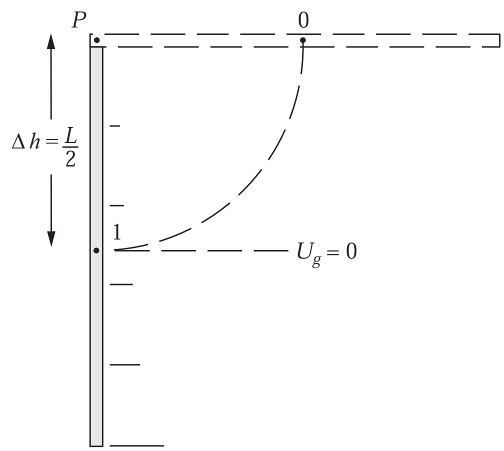
$$\Delta\theta = \tfrac{1}{2}(66.7\,\text{rad/s}^2)(5.0\,\text{s})^2 = \boxed{8.3\times10^2\ \text{rad}}$$

(*f*) Substitute for I and ω^2 in the expression for K and simplify to obtain:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\frac{\tau}{\alpha}\right)(\alpha\Delta t)^2 = \tfrac{1}{2}\alpha\tau(\Delta t)^2 = \boxed{\tau\Delta\theta}$$

111 •• A uniform 0.25-kg thin rod that has a 80-cm length is free to rotate about a fixed horizontal axis perpendicular to, and through one end, of the rod. It is held horizontal and released. Immediately after it is released, what is (*a*) the acceleration of the center of the rod, and (*b*) the initial acceleration of the free end of the rod? (*c*) What is the speed of the center of mass of the rod when the rod is (momentarily) vertical.

Picture the Problem The diagram shows the rod in its initial horizontal position and then, later, as it swings through its vertical position. The center of mass is denoted by the numerals 0 and 1. Let the length of the rod be represented by L and its mass by m. We can use Newton's 2nd law in rotational form to find, first, the angular acceleration of the rod and then, from α, the acceleration of any point on the rod. We can use conservation of energy to find the angular speed of the center of mass of the rod when it is vertical and then use this value to find its linear velocity.



(a) Relate the acceleration of the center of the rod to the angular acceleration of the rod:

$$a = \ell\alpha = \frac{L}{2}\alpha$$

Use Newton's 2nd law to relate the torque about the suspension point of the rod (exerted by the weight of the rod) to the rod's angular acceleration:

$$\boldsymbol{\alpha} = \frac{\boldsymbol{\tau}}{\boldsymbol{I}_{\mathrm{P}}} = \frac{\boldsymbol{Mg}\frac{\boldsymbol{L}}{2}}{\frac{1}{3}\boldsymbol{ML}^2} = \frac{3\boldsymbol{g}}{2\boldsymbol{L}}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{3(9.81\,\mathrm{m/s^2})}{2(0.80\,\mathrm{m})} = 18.4\,\mathrm{rad/s^2}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \tfrac{1}{2}(0.80\,\mathrm{m})(18.4\,\mathrm{rad/s^2}) = \boxed{7.4\,\mathrm{m/s^2}}$$

(b) Relate the acceleration of the end of the rod to α:

$$\boldsymbol{a}_{\mathrm{end}} = \boldsymbol{L\alpha}$$

Substitute numerical values and evaluate a_{end}:

$$\boldsymbol{a}_{\mathrm{end}} = (0.80\,\mathrm{m})(18.4\,\mathrm{rad/s^2}) = \boxed{15\,\mathrm{m/s^2}}$$

(c) Relate the linear velocity of the center of mass of the rod to its angular speed as it passes through the vertical:

$$v = \omega\Delta h = \tfrac{1}{2}\omega L \qquad (1)$$

Solving for Δh yields:

$$\Delta\boldsymbol{h} = \tfrac{1}{2}\boldsymbol{L}$$

Use conservation of energy to relate the changes in the kinetic and potential energies of the rod as it swings from its initial horizontal orientation through its vertical orientation:

$$\Delta K + \Delta U = K_1 - K_0 + U_1 - U_0 = 0$$

or, because $K_0 = U_1 = 0$,

$$K_1 - U_0 = 0$$

Substitute for K_1 and U_0 to obtain:

$$\tfrac{1}{2} \boldsymbol{I}_{\mathrm{P}} \boldsymbol{\omega}^2 - \boldsymbol{mg}\Delta \boldsymbol{h} = 0$$

Substituting for I_{P} and Δh yields:

$$\tfrac{1}{2}\left(\tfrac{1}{3}\boldsymbol{mL}^2\right)\boldsymbol{\omega}^2 - \boldsymbol{mg}\left(\tfrac{1}{2}\boldsymbol{L}\right) = 0 \Rightarrow \omega = \sqrt{\frac{3g}{L}}$$

Substituting for ω in equation (1) yields:

$$v = \tfrac{1}{2} L\sqrt{\frac{3g}{L}} = \tfrac{1}{2}\sqrt{3gL}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \tfrac{1}{2}\sqrt{3\left(9.81\,\mathrm{m/s^2}\right)\left(0.80\,\mathrm{m}\right)} = \boxed{2.4\,\mathrm{m/s}}$$

112 •• A marble of mass M and radius R rolls without slipping down the track on the left from a height h_1, as shown in Figure 9-71. The marble then goes up the *frictionless* track on the right to a height h_2. Find h_2.

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the track. The initial potential energy of the marble is transformed into translational and rotational kinetic energy and gravitational potential energy (note that its center of mass is a distance R above the bottom of the track when it reaches the lowest point on the track) as it rolls down the track to its lowest point and then, because the portion of the track to the right is frictionless, into translational kinetic energy and, eventually, into gravitational potential energy.

Using conservation of energy, relate h_2 to the kinetic energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_{\mathrm{f}} = U_{\mathrm{i}} = 0$,

$$-K_{\mathrm{i}} + U_{\mathrm{f}} = 0$$

Substitute for K_{i} and U_{f} to obtain:

$$-\tfrac{1}{2}Mv^2 - Mgh_2 = 0 \Rightarrow h_2 = \frac{v^2}{2g} \quad (1)$$

Using conservation of energy, relate h_1 to the energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_{\mathrm{i}} = 0$,

$$K_{\mathrm{f,\,trans}} + K_{\mathrm{f,\,rot}} + U_{\mathrm{f}} - U_{\mathrm{i}} = 0$$

Substitute for the energies to obtain: $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + MgR - Mgh_1 = 0$

The moment of inertia of a sphere of mass M and radius R, about an axis through its center, is $\frac{2}{5}MR^2$: $\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 + MgR - Mgh_1 = 0$

Because the marble is rolling without slipping, $v = R\omega$ and:

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 + MgR - Mgh_1 = 0$$

Solve for v^2 to obtain: $v^2 = \frac{10}{7}g(h_1 - R)$

Substitute for v^2 in equation (1) and simplify to obtain:

$$h_2 = \frac{\frac{10}{7}g(h_1 - R)}{2g} = \boxed{\frac{5}{7}(h_1 - R)}$$

Remarks: If h_1, h_2 >> R, then our result becomes $h_2 = \frac{5}{7}h_1$.

113 •• **[SSM]** A uniform 120-kg disk with a radius equal to 1.4 m initially rotates with an angular speed of 1100 rev/min. A constant tangential force is applied at a radial distance of 0.60 m from the axis. (*a*) How much work must this force do to stop the wheel? (*b*) If the wheel is brought to rest in 2.5 min, what torque does the force produce? What is the magnitude of the force? (*c*) How many revolutions does the wheel make in these 2.5 min?

Picture the Problem To stop the wheel, the tangential force will have to do an amount of work equal to the initial rotational kinetic energy of the wheel. We can find the stopping torque and the force from the average power delivered by the force during the slowing of the wheel. The number of revolutions made by the wheel as it stops can be found from a constant-acceleration equation.

(*a*) Relate the work that must be done to stop the wheel to its kinetic energy: $W = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{1}{4}mr^2\omega^2$

Substitute numerical values and evaluate W:

$$W = \frac{1}{4}(120\,\text{kg})(1.4\,\text{m})^2\left[1100\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right]^2 = 780\,\text{kJ} = \boxed{7.8\times10^2\,\text{kJ}}$$

(*b*) Express the stopping torque in terms of the average power required:

$$P_{\text{av}} = \tau\omega_{\text{av}} \Rightarrow \tau = \frac{P_{\text{av}}}{\omega_{\text{av}}}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{\dfrac{780\,\text{kJ}}{(2.5\,\text{min})(60\,\text{s/min})}}{\dfrac{(1100\,\text{rev/min})(2\pi\,\text{rad/rev})(1\,\text{min}/60\,\text{s})}{2}} = 90.3\,\text{N}\cdot\text{m} = \boxed{90\,\text{N}\cdot\text{m}}$$

Relate the stopping torque to the magnitude of the required force and evaluate *F*:

$$\boldsymbol{F} = \frac{\boldsymbol{\tau}}{\boldsymbol{R}} = \frac{90.3\,\text{N}\cdot\text{m}}{0.60\,\text{m}} = \boxed{0.15\text{ kN}}$$

(*c*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its average angular speed and the stopping time:

$$\Delta\theta = \omega_{\text{av}}\Delta t$$

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\boldsymbol{\theta} = \left(\frac{1100\,\text{rev/min}}{2}\right)(2.5\,\text{min})$$
$$= \boxed{1.4\times10^{3}\text{ rev}}$$

114 •• A day-care center has a merry-go-round that consists of a uniform 240-kg circular wooden platform 4.00 m in diameter. Four children run alongside the merry-go-round and push tangentially along the platform's circumference until, starting from rest, the merry-go-round is spinning at 2.14 rev/min. During the spin up: (*a*) If each child exerts a sustained force equal to 26 N how far does each child run? (*b*) What is the angular acceleration of the merry-go-round during spin up? (*c*) How much work does each child do? (*d*) What is the increase in the kinetic energy of the merry-go-round?

Picture the Problem The work done by the four children on the merry-go-round will change its kinetic energy. We can use the work-energy theorem to relate the work done by the children to the distance they ran and Newton's 2nd law to find the angular acceleration of the merry-go-round.

(*a*) Use the work-kinetic energy theorem to relate the work done by the children to the kinetic energy of the merry-go-round:

$$\boldsymbol{W}_{\text{net force}} = \Delta\boldsymbol{K} = \boldsymbol{K}_{\text{f}}$$

or

$$4F\Delta s = \tfrac{1}{2}I\omega^2$$

Substitute for I to obtain:

$$4\boldsymbol{F}\Delta\boldsymbol{s} = \tfrac{1}{2}\left(\tfrac{1}{2}\boldsymbol{mr}^2\right)\boldsymbol{\omega}^2 = \tfrac{1}{4}\boldsymbol{mr}^2\boldsymbol{\omega}^2$$

Solving for Δs yields:

$$\Delta\boldsymbol{s} = \frac{\boldsymbol{mr}^2\boldsymbol{\omega}^2}{16\boldsymbol{F}}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(240\,\text{kg})(2.00\,\text{m})^2\left[\dfrac{1\,\text{rev}}{2.8\,\text{s}}\times\dfrac{2\pi\,\text{rad}}{\text{rev}}\right]^2}{16(26\,\text{N})}$$

$$= 11.6\,\text{m} = \boxed{12\,\text{m}}$$

(b) Apply Newton's 2nd law to express the angular acceleration of the merry-go-round:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{4Fr}{\frac{1}{2}mr^2} = \frac{8F}{mr}$$

Substitute numerical values and evaluate α:

$$\boldsymbol{\alpha} = \frac{8(26\,\text{N})}{(240\,\text{kg})(2.00\,\text{m})} = \boxed{0.43\,\text{rad/s}^2}$$

(c) Use the definition of work to relate the force exerted by each child to the distance over which that force is exerted:

$$\boldsymbol{W} = \boldsymbol{F}\Delta\boldsymbol{s} = (26\,\text{N})(11.6\,\text{m}) = \boxed{0.30\,\text{kJ}}$$

(d) Relate the kinetic energy of the merry-go-round to the work that was done on it:

$$W_{\text{net force}} = \Delta K = K_{\text{f}} - 0 = 4F\Delta s$$

Substitute numerical values and evaluate $W_{\text{net force}}$:

$$\boldsymbol{W}_{\text{net force}} = 4(26\,\text{N})(11.6\,\text{m}) = \boxed{1.2\,\text{kJ}}$$

115 •• A uniform 1.5-kg hoop with a 65-cm radius has a string wrapped around its circumference and lies flat on a horizontal frictionless table. The free end of the string is pulled with a constant horizontal force equal to 5.0 N and the string does not slip on the hoop. (a) How far does the center of the hoop travel in 3.0 s? (b) What is the angular speed of the hoop after 3.0 s?

Picture the Problem Because the center of mass of the hoop is at its center, we can use Newton's second law to relate the acceleration of the hoop to the net force acting on it. The distance moved by the center of the hoop can be determined using a constant-acceleration equation, as can the angular speed of the hoop.

(*a*) Using a constant-acceleration equation, relate the distance the center of the hoop travels in 3.0 s to the acceleration of its center of mass:

$$\Delta s = \tfrac{1}{2} a_{\text{cm}} (\Delta t)^2$$

Relate the acceleration of the center of mass of the hoop to the net force acting on it:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{m}$$

Substitute for a_{cm} to obtain:

$$\Delta s = \frac{F(\Delta t)^2}{2m}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(5.0\,\text{N})(3.0\,\text{s})^2}{2(1.5\,\text{kg})} = \boxed{15\,\text{m}}$$

(*b*) Relate the angular speed of the hoop to its angular acceleration and the elapsed time:

$$\omega = \alpha\,\Delta t$$

Use Newton's 2nd law to relate the angular acceleration of the hoop to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{FR}{mR^2} = \frac{F}{mR}$$

Substitute for α in the expression for ω to obtain:

$$\omega = \frac{F\Delta t}{mR}$$

Substitute numerical values and evaluate ω:

$$\boldsymbol{\omega} = \frac{(5.0\,\text{N})(3.0\,\text{s})}{(1.5\,\text{kg})(0.65\,\text{m})} = \boxed{15\,\text{rad/s}}$$

116 •• A hand driven grinding wheel is a uniform 60-kg disk with a 45-cm radius. It has a handle of negligible mass 65 cm from the rotation axis. A compact 25-kg load is attached to the handle when it is at the same height as the horizontal rotation axis. Neglecting the effects of friction, find (*a*) the initial angular acceleration of the wheel, and (*b*) the maximum angular speed of the wheel.

Picture the Problem Let *R* represent the radius of the grinding wheel, *M* its mass, *r* the radius of the handle, and *m* the mass of the load attached to the handle. In the absence of information to the contrary, we'll treat the 25-kg load as though it were concentrated at a point. Let the zero of gravitational potential energy be where the 25-kg load is at its lowest point. We'll apply Newton's 2nd law and the conservation of mechanical energy to determine the initial angular acceleration and the maximum angular speed of the wheel.

(*a*) Use Newton's 2nd law to relate the acceleration of the wheel to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{mgr}{I}$$

The moment of inertia of the wheel-and-load system is the sum of their moments of inertia. Substituting for I yields:

$$\alpha = \frac{mgr}{\frac{1}{2}MR^2 + mr^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{\frac{1}{2}(60\,\text{kg})(0.45\,\text{m})^2 + (25\,\text{kg})(0.65\,\text{m})^2} = \boxed{9.6\,\text{rad/s}^2}$$

(*b*) Use conservation of mechanical energy to relate the initial potential energy of the load to its kinetic energy and the rotational kinetic energy of the wheel when the load is directly below the center of mass of the wheel:

$$\Delta K + \Delta U = 0$$

or, because $K_\text{i} = U_\text{f} = 0$,

$$K_{\text{f,trans}} + K_{\text{f,rot}} - U_\text{i} = 0\,.$$

Substitute for $K_{\text{f,trans}}$, $K_{\text{f,rot}}$, and U_i to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 - mgr = 0$$

Solving for ω yields:

$$\omega = \sqrt{\frac{4mgr}{2mr^2 + MR^2}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{4(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{2(25\,\text{kg})(0.65\,\text{m})^2 + (60\,\text{kg})(0.45\,\text{m})^2}} = \boxed{4.4\,\text{rad/s}}$$

117 •• A uniform disk of radius R and mass M is pivoted about a horizontal axis parallel to its symmetry axis and passing through a point on its perimeter, so that it can swing freely in a vertical plane (Figure 9-72). It is released from rest with its center of mass at the same height as the pivot. (*a*) What is the angular speed of the disk when its center of mass is directly below the pivot? (*b*) What force is exerted by the pivot on the disk at this moment?

Picture the Problem Let the zero of gravitational potential energy be at the center of the disk when it is directly below the pivot. The initial gravitational potential energy of the disk is transformed into rotational kinetic energy when its center of mass is directly below the pivot. We can use Newton's 2nd law to relate the force exerted by the pivot to the weight of the disk and the centripetal force acting on it at its lowest point.

(*a*) Use conservation of mechanical energy to relate the initial potential energy of the disk to its kinetic energy when its center of mass is directly below the pivot:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{f,rot} - U_i = 0$$

Substitute for $K_{f,rot}$ and U_i to obtain:

$$\tfrac{1}{2}I\omega^2 - Mgr = 0 \qquad (1)$$

Use the parallel-axis theorem to relate the moment of inertia of the disk about the pivot to its moment of inertia with respect to an axis through its center of mass:

$$I = I_{cm} + Mh^2$$

or

$$I = \tfrac{1}{2}Mr^2 + Mr^2 = \tfrac{3}{2}Mr^2$$

Substituting for I in equation (1) yields:

$$\tfrac{1}{2}\left(\tfrac{3}{2}Mr^2\right)\omega^2 - Mgr = 0 \Rightarrow \omega = \boxed{\sqrt{\frac{4g}{3r}}}$$

(*b*) Letting F represent the force exerted by the pivot, use Newton's 2nd law to express the net force acting on the swinging disk as it passes through its lowest point:

$$F_{net} = F - Mg = Mr\omega^2$$

Solve for F to obtain:

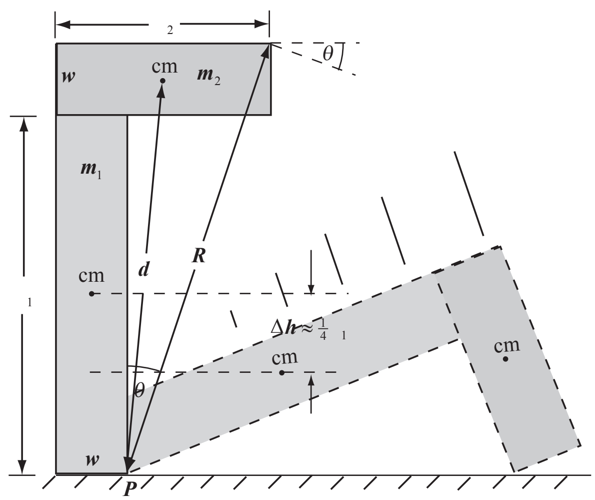
$$F = Mg + Mr\omega^2$$

Substituting for ω^2 and simplifying yields:

$$F = Mg + Mr\frac{4g}{3r} = \boxed{\tfrac{7}{3}Mg}$$

118 •• The roof of the student dining hall at your college will be supported by high cross-braced wooden beams attached in the shape of an upside-down *L* (Figure 9-73). Each vertical beam is 12.0 ft high and 2.0 ft wide, and the horizontal cross-member is 6.0 ft long. The mass of the vertical beam is 350 kg, and the mass of the horizontal beam is 175 kg. As the workers were building the hall, one of these structures started to fall over before it was anchored into place. (Luckily, they stopped it before it fell.) (*a*) If it started falling from an upright position, what was the initial angular acceleration of the structure? Assume that the bottom did not slide across the floor and that it did not fall "out of plane," that is, that during the fall the structure remained in the vertical plane defined by the initial position of the structure. (*b*) What would be the magnitude of the initial linear acceleration of the right end of the horizontal beam? (*c*) What would the horizontal component of the initial linear acceleration be at this same location? (*d*) Assuming the workers caught the cross-braced beam just before it hit the floor, estimate the beam's rotational speed when they caught it.

Picture the Problem The pictorial representation shows one of the structures initially in its upright position and, later, as it is about to strike the floor. Because we need to take moments about the axis of rotation (a line through point *P*), we'll need to use the parallel-axis theorem to find the moments of inertia of the two parts of this composite structure. Let the numeral 1 denote the vertical member and the numeral 2 the horizontal member. We can apply Newton's 2nd law in rotational form to the structure to express its angular acceleration in terms of the net torque causing it to fall and its moment of inertia with respect to point *P*.



(*a*) Taking clockwise rotation to be positive (this is the direction the structure is going to rotate), apply $\sum \tau = I_P\alpha$:

$$m_2 g\left(\frac{\ell_2}{2}\right) - m_1 g\left(\frac{w}{2}\right) = I_P\alpha$$

Solving for α yields:

$$\alpha = \frac{m_2 g\ell_2 - m_1 g w}{2I_P}$$

Because $I_P = I_{1P} + I_{2P}$:

$$\alpha = \frac{g(m_2\ell_2 - m_1 w)}{2(I_{1P} + I_{2P})} \qquad (1)$$

Convert ℓ_1, ℓ_2, and w to SI units:

$$\ell_1 = 10.0\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 3.05\,\text{m},$$

$$\ell_2 = 6.0\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 1.83\,\text{m}, \text{ and}$$

$$w = 2.0\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 0.610\,\text{m}$$

Using Table 9-1 and the parallel-axis theorem, express the moment of inertia of the vertical member about an axis through point P:

$$I_{1P} = \tfrac{1}{3} m_1 \ell_1^2 + m_1 \left(\frac{w}{2}\right)^2 = m_1\left(\tfrac{1}{3}\ell_1^2 + \tfrac{1}{4}w^2\right)$$

Substitute numerical values and evaluate I_{1P}:

$$I_{1P} = (350\,\text{kg})\left[\tfrac{1}{3}(3.05\,\text{m})^2 + \tfrac{1}{4}(0.610\,\text{m})^2\right] = 1.12\times10^3\,\text{kg}\cdot\text{m}^2$$

Using the parallel-axis theorem, express the moment of inertia of the horizontal member about an axis through point P:

$$I_{2P} = I_{2,\text{cm}} + m_2 d^2 \qquad (2)$$

where

$$d^2 = \left(\ell_1 + \tfrac{1}{2}w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2$$

Solving for d yields:

$$d = \sqrt{\left(\ell_1 + \tfrac{1}{2}w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2}$$

Substitute numerical values and evaluate d:

$$d = \sqrt{\left[3.05\,\text{m} + \tfrac{1}{2}(0.610\,\text{m})\right]^2 + \left[\tfrac{1}{2}(1.83\,\text{m}) - 0.610\,\text{m}\right]^2} = 3.37\,\text{m}$$

From Table 9-1 we have:

$$I_{2,\text{cm}} = \tfrac{1}{12} m_2 \ell_2^2$$

Substitute in equation (2) to obtain:

$$I_{2P} = \tfrac{1}{12} m_2 \ell_2^2 + m_2 d^2 = m_2\left(\tfrac{1}{12}\ell_2^2 + d^2\right)$$

Evaluate I_{2P}:

$$I_{2P} = (175\,\text{kg})\left[\tfrac{1}{12}(1.83\,\text{m})^2 + (3.37\,\text{m})^2\right] = 2.036\times10^3\,\text{kg}\cdot\text{m}^2$$

Substitute numerical values in equation (1) and evaluate α:

$$\alpha = \frac{(9.81\,\text{m/s}^2)\left[(175\,\text{kg})(1.83\,\text{m}) - (350\,\text{kg})(0.61\,\text{m})\right]}{2(1.12 + 2.036)\times10^3\,\text{kg}\cdot\text{m}^2} = 0.166\,\text{rad/s}^2 = \boxed{0.17\,\text{rad/s}^2}$$

(b) Express the magnitude of the acceleration of the top right-corner of the cross member:

$a = \alpha R$
where $R^2 = (\ell_1 + w)^2 + (\ell_2 - w)^2$.

Solving for R yields:

$$R = \sqrt{(\ell_1 + w)^2 + (\ell_2 - w)^2}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{(3.05\,\text{m} + 0.610\,\text{m})^2 + (1.83\,\text{m} - 0.610\,\text{m})^2} = 3.86\,\text{m}$$

Substitute numerical values and evaluate a:

$$a = (0.166\,\text{rad/s}^2)(3.86\,\text{m}) = 0.641\,\text{m/s}^2 = \boxed{0.64\,\text{m/s}^2}$$

(c) Refer to the diagram to express a_x in terms of a:

$$a_x = a\cos\theta = a\frac{\ell_1 + w}{R}$$

Substitute numerical values and evaluate a_x:

$$a_x = (0.641\,\text{m/s}^2)\frac{3.05\,\text{m} + 0.610\,\text{m}}{3.86\,\text{m}} = 0.608\,\text{m/s}^2 = \boxed{0.61\,\text{m/s}^2}$$

(d) Letting the system include the beam and the earth, apply conservation of mechanical energy to the beam to obtain:

$$\Delta K + \Delta U_{\text{g}} = K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

where the subscript "f" refers to the beam just before it is caught and the subscript "i" refers to its initial vertical position.

Letting $U_{\text{g}} = 0$ in the final state:

$$K_{\text{f}} - U_{\text{i}} = 0$$

Substituting for K_{f} and U_{i} yields:

$$\tfrac{1}{2}I_P\omega_{\text{f}}^2 - mg(\Delta h) = 0$$

From the pictorial representation:

$$\Delta h \approx \tfrac{1}{4}\ell_1$$

Substituting for Δh yields:

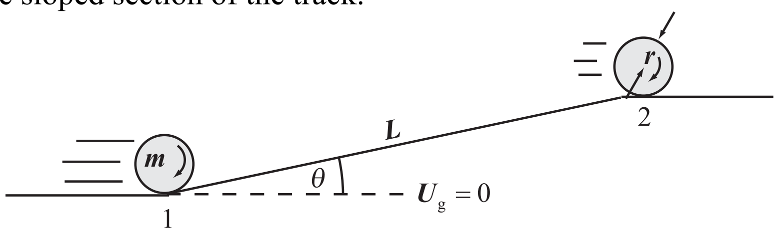
$$\tfrac{1}{2}I_P\omega_{\text{f}}^2 - \tfrac{1}{4}mg\ell_1 = 0 \Rightarrow \omega_{\text{f}} = \sqrt{\frac{mg\ell_1}{2I_P}}$$

Substitute numerical values and evaluate ω_{f}:

$$\omega_{\text{f}} = \sqrt{\frac{(350\,\text{kg})(9.81\,\text{m/s}^2)(3.05\,\text{m})}{2(1.12 + 2.04)\times 10^3\,\text{kg}\cdot\text{m}^2}} \approx \boxed{1.3\,\text{rad/s}}$$

119 •• **[SSM]** You are participating in league bowling with your friends. Time after time, you notice that your bowling ball rolls back to you without slipping on the flat section of track. When the ball encounters the slope that brings it up to the ball return, it is moving at 3.70 m/s. The length of the sloped part of the track is 2.50 m. The radius of the bowling ball is 11.5 cm. (*a*) What is the angular speed of the ball before it encounters the slope? (*b*) If the speed with which the ball emerges at the top of the incline is 0.40 m/s, what is the angle (assumed constant), that the sloped section of the track makes with the horizontal? (*c*) What is the magnitude of the angular acceleration of the ball while it is on the slope?

Picture the Problem The pictorial representation shows the bowling ball slowing down as it rolls up the slope. Let the system include the ball, the incline, and the earth. Then $W_{ext} = 0$ and we can use conservation of mechanical energy to find the angle of the sloped section of the track.



(*a*) Because the bowling ball rolls without slipping, its angular speed is directly proportional to its linear speed:

$$\boldsymbol{\omega} = \frac{\boldsymbol{v}}{\boldsymbol{r}}$$

where r is the radius of the bowling ball.

Substitute numerical values and evaluate ω:

$$\boldsymbol{\omega} = \frac{3.70\text{ m/s}}{0.115\text{ m}} = 32.17\text{ rad/s} = \boxed{32.2\text{ rad/s}}$$

(*b*) Apply conservation of mechanical energy to the system as the bowling ball rolls up the incline:

$$\boldsymbol{W}_{ext} = \Delta\boldsymbol{K} + \Delta\boldsymbol{U}$$

or, because $W_{ext} = 0$,

$$\boldsymbol{K}_{t,2} - \boldsymbol{K}_{t,1} + \boldsymbol{K}_{r,2} - \boldsymbol{K}_{r,1} + \boldsymbol{U}_2 - \boldsymbol{U}_1 = 0$$

Substituting for the kinetic and potential energies yields:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I_{ball}\omega_2^2 - \tfrac{1}{2}I_{ball}\omega_1^2 + mgL\sin\theta = 0$$

Solving for θ yields:

$$\theta = \sin^{-1}\left[\frac{m\left(v_1^2 - v_2^2\right) + I_{ball}\left(\omega_1^2 - \omega_2^2\right)}{2mgL}\right]$$

Because $I_{\text{ball}} = \frac{2}{5}mr^2$:

$$\theta = \sin^{-1}\left[\frac{m\left(v_1^2 - v_2^2\right) + \frac{2}{5}mr^2\left(\omega_1^2 - \omega_2^2\right)}{2mgL}\right]$$
$$= \sin^{-1}\left[\frac{7\left(v_1^2 - v_2^2\right)}{10gL}\right]$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{7\left((3.70\ \text{m/s})^2 - (0.40\ \text{m/s})^2\right)}{10\left(9.81\ \text{m/s}^2\right)(2.50\ \text{m})}\right]$$
$$= \boxed{23°}$$

(*c*) The angular acceleration of the bowling ball is directly proportional to its translational acceleration:

$$\boldsymbol{\alpha} = \frac{\boldsymbol{a}}{\boldsymbol{r}} \qquad (1)$$

Use a constant-acceleration equation to relate the speeds of the ball at points 1 and 2 to its acceleration:

$$\boldsymbol{v}_2^2 = \boldsymbol{v}_1^2 + 2\boldsymbol{aL} \Rightarrow \boldsymbol{a} = \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2\boldsymbol{L}}$$

Substitute in equation (1) to obtain:

$$\boldsymbol{\alpha} = \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2\boldsymbol{rL}}$$

Substitute numerical values and evaluate $|\boldsymbol{\alpha}|$:

$$|\boldsymbol{\alpha}| = \left|\frac{(0.40\ \text{m/s})^2 - (3.70\ \text{m/s})^2}{2(0.115\ \text{m})(2.50\ \text{m})}\right|$$
$$= \boxed{24\ \text{rad/s}^2}$$

120 •• Figure 9-74 shows a hollow cylinder that has a length equal to 1.80 m, a mass equal to 0.80 kg, and radius equal to 0.20 m. The cylinder is free to rotate about a vertical axis that passes through its center and is perpendicular to the cylinder. Two objects are inside the cylinder. Each object has a mass equal to 0.20 kg, is attached to springs that have a force constant k and unstressed lengths equal to 0.40 m. The inside walls of the cylinder are frictionless. (*a*) Determine the value of the force constant if the objects are located 0.80 m from the center of the cylinder when the cylinder rotates at 24 rad/s. (*b*) How much work is required to bring the system from rest to an angular speed of 24 rad/s?

Picture the Problem Let m represent the mass of the 0.20-kg cylinder, M the mass of the 0.80-kg cylinder, L the 1.8-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is m. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's force constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

(*a*) Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x+\Delta x)\omega^2$$

Solving for *k* yields:

$$k = \frac{m(x+\Delta x)\omega^2}{\Delta x}$$

Substitute numerical values and evaluate *k*:

$$k = \frac{(0.20\,\text{kg})(0.80\,\text{m})(24\,\text{rad/s})^2}{0.40\,\text{m}} = 230\,\text{N/m} = \boxed{0.23\,\text{kN/m}}$$

(*b*) Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express *I* as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where L is the length of the cylinder.

For a disk (thin cylinder), L is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right) = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate *I*:

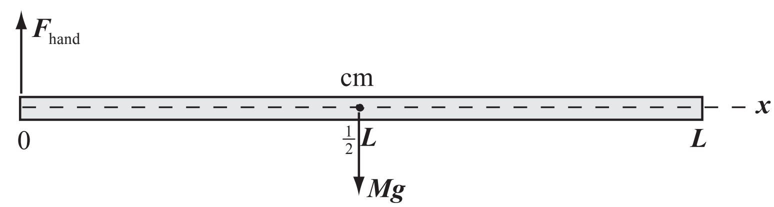
$$\boldsymbol{I} = \tfrac{1}{2}(0.80\,\text{kg})(0.20\,\text{m})^2 + \tfrac{1}{12}(0.80\,\text{kg})(1.8\,\text{m})^2 + 2(0.20\,\text{kg})\left[\tfrac{1}{4}(0.20\,\text{m})^2 + (0.80\,\text{m})^2\right] = 0.492\,\text{N}\cdot\text{m}^2$$

Substitute in equation (1) to obtain:

$$\boldsymbol{W} = \tfrac{1}{2}\left(0.492\,\text{N}\cdot\text{m}^2\right)(24\,\text{rad/s})^2 + \tfrac{1}{2}(230\,\text{N/m})(0.40\,\text{m})^2 = \boxed{0.16\,\text{kJ}}$$

121 •• **[SSM]** A popular classroom demonstration involves taking a meterstick and holding it horizontally at the 0.0-cm end with a number of pennies spaced evenly along its surface. If the hand is suddenly relaxed so that the meterstick pivots freely about the 0.0-cm mark under the influence of gravity, an interesting thing is seen during the first part of the stick's rotation: the pennies nearest the 0.0-cm mark remain on the meterstick, while those nearest the 100-cm mark are left behind by the falling meterstick. (This demonstration is often called the ″faster than gravity″ demonstration.) Suppose this demonstration is repeated without any pennies on the meterstick. (*a*) What would the initial acceleration of the 100.0-cm mark then be? (The initial acceleration is the acceleration just after the release.) (*b*) What point on the meterstick would then have an initial acceleration greater than *g*?

Picture the Problem The diagram shows the force the hand supporting the meterstick exerts at the pivot point and the force the earth exerts on the meterstick acting at the center of mass. We can relate the angular acceleration to the acceleration of the end of the meterstick using $a = L\alpha$ and use Newton's 2nd law in rotational form to relate α to the moment of inertia of the meterstick.



(*a*) Relate the acceleration of the far end of the meterstick to the angular acceleration of the meterstick:

$$a = L\alpha \qquad (1)$$

Apply $\sum \tau_P = I_P\alpha$ to the meterstick:

$$Mg\left(\frac{L}{2}\right) = I_P\alpha \Rightarrow \alpha = \frac{MgL}{2I_P}$$

From Table 9-1, for a rod pivoted at one end, we have:

$$I_P = \frac{1}{3}ML^2$$

Substitute for I_P in the expression for α to obtain:

$$\alpha = \frac{3MgL}{2ML^2} = \frac{3g}{2L}$$

Substitute for α in equation (1) to obtain:

$$a = \frac{3g}{2}$$

Substitute numerical values and evaluate *a*:

$$\boldsymbol{a} = \frac{3\left(9.81\,\text{m/s}^2\right)}{2} = \boxed{14.7\,\text{m/s}^2}$$

(*b*) Express the acceleration of a point on the meterstick a distance x from the pivot point:

$$a = \alpha x = \frac{3g}{2L}x$$

Express the condition that the meterstick have an initial acceleration greater than g:

$$\frac{3g}{2L}x > g \Rightarrow \boldsymbol{x} > \frac{2\boldsymbol{L}}{3}$$

Substitute the numerical value of L and evaluate x:

$$\boldsymbol{x} > \frac{2(100.0\,\text{cm})}{3} = \boxed{66.7\,\text{cm}}$$

122 •• A solid metal rod, 1.5 m long, is free to pivot without friction about a fixed horizontal axis perpendicular to the rod and passing through one of its ends. The rod is held in a horizontal position. Small coins, each of mass *m,* are placed on the rod 25 cm, 50 cm, 75 cm, 1 m, 1.25 m, and 1.5 m from the pivot. If the free end is now released, calculate the initial force exerted on each coin by the rod. Assume that the masses of the coins can be neglected in comparison to the mass of the rod.

Picture the Problem While the angular acceleration of the rod is the same at each point along its length, the linear acceleration and, hence, the force exerted on each coin by the rod, varies along its length. We can relate this force the linear acceleration of the rod through Newton's 2nd law and the angular acceleration of the rod.

Letting x be the distance from the pivot, use Newton's 2nd law to express the force F acting on a coin:

$$F_{\text{net}} = mg - F(x) = ma(x)$$

or

$$F(x) = m(g - a(x)) \qquad (1)$$

Use Newton's 2nd law to relate the angular acceleration of the system to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Mg\dfrac{L}{2}}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

Relate $a(x)$ and α:

$$a(x) = x\alpha = x\frac{3g}{2(1.5\,\text{m})} = gx$$

Substitute in equation (1) to obtain:

$$F(x) = m(g - gx) = mg(1 - x)$$

Evaluate $F(0.25\text{ m})$:

$$\boldsymbol{F}(0.25\,\text{m}) = \boldsymbol{mg}(1 - 0.25\,\text{m}) = \boxed{0.75\boldsymbol{mg}}$$

Evaluate $F(0.50\text{ m})$:

$$\boldsymbol{F}(0.50\,\text{m}) = \boldsymbol{mg}(1 - 0.50\,\text{m}) = \boxed{0.50\boldsymbol{mg}}$$

Evaluate $F(0.75\text{ m})$:

$$\boldsymbol{F}(0.75\,\text{m}) = \boldsymbol{mg}(1 - 0.75\,\text{m}) = \boxed{0.25\boldsymbol{mg}}$$

Evaluate $F(1.0\text{ m})$:

$$\boldsymbol{F}(1.0\,\text{m}) = \boldsymbol{F}(1.25\,\text{m}) = \boldsymbol{F}(1.5\,\text{m}) = \boxed{0}$$

123 •• Suppose that for the system described in Problem 120, the force constants are each 60 N/m. The system starts from rest and slowly accelerates until the masses are 0.80 m from the center of the cylinder. How much work was done in the process?

Picture the Problem Let m represent the mass of the 0.20-kg cylinder, M the mass of the 0.80-kg cylinder, L the 1.80-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is m. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's force constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express I as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where L is the length of the cylinder.

For a disk (thin cylinder), L is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute for the moments of inertia and simplify to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right)$$
$$= \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I} = \tfrac{1}{2}(0.80\,\text{kg})(0.20\,\text{m})^2 + \tfrac{1}{12}(0.80\,\text{kg})(1.80\,\text{m})^2 + 2(0.20\,\text{kg})\left[\tfrac{1}{4}(0.20\,\text{m})^2 + (0.80\,\text{m})^2\right]$$
$$= 0.492\,\text{N}\cdot\text{m}^2$$

Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x + \Delta x)\omega^2$$

Solving for ω yields:

$$\omega = \sqrt{\frac{k\Delta x}{m(x+\Delta x)}}$$

Substitute numerical values and evaluate ω:

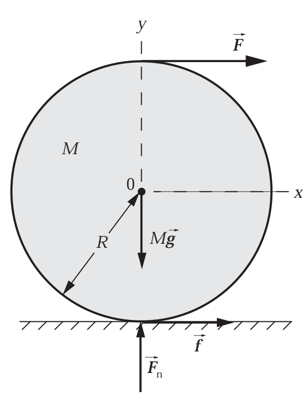
$$\boldsymbol{\omega} = \sqrt{\frac{(60\,\text{N/m})(0.40\,\text{m})}{(0.20\,\text{kg})(0.80\,\text{m})}} = 12.25\,\text{rad/s}$$

Substitute numerical values in equation (1) and evaluate W:

$$\boldsymbol{W} = \tfrac{1}{2}\left(0.492\,\text{N}\cdot\text{m}^2\right)(12.25\,\text{rad/s})^2 + \tfrac{1}{2}(60\,\text{N/m})(0.40\,\text{m})^2 = \boxed{42\,\text{J}}$$

124 ••• A string is wrapped around a uniform cylinder of radius R and mass M that rests on a horizontal frictionless surface (The string does not touch the surface because there is a groove cut in the surface to provide space for the string to clear). The string is pulled horizontally from the top with force F. (*a*) Show that the magnitude of the angular acceleration of the cylinder is twice the magnitude of the angular acceleration needed for rolling without slipping, so that the bottom point on the cylinder slides backward against the table. (*b*) Find the magnitude and direction of the frictional force between the table and cylinder that would be needed for the cylinder to roll without slipping. What would be the magnitude of the acceleration of the cylinder in this case?

Picture the Problem The force diagram shows the forces acting on the cylinder. Because F causes the cylinder to rotate clockwise, f, which opposes this motion, is to the right. We can use Newton's 2$^{\text{nd}}$ law in both translational and rotational forms to relate the linear and angular accelerations to the forces acting on the cylinder.



(*a*) Use Newton's 2nd law to relate the angular acceleration of the center of mass of the cylinder to *F*:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

Use Newton's 2nd law to relate the acceleration of the center of mass of the cylinder to *F*:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{M} = \frac{F}{M}$$

Apply the rolling-without-slipping condition to the linear and angular accelerations:

$$\alpha' = \frac{a_{\text{cm}}}{R}$$

Substituting for a_{cm} yields:

$$\alpha' = \frac{F}{MR} = \boxed{\tfrac{1}{2}\alpha}$$

(*b*) Take the point of contact with the floor as the "pivot" point, express the net torque about that point:

$$\tau_{\text{net}} = 2FR = I\alpha \Rightarrow \alpha = \frac{2FR}{I} \qquad (1)$$

Express the moment of inertia of the cylinder with respect to the pivot point:

$$I = \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

Substitute for *I* in equation (1) to obtain:

$$\alpha = \frac{2FR}{\frac{3}{2}MR^2} = \frac{4F}{3MR}$$

The linear acceleration of the cylinder is:

$$a_{\text{cm}} = R\alpha = \boxed{\frac{4F}{3M}}$$

Apply Newton's 2nd law to the cylinder to obtain:

$$\sum F_x = F + f = Ma_{\text{cm}}$$

Solving for *f* and substituting for a_{cm} yields:

$$f = Ma_{\text{cm}} - F = \frac{4F}{3} - F$$
$$= \boxed{\tfrac{1}{3}F \text{ in the positive } x \text{ direction.}}$$

125 ••• **[SSM]** Let's calculate the position *y* of the falling load attached to the winch in Example 9-8 as a function of time by numerical integration. Let the +*y* direction be straight downward. Then, $v(y) = dy/dt$, or

$$t=\int_0^y \frac{1}{v(y')}dy' \approx \sum_{i=0}^{N}\frac{1}{v(y'_i)}\Delta y'$$

where t is the time taken for the bucket to fall a distance y, $\Delta y'$ is a small increment of y', and $y' = N\Delta y'$. Hence, we can calculate t as a function of d by numerical summation. Make a graph of y versus t between 0 s and 2.00 s. Assume m_w = 10.0 kg, R = 0.50 m, m_b = 5.0 kg, L = 10.0 m, and m_c = 3.50 kg. Use $\Delta y'$ = 0.10 m. Compare this position to the position of the falling load if it were in free-fall.

Picture the Problem As the load falls, mechanical energy is conserved. As in Example 9-7, choose the initial potential energy to be zero and let the system include the winch, the bucket, and the earth. Apply conservation of mechanical energy to obtain an expression for the speed of the bucket as a function of its position and use the given expression for t to determine the time required for the bucket to travel a distance y.

Apply conservation of mechanical energy to the system to obtain:

$$U_f + K_f = U_i + K_i = 0 + 0 = 0 \quad (1)$$

Express the total potential energy when the bucket has fallen a distance y:

$$U_f = U_{bf} + U_{cf} + U_{wf}$$
$$= -mgy - m_c'g\left(\frac{y}{2}\right)$$

where m_c' is the mass of the hanging part of the cable.

Assume the cable is uniform and express m_c' in terms of m_c, y, and L:

$$\frac{m_c'}{y} = \frac{m_c}{L} \text{ or } m_c' = \frac{m_c}{L}y$$

Substitute for m_c' to obtain:

$$U_f = -mgy - \frac{m_c gy^2}{2L}$$

Noting that bucket, cable, and rim of the winch have the same speed v, express the total kinetic energy when the bucket is falling with speed v:

$$K_f = K_{bf} + K_{cf} + K_{wf}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}I\omega_f^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2$$

Substituting in equation (1) yields:

$$-mgy - \frac{m_c gy^2}{2L} + \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2 = 0$$

Solving for v yields:

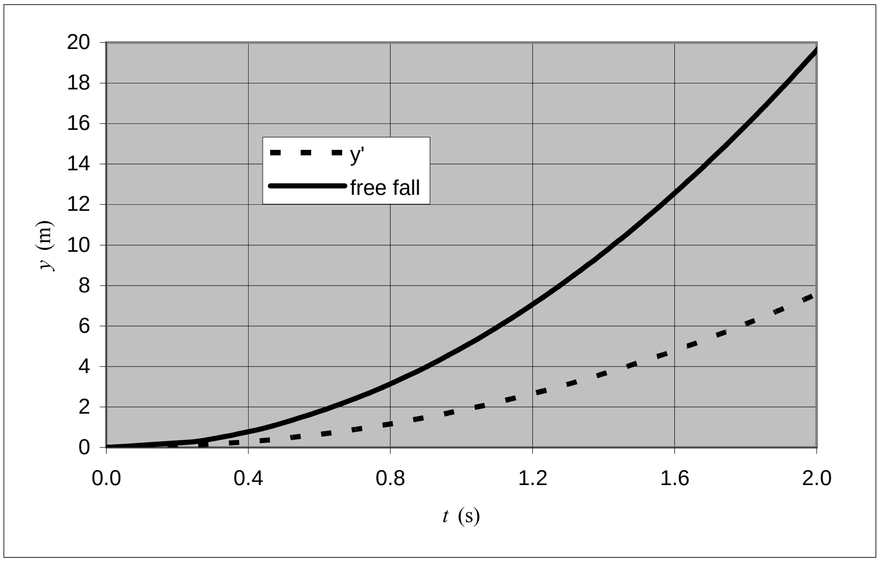
$$v=\sqrt{\frac{4mgy+\dfrac{2m_c gy^2}{L}}{M+2m+2m_c}}$$

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D9	0	y_0
D10	D9+$B$8	$y+\Delta y$
E9	0	v_0
E10	((4*$B$3*$B$7*D10+2*$B$7*D10^2/(2*$B$5))/ ($B$1+2*$B$3+2*$B$4))^0.5	$\sqrt{\frac{4mgy+\dfrac{2m_c gy^2}{L}}{M+2m+2m_c}}$
F10	F9+$B$8/((E10+E9)/2)	$t_{n-1}+\left(\dfrac{v_{n-1}+v_n}{2}\right)\Delta y$
J9	0.5*$B$7*H9^2	$\frac{1}{2}gt^2$

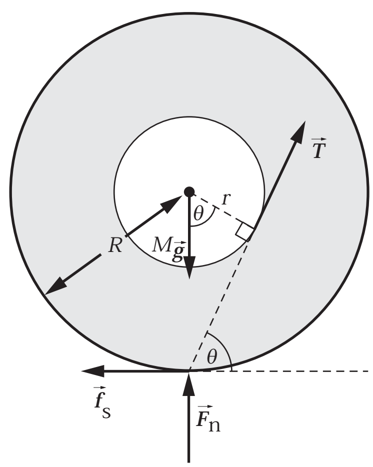
	A	B	C	D	E	F	G	H	I	J
1	M=	10	kg							
2	R=	0.5	m							
3	m=	5	kg							
4	m_c=	3.5	kg							
5	L=	10	m							
6										
7	g=	9.81	m/s^2							
8	dy=	0.1	m	y	$v(y)$	$t(y)$		$t(y)$	y	$0.5gt^2$
9				0.0	0.00	0.00		0.00	0.0	0.00
10				0.1	0.85	0.23		0.23	0.1	0.27
11				0.2	1.21	0.33		0.33	0.2	0.54
12				0.3	1.48	0.41		0.41	0.3	0.81
13				0.4	1.71	0.47		0.47	0.4	1.08
15				0.5	1.91	0.52		0.52	0.5	1.35
105				9.6	9.03	2.24		2.24	9.6	24.61
106				9.7	9.08	2.25		2.25	9.7	24.85
107				9.8	9.13	2.26		2.26	9.8	25.09
108				9.9	9.19	2.27		2.27	9.9	25.34
109				10.0	9.24	2.28		2.28	10.0	25.58

The solid line on the following graph shows the position of the bucket as a function of time when it is in free fall and the dashed line shows its position as a function of time under the conditions modeled in this problem.



126 ••• Figure 9-75 shows a solid cylinder that has mass M and radius R to which a second solid cylinder that has mass m and radius r is attached. A string is wound about the smaller cylinder. The solid cylinder rests on a horizontal surface. The coefficient of static friction between the larger cylinder and the surface is μ_s. If a light tension is applied to the string in the vertical direction, the cylinder will roll to the left; if the tension is applied with the string horizontally to the right, the cylinder rolls to the right. Find the angle between the string and with the horizontal that will allow the cylinder to remain stationary when a light tension is applied to the string.

Picture the Problem The pictorial representation shows the forces acting on the cylinder when it is stationary. First, we note that if the tension is small, then there can be no slipping, and the system must roll. Now consider the point of contact of the cylinder with the surface as the ″pivot″ point. If τ about that point is zero, the system will not roll. This will occur if the line of action of the tension passes through the pivot point.

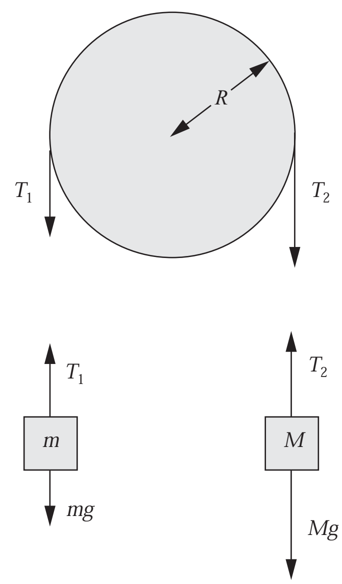


From the diagram we see that:

$$\theta = \boxed{\cos^{-1}\left(\frac{r}{R}\right)}$$

127 ••• **[SSM]** In problems dealing with a pulley with a nonzero moment of inertia, the magnitude of the tensions in the ropes hanging on either side of the pulley are not equal. The difference in the tension is due to the static frictional force between the rope and the pulley; however, the static frictional force cannot be made arbitrarily large. Consider a massless rope wrapped partly around a cylinder through an angle $\Delta\theta$ (measured in radians). It can be shown that if the tension on one side of the pulley is T, while the tension on the other side is T' ($T' > T$), the maximum value of T' that can be maintained without the rope slipping is $T'_{\text{max}} = Te^{\mu_s \Delta\theta}$, where μ_s is the coefficient of static friction. Consider the Atwood's machine in Figure 9-77: the pulley has a radius $R = 0.15$ m, the moment of inertia is $I = 0.35\ \text{kg}\cdot\text{m}^2$, and the coefficient of static friction between the wheel and the string is $\mu_s = 0.30$. (*a*) If the tension on one side of the pulley is 10 N, what is the maximum tension on the other side that will prevent the rope from slipping on the pulley? (*b*) What is the acceleration of the blocks in this case? (c) If the mass of one of the hanging blocks is 1.0 kg, what is the maximum mass of the other block if, after the blocks are released, the pulley is to rotate without slipping?

Picture the Problem Free-body diagrams for the pulley and the two blocks are shown to the right. Choose a coordinate system in which the direction of motion of the block whose mass is M (downward) is the positive y direction. We can use the given relationship $T'_{\text{max}} = Te^{\mu_s \Delta\theta}$ to relate the tensions in the rope on either side of the pulley and apply Newton's 2nd law in both rotational form (to the pulley) and translational form (to the blocks) to obtain a system of equations that we can solve simultaneously for a, T_1, T_2, and M.



(*a*) Use $T'_{\text{max}} = Te^{\mu_s \Delta\theta}$ to evaluate the maximum tension required to prevent the rope from slipping on the pulley:

$$T'_{\text{max}} = (10\ \text{N})e^{(0.30)\pi} = 25.66\ \text{N} = \boxed{26\ \text{N}}$$

(*b*) Given that the angle of wrap is π radians, express T_2 in terms of T_1:

$$T_2 = T_1 e^{(0.30)\pi} \qquad (1)$$

Because the rope doesn't slip, we can relate the angular acceleration, α, of the pulley to the acceleration, a, of the hanging masses by:

$$\alpha = \frac{a}{R}$$

Apply $\sum \tau = I\alpha$ to the pulley to obtain:

$$(T_2 - T_1)R = I\frac{a}{R} \qquad (2)$$

Substitute for T_2 from equation (1) in equation (2) to obtain:

$$\left(T_1 e^{(0.30)\pi} - T_1\right)R = I\frac{a}{R}$$

Solving for T_1 yields:

$$T_1 = \frac{I}{\left(e^{(0.30)\pi} - 1\right)R^2}a$$

Apply $\sum F_y = ma_y$ to the block whose mass is m to obtain:

$$T_1 - mg = ma$$

and

$$T_1 = ma + mg \qquad (3)$$

Equating these two expressions for T_1 and solving for a yields:

$$a = \frac{g}{\dfrac{I}{\left(e^{(0.30)\pi} - 1\right)mR^2} - 1}$$

Substitute numerical values and evaluate a:

$$a = \frac{9.81\,\text{m/s}^2}{\dfrac{0.35\,\text{kg}\cdot\text{m}^2}{\left(e^{(0.30)\pi} - 1\right)(1.0\,\text{kg})(0.15\,\text{m})^2} - 1}$$

$$= 1.098\,\text{m/s}^2$$

$$= \boxed{1.1\,\text{m/s}^2}$$

(*c*) Apply $\sum F_y = ma_y$ to the block whose mass is M to obtain:

$$Mg - T_2 = Ma \Rightarrow M = \frac{T_2}{g - a}$$

Substitute for T_2 (from equation (1)) and T1 (from equation (3)) yields:

$$M = \frac{m(a + g)e^{(0.30)\pi}}{g - a}$$

Substitute numerical values and evaluate M:

$$M = \frac{(1.0\,\text{kg})\left(1.098\,\text{m/s}^2 + 9.81\,\text{m/s}^2\right)e^{(0.30)\pi}}{9.81\,\text{m/s}^2 - 1.098\,\text{m/s}^2} = \boxed{3.2\,\text{kg}}$$

128 ••• A massive, uniform cylinder has a mass m and a radius R (Figure 9-77). It is accelerated by a tension force $\vec{T}$ that is applied through a rope wound around a light drum of radius r that is attached to the cylinder. The coefficient of static friction is sufficient for the cylinder to roll without slipping. (*a*) Find the frictional force. (*b*) Find the acceleration a of the center of the cylinder. (*c*) Show that it is possible to choose r so that a is greater than T/m. (*d*) What is the direction of the frictional force in the circumstances of Part (*c*)?

Picture the Problem When the tension is horizontal, the cylinder will roll forward and the friction force will be in the direction of $\vec{T}$. We can use Newton's 2nd law to obtain equations that we can solve simultaneously for a and f.

(*a*) Apply Newton's 2nd law to the cylinder:

$$\sum F_x = T + f = ma \qquad (1)$$

and

$$\sum \tau = Tr - fR = I\alpha \qquad (2)$$

Substitute for I and α in equation (2) to obtain:

$$Tr - fR = \tfrac{1}{2}mR^2\frac{a}{R} = \tfrac{1}{2}mRa \qquad (3)$$

Solve equation (3) for f:

$$f = \frac{Tr}{R} - \tfrac{1}{2}ma \qquad (4)$$

Substitute equation (4) in equation (1) to obtain:

$$T + \frac{Tr}{R} - \tfrac{1}{2}ma = ma$$

Solving for a yields:

$$a = \frac{2T}{3m}\left(1 + \frac{r}{R}\right) \qquad (5)$$

Substitute equation (5) in equation (4) to obtain:

$$f = \frac{Tr}{R} - \frac{1}{2}m\left[\frac{2T}{3m}\left(1 + \frac{r}{R}\right)\right]$$

Simplifying yields:

$$f = \boxed{\frac{T}{3}\left(\frac{2r}{R} - 1\right)}$$

(*b*) Solve equation (4) for a:

$$a = \frac{2\left(f - \frac{Tr}{R}\right)}{m}$$

Substituting for *f* yields:

$$a=\frac{2\left(\frac{T}{3}\left(\frac{2r}{R}-1\right)-\frac{Tr}{R}\right)}{m}=\boxed{-\frac{2T}{3m}\left(1+\frac{r}{R}\right)}$$

(*c*) Express the condition that $a>\frac{T}{m}$:

$$\frac{2T}{3m}\left(1+\frac{r}{R}\right)>\frac{T}{m}\Rightarrow\frac{2}{3}\left(1+\frac{r}{R}\right)>1$$

or

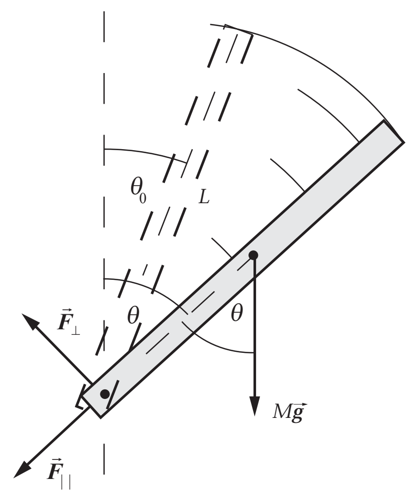
$$r>\boxed{\tfrac{1}{2}R}$$

(*d*) If $r>\frac{1}{2}R$:

$$\boxed{f>0\text{, i.e., in the direction of }\vec{T}.}$$

129 ••• A uniform rod that has a length L and a mass M is free to rotate about a horizontal axis through one end, as shown in Figure 9-78. The rod is released from rest at $\theta=\theta_0$. Show that the parallel and perpendicular components of the force exerted by the axis on the rod are given by $F_{\mathrm{P}}=\frac{1}{2}Mg(5\cos\theta-3\cos\theta_0)$ and $F_{\perp}=\frac{1}{4}Mg\sin\theta$, where F_{P} is the component parallel with the rod and $F_{\perp}$ is the component perpendicular to the rod.

Picture the Problem The system is shown in the drawing in two positions, with angles θ_0 and θ with the vertical. The drawing also shows all the forces that act on the stick. These forces result in a rotation of the stick—and its center of mass—about the pivot, and a tangential acceleration of the center of mass. We'll apply the conservation of mechanical energy and Newton's 2nd law to relate the radial and tangential forces acting on the stick.



Apply conservation of mechanical energy to relate the kinetic energy of the stick when it makes an angle θ with the vertical and its initial potential energy:

$$K_{\mathrm{f}}-K_{\mathrm{i}}+U_{\mathrm{f}}-U_{\mathrm{i}}=0$$

or, because $K_{\mathrm{i}}=0$,

$$-\tfrac{1}{2}I\omega^2+Mg\frac{L}{2}\cos\theta-Mg\frac{L}{2}\cos\theta_0=0$$

Substitute for I to obtain:

$$-\tfrac{1}{2}\left(\tfrac{1}{3}ML^2\right)\omega^2 + Mg\frac{L}{2}\cos\theta - Mg\frac{L}{2}\cos\theta_0 = 0$$

Solving for ω^2 yields:

$$\omega^2 = \frac{3g}{L}\left(\cos\theta - \cos\theta_0\right)$$

Express the centripetal force acting on the center of mass:

$$F_c = M\frac{L}{2}\omega^2$$

Substitute for ω^2 and simplify to obtain:

$$F_c = M\frac{L}{2}\left[\frac{3g}{L}\left(\cos\theta - \cos\theta_0\right)\right] = \frac{3Mg}{2}\left(\cos\theta - \cos\theta_0\right)$$

Express the radial component of $M\vec{g}$:

$$\left(Mg\right)_{\text{radial}} = Mg\cos\theta$$

The total radial force at the hinge is:

$$F_{\parallel} = F_c + \left(Mg\right)_{\text{radial}}$$

Substitute for F_c and $(Mg)_{\text{radial}}$ and simplify to obtain:

$$F_{\parallel} = \frac{3Mg}{2}\left(\cos\theta - \cos\theta_0\right) + Mg\cos\theta = \boxed{\tfrac{1}{2}Mg\left(5\cos\theta - 3\cos\theta_0\right)}$$

Relate the tangential acceleration of the center of mass to its angular acceleration:

$$a_{\perp} = \tfrac{1}{2}L\alpha$$

Use Newton's 2nd law to relate the angular acceleration of the stick to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Mg\frac{L}{2}\sin\theta}{\frac{1}{3}ML^2} = \frac{3g\sin\theta}{2L}$$

Express $a_{\perp}$ in terms of α:⊥

$$\boldsymbol{a}_{\perp} = \tfrac{1}{2}\boldsymbol{L\alpha} = \tfrac{3}{4}\boldsymbol{g}\sin\boldsymbol{\theta} = \boldsymbol{g}\sin\boldsymbol{\theta} + \frac{\boldsymbol{F}_{\perp}}{\boldsymbol{M}}$$

Solving for $F_\perp$ yields:

$F_\perp = \boxed{-\frac{1}{4}\boldsymbol{Mg}\sin\boldsymbol{\theta}}$ where the minus sign indicates that the force is directed oppositely to the tangential component of $M\vec{\boldsymbol{g}}$.