Chapter 8
Conservation of Linear Momentum

Conceptual Problems

1 • **[SSM]** Show that if two particles have equal kinetic energies, the magnitudes of their momenta are equal only if they have the same mass.

Determine the Concept The kinetic energy of a particle, as a function of its momentum, is given by $K = p^2/2m$.

The kinetic energy of the particles is given by:

$$K_1 = \frac{p_1^2}{2m_1} \text{ and } K_2 = \frac{p_2^2}{2m_2}$$

Equate these kinetic energies to obtain:

$$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2}$$

Because the magnitudes of their momenta are equal:

$$\frac{1}{m_1} = \frac{1}{m_2} \text{ and } m_1 = \boxed{m_2}$$

2 • Particle A has twice the (magnitude) momentum and four times the kinetic energy of particle B. A also has four times the kinetic energy of B. What is the ratio of their masses (the mass of particle A to that of particle B)? Explain your reasoning.

Determine the Concept The kinetic energy of a particle, as a function of its momentum, is given by $K = p^2/2m$.

The kinetic energy of particle A is given by:

$$K_A = \frac{p_A^2}{2m_A} \Rightarrow m_A = \frac{p_A^2}{2K_A}$$

The kinetic energy of particle B is given by:

$$K_B = \frac{p_B^2}{2m_B} \Rightarrow m_B = \frac{p_B^2}{2K_B}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{m_A}{m_B} = \frac{\dfrac{p_A^2}{2K_A}}{\dfrac{p_B^2}{2K_B}} = \frac{K_B p_A^2}{K_A p_B^2} = \frac{K_B}{K_A}\left(\frac{p_A}{p_B}\right)^2$$

Because particle A has twice the (magnitude) momentum of particle B and four times as much kinetic energy:

$$\frac{\boldsymbol{m}_\text{A}}{\boldsymbol{m}_\text{B}}=\frac{\boldsymbol{K}_\text{B}}{4\boldsymbol{K}_\text{B}}\left(\frac{2\boldsymbol{p}_\text{B}}{\boldsymbol{p}_\text{B}}\right)^2=\boxed{1}$$

3 • Using SI units, show that the units of momentum squared divided by those of mass is equivalent to the joule.

Determine the Concept The SI units of momentum are kg·m/s.

Express the ratio of the square of the units of momentum to the units of mass:

$$\frac{\left(\text{kg}\cdot\dfrac{\text{m}}{\text{s}}\right)^2}{\text{kg}}$$

Simplify to obtain:

$$\frac{\left(\text{kg}\cdot\dfrac{\text{m}}{\text{s}}\right)^2}{\text{kg}}=\frac{\text{kg}^2\cdot\dfrac{\text{m}^2}{\text{s}^2}}{\text{kg}}=\text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}=\left(\text{kg}\cdot\frac{\text{m}}{\text{s}^2}\right)\cdot\text{m}=\text{N}\cdot\text{m}=\boxed{\text{J}}$$

4 • True or false:

(*a*) The momentum of a 1.00-kg object is greater than that of a 0.25-kg object moving at the same speed.
(*b*) The total linear momentum of a system may be conserved even when the mechanical energy of the system is not.
(*c*) For the total linear momentum of a system to be conserved, there must be no external forces acting on the system.
(*d*) The velocity of the center of mass of a system changes only when there is a net external force on the system.

(*a*) True. The momentum of an object is the product of its mass and velocity. Therefore, if we are considering just the magnitudes of the momenta, the momentum of a heavy object is greater than that of a light object moving at the same speed.

(*b*) True. Consider the collision of two objects of equal mass traveling in opposite directions with the same speed. Assume that they collide inelastically. The mechanical energy of the system is not conserved (it is transformed into other forms of energy), but the momentum of the system is the same after the collision as before the collision; that is, zero. Therefore, for any inelastic collision, the momentum of a system may be conserved even when mechanical energy is not.

(*c*) False. The *net* external force must be zero if the linear momentum of the system is to be conserved.

(*d*) True. This non-zero net force accelerates the center of mass. Hence its velocity changes.

5 • If a bullet is fired due west, explain how conservation of linear momentum enables you to predict that the recoil of the rifle be exactly due east. Is kinetic energy conserved here?

Determine the Concept The momentum of the bullet-gun system is initially zero. After firing, the bullet's momentum is directed west. Momentum conservation requires that the system's total momentum does not change, so the gun's momentum must be directed east.

6 • A child jumps from a small boat to a dock. Why does she have to jump with more effort than she would need if she were jumping through an identical displacement, but from a boulder to a tree stump?

Determine the Concept When she jumps from a boat to a dock, she must, in order for momentum to be conserved, give the boat a recoil momentum, i.e., her forward momentum must be the same as the boat's backward momentum. When she jumps through an identical displacement from a boulder to a tree stump, the mass of the boulder plus the Earth is so large that the momentum she imparts to them is essentially zero.

7 •• **[SSM]** Much early research in rocket motion was done by Robert Goddard, physics professor at Clark College in Worcester, Massachusetts. A quotation from a 1920 editorial in the *New York Times* illustrates the public opinion of his work: ″That Professor Goddard with his ′chair′ at Clark College and the countenance of the Smithsonian Institution does not know the relation between action and reaction, and the need to have something better than a vacuum against which to react—to say that would be absurd. Of course, he only seems to lack the knowledge ladled out daily in high schools.″ The belief that a rocket needs something to push against was a prevalent misconception before rockets in space were commonplace. Explain why that belief is wrong.

Determine the Concept In a way, the rocket does need something to push upon. It pushes the exhaust in one direction, and the exhaust pushes it in the opposite direction. However, the rocket does not push against the air.

8 • Two identical bowling balls are moving with the same center-of-mass velocity, but one just slides down the alley without rotating, whereas the other rolls down the alley. Which ball has more kinetic energy? Which one has more total momentum (magnitude)? Because of the relationship between kinetic energy and momentum of a particle ($\boldsymbol{K} = \boldsymbol{p}^2/2\boldsymbol{m}$), it would seem there is something wrong with your answer. Explain why there is nothing wrong with your answer.

Determine the Concept The kinetic energy of the sliding ball is $\frac{1}{2}mv_{\text{cm}}^2$. The kinetic energy of the rolling ball is $\frac{1}{2}mv_{\text{cm}}^2 + K_{\text{rel}}$, where K_{rel} is its kinetic energy relative to its center of mass. Because the bowling balls are identical and have the same velocity, the rolling ball has more energy. There is no problem here because the relationship $\boldsymbol{K} = \boldsymbol{p}^2/2\boldsymbol{m}$ is between the center of mass kinetic energy of the ball and its linear momentum.

9 • A philosopher tells you, ″Changing motion of objects is impossible. Forces always come in equal but pairs. Therefore, all forces cancel out. Since forces cancel, the momenta of objects can never be changed.″ Answer his argument.

Determine the Concept Think of someone pushing a box across a floor. Her push on the box is equal but opposite to the push of the box on her, but the action and reaction forces act on *different objects*. Newton’s second law is that the sum of the forces acting on the box equals the rate of change of momentum of the box. This sum does not include the force of the box on her.

10 • A moving objects collides with an arbitrary. Is it possible for both objects to be at rest immediately after the collision? (Assume any external forces acting on this two-object system are negligibly small.) Is it possible for one object to be at rest immediately after the collision? Explain.

Determine the Concept It’s not possible for both to remain at rest after the collision, as that wouldn't satisfy the requirement that momentum is conserved. It is possible for one to remain at rest: This is what happens for a one-dimensional collision of two identical particles colliding elastically.

11 • Several researchers in physics education claim that part of the cause of physical misconceptions amongst students comes from special effects they observe in cartoons and movies. Using the conservation of linear momentum, how would you explain to a class of high school physics students what is conceptually wrong with a superhero hovering at rest in midair while tossing massive objects such as cars at villains? Does this action violate conservation of energy as well? Explain.

Determine the Concept Hovering in midair while tossing objects violates the conservation of linear momentum! To throw something forward requires being pushed backward. Superheroes are not depicted as experiencing this backward motion that is predicted by conservation of linear momentum. This action also violates conservation of energy in that, with no change in the superheroes potential or kinetic energy resulting from the tossing of objects, the mechanical energy of the hero-object-Earth system is greater after the toss than it was before the toss.

12 •• A struggling physics student asks ″If only external forces can cause the center of mass of a system of particles to accelerate, how can a car move? Doesn't the car's engine supply the force needed to accelerate the car? ″ Explain what external agent produces the force that accelerates the car, and explain how the engine makes that agent do so.

Determine the Concept There is only one force which can cause the car to move forward–the friction of the road! The car's engine causes the tires to rotate, but if the road were frictionless (as is closely approximated by icy conditions) the wheels would simply spin without the car moving anywhere. Because of friction, the car's tire pushes backwards against the road and the frictional force acting on the tire pushes it forward. This may seem odd, as we tend to think of friction as being a retarding force only, but it is true.

13 •• When we push on the brake pedal to slow down a car, a brake pad is pressed against the rotor so that the friction of the pad slows the rotor's, and thus the wheel's rotation. However, the friction of the pad against the rotor can't be the force that slows the car down, because it is an internal force—both the rotor and the wheel are parts of the car, so any forces between them are internal, not external, forces. What external agent exerts the force that slows down the car? Give a detailed explanation of how this force operates.

Determine the Concept The frictional force by the road on the tire causes the car to slow. Normally the wheel is rotating at just the right speed so both the road and the tread in contact with the road are moving backward at the same speed relative to the car. By stepping on the brake pedal, you slow the rotation rate of the wheel. The tread in contact with the road is no longer moving as fast, relative to the car, as the road. To oppose the tendency to skid, the tread exerts a forward frictional force on the road and the road exerts an equal and opposite force on the tread.

14 • Explain why a circus performer falling into a safety net can survive unharmed, while a circus performer falling from the same height onto the hard concrete floor suffers serious injury or death. Base your explanation on the impulse-momentum theorem.

Determine the Concept Because $\Delta p = F\Delta t$ is constant, the safety net reduces the force acting on the performer by increasing the time Δt during which the slowing force acts.

15 •• **[SSM]** In Problem 14 for the performer falling from a height of 25 m, estimate the ratio of the collision time with the safety net to the collision time with the concrete. *Hint: Use the procedure outlined in Step 4 of the Problem-Solving Strategy located in Section 8-3.*

Determine the Concept The stopping time for the performer is the ratio of the distance traveled during stopping to the average speed during stopping.

Letting d_{net} be the distance the net gives on impact, $d_{concrete}$ the distance the concrete gives, and $v_{av,\text{ with net}}$ and $v_{av,\text{without net}}$ the average speeds during stopping, express the ratio of the impact times:

$$r = \frac{\Delta t_{net}}{\Delta t_{concrete}} = \frac{\dfrac{d_{net}}{v_{av,\text{ with net}}}}{\dfrac{d_{concrete}}{v_{av,\text{ without net}}}} \qquad (1)$$

Assuming constant acceleration, the average speed of the performer during stopping is given by:

$$v_{av} = \frac{v_f + v}{2}$$

or, because $v_f = 0$ in both cases,

$$v_{av} = \tfrac{1}{2}v$$

where v is the impact speed.

Substituting in equation (1) and simplifying yields:

$$r = \frac{\dfrac{d_{net}}{\frac{1}{2}v}}{\dfrac{d_{concrete}}{\frac{1}{2}v}} = \frac{d_{net}}{d_{concrete}}$$

Assuming that the net gives about 1 m and concrete about 0.1 mm yields:

$$r = \frac{1\text{ m}}{0.1\text{ mm}} \approx \boxed{10^4}$$

16 •• (*a*) Why does a drinking glass survive a fall onto a carpet but not onto a concrete floor? (*b*) On many race tracks, dangerous curves are surrounded by massive bails of hay. Explain how this setup reduces the chances of car damage and driver injury.

Determine the Concept In both (*a*) and (*b*), longer impulse times (Impulse = $F_{av}\Delta t$) are the result of collisions with a carpet and bails of hay. The average force on a drinking glass or a car is reduced (nothing can be done about the impulse, or change in linear momentum, during a collision but increasing the impulse time decreases the average force acting on an object) and the likelihood of breakage, damage or injury is reduced.

17 • True or false:

(*a*) Following any perfectly inelastic collision, the kinetic energy of the system is zero after the collision in all inertial reference frames.
(*b*) For a head-on elastic collision, the relative speed of recession equals the relative speed of approach.
(*c*) During a perfectly inelastic head-on collision with one object initially at rest, only some of the system's kinetic energy is dissipated.
(*d*) After a perfectly inelastic head-on collision along the east-west direction, the two objects are observed to be moving west. The initial total system momentum was, therefore, to the west.

(*a*) False. Following a perfectly inelastic collision, the colliding bodies stick together but may or may not continue moving, depending on the momentum each brings to the collision.

(*b*) True. For a head-on elastic collision both kinetic energy and momentum are conserved and the relative speeds of approach and recession are equal.

(*c*) True. This is the definition of an inelastic collision.

(*d*) True. The linear momentum of the system before the collision must be in the same direction as the linear momentum of the system after the collision.

18 •• Under what conditions can all the initial kinetic energy of an isolated system consisting of two colliding objects be lost in a collision? Explain how this result can be, and yet the momentum of the system can be conserved.

Determine the Concept If the collision is perfectly inelastic, the objects stick together and neither will be moving after the collision. Therefore the final kinetic energy will be zero and all of it will have been lost (that is, transformed into some other form of energy). Momentum is conserved because in an isolated system the net external force is zero.

19 •• Consider a perfectly inelastic collision of two objects of equal mass. (*a*) Is the loss of kinetic energy greater if the two objects are moving in opposite directions, each moving with at speed $v/2$, or if one of the two objects is initially at rest and the other has an initial speed of v? (*b*) In which of these situations is the percentage loss in kinetic energy the greatest?

Determine the Concept We can find the loss of kinetic energy in these two collisions by finding the initial and final kinetic energies. We'll use conservation of momentum to find the final velocities of the two masses in each perfectly elastic collision.

(*a*) Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv - mv = 2mV \Rightarrow V = 0$$

Express the loss of kinetic energy for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\Delta \boldsymbol{K} = \boldsymbol{K}_{\text{f}} - \boldsymbol{K}_{\text{i}} = 0 - 2\left(\tfrac{1}{2}\boldsymbol{m}\left(\frac{\boldsymbol{v}}{2}\right)^2\right)$$

$$= -\tfrac{1}{4}\boldsymbol{mv}^2$$

Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv = 2mV \Rightarrow V = \tfrac{1}{2}v$$

Express the loss of kinetic energy for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\Delta \boldsymbol{K} = \boldsymbol{K}_{\text{f}} - \boldsymbol{K}_{\text{i}}$$

$$= \tfrac{1}{2}(2\boldsymbol{m})\left(\frac{\boldsymbol{v}}{2}\right)^2 - \tfrac{1}{2}\boldsymbol{mv}^2 = -\tfrac{1}{4}\boldsymbol{mv}^2$$

The loss of kinetic energy is the same in both cases.

(*b*) Express the percentage loss for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\frac{\Delta K}{K_{\text{before}}} = \frac{\tfrac{1}{4}mv^2}{\tfrac{1}{4}mv^2} = 100\%$$

Express the percentage loss for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\frac{\Delta K}{K_{\text{before}}} = \frac{\tfrac{1}{4}mv^2}{\tfrac{1}{2}mv^2} = 50\%$$

The percentage loss is greatest for the case in which the two objects have oppositely directed velocities of magnitude *v*/2.

20 •• A double-barreled pea shooter is shown in Figure 8-41. Air is blown into the left end of the pea shooter, and identical peas A and B are positioned inside the straw as shown. If the shooter is held horizontally while the peas are shot off, which pea, A or B, will travel farther after leaving the straw? Explain. Base your explanation on the impulse–momentum theorem.

Determine the Concept A will travel farther. Both peas are acted on by the same force, but pea A is acted on by that force for a longer time. By the impulse-momentum theorem, its momentum (and, hence, speed) will be higher than pea B's speed on leaving the shooter.

21 •• A particle of mass m_1 traveling with a speed v makes a head-on elastic collision with a stationary particle of mass m_2. In which scenario will the largest amount of energy be imparted to the particle of mass m_2? (*a*) $m_2 << m_1$, (*b*) $m_2 = m_1$, (*c*) $m_2 >> m_1$, (*d*) None of the above.

Determine the Concept Refer to the particles as particle 1 and particle 2. Let the direction particle 1 is moving before the collision be the positive x direction. We'll use both conservation of momentum and conservation of mechanical energy to obtain an expression for the velocity of particle 2 after the collision. Finally, we'll examine the ratio of the final kinetic energy of particle 2 to that of particle 1 to determine the condition under which there is maximum energy transfer from particle 1 to particle 2.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_1 v_{1,\mathrm{i}} = m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} \quad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{2,\mathrm{f}} - v_{1,\mathrm{f}} = -\left(v_{2,\mathrm{i}} - v_{1,\mathrm{i}}\right) = v_{1,\mathrm{i}} \quad (2)$$

To eliminate $v_{1,\mathrm{f}}$, solve equation (2) for $v_{1,\mathrm{f}}$, and substitute the result in equation (1):

$$v_{1,\mathrm{f}} = v_{2,\mathrm{f}} + v_{1,\mathrm{i}}$$

$$m_1 v_{1,\mathrm{i}} = m_1\left(v_{2,\mathrm{f}} - v_{1,\mathrm{i}}\right) + m_2 v_{2,\mathrm{f}}$$

Solve for $v_{2,\mathrm{f}}$ to obtain:

$$v_{2,\mathrm{f}} = \frac{2m_1}{m_1 + m_2} v_{1,\mathrm{i}}$$

Express the ratio R of $K_{2,\mathrm{f}}$ to $K_{1,\mathrm{i}}$ in terms of m_1 and m_2:

$$R = \frac{K_{2,\mathrm{f}}}{K_{1,\mathrm{i}}} = \frac{\frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_{1,\mathrm{i}}^2}{\frac{1}{2} m_1 v_{1,\mathrm{i}}^2}$$

$$= \frac{m_2}{m_1} \frac{4m_1^2}{\left(m_1 + m_2\right)^2}$$

Differentiate this ratio with respect to m_2, set the derivative equal to zero, and obtain the quadratic equation:

$$-\frac{m_2^2}{m_1^2}+1=0$$

Solve this equation for m_2 to determine its value for maximum energy transfer:

$$m_2 = m_1$$

$\boxed{(b)}$ is correct because all of particle 1's kinetic energy is transferred to particle 2 when $\boldsymbol{m}_2 = \boldsymbol{m}_1$.

22 •• Suppose you are in charge of an accident-reconstruction team which has reconstructed an accident in which a car was ″rear-ended″ and the two cars locked bumpers and skidded to a halt. During the trial, you are on the stand as an expert witness for the prosecution and the defense lawyer claims that you wrongly neglected friction and the force of gravity during the fraction of a second while the cars collided. Defend your report. Why were you correct in ignoring these forces? You did not ignore these two forces in your skid analysis both before and after the collision. Can you explain to the jury why you did not ignore these two forces during the pre- and post-collision skids?

Determine the Concept You only used conservation of linear momentum for the few fractions of a second of actual contact between the cars. Over that short time, friction and other external forces can be neglected. In the long run, over the duration of the accident, they cannot.

23 •• Nozzles for a garden hose are often made with a right-angle shape as shown in Figure 8-41. If you open the nozzle and spray water out, you will find that the nozzle presses against your hand with a pretty strong force—much stronger than if you used a nozzle not bent into a right angle. Why is this situation true?

Determine the Concept The water is changing direction when it rounds the corner in the nozzle. Therefore, the nozzle must exert a force on the stream of water to change its momentum, and this requires a net force in the direction of the momentum change.

Conceptual Problems from Optional Sections

24 •• Describe a perfectly inelastic head-on collision between two stunt cars as viewed in the center-of-mass reference frame.

Determine the Concept In the center-of-mass reference frame the two objects approach with equal but opposite momenta and remain at rest after the collision.

25 •• One air-hockey puck is initially at rest. An identical air-hockey puck collides with it, striking it with a glancing blow. Assume the collision was elastic and neglect any rotational motion of the pucks. Describe the collision in the center-of-mass frame of the pucks.

Determine the Concept Before the collision, the center-of-mass is midway between the two pucks and continues on a straight line throughout the interaction between the pucks. As viewed from the center of mass, the two pucks approach each other and then recede in a different direction, but with the same relative speed before and after the collision.

26 •• A baton with one end more massive than the other is tossed at an angle into the air. (*a*) Describe the trajectory of the center of mass of the baton in the reference frame of the ground. (*b*) Describe the motion of the two ends of the baton in the center-of-mass frame of the baton.

Determine the Concept
(*a*) In the center-of-mass frame of the ground, the center of mass moves in a parabolic arc.

(*b*) Relative to the center of mass, each end of the baton would describe a circular path. The more massive end of the baton would travel in the circle with the smaller radius because it is closer to the location of the center of mass.

27 •• Describe the forces acting on a descending Lunar lander as it fires its retrorockets to slow it down for a safe landing. (Assume its mass loss during the rocket firing is not negligible.)

Determine the Concept The forces acting on a descending Lunar lander are the downward force of lunar gravity and the upward thrust provided by the rocket engines.

28 •• A railroad car rolling along by itself is passing by a grain elevator, which is dumping grain into it at a constant rate. (*a*) Does momentum conservation imply that the railroad car should be slowing down as it passes the grain elevator? Assume that the track is frictionless and perfectly level and that the grain is falling vertically. (*b*) If the car is slowing down, this situation implies that there is some external force acting on the car to slow it down. Where does this force come from? (*c*) After passing the elevator, the railroad car springs a leak, and grain starts leaking out of a vertical hole in its floor at a constant rate. Should the car speed up as it loses mass?

Determine the Concept We can apply conservation of linear momentum and Newton's laws of motion to each of these scenarios.

(*a*) Yes, the car should slow down. An easy way of seeing this is to imagine a "packet" of grain being dumped into the car all at once: This is a completely inelastic collision, with the packet having an initial horizontal velocity of 0. After the collision, it is moving with the same horizontal velocity that the car does, so the car must slow down.

(*b*) When the packet of grain lands in the car, it initially has a horizontal velocity of 0, so it must be accelerated to come to the same speed as the car of the train. Therefore, the train must exert a force on it to accelerate it. By Newton's 3rd law, the grain exerts an equal but opposite force on the car, slowing it down. In general, this is a frictional force which causes the grain to come to the same speed as the car.

(*c*) No it doesn't speed up. Imagine a packet of grain being "dumped" out of the railroad car. This can be treated as a collision, too. It has the same horizontal speed as the railroad car when it leaks out, so the train car doesn't have to speed up or slow down to conserve momentum.

29 ••• **[SSM]** To show that even really intelligent people can make mistakes, consider the following problem which was asked of a freshman class at Caltech on an exam (paraphrased): *A sailboat is sitting in the water on a windless day. In order to make the boat move, a misguided sailor sets up a fan in the back of the boat to blow into the sails to make the boat move forward. Explain why the boat won't move.* The idea was that the net force of the wind pushing the sail forward would be counteracted by the force pushing the fan back (Newton's third law). However, as one of the students pointed out to his professor, the sailboat *could* in fact move forward. Why is that?

Determine the Concept Think of the sail facing the fan (like the sail on a square rigger might), and think of the stream of air molecules hitting the sail. Imagine that they bounce off the sail elastically−their net change in momentum is then roughly twice the change in momentum that they experienced going through the fan. Thus the change in momentum of the air is backward, so to conserve momentum of the air-fan-boat system the change in momentum of the fan-boat system will be forward.

Estimation and Approximation

30 •• A 2000-kg car traveling at 90 km/h crashes into an immovable concrete wall. (*a*) Estimate the time of collision, assuming that the center of the car travels halfway to the wall with constant acceleration. (Use any plausible length for the car.) (*b*) Estimate the average force exerted by the wall on the car.

Picture the Problem We can estimate the time of collision from the average speed of the car and the distance traveled by the center of the car during the collision. We'll assume a car length of 6.0 m. We can calculate the average force exerted by the wall on the car from the car's change in momentum and its stopping time.

(*a*) Relate the stopping time to the assumption that the center of the car travels halfway to the wall with constant deceleration:

$$\Delta t=\frac{d_{\text{stopping}}}{v_{\text{av}}}=\frac{\frac{1}{2}\left(\frac{1}{2}L_{\text{car}}\right)}{v_{\text{av}}}=\frac{\frac{1}{4}L_{\text{car}}}{v_{\text{av}}}\quad(1)$$

Because *a* is constant, the average speed of the car is given by:

$$\boldsymbol{v}_{\text{av}}=\frac{\boldsymbol{v}_{\text{i}}+\boldsymbol{v}_{\text{f}}}{2}$$

Substitute numerical values and evaluate v_{av}:

$$\boldsymbol{v}_{\text{av}}=\frac{0+90\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}}}{2}=12.5\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate Δt:

$$\Delta \boldsymbol{t}=\frac{\frac{1}{4}(6.0\,\text{m})}{12.5\,\text{m/s}}=0.120\,\text{s}=\boxed{0.12\,\text{s}}$$

(*b*) Relate the average force exerted by the wall on the car to the car's change in momentum:

$$\boldsymbol{F}_{\text{av}}=\frac{\Delta\boldsymbol{p}}{\Delta\boldsymbol{t}}=\frac{(2000\,\text{kg})\left(90\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}}\right)}{0.120\,\text{s}}=\boxed{4.2\times10^{5}\,\text{N}}$$

31 •• During hand-pumped railcar races, a speed of 32.0 km/h has been achieved by teams of four people. A car that has a mass equal to 350 kg is moving at that speed toward a river when Carlos, the chief pumper, notices that the bridge ahead is out. All four people (each with a mass of 75.0 kg) simultaneously jump backward off the car with a velocity that has a horizontal component of 4.00 m/s relative to the car. The car proceeds off the bank and falls into the water a horizontal distance of 25.0 m from the bank. (*a*) Estimate the time of the fall of the railcar. (*b*) What is the horizontal component of the velocity of the pumpers when they hit the ground?

Picture the Problem Let the direction the railcar is moving be the positive *x* direction and the system include the earth, the pumpers, and the railcar. We'll also denote the railcar with the letter c and the pumpers with the letter p. We'll use conservation of linear momentum to relate the center of mass frame velocities of the car and the pumpers and then transform to the earth frame of reference to find the time of fall of the car.

(*a*) Relate the time of fall of the railcar to the distance it falls and its velocity as it leaves the bank:

$$\Delta t = \frac{\Delta y}{v_c} \quad (1)$$

Use conservation of momentum to find the speed of the car relative to the velocity of its center of mass:

$$\vec{p}_i = \vec{p}_f$$
or
$$m_c u_c + m_p u_p = 0$$

Relate u_c to u_p and solve for u_c:

$$\boldsymbol{u}_c - \boldsymbol{u}_p = 4.00\,\text{m/s}$$
and
$$\boldsymbol{u}_p = \boldsymbol{u}_c - 4.00\,\text{m/s}$$

Substitute for u_p to obtain:

$$\boldsymbol{m}_c\boldsymbol{u}_c + \boldsymbol{m}_p(\boldsymbol{u}_c - 4.00\,\text{m/s}) = 0$$

Solving for u_c yields:

$$\boldsymbol{u}_c = \frac{4.00\,\text{m/s}}{1 + \dfrac{\boldsymbol{m}_c}{\boldsymbol{m}_p}}$$

Substitute numerical values and evaluate u_c:

$$\boldsymbol{u}_c = \frac{4.00\,\text{m/s}}{1 + \dfrac{350\,\text{kg}}{4(75.0\,\text{kg})}} = 1.87\,\text{m/s}$$

Relate the speed of the car to its speed relative to the center of mass of the system:

$$\boldsymbol{v}_c = \boldsymbol{u}_c + \boldsymbol{v}_{cm}$$

Substitute numerical values and evaluate v_c:

$$\boldsymbol{v}_c = 1.87\,\frac{\text{m}}{\text{s}} + \left(32.0\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right) = 10.7\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{25.0\,\text{m}}{10.7\,\text{m/s}} = \boxed{2.34\,\text{s}}$$

(*b*) The horizontal velocity of the pumpers when they hit the ground is:

$$\boldsymbol{v}_\text{p} = \boldsymbol{v}_\text{c} - \boldsymbol{u}_\text{p} = 10.7\,\text{m/s} - 4.00\,\text{m/s}$$
$$= \boxed{6.7\,\text{m/s}}$$

32 •• A wooden block and a gun are firmly fixed to opposite ends of a long glider mounted on a frictionless air track (Figure 8-43). The block and gun are a distance L apart. The system is initially at rest. The gun is fired and the bullet leaves the gun with a velocity v_b and impacts the block, becoming imbedded in it. The mass of the bullet is m_b and the mass of the gun–glider–block system is m_p. (*a*) What is the velocity of the glider immediately after the bullet leaves the gun? (*b*) What is the velocity of the glider immediately after the bullet comes to rest in the block? (*c*) How far does the glider move while the bullet is in transit between the gun and the block?

Picture the Problem Let the system include the earth, platform, gun, bullet, and block. Then $\vec{\boldsymbol{F}}_\text{net,ext} = 0$ and momentum is conserved within the system. Choose a coordinate system in which the $+x$ direction is the direction of the bullet and let b and p denote the bullet and platform, respectively.

(*a*) Apply conservation of linear momentum to the system just before and just after the bullet leaves the gun:

$$\vec{p}_\text{before} = \vec{p}_\text{after}$$
Or
$$0 = \vec{p}_\text{bullet} + \vec{p}_\text{glider}$$

Substitute for $\vec{p}_\text{bullet}$ and $\vec{\boldsymbol{p}}_\text{glider}$ to obtain:

$$0 = m_\text{b} v_\text{b} \hat{i} + m_\text{p} \vec{v}_\text{p}$$

Solving for $\vec{\boldsymbol{v}}_\text{p}$ yields:

$$\vec{v}_\text{p} = \boxed{-\frac{m_\text{b}}{m_\text{p}} v_\text{b} \hat{i}}$$

(*b*) Apply conservation of momentum to the system just before the dart leaves the gun and just after it comes to rest in the block:

$$\vec{\boldsymbol{p}}_\text{before} = \vec{\boldsymbol{p}}_\text{after}$$
or
$$0 = \vec{\boldsymbol{p}}_\text{glider} \Rightarrow \vec{\boldsymbol{v}}_\text{glider} = \boxed{0}$$

(*c*) Express the distance Δs traveled by the glider:

$$\Delta \boldsymbol{s} = \boldsymbol{v}_\text{p} \Delta \boldsymbol{t}$$

Express the velocity of the bullet relative to the glider:

$$v_{\text{rel}} = v_{\text{b}} - v_{\text{p}} = v_{\text{b}} + \frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}$$

$$= \left(1 + \frac{m_{\text{b}}}{m_{\text{p}}}\right) v_{\text{b}} = \frac{m_{\text{p}} + m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}$$

Relate the time of flight Δt to L and v_{rel}:

$$\Delta t = \frac{L}{v_{\text{rel}}}$$

Substitute and simplify to find the distance Δs moved by the glider in time Δt:

$$\Delta s = v_{\text{p}} \Delta t = \left(\frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}\right)\left(\frac{L}{v_{\text{rel}}}\right) = \left(\frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}\right)\left(\frac{L}{\dfrac{m_{\text{p}} + m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}}\right) = \boxed{\frac{m_{\text{b}}}{m_{\text{p}} + m_{\text{b}}} L}$$

Conservation of Linear Momentum

33 • **[SSM]** Tyrone, an 85-kg teenager, runs off the end of a horizontal pier and lands on a free-floating 150-kg raft that was initially at rest. After he lands on the raft, the raft, with him on it, moves away from the pier at 2.0 m/s. What was Tyrone's speed as he ran off the end of the pier?

Picture the Problem Let the system include the raft, the earth, and Tyrone and apply conservation of linear momentum to find Tyrone's speed when he ran off the end of the pier.

Apply conservation of linear momentum to the system consisting of the raft and Tyrone to obtain:

$$\Delta \vec{p}_{\text{system}} = \Delta \vec{p}_{\text{Tyrone}} + \Delta \vec{p}_{\text{raft}} = 0$$

or, because the motion is one-dimensional,

$$\boldsymbol{p}_{\text{f,Tyrone}} - \boldsymbol{p}_{\text{i, Tyrone}} + \boldsymbol{p}_{\text{f,raft}} - \boldsymbol{p}_{\text{i,raft}} = 0$$

Because the raft is initially at rest:

$$\boldsymbol{p}_{\text{f,Tyrone}} - \boldsymbol{p}_{\text{i, Tyrone}} + \boldsymbol{p}_{\text{f,raft}} = 0$$

Use the definition of linear momentum to obtain:

$$\boldsymbol{m}_{\text{Tyrone}} \boldsymbol{v}_{\text{f,Tyrone}} - \boldsymbol{m}_{\text{Tyrone}} \boldsymbol{v}_{\text{i,Tyrone}} + \boldsymbol{m}_{\text{raft}} \boldsymbol{v}_{\text{f,raft}} = 0$$

Solve for $\boldsymbol{v}_{\text{i,Tyrone}}$ to obtain:

$$\boldsymbol{v}_{\text{i,Tyrone}} = \frac{\boldsymbol{m}_{\text{raft}}}{\boldsymbol{m}_{\text{Tyrone}}} \boldsymbol{v}_{\text{f,raft}} + \boldsymbol{v}_{\text{f,Tyrone}}$$

Letting *v* represent the common final speed of the raft and Tyrone yields:

$$\boldsymbol{v}_{\text{i,Tyrone}} = \left(1 + \frac{\boldsymbol{m}_{\text{raft}}}{\boldsymbol{m}_{\text{Tyrone}}}\right)\boldsymbol{v}$$

Substitute numerical values and evaluate $\boldsymbol{v}_{\text{i,Tyrone}}$:

$$\boldsymbol{v}_{\text{i, Tyrone}} = \left(1 + \frac{150\text{ kg}}{85\text{ kg}}\right)(2.0\text{ m/s}) = \boxed{5.5\text{ m/s}}$$

34 •• A 55-kg woman contestant on a reality television show is at rest at the south end of a horizontal 150-kg raft that is floating in crocodile-infested waters. She and the raft are initially at rest. She needs to jump from the raft to a platform that is several meters off the north end of the raft. She takes a running start. When she reaches the north end of the raft she is running at 5.0 m/s relative to the raft. At that instant, what is her velocity relative to the water?

Picture the Problem Let the system include the woman, the canoe, and the earth. Then the *net* external force is zero and linear momentum is conserved as she jumps off the canoe. Let the direction she jumps be the positive *x* direction.

Apply conservation of linear momentum to the system:

$$\sum \boldsymbol{m}_{\text{i}}\vec{\boldsymbol{v}}_{\text{i}} = \boldsymbol{m}_{\text{woman}}\vec{\boldsymbol{v}}_{\text{woman}} + \boldsymbol{m}_{\text{raft}}\vec{\boldsymbol{v}}_{\text{raft}} = 0$$

Solving for $\vec{\boldsymbol{v}}_{\text{raft}}$ yields:

$$\vec{\boldsymbol{v}}_{\text{raft}} = -\frac{\boldsymbol{m}_{\text{woman}}\vec{\boldsymbol{v}}_{\text{woman}}}{\boldsymbol{m}_{\text{raft}}}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{raft}}$:

$$\vec{\boldsymbol{v}}_{\text{raft}} = -\frac{(55\text{ kg})(5.0\text{ m/s})\hat{\boldsymbol{i}}}{150\text{ kg}} = \boxed{(-1.8\text{ m/s})\hat{\boldsymbol{i}}}$$

35 • A 5.0-kg object and a 10-kg object are connected by a massless compressed spring and rest on a frictionless table. The spring is released and the objects fly off in opposite directions. The 5.0-kg object has a velocity of 8.0 m/s to the left. What is the velocity of the 10-kg object?

Picture the Problem If we include the earth in our system, then the net external force is zero and linear momentum is conserved as the spring delivers its energy to the two objects. Choose a coordinate system in which the +*x* direction is to the right.

Apply conservation of linear momentum to the system:

$$\sum m_{\text{i}}\vec{\boldsymbol{v}}_{\text{i}} = m_5\vec{\boldsymbol{v}}_5 + m_{10}\vec{\boldsymbol{v}}_{10} = 0$$

Solving for $\vec{v}_{10}$ yields:

$$\vec{v}_{10} = -\frac{m_5\vec{v}_5}{m_{10}}$$

Substitute numerical values and evaluate $\vec{v}_{10}$:

$$\vec{v}_{10} = -\frac{(5.0\,\text{kg})(-8.0\,\text{m/s})\hat{i}}{10\,\text{kg}} = \boxed{(4.0\,\text{m/s})\hat{i}}$$

or 4.0 m/s to the right.

36 • Figure 8-44 shows the behavior of a projectile just after it has broken up into three pieces. What was the speed of the projectile the instant before it broke up? (*a*) v_3. (*b*) $v_3/3$. (*c*) $v_3/4$. (*d*) $4v_3$. (*e*) $(v_1 + v_2 + v_3)/4$.

Picture the Problem This is an explosion-like event in which linear momentum is conserved. Thus we can equate the initial and final momenta in the *x* direction and the initial and final momenta in the *y* direction. Choose a coordinate system in the +*x* direction is to the right and the +*y* direction is upward.

Equate the momenta in the *y* direction before and after the explosion:

$$\sum p_{y,i} = \sum p_{y,f} = mv_2 - 2mv_1 = m(2v_1) - 2mv_1 = 0$$

We can conclude that the momentum was entirely in the *x* direction before the particle exploded.

Equate the momenta in the *x* direction before and after the explosion:

$$\sum \boldsymbol{p}_{x,i} = \sum \boldsymbol{p}_{x,f}$$

or

$$4m\boldsymbol{v}_{\text{projectile}} = m\boldsymbol{v}_3$$

Solving for $v_{\text{projectile}}$ yields:

$v_{\text{projectile}} = \frac{1}{4}v_3$ and $\boxed{(c)}$ is correct.

37 • A shell of mass *m* and speed *v* explodes into two identical fragments. If the shell was moving horizontally with respect to Earth, and one of the fragments is subsequently moving vertically with speed *v*, find the velocity $\vec{v}'$ of the other fragment immediately following the explosion.

Picture the Problem Choose the direction the shell is moving just before the explosion to be the positive *x* direction and apply conservation of momentum.

Use conservation of momentum to relate the masses of the fragments to their velocities:

$$\vec{p}_i = \vec{p}_f$$

or

$$mv\hat{i} = \tfrac{1}{2}mv\hat{j} + \tfrac{1}{2}m\vec{v}' \Rightarrow \vec{v}' = \boxed{2v\hat{i} - v\hat{j}}$$

38 •• During this week's physics lab, the experimental setup consists of two gliders on a horizontal frictionless air track (see Figure 8-45). Each glider supports a strong magnet centered on top of it, and the magnets are oriented so they attract each other. The mass of glider 1 and its magnet is 0.100 kg and the mass of glider 2 and its magnet is 0.200 kg. You and your lab partners take the origin to be at the left end of the track and to center glider 1 at x_1 = 0.100 m and glider 2 at x_2 = 1.600 m. Glider 1 is 10.0 cm long, while glider 2 is 20.0 cm long and each glider has its center of mass at its geometric center. When the two are released from rest, they will move toward each other and stick. (*a*) Predict the position of the center of each glider when they first touch. (*b*) Predict the velocity the two gliders will continue to move with after they stick. Explain the reasoning behind this prediction for your lab partners.

Picture the Problem Because no external forces act on either glider, the center of mass of the two-glider system can't move. We can use the data concerning the masses and separation of the gliders initially to calculate its location and then apply the definition of the center of mass a second time to relate the positions x_1 and x_2 of the centers of the carts when they first touch. We can also use the separation of the centers of the gliders when they touch to obtain a second equation in x_1 and x_2 that we can solve simultaneously with the equation obtained from the location of the center of mass.

(*a*) The x coordinate of the center of mass of the 2-glider system is given by:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(0.100\,\text{kg})(0.100\,\text{m}) + (0.200\,\text{kg})(1.600\,\text{m})}{0.100\,\text{kg} + 0.200\,\text{kg}} = 1.10\,\text{m}$$

from the left end of the air track.

Because the location of the center of mass has not moved when two gliders first touch:

$$1.10\,\text{m} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$$

Substitute numerical values and simplify to obtain:

$$1.10\,\text{m} = \tfrac{1}{3} X_1 + \tfrac{2}{3} X_2$$

Also, when they first touch, their centers are separated by half their combined lengths:

$$X_2 - X_1 = \tfrac{1}{2}(10.0\,\text{cm} + 20.0\,\text{cm}) = 0.150\,\text{m}$$

Thus we have:

$\frac{1}{3}X_1 + \frac{2}{3}X_2 = 1.10\,\text{m}$

and

$X_2 - X_1 = 0.150\,\text{m}$

Solving these equations simultaneously yields:

$X_1 = \boxed{1.00\,\text{m}}$ and $X_2 = \boxed{1.15\,\text{m}}$

(*b*) Because the momentum of the system was zero initially, it must be zero just before the collision and after the collision in which the gliders stick together. Hence their velocity after the collision must be $\boxed{0}$.

39 •• Bored, a boy shoots his pellet gun at a piece of cheese that sits, keeping cool for dinner guests, on a massive block of ice. On one particular shot, his 1.2 g pellet gets stuck in the cheese, causing it to slide 25 cm before coming to a stop. If the muzzle velocity of the gun is 65 m/s and the cheese has a mass of 120 g, what is the coefficient of friction between the cheese and ice?

Picture the Problem Let the system consist of the pellet and the cheese. Then we can apply the conservation of linear momentum and the conservation of energy with friction to this inelastic collision to find the coefficient of friction between the cheese and the ice.

Apply conservation of linear momentum to the system to obtain:

$\Delta\vec{p}_{\text{system}} = \Delta\vec{p}_{\text{pellet}} + \Delta\vec{p}_{\text{cheese}} = 0$

or, because the motion is one-dimensional,

$p_{\text{f,pellet}} - p_{\text{i,pellet}} + p_{\text{f,cheese}} - p_{\text{i,cheese}} = 0$

Because the cheese is initially at rest:

$p_{\text{f,pellet}} - p_{\text{i,pellet}} + p_{\text{f,cheeset}} = 0$

Letting *m* represent the mass of the pellet, *M* the mass of the cheese, and *v* the common final speed of the pellet and the cheese, use the definition of linear momentum to obtain:

$mv - mv_{\text{i,pellet}} + Mv = 0$

Solving for *v* yields:

$$v = \frac{m}{m+M}v_{\text{i,pellet}} \quad (1)$$

Apply the conservation of energy with friction to the system to obtain:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$$

or, because $W_{\text{ext}} = 0$,

$$\Delta K + \Delta U_{\text{g}} + \Delta E_{\text{therm}} = 0$$

Because $\Delta U_{\text{g}} = K_{\text{f}} = 0$, and $\Delta E_{\text{therm}} = f\Delta s$ (where Δs is the distance the cheese slides on the ice):

$$-\tfrac{1}{2}(m+M)v^2 + f\Delta s = 0$$

f is given by:

$$f = \mu_{\text{k}}(m+M)g$$

Substituting for f yields:

$$-\tfrac{1}{2}(m+M)v^2 + \mu_{\text{k}}(m+M)g\Delta s = 0$$

Substitute for v from equation (1) to obtain:

$$-\tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_{\text{i,pellet}}\right)^2 + \mu_{\text{k}}(m+M)g\Delta s = 0$$

Solving for μ_{k} yields:

$$\mu_{\text{k}} = \frac{1}{2g\Delta s}\left(\frac{mv_{\text{i,pellet}}}{m+M}\right)^2$$

Substitute numerical values and evaluate μ_{k}:

$$\mu_{\text{k}} = \frac{1}{2(9.81\,\text{m/s}^2)(0.25\,\text{m})}\left(\frac{(0.0012\,\text{kg})(65\,\text{m/s})}{0.0012\,\text{kg} + 0.120\,\text{kg}}\right)^2 = \boxed{0.084}$$

40 ••• A wedge of mass *M*, as shown in Figure 8-46, is placed on a frictionless, horizontal surface, and a block of mass *m* is placed on the wedge, whose surface is also frictionless. The center of mass of the block moves downward a distance *h*, as the block slides from its initial position to the horizontal floor. (*a*) What are the speeds of the block and of the wedge, as they separate from each other and each go their own way? (*b*) Check your calculation plausibility by considering the limiting case when $M >> m$.

Picture the Problem Let the system include the earth, block, and wedge and apply conservation of energy and conservation of linear momentum.

(*a*) Apply conservation of energy with no frictional forces to the system to obtain:

$$W_{\text{ext}} = \Delta K + \Delta U$$

or, because $W_{\text{ext}} = 0$,

$$\Delta K + \Delta U = 0$$

Substituting for ΔK and ΔU yields: $K_f - K_i + U_f - U_i = 0$

Because $K_i = U_f = 0$: $K_f - U_i = 0$

Letting "b" refer to the block and "w" to the wedge yields: $K_{b,f} + K_{w,f} - U_{b,i} = 0$

Substitute for $K_{b,f}$, $K_{w,f}$, and $U_{b,i}$ to obtain:

$$\tfrac{1}{2}mv_b^2 + \tfrac{1}{2}Mv_w^2 - mgh = 0 \qquad (1)$$

Applying conservation of linear momentum to the system yields:

$$\Delta\vec{p}_{sys} = \Delta\vec{p}_b + \Delta\vec{p}_w = 0$$

or

$$\vec{p}_{b,f} - \vec{p}_{b,i} + \vec{p}_{w,f} - \vec{p}_{w,i} = 0$$

Because $\vec{p}_{b,i} = \vec{p}_{w,i} = 0$: $\vec{p}_{b,f} + \vec{p}_{w,f} = 0$

Substituting for $\vec{p}_{b,f}$ and $\vec{p}_{w,f}$ yields:

$$-mv_b\hat{i} + Mv_w\hat{i} = 0$$

or

$$-mv_b + Mv_w = 0$$

Solve for v_w to obtain:

$$v_w = \frac{m}{M}v_b \qquad (2)$$

Substituting for v_w in equation (1) yields:

$$\tfrac{1}{2}mv_b^2 + \tfrac{1}{2}M\left(\frac{m}{M}v_b\right)^2 - mgh = 0$$

Solve for v_b to obtain:

$$v_b = \boxed{\sqrt{\frac{2ghM}{M+m}}} \qquad (3)$$

Substitute for v_b in equation (2) and simplify to obtain:

$$v_w = \frac{m}{M}\sqrt{\frac{2ghM}{M+m}} = \boxed{\sqrt{\frac{2ghm^2}{M(M+m)}}} \qquad (4)$$

(*b*) Rewriting equation (3) by dividing the numerator and denominator of the radicand by M yields:

$$v_b = \sqrt{\frac{2gh}{1+\dfrac{m}{M}}}$$

When $M >> m$: $v_b = \sqrt{2gh}$

Rewriting equation (4) by dividing the numerator and denominator of the radicand by M yields:

$$v_w = \sqrt{\frac{2gh\left(\frac{m}{M}\right)^2}{1+\frac{m}{M}}}$$

When $M >> m$: $v_w = 0$

These results are exactly what we'd expect in this case: the physics is that of a block sliding down a fixed wedge incline with no movement of the incline.

Kinetic Energy of a System of Particles

41 •• **[SSM]** A 3.0-kg block is traveling to the right (the +*x* direction) at 5.0 m/s, and a second 3.0-kg block is traveling to the left at 2.0 m/s. (*a*) Find the total kinetic energy of the two blocks. (*b*) Find the velocity of the center of mass of the two-block system. (*c*) Find the velocity of each block relative to the center of mass. (*d*) Find the kinetic energy of the blocks relative to the center of mass. (*e*) Show that your answer for Part (*a*) is greater than your answer for Part (*d*) by an amount equal to the kinetic energy associated with the motion of the center of mass.

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) The total kinetic energy is the sum of the kinetic energies of the blocks:

$$K = K_1 + K_2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}(3.0\,\text{kg})(5.0\,\text{m/s})^2 + \tfrac{1}{2}(3.0\,\text{kg})(2.0\,\text{m/s})^2 = 43.5\,\text{J} = \boxed{44\,\text{J}}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Solving for $\vec{v}_{cm}$ yields:

$$\vec{\boldsymbol{v}}_{cm} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{cm}$:

$$\vec{\boldsymbol{v}}_{cm} = \frac{(3.0\,\text{kg})(5.0\,\text{m/s})\hat{\boldsymbol{i}} + (3.0\,\text{kg})(2.0\,\text{m/s})\hat{\boldsymbol{i}}}{3.0\,\text{kg} + 3.0\,\text{kg}} = \boxed{(1.5\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{\boldsymbol{v}}_{rel} = \vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_{cm}$$

Substitute numerical values to obtain:

$$\vec{\boldsymbol{v}}_{1,rel} = (5.0\,\text{m/s})\hat{\boldsymbol{i}} - (1.5\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(3.5\,\text{m/s})\hat{\boldsymbol{i}}}$$

$$\vec{\boldsymbol{v}}_{2,rel} = (-2.0\,\text{m/s})\hat{\boldsymbol{i}} - (1.5\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(-3.5\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{rel} = K_{1,rel} + K_{2,rel} = \tfrac{1}{2}m_1v_{1,rel}^2 + \tfrac{1}{2}m_2v_{2,rel}^2$$

Substitute numerical values and evaluate K_{rel}:

$$K_{rel} = \tfrac{1}{2}(3.0\,\text{kg})(3.5\,\text{m/s})^2 + \tfrac{1}{2}(3.0\,\text{kg})(-3.5\,\text{m/s})^2 = \boxed{37\,\text{J}}$$

(*e*) K_{cm} is given by:

$$K_{cm} = \tfrac{1}{2}m_{tot}v_{cm}^2$$

Substitute numerical values and evaluate K_{cm} :

$$K_{cm} = \tfrac{1}{2}(6.0\,\text{kg})(1.5\,\text{m/s})^2 = 6.75\,\text{J} = 43.5\,\text{J} - 36.75\,\text{J} = \boxed{K - K_{rel}}$$

42 •• Repeat Problem 41 with the second 3.0-kg block replaced by a 5.0-kg block moving to the right at 3.0 m/s.

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) The total kinetic energy is the sum of the kinetic energies of the blocks:

$$\boldsymbol{K} = \boldsymbol{K}_1 + \boldsymbol{K}_2 = \tfrac{1}{2}\boldsymbol{m}_1\boldsymbol{v}_1^2 + \tfrac{1}{2}\boldsymbol{m}_2\boldsymbol{v}_2^2$$

Substitute numerical values and evaluate *K*:

$$\boldsymbol{K} = \tfrac{1}{2}(3.0\,\text{kg})(5.0\,\text{m/s})^2 + \tfrac{1}{2}(5.0\,\text{kg})(3.0\,\text{m/s})^2 = 60.0\,\text{J} = \boxed{60\,\text{J}}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{\boldsymbol{v}}_{\text{cm}} = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2$$

Solving for $\vec{v}_{\text{cm}}$ yields:

$$\vec{\boldsymbol{v}}_{\text{cm}} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{cm}}$:

$$\vec{\boldsymbol{v}}_{\text{cm}} = \frac{(3.0\,\text{kg})(5.0\,\text{m/s})\hat{\boldsymbol{i}} + (5.0\,\text{kg})(3.0\,\text{m/s})\hat{\boldsymbol{i}}}{3.0\,\text{kg} + 5.0\,\text{kg}} = (3.75\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(3.8\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{\boldsymbol{v}}_{\text{rel}} = \vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_{\text{cm}}$$

Substitute numerical values to obtain:

$$\vec{\boldsymbol{v}}_{1,\text{rel}} = (5.0\,\text{m/s})\hat{\boldsymbol{i}} - (3.75\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(1.3\,\text{m/s})\hat{\boldsymbol{i}}}$$

$$\vec{\boldsymbol{v}}_{2,\text{rel}} = (3.0\,\text{m/s})\hat{\boldsymbol{i}} - (3.75\,\text{m/s})\hat{\boldsymbol{i}} = (-0.75\,\text{m/s})\hat{\boldsymbol{i}} = \boxed{(-0.8\,\text{m/s})\hat{\boldsymbol{i}}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{\text{rel}} = K_{1,\text{rel}} + K_{2,\text{rel}} = \tfrac{1}{2}m_1v_{1,\text{rel}}^2 + \tfrac{1}{2}m_2v_{2,\text{rel}}^2$$

Substitute numerical values and evaluate K_{rel}:

$$\boldsymbol{K}_{\text{rel}} = \tfrac{1}{2}(3.0\,\text{kg})(1.25\,\text{m/s})^2 + \tfrac{1}{2}(5.0\,\text{kg})(-0.75\,\text{m/s})^2 = 3.75\,\text{J} = \boxed{4\,\text{J}}$$

(*e*) K_{cm} is given by:

$$\boldsymbol{K}_{cm} = \tfrac{1}{2}\boldsymbol{m}_{tot}\boldsymbol{v}_{cm}^2$$

Substitute numerical values and evaluate K_{cm}:

$$\boldsymbol{K}_{cm} = \tfrac{1}{2}(8.0\,\text{kg})(3.75\,\text{m/s})^2 \approx 56.3\,\text{J} \approx \boxed{\boldsymbol{K} - \boldsymbol{K}_{rel}}$$

Impulse and Average Force

43 • **[SSM]** You kick a soccer ball whose mass is 0.43 kg. The ball leaves your foot with an initial speed of 25 m/s. (*a*) What is the magnitude of the impulse associated with the force of your foot on the ball? (*b*) If your foot is in contact with the ball for 8.0 ms, what is the magnitude of the average force exerted by your foot on the ball?

Picture the Problem The impulse imparted to the ball by the kicker equals the *change* in the ball's momentum. The impulse is also the product of the average force exerted on the ball by the kicker and the time during which the average force acts.

(*a*) Relate the magnitude of the impulse delivered to the ball to its change in momentum:

$$\boldsymbol{I} = |\Delta\vec{\boldsymbol{p}}| = \boldsymbol{p}_f - \boldsymbol{p}_i$$

or, because $v_i = 0$,

$$\boldsymbol{I} = \boldsymbol{m}\boldsymbol{v}_f$$

Substitute numerical values and evaluate *I*:

$$\boldsymbol{I} = (0.43\,\text{kg})(25\,\text{m/s}) = 10.8\,\text{N}\cdot\text{s} = \boxed{11\,\text{N}\cdot\text{s}}$$

(*b*) The impulse delivered to the ball as a function of the average force acting on it is given by:

$$I = F_{av}\Delta t \Rightarrow \boldsymbol{F}_{av} = \frac{\boldsymbol{I}}{\Delta\boldsymbol{t}}$$

Substitute numerical values and evaluate F_{av}:

$$\boldsymbol{F}_{av} = \frac{10.8\,\text{N}\cdot\text{s}}{0.0080\,\text{s}} = \boxed{1.3\,\text{kN}}$$

44 • A 0.30-kg brick is dropped from a height of 8.0 m. It hits the ground and comes to rest. (*a*) What is the impulse exerted by the ground on the brick during the collision? (*b*) If it takes 0.0013 s from the time the brick first touches the ground until it comes to rest, what is the average force exerted by the ground on the brick during impact?

Picture the Problem The impulse exerted by the ground on the brick equals the *change* in momentum of the brick and is also the product of the average force exerted by the ground on the brick and the time during which the average force acts.

(*a*) Express the magnitude of the impulse exerted by the ground on the brick:

$$I = \left|\Delta p_{\text{brick}}\right| = \left|p_{\text{f,brick}} - p_{\text{i,brick}}\right|$$

Because $p_{\text{f,brick}} = 0$:

$$I = p_{\text{i,brick}} = m_{\text{brick}} v \qquad (1)$$

Use conservation of energy to determine the speed of the brick at impact:

$$\Delta K + \Delta U = 0$$
or
$$K_\text{f} - K_\text{i} + U_\text{f} - U_\text{i} = 0$$

Because $U_\text{f} = K_\text{i} = 0$:

$$\boldsymbol{K}_\text{f} - \boldsymbol{U}_\text{i} = 0$$
and
$$\tfrac{1}{2}\boldsymbol{m}_{\text{brick}}\boldsymbol{v}^2 - \boldsymbol{m}_{\text{brick}}\boldsymbol{gh} = 0 \Rightarrow v = \sqrt{2gh}$$

Substitute in equation (1) to obtain:

$$I = m_{\text{brick}}\sqrt{2gh}$$

Substitute numerical values and evaluate I:

$$I = (0.30\,\text{kg})\sqrt{2(9.81\,\text{m/s}^2)(8.0\,\text{m})}$$
$$= 3.76\,\text{N}\cdot\text{s} = \boxed{3.8\,\text{N}\cdot\text{s}}$$

(*c*) The average force acting on the brick is:

$$\boldsymbol{F}_{\text{av}} = \frac{\boldsymbol{I}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate F_{av}:

$$\boldsymbol{F}_{\text{av}} = \frac{3.76\,\text{N}\cdot\text{s}}{0.0013\,\text{s}} = \boxed{2.9\,\text{kN}}$$

45 • A meteorite that has a mass equal to 30.8 tonne (1 tonne = 1000 kg) is exhibited in the American Museum of Natural History in New York City. Suppose that the kinetic energy of the meteorite as it hit the ground was 617 MJ. Find the magnitude of the impulse I experienced by the meteorite up to the time its kinetic energy was halved (which took about t = 3.0 s). Find also the average force F exerted on the meteorite during this time interval.

Picture the Problem The impulse exerted by the ground on the meteorite equals the *change* in momentum of the meteorite and is also the product of the average force exerted by the ground on the meteorite and the time during which the average force acts.

Express the magnitude of the impulse exerted by the ground on the meteorite:

$$\boldsymbol{I} = \left|\Delta\vec{\boldsymbol{p}}_{\text{meteorite}}\right| = \boldsymbol{p}_{\text{f}} - \boldsymbol{p}_{\text{i}}$$

Relate the kinetic energy of the meteorite to its initial momentum and solve for its initial momentum:

$$K_{\text{i}} = \frac{p_{\text{i}}^2}{2m} \Rightarrow p_{\text{i}} = \sqrt{2mK_{\text{i}}}$$

Express the ratio of the initial and final kinetic energies of the meteorite:

$$\frac{K_{\text{i}}}{K_{\text{f}}} = \frac{\dfrac{p_{\text{i}}^2}{2m}}{\dfrac{p_{\text{f}}^2}{2\text{m}}} = \frac{p_{\text{i}}^2}{p_{\text{f}}^2} = 2 \Rightarrow p_{\text{f}} = \frac{p_{\text{i}}}{\sqrt{2}}$$

Substitute in our expression for I and simplify:

$$I = \frac{p_{\text{i}}}{\sqrt{2}} - p_{\text{i}} = p_{\text{i}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= \sqrt{2mK_{\text{i}}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

Because our interest is in its magnitude, substitute numerical values and evaluate the absolute value of I:

$$\boldsymbol{I} = \left|\sqrt{2\left(30.8\times10^3\ \text{kg}\right)\left(617\times10^6\ \text{J}\right)}\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \boxed{1.81\,\text{MN}\cdot\text{s}}$$

The average force acting on the meteorite is:

$$\boldsymbol{F}_{\text{av}} = \frac{\boldsymbol{I}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate F_{av}:

$$\boldsymbol{F}_{\text{av}} = \frac{1.81\,\text{MN}\cdot\text{s}}{3.0\,\text{s}} = \boxed{0.60\,\text{MN}}$$

46 •• A 0.15-kg baseball traveling horizontally is hit by a bat and its direction exactly reversed. Its velocity changes from +20 m/s to –20 m/s. (*a*) What is the magnitude of the impulse delivered by the bat to the ball? (*b*) If the baseball is in contact with the bat for 1.3 ms, what is the average force exerted by the bat on the ball?

Picture the Problem The impulse exerted by the bat on the ball equals the *change* in momentum of the ball and is also the product of the average force exerted by the bat on the ball and the time during which the bat and ball were in contact.

(*a*) Express the impulse exerted by the bat on the ball in terms of the change in momentum of the ball:

$$\vec{I} = \Delta\vec{p}_{\text{ball}} = \vec{p}_{\text{f}} - \vec{p}_{\text{i}} = mv_{\text{f}}\hat{i} - \left(-mv_{\text{i}}\hat{i}\right) = 2mv\,\hat{i}$$

where $v = v_{\text{f}} = v_{\text{i}}$

Substitute for m and v and evaluate $\left|\vec{I}\right|$:

$$I = 2(0.15\,\text{kg})(20\,\text{m/s}) = 6.00\,\text{N}\cdot\text{s} = \boxed{6.0\,\text{N}\cdot\text{s}}$$

(*b*) The average force acting on the ball is:

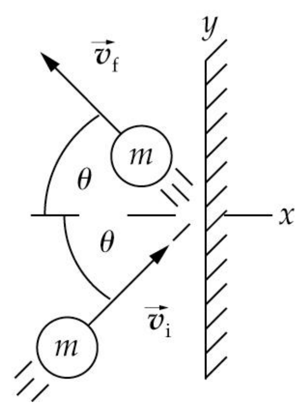
$$F_{\text{av}} = \frac{I}{\Delta t}$$

Substitute numerical values and evaluate F_{av}:

$$F_{\text{av}} = \frac{6.00\,\text{N}\cdot\text{s}}{1.3\,\text{ms}} = \boxed{4.6\,\text{kN}}$$

47 •• A 60-g handball moving with a speed of 5.0 m/s strikes the wall at an angle of 40° with the normal, and then bounces off with the same speed at the same angle with the normal. It is in contact with the wall for 2.0 ms. What is the average force exerted by the ball on the wall?

Picture the Problem The figure shows the handball just before and immediately after its collision with the wall. Choose a coordinate system in which the positive x direction is to the right. The wall changes the momentum of the ball by exerting a force on it during the ball's collision with it. The reaction to this force is the force the ball exerts on the wall. Because these action and reaction forces are equal in magnitude, we can find the average force exerted on the ball by finding the change in momentum of the ball.



Using Newton's 3rd law, relate the average force exerted by the ball on the wall to the average force exerted by the wall on the ball:

$$\vec{F}_{\text{av on wall}} = -\vec{F}_{\text{av on ball}}$$

and

$$F_{\text{av on wall}} = F_{\text{av on ball}} \qquad (1)$$

Relate the average force exerted by the wall on the ball to its change in momentum:

$$\vec{F}_{\text{av on ball}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t}$$

Express $\Delta\vec{v}$ in terms of its components:

$$\Delta\vec{v} = \Delta\vec{v}_x + \Delta\vec{v}_y$$

or, because

$\Delta\vec{v}_y = v_{f,y}\hat{j} - v_{i,y}\hat{j}$ and $v_{f,y} = v_{i,y}$.

$$\Delta\vec{v} = \Delta\vec{v}_x$$

Express $\Delta\vec{v}_x$ for the ball:

$$\Delta\vec{v}_x = v_{f,x}\hat{i} - v_{i,x}\hat{i}$$

or, because $v_{i,x} = v\cos\theta$ and

$v_{f,x} = -v\cos\theta$,

$$\Delta\vec{v}_x = -v\cos\theta\,\hat{i} - v\cos\theta\,\hat{i} = -2v\cos\theta\,\hat{i}$$

Substituting in the expression for $\vec{F}_{\text{av on ball}}$ yields:

$$\vec{F}_{\text{av on ball}} = \frac{m\Delta\vec{v}}{\Delta t} = -\frac{2mv\cos\theta}{\Delta t}\hat{i}$$

The magnitude of $\vec{F}_{\text{av on ball}}$ is:

$$F_{\text{av on ball}} = \frac{2mv\cos\theta}{\Delta t}$$

Substitute numerical values and evaluate $F_{\text{av on ball}}$:

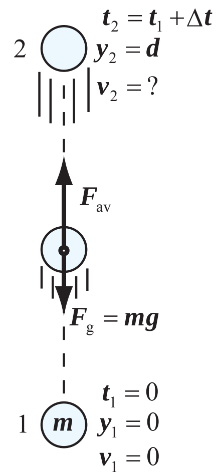
$$F_{\text{av on ball}} = \frac{2(0.060\,\text{kg})(5.0\,\text{m/s})\cos 40°}{2.0\,\text{ms}} = 0.23\,\text{kN}$$

Substitute in equation (1) to obtain:

$$F_{\text{av on wall}} = \boxed{0.23\,\text{kN}}$$

48 •• You throw a 150-g ball straight up to a height of 40.0 m. (*a*) Use a reasonable value for the displacement of the ball while it is in your hand to estimate the time the ball is in your hand while you are throwing it. (*b*) Calculate the average force exerted by your hand while you are throwing it. (Is it OK to neglect the gravitational force on the ball while it is being thrown?)

Picture the Problem The pictorial representation shows the ball during the interval of time during which you are exerting a force on it to accelerate it upward. The average force you exert can be determined from the change in momentum of the ball. The change in the velocity of the ball can be found by applying conservation of mechanical energy to the ball-earth system once it has left your hand.



(*a*) Relate the time the ball is in your hand to its average speed while it is in your hand and the displacement of your hand:

$$\Delta t = \frac{\Delta y}{v_{\text{av, in your hand}}}$$

Letting $U_g = 0$ at the initial elevation of your hand, use conservation of mechanical energy to relate the initial kinetic energy of the ball to its potential energy when it is at its highest point:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_f *and* U_i and solve for v_2:

$$-\tfrac{1}{2}mv_2^2 + mgh = 0 \Rightarrow v_2 = \sqrt{2gh}$$

Because $v_{\text{av, in your hand}} = \tfrac{1}{2}v_2$:

$$\Delta t = \frac{\Delta y}{\tfrac{1}{2}v_2} = \frac{2\Delta y}{\sqrt{2gh}}$$

Assuming the displacement of your hand is 0.70 m as you throw the ball straight up, substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(0.70\ \text{m})}{\sqrt{2(9.81\ \text{m/s}^2)(40\ \text{m})}} = 50.0\ \text{ms}$$

$$= \boxed{50\ \text{ms}}$$

(*b*) Relate the average force exerted by your hand on the ball to the change in momentum of the ball:

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{\Delta t}$$

or, because $v_1 = p_1 = 0$,

$$F_{av} = \frac{mv_2}{\Delta t}$$

Substitute for v_2 to obtain:

$$\boldsymbol{F}_{\text{av}} = \frac{\boldsymbol{m}\sqrt{2\boldsymbol{gh}}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate F_{av}:

$$F_{\text{av}} = \frac{(0.15\,\text{kg})\sqrt{2(9.81\,\text{m/s}^2)(40\,\text{m})}}{50.0\,\text{ms}} = 84.1\,\text{N} = \boxed{84\,\text{N}}$$

Express the ratio of the gravitational force on the ball to the average force acting on it:

$$\frac{\boldsymbol{F}_{\text{g}}}{\boldsymbol{F}_{\text{av}}} = \frac{\boldsymbol{mg}}{\boldsymbol{F}_{\text{av}}}$$

Substitute numerical values and evaluate $F_{\text{g}}/F_{\text{av}}$:

$$\frac{\boldsymbol{F}_{\text{g}}}{\boldsymbol{F}_{\text{av}}} = \frac{(0.15\,\text{kg})(9.81\,\text{m/s}^2)}{84.1\,\text{N}} < 2\%$$

Because the gravitational force acting on the ball is less than 2% of the average force exerted by your hand on the ball, it is reasonable to have neglected the gravitational force.

49 •• A 0.060-kg handball is thrown straight toward a wall with a speed of 10 m/s. It rebounds straight backward at a speed of 8.0 m/s. (*a*) What impulse is exerted on the wall? (*b*) If the ball is in contact with the wall for 3.0 ms, what average force is exerted on the wall by the ball? (*c*) The rebounding ball is caught by a player who brings it to rest. During the process, her hand moves back 0.50 m. What is the impulse received by the player? (*d*) What average force was exerted on the player by the ball?

Picture the Problem Choose a coordinate system in which the direction the ball is moving *after* its collision with the wall is the +*x* direction. The impulse delivered to the wall or received by the player equals the change in the momentum of the ball during these two collisions. We can find the average forces from the rate of change in the momentum of the ball.

(*a*) The impulse delivered to the wall is the change in momentum of the handball:

$$\vec{\boldsymbol{I}} = \Delta\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}_{\text{f}} - \boldsymbol{m}\vec{\boldsymbol{v}}_{\text{i}}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{I}}$:

$$\vec{\boldsymbol{I}} = (0.060\,\text{kg})(8.0\,\text{m/s})\hat{\boldsymbol{i}} - \left[-(0.060\,\text{kg})(10\,\text{m/s})\hat{\boldsymbol{i}}\right] = (1.08\,\text{N}\cdot\text{s})\hat{\boldsymbol{i}} = \boxed{(1.1\,\text{N}\cdot\text{s})\hat{\boldsymbol{i}}}$$

or 1.1 N·s directed into the wall.

(*b*) F_{av} is the rate of change of the ball's momentum:

$$\boldsymbol{F}_{av} = \frac{\Delta \boldsymbol{p}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate F_{av}:

$$\boldsymbol{F}_{av} = \frac{1.08\,\text{N}\cdot\text{s}}{0.0030\,\text{s}} = 360\,\text{N}$$
$$= \boxed{0.36\,\text{kN, into the wall.}}$$

(*c*) The impulse received by the player from the change in momentum of the ball is given by:

$$\boldsymbol{I} = \Delta \boldsymbol{p}_{ball} = \boldsymbol{m}\Delta\boldsymbol{v}$$

Substitute numerical values and evaluate I:

$$\boldsymbol{I} = (0.060\,\text{kg})(8.0\,\text{m/s}) = 0.480\,\text{N}\cdot\text{s}$$
$$= \boxed{0.48\,\text{N}\cdot\text{s, away from the wall.}}$$

(*d*) Relate F_{av} to the change in the ball's momentum:

$$F_{av} = \frac{\Delta p_{ball}}{\Delta t}$$

Express the stopping time in terms of the average speed v_{av} of the ball and its stopping distance d:

$$\Delta t = \frac{d}{v_{av}}$$

Substitute for Δt and simplify to obtain:

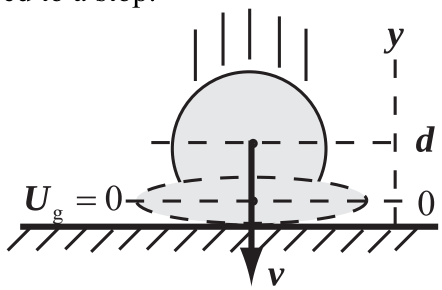
$$F_{av} = \frac{v_{av}\Delta p_{ball}}{d}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(4.0\,\text{m/s})(0.480\,\text{N}\cdot\text{s})}{0.50\,\text{m}}$$
$$= \boxed{3.8\,\text{N, away from the wall.}}$$

50 •• A spherical 0.34-kg orange, 2.0 cm in radius, is dropped from the top of a 35 m-tall building. After striking the pavement, the shape of the orange is a 0.50 cm thick pancake. Neglect air resistance and assume that the collision is completely inelastic. (*a*) How much time did the orange take to completely "squish" to a stop? (*b*) What average force did the pavement exert on the orange during the collision?

Picture the Problem The following pictorial representation shows the orange moving with velocity $\vec{v}$, just before impact, after falling from a height of 35 m. Let the system be the orange and let the zero of gravitational potential energy be at the center of mass of the squished orange. The external forces are gravity, acting on the orange throughout its fall, and the normal force exerted by the ground that acts on the orange as it is squished. We can find the squishing time from the displacement of the center-of-mass of the orange as it stops and its average speed during this period of (assumed) constant acceleration. We can use the impulse-momentum theorem to find the average force exerted by the ground on the orange as it slowed to a stop.



(*a*) Express the stopping time for the orange in terms of its average speed and the distance traveled by its center of mass:

$$\Delta t = \frac{d}{v_{av}} \qquad (1)$$

In order to find v_{av}, apply the conservation of mechanical energy to the free-fall portion of the orange's motion:

$$K_f - K_i + U_{g,f} - U_{g,i} = 0$$

or, because $K_i = 0$,

$$K_f + U_{g,f} - U_{g,i} = 0$$

Substituting for K_f, $U_{g,f}$, and $U_{g,i}$ yields:

$$\tfrac{1}{2}mv^2 + mgd - mg(h-d) = 0$$

Solving for v yields:

$$v = \sqrt{2gh\left(1 - 2\frac{d}{h}\right)}$$

or, because $d << h$,

$$v \approx \sqrt{2gh}$$

Assuming constant acceleration as the orange squishes:

$$v_{av} = \tfrac{1}{2}v = \tfrac{1}{2}\sqrt{2gh}$$

Substituting for v_{av} in equation (1) and simplifying yields:

$$\Delta t = \frac{2d}{\sqrt{2gh}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(2.25\text{ cm})}{\sqrt{2(9.81\text{ m/s}^2)(35\text{ m})}} = 4.43\times10^{-4}\text{ s} = \boxed{0.44\text{ ms}}$$

(*b*) Apply the impulse-momentum theorem to the squishing orange to obtain:

$$\left|\vec{F}_{av}\Delta t\right| = \left|\Delta\vec{p}\right| = \left|\vec{p}_f - \vec{p}_i\right|$$

or, because $p_f = 0$,

$$F_{av}\Delta t = p_i \Rightarrow F_{av} = \frac{p_i}{\Delta t} = \frac{mv_i}{\Delta t}$$

In Part (*a*) we showed that $v_i = v \approx \sqrt{2gh}$. Therefore:

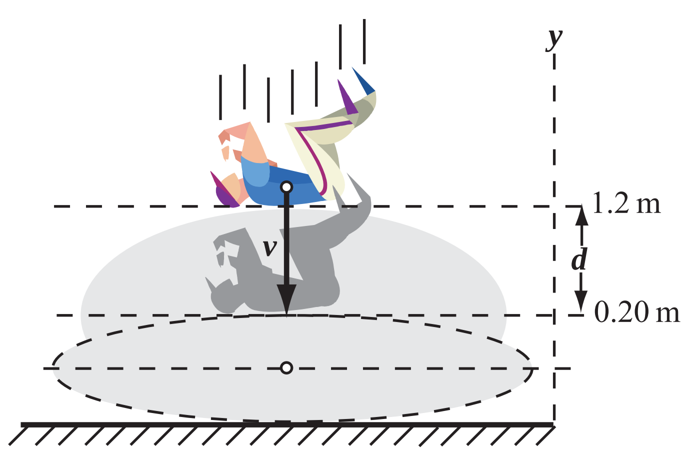
$$F_{av} = \frac{m\sqrt{2gh}}{\Delta t}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(0.34\text{ kg})\sqrt{2(9.81\text{ m/s}^2)(35\text{ m})}}{4.43\times10^{-4}\text{ s}} \approx \boxed{54\text{ kN}}$$

51 •• The pole-vault landing pad at an Olympic competition contains what is essentially a bag of air that compresses from its ″resting″ height of 1.2 m down to 0.20 m as the vaulter is slowed to a stop. (*a*) What is the time interval during which a vaulter who has just cleared a height of 6.40 m slows to a stop? (*b*) What is the time interval if instead the vaulter is brought to rest by a 20 cm layer of sawdust that compresses to 5.0 cm when he lands? (*c*) Qualitatively discuss the difference in the average force the vaulter experiences from the two different landing pads. That is, which landing pad exerts the least force on the vaulter and why?

Picture the Problem The pictorial representation shows the vaulter moving with velocity $\vec{v}$, just before impact on the landing pad after falling from a height of 6.40 m. In order to determine the time interval during which the vaulter stops, we have to know his momentum change and the average net force acting on him. With knowledge of these quantities, we can use the impulse-momentum equation, $F_{net}\Delta t = \Delta p$. We can determine the average force by noting that as the vaulter comes to a stop on the landing pad, work is done on him by the airbag.



(*a*) Use the impulse-momentum theorem to relate the stopping time to the average force acting on the vaulter:

$$F_{\text{net}}\Delta t = \Delta p$$

Use the work-kinetic energy theorem to obtain:

$$W_{\text{net}} = F_{\text{net}}d = (F_{\text{airbag}} - mg)d = \Delta K$$

where d is the distance the vaulter moves while being decelerated.

Substituting for F_{net} in equation (1) yields:

$$(F_{\text{airbag}} - mg)\Delta t = \Delta p$$

or

$$\frac{\Delta K}{d}\Delta t = \Delta p$$

Solve for Δt to obtain:

$$\Delta t = \frac{d\Delta p}{\Delta K} = \frac{d(p_{\text{f}} - p_{\text{i}})}{K_{\text{f}} - K_{\text{i}}}$$

or, because $K_{\text{f}} = p_{\text{f}} = 0$,

$$\Delta t = \frac{d(-p_{\text{i}})}{-K_{\text{i}}} = \frac{p_{\text{i}}d}{K_{\text{i}}}$$

Use $K = \frac{p^2}{2m}$ to obtain:

$$\Delta t = \frac{2mp_{\text{i}}d}{p_{\text{i}}^2} = \frac{2md}{p_{\text{i}}} = \frac{2md}{\sqrt{2mK_{\text{i}}}}$$

Rationalizing the denominator of this expression and simplifying yields:

$$\Delta t = d\sqrt{\frac{2m}{K_{\text{i}}}} \qquad (1)$$

K_{i} is equal to the change in the gravitational potential energy of the vaulter as he falls a distance Δy before hitting the airbag:

$$K_{\text{i}} = mg\Delta y$$

Substituting for K_{i} in equation (1) and simplifying yields:

$$\Delta t = d\sqrt{\frac{2}{g\Delta y}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = (1.2\text{ m} - 0.2\text{ m})\sqrt{\frac{2}{(9.81\text{ m/s}^2)(6.4\text{ m} - 1.2\text{ m})}} = 0.198\text{ s} = \boxed{0.20\text{ s}}$$

(*b*) In this case, d_1 = 20 cm, d_2 = 5.0 cm. Substitute numerical values and evaluate Δt:

$$\Delta t = (0.20\text{ m} - 0.05\text{ m})\sqrt{\frac{2}{(9.81\text{ m/s}^2)(6.4\text{ m} - 0.20\text{ m})}} = 27.2\text{ ms} = \boxed{27\text{ ms}}$$

The average force exerted on the vaulter by the airbag is much less than the average force the sawdust exerts on him because the collision time is much shorter for the sawdust landing.

52 ••• Great limestone caverns have been formed by dripping water. (*a*) If water droplets of 0.030 mL fall from a height of 5.0 m at a rate of 10 droplets per minute, what is the average force exerted on the limestone floor by the droplets of water during a 1.0-min period? (Assume the water does not accumulate on the floor.) (*b*) Compare this force to the weight of one water droplet.

Picture the Problem The average force exerted on the limestone by the droplets of water equals the rate at which momentum is being delivered to the floor. We're given the number of droplets that arrive per minute and can use conservation of mechanical energy to determine their velocity as they reach the floor.

(*a*) Letting N represent the rate at which droplets fall, relate F_{av} to the change in the droplet's momentum:

$$F_{av} = \frac{\Delta p_{\text{droplets}}}{\Delta t} = N\frac{m\Delta v}{\Delta t}$$

or, because the droplets fall from rest,

$$F_{av} = \frac{N}{\Delta t}mv \qquad (1)$$

where v is their speed after falling 5.0 m.

The mass of the droplets is the product of their density and volume:

$$m = \rho V$$

Letting $U_g = 0$ at the point of impact of the droplets, use conservation of mechanical energy to relate their speed at impact to their fall distance:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2}mv^2 - mgh = 0 \Rightarrow v = \sqrt{2gh}$$

Substitute for m and v in equation (1) to obtain:

$$F_{av} = \frac{N}{\Delta t}\rho V\sqrt{2gh}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = 4.95\times10^{-5}\ \text{N} = \boxed{50\ \mu\text{N}}$$

(*b*) Express the ratio of the weight of a droplet to F_{av}:

$$\frac{w}{F_{av}} = \frac{mg}{F_{av}}$$

Substitute numerical values and evaluate w/F_{av}:

$$\frac{w}{F_{av}} = \frac{(3\times10^{-5}\ \text{kg})(9.81\ \text{m/s}^2)}{4.95\times10^{-5}\ \text{N}} \approx \boxed{6}$$

Collisions in One Dimension

53 • **[SSM]** A 2000-kg car traveling to the right at 30 m/s is chasing a second car of the same mass that is traveling in the same direction at 10 m/s. (*a*) If the two cars collide and stick together, what is their speed just after the collision? (*b*) What fraction of the initial kinetic energy of the cars is lost during this collision? Where does it go?

Picture the Problem We can apply conservation of linear momentum to this perfectly inelastic collision to find the after-collision speed of the two cars. The ratio of the transformed kinetic energy to kinetic energy before the collision is the fraction of kinetic energy lost in the collision.

(*a*) Letting V be the velocity of the two cars after their collision, apply conservation of linear momentum to their perfectly inelastic collision:

$$p_{\text{initial}} = p_{\text{final}}$$

or

$$mv_1 + mv_2 = (m+m)V \Rightarrow V = \frac{v_1+v_2}{2}$$

Substitute numerical values and evaluate V:

$$V = \frac{30\ \text{m/s}+10\ \text{m/s}}{2} = \boxed{20\ \text{m/s}}$$

(*b*) The ratio of the kinetic energy that is lost to the kinetic energy of the two cars before the collision is:

$$\frac{\Delta K}{K_{\text{initial}}} = \frac{K_{\text{final}} - K_{\text{initial}}}{K_{\text{initial}}} = \frac{K_{\text{final}}}{K_{\text{initial}}} - 1$$

Substitute for the kinetic energies and simplify to obtain:

$$\frac{\Delta K}{K_{\text{initial}}} = \frac{\frac{1}{2}(2m)V^2}{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2} - 1$$

$$= \frac{2V^2}{v_1^2 + v_2^2} - 1$$

Substitute numerical values and evaluate $\Delta K/K_{\text{initial}}$:

$$\frac{\Delta \boldsymbol{K}}{\boldsymbol{K}_{\text{initial}}}=\frac{2(20\,\text{m/s})^2}{(30\,\text{m/s})^2+(10\,\text{m/s})^2}-1$$
$$=-0.20$$

20% of the initial kinetic energy is transformed into heat, sound, and the deformation of the materials from which the car is constructed.

54 • An 85-kg running back moving at 7.0 m/s makes a perfectly inelastic head-on collision with a 105-kg linebacker who is initially at rest. What is the speed of the players just after their collision?

Picture the Problem We can apply conservation of linear momentum to this perfectly inelastic collision to find the after-collision speed of the two players.

Letting the subscript 1 refer to the running back and the subscript 2 refer to the linebacker, apply conservation of momentum to their perfectly inelastic collision:

$$p_i=p_f$$
or
$$m_1v_1=(m_1+m_2)V\Rightarrow V=\frac{m_1}{m_1+m_2}v_1$$

Substitute numerical values and evaluate V:

$$\boldsymbol{V}=\frac{85\,\text{kg}}{85\,\text{kg}+105\,\text{kg}}(7.0\,\text{m/s})=\boxed{3.1\,\text{m/s}}$$

55 • A 5.0-kg object with a speed of 4.0 m/s collides head-on with a 10-kg object moving toward it with a speed of 3.0 m/s. The 10-kg object stops dead after the collision. (*a*) What is the post-collision speed of the 5.0-kg object? (*b*) Is the collision elastic?

Picture the Problem We can apply conservation of linear momentum to this collision to find the post-collision speed of the 5.0-kg object. Let the direction the 5.0-kg object is moving before the collision be the positive direction. We can decide whether the collision was elastic by examining the initial and final kinetic energies of the system.

(*a*) Letting the subscript 5 refer to the 5.0-kg object and the subscript 10 refer to the 10-kg object, apply conservation of momentum to obtain:

$$p_i=p_f$$
or
$$m_5v_{i,5}-m_{10}v_{i,10}=m_5v_{f,5}$$

Solve for $v_{f,5}$:

$$v_{f,5}=\frac{m_5v_{i,5}-m_{10}v_{i,10}}{m_5}$$

Substitute numerical values and evaluate $v_{f,5}$:

$$\boldsymbol{v}_{f,5} = \frac{(5.0\,\text{kg})(4.0\,\text{m/s}) - (10\,\text{kg})(3.0\,\text{m/s})}{5\,\text{kg}} = \boxed{-2.0\,\text{m/s}}$$

where the minus sign means that the 5.0-kg object is moving to the left after the collision.

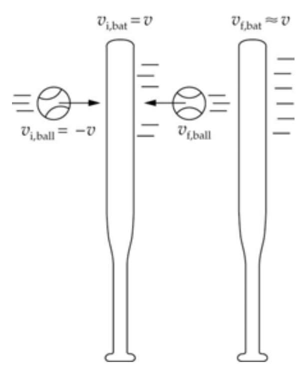
(*b*) Evaluate ΔK for the collision:

$$\Delta K = K_f - K_i = \tfrac{1}{2}(5.0\,\text{kg})(2.0\,\text{m/s})^2 - \left[\tfrac{1}{2}(5.0\,\text{kg})(4.0\,\text{m/s})^2 + \tfrac{1}{2}(10\,\text{kg})(3.0\,\text{m/s})^2\right]$$
$$= -75\,\text{J}$$

Because $\Delta K \neq 0$, the collision was not elastic.

56 • A small superball of mass *m* moves with speed *v* to the right toward a much more massive bat that is moving to the left with speed *v*. Find the speed of the ball after it makes an elastic head-on collision with the bat.

Picture the Problem The pictorial representation shows the ball and bat just before and just after their collision. Take the direction the bat is moving to be the positive direction. Because the collision is elastic, we can equate the speeds of recession and approach, with the approximation that $v_{i,bat} \approx v_{f,bat}$ to find $v_{f,ball}$.



Express the speed of approach of the bat and ball:

$$v_{f,bat} - v_{f,ball} = -(v_{i,bat} - v_{i,ball})$$

Because the mass of the bat is much greater than that of the ball:

$$v_{i,bat} \approx v_{f,bat}$$

Substitute to obtain:

$$v_{f,bat} - v_{f,ball} = -(v_{f,bat} - v_{i,ball})$$

Solve for and evaluate $v_{f,ball}$:

$$v_{f,ball} = v_{f,bat} + (v_{f,bat} - v_{i,ball})$$
$$= -v_{i,ball} + 2v_{f,bat} = v + 2v$$
$$= \boxed{3v}$$

57 •• A proton that has a mass *m* and is moving at 300 m/s undergoes a head-on elastic collision with a stationary carbon nucleus of mass 12*m*. Find the velocity of the proton and the carbon nucleus after the collision.

Picture the Problem Let the direction the proton is moving before the collision be the +*x* direction. We can use both conservation of momentum and conservation of mechanical energy to obtain an expression for velocities of the proton and the carbon nucleus after the collision.

Use conservation of linear momentum to obtain one relation for the final velocities:

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc} v_{nuc,f} \quad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{nuc,f} - v_{p,f} = -(v_{nuc,i} - v_{p,i}) = v_{p,i} \quad (2)$$

To eliminate $v_{nuc,f}$, solve equation (2) for $v_{nuc,f}$, and substitute the result in equation (1):

$$v_{nuc,f} = v_{p,i} + v_{p,f}$$

and

$$m_p v_{p,i} = m_p v_{p,f} + m_{nuc}(v_{p,i} + v_{p,f})$$

Solving for $v_{p,f}$ yields:

$$\boldsymbol{v}_{p,f} = \frac{\boldsymbol{m}_p - \boldsymbol{m}_{nuc}}{\boldsymbol{m}_p + \boldsymbol{m}_{nuc}} \boldsymbol{v}_{p,i}$$

Substituting for m_p and m_{nuc} and simplifying yields:

$$\boldsymbol{v}_{p,f} = \frac{\boldsymbol{m} - 12\boldsymbol{m}}{\boldsymbol{m} + 12\boldsymbol{m}} \boldsymbol{v}_{p,i} = -\frac{11}{13} \boldsymbol{v}_{p,i}$$

Substitute the numerical value of $v_{p,i}$ and evaluate $v_{p,f}$:

$$\boldsymbol{v}_{p,f} = -\frac{11}{13}(300\,\text{m/s}) = \boxed{-254\,\text{m/s}}$$

where the minus sign tells us that the velocity of the proton was reversed in the collision.

Solving equation (2) for $v_{nuc,f}$ yields:

$$\boldsymbol{v}_{nuc,f} = \boldsymbol{v}_{p,i} + \boldsymbol{v}_{p,f}$$

Substitute numerical values and evaluate $v_{nuc,f}$:

$$\boldsymbol{v}_{nuc,f} = 300\,\text{m/s} - 254\,\text{m/s} = \boxed{46\,\text{m/s, forward}}$$

58 •• A 3.0-kg block moving at 4.0 m/s makes a head-on elastic collision with a stationary block of mass 2.0 kg. Use conservation of momentum and the fact that the relative speed of recession equals the relative speed of approach to find the velocity of each block after the collision. Check your answer by calculating the initial and final kinetic energies of each block.

Picture the Problem We can use conservation of momentum and the definition of an elastic collision to obtain two equations in v_{2f} and v_{3f} that we can solve simultaneously.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_3 v_{3i} = m_3 v_{3f} + m_2 v_{2f} \quad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{2f} - v_{3f} = -(v_{2i} - v_{3i}) = v_{3i} \quad (2)$$

Solve equation (2) for v_{3f}, substitute in equation (1) to eliminate v_{3f}, and solve for v_{2f} to obtain:

$$\boldsymbol{v}_{2f} = \frac{2\boldsymbol{m}_3\boldsymbol{v}_{3i}}{\boldsymbol{m}_2 + \boldsymbol{m}_3}$$

Substitute numerical values and evaluate v_{2f}:

$$\boldsymbol{v}_{2f} = \frac{2(3.0\,\text{kg})(4.0\,\text{m/s})}{2.0\,\text{kg} + 3.0\,\text{kg}} = 4.80\,\text{m/s} = \boxed{4.8\,\text{m/s}}$$

Use equation (2) to find v_{3f}:

$$\boldsymbol{v}_{3f} = \boldsymbol{v}_{2f} - \boldsymbol{v}_{3i} = 4.8\,\text{m/s} - 4.0\,\text{m/s} = \boxed{0.8\,\text{m/s}}$$

Evaluate K_i and K_f:

$$\boldsymbol{K}_i = \boldsymbol{K}_{3i} = \tfrac{1}{2}\boldsymbol{m}_3\boldsymbol{v}_{3i}^2 = \tfrac{1}{2}(3.0\,\text{kg})(4.0\,\text{m/s})^2 = 24\,\text{J}$$

and

$$\boldsymbol{K}_f = \boldsymbol{K}_{3f} + \boldsymbol{K}_{2f} = \tfrac{1}{2}\boldsymbol{m}_3\boldsymbol{v}_{3f}^2 + \tfrac{1}{2}\boldsymbol{m}_2\boldsymbol{v}_{2f}^2$$
$$= \tfrac{1}{2}(3.0\,\text{kg})(0.8\,\text{m/s})^2 + \tfrac{1}{2}(2.0\,\text{kg})(4.8\,\text{m/s})^2 = 24\,\text{J}$$

Because $K_i = K_f$, we can conclude that the values obtained for v_{2f} and v_{3f} are consistent with the collision having been elastic.

59 •• A block of mass m_1 = 2.0 kg slides along a frictionless table with a speed of 10 m/s. Directly in front of it, and moving in the same direction with a speed of 3.0 m/s, is a block of mass m_2 = 5.0 kg. A massless spring that has a force constant k = 1120 N/m is attached to the second block as in Figure 8-47. (*a*) What is the velocity of the center of mass of the system? (*b*) During the collision, the spring is compressed by a maximum amount Δx. What is the value of Δx? (*c*) The blocks will eventually separate again. What are the final velocities of the two blocks measured in the reference frame of the table, after they separate?

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the maximum compression of the spring and express the initial (i.e., before collision) and final (i.e., at separation) velocities. Finally, we'll transform the velocities from the center-of-mass frame of reference to the table frame of reference.

(*a*) Use the definition of the total momentum of a system to relate the initial momenta to the velocity of the center of mass:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

or

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(2.0\,\text{kg})(10\,\text{m/s}) + (5.0\,\text{kg})(3.0\,\text{m/s})}{2.0\,\text{kg} + 5.0\,\text{kg}}$$

$$= 5.00\,\text{m/s} = \boxed{5.0\,\text{m/s}}$$

(*b*) Find the kinetic energy of the system at maximum compression ($u_1 = u_2 = 0$):

$$K = K_{cm} = \tfrac{1}{2} M v_{cm}^2$$

$$= \tfrac{1}{2}(7.0\,\text{kg})(5.00\,\text{m/s})^2 = 87.5\,\text{J}$$

Use conservation of mechanical energy to relate the kinetic energy of the system to the potential energy stored in the spring at maximum compression:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{sf} - U_{si} = 0$$

Because $K_f = K_{cm}$ and $U_{si} = 0$:

$$K_{cm} - K_i + \tfrac{1}{2} k (\Delta x)^2 = 0$$

Solving for Δx yields:

$$\Delta \boldsymbol{x}=\sqrt{\frac{2(\boldsymbol{K}_{\mathrm{i}}-\boldsymbol{K}_{\mathrm{cm}})}{\boldsymbol{k}}}=\sqrt{\frac{2\left[\frac{1}{2}\boldsymbol{m}_1\boldsymbol{v}_{1\mathrm{i}}^2+\frac{1}{2}\boldsymbol{m}_2\boldsymbol{v}_{2\mathrm{i}}^2-\boldsymbol{K}_{\mathrm{cm}}\right]}{\boldsymbol{k}}}=\sqrt{\frac{\boldsymbol{m}_1\boldsymbol{v}_{1\mathrm{i}}^2+\boldsymbol{m}_2\boldsymbol{v}_{2\mathrm{i}}^2-2\boldsymbol{K}_{\mathrm{cm}}}{\boldsymbol{k}}}$$

Substitute numerical values and evaluate Δx:

$$\Delta \boldsymbol{x}=\sqrt{\frac{(2.0\,\mathrm{kg})(10\,\mathrm{m/s})^2+(5.0\,\mathrm{kg})(3.0\,\mathrm{m/s})^2}{1120\,\mathrm{N/m}}-\frac{2(87.5\,\mathrm{J})}{1120\,\mathrm{N/m}}}=\boxed{0.25\,\mathrm{m}}$$

(*c*) Find $u_{1\mathrm{i}}$, $u_{2\mathrm{i}}$, and $u_{1\mathrm{f}}$ for this elastic collision:

$$\boldsymbol{u}_{1\mathrm{i}}=\boldsymbol{v}_{1\mathrm{i}}-\boldsymbol{v}_{\mathrm{cm}}=10\,\mathrm{m/s}-5\,\mathrm{m/s}=5\,\mathrm{m/s},$$
$$\boldsymbol{u}_{2\mathrm{i}}=\boldsymbol{v}_{2\mathrm{i}}-\boldsymbol{v}_{\mathrm{cm}}=3\,\mathrm{m/s}-5\,\mathrm{m/s}=-2\,\mathrm{m/s},$$
and
$$\boldsymbol{u}_{1\mathrm{f}}=\boldsymbol{v}_{1\mathrm{f}}-\boldsymbol{v}_{\mathrm{cm}}=0-5\,\mathrm{m/s}=-5\,\mathrm{m/s}$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach and solve for $u_{2\mathrm{f}}$:

$$u_{2\mathrm{f}}-u_{1\mathrm{f}}=-(u_{2\mathrm{i}}-u_{1\mathrm{i}})$$
and
$$\boldsymbol{u}_{2\mathrm{f}}=-\boldsymbol{u}_{2\mathrm{i}}+\boldsymbol{u}_{1\mathrm{i}}+\boldsymbol{u}_{1\mathrm{f}}$$

Substitute numerical values and evaluate $u_{2\mathrm{f}}$:

$$\boldsymbol{u}_{2\mathrm{f}}=-(-2.0\,\mathrm{m/s})+5.0\,\mathrm{m/s}-5.0\,\mathrm{m/s}$$
$$=2.0\,\mathrm{m/s}$$

Transform $u_{1\mathrm{f}}$ and $u_{2\mathrm{f}}$ to the table frame of reference:

$$\boldsymbol{v}_{1\mathrm{f}}=\boldsymbol{u}_{1\mathrm{f}}+\boldsymbol{v}_{\mathrm{cm}}=-5.0\,\mathrm{m/s}+5.0\,\mathrm{m/s}$$
$$=\boxed{0}$$
and
$$\boldsymbol{v}_{2\mathrm{f}}=\boldsymbol{u}_{2\mathrm{f}}+\boldsymbol{v}_{\mathrm{cm}}=2.0\,\mathrm{m/s}+5.0\,\mathrm{m/s}$$
$$=\boxed{7.0\,\mathrm{m/s}}$$

60 •• A bullet of mass *m* is fired vertically from below into a thin horizontal sheet of plywood of mass *M* that is initially at rest, supported by a thin sheet of paper (Figure 8-48). The bullet punches through the plywood, which rises to a height *H* above the paper before falling back down. The bullet continues rising to a height *h* above the paper. (*a*) Express the upward velocity of the bullet and the plywood immediately after the bullet exits the plywood in terms of *h* and *H*. (*b*) What is the speed of the bullet? (*c*) What is the mechanical energy of the system before and after the inelastic collision? (*d*) How much mechanical energy is dissipated during the collision?

Picture the Problem Let the system include the earth, the bullet, and the sheet of plywood. Then $W_{ext} = 0$. Choose the zero of gravitational potential energy to be where the bullet enters the plywood. We can apply both conservation of energy and conservation of momentum to obtain the various physical quantities called for in this problem.

(*a*) Use conservation of mechanical energy after the bullet exits the sheet of plywood to relate its exit speed to the height to which it rises:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-\tfrac{1}{2}mv_m^2 + mgh = 0 \Rightarrow v_m = \boxed{\sqrt{2gh}}$$

Proceed similarly to relate the initial velocity of the plywood to the height to which it rises:

$$v_M = \boxed{\sqrt{2gH}}$$

(*b*) Apply conservation of momentum to the collision of the bullet and the sheet of plywood:

$$\vec{p}_i = \vec{p}_f$$

or

$$mv_{mi} = mv_m + Mv_M$$

Substitute for v_m and v_M and solve for v_{mi}:

$$v_{mi} = \boxed{\sqrt{2gh} + \frac{M}{m}\sqrt{2gH}}$$

(*c*) Express the initial mechanical energy of the system (i.e., just before the collision):

$$E_i = \tfrac{1}{2}mv_{mi}^2 = \boxed{mg\left[h + \frac{2M}{m}\sqrt{hH} + \left(\frac{M}{m}\right)^2 H\right]}$$

Express the final mechanical energy of the system (that is, when the bullet and block have reached their maximum heights):

$$\boldsymbol{E_f = mgh + MgH = \boxed{g(mh + MH)}}$$

(*d*) Use the work-energy theorem with $W_{ext} = 0$ to find the energy dissipated by friction in the inelastic collision:

$$E_f - E_i + W_{friction} = 0$$

and

$$W_{friction} = E_i - E_f = \boxed{gMH\left[2\sqrt{\frac{h}{H}} + \frac{M}{m} - 1\right]}$$

61 •• A proton of mass m is moving with initial speed v_0 directly toward the center of an α particle of mass $4m$, which is initially at rest. Both particles carry positive charge, so they repel each other. (The repulsive forces are sufficient to prevent the two particles from coming into direct contact.) Find the speed v' of the α particle (a) when the distance between the two particles is a minimum, and (b) later when the two particles are far apart.

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the speeds of the particles when their separation is a minimum and when they are far apart.

(a) Noting that when the distance between the two particles is a minimum, both move at the same speed, namely v_{cm}, use the definition of the total momentum of a system to relate the initial momenta to the velocity of the center of mass:

$$\vec{\boldsymbol{P}} = \sum_i m_i\vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$$

or

$$m_p v_{pi} = (m_p + m_\alpha)v_{cm}.$$

Solve for v_{cm} to obtain:

$$v_{cm} = v' = \frac{m_p v_{pi} + m_\alpha v_{\alpha i}}{m_1 + m_2}$$

Additional simplification yields:

$$\boldsymbol{v}_{cm} = \boldsymbol{v}' = \frac{m\boldsymbol{v}_0 + 0}{\boldsymbol{m} + 4\boldsymbol{m}} = \boxed{0.2\boldsymbol{v}_0}$$

(b) Use conservation of linear momentum to obtain one relation for the final velocities:

$$m_p v_0 = m_p v_{pf} + m_\alpha v_{\alpha f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{pf} - v_{\alpha f} = -(v_{pi} - v_{\alpha i}) = -v_{pi} \qquad (2)$$

Solve equation (2) for v_{pf}, substitute in equation (1) to eliminate v_{pf}, and solve for $v_{\alpha f}$:

$$\boldsymbol{v}_{\alpha f} = \frac{2\boldsymbol{m}_p\boldsymbol{v}_0}{\boldsymbol{m}_p + \boldsymbol{m}_\alpha}$$

Simplifying further yields:

$$\boldsymbol{v}_{\alpha f} = \frac{2\boldsymbol{m}\boldsymbol{v}_0}{\boldsymbol{m} + 4\boldsymbol{m}} = \boxed{0.4\boldsymbol{v}_0}$$

62 •• An electron collides elastically with a hydrogen atom initially at rest. Assume all the motion occurs along a straight line. What fraction of the electron's initial kinetic energy is transferred to the atom? (Take the mass of the hydrogen atom to be 1840 times the mass of an electron.)

Picture the Problem Let the numeral 1 denote the electron and the numeral 2 the hydrogen atom. We can find the final velocity of the electron and, hence, the fraction of its initial kinetic energy that is transferred to the atom, by transforming to the center-of-mass reference frame, calculating the post-collision velocity of the electron, and then transforming back to the laboratory frame of reference.

Express f, the fraction of the electron's initial kinetic energy that is transferred to the atom:

$$f = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{\frac{1}{2}m_1 v_{1f}^2}{\frac{1}{2}m_1 v_{1i}^2} = 1 - \left(\frac{v_{1f}}{v_{1i}}\right)^2 \quad (1)$$

Find the velocity of the center of mass:

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + m_2}$$

or, because $m_2 = 1840m_1$,

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + 1840m_1} = \frac{1}{1841}v_{1i}$$

Find the initial velocity of the electron in the center-of-mass reference frame:

$$u_{1i} = v_{1i} - v_{cm} = v_{1i} - \frac{1}{1841}v_{1i} = \left(1 - \frac{1}{1841}\right)v_{1i}$$

Find the post-collision velocity of the electron in the center-of-mass reference frame by reversing its velocity:

$$u_{1f} = -u_{1i} = \left(\frac{1}{1841} - 1\right)v_{1i}$$

To find the final velocity of the electron in the original frame, add v_{cm} to its final velocity in the center-of-mass reference frame:

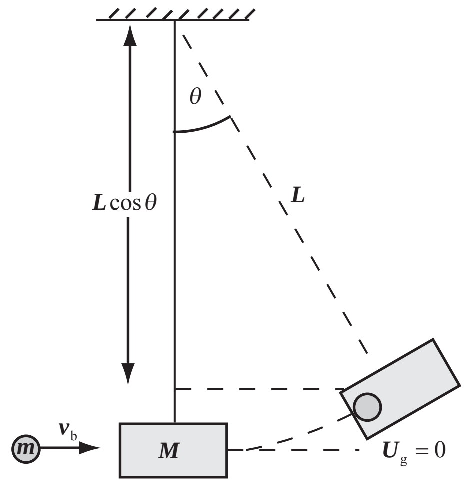
$$v_{1f} = u_{1f} + v_{cm} = \left(\frac{2}{1841} - 1\right)v_{1i}$$

Substituting in equation (1) and simplifying yields:

$$f = 1 - \left(\frac{2}{1841} - 1\right)^2 = \boxed{0.217\%}$$

63 •• **[SSM]** A16-g bullet is fired into the bob of a 1.5-kg ballistic pendulum (Figure 8-18). When the bob is at its maximum height, the strings make an angle of 60° with the vertical. The pendulum strings are 2.3 m long. Find the speed of the bullet prior to impact.

Picture the Problem The pictorial representation shows the bullet about to imbed itself in the bob of the ballistic pendulum and then, later, when the bob plus bullet have risen to their maximum height. We can use conservation of momentum during the collision to relate the speed of the bullet to the initial speed of the bob plus bullet (V). The initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy when they reach their maximum height. Hence we apply conservation of mechanical energy to relate V to the angle through which the bullet plus bob swings and then solve the momentum and energy equations simultaneously for the speed of the bullet.



Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob-bullet:

$$mv_b = (m+M)V \Rightarrow v_b = \left(1+\frac{M}{m}\right)V \quad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bob-bullet to their final potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f to obtain:

$$-\tfrac{1}{2}(m+M)V^2 + (m+M)gL(1-\cos\theta) = 0$$

Solving for V yields:

$$V = \sqrt{2gL(1-\cos\theta)}$$

Substitute for V in equation (1) to obtain:

$$v_b = \left(1+\frac{M}{m}\right)\sqrt{2gL(1-\cos\theta)}$$

Substitute numerical values and evaluate v_b:

$$v_b = \left(1+\frac{1.5\,\text{kg}}{0.016\,\text{kg}}\right)\sqrt{2(9.81\,\text{m/s}^2)(2.3\,\text{m})(1-\cos 60°)} = \boxed{0.45\,\text{km/s}}$$

64 •• Show that in a one-dimensional elastic collision, if the mass and velocity of object 1 are m_1 and v_{1i}, and if the mass and velocity of object 2 are m_2

and v_{2i}, then their final velocities v_{1f} and v_{2f} are given by

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \text{ and } v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

Picture the Problem We can apply conservation of linear momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the colliding objects that we can solve for v_{1f} and v_{2f}.

Apply conservation of momentum to the elastic collision of the particles to obtain:

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \quad (1)$$

Relate the initial and final kinetic energies of the particles in an elastic collision:

$$\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) \quad (2)$$

Rearrange equation (1) to obtain:

$$m_2\left(v_{2f} - v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right) \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \quad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$\left(m_1 + m_2\right) v_{1f} = \left(m_1 - m_2\right) v_{1i} + 2m_2 v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \boxed{\frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}}$$

Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$\left(m_1 + m_2\right) v_{2f} = \left(m_2 - m_1\right) v_{2i} + 2m_1 v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \boxed{\frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}}$$

Remarks: Note that the velocities satisfy the condition that $\boldsymbol{v}_{2f} - \boldsymbol{v}_{1f} = -(\boldsymbol{v}_{2i} - \boldsymbol{v}_{1i})$. This verifies that the speed of recession equals the speed of approach.

65 •• Investigate the plausibility of the results of Problem 64 by calculating the final velocities in the following limits: (*a*) When the two masses are equal, show that the particles ″swap″ velocities: $\boldsymbol{v}_{1f} = \boldsymbol{v}_{2i}$ i and $\boldsymbol{v}_{2f} = \boldsymbol{v}_{1i}$ (*b*) If $m_2 >> m_1$, and $\boldsymbol{v}_{2i} = 0$, show that $\boldsymbol{v}_{1f} \approx -\boldsymbol{v}_{1i}$ and $\boldsymbol{v}_{2f} \approx 0$. (*c*) If $m_1 >> m_2$, and $\boldsymbol{v}_{2i} = 0$, show that $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$.

Picture the Problem As in this problem, Problem 74 involves an elastic, one-dimensional collision between two objects. Both solutions involve using the conservation of momentum equation $m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$ and the elastic collision equation $v_{1f} - v_{2f} = v_{2i} - v_{1i}$. In Part (*a*) we can simply set the masses equal to each other and substitute in the equations in Problem 64 to show that the particles "swap" velocities. In Part (*b*) we can divide the numerator and denominator of the equations in Problem 64 by m_2 and use the condition that $m_2 >> m_1$ to show that $v_{1f} \approx -v_{1i}+2v_{2i}$ and $v_{2f} \approx v_{2i}$.

(*a*) From Problem 64 we have:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \quad (1)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \quad (2)$$

Set $m_1 = m_2 = m$ to obtain:

$$\boldsymbol{v}_{1f} = \frac{2\boldsymbol{m}}{\boldsymbol{m} + \boldsymbol{m}}\boldsymbol{v}_{2i} = \boxed{\boldsymbol{v}_{2i}}$$

and

$$\boldsymbol{v}_{2f} = \frac{2\boldsymbol{m}}{\boldsymbol{m} + \boldsymbol{m}}\boldsymbol{v}_{1i} = \boxed{\boldsymbol{v}_{1i}}$$

(*b*) Divide the numerator and denominator of both terms in equation (1) by m_2 to obtain:

$$v_{1f} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}v_{1i} + \frac{2}{\frac{m_1}{m_2} + 1}v_{2i}$$

If $m_2 >> m_1$ and $v_{2i} = 0$:

$$\boldsymbol{v}_{1f} \approx \boxed{-\boldsymbol{v}_{1i}}$$

Divide the numerator and denominator of both terms in equation (2) by m_2 to obtain:

$$v_{2f} = \frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}v_{1i} + \frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}v_{2i}$$

If $m_2 >> m_1$:

$$v_{2f} \approx \boxed{v_{2i}}$$

(*c*) Divide the numerator and denominator of equation (1) by m_1 to obtain:

$$v_{1f} = \frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}v_{1i} + \frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}v_{2i}$$

If $m_1 >> m_2$ and $v_{2i} = 0$:

$$v_{1f} \approx \boxed{v_{1i}}$$

Divide the numerator and denominator of equation (2) by m_1 to obtain:

$$v_{2f} = \frac{2}{1+\frac{m_2}{m_1}}v_{1i} + \frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}v_{2i}$$

If $m_1 >> m_2$ and $v_{2i} = 0$:

$$v_{2f} \approx \boxed{2v_{1i}}$$

Remarks: Note that, in both parts of this problem, the velocities satisfy the condition that $\boldsymbol{v}_{2f} - \boldsymbol{v}_{1f} = -(\boldsymbol{v}_{2i} - \boldsymbol{v}_{1i})$. This verifies that the speed of recession equals the speed of approach.

66 •• A bullet of mass m_1 is fired horizontally with a speed v_0 into the bob of a ballistic pendulum of mass m_2. The pendulum consists of a bob attached to one end of a very light rod of length *L*. The rod is free to rotate about a horizontal axis through its other end. The bullet is stopped in the bob. Find the minimum v_0 such that the bob will swing through a complete circle.

Picture the Problem Choose $U_g = 0$ at the bob's equilibrium position. Momentum is conserved in the collision of the bullet with bob and the initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy as it swings up to the top of the circle. If the bullet plus bob just makes it to the top of the circle with zero speed, it will swing through a complete circle.

Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$\boldsymbol{m}_1\boldsymbol{v}_0 - (\boldsymbol{m}_1 + \boldsymbol{m}_2)\boldsymbol{V} = 0$$

Solve for the speed of the bullet to obtain:

$$v_0 = \left(1+\frac{m_2}{m_1}\right)V \qquad (1)$$

Use conservation of mechanical energy to relate the initial kinetic energy of the bob plus bullet to their potential energy at the top of the circle:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

$$-\tfrac{1}{2}(m_1 + m_2)V^2 + (m_1 + m_2)g(2L) = 0$$

Solving for V yields:

$$V = 2\sqrt{gL}$$

Substitute for V in equation (1) and simplify to obtain:

$$\boxed{v_0 = 2\left(1 + \frac{m_2}{m_1}\right)\sqrt{gL}}$$

67 •• A bullet of mass m_1 is fired horizontally with a speed v into the bob of a ballistic pendulum of mass m_2 (Figure 8-19). Find the maximum height h attained by the bob if the bullet passes through the bob and emerges with a speed $v/3$.

Picture the Problem Choose $U_g = 0$ at the equilibrium position of the ballistic pendulum. Momentum is conserved in the collision of the bullet with the bob and kinetic energy is transformed into gravitational potential energy as the bob swings up to its maximum height.

Letting V represent the initial speed of the bob as it begins its upward swing, use conservation of momentum to relate this speed to the speeds of the bullet just before and after its collision with the bob:

$$m_1 v = m_1\left(\tfrac{1}{3}v\right) + m_2 V \Rightarrow V = \frac{2m_1}{3m_2}v$$

Use conservation of energy to relate the initial kinetic energy of the bob to its potential energy at its maximum height:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

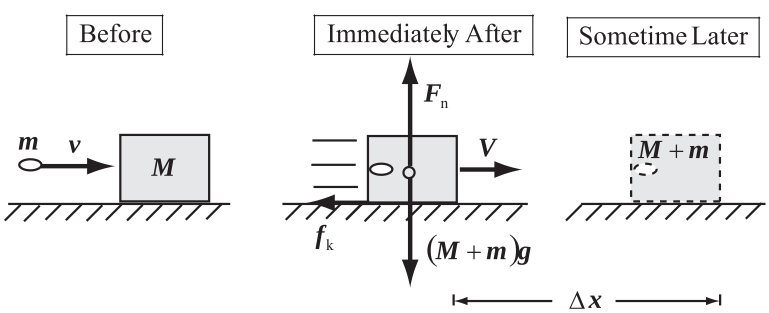
$$-\tfrac{1}{2}m_2 V^2 + m_2 gh = 0 \Rightarrow h = \frac{V^2}{2g}$$

Substitute for V in the expression for h and simplify to obtain:

$$h=\frac{\left(\frac{2m_1}{3m_2}v\right)^2}{2g}=\boxed{\frac{2m_1^2v^2}{9m_2^2g}}$$

68 •• A heavy wooden block rests on a flat table and a high-speed bullet is fired horizontally into the block, the bullet stopping in it. How far will the block slide before coming to a stop? The mass of the bullet is 10.5 g, the mass of the block is 10.5 g, the bullet's impact speed is 750 m/s, and the coefficient of kinetic friction between the block and the table is 0.220. (Assume that the bullet does not cause the block to spin.)

Picture the Problem Let the mass of the bullet be *m*, that of the wooden block *M*, the pre-collision velocity of the bullet *v*, and the post-collision velocity of the block+bullet be *V*. We can use conservation of momentum to find the velocity of the block with the bullet imbedded in it immediately after their perfectly inelastic collision. We can use Newton's 2nd law to find the acceleration of the sliding block and a constant-acceleration equation to find the distance the block slides.



Using a constant-acceleration equation, relate the velocity of the block+bullet just after their collision to their acceleration and displacement before stopping:

$$0=V^2+2a\Delta x \Rightarrow \Delta x=-\frac{V^2}{2a}$$

because the final velocity of the block+bullet is zero.

Use conservation of momentum to relate the pre-collision velocity of the bullet to the post-collision velocity of the block+bullet:

$$mv=(m+M)V \Rightarrow V=\frac{m}{m+M}v$$

Substitute for V in the expression for Δx to obtain:

$$\Delta x=-\frac{1}{2a}\left(\frac{m}{m+M}v\right)^2$$

Apply $\sum\vec{F}=m\vec{a}$ to the block+bullet (see the force diagram above):

$$\sum F_x=-f_k=(m+M)a \qquad (1)$$

and

$$\sum F_y=F_n-(m+M)g=0 \qquad (2)$$

Use the definition of the coefficient of kinetic friction and equation (2) to obtain:

$$f_k = \mu_k F_n = \mu_k (m+M)g$$

Substituting for f_k in equation (1) yields:

$$-\mu_k (m+M)g = (m+M)a$$

Solve for a to obtain:

$$a = -\mu_k g$$

Substituting for a in the expression for Δx yields:

$$\Delta x = \frac{1}{2\mu_k g}\left(\frac{m}{m+M}v\right)^2$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{1}{2(0.220)(9.81\,\text{m/s}^2)}\left(\frac{0.0105\,\text{kg}}{0.0105\,\text{kg}+10.5\,\text{kg}}(750\,\text{m/s})\right)^2 = \boxed{13.0\,\text{cm}}$$

69 •• **[SSM]** A 0.425-kg ball with a speed of 1.30 m/s rolls across a level surface toward an open 0.327-kg box that is resting on its side. The ball enters the box, and the box (with the ball inside slides across the surface a distance of x = 0.520 m. What is the coefficient of kinetic friction between the box and the table?

Picture the Problem The collision of the ball with the box is perfectly inelastic and we can find the speed of the box-and-ball immediately after their collision by applying conservation of momentum. If we assume that the kinetic friction force is constant, we can use a constant-acceleration equation to find the acceleration of the box and ball combination and the definition of μ_k to find its value.

Using its definition, express the coefficient of kinetic friction of the table:

$$\mu_k = \frac{f_k}{F_n} = \frac{(M+m)|a|}{(M+m)g} = \frac{|a|}{g} \quad (1)$$

Use conservation of momentum to relate the speed of the ball just before the collision to the speed of the ball+box immediately after the collision:

$$MV = (m+M)v \Rightarrow v = \frac{MV}{m+M} \quad (2)$$

Use a constant-acceleration equation to relate the sliding distance of the ball+box to its initial and final velocities and its acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x$$

or, because $v_f = 0$ and $v_i = v$,

$$0 = v^2 + 2a\Delta x \Rightarrow a = -\frac{v^2}{2\Delta x}$$

Substitute for a in equation (1) to obtain:

$$\mu_k = \frac{v^2}{2g\Delta x}$$

Use equation (2) to eliminate v:

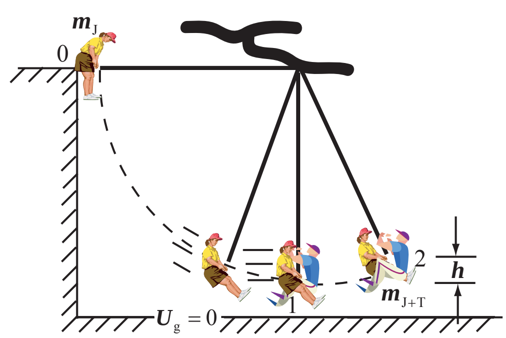
$$\mu_k = \frac{1}{2g\Delta x}\left(\frac{MV}{m+M}\right)^2 = \frac{1}{2g\Delta x}\left(\frac{V}{\frac{m}{M}+1}\right)^2$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{1}{2(9.81\,\text{m/s}^2)(0.520\,\text{m})}\left(\frac{1.30\,\text{m/s}}{\frac{0.327\,\text{kg}}{0.425\,\text{kg}}+1}\right)^2 = \boxed{0.0529}$$

70 •• Tarzan is in the path of a pack of stampeding elephants when Jane swings in to the rescue on a rope vine, hauling him off to safety. The length of the vine is 25 m, and Jane starts her swing with the rope horizontal. If Jane's mass is 54 kg, and Tarzan's is 82 kg, to what height above the ground will the pair swing after she rescues him? (Assume that the rope is vertical when she grabs him.)

Picture the Problem Jane's collision with Tarzan is a perfectly inelastic collision. We can find her speed v_1 just before she grabs Tarzan from conservation of energy and their speed V just after she grabs him from conservation of momentum. Their kinetic energy just after their collision will be transformed into gravitational potential energy when they have reached their greatest height h.



Use conservation of energy to relate the potential energy of Jane and Tarzan at their highest point (2) to their kinetic energy immediately after Jane grabbed Tarzan:

$$U_2 = K_1$$

or

$$m_{J+T}gh = \tfrac{1}{2}m_{J+T}V^2 \Rightarrow h = \frac{V^2}{2g} \qquad (1)$$

Apply conservation of linear momentum to relate Jane's velocity just before she collides with Tarzan to their velocity just after their perfectly inelastic collision:

$$\boldsymbol{m}_{\mathrm{J}}\boldsymbol{v}_1 - \boldsymbol{m}_{\mathrm{J+T}}\boldsymbol{V} = 0 \Rightarrow V = \frac{m_{\mathrm{J}}}{m_{\mathrm{J+T}}}v_1 \quad (2)$$

Apply conservation of mechanical energy to relate Jane's kinetic energy at 1 to her potential energy at 0:

$$K_1 = U_0$$

or

$$\tfrac{1}{2}m_{\mathrm{J}}v_1^2 = m_{\mathrm{J}}gL \Rightarrow v_1 = \sqrt{2gL}$$

Substitute for v_1 in equation (2) to obtain:

$$V = \frac{m_{\mathrm{J}}}{m_{\mathrm{J+T}}}\sqrt{2gL}$$

Substitute for V in equation (1) and simplify:

$$h = \frac{1}{2g}\left(\frac{m_{\mathrm{J}}}{m_{\mathrm{J+T}}}\right)^2 2gL = \left(\frac{m_{\mathrm{J}}}{m_{\mathrm{J+T}}}\right)^2 L$$

Substitute numerical values and evaluate h:

$$\boldsymbol{h} = \left(\frac{54\,\mathrm{kg}}{54\,\mathrm{kg} + 82\,\mathrm{kg}}\right)^2 (25\,\mathrm{m}) = \boxed{3.9\,\mathrm{m}}$$

71 •• **[SSM]** Scientists estimate that the meteorite responsible for the creation of Barringer Meteorite Crater in Arizona weighed roughly 2.72×10^5 tonne (1 tonne = 1000 kg) and was traveling at a speed of 17.9 km/s. Take Earth's orbital speed to be about 30.0 km/s. (*a*) What should the direction of impact be if Earth's orbital speed is to be changed by the maximum possible amount? (*b*) Assuming the condition of collision in Part (*a*), estimate the maximum percentage change in Earth's orbital speed as a result of this collision. (*c*) What mass asteroid, having a speed equal to Earth's orbital speed, would be necessary to change Earth's orbital speed by 1.00%?

Picture the Problem Let the system include Earth and the asteroid. Choose a coordinate system in which the direction of Earth's orbital speed is the +*x* direction. We can apply conservation of linear momentum to the perfectly inelastic collision of Earth and the asteroid to find the percentage change in Earth's orbital speed as well as the mass of an asteroid that would change Earth's orbital speed by 1.00%. Note that the following solution neglects the increase in Earth's orbital speed due to the gravitational pull of the asteroid during descent.

(*a*) For maximum slowing of Earth, the collision would have to have taken place with the meteorite impacting Earth along a line exactly opposite Earth's orbital velocity vector. In this case, we have a head-on inelastic collision.

(*b*) Express the percentage change in the Earth's orbital speed as a result of the collision:

$$\left|\frac{\Delta \boldsymbol{v}}{\boldsymbol{v}_{\text{Earth}}}\right| = \left|\frac{\boldsymbol{v}_{\text{Earth}} - \boldsymbol{v}_{\text{f}}}{\boldsymbol{v}_{\text{Earth}}}\right| = \left|1 - \frac{\boldsymbol{v}_{\text{f}}}{\boldsymbol{v}_{\text{Earth}}}\right| \qquad (1)$$

where v_{f} is Earth's orbital speed after the collision.

Apply the conservation of linear momentum to the system to obtain:

$$\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_{\text{f}} - \vec{\boldsymbol{p}}_{\text{i}} = 0$$

or, because the asteroid and the earth are moving horizontally,

$$\boldsymbol{p}_{\text{f},x} - \boldsymbol{p}_{\text{i},x} = 0$$

Because the collision is perfectly inelastic:

$$\left(\boldsymbol{m}_{\text{Earth}} + \boldsymbol{m}_{\text{asteroid}}\right)\boldsymbol{v}_{\text{f}} - \left(\boldsymbol{m}_{\text{Earth}}\boldsymbol{v}_{\text{Earth}} - \boldsymbol{m}_{\text{asteroid}}\boldsymbol{v}_{\text{asteroid}}\right) = 0$$

Solving for v_{f} yields:

$$v_{\text{f}} = \frac{m_{\text{Earth}}v_{\text{Earth}} - m_{\text{asteroid}}v_{\text{asteroid}}}{m_{\text{Earth}} + m_{\text{asteroid}}} = \frac{m_{\text{Earth}}v_{\text{Earth}}}{m_{\text{Earth}} + m_{\text{asteroid}}} - \frac{m_{\text{asteroid}}v_{\text{asteroid}}}{m_{\text{Earth}} + m_{\text{asteroid}}}$$

$$= \frac{v_{\text{Earth}}}{1 + \dfrac{m_{\text{asteroid}}}{m_{\text{Earth}}}} - \frac{\dfrac{m_{\text{asteroid}}}{m_{\text{Earth}}}v_{\text{asteroid}}}{1 + \dfrac{m_{\text{asteroid}}}{m_{\text{Earth}}}}$$

Because $m_{\text{asteroid}} << m_{\text{Earth}}$:

$$v_{\text{f}} \approx v_{\text{Earth}} - \frac{\dfrac{m_{\text{asteroid}}}{m_{\text{Earth}}}v_{\text{asteroid}}}{1 + \dfrac{m_{\text{asteroid}}}{m_{\text{Earth}}}} = v_{\text{Earth}} - \frac{m_{\text{asteroid}}}{m_{\text{Earth}}}v_{\text{asteroid}}\left(1 + \frac{m_{\text{asteroid}}}{m_{\text{Earth}}}\right)^{-1}$$

Expanding $\left(1 + \dfrac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\right)^{-1}$ binomially yields:

$$\left(1 + \frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\right)^{-1} = 1 - \frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}} + \text{higher order terms}$$

Substitute for $\left(1 + \dfrac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\right)^{-1}$ in the expression for v_{f} to obtain:

$$\boldsymbol{v}_{\text{f}} \approx \boldsymbol{v}_{\text{Earth}} - \frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\boldsymbol{v}_{\text{asteroid}}\left(1 - \frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\right)$$

$$\approx \boldsymbol{v}_{\text{Earth}} - \frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\boldsymbol{v}_{\text{asteroid}}$$

Substitute for v_f in equation (1) to obtain:

$$\left|\frac{\Delta v}{v_{\text{Earth}}}\right| = \left|1 - \frac{v_{\text{Earth}} - \frac{m_{\text{asteroid}}}{m_{\text{Earth}}} v_{\text{asteroid}}}{v_{\text{Earth}}}\right| = \left|\frac{\frac{m_{\text{asteroid}}}{m_{\text{Earth}}} v_{\text{asteroid}}}{v_{\text{Earth}}}\right|$$

Using data found in the appendices of your text or given in the problem statement, substitute numerical values and evaluate $\left|\frac{\Delta \boldsymbol{v}}{\boldsymbol{v}_{\text{Earth}}}\right|$:

$$\left|\frac{\Delta v}{v_{\text{Earth}}}\right| = \left|\frac{\frac{\left(2.72\times10^{5}\ \text{tonne}\times\frac{10^{3}\ \text{kg}}{1\ \text{tonne}}\right)(17.9\ \text{km/s})}{5.98\times10^{24}\ \text{kg}}}{30.0\ \text{km/s}}\right| = \boxed{2.71\times10^{-15}\%}$$

(*c*) If the asteroid is to change the earth's orbital speed by 1%:

$$\frac{\frac{\boldsymbol{m}_{\text{asteroid}}}{\boldsymbol{m}_{\text{Earth}}}\boldsymbol{v}_{\text{asteroid}}}{\boldsymbol{v}_{\text{Earth}}} = \frac{1}{100}$$

Solve for m_{asteroid} to obtain:

$$\boldsymbol{m}_{\text{asteroid}} = \frac{\boldsymbol{v}_{\text{Earth}}\boldsymbol{m}_{\text{Earth}}}{100\boldsymbol{v}_{\text{asteroid}}}$$

Substitute numerical values and evaluate m_{asteroid}:

$$\boldsymbol{m}_{\text{asteroid}} = \frac{(30.0\ \text{km/s})(5.98\times10^{24}\ \text{kg})}{100(17.9\ \text{km/s})} = \boxed{1.00\times10^{23}\ \text{kg}}$$

Remarks: The mass of this asteroid is approximately that of the moon!

72 ••• William Tell shoots an apple from his son's head. The speed of the 125-g arrow just before it strikes the apple is 25.0 m/s, and at the time of impact it is traveling horizontally. If the arrow sticks in the apple and the arrow/apple combination strikes the ground 8.50 m behind the son's feet, how massive was the apple? Assume the son is 1.85 m tall.

Picture the Problem Let the system include Earth, the apple, and the arrow. Choose a coordinate system in which the direction the arrow is traveling before imbedding itself in the apple is the $+x$ direction. We can apply conservation of linear momentum to express the mass of the apple in terms of the speed of the arrow-apple combination just after the collision and then use constant-acceleration equations to find this post-collision speed.

Apply conservation of linear momentum to the system to obtain:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0$$

or, because the arrow and apple are moving horizontally,

$$p_{f,x} - p_{i,x} = 0$$

Because the collision is perfectly inelastic (the arrow is imbedded in the apple):

$$(m_{arrow} + m_{apple})v_x - m_{arrow}v_{i,arrow} = 0$$

Solving for m_{apple} yields:

$$m_{apple} = m_{arrow}\left(\frac{v_{i,arrow}}{v_x} - 1\right) \quad (1)$$

Using constant-acceleration equations, express the horizontal and vertical displacements of the apple-arrow after their collision:

$$\Delta x = v_x \Delta t \quad (2)$$

and

$$\Delta y = \tfrac{1}{2} g (\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Substituting for Δt in equation (2) yields:

$$\Delta x = v_x \sqrt{\frac{2\Delta y}{g}} \Rightarrow v_x = \Delta x \sqrt{\frac{g}{2\Delta y}}$$

Substitute for v_x in equation (1) to obtain:

$$m_{apple} = m_{arrow}\left(\frac{v_{i,arrow}}{\Delta x \sqrt{\dfrac{g}{2\Delta y}}} - 1\right)$$

Substitute numerical values and evaluate m_{apple}:

$$\boldsymbol{m}_{\text{apple}} = (0.125\ \text{kg})\left(\frac{25.0\ \text{m/s}}{(8.50\ \text{m})\sqrt{\dfrac{9.81\ \text{m/s}^2}{2(1.85\ \text{m})}}} - 1\right) = \boxed{101\ \text{g}}$$

Explosions and Radioactive Decay

73 •• **[SSM]** The beryllium isotope ^{8}Be is unstable and decays into two α particles ($m_\alpha = 6.64 \times 10^{-27}$ kg) and releases 1.5×10^{-14} J of energy. Determine the velocities of the two α particles that arise from the decay of a ^{8}Be nucleus at rest, assuming that all the energy appears as kinetic energy of the particles.

Picture the Problem This nuclear reaction is $^4\text{Be} \rightarrow 2\alpha + 1.5 \times 10^{-14}$ J. In order to conserve momentum, the alpha particles will have move in opposite directions with the same velocities. We'll use conservation of energy to find their speeds.

Letting E represent the energy released in the reaction, express conservation of energy for this process:

$$2K_\alpha = 2\left(\tfrac{1}{2} m_\alpha v_\alpha^2\right) = E \Rightarrow v_\alpha = \sqrt{\frac{E}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$\boldsymbol{v}_\alpha = \sqrt{\frac{1.5\times10^{-14}\ \text{J}}{6.64\times10^{-27}\ \text{kg}}} = \boxed{1.5\times10^{6}\ \text{m/s}}$$

74 •• The light isotope of lithium, ^{5}Li, is unstable and breaks up spontaneously into a proton and an α particle. During this process, 3.15×10^{-13} J of energy are released, appearing as the kinetic energy of the two decay products. Determine the velocities of the proton and a particle that arise from the decay of a ^{5}Li nucleus at rest. (*Note*: The masses of the proton and alpha particle are $m_p = 1.67 \times 10^{-27}$ kg and $m_\alpha = 6.64 \times 10^{-27}$ kg.)

Picture the Problem This nuclear reaction is $^5\text{Li} \rightarrow \alpha + \text{p} + 3.15 \times 10^{-13}$ J. To conserve momentum, the alpha particle and proton must move in opposite directions. We'll apply both conservation of energy and conservation of momentum to find the speeds of the proton and alpha particle.

Use conservation of momentum in this process to express the alpha particle's speed in terms of the proton's:

$p_i = p_f = 0$

and

$0 = m_p v_p - m_\alpha v_\alpha$

Solve for v_α and substitute for m_α to obtain:

$$v_\alpha = \frac{m_p}{m_\alpha} v_p = \frac{m_p}{4m_p} v_p = \tfrac{1}{4} v_p$$

Letting E represent the energy released in the reaction, apply conservation of energy to the process:

$K_p + K_\alpha = E$

or

$\tfrac{1}{2} m_p v_p^2 + \tfrac{1}{2} m_\alpha v_\alpha^2 = E$

Substitute for v_α:

$\tfrac{1}{2} m_p v_p^2 + \tfrac{1}{2} m_\alpha \left(\tfrac{1}{4} v_p\right)^2 = E$

Solve for v_p and substitute for m_α to obtain:

$$\boldsymbol{v}_p = \sqrt{\frac{32\boldsymbol{E}}{16\boldsymbol{m}_p + \boldsymbol{m}_{\boldsymbol{\alpha}}}}$$

Substitute numerical values and evaluate v_p:

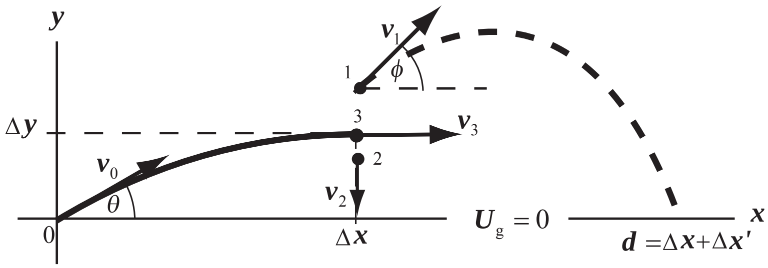
$$\boldsymbol{v}_p = \sqrt{\frac{32\left(3.15\times10^{-13}\ \text{J}\right)}{16\left(1.67\times10^{-27}\ \text{kg}\right) + 6.64\times10^{-27}\ \text{kg}}} = \boxed{1.74\times10^{7}\ \text{m/s}}$$

Use the relationship between v_p and v_α to obtain v_α:

$$v_\alpha = \tfrac{1}{4} v_p = \tfrac{1}{4}\left(1.74\times10^{7}\ \text{m/s}\right) = \boxed{4.34\times10^{6}\ \text{m/s}}$$

75 ••• A 3.00-kg projectile is fired with an initial speed of 120 m/s at an angle of 30.0º with the horizontal. At the top of its trajectory, the projectile explodes into two fragments of masses 1.00 kg and 2.00 kg. At 3.60 s after the explosion the 2.00-kg fragment lands on the ground directly below the point of explosion. (*a*) Determine the velocity of the 1.00-kg fragment immediately after the explosion. (*b*) Find the distance between the point of firing and the point at which the 1.00-kg fragment strikes the ground. (*c*) Determine the energy released in the explosion.

Picture the Problem The pictorial representation shows the projectile at its maximum elevation and is moving horizontally. It also shows the two fragments resulting from the explosion. We'll choose the system to include the projectile and the earth so that no external forces act to change the momentum of the system during the explosion. With this choice of system we can also use conservation of energy to determine the elevation of the projectile when it explodes. We'll also find it useful to use constant-acceleration equations in our description of the motion of the projectile and its fragments. Neglect air resistance.



(*a*) Use conservation of linear momentum to relate the velocity of the projectile before its explosion to the velocities of its two parts after the explosion:

$$\vec{p}_i = \vec{p}_f$$
$$m_3\vec{v}_3 = m_1\vec{v}_1 + m_2\vec{v}_2$$
$$m_3 v_3\hat{i} = m_1 v_{x1}\hat{i} + m_1 v_{y1}\hat{j} - m_2 v_{y2}\hat{j}$$

The only way this equality can hold is if the *x* and *y* components are equal:

$$m_3 v_3 = m_1 v_{x1}$$
and
$$m_1 v_{y1} = m_2 v_{y2}$$

Express v_3 in terms of v_0 and substitute for the masses to obtain:

$$\boldsymbol{v}_{x1} = 3\boldsymbol{v}_3 = 3\boldsymbol{v}_0\cos\boldsymbol{\theta} = 3(120\,\text{m/s})\cos 30.0° = 312\,\text{m/s}$$
and
$$v_{y1} = 2v_{y2} \quad (1)$$

Using a constant-acceleration equation with the downward direction positive, relate v_{y2} to the time it takes the 2.00-kg fragment to hit the ground:

$$\Delta y = v_{y2}\Delta t + \tfrac{1}{2}g(\Delta t)^2$$
$$v_{y2} = \frac{\Delta y - \frac{1}{2}g(\Delta t)^2}{\Delta t} \quad (2)$$

With $U_g = 0$ at the launch site, apply conservation of energy to the climb of the projectile to its maximum elevation:

$$\Delta K + \Delta U = 0$$
Because $K_f = U_i = 0$, $-K_i + U_f = 0$
or
$$-\tfrac{1}{2}m_3 v_{y0}^2 + m_3 g\Delta y = 0$$

Solving for Δ*y* yields:

$$\Delta\boldsymbol{y} = \frac{\boldsymbol{v}_{y0}^2}{2\boldsymbol{g}} = \frac{(\boldsymbol{v}_0\sin\boldsymbol{\theta})^2}{2\boldsymbol{g}}$$

Substitute numerical values and evaluate Δy:

$$\Delta \boldsymbol{y} = \frac{\left[(120\,\text{m/s})\sin 30.0°\right]^2}{2(9.81\,\text{m/s}^2)} = 183.5\,\text{m}$$

Substitute in equation (2) and evaluate v_{y2}:

$$\boldsymbol{v}_{y2} = \frac{183.5\,\text{m} - \frac{1}{2}(9.81\,\text{m/s}^2)(3.60\,\text{s})^2}{3.60\,\text{s}} = 33.31\,\text{m/s}$$

Substitute in equation (1) and evaluate v_{y1}:

$$\boldsymbol{v}_{y1} = 2(33.31\,\text{m/s}) = 66.62\,\text{m/s}$$

Express $\vec{\boldsymbol{v}}_1$ in vector form:

$$\vec{\boldsymbol{v}}_1 = v_{x1}\hat{\boldsymbol{i}} + v_{y1}\hat{\boldsymbol{j}} = \boxed{(312\,\text{m/s})\hat{\boldsymbol{i}} + (66.6\,\text{m/s})\hat{\boldsymbol{j}}}$$

(*b*) Express the total distance d traveled by the 1.00-kg fragment:

$$d = \Delta x + \Delta x' \qquad (3)$$

Relate Δx to v_0 and the time-to-explosion:

$$\Delta \boldsymbol{x} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta})(\Delta \boldsymbol{t}_{\text{exp}}) \qquad (4)$$

Using a constant-acceleration equation, express Δt_{exp}:

$$\Delta t_{\text{exp}} = \frac{v_{y0}}{g} = \frac{v_0 \sin\theta}{g}$$

Substitute numerical values and evaluate Δt_{exp}:

$$\Delta \boldsymbol{t}_{\text{exp}} = \frac{(120\,\text{m/s})\sin 30.0°}{9.81\,\text{m/s}^2} = 6.116\,\text{s}$$

Substitute in equation (4) and evaluate Δx:

$$\Delta \boldsymbol{x} = (120\,\text{m/s})(\cos 30.0°)(6.116\,\text{s}) = 635.6\,\text{m}$$

Relate the distance traveled by the 1.00-kg fragment after the explosion to the time it takes it to reach the ground:

$$\Delta x' = v_{x1}\Delta t'$$

Using a constant-acceleration equation, relate the time $\Delta t'$ for the 1.00-kg fragment to reach the ground to its initial speed in the y direction and the distance to the ground:

$$\Delta y = v_{y1}\Delta t' - \tfrac{1}{2} g (\Delta t')^2$$

Substitute to obtain the quadratic equation:

$$(\Delta t')^2 - (13.6\,\text{s})\Delta t' - 37.4\,\text{s}^2 = 0$$

Solve the quadratic equation to find $\Delta t'$:

$$\Delta t' = 15.945\ \text{s}$$

Substitute in equation (3) and evaluate d:

$$\begin{aligned} \boldsymbol{d} &= \Delta \boldsymbol{x} + \Delta \boldsymbol{x}' = \Delta \boldsymbol{x} + \boldsymbol{v}_{x1}\Delta \boldsymbol{t}' \\ &= 635.6\,\text{m} + (312\,\text{m/s})(15.945\,\text{s}) \\ &= \boxed{5.6\,\text{km}} \end{aligned}$$

(*c*) Express the energy released in the explosion:

$$E_{\text{exp}} = \Delta K = K_\text{f} - K_\text{i} \qquad (5)$$

Find the kinetic energy of the fragments after the explosion:

$$\begin{aligned} \boldsymbol{K}_\text{f} &= \boldsymbol{K}_1 + \boldsymbol{K}_2 = \tfrac{1}{2}\boldsymbol{m}_1\boldsymbol{v}_1^2 + \tfrac{1}{2}\boldsymbol{m}_2\boldsymbol{v}_2^2 \\ &= \tfrac{1}{2}(1.00\,\text{kg})\left[(312\,\text{m/s})^2 + (66.6\,\text{m/s})^2\right] \\ &\quad + \tfrac{1}{2}(2.00\,\text{kg})(33.3\,\text{m/s})^2 \\ &= 52.0\,\text{kJ} \end{aligned}$$

Find the kinetic energy of the projectile before the explosion:

$$\begin{aligned} \boldsymbol{K}_\text{i} &= \tfrac{1}{2}\boldsymbol{m}_3\boldsymbol{v}_3^2 = \tfrac{1}{2}\boldsymbol{m}_3(\boldsymbol{v}_0\cos\boldsymbol{\theta})^2 \\ &= \tfrac{1}{2}(3.00\,\text{kg})[(120\,\text{m/s})\cos 30°]^2 \\ &= 16.2\,\text{kJ} \end{aligned}$$

Substitute in equation (5) to determine the energy released in the explosion:

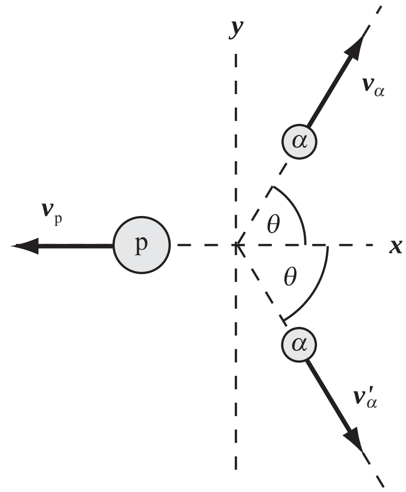
$$\begin{aligned} \boldsymbol{E}_{\text{exp}} &= \boldsymbol{K}_\text{f} - \boldsymbol{K}_\text{i} = 52.0\,\text{kJ} - 16.2\,\text{kJ} \\ &= \boxed{35.8\,\text{kJ}} \end{aligned}$$

76 ••• The boron isotope ^{9}B is unstable and disintegrates into a proton and two α particles. The total energy released as kinetic energy of the decay products is 4.4×10^{-14} J. After one such event, with the ^{9}B nucleus at rest prior to decay, the velocity of the proton is measured as 6.0×10^6 m/s. If the two a particles have equal energies, find the magnitude and the direction of their velocities with respect to the direction of the proton.

Picture the Problem This nuclear reaction is

$$^{9}\text{B} \rightarrow 2\alpha + \text{p} + 4.4\times10^{-14}\ \text{J}.$$

Assume that the proton moves in the –*x* direction as shown in the diagram. The sum of the kinetic energies of the decay products equals the energy released in the decay. We'll use conservation of momentum to find the angle between the velocities of the proton and the alpha particles. Note that $\boldsymbol{v}_\alpha = \boldsymbol{v}'_\alpha$.



Express the energy released to the kinetic energies of the decay products:

$$K_\text{p} + 2K_\alpha = E_\text{rel}$$

or

$$\tfrac{1}{2}m_\text{p}v_\text{p}^2 + 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E_\text{rel}$$

Solving for v_α yields:

$$v_\alpha = \sqrt{\frac{E_\text{rel} - \tfrac{1}{2}m_\text{p}v_\text{p}^2}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$\boldsymbol{v}_\alpha = \sqrt{\frac{4.4\times10^{-14}\ \text{J}}{6.64\times10^{-27}\ \text{kg}} - \frac{\tfrac{1}{2}\left(1.67\times10^{-27}\ \text{kg}\right)\left(6.0\times10^{6}\ \text{m/s}\right)^2}{6.64\times10^{-27}\ \text{kg}}} = 1.44\times10^{6}\ \text{m/s}$$

$$= \boxed{1.4\times10^{6}\ \text{m/s}}$$

Given that the boron isotope was at rest prior to the decay, use conservation of momentum to relate the momenta of the decay products:

$$\vec{\boldsymbol{p}}_\text{f} = \vec{\boldsymbol{p}}_\text{i} = 0$$

or, because $p_{x\text{f}} = 0$,

$$2\left(\boldsymbol{m}_\alpha \boldsymbol{v}_\alpha \cos\boldsymbol{\theta}\right) - \boldsymbol{m}_\text{p}\boldsymbol{v}_\text{p} = 0$$

Substituting for m_α to obtain:

$$2\left(4m_\text{p}v_\alpha \cos\theta\right) - m_\text{p}v_\text{p} = 0$$

Solving for θ yields:

$$\boldsymbol{\theta} = \cos^{-1}\left[\frac{\boldsymbol{v}_\text{p}}{8\boldsymbol{v}_\alpha}\right]$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \cos^{-1}\left[\frac{6.0\times10^{6}\text{ m/s}}{8(1.44\times10^{6}\text{ m/s})}\right] = \pm59°$$

Let θ' equal the angle the velocities of the alpha particles make with that of the proton:

$$\boldsymbol{\theta'} = \pm(180° - 59°) = \boxed{\pm121°}$$

Coefficient of Restitution

77 • **[SSM]** During the design of a new alloy of steel, you are in charge of measuring its coefficient of restitution. You convince your engineering team to accomplish this task by simply dropping a small ball onto a plate, with both the ball and the plate made from the experimental alloy. If the ball is dropped from a height of 3.0 m and rebounds to a height of 2.5 m, what is the coefficient of restitution?

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}}$$

Letting $U_g = 0$ at the surface of the steel plate, apply conservation of energy to obtain:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\boldsymbol{K}_f - \boldsymbol{U}_i = 0$$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_{\text{app}}^2 - \boldsymbol{mgh}_{\text{app}} = 0$$

Solving for v_{app} yields:

$$v_{\text{app}} = \sqrt{2gh_{\text{app}}}$$

In like manner, show that:

$$v_{\text{rec}} = \sqrt{2gh_{\text{rec}}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{\text{rec}}}}{\sqrt{2gh_{\text{app}}}} = \sqrt{\frac{h_{\text{rec}}}{h_{\text{app}}}}$$

Substitute numerical values and evaluate e:

$$\boldsymbol{e} = \sqrt{\frac{2.5\,\text{m}}{3.0\,\text{m}}} = \boxed{0.91}$$

78 • According to the official rules of racquetball, a ball acceptable for tournament play must bounce to a height of between 173 and 183 cm when dropped from a height of 254 cm at room temperature. What is the acceptable range of values for the coefficient of restitution for the racquetball–floor system?

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}}$$

Letting $U_g = 0$ at the surface of the steel plate, the mechanical energy of the ball-Earth system is:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\boldsymbol{K}_{\text{f}} - \boldsymbol{U}_{\text{i}} = 0$$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}\boldsymbol{mv}_{\text{app}}^2 - \boldsymbol{mgh}_{\text{app}} = 0$$

Solve for v_{app}:

$$v_{\text{app}} = \sqrt{2gh_{\text{app}}}$$

In like manner, show that:

$$v_{\text{rec}} = \sqrt{2gh_{\text{rec}}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{\text{rec}}}}{\sqrt{2gh_{\text{app}}}} = \sqrt{\frac{h_{\text{rec}}}{h_{\text{app}}}}$$

Substitute numerical values and evaluate e_{min}:

$$e_{\text{min}} = \sqrt{\frac{173\,\text{cm}}{254\,\text{cm}}} = 0.825$$

Substitute numerical values and evaluate e_{max}:

$$e_{\text{max}} = \sqrt{\frac{183\,\text{cm}}{254\,\text{cm}}} = 0.849$$

and $\boxed{0.825 \le e \le 0.849}$

79 •• A ball bounces to 80 percent of its original height. (*a*) What fraction of its mechanical energy is lost each time it bounces? (*b*) What is the coefficient of restitution of the ball–floor system?

Picture the Problem Because the rebound kinetic energy is proportional to the rebound height, the percentage of mechanical energy lost in one bounce can be inferred from knowledge of the rebound height. The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which an object was dropped and the height to which it rebounded by using conservation of mechanical energy.

(*a*) We know, because the mechanical energy of the ball-earth system is constant, that the kinetic energy of an object dropped from a given height h is proportional to h. If, for each bounce of the ball, $h_{rec} = 0.80h_{app}$, 20% of its mechanical energy is lost.

(*b*) Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{rec}}{v_{app}}$$

Letting $U_g = 0$ at the surface from which the ball is rebounding, the mechanical energy of the ball is:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\boldsymbol{K}_f - \boldsymbol{U}_i = 0$$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}\boldsymbol{mv}_{app}^2 - \boldsymbol{mgh}_{app} = 0$$

Solve for v_{app}:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute for $\frac{h_{rec}}{h_{app}}$ to obtain:

$$\boldsymbol{e} = \sqrt{0.80} = \boxed{0.89}$$

80 •• A 2.0-kg object moving to the right at 6.0 m/s collides head-on with a 4.0-kg object that is initially at rest. After the collision, the 2.0-kg object is moving to the left at 1.0 m/s. (*a*) Find the velocity of the 4.0-kg object after the collision. (*b*) Find the energy lost in the collision. (*c*) What is the coefficient of restitution for these objects?

Picture the Problem Let the numerals 2 and 4 refer, respectively, to the 2.0-kg object and the 4.0-kg object. Choose a coordinate system in which the direction the 2.0-kg object is moving before the collision is the positive x direction and let the system consist of Earth, the surface on which the objects slide, and the objects. Then we can use conservation of momentum to find the velocity of the recoiling 4.0-kg object. We can find the energy transformed in the collision by calculating the difference between the pre- and post-collision kinetic energies and find the coefficient of restitution from its definition.

(*a*) Use conservation of linear momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_2 v_{2i} = m_4 v_{4f} - m_2 v_{2f}$$

Solve for the final velocity of the 4.0-kg object:

$$v_{4f} = \frac{m_2 v_{2i} + m_2 v_{2f}}{m_4}$$

Substitute numerical values and evaluate v_{4f}:

$$v_{4f} = \frac{(2.0\,\text{kg})(6.0\,\text{m/s} + 1.0\,\text{m/s})}{4.0\,\text{kg}} = 3.50\,\text{m/s} = \boxed{3.5\,\text{m/s}}$$

(*b*) Express the energy lost in terms of the kinetic energies before and after the collision:

$$E_{\text{lost}} = K_i - K_f = \tfrac{1}{2} m_2 v_{2i}^2 - \left(\tfrac{1}{2} m_2 v_{2f}^2 + \tfrac{1}{2} m_4 v_{4f}^2\right) = \tfrac{1}{2}\left[m_2\left(v_{2i}^2 - v_{2f}^2\right) - m_4 v_{4f}^2\right]$$

Substitute numerical values and evaluate E_{lost}:

$$E_{\text{lost}} = \tfrac{1}{2}\left[\left((2.0\,\text{kg})\left\{(6.0\,\text{m/s})^2 - (1.0\,\text{m/s})^2\right\}\right) - (4.0\,\text{kg})(3.50\,\text{m/s})^2\right] = \boxed{11\,\text{J}}$$

(*c*) From the definition of the coefficient of restitution we have:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}} = \frac{v_{4f} - v_{2f}}{v_{2i}}$$

Substitute numerical values and evaluate *e*:

$$e = \frac{3.50\,\text{m/s} - (-1.0\,\text{m/s})}{6.0\,\text{m/s}} = \boxed{0.75}$$

81 •• A 2.0-kg block moving to the right with speed of 5.0 m/s collides with a 3.0-kg block that is moving in the same direction at 2.0 m/s, as in Figure 8-49. After the collision, the 3.0-kg block moves to the right at 4.2 m/s. Find (*a*) the velocity of the 2.0-kg block after the collision and (*b*) the coefficient of restitution between the two blocks.

Picture the Problem Let the numeral 2 refer to the 2.0-kg block and the numeral 3 to the 3.0-kg block. Choose a coordinate system in which the direction the blocks are moving before the collision is the +*x* direction and let the system consist of Earth, the surface on which the blocks move, and the blocks. Then we can use conservation of momentum to find the velocity of the 2.0-kg block after the collision. We can find the coefficient of restitution from its definition.

(*a*) Use conservation of linear momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_2 v_{2i} + m_3 v_{3i} = m_2 v_{2f} + m_3 v_{3f}$$

Solve for the final velocity of the 2.0-kg object:

$$v_{2f} = \frac{m_2 v_{2i} + m_3 v_{3i} - m_3 v_{3f}}{m_2}$$

Substitute numerical values and evaluate v_{2f}:

$$\boldsymbol{v}_{2f} = \frac{(2.0\,\text{kg})(5.0\,\text{m/s}) + (3.0\,\text{kg})(2.0\,\text{m/s} - 4.2\,\text{m/s})}{2.0\,\text{kg}} = 1.70\,\text{m/s} = \boxed{1.7\,\text{m/s}}$$

(*b*) From the definition of the coefficient of restitution we have:

$$\boldsymbol{e} = \frac{\boldsymbol{v}_{rec}}{\boldsymbol{v}_{app}} = \frac{\boldsymbol{v}_{3f} - \boldsymbol{v}_{2f}}{\boldsymbol{v}_{2i} - \boldsymbol{v}_{3i}}$$

Substitute numerical values and evaluate *e*:

$$\boldsymbol{e} = \frac{4.2\,\text{m/s} - 1.7\,\text{m/s}}{5.0\,\text{m/s} - 2.0\,\text{m/s}} = \boxed{0.83}$$

82 ••• To keep homerun records and distances consistent from year to year, organized baseball randomly checks the coefficient of restitution between new baseballs and wooden surfaces similar to that of an average bat. Suppose you are in charge of making sure that no ″juiced″ baseballs are produced. (*a*) In a random test, you find one that when dropped from 2.0 m rebounds 0.25 m. What is the coefficient of restitution for this ball? (*b*) What is the maximum distance home run shot you would expect from this ball, neglecting any effects due to air resistance and making reasonable assumption for bat speeds and incoming pitch speeds? Is this a ″juiced″ ball, a ″normal″ ball, or a ″dead″ ball?

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy. We can use the same elevation range equation to find the maximum home run you would expect from the ball with the experimental coefficient of restitution.

(*a*) The coefficient of restitution is the ratio of the speeds of approach and recession:

$$e = \frac{v_{rec}}{v_{app}} \quad (1)$$

Letting $U_g = 0$ at the surface of from which the ball rebounds, the mechanical energy of the ball-earth system is:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$, $\boldsymbol{K}_f - \boldsymbol{U}_i = 0$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}\boldsymbol{mv}_{app}^2 - \boldsymbol{mgh}_{app} = 0$$

Solve for v_{app} to obtain:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute for v_{rec} and v_{app} in equation (1) and simplify to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute numerical values and evaluate e:

$$\boldsymbol{e} = \sqrt{\frac{0.25\,\text{m}}{2.0\,\text{m}}} = 0.3536 = \boxed{0.35}$$

(*b*) The "same-elevation" range equation is:

$$\boldsymbol{R} = \frac{\boldsymbol{v}_{rec}^2 \sin 2\boldsymbol{\theta}}{\boldsymbol{g}} = \frac{\boldsymbol{e}^2\boldsymbol{v}_{app}^2 \sin 2\boldsymbol{\theta}}{\boldsymbol{g}} \quad (2)$$

v_{app} is the sum of the speed of the ball and the speed of the bat:

$$\boldsymbol{v}_{app} = \boldsymbol{v}_{ball} + \boldsymbol{v}_{bat}$$

Assuming that the bat travels about 1 m in 0.2 s yields:

$$\boldsymbol{v}_{bat} = \frac{1\,\text{m}}{0.2\,\text{s}} = 5\,\text{m/s}$$

Assuming that the speed of the baseball thrown by the pitcher is close to 100 mi/h yields:

$$\boldsymbol{v}_{ball} \approx 45\,\text{m/s}$$

Evaluate $\boldsymbol{v}_{app}$ to obtain:

$$\boldsymbol{v}_{app} = 45\,\text{m/s} + 5\,\text{m/s} = 50\,\text{m/s}$$

Assuming a 45° launch angle, substitute numerical values in equation (2) and evaluate R:

$$\boldsymbol{R} = \frac{(0.3536)^2 (50\ \text{m/s})^2 \sin 2(45°)}{9.81\ \text{m/s}^2} \approx \boxed{32\ \text{m}}$$

Because home runs must travel at least 100 m in modern major league ballparks, this is a ″dead″ ball and should be tossed out.

83 •• **[SSM]** To make puck handling easy, hockey pucks are kept frozen until they are used in the game. (*a*) Explain why room temperature pucks would be more difficult to handle on the end of a stick than a frozen puck. (*Hint: Hockey pucks are made of rubber*.) (*b*) A room-temperature puck rebounds 15 cm when dropped onto a wooden surface from 100 cm. If a frozen puck has only half the coefficient of restitution of a room-temperature one, predict how high the frozen puck would rebound under the same conditions.

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy.

(*a*) At room-temperature rubber will bounce more when it hits a stick than it will at freezing temperatures.

(*b*) The mechanical energy of the rebounding puck is constant:

$$\Delta\boldsymbol{K} + \Delta\boldsymbol{U} = 0$$

or, because $K_\text{f} = U_\text{i} = 0$,

$$-\boldsymbol{K}_\text{i} + \boldsymbol{U}_\text{f} = 0$$

If the puck's speed of recession is v_rec and it rebounds to a height h, then:

$$-\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_\text{rec}^2 + \boldsymbol{mgh} = 0 \Rightarrow \boldsymbol{h} = \frac{\boldsymbol{v}_\text{rec}^2}{2\boldsymbol{g}}$$

The coefficient of restitution is the ratio of the speeds of approach and recession:

$$e = \frac{v_\text{rec}}{v_\text{app}} \Rightarrow \boldsymbol{v}_\text{rec} = \boldsymbol{e}\boldsymbol{v}_\text{app} \qquad (1)$$

Substitute for v_rec to obtain:

$$\boldsymbol{h} = \frac{\boldsymbol{e}^2\boldsymbol{v}_\text{app}^2}{2\boldsymbol{g}} \qquad (2)$$

Letting $U_g = 0$ at the surface of from which the puck rebounds, the mechanical energy of the puck-Earth system is:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$\boldsymbol{K}_f - \boldsymbol{U}_i = 0$$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}mv_{app}^2 - mgh_{app} = 0$$

Solve for v_{app} to obtain:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute for v_{rec} and v_{app} in equation (1) and simplify to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute numerical values and evaluate $e_{room\ temp}$:

$$e_{room\ temp} = \sqrt{\frac{15\,\text{cm}}{100\,\text{cm}}} = 0.387$$

For the falling puck, v_{app} is given by:

$$v_{app} = \sqrt{2gH}$$

where H is the height from which the puck was dropped.

Substituting for v_{app} in equation (2) and simplifying yields:

$$h = \frac{2e^2 gH}{2g} = e^2 H \qquad (3)$$

For the room-temperature puck:

$$e_{frozen} = \tfrac{1}{2}e_{room\ temp}$$

Substituting for e in equation (3) yields:

$$h = \tfrac{1}{4}e_{room\ temp}^2 H$$

Substitute numerical values and evaluate h:

$$\boldsymbol{h} = \tfrac{1}{4}(0.387)^2(100\,\text{cm}) = \boxed{3.8\,\text{cm}}$$

Remarks: The puck that rebounds only 3.8 cm is a much ″deader″ and, therefore, much better puck.

Collisions in More Than One Dimension

84 •• In Section 8-3 it was proven by using geometry that when a particle elastically collides with another particle of equal mass that is initially at rest, the two post-collision velocities are perpendicular. Here we examine another way of proving this result that illustrates the power of vector notation. (*a*) Given that $\vec{A} = \vec{B} + \vec{C}$, square both sides of this equation (obtain the scalar product of each side with itself) to show that $A^2 = B^2 + C^2 + 2\vec{B}\cdot\vec{C}$. (*b*) Let the momentum of the initially moving particle be $\vec{P}$ and the momenta of the particles after the collision be $\vec{p}_1$ and $\vec{p}_2$. Write the vector equation for the conservation of linear momentum and square both sides (obtain the dot product of each side with itself). Compare it to the equation gotten from the elastic-collision condition (kinetic energy is conserved) and finally show that these two equations imply that $\vec{p}_1 \cdot \vec{p}_2 = 0$.

Picture the Problem We can use the definition of the magnitude of a vector and the definition of the scalar product to establish the result called for in (*a*). In Part (*b*) we can use the result of Part (*a*), the conservation of momentum, and the definition of an elastic collision (kinetic energy is conserved) to show that the particles separate at right angles.

(*a*) Find the dot product of $\vec{B}+\vec{C}$ with itself:

$$\left(\vec{B}+\vec{C}\right)\cdot\left(\vec{B}+\vec{C}\right)=B^2+C^2+2\vec{B}\cdot\vec{C}$$

Because $\vec{A}=\vec{B}+\vec{C}$:

$$A^2=\left|\vec{B}+\vec{C}\right|^2=\left(\vec{B}+\vec{C}\right)\cdot\left(\vec{B}+\vec{C}\right)$$

Substitute for $\left(\vec{B}+\vec{C}\right)\cdot\left(\vec{B}+\vec{C}\right)$ to obtain:

$$\boxed{A^2=B^2+C^2+2\vec{B}\cdot\vec{C}}$$

(*b*) Apply conservation of momentum to the collision of the particles:

$$\vec{p}_1+\vec{p}_2=\vec{p}$$

Form the scalar product of each side of this equation with itself to obtain:

$$\left(\vec{p}_1+\vec{p}_2\right)\cdot\left(\vec{p}_1+\vec{p}_2\right)=\vec{p}\cdot\vec{p}$$

or

$$p_1^2+p_2^2+2\vec{p}_1\cdot\vec{p}_2=p^2 \qquad (1)$$

Use the definition of an elastic collision to obtain:

$$\frac{p_1^2}{2m}+\frac{p_2^2}{2m}=\frac{p^2}{2m}$$

or

$$p_1^2+p_2^2=p^2 \qquad (2)$$

Subtract equation (1) from equation (2) to obtain:

$$2\vec{p}_1 \cdot \vec{p}_2 = 0 \text{ or } \boxed{\vec{p}_1 \cdot \vec{p}_2 = 0}$$

i.e., the particles move apart along paths that are at right angles to each other.

85 •• During a pool game, the cue ball, which has an initial speed of 5.0 m/s, makes an elastic collision with the eight ball, which is initially at rest. After the collision, the eight ball moves at an angle of 30º to the right of the original direction of the cue ball. Assume that the balls have equal masses. (*a*) Find the direction of motion of the cue ball immediately after the collision. (*b*) Find the speed of each ball immediately after the collision.

Picture the Problem Let the initial direction of motion of the cue ball be the +*x* direction. We can apply conservation of energy to determine the angle the cue ball makes with the +*x* direction and the conservation of momentum to find the final velocities of the cue ball and the eight ball.

(*a*) Use conservation of energy to relate the velocities of the collision participants before and after the collision:

$$\tfrac{1}{2}mv_{\text{ci}}^2 = \tfrac{1}{2}mv_{\text{cf}}^2 + \tfrac{1}{2}mv_8^2$$

or

$$v_{\text{ci}}^2 = v_{\text{cf}}^2 + v_8^2$$

This Pythagorean relationship tells us that $\vec{v}_{\text{ci}}$, $\vec{v}_{\text{cf}}$, and $\vec{v}_8$ form a right triangle. Hence:

$$\theta_{\text{cf}} + \theta_8 = 90°$$

and

$$\theta_{\text{cf}} = \boxed{60°}$$

(*b*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$\vec{p}_{\text{xi}} = \vec{p}_{\text{xf}}$$

or

$$mv_{\text{ci}} = mv_{\text{cf}}\cos\theta_{\text{cf}} + mv_8\cos\theta_8$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\vec{p}_{\text{yi}} = \vec{p}_{\text{yf}}$$

or

$$0 = mv_{\text{cf}}\sin\theta_{\text{cf}} + mv_8\sin\theta_8$$

Solve these equations simultaneously to obtain:

$$\boldsymbol{v}_{\text{cf}} = \boxed{2.50\,\text{m/s}} \text{ and } \boldsymbol{v}_8 = \boxed{4.33\,\text{m/s}}$$

86 •• Object A that has a mass m and a velocity $v_0\hat{\boldsymbol{i}}$ collides head-on with object B that has a mass $2m$ and a velocity $\frac{1}{2}\boldsymbol{v}_0\hat{\boldsymbol{i}}$. Following the collision, object B has a velocity of $\frac{1}{4}\boldsymbol{v}_0\hat{\boldsymbol{i}}$. (*a*) Determine the velocity of object A after the collision. (*b*) Is the collision elastic? If not, express the change in the kinetic energy in terms of m and v_0.

Picture the Problem We can find the final velocity of the object whose mass is m by using the conservation of momentum. Whether the collision was elastic can be decided by examining the difference between the initial and final kinetic energy of the interacting objects.

(*a*) Use conservation of linear momentum to relate the initial and final velocities of the two objects:

$$\vec{\boldsymbol{p}}_\text{i} = \vec{\boldsymbol{p}}_\text{f}$$

or

$$mv_0\hat{\boldsymbol{i}} + 2m\left(\tfrac{1}{2}v_0\hat{\boldsymbol{j}}\right) = 2m\left(\tfrac{1}{4}v_0\hat{\boldsymbol{i}}\right) + m\vec{\boldsymbol{v}}_{1\text{f}}$$

Simplify to obtain:

$$v_0\hat{\boldsymbol{i}} + v_0\hat{\boldsymbol{j}} = \tfrac{1}{2}v_0\hat{\boldsymbol{i}} + \vec{\boldsymbol{v}}_{1\text{f}}$$

Solving for $\vec{\boldsymbol{v}}_{1\text{f}}$ yields:

$$\vec{\boldsymbol{v}}_{1\text{f}} = \boxed{\tfrac{1}{2}v_0\hat{\boldsymbol{i}} + v_0\hat{\boldsymbol{j}}}$$

(*b*) Express the difference between the kinetic energy of the system before the collision and its kinetic energy after the collision:

$$\Delta \boldsymbol{E} = \boldsymbol{K}_\text{i} - \boldsymbol{K}_\text{f} = \boldsymbol{K}_{1\text{i}} + \boldsymbol{K}_{2\text{i}} - \left(\boldsymbol{K}_{1\text{f}} + \boldsymbol{K}_{2\text{f}}\right)$$

Substituting for the kinetic energies yields:

$$\Delta \boldsymbol{E} = \tfrac{1}{2}\left(\boldsymbol{m}\boldsymbol{v}_{1\text{i}}^2 + 2\boldsymbol{m}\boldsymbol{v}_{2\text{i}}^2 - \boldsymbol{m}\boldsymbol{v}_{1\text{f}}^2 - 2\boldsymbol{m}\boldsymbol{v}_{2\text{f}}^2\right)$$

Substitute for speeds and simplify to obtain:

$$\Delta \boldsymbol{E} = \tfrac{1}{2}\boldsymbol{m}\left[\boldsymbol{v}_0^2 + 2\left(\tfrac{1}{4}\boldsymbol{v}_0^2\right) - \tfrac{5}{4}\boldsymbol{v}_0^2 - 2\left(\tfrac{1}{16}\boldsymbol{v}_0^2\right)\right] = \boxed{\tfrac{1}{16}\boldsymbol{m}\boldsymbol{v}_0^2}$$

Because $\Delta E \neq 0$, the collision is **inelastic**.

87 •• **[SSM]** A puck of mass 5.0 kg moving at 2.0 m/s approaches an identical puck that is stationary on frictionless ice. After the collision, the first puck leaves with a speed v_1 at 30º to the original line of motion; the second puck leaves with speed v_2 at 60º, as in Figure 8-50. (*a*) Calculate v_1 and v_2. (*b*) Was the collision elastic?

Picture the Problem Let the direction of motion of the puck that is moving before the collision be the +*x* direction. Applying conservation of momentum to the collision in both the *x* and *y* directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously. We can decide whether the collision was elastic by either calculating the system's kinetic energy before and after the collision or by determining whether the angle between the final velocities is 90°.

(*a*) Use conservation of linear momentum in the *x* direction to obtain:

$$\boldsymbol{p}_{\text{xi}} = \boldsymbol{p}_{\text{xf}}$$

or

$$m\boldsymbol{v} = m\boldsymbol{v}_1 \cos 30° + m\boldsymbol{v}_2 \cos 60°$$

Simplify further to obtain:

$$\boldsymbol{v} = \boldsymbol{v}_1 \cos 30° + \boldsymbol{v}_2 \cos 60° \qquad (1)$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\boldsymbol{p}_{\text{yi}} = \boldsymbol{p}_{\text{yf}}$$

or

$$0 = m\boldsymbol{v}_1 \sin 30° - m\boldsymbol{v}_2 \sin 60°$$

Simplifying further yields:

$$0 = \boldsymbol{v}_1 \sin 30° - \boldsymbol{v}_2 \sin 60° \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\boldsymbol{v}_1 = \boxed{1.7\,\text{m/s}} \text{ and } \boldsymbol{v}_2 = \boxed{1.0\,\text{m/s}}$$

(*b*) Because the angle between $\vec{\boldsymbol{v}}_1$ and $\vec{\boldsymbol{v}}_2$ is 90°, the collision was elastic.

88 •• Figure 8-51 shows the result of a collision between two objects of unequal mass. (*a*) Find the speed v_2 of the larger mass after the collision and the angle θ_2. (*b*) Show that the collision is elastic.

Picture the Problem Let the direction of motion of the object that is moving before the collision be the +*x* direction. Applying conservation of momentum to the motion in both the *x* and *y* directions will lead us to two equations in the unknowns v_2 and θ_2 that we can solve simultaneously. We can show that the collision was elastic by showing that the system's kinetic energy before and after the collision is the same.

(*a*) Use conservation of linear momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$p_{xi} = p_{xf}$$

or

$$3mv_0 = \sqrt{5}mv_0 \cos\theta_1 + 2mv_2 \cos\theta_2$$

or

$$3v_0 = \sqrt{5}v_0 \cos\theta_1 + 2v_2 \cos\theta_2$$

Use conservation of linear momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\boldsymbol{p}_{yi} = \boldsymbol{p}_{yf}$$

or

$$0 = \sqrt{5}\boldsymbol{mv}_0 \sin\boldsymbol{\theta}_1 - 2\boldsymbol{mv}_2 \sin\boldsymbol{\theta}_2$$

Simplifying further yields:

$$0 = \sqrt{5}\boldsymbol{v}_0 \sin\boldsymbol{\theta}_1 - 2\boldsymbol{v}_2 \sin\boldsymbol{\theta}_2$$

Note that if $\tan\theta_1 = 2$, then:

$$\cos\theta_1 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_1 = \frac{2}{\sqrt{5}}$$

Substitute in the *x*-direction momentum equation and simplify to obtain:

$$3v_0 = \sqrt{5}v_0 \frac{1}{\sqrt{5}} + 2v_2 \cos\theta_2$$

or

$$v_0 = v_2 \cos\theta_2 \qquad (1)$$

Substitute in the *y*-direction momentum equation and simplify to obtain:

$$0 = \sqrt{5}v_0 \frac{2}{\sqrt{5}} - 2v_2 \sin\theta_2$$

or

$$0 = v_0 - v_2 \sin\theta_2 \qquad (2)$$

Solve equations (1) and (2) simultaneously for θ_2 :

$$\boldsymbol{\theta}_2 = \tan^{-1}(1) = \boxed{45.0^\circ}$$

Substitute in equation (1) to find v_2:

$$\boldsymbol{v}_2 = \frac{\boldsymbol{v}_0}{\cos\boldsymbol{\theta}_2} = \frac{\boldsymbol{v}_0}{\cos 45^\circ} = \boxed{\sqrt{2}\boldsymbol{v}_0}$$

(*b*) To show that the collision was elastic, find the before-collision and after-collision kinetic energies:

$$K_i = \tfrac{1}{2}m(3v_0)^2 = 4.5mv_0^2$$

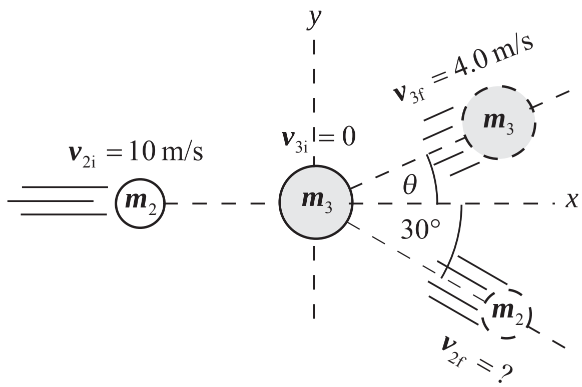
and

$$K_f = \tfrac{1}{2}m\left(\sqrt{5}v_0\right)^2 + \tfrac{1}{2}(2m)\left(\sqrt{2}v_0\right)^2 = 4.5mv_0^2$$

Because $K_i = K_f$, the collision is elastic.

89 •• A 2.0-kg ball moving at 10 m/s makes an off-center collision with a 3.0-kg ball that is initially at rest. After the collision, the 2.0-kg ball is deflected at an angle of 30º from its original direction of motion and the 3.0-kg ball is moving at 4.0 m/s. Find the speed of the 2.0-kg ball and the direction of the 3.0-kg ball after the collision. *Hint*: $\sin^2\alpha + \cos^2\alpha = 1$.

Picture the Problem Let the direction of motion of the ball that is moving before the collision be the +*x* direction and use the subscripts 2 and 3 to designate the 2.0-kg and 3.0-kg balls, respectively. Applying conservation of momentum to the collision in both the *x* and *y* directions will lead us to two equations in the unknowns v_{2f} and θ that we can solve simultaneously.



Use conservation of momentum in the *x* and *y* directions to relate the speeds and directions of the balls before and after the collision:

$$m_2 v_{2i} = m_2 v_{2f}\cos 30° + m_3 v_{3f}\sin\theta$$

and

$$0 = m_3 v_{3f}\sin\theta - m_2 v_{2f}\sin 30°$$

Solve the first of these equations for $\cos\theta$ to obtain:

$$\cos\theta = \frac{m_2 v_{2i} - m_2 v_{2f}\cos 30°}{m_3 v_{3f}} \quad (1)$$

Solve the second of these equations for $\sin\theta$ to obtain:

$$\sin\theta = \frac{m_2 v_{2f}\sin 30°}{m_3 v_{3f}} \quad (2)$$

Using the hint given in the problem statement, square and add equations (1) and (2) and simplify the result to obtain the quadratic equation:

$$v_{2f}^2\sin^2 30° + \left(v_{2i} - v_{2f}\cos 30°\right)^2 = \frac{m_3^2 v_{3f}^2}{m_2^2}$$

Substituting numerical values and simplifying yields:

$$v_{2f}^2 + \left(10\text{ m/s} - 0.866 v_{2f}\right)^2 = 144\text{ m}^2/\text{s}^2$$

Use the quadratic formula or your graphing calculator to obtain:

$$\boldsymbol{v}_{2f} = 5.344\text{ m/s or } 11.977\text{ m/s}$$

Because the larger of these values corresponds to there being more kinetic energy in the system after the collision than there was before the collision:

$$\boldsymbol{v}_{2f} = \boxed{5.3\text{ m/s}}$$

Solving equation (2) for θ yields:

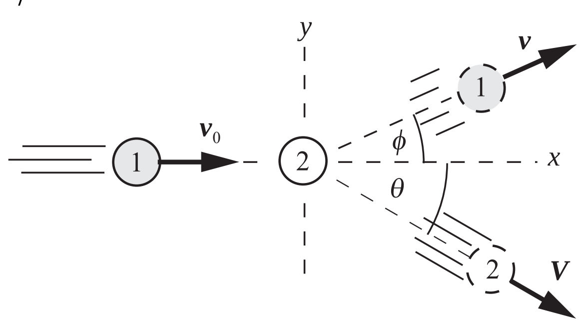
$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{\boldsymbol{m}_2\boldsymbol{v}_{2f}\sin 30°}{\boldsymbol{m}_3\boldsymbol{v}_{3f}}\right]$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{(2.0\text{ kg})(5.344\text{ m/s})\sin 30°}{(3.0\text{ kg})(4.0\text{ m/s})}\right] = \boxed{26°}$$

90 •• A particle has initial speed v_0. It collides with a second particle with the same mass that is initially at rest, and is deflected through an angle ϕ. Its speed after the collision is v. The second particle recoils, and its velocity makes an angle θ with the initial direction of the first particle. (*a*) Show that $\tan\theta = \dfrac{v\sin\phi}{(v_0 - v\cos\phi)}$. (*b*) Show that if the collision is elastic, then $v = v_0\cos\phi$.

Picture the Problem Choose the coordinate system shown in the following diagram with the +*x* direction the direction of the initial approach of the projectile particle. Call *V* the speed of the target particle after the collision. In Part (*a*) we can apply conservation of momentum in the *x* and *y* directions to obtain two equations that we can solve simultaneously for tanθ. In Part (*b*) we can use conservation of momentum in vector form and the elastic-collision equation to show that $v = v_0\cos\phi$.



(*a*) Apply conservation of linear momentum in the *x* direction to obtain:

$$v_0 = v\cos\phi + V\cos\theta \qquad (1)$$

Apply conservation of linear momentum in the y direction to obtain:

$$v\sin\phi = V\sin\theta \qquad (2)$$

Solve equation (1) for $V\cos\theta$:

$$V\cos\theta = v_0 - v\cos\phi \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{V\sin\theta}{V\cos\theta} = \frac{v\sin\phi}{v_0 - v\cos\phi}$$

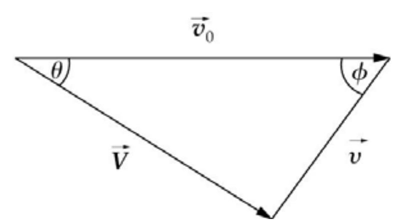
or

$$\tan\theta = \boxed{\frac{v\sin\phi}{v_0 - v\cos\phi}}$$

(*b*) Noting that the masses of the particles are equal, apply conservation of linear momentum to obtain:

$$\vec{v}_0 = \vec{v} + \vec{V}$$

Draw the vector diagram representing this equation:



Use the definition of an elastic collision to obtain:

$$v_0^2 = v^2 + V^2$$

If this Pythagorean condition is to hold, the third angle of the triangle must be a right angle and, using the definition of the cosine function:

$$v = \boxed{v_0\cos\phi}$$

*Center-of-Mass Reference Frame

91 •• In the center-of-mass reference frame a particle with mass m_1 and momentum p_1 makes an elastic head-on collision with a second particle of mass m_2 and momentum $p_2 = -p_1$. After the collision its momentum is p_1'. Write the total kinetic energy in terms of m_1, m_2, and p_1 and the total final energy in terms of m_1, m_2, and p_1', and show that $\boldsymbol{p}_1' = \pm\boldsymbol{p}_1$. If $\boldsymbol{p}_1' = -\boldsymbol{p}_1$, the particle is merely turned around by the collision and leaves with the speed it had initially. What is the situation for the $\boldsymbol{p}_1' = +\boldsymbol{p}_1$ solution?

Picture the Problem The total kinetic energy of a system of particles is the sum of the kinetic energy of the center of mass and the kinetic energy relative to the center of mass. The kinetic energy of a particle of mass m is related to its momentum according to $K = p^2/2m$.

Express the total kinetic energy of the system:

$$K = K_{\text{rel}} + K_{\text{cm}} \quad (1)$$

Relate the kinetic energy relative to the center of mass to the momenta of the two particles:

$$K_{\text{rel}} = \frac{p_1^2}{2m_1} + \frac{p_1^2}{2m_2} = \frac{p_1^2(m_1 + m_2)}{2m_1 m_2}$$

Express the kinetic energy of the center of mass of the two particles:

$$K_{\text{cm}} = \frac{(2p_1)^2}{2(m_1 + m_2)} = \frac{2p_1^2}{m_1 + m_2}$$

Substitute in equation (1) and simplify to obtain:

$$K = \frac{p_1^2(m_1 + m_2)}{2m_1 m_2} + \frac{2p_1^2}{m_1 + m_2} = \frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]$$

In an elastic collision:

$$K_{\text{i}} = K_{\text{f}}$$

$$= \boxed{\frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]}$$

$$= \boxed{\frac{p_1'^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]}$$

Simplify to obtain:

$$(p_1')^2 = (p_1)^2 \Rightarrow \boxed{p_1' = \pm p_1}$$

and if $\boldsymbol{p}_1' = +\boldsymbol{p}_1$, the particles do not collide.

92 •• A 3.0-kg block is traveling in the −*x* direction at 5.0 m/s, and a 1.0-kg block is traveling in the +*x* direction at 3.0 m/s. (*a*) Find the velocity v_{cm} of the center of mass. (*b*) Subtract v_{cm} from the velocity of each block to find the velocity of each block in the center-of-mass reference frame. (*c*) After they make a head-on elastic collision, the velocity of each block is reversed (in the center-of-mass frame). Find the velocity of each block in the center-of-mass frame after the collision. (*d*) Transform back into the original frame by adding v_{cm} to the velocity of each block. (*e*) Check your result by finding the initial and final kinetic energies of the blocks in the original frame and comparing them.

Picture the Problem Let the numerals 3 and 1 denote the blocks whose masses are 3.0 kg and 1.0 kg respectively. We can use $\sum_i m_i\vec{v}_i = M\vec{v}_{\rm cm}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = m_1\vec{v}_1 + m_3\vec{v}_3 = M\vec{v}_{\rm cm} = (m_1 + m_3)\vec{v}_{\rm cm}$$

Solve for $\vec{v}_{\rm cm}$:

$$\vec{v}_{\rm cm} = \frac{m_3\vec{v}_3 + m_1\vec{v}_1}{m_3 + m_1}$$

Substitute numerical values and evaluate $\vec{v}_{\rm cm}$:

$$\vec{v}_{\rm cm} = \frac{(3.0\,\text{kg})(-5.0\,\text{m/s})\hat{i} + (1.0\,\text{kg})(3.0\,\text{m/s})\hat{i}}{3.0\,\text{kg} + 1.0\,\text{kg}} = \boxed{(-3.0\,\text{m/s})\hat{i}}$$

(*b*) Find the velocity of the 3-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{\rm cm} = (-5.0\,\text{m/s})\hat{i} - (-3.0\,\text{m/s})\hat{i} = \boxed{(-2.0\,\text{m/s})\hat{i}}$$

Find the velocity of the 1-kg block in the center of mass reference frame:

$$\vec{u}_1 = \vec{v}_1 - \vec{v}_{\rm cm} = (3.0\,\text{m/s})\hat{i} - (-3.0\,\text{m/s})\hat{i} = \boxed{(6.0\,\text{m/s})\hat{i}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(2.0\,\text{m/s})\hat{i}}$$

and

$$\vec{u}_1' = \boxed{(-6.0\,\text{m/s})\hat{i}}$$

(*d*) Transform the after-collision velocity of the 3-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{\rm cm} = (2.0\,\text{m/s})\hat{i} + (-3.0\,\text{m/s})\hat{i} = \boxed{(-1.0\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 1-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_1' = \vec{u}_1' + \vec{v}_{cm} = (-6.0\,\text{m/s})\hat{i} + (-3.0\,\text{m/s})\hat{i} = \boxed{(-9.0\,\text{m/s})\hat{i}}$$

(*e*) Express K_i in the original frame of reference:

$$K_i = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_1v_1^2$$

Substitute numerical values and evaluate K_i:

$$K_i = \tfrac{1}{2}\left[(3.0\,\text{kg})(5.0\,\text{m/s})^2 + (1.0\,\text{kg})(3.0\,\text{m/s})^2\right] = \boxed{42\,\text{J}}$$

Express K_i in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3v_3'^2 + \tfrac{1}{2}m_1v_1'^2$$

Substitute numerical values and evaluate K_f:

$$K_f = \tfrac{1}{2}\left[(3.0\,\text{kg})(1.0\,\text{m/s})^2 + (1.0\,\text{kg})(9.0\,\text{m/s})^2\right] = \boxed{42\,\text{J}}$$

93 •• **[SSM]** Repeat Problem 92 with the second block having a mass of 5.0 kg and moving to the right at 3.0 m/s.

Picture the Problem Let the numerals 3 and 5 denote the blocks whose masses are 3.0 kg and 5.0 kg respectively. We can use $\sum_i m_i\vec{v}_i = M\vec{v}_{cm}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = m_3\vec{v}_3 + m_5\vec{v}_5 = M\vec{v}_{cm} = (m_3 + m_5)\vec{v}_{cm}$$

Solve for $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{m_3\vec{v}_3 + m_5\vec{v}_5}{m_3 + m_5}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3.0\,\text{kg})(-5.0\,\text{m/s})\hat{i} + (5.0\,\text{kg})(3.0\,\text{m/s})\hat{i}}{3.0\,\text{kg} + 5.0\,\text{kg}} = \boxed{0}$$

(*b*) Find the velocity of the 3.0-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{cm} = (-5.0\,\text{m/s})\,\hat{i} - 0$$
$$= \boxed{(-5.0\,\text{m/s})\,\hat{i}}$$

Find the velocity of the 5.0-kg block in the center of mass reference frame:

$$\vec{u}_5 = \vec{v}_5 - \vec{v}_{cm} = (3.0\,\text{m/s})\hat{i} - 0$$
$$= \boxed{(3.0\,\text{m/s})\hat{i}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(5.0\,\text{m/s})\hat{i}}$$
and
$$u_5' = \boxed{0.75\,\text{m/s}}$$

(*d*) Transform the after-collision velocity of the 3.0-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{cm} = (5.0\,\text{m/s})\hat{i} + 0$$
$$= \boxed{(5.0\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 5.0-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_5' = \vec{u}_5' + \vec{v}_{cm} = (-3.0\,\text{m/s})\hat{i} + 0$$
$$= \boxed{(-3.0\,\text{m/s})\hat{i}}$$

(*e*) Express K_i in the original frame of reference:

$$K_i = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_5v_5^2$$

Substitute numerical values and evaluate K_i:

$$\boldsymbol{K}_i = \tfrac{1}{2}\left[(3.0\,\text{kg})(5.0\,\text{m/s})^2 + (5.0\,\text{kg})(3.0\,\text{m/s})^2\right] = \boxed{60\,\text{J}}$$

Express K_f in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3v'^2_3 + \tfrac{1}{2}m_5v'^2_5$$

Substitute numerical values and evaluate K_f:

$$\boldsymbol{K}_f = \tfrac{1}{2}\left[(3.0\,\text{kg})(5.0\,\text{m/s})^2 + (5.0\,\text{kg})(3.0\,\text{m/s})^2\right] = \boxed{60\,\text{J}}$$

*Systems With Continuously Varying Mass: Rocket Propulsion

94 • A rocket burns fuel at a rate of 200 kg/s and exhausts the gas at a relative speed of 6.00 km/s relative to the rocket. Find the magnitude of the thrust of the rocket.

Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel dm/dt and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt| u_{ex}$.

Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{th} = \left|\frac{dm}{dt}\right| u_{ex}$$

Substitute numerical values and evaluate F_{th}:

$$\boldsymbol{F}_{th} = (200\,\text{kg/s})(6.00\,\text{km/s}) = \boxed{1.20\,\text{MN}}$$

95 •• A rocket has an initial mass of 30,000 kg, of which 80 percent is the fuel. It burns fuel at a rate of 200 kg/s and exhausts its gas at a relative speed of 1.80 km/s. Find (*a*) the thrust of the rocket, (*b*) the time until burnout, and (*c*) its speed at burnout assuming it moves straight upward near the surface of Earth. Assume that *g* is constant and neglect any effects of air resistance.

Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel dm/dt and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt| u_{ex}$. The final velocity v_f of a rocket depends on the relative speed of its exhaust gases u_{ex}, its payload to initial mass ratio m_f/m_0 and its burn time according to $v_f = -u_{ex}\ln(m_f/m_0) - gt_b$.

(*a*) Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{th} = \left|\frac{dm}{dt}\right| u_{ex}$$

Substitute numerical values and evaluate F_{th}:

$$\boldsymbol{F}_{th} = (200\,\text{kg/s})(1.80\,\text{km/s}) = \boxed{360\,\text{kN}}$$

(*b*) Relate the time to burnout to the mass of the fuel and its burn rate:

$$t_b = \frac{m_{fuel}}{dm/dt} = \frac{0.8m_0}{dm/dt}$$

Substitute numerical values and evaluate t_b:

$$t_b = \frac{(0.80)(30{,}000\,\text{kg})}{200\,\text{kg/s}} = \boxed{120\,\text{s}}$$

(*c*) Relate the final velocity of a rocket to its initial mass, exhaust velocity, and burn time:

$$v_f = -u_{ex}\ln\left(\frac{m_f}{m_0}\right) - gt_b$$

Substitute numerical values and evaluate v_f:

$$\boldsymbol{v}_f = -(1.80\,\text{km/s})\ln\left(\frac{1}{5}\right) - (9.81\,\text{m/s}^2)(120\,\text{s}) = \boxed{1.72\,\text{km/s}}$$

96 •• The *specific impulse* of a rocket propellant is defined as $I_{sp} = F_{th}/(Rg)$, where F_{th} is the thrust of the propellant, g the magnitude of free-fall acceleration, and R the rate at which the propellant is burned. The rate depends predominantly on the type and exact mixture of the propellant. (*a*) Show that the specific impulse has the dimension of time. (*b*) Show that $u_{ex} = gI_{sp}$, where u_{ex} is the relative speed of the exhaust. (*c*) What is the specific impulse (in seconds) of the propellant used in the Saturn V rocket of Example 8-16.

Picture the Problem We can use the dimensions of thrust, burn rate, and acceleration to show that the dimension of specific impulse is time. Combining the definitions of rocket thrust and specific impulse will lead us to $u_{ex} = gI_{sp}$.

(*a*) Express the dimension of specific impulse in terms of the dimensions of F_{th}, R, and g:

$$[I_{sp}] = \frac{[F_{th}]}{[R][g]} = \frac{\frac{\text{M}\cdot\text{L}}{\text{T}^2}}{\frac{\text{M}}{\text{T}}\cdot\frac{\text{L}}{\text{T}^2}} = \boxed{\text{T}}$$

(*b*) From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Solve for u_{ex}:

$$u_{ex} = \frac{F_{th}}{R}$$

Substitute for F_{th} to obtain:

$$\boldsymbol{u}_{ex} = \frac{\boldsymbol{RgI}_{sp}}{\boldsymbol{R}} = \boxed{\boldsymbol{gI}_{sp}} \qquad (1)$$

(*c*) Solve equation (1) for I_{sp} and substitute for u_{ex} to obtain:

$$I_{sp} = \frac{F_{th}}{Rg}$$

From Example 8-21 we have:

$R = 1.384\times10^4$ kg/s
and
$F_{th} = 3.4\times10^6$ N

Substitute numerical values and evaluate I_{sp}:

$$\boldsymbol{I}_{sp} = \frac{3.4\times10^{6}\ \text{N}}{(1.384\times10^{4}\ \text{kg/s})(9.81\,\text{m/s}^2)} = \boxed{25\,\text{s}}$$

97 ••• **[SSM]** The initial *thrust-to-weight ratio* τ_0 of a rocket is $\tau_0 = F_{th}/(m_0 g)$, where F_{th} is the rocket's thrust and m_0 the initial mass of the rocket, including the propellant. (*a*) For a rocket launched straight up from the earth's surface, show that $\tau_0 = 1 + (a_0/g)$, where a_0 is the initial acceleration of the rocket. For manned rocket flight, τ_0 cannot be made much larger than 4 for the comfort and safety of the astronauts. (The astronauts will feel that their weight as the rocket lifts off is equal to τ_0 times their normal weight.) (*b*) Show that the final velocity of a rocket launched from the earth's surface, in terms of τ_0 and I_{sp} (see Problem 96) can be written as

$$v_f = gI_{sp}\left[1n\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau_0}\left(1-\frac{m_f}{m_0}\right)\right]$$

where m_f is the mass of the rocket (not including the spent propellant). (*c*) Using a **spreadsheet** program or **graphing calculator**, graph v_f as a function of the mass ratio m_0/m_f for I_{sp} = 250 s and τ_0 = 2 for values of the mass ratio from 2 to 10. (Note that the mass ratio cannot be less than 1.) (*d*) To lift a rocket into orbit, a final velocity after burnout of v_f = 7.0 km/s is needed. Calculate the mass ratio required of a single stage rocket to do this, using the values of specific impulse and thrust ratio given in Part (*b*). For engineering reasons, it is difficult to make a rocket with a mass ratio much greater than 10. Can you see why multistage rockets are usually used to put payloads into orbit around the earth?

Picture the Problem We can use the rocket equation and the definition of rocket thrust to show that $\tau_0 = 1 + a_0/g$. In Part (*b*) we can express the burn time t_b in terms of the initial and final masses of the rocket and the rate at which the fuel burns, and then use this equation to express the rocket's final velocity in terms of I_{sp}, τ_0, and the mass ratio m_0/m_f. In Part (*d*) we'll need to use trial-and-error methods or a graphing calculator to solve the transcendental equation giving v_f as a function of m_0/m_f.

(*a*) Express the rocket equation:

$$-mg + Ru_{ex} = ma$$

From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Substitute for Ru_{ex} to obtain:

$$-mg + F_{th} = ma$$

Solve for F_{th} at takeoff:

$$F_{th} = m_0 g + m_0 a_0$$

Divide both sides of this equation by $m_0 g$ to obtain:

$$\frac{F_{\text{th}}}{m_0 g} = 1 + \frac{a_0}{g}$$

Because $\tau_0 = F_{\text{th}}/(m_0 g)$:

$$\tau_0 = \boxed{1 + \frac{a_0}{g}}$$

(*b*) Use Equation 8-39 to express the final speed of a rocket that starts from rest with mass m_0:

$$v_{\text{f}} = u_{\text{ex}} \ln\left(\frac{m_0}{m_{\text{f}}}\right) - g t_{\text{b}}, \qquad (1)$$

where t_{b} is the burn time.

Express the burn time in terms of the burn rate R (assumed constant):

$$t_{\text{b}} = \frac{m_0 - m_{\text{f}}}{R} = \frac{m_0}{R}\left(1 - \frac{m_{\text{f}}}{m_0}\right)$$

Multiply t_{b} by one in the form $\frac{gF_{\text{th}}}{gF_{\text{th}}}$ and simplify to obtain:

$$t_{\text{b}} = \frac{gF_{\text{th}}}{gF_{\text{th}}}\frac{m_0}{R}\left(1 - \frac{m_{\text{f}}}{m_0}\right) = \frac{gm_0}{F_{\text{th}}}\frac{F_{\text{th}}}{gR}\left(1 - \frac{m_{\text{f}}}{m_0}\right) = \frac{I_{\text{sp}}}{\tau_0}\left(1 - \frac{m_{\text{f}}}{m_0}\right)$$

Substitute in equation (1):

$$v_{\text{f}} = u_{\text{ex}} \ln\left(\frac{m_0}{m_{\text{f}}}\right) - \frac{gI_{\text{sp}}}{\tau_0}\left(1 - \frac{m_{\text{f}}}{m_0}\right)$$

From Problem 96 we have:

$$u_{\text{ex}} = gI_{\text{sp}},$$

where u_{ex} is the exhaust velocity of the propellant.

Substitute for u_{ex} and factor to obtain:

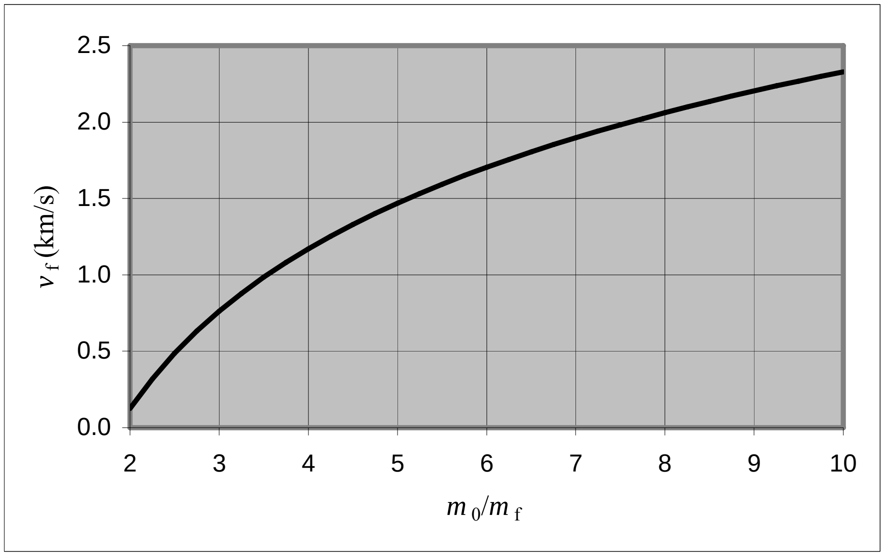
$$v_{\text{f}} = gI_{\text{sp}} \ln\left(\frac{m_0}{m_{\text{f}}}\right) - \frac{gI_{\text{sp}}}{\tau_0}\left(1 - \frac{m_{\text{f}}}{m_0}\right) = \boxed{gI_{\text{sp}}\left[\ln\left(\frac{m_0}{m_{\text{f}}}\right) - \frac{1}{\tau_0}\left(1 - \frac{m_{\text{f}}}{m_0}\right)\right]}$$

(*c*) A spreadsheet program to calculate the final velocity of the rocket as a function of the mass ratio m_0/m_{f} is shown below. The constants used in the velocity function and the formulas used to calculate the final velocity are as follows:

Cell	Content/Formula	Algebraic Form
B1	250	I_{sp}
B2	9.81	g
B3	2	τ_0
D9	D8 + 0.25	m_0/m_f
E8	B2*B1*(LOG(D8) – (1/B3)*(1/D8))	$gI_{sp}\left[\ln\left(\frac{m_0}{m_f}\right)-\frac{1}{\tau_0}\left(1-\frac{m_f}{m_0}\right)\right]$

	A	B	C	D	E
1	I_{sp} =	250	s		
2	g =	9.81	m/s^2		
3	τ_0=	2			
4					
5					
6					
7				mass ratio	v_f
8				2.00	1.252E+02
9				2.25	3.187E+02
10				2.50	4.854E+02
11				2.75	6.316E+02
12				3.00	7.614E+02
36				9.00	2.204E+03
37				9.25	2.237E+03
38				9.50	2.269E+03
39				9.75	2.300E+03
40				10.00	2.330E+03
41				725.00	7.013E+03

A graph of final velocity as a function of mass ratio follows.



(*d*) Substitute the data given in part (*c*) in the equation derived in Part (*b*) to obtain:

$$7.00\,\text{km/s} = \left(9.81\,\text{m/s}^2\right)\left(250\,\text{s}\right)\left(\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{2}\left(1 - \frac{m_f}{m_0}\right)\right)$$

or

$$2.854 = \ln x - 0.5 + \frac{0.5}{x} \quad \text{where } x = m_0/m_f.$$

Use trial-and-error methods or a graphing calculator to solve this transcendental equation for the root greater than 1:

$\boldsymbol{x} \approx \boxed{28}$, a value considerably larger than the practical limit of 10 for single-stage rockets.

98 •• The height that a model rocket launched from Earth's surface can reach can be estimated by assuming that the burn time is short compared to the total flight time, so for most of the flight the rocket is in free-fall. (This estimate neglects the burn time in calculations of both time and displacement.) For a model rocket with specific impulse I_{sp} = 100 s, mass ratio m_0/m_f = 1.20, and initial thrust-to-weight ratio τ_0 = 5.00 (these parameters are defined in Problems 96 and 97), estimate (*a*) the height the rocket can reach, and (*b*) the total flight time. (*c*) Justify the assumption used in the estimates by comparing the flight time from Part (*b*) to the time it takes for the fuel to be spent.

Picture the Problem We can use the velocity-at-burnout equation from Problem 96 to find v_f and constant-acceleration equations to approximate the maximum height the rocket will reach and its total flight time.

(*a*) Assuming constant acceleration, relate the maximum height reached by the model rocket to its time-to-top-of-trajectory:

$$h=\tfrac{1}{2}gt_{\text{top}}^2 \quad (1)$$

From Problem 96 we have:

$$v_{\text{f}}=gI_{\text{sp}}\left(\ln\left(\frac{m_0}{m_{\text{f}}}\right)-\frac{1}{\tau}\left(1-\frac{m_{\text{f}}}{m_0}\right)\right)$$

Evaluate the velocity at burnout v_{f} for I_{sp} = 100 s, m_0/m_{f} = 1.2, and τ = 5:

$$v_{\text{f}}=\left(9.81\,\text{m/s}^2\right)\left(100\,\text{s}\right)\left[\ln(1.2)-\frac{1}{5}\left(1-\frac{1}{1.2}\right)\right]=146\,\text{m/s}$$

Assuming that the time for the fuel to burn up is short compared to the total flight time, find the time to the top of the trajectory:

$$t_{\text{top}}=\frac{v_{\text{f}}}{g}=\frac{146\,\text{m/s}}{9.81\,\text{m/s}^2}=14.9\,\text{s}$$

Substitute in equation (1) and evaluate *h*:

$$\boldsymbol{h}=\tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(14.9\,\text{s}\right)^2=\boxed{1.09\,\text{km}}$$

(*b*) Find the total flight time from the time it took the rocket to reach its maximum height:

$$\boldsymbol{t}_{\text{flight}}=2\boldsymbol{t}_{\text{top}}=2\left(14.9\,\text{s}\right)=\boxed{29.8\,\text{s}}$$

(*c*) The fuel burn time t_{b} is:

$$t_{\text{b}}=\frac{I_{sp}}{\tau}\left(1-\frac{m_f}{m_0}\right)=\frac{100\,\text{s}}{5}\left(1-\frac{1}{1.2}\right)$$
$$=3.33\,\text{s}$$

Because this burn time is approximately 1/5 of the total flight time, we can't expect the answer we obtain in Part (*b*) to be very accurate. It should, however, be good to about 30% accuracy, as the maximum distance the model rocket could possibly move in this time is $\tfrac{1}{2}\boldsymbol{v}\boldsymbol{t}_{\text{b}}=244\,\text{m}$, assuming constant acceleration until burnout.

General Problems

99 • **[SSM]** A 250-g model-train car traveling at 0.50 m/s links up with a 400-g car that is initially at rest. What is the speed of the cars immediately after they link up? Find the pre- and post-collision kinetic energies of the two-car system.

Picture the Problem Let the direction the 250-g car is moving before the collision be the +*x* direction. Let the numeral 1 refer to the 250-kg car, the numeral 2 refer to the 400-kg car, and *V* represent the velocity of the linked cars. Let the system include Earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their pre- and post-collision kinetic energies.

Use conservation of momentum to relate the speeds of the cars immediately before and immediately after their collision:

$$p_{ix} = p_{fx}$$

or

$$m_1 v_1 = (m_1 + m_2)V \Rightarrow V = \frac{m_1 v_1}{m_1 + m_2}$$

Substitute numerical values and evaluate *V*:

$$\boldsymbol{V} = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = 0.192\,\text{m/s}$$

$$= \boxed{0.19\,\text{m/s}}$$

Find the pre-collision kinetic energy of the cars:

$$\boldsymbol{K}_{\text{pre}} = \tfrac{1}{2}\boldsymbol{m}_1\boldsymbol{v}_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2$$

$$= \boxed{31\,\text{mJ}}$$

Find the post-collision kinetic energy of the coupled cars:

$$\boldsymbol{K}_{\text{post}} = \tfrac{1}{2}(\boldsymbol{m}_1 + \boldsymbol{m}_2)\boldsymbol{V}^2$$

$$= \tfrac{1}{2}(0.250\,\text{kg} + 0.400\,\text{kg})(0.192\,\text{m/s})^2$$

$$= \boxed{12\,\text{mJ}}$$

100 • A 250-g model train car traveling at 0.50 m/s heads toward a 400-g car that is initially at rest. (*a*) Find the kinetic energy of the two-car system. (*b*) Find the velocity of each car in the center-of-mass reference frame, and use these velocities to calculate the kinetic energy of the two-car system in the center-of-mass reference. (*c*) Find the kinetic energy associated with the motion of the center of mass of the system. (*d*) Compare your answer for Part (*a*) with the sum of your answers for Parts (*b*) and (*c*).

Picture the Problem Let the direction the 250-g car is moving before the collision be the +*x* direction. Let the numeral 1 refer to the 250-kg car and the numeral 2 refer to the 400-g car and the system include Earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their pre- and post-collision kinetic energies.

(*a*) The pre-collision kinetic energy of the two-car system is:

$$K_{pre} = \tfrac{1}{2}m_1 v_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2$$
$$= 31.3\,\text{mJ} = \boxed{31\,\text{mJ}}$$

(*b*) Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_i m_i \vec{\boldsymbol{v}}_i = m\vec{\boldsymbol{v}}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = 0.192\,\text{m/s}$$

Find the initial velocity of the 250-g car relative to the velocity of the center of mass:

$$\boldsymbol{u}_1 = \boldsymbol{v}_1 - \boldsymbol{v}_{cm} = 0.50\,\text{m/s} - 0.192\,\text{m/s}$$
$$= \boxed{0.31\,\text{m/s}}$$

Find the initial velocity of the 400-g car relative to the velocity of the center of mass:

$$\boldsymbol{u}_2 = \boldsymbol{v}_2 - \boldsymbol{v}_{cm} = 0\,\text{m/s} - 0.192\,\text{m/s}$$
$$= \boxed{-0.19\,\text{m/s}}$$

Express the pre-collision kinetic energy of the system relative to the center of mass:

$$\boldsymbol{K}_{pre,rel} = \tfrac{1}{2}\boldsymbol{m}_1\boldsymbol{u}_1^2 + \tfrac{1}{2}\boldsymbol{m}_2\boldsymbol{u}_2^2$$

Substitute numerical values and evaluate $\boldsymbol{K}_{pre,rel}$:

$$\boldsymbol{K}_{pre,rel} = \tfrac{1}{2}(0.250\,\text{kg})(0.308\,\text{m/s})^2$$
$$+ \tfrac{1}{2}(0.400\,\text{kg})(-0.192\,\text{m/s})^2$$
$$= \boxed{19\,\text{mJ}}$$

(*c*) Express the kinetic energy of the center of mass:

$$K_{cm} = \tfrac{1}{2}Mv_{cm}^2$$

Substitute numerical values and evaluate K_{cm}:

$$\boldsymbol{K}_{cm} = \tfrac{1}{2}(0.650\,\text{kg})(0.192\,\text{m/s})^2$$
$$= \boxed{12\,\text{mJ}}$$

(*d*) Relate the pre-collision kinetic energy of the system to its pre-collision kinetic energy relative to the center of mass and the kinetic energy of the center of mass:

$$K_{\rm i} = K_{\rm i,rel} + K_{\rm cm} = 19.2\,{\rm mJ} + 12.0\,{\rm mJ} = 31.2\,{\rm mJ}$$

and

$$\boldsymbol{K}_{\rm i} = \boxed{\boldsymbol{K}_{\rm i,rel} + \boldsymbol{K}_{\rm cm}}$$

101 •• A 1500-kg car traveling north at 70 km/h collides at an intersection with a 2000-kg car traveling west at 55 km/h. The two cars stick together. (*a*) What is the total momentum of the system before the collision? (*b*) Find the magnitude and direction of the velocity of the wreckage just after the collision.

Picture the Problem Let east be the positive *x* direction and north the positive *y* direction. Include both cars and the earth in the system and let the numeral 1 denote the 1500-kg car and the numeral 2 the 2000-kg car. Because the net external force acting on the system is zero, momentum is conserved in this perfectly inelastic collision.

(*a*) Express the total momentum of the system:

$$\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2 = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2 = m_1 v_1\,\hat{\boldsymbol{j}} - m_2 v_2\,\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{p}}$:

$$\begin{aligned}\vec{\boldsymbol{p}} &= (1500\,{\rm kg})(70\,{\rm km/h})\hat{\boldsymbol{j}} - (2000\,{\rm kg})(55\,{\rm km/h})\hat{\boldsymbol{i}}\\ &= -\left(1.10\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{i}} + \left(1.05\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{j}}\\ &= \boxed{-\left(1.1\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{i}} + \left(1.1\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{j}}}\end{aligned}$$

(*b*) The velocity of the wreckage in terms of the total momentum of the system is given by:

$$\vec{\boldsymbol{v}}_{\rm f} = \vec{\boldsymbol{v}}_{\rm cm} = \frac{\vec{\boldsymbol{p}}}{M}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\rm f}$:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\rm f} &= \frac{-\left(1.10\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{i}}}{1500\,{\rm kg} + 2000\,{\rm kg}} + \frac{\left(1.05\times10^5\,{\rm kg\cdot km/h}\right)\hat{\boldsymbol{j}}}{1500\,{\rm kg} + 2000\,{\rm kg}}\\ &= -(31.4\,{\rm km/h})\,\hat{\boldsymbol{i}} + (30.0\,{\rm km/h})\hat{\boldsymbol{j}}\end{aligned}$$

Find the magnitude of the velocity of the wreckage:

$$v_{\rm f} = \sqrt{(31.4\,{\rm km/h})^2 + (30.0\,{\rm km/h})^2} = \boxed{43\,{\rm km/h}}$$

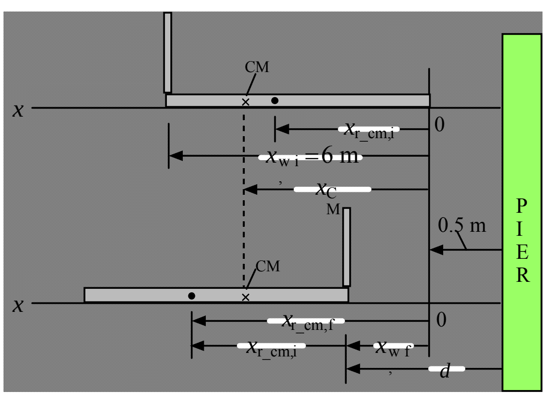
Find the direction the wreckage moves:

$$\theta = \tan^{-1}\left[\frac{30.0\,\text{km/h}}{-31.4\,\text{km/h}}\right] = -43.7°$$

The direction of the wreckage is 46° west of north.

102 •• A 60-kg woman stands on the back of a 6.0-m-long, 120-kg raft that is floating at rest in still water. The raft is 0.50 m from a fixed pier, as shown in Figure 8-52. (*a*) The woman walks to the front of the raft and stops. How far is the raft from the pier now? (*b*) While the woman walks, she maintains a constant speed of 3.0 m/s relative to the raft. Find the total kinetic energy of the system (woman plus raft), and compare with the kinetic energy if the woman walked at 3.0 m/s on a raft tied to the pier. (*c*) Where does these kinetic energies come from, and where do they go when the woman stops at the front of the raft? (*d*) On land, the woman puts a lead shot 6.0 m. She stands at the back of the raft, aims forward, and puts the shot so that just after it leaves her hand, it has the same velocity relative to her as it did when she threw it from the ground. Approximately, where does her shot land?

Picture the Problem Take the origin to be at the initial position of the right-hand end of raft and let the positive *x* direction be to the left. Let "*w*" denote the woman and "*r*" the raft, *d* be the distance of the end of the raft from the pier after the woman has walked to its front. The raft moves to the left as the woman moves to the right; with the center of mass of the woman-raft system remaining fixed (because $F_{\text{ext,net}} = 0$). The diagram shows the initial ($x_{\text{w,i}}$) and final ($x_{\text{w,f}}$) positions of the woman as well as the initial ($x_{\text{r\_cm,i}}$) and final ($x_{\text{r\_cm,f}}$) positions of the center of mass of the raft both before and after the woman has walked to the front of the raft.



(*a*) Express the distance of the raft from the pier after the woman has walked to the front of the raft:

$$\boldsymbol{d} = 0.50\,\text{m} + \boldsymbol{x}_{\text{w, f}} \tag{1}$$

Express x_{cm} before the woman has walked to the front of the raft:

$$x_{cm} = \frac{m_w x_{w,i} + m_r x_{r\_cm,i}}{m_w + m_r}$$

Express x_{cm} after the woman has walked to the front of the raft:

$$x_{cm} = \frac{m_w x_{w,f} + m_r x_{r\_cm,f}}{m_w + m_r}$$

Because $F_{ext,net} = 0$, the center of mass remains fixed and we can equate these two expressions for x_{cm} to obtain:

$$m_w x_{w,i} + m_r x_{r\_cm,i} = m_w x_{w,f} + m_r x_{r\_cm,f}$$

Solve for $\boldsymbol{x}_{w,f}$:

$$x_{w,f} = x_{w,i} - \frac{m_r}{m_w}\left(x_{r\_cm,f} - x_{r\_cm,i}\right)$$

From the figure it can be seen that $\boldsymbol{x}_{r\_cm,f} - \boldsymbol{x}_{r\_cm,i} = \boldsymbol{x}_{w,f}$. Substitute $\boldsymbol{x}_{w,f}$ for $\boldsymbol{x}_{r\_cm,f} - \boldsymbol{x}_{r\_cm,i}$ to obtain:

$$x_{w,f} = \frac{m_w x_{w,i}}{m_w + m_r}$$

Substitute numerical values and evaluate $\boldsymbol{x}_{w,f}$:

$$\boldsymbol{x}_{w,f} = \frac{(60\,\text{kg})(6.0\,\text{m})}{60\,\text{kg} + 120\,\text{kg}} = 2.0\,\text{m}$$

Substitute in equation (1) to obtain:

$$\boldsymbol{d} = 2.0\,\text{m} + 0.50\,\text{m} = \boxed{2.5\,\text{m}}$$

(*b*) Express the total kinetic energy of the system:

$$K_{tot} = \tfrac{1}{2} m_w v_w^2 + \tfrac{1}{2} m_r v_r^2$$

Noting that the elapsed time is 2.0 s, find v_w and v_r:

$$\boldsymbol{v}_w = \frac{\boldsymbol{x}_{w,f} - \boldsymbol{x}_{w,i}}{\Delta \boldsymbol{t}} = \frac{2.0\,\text{m} - 6.0\,\text{m}}{2.0\,\text{s}} = -2.0\,\text{m/s}$$

relative to the dock, and

$$\boldsymbol{v}_r = \frac{\boldsymbol{x}_{r,f} - \boldsymbol{x}_{r,i}}{\Delta \boldsymbol{t}} = \frac{2.50\,\text{m} - 0.50\,\text{m}}{2.0\,\text{s}} = 1.0\,\text{m/s}$$

also relative to the dock.

Substitute numerical values and evaluate K_{tot}:

$$K_{\text{tot}} = \tfrac{1}{2}(60\,\text{kg})(-2.0\,\text{m/s})^2 + \tfrac{1}{2}(120\,\text{kg})(1.0\,\text{m/s})^2 = \boxed{0.18\,\text{kJ}}$$

Evaluate K with the raft tied to the pier:

$$K_{\text{tot}} = \tfrac{1}{2}m_{\text{w}}v_{\text{w}}^2 = \tfrac{1}{2}(60\,\text{kg})(3.0\,\text{m/s})^2 = \boxed{0.27\,\text{kJ}}$$

(*c*) All the kinetic energy derives from the chemical energy of the woman and, assuming she stops via static friction, the kinetic energy is transformed into her internal energy.

(*d*) After the shot leaves the woman's hand, the raft-woman system constitutes an inertial reference frame. In that frame, the shot has the same initial velocity as did the shot that had a range of 6.0 m in the reference frame of the land. Thus, in the raft-woman frame, the shot also has a range of 6.0 m and lands at the front of the raft.

103 •• A 1.0-kg steel ball and a 2.0-m cord of negligible mass make up a simple pendulum that can pivot without friction about the point *O*, as in Figure 8-53. This pendulum is released from rest in a horizontal position and when the ball is at its lowest point it strikes a 1.0-kg block sitting at rest on a shelf. Assume that the collision is perfectly elastic and take the coefficient of kinetic friction between the block and shelf to be 0.10. (*a*) What is the velocity of the block just after impact? (*b*) How far does the block slide before coming to rest (assuming the shelf is long enough)?

Picture the Problem Let the zero of gravitational potential energy be at the elevation of the 1.0-kg block. We can use conservation of energy to find the speed of the bob just before its perfectly elastic collision with the block and conservation of momentum to find the speed of the block immediately after the collision. We'll apply Newton's 2$^{\text{nd}}$ law to find the acceleration of the sliding block and use a constant-acceleration equation to find how far it slides before coming to rest.

(*a*) Use conservation of energy to find the speed of the bob just before its collision with the block:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2} m_{ball} v_{ball}^2 + m_{ball} g\Delta h = 0$$

and

$$v_{ball} = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_{ball}:

$$\boldsymbol{v}_{ball} = \sqrt{2(9.81\,\text{m/s}^2)(2.0\,\text{m})} = 6.26\ \text{m/s}$$

Because the collision is perfectly elastic and the ball and block have the same mass:

$$\boldsymbol{v}_{block} = \boldsymbol{v}_{ball} = \boxed{6.3\,\text{m/s}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the block to its acceleration and initial speed:

$$\boldsymbol{v}_f^2 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}_{block}\Delta\boldsymbol{x}$$

or, because $v_f = 0$,

$$0 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}_{block}\Delta\boldsymbol{x}$$

Solving for Δx yields:

$$\Delta\boldsymbol{x} = \frac{-\boldsymbol{v}_i^2}{2\boldsymbol{a}_{block}} = \frac{-\boldsymbol{v}_{block}^2}{2\boldsymbol{a}_{block}}$$

Apply $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the sliding block:

$$\sum F_x = -f_k = ma_{block}$$

and

$$\sum F_y = F_n - m_{block}g = 0$$

Using the definition of f_k ($=\mu_k F_n$) eliminate f_k and F_n between the two equations and solve for a_{block}:

$$a_{block} = -\mu_k g$$

Substitute for a_{block} to obtain:

$$\Delta x = \frac{-v_{block}^2}{-2\mu_k g} = \frac{v_{block}^2}{2\mu_k g}$$

Substitute numerical values and evaluate Δx:

$$\Delta\boldsymbol{x} = \frac{(6.26\,\text{m/s})^2}{2(0.10)(9.81\,\text{m/s}^2)} = \boxed{20\,\text{m}}$$

104 •• Figure 8-54 shows a World War I cannon mounted on a railcar so that it will project a shell at an angle of 30º. With the car initially at rest, the cannon fires a 200-kg projectile at 125 m/s. (All values are for the frame of reference of the track.) Now consider a system composed of a cannon, shell, and railcar, all on the frictionless track. (*a*) Will the total vector momentum of that system be the same just before and just after the shell is fired? Explain your answer. (*b*) If the mass of the railcar plus cannon is 5000 kg, what will be the recoil velocity of the car along the track after the firing? (*c*) The shell is observed to rise to a maximum height of 180 m as it moves through its trajectory. At this point, its speed is 80.0 m/s. On the basis of this information, calculate the amount of thermal energy produced by air friction on the shell on its way from firing to this maximum height.

Picture the Problem We can use conservation of momentum in the horizontal direction to find the recoil velocity of the car along the track after the firing. Because the shell will neither rise as high nor be moving as fast at the top of its trajectory as it would be in the absence of air friction, we can apply the work-energy theorem to find the amount of thermal energy produced by the air friction.

(*a*) No. The vertical reaction force of the rails is an external force and so the momentum of the system will not be conserved.

(*b*) Use conservation of momentum in the horizontal (*x*) direction to obtain:

$$\Delta p_x = 0$$

or

$$mv\cos 30° - Mv_{\text{recoil}} = 0$$

Solving for v_{recoil} yields:

$$v_{\text{recoil}} = \frac{mv\cos 30°}{M}$$

Substitute numerical values and evaluate v_{recoil}:

$$\boldsymbol{v}_{\text{recoil}} = \frac{(200\,\text{kg})(125\,\text{m/s})\cos 30°}{5000\,\text{kg}} = \boxed{4.3\,\text{m/s}}$$

(*c*) Using the work-energy theorem, relate the thermal energy produced by air friction to the change in the energy of the system:

$$W_{\text{ext}} = W_{\text{f}} = \Delta E_{\text{sys}} = \Delta U + \Delta K$$

Substitute for ΔU and ΔK to obtain:

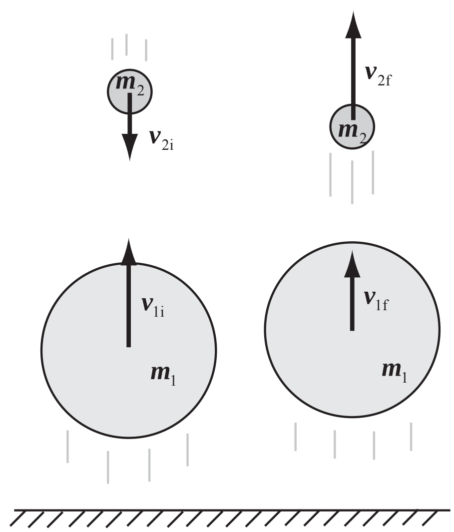
$$W_{\text{ext}} = mgy_{\text{f}} - mgy_{\text{i}} + \tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_i^2 = mg(y_{\text{f}} - y_{\text{i}}) + \tfrac{1}{2}m(v_{\text{f}}^2 - v_i^2)$$

Substitute numerical values and evaluate W_{ext}:

$$\boldsymbol{W}_{\text{ext}} = (200\,\text{kg})(9.81\,\text{m/s}^2)(180\,\text{m}) + \tfrac{1}{2}(200\,\text{kg})\left[(80.0\,\text{m/s})^2 - (125\,\text{m/s})^2\right] = \boxed{-569\,\text{kJ}}$$

105 ••• [SSM] One popular, if dangerous, classroom demonstration involves holding a baseball an inch or so directly above a basketball, holding the basketball a few feet above a hard floor, and dropping the two balls simultaneously. The two balls will collide just after the basketball bounces from the floor; the baseball will then rocket off into the ceiling tiles with a hard "thud" while the basketball will stop in midair. (The author of this problem once broke a light doing this.)
(*a*) Assuming that the collision of the basketball with the floor is elastic, what is the relation between the velocities of the balls just before they collide?
(*b*) Assuming the collision between the two balls is elastic, use the result of Part (*a*) and the conservation of momentum and energy to show that, if the basketball is three times as heavy as the baseball, the final velocity of the basketball will be zero. (This is approximately the true mass ratio, which is why the demonstration is so dramatic.) (*c*) If the speed of the baseball is *v* just before the collision, what is its speed just after the collision?

Picture the Problem Let the numeral 1 refer to the basketball and the numeral 2 to the baseball. The left-hand side of the diagram shows the balls after the basketball's elastic collision with the floor and just before they collide. The right-hand side of the diagram shows the balls just after their collision. We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the masses of the colliding objects that we can solve for v_{1f} and v_{2f}.



(*a*) Because both balls are in free-fall, and both are in the air for the same amount of time, they have the same velocity just before the basketball rebounds. After the basketball rebounds elastically, its velocity will have the same magnitude, but the opposite direction than just before it hit the ground. The velocity of the basketball will be equal in magnitude but opposite in direction to the velocity of the baseball.

(*b*) Apply conservation of linear momentum to the collision of the balls to obtain:

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \quad (1)$$

Relate the initial and final kinetic energies of the balls in their elastic collision:

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) \quad (2)$$

Rearrange equation (1) to obtain:

$$m_2\left(v_{2f} - v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right) \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \quad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$\left(m_1 + m_2\right)v_{1f} = \left(m_1 - m_2\right)v_{1i} + 2m_2v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$

or, because $v_{2i} = -v_{1i}$,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} - \frac{2m_2}{m_1 + m_2}v_{1i} = \frac{m_1 - 3m_2}{m_1 + m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$\boldsymbol{v}_{1f} = \frac{3\boldsymbol{m}_2 - 3\boldsymbol{m}_2}{3\boldsymbol{m}_2 + \boldsymbol{m}_2}\boldsymbol{v} = \boxed{0}$$

(*c*) Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$\left(m_1 + m_2\right)v_{2f} = \left(m_2 - m_1\right)v_{2i} + 2m_1v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

or, because $v_{2i} = -v_{1i}$,

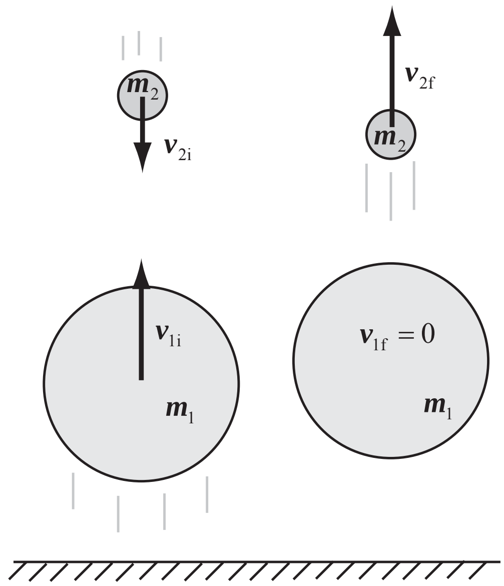
$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} - \frac{m_2 - m_1}{m_1 + m_2}v_{1i} = \frac{3m_1 - m_2}{m_1 + m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$v_{2f} = \frac{3\left(3m_2\right) - m_2}{3m_2 + m_2}v = \boxed{2v}$$

106 ••• (*a*) Referring to Problem 105, if we held a third ball above the baseball and basketball, and wanted both the basketball and baseball to stop in mid-air, what should the ratio of the mass of the top ball to the mass of the baseball be? (*b*) If the speed of the top ball is *v* just before the collision, what is its speed just after the collision?

Picture the Problem In Problem 105 only two balls are dropped. They collide head on, each moving at speed *v*, and the collision is elastic. In this problem, as it did in Problem 105, the solution involves using the conservation of momentum equation $m_1v_{1f}+m_2v_{2f}=m_1v_{1i}+m_2v_{2i}$ and the elastic collision equation $v_{1f}-v_{2f}=v_{2i}-v_{1i}$ where the numeral 1 refers to the baseball, and the numeral 2 to the top ball. The diagram shows the balls just before and just after their collision. From Problem 105 we know that $v_{1i}=2v$ and $v_{2i}=-v$.



(*a*) Express the final speed v_{1f} of the baseball as a function of its initial speed v_{1i} and the initial speed of the top ball v_{2i} (see Problem 64):

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$$

Substitute for v_{1i} and , v_{2i} to obtain:

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}(2v)+\frac{2m_2}{m_1+m_2}(-v)$$

Divide the numerator and denominator of each term by m_2 to introduce the mass ratio of the upper ball to the lower ball:

$$v_{1f}=\frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1}(2v)+\frac{2}{\frac{m_1}{m_2}+1}(-v)$$

Set the final speed of the baseball v_{1f} equal to zero and let *x* represent the mass ratio m_1/m_2 to obtain:

$$0=\frac{x-1}{x+1}(2v)+\frac{2}{x+1}(-v)$$

Solving for *x* yields:

$$x=\frac{m_1}{m_2}=\boxed{\tfrac{1}{2}}$$

(*b*) Apply the second of the two equations in Problem 64 to the collision between the top ball and the baseball:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Substitute $v_{1i} = 2v$ and $v_{2i} = -v$ to obtain:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}(2v) + \frac{m_2 - m_1}{m_1 + m_2}(-v)$$

In part (*a*) we showed that $m_2 = 2m_1$. Substitute and simplify to obtain:

$$v_{3f} = \frac{2(2\boldsymbol{m}_1)}{\boldsymbol{m}_1 + 2\boldsymbol{m}_1}(2\boldsymbol{v}) - \frac{2\boldsymbol{m}_1 - \boldsymbol{m}_1}{\boldsymbol{m}_1 + 2\boldsymbol{m}_1}\boldsymbol{v} = \boxed{\tfrac{7}{3}\boldsymbol{v}}$$

107 ••• [SSM] In the "slingshot effect," the transfer of energy in an elastic collision is used to boost the energy of a space probe so that it can escape from the solar system. All speeds are relative to an inertial frame in which the center of the sun remains at rest. Figure 8-55 shows a space probe moving at 10.4 km/s toward Saturn, which is moving at 9.6 km/s toward the probe. Because of the gravitational attraction between Saturn and the probe, the probe swings around Saturn and heads back in the opposite direction with speed v_f. (*a*) Assuming this collision to be a one-dimensional elastic collision with the mass of Saturn much greater than that of the probe, find v_f. (*b*) By what factor is the kinetic energy of the probe increased? Where does this energy come from?

Picture the Problem Let the direction the probe is moving after its elastic collision with Saturn be the positive direction. The probe gains kinetic energy at the expense of the kinetic energy of Saturn. We'll relate the velocity of approach relative to the center of mass to u_{rec} and then to v. Let the +x direction be in the direction of the motion of Saturn.

(*a*) Relate the velocity of recession to the velocity of recession relative to the center of mass:

$$v = u_{rec} + v_{cm} \qquad (1)$$

Find the velocity of approach:

$$u_{app} = -9.6\,\text{km/s} - 10.4\,\text{km/s} = -20.0\,\text{km/s}$$

Relate the relative velocity of approach to the relative velocity of recession for an elastic collision:

$$u_{rec} = -u_{app} = 20.0\,\text{km/s}$$

Because Saturn is so much more massive than the space probe:

$$v_{cm} = v_{Saturn} = 9.6\,\text{km/s}$$

Substitute numerical values in equation (1) and evaluate v:

$$\boldsymbol{v} = 20\,\text{km/s} + 9.6\,\text{km/s} = \boxed{30\,\text{km/s}}$$

(b) Express the ratio of the final kinetic energy to the initial kinetic energy and simplify:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}Mv_{rec}^2}{\frac{1}{2}Mv_i^2} = \left(\frac{v_{rec}}{v_i}\right)^2$$

Substitute numerical values and evaluate K_f/K_i:

$$\frac{K_f}{K_i} = \left(\frac{29.6\,\text{km/s}}{10.4\,\text{km/s}}\right)^2 = \boxed{8.1}$$

The energy comes from an immeasurably small slowing of Saturn.

108 •• A 13-kg block is at rest on a level floor. A 400-g glob of putty is thrown at the block so that the putty travels horizontally, hits the block, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?

Picture the Problem Let the system include the block, the putty, and the earth. Then $F_{ext,net} = 0$ and momentum is conserved in this perfectly inelastic collision. We'll use conservation of momentum to relate the after-collision velocity of the block plus blob and conservation of energy to find their after-collision velocity.

Noting that, because this is a perfectly elastic collision, the final velocity of the block plus blob is the velocity of the center of mass, use conservation of momentum to relate the velocity of the center of mass to the velocity of the glob before the collision:

$$p_i = p_f$$

or

$$m_{gl}v_{gl} = Mv_{cm} \Rightarrow v_{gl} = \frac{M}{m_{gl}}v_{cm} \quad (1)$$

where $M = m_{gl} + m_{bl}$.

Use conservation of energy to find the initial energy of the block plus glob:

$$\Delta K + \Delta U + W_f = 0$$

Because $\Delta U = K_f = 0$,

$$-\tfrac{1}{2}Mv_{cm}^2 + f_k\Delta x = 0$$

Because $f_k = \mu_k Mg$:

$$-\tfrac{1}{2}Mv_{cm}^2 + \mu_k Mg\Delta x = 0$$

Solve for v_{cm} to obtain:

$$v_{cm} = \sqrt{2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_{cm}:

$$\boldsymbol{v}_{cm} = \sqrt{2(0.40)(9.81\,\text{m/s}^2)(0.15\,\text{m})} = 1.08\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate v_{gl}:

$$v_{gl}=\frac{13\,\text{kg}+0.400\,\text{kg}}{0.400\,\text{kg}}(1.08\,\text{m/s})=\boxed{36\,\text{m/s}}$$

109 ••• [SSM] Your accident reconstruction team has been hired by the local police to analyze the following accident. A careless driver rear-ended a car that was halted at a stop sign. Just before impact, the driver slammed on his brakes, locking the wheels. The driver of the struck car had his foot solidly on the brake pedal, locking his brakes. The mass of the struck car was 900 kg, and that of the initially moving vehicle was 1200 kg. On collision, the bumpers of the two cars meshed. Police determine from the skid marks that after the collision the two cars moved 0.76 m together. Tests revealed that the coefficient of kinetic friction between the tires and pavement was 0.92. The driver of the moving car claims that he was traveling at less than 15 km/h as he approached the intersection. Is he telling the truth?

Picture the Problem Let the direction the moving car was traveling before the collision be the +*x* direction. Let the numeral 1 denote this car and the numeral 2 the car that is stopped at the stop sign and the system include both cars and Earth. We can use conservation of momentum to relate the speed of the initially-moving car to the speed of the meshed cars immediately after their perfectly inelastic collision and conservation of energy to find the initial speed of the meshed cars.

Using conservation of momentum, relate the before-collision velocity to the after-collision velocity of the meshed cars:

$$p_i=p_f$$

or

$$m_1v_1=(m_1+m_2)V$$

Solving for v_1 and simplifying yields:

$$v_1=\frac{m_1+m_2}{m_1}V=\left(1+\frac{m_2}{m_1}\right)V \qquad (1)$$

Using conservation of energy, relate the initial kinetic energy of the meshed cars to the work done by friction in bringing them to a stop:

$$\Delta K+\Delta E_{\text{thermal}}=0$$

or, because $K_f=0$ and $\Delta E_{\text{thermal}}=f\Delta s$,

$$-K_i+f_k\Delta s=0$$

Substitute for K_i and, using $f_k=\mu_kF_n=\mu_kMg$, eliminate f_k to obtain:

$$-\tfrac{1}{2}MV^2+\mu_kMg\Delta x=0$$

Solving for V yields:

$$V=\sqrt{2\mu_kg\Delta x}$$

Substitute for V in equation (1) to obtain:

$$v_1 = \left(1 + \frac{m_2}{m_1}\right)\sqrt{2\mu_k g \Delta x}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \left(1 + \frac{900\,\text{kg}}{1200\,\text{kg}}\right)\sqrt{2(0.92)(9.81\,\text{m/s}^2)(0.76\,\text{m})} = 6.48\,\text{m/s} = 23\,\text{km/h}$$

The driver was not telling the truth. He was traveling at 23 km/h.

110 •• A pendulum consists of a compact 0.40-kg bob attached to a string of length 1.6 m. A block of mass *m* rests on a horizontal frictionless surface. The pendulum is released from rest at an angle of 53º with the vertical. The bob collides elastically with the block at the lowest point in its arc. Following the collision, the maximum angle of the pendulum with the vertical is 5.73º. Determine the mass *m*.

Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the bob's swing and note that the bob can swing either forward or backward after the collision. We'll use both conservation of momentum and conservation of energy to relate the velocities of the bob and the block before and after their collision. Choose the positive *x* direction to be in the direction of the motion of the block.

Express the kinetic energy of the block in terms of its after-collision momentum:

$$K_m = \frac{p_m^2}{2m} \Rightarrow m = \frac{p_m^2}{2K_m} \qquad (1)$$

Use conservation of energy to relate K_m to the change in the potential energy of the bob:

$$\Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K_m + U_f - U_i = 0$$

Solve for K_m, substitute for U_f and U_i and simplify to obtain:

$$\begin{aligned} K_m &= -U_f + U_i \\ &= m_{bob} g\left[L(1-\cos\theta_i) - L(1-\cos\theta_f)\right] \\ &= m_{bob} gL\left[\cos\theta_f - \cos\theta_i\right] \end{aligned}$$

Substitute numerical values and evaluate K_m:

$$K_m = (0.40\,\text{kg})(9.81\,\text{m/s}^2)(1.6\,\text{m})[\cos 5.73° - \cos 53°] = 2.47\,\text{J}$$

Use conservation of energy to find the velocity of the bob just before its collision with the block:

$\Delta K + \Delta U = 0$
or, because $K_i = U_f = 0$,
$K_f - U_i = 0$

Substitute for K_f and U_i to obtain:

$\frac{1}{2} m_{bob} v^2 - m_{bob} g L (1 - \cos\theta_i) = 0$

Solving for v yields:

$v = \sqrt{2gL(1-\cos\theta_i)}$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1-\cos 53°)} = 3.536\,\text{m/s}$$

Use conservation of energy to find the velocity of the bob just after its collision with the block:

$\Delta K + \Delta U = 0$
or, because $K_f = U_i = 0$,
$-K_i + U_f = 0$

Substitute for K_i and U_f to obtain:

$-\frac{1}{2} m_{bob} v'^2 + m_{bob} g L (1 - \cos\theta_f) = 0$

Solve for v':

$v' = \sqrt{2gL(1-\cos\theta_f)}$

Substitute numerical values and evaluate v':

$$v' = \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1-\cos 5.73°)} = 0.396\,\text{m/s}$$

Use conservation of momentum to relate p_m after the collision to the momentum of the bob just before and just after the collision:

$\boldsymbol{p}_i = \boldsymbol{p}_f$
or
$\boldsymbol{m}_{bob}\boldsymbol{v} = \pm \boldsymbol{m}_{bob}\boldsymbol{v}' + \boldsymbol{p}_{\boldsymbol{m}}$

Solve for and evaluate p_m:

$$\boldsymbol{p}_{\boldsymbol{m}} = \boldsymbol{m}_{bob}\boldsymbol{v} \pm \boldsymbol{m}_{bob}\boldsymbol{v}' = (0.40\,\text{kg})(3.536\,\text{m/s} \pm 0.396\,\text{m/s}) = 1.414\,\text{kg}\cdot\text{m/s} \pm 0.158\,\text{kg}\cdot\text{m/s}$$

Find the larger value for p_m:

$$\boldsymbol{p}_{\boldsymbol{m}} = 1.414\,\text{kg}\cdot\text{m/s} + 0.158\,\text{kg}\cdot\text{m/s} = 1.573\,\text{kg}\cdot\text{m/s}$$

Find the smaller value for p_m:

$$\boldsymbol{p}_{\boldsymbol{m}} = 1.414\,\text{kg}\cdot\text{m/s} - 0.158\,\text{kg}\cdot\text{m/s} = 1.256\,\text{kg}\cdot\text{m/s}$$

Substitute numerical values in equation (1) to determine the two values for *m*:

$$\boldsymbol{m} = \frac{(1.573\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.50\,\text{kg}}$$

or

$$\boldsymbol{m} = \frac{(1.256\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.32\,\text{kg}}$$

111 ••• **[SSM]** A 1.00-kg block and a second block of mass *M* are both initially at rest on a frictionless inclined plane (Figure 8-56) Mass *M* rests against a spring that has a force constant of 11.0 kN/m. The distance along the plane between the two blocks is 4.00 m. The 1.00-kg block is released, making an elastic collision with the unknown block. The 1.00-kg block then rebounds a distance of 2.56 m back up the inclined plane. The block of mass *M* comes momentarily comes to rest 4.00 cm from its initial position. Find *M*.

Picture the Problem Choose the zero of gravitational potential energy at the location of the spring's maximum compression. Let the system include the spring, the blocks, and Earth. Then the net external force is zero as is work done against friction. We can use conservation of energy to relate the energy transformations taking place during the evolution of this system.

Apply conservation of energy to the system:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = 0$:

$$\Delta U_g + \Delta U_s = 0$$

Express the change in the gravitational potential energy:

$$\Delta U_g = -mg\Delta h - Mgx\sin\theta$$

Express the change in the potential energy of the spring:

$$\Delta U_s = \tfrac{1}{2}kx^2$$

Substitute to obtain:

$$-mg\Delta h - Mgx\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Solving for *M* and simplifying yields:

$$M = \frac{\tfrac{1}{2}kx^2 - mg\Delta h}{gx\sin 30°} = \frac{kx}{g} - \frac{2m\Delta h}{x}$$

Relate Δh to the initial and rebound positions of the block whose mass is *m*:

$$\Delta \boldsymbol{h} = (4.00\,\text{m} - 2.56\,\text{m})\sin 30° = 0.72\,\text{m}$$

Substitute numerical values and evaluate M:

$$\boldsymbol{M} = \frac{(11.0\times10^3\ \text{N/m})(0.0400\,\text{m})}{9.81\,\text{m/s}^2} - \frac{2(1.00\,\text{kg})(0.72\,\text{m})}{0.0400\,\text{m}} = \boxed{8.9\,\text{kg}}$$

112 ••• A neutron of mass m makes an elastic head-on collision with a stationary nucleus of mass M. (*a*) Show that the kinetic energy of the nucleus after the collision is given by $K_{\text{nucleus}} = [4mM/(m + M)^2]K_{\text{n}}$, where K_{n} is the initial kinetic energy of the neutron. (*b*) Show that the *fractional* change in the kinetic energy of the neutron is given by

$$\frac{\Delta K_{\text{n}}}{K_{\text{n}}} = -\frac{4(m/M)}{(1+[m/M])^2}.$$

(*c*) Show that this expression gives plausible results both if $m << M$ and $m = M$. What is the best stationary nucleus for the neutron to collide head-on with if the objective is to produce a maximum loss in the kinetic energy of the neutron?

Picture the Problem In this elastic head-on collision, the kinetic energy of recoiling nucleus is the difference between the initial and final kinetic energies of the neutron. We can derive the indicated results by using both conservation of energy and conservation of momentum and writing the kinetic energies in terms of the momenta of the particles before and after the collision.

(*a*) Use conservation of energy to relate the kinetic energies of the particles before and after the collision:

$$\frac{p_{\text{ni}}^2}{2m} = \frac{p_{\text{nf}}^2}{2m} + \frac{p_{\text{nucleus}}^2}{2M} \quad (1)$$

Apply conservation of momentum to obtain a second relationship between the initial and final momenta:

$$p_{\text{ni}} = p_{\text{nf}} + p_{\text{nucleus}} \quad (2)$$

Eliminate p_{nf} in equation (1) using equation (2):

$$\frac{p_{\text{nucleus}}}{2M} + \frac{p_{\text{nucleus}}}{2m} - \frac{p_{\text{ni}}}{m} = 0 \quad (3)$$

Use equation (3) to write $p_{\text{ni}}^2/2m$ in terms of p_{nucleus}:

$$\frac{p_{\text{ni}}^2}{2m} = K_{\text{n}} = \frac{p_{\text{nucleus}}^2(M+m)^2}{8M^2m} \quad (4)$$

Use equation (4) to express $K_{\text{nucleus}} = p_{\text{nucleus}}^2/2M$ in terms of K_{n}:

$$K_{\text{nucleus}} = \boxed{K_{\text{n}}\left[\frac{4Mm}{(M+m)^2}\right]} \quad (5)$$

(*b*) Relate the *change* in the kinetic energy of the neutron to the after-collision kinetic energy of the nucleus:

$$\Delta K_n = -K_{nucleus}$$

Using equation (5), express the fraction of the energy lost in the collision:

$$\frac{\Delta K_n}{K_n} = -\frac{4Mm}{(M+m)^2} = \boxed{-\frac{4(m/M)}{(1+(m/M))^2}}$$

(*c*) If $m << M$:

$$\frac{\Delta \boldsymbol{K}_n}{\boldsymbol{K}_n} \to \boxed{0} \text{ as expected.}$$

If $m = M$:

$$\frac{\Delta \boldsymbol{K}_n}{\boldsymbol{K}_n} = -\frac{4}{(1+1)^2} = \boxed{-1} \text{ as expected.}$$

113 ••• The mass of a carbon nucleus is approximately 12 times the mass of a neutron. (*a*) Use the results of Problem 112 to show that after N head-on collisions of a neutron with carbon nuclei at rest, the kinetic energy of the neutron is approximately $0.716^N K_0$, where K_0 is its initial kinetic energy. (*b*) Neutrons emitted during the fission of a uranium nucleus have kinetic energies of about 2.0 MeV. For such a neutron to cause the fission of another uranium nucleus in a reactor, its kinetic energy must be reduced to about 0.020 eV. How many head-on collisions are needed to reduce the kinetic energy of a neutron from 2.0 MeV to 0.020 eV, assuming elastic head-on collisions with stationary carbon nuclei?

Picture the Problem Problem 112 (*b*) provides an expression for the fractional loss of kinetic energy per collision.

(*a*) Using the result of Problem 112 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{(M-m)^2}{(M+m)^2}$$

Evaluate this fraction to obtain:

$$\frac{K_{nf}}{E_0} = \frac{(12m-m)^2}{(12m+m)^2} = 0.716$$

Express the kinetic energy of one neutron after N collisions:

$$K_{nf} = \boxed{0.716^N E_0}$$

(*b*) Substitute for K_{nf} and E_0 to obtain:

$$0.716^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.716} \approx \boxed{55}$$

114 ••• On average, a neutron actually loses only 63 percent of its energy in an elastic collision with a hydrogen atom (not 100 percent) and 11 percent of its energy during an elastic collision with a carbon atom (not 18 percent). (These numbers are an average over all types of collisions, not just head-on ones. Thus the results are lower than the ones determined from analyses like that in Problem 113 because most collisions are not head-on.) Calculate the actual number of collisions, on average, needed to reduce the energy of a neutron from 2.0 MeV to 0.020 eV if the neutron collides with stationary (*a*) hydrogen atoms and (*b*) carbon atoms.

Picture the Problem We can relate the number of collisions needed to reduce the energy of a neutron from 2 MeV to 0.02 eV to the fractional energy loss per collision and solve the resulting exponential equation for *N*.

(*a*) Using the result of Problem 113 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.63K_{ni}}{K_{ni}}$$
$$= 0.37$$

Express the kinetic energy of one neutron after *N* collisions:

$$\boldsymbol{K}_{nf} = 0.37^N \boldsymbol{K}_0$$

Substitute for K_{nf} and K_0 to obtain:

$$0.37^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$\boldsymbol{N} = \frac{-8}{\log 0.37} \approx \boxed{19}$$

(*b*) Proceed as in (*a*) to obtain:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.11K_{ni}}{K_{ni}}$$
$$= 0.89$$

Express the kinetic energy of one neutron after *N* collisions:

$$\boldsymbol{K}_{nf} = 0.89^N \boldsymbol{K}_0$$

Substitute for K_{nf} and K_0 to obtain:

$$0.89^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$\boldsymbol{N} = \frac{-8}{\log 0.89} \approx \boxed{158}$$

115 ••• **[SSM]** Two astronauts at rest face each other in space. One, with mass m_1, throws a ball of mass m_b to the other, whose mass is m_2. She catches the ball and throws it back to the first astronaut. Following each throw the ball has a speed of v relative to the thrower. After each has made one throw and one catch, (*a*) How fast are the astronauts moving? (*b*) How much has the two-astronaut system's kinetic energy changed and where did this energy come from?

Picture the Problem Let the direction that astronaut 1 first throws the ball be the positive direction and let v_b be the initial speed of the ball in the laboratory frame. Note that each collision is perfectly inelastic. We can apply conservation of momentum and the definition of the speed of the ball relative to the thrower to each of the perfectly inelastic collisions to express the final speeds of each astronaut after one throw and one catch.

(*a*) Use conservation of linear momentum to relate the speeds of astronaut 1 and the ball after the first throw:

$$m_1v_1 + m_bv_b = 0 \quad (1)$$

Relate the speed of the ball in the laboratory frame to its speed relative to astronaut 1:

$$v = v_b - v_1 \quad (2)$$

Eliminate v_b between equations (1) and (2) and solve for v_1:

$$v_1 = -\frac{m_b}{m_1 + m_b}v \quad (3)$$

Substitute equation (3) in equation (2) and solve for v_b:

$$v_b = \frac{m_1}{m_1 + m_b}v \quad (4)$$

Apply conservation of linear momentum to express the speed of astronaut 2 and the ball after the first catch:

$$0 = m_bv_b = (m_2 + m_b)v_2 \quad (5)$$

Solving for v_2 yields:

$$v_2 = \frac{m_b}{m_2 + m_b}v_b \quad (6)$$

Express v_2 in terms of v by substituting equation (4) in equation (6):

$$v_2 = \frac{m_b}{m_2 + m_b}\frac{m_1}{m_1 + m_b}v = \left[\frac{m_bm_1}{(m_2 + m_b)(m_1 + m_b)}\right]v \quad (7)$$

Use conservation of momentum to express the speed of astronaut 2 and the ball after she throws the ball:

$$(m_2 + m_b)v_2 = m_b v_{bf} + m_2 v_{2f} \qquad (8)$$

Relate the speed of the ball in the laboratory frame to its speed relative to astronaut 2:

$$v = v_{2f} - v_{bf} \qquad (9)$$

Eliminate v_{bf} between equations (8) and (9) and solve for v_{2f}:

$$\boldsymbol{v}_{2f} = \boxed{\left(\frac{\boldsymbol{m}_b}{\boldsymbol{m}_2 + \boldsymbol{m}_b}\right)\left(1 + \frac{\boldsymbol{m}_1}{\boldsymbol{m}_1 + \boldsymbol{m}_b}\right)\boldsymbol{v}} \qquad (10)$$

Substitute equation (10) in equation (9) and solve for v_{bf}:

$$\boldsymbol{v}_{bf} = \left[\frac{\boldsymbol{m}_b}{\boldsymbol{m}_2 + \boldsymbol{m}_b} - 1\right]\left[1 + \frac{\boldsymbol{m}_1}{\boldsymbol{m}_1 + \boldsymbol{m}_b}\right]\boldsymbol{v} \qquad (11)$$

Apply conservation of momentum to express the speed of astronaut 1 and the ball after she catches the ball:

$$(m_1 + m_b)v_{1f} = m_b v_{bf} + m_1 v_1 \qquad (12)$$

Using equations (3) and (11), eliminate v_{bf} and v_1 in equation (12) and solve for v_{1f}:

$$\boldsymbol{v}_{1f} = \boxed{-\frac{\boldsymbol{m}_2 \boldsymbol{m}_b (2\boldsymbol{m}_1 + \boldsymbol{m}_b)}{(\boldsymbol{m}_1 + \boldsymbol{m}_b)^2 (\boldsymbol{m}_2 + \boldsymbol{m}_b)}\boldsymbol{v}}$$

(*b*) The change in the kinetic energy of the system is:

$$\Delta \boldsymbol{K} = \boldsymbol{K}_f - \boldsymbol{K}_i$$

or, because $K_i = 0$,

$$\Delta \boldsymbol{K} = \boldsymbol{K}_f = \boldsymbol{K}_{1f} + \boldsymbol{K}_{2f} = \tfrac{1}{2}\boldsymbol{m}_1 \boldsymbol{v}_{1f}^2 + \tfrac{1}{2}\boldsymbol{m}_2 \boldsymbol{v}_{2f}^2$$

Substitute for v_{1f} and v_{2f} to obtain:

$$\Delta K = \tfrac{1}{2} m_1 \left(-\frac{m_2 m_b (2m_1 + m_b)}{(m_1 + m_b)^2 (m_2 + m_b)}\right)^2 v^2 + \tfrac{1}{2} m_2 \left(\frac{m_b}{m_2 + m_b}\right)^2 \left(1 + \frac{m_1}{m_1 + m_b}\right)^2 v^2$$

Simplify to obtain:

$$\Delta K = \boxed{\tfrac{1}{2}\frac{m_2 m_b^2 (2m_1 + m_b)^2}{(m_2 + m_b)^2 (m_1 + m_b)^2}\left(1 + \frac{m_1 m_2}{(m_1 + m_b)^2}\right)v^2}$$

This additional energy came from chemical energy in the astronaut's bodies.

116 ••• A stream of elastic glass beads, each with a mass of 0.50 g, comes out of a horizontal tube at a rate of 100 per second (see Figure 8-57). The beads fall a distance of 0.50 m to a balance pan and bounce back to their original height. How much mass must be placed in the other pan of the balance to keep the pointer at zero?

Picture the Problem Take the zero of gravitational potential energy to be at the elevation of the pan and let the system include the balance, the beads, and the earth. We can use conservation of energy to find the vertical component of the velocity of the beads as they hit the pan and then calculate the net downward force on the pan from Newton's 2nd law. Let the positive *y* direction be upward.

Use conservation of energy to relate the *y* component of the bead's velocity as it hits the pan to its height of fall:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}mv_y^2 - mgh = 0 \Rightarrow v_y = \sqrt{2gh}$$

Substitute numerical values and evaluate v_y:

$$\boldsymbol{v}_y = \sqrt{2(9.81\,\text{m/s}^2)(0.50\,\text{m})} = 3.13\,\text{m/s}$$

Express the change in momentum in the *y* direction per bead:

$$\Delta p_y = p_{yf} - p_{yi} = mv_y - (-mv_y) = 2mv_y$$

Use Newton's 2nd law to express the net force in the *y* direction exerted on the pan by the beads:

$$\boldsymbol{F}_{\text{net},y} = -\boldsymbol{N}\frac{\Delta \boldsymbol{p}_y}{\Delta \boldsymbol{t}}$$

Letting *M* represent the mass to be placed on the other pan, equate its weight to the net force exerted by the beads, substitute for Δp_y, and solve for *M*:

$$-\boldsymbol{Mg} = -\boldsymbol{N}\frac{\Delta \boldsymbol{p}_y}{\Delta \boldsymbol{t}}$$

and

$$M = \frac{N}{\Delta t}\left(\frac{2mv_y}{g}\right)$$

Substitute numerical values and evaluate *M*:

$$\boldsymbol{M} = (100/\text{s})\frac{[2(0.00050\,\text{kg})(3.13\,\text{m/s})]}{9.81\,\text{m/s}^2} = \boxed{32\,\text{g}}$$

117 ••• A dumbbell, consisting of two balls of mass *m* connected by a massless 1.00-m-long rod, rests on a frictionless floor against a frictionless wall until it begins to slide down the wall as in Figure 8-58. Find the speed of the bottom ball at the moment when it equals the speed of the top ball.

Picture the Problem Assume that the connecting rod goes halfway through both balls, i.e., the centers of mass of the balls are separated by 1.00 m. Let the system include the dumbbell, the wall and floor, and the earth. Let the zero of gravitational potential be at the center of mass of the lower ball and use conservation of energy to relate the speeds of the balls to the potential energy of the system. By symmetry, the speeds will be equal when the angle with the vertical is 45°.

Use conservation of energy to express the relationship between the initial and final energies of the system:

$$E_i = E_f$$

Express the initial energy of the system:

$$E_i = mgL$$

where L is the length of the rod.

Express the energy of the system when the angle with the vertical is 45°:

$$E_f = mgL\sin 45° + \tfrac{1}{2}(2m)v^2$$

Substitute to obtain:

$$gL = gL\left(\frac{1}{\sqrt{2}}\right) + v^2$$

Solving for v yields:

$$v = \sqrt{gL\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{(9.81\,\text{m/s}^2)(1.00\text{ m})\left(1 - \frac{1}{\sqrt{2}}\right)}$$

$$= \boxed{4.53\text{ m/s}}$$