Chapter 7
Conservation of Energy

Conceptual Problems

1 • **[SSM]** Two cylinders of unequal mass are connected by a massless cord that passes over a frictionless peg (Figure 7-34). After the system is released from rest, which of the following statements are true? (U is the gravitational potential energy and K is the kinetic energy of the system.) (*a*) $\Delta U < 0$ and $\Delta K > 0$, (*b*) $\Delta U = 0$ and $\Delta K > 0$, (*c*) $\Delta U < 0$ and $\Delta K = 0$, (*d*) $\Delta U = 0$ and $\Delta K = 0$, (*e*) $\Delta U > 0$ and $\Delta K < 0$.

Determine the Concept Because the peg is frictionless, mechanical energy is conserved as this system evolves from one state to another. The system moves and so we know that $\Delta K > 0$. Because $\Delta K + \Delta U = constant$, $\Delta U < 0$. $\boxed{(\boldsymbol{a})}$ is correct.

2 • Two stones are simultaneously thrown with the same initial speed from the roof of a building. One stone is thrown at an angle of 30° above the horizontal, the other is thrown horizontally. (Neglect effects due to air resistance.) Which statement below is true?

(*a*) The stones strike the ground at the same time and with equal speeds.
(*b*) The stones strike the ground at the same time with different speeds.
(*c*) The stones strike the ground at different times with equal speeds.
(*d*) The stones strike the ground at different times with different speeds.

Determine the Concept Choose the zero of gravitational potential energy to be at ground level. The two stones have the same initial energy because they are thrown from the same height with the same initial speeds. Therefore, they will have the same total energy at all times during their fall. When they strike the ground, their gravitational potential energies will be zero and their kinetic energies will be equal. Thus, their speeds at impact will be equal. The stone that is thrown at an angle of 30° above the horizontal has a longer flight time due to its initial upward velocity and so they do not strike the ground at the same time. $\boxed{(\boldsymbol{c})}$ is correct.

3 • True or false:

(*a*) The total energy of a system cannot change.
(*b*) When you jump into the air, the floor does work on you increasing your mechanical energy.
(*c*) Work done by frictional forces must always decrease the total mechanical energy of a system.
(*d*) Compressing a given spring 2.0 cm from its unstressed length takes more work than stretching it 2.0 cm from its unstressed length.

(*a*) False. Forces that are external to a system can do work on the system to change its energy.

(*b*) False. In order for some object to do work, it must exert a force *over some distance*. Your muscles increase the force exerted on the floor by your feet and, in turn, the normal force of the floor on your feet increases and launches you into the air.

(*c*) False. The frictional force that accelerates a sprinter increases the total mechanical energy of the sprinter.

(*d*) False. Because the work required to stretch a spring a given distance varies as the square of that distance, the work is the same regardless of whether the spring is stretched or compressed.

4 • As a novice ice hockey player (assume frictionless situation), you have not mastered the art of stopping except by coasting straight for the boards of the rink (assumed to be a rigid wall). Discuss the energy changes that occur as you use the boards to slow your motion to a stop.

Determine the Concept The boards don't do any work on you. Your loss of kinetic energy is converted into thermal energy of your body and the boards.

5 • True or false (The particle in this question can move only along the *x* axis and is acted on by only one force, and $U(x)$ is the potential-energy function associated with this force.):

(*a*) The particle will be in equilibrium if it is at a location where $dU/dx = 0$.
(*b*) The particle will accelerate in the $-x$ direction if it is at a location where $dU/dx > 0$.
(*c*) The particle will both be in equilibrium and have constant speed if it is at a section of the *x* axis where $dU/dx = 0$ throughout the section.
(*d*) The particle will be in stable equilibrium if it is at a location where both $dU/dx = 0$ and $d^2U/dx^2 > 0$.
(*e*) The particle will be in neutral equilibrium if it is at a location where both $dU/dx = 0$ and $d^2U/dx^2 > 0$.

Determine the Concept $\boldsymbol{dU/dx}$ is the slope of the graph of $U(x)$ and $\boldsymbol{d^2U/dx^2}$ is the rate at which the slope is changing. The force acting on the object is given by $\boldsymbol{F} = -\boldsymbol{dU/dx}$.

(*a*) True. If $\boldsymbol{dU/dx} = 0$, then the net force acting on the object is zero (the condition for equilibrium).

(*b*) True. If $dU/dx > 0$ at a given location, then the net force acting on the object is negative and its acceleration is to the left.

(*c*) True. If $dU/dx = 0$ over a section of the *x* axis, then the net force acting on the object is zero and its acceleration is zero.

(*d*) True. If $dU/dx = 0$ and $d^2U/dx^2 > 0$ at a given location, then *U*(*x*) is concave upward at that location (the condition for stable equilibrium).

(*e*) False. If $dU/dx = 0$ and $d^2U/dx^2 > 0$ at a given location, then *U*(*x*) is concave upward at that location (the condition for stable equilibrium).

6 • Two knowledge seekers decide to ascend a mountain. Sal chooses a short, steep trail, while Joe, who weighs the same as Sal, chooses a long, gently sloped trail. At the top, they get into an argument about who gained more potential energy. Which of the following is true:

(*a*) Sal gains more gravitational potential energy than Joe.
(*b*) Sal gains less gravitational potential energy than Joe.
(*c*) Sal gains the same gravitational potential energy as Joe.
(*d*) To compare the gravitational potential energies, we must know the height of the mountain.
(*e*) To compare the gravitational potential energies, we must know the length of the two trails.

Determine the Concept The change in gravitational potential energy, over elevation changes that are small enough so that the gravitational field can be considered constant, is $mg\Delta h$, where Δh is the elevation change. Because Δh is the same for both Sal and Joe, their gains in gravitational potential energy are the same. $\boxed{(c)}$ is correct.

7 • True or false:

(*a*) Only conservative forces can do work.
(*b*) If only conservative forces act on a particle, the kinetic energy of the particle can not change.
(*c*) The work done by a conservative force equals the change in the potential energy associated with that force.
(*d*) If, for a particle constrained to the *x* axis, the potential energy associated with a conservative force decreases as the particle moves to the right, then the force points to the left.
(*e*) If, for a particle constrained to the *x* axis, a conservative force points to the right, then the potential energy associated with the force increases as the particle moves to the left.

(*a*) False. The definition of work is not limited to displacements caused by conservative forces.

(*b*) False. Consider the work done by the gravitational force on an object in freefall.

(*c*) False. The work done may change the kinetic energy of the system.

(*d*) False. The direction of the force is given by $\boldsymbol{F} = -\boldsymbol{dU/dx}$, so if the potential energy is decreasing to the right (the slope of $U(x)$ is negative), F must be positive (that is, points to the right).

(*e*) True. The direction of the force is given by $\boldsymbol{F} = -\boldsymbol{dU/dx}$, so if F points to the right, the potential energy function must increase to the left.

8 • Figure 7-35 shows the plot of a potential-energy function U versus x. (*a*) At each point indicated, state whether the x component of the force associated with this function is positive, negative, or zero. (*b*) At which point does the force have the greatest magnitude? (*c*) Identify any equilibrium points, and state whether the equilibrium is stable, unstable, or neutral

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x; that is, $F_x = -dU/dx$.

(*a*) Examine the slopes of the curve at each of the lettered points, remembering that F_x is the negative of the slope of the potential energy graph, to complete the table:

Point	dU/dx	F_x
A	+	–
B	0	0
C	–	+
D	0	0
E	+	–
F	0	0

(*b*) Find the point where the slope is steepest: $|F_x|$ is greatest at point *C*.

(*c*) If $\boldsymbol{d^2U/dx^2} < 0$, then the curve is concave downward and the equilibrium is *unstable*.

The equilibrium is unstable at point *B*.

If $d^2U/dx^2 > 0$, then the curve is concave upward and the equilibrium is *stable*.

The equilibrium is stable at point *D*.

Remarks: At point *F*, if $d^2U/dx^2 = 0$ while the fourth derivative is positive, then the equilibrium is *stable.*

9 • Assume that, when the brakes are applied, a constant frictional force is exerted on the wheels of a car by the road. If that is so, then which of the following are necessarily true? (*a*) The distance the car travels before coming to rest is proportional to the speed of the car just as the brakes are first applied, (*b*) the car's kinetic energy diminishes at a constant rate, (*c*) the kinetic energy of the car is inversely proportional to the time that has elapsed since the application of the brakes, (*d*) none of the above.

Picture the Problem Because the constant friction force is responsible for a constant acceleration, we can apply the constant-acceleration equations to the analysis of these statements. We can also apply the work-energy theorem with friction to obtain expressions for the kinetic energy of the car and the rate at which it is changing. Choose the system to include the earth and car and assume that the car is moving on a horizontal surface so that $\Delta U = 0$.

(*a*) A constant frictional force causes a constant acceleration. The stopping distance of the car is related to its speed before the brakes were applied through a constant-acceleration equation.

$$\boldsymbol{v}^2 = \boldsymbol{v}_0^2 + 2\boldsymbol{a}\Delta\boldsymbol{s}$$

or, because $v = 0$,

$$0 = \boldsymbol{v}_0^2 + 2\boldsymbol{a}\Delta\boldsymbol{s} \Rightarrow \Delta s = \frac{-v_0^2}{2a}$$

where $a < 0$.

Thus, $\Delta s \propto v_0^2$:

Statement (*a*) is *false*.

(*b*) Apply the work-energy theorem with friction to obtain:

$$\Delta K = -W_\text{f} = -\mu_\text{k} mg\Delta s$$

Express the rate at which *K* is dissipated:

$$\frac{\Delta K}{\Delta t} = -\mu_\text{k} mg\frac{\Delta s}{\Delta t}$$

Thus, $\frac{\Delta K}{\Delta t} \propto v$ and therefore not constant.

Statement (*b*) is *false*.

(*c*) In Part (*b*) we saw that:	$K \propto \Delta s$
Because $\Delta s \propto \Delta t$ and $K \propto \Delta t$:	Statement (*c*) is *false.*
Because none of the above are correct:	$\boxed{(\boldsymbol{d})}$ is correct.

10 •• If a rock is attached to a massless, rigid rod and swung in a vertical circle (Figure 7-36) at a constant speed, the total mechanical energy of the rock-Earth system does not remain constant. The kinetic energy of the rock remains constant, but the gravitational potential energy is continually changing. Is the total work done on the rock equal zero during all time intervals? Does the force by the rod on the rock ever have a nonzero tangential component?

Determine the Concept No. From the work-kinetic energy theorem, no total work is being done on the rock, as its kinetic energy is constant. However, the rod must exert a tangential force on the rock to keep the speed constant. The effect of this force is to cancel the component of the force of gravity that is tangential to the trajectory of the rock.

11 •• Use the rest energies given in Table 7-1 to answer the following questions. (*a*) Can the triton naturally decay into a helion? (*b*) Can the alpha particle naturally decay into helion plus a neutron? (*c*) Can the proton naturally decay into a neutron and a positron?

Determine the Concept
(*a*) Yes, because the triton mass is slightly more than that of the helion (^{3}He) mass.

(*b*) No, because the total of the neutron and helion masses is 3747.96 MeV which is larger than the alpha particle mass.

(*c*) No, because the neutron mass is already larger than that of the proton.

Estimation and Approximation

12 • Estimate (*a*) the change in your potential energy on taking an elevator from the ground floor to the top of the Empire State building, (*b*) the average force acting on you by the elevator to bring you to the top, and (*c*) the average power due to that force. The building is 102 stories high.

Picture the Problem You can estimate your change in potential energy due to this change in elevation from the definition of ΔU. You'll also need to estimate the height of one story of the Empire State building. We'll assume your mass is 70.0 kg and the height of one story to be 3.50 m. This approximation gives us a height of 1170 ft (357 m), a height that agrees to within 7% with the actual height of 1250 ft from the ground floor to the observation deck. We'll also assume that it takes 3 min to ride non-stop to the top floor in one of the high-speed elevators.

(*a*) Express the change in your gravitational potential energy as you ride the elevator to the 102nd floor:

$$\Delta U = mg\Delta h$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (70.0\,\text{kg})(9.81\,\text{m/s}^2)(357\,\text{m}) = 245.2\text{ kJ} = \boxed{245\,\text{kJ}}$$

(*b*) Ignoring the acceleration intervals at the beginning and the end of your ride, express the work done on you by the elevator in terms of the change in your gravitational potential energy:

$$W = Fh = \Delta U \Rightarrow \boldsymbol{F} = \frac{\Delta \boldsymbol{U}}{\boldsymbol{h}}$$

Substitute numerical values and evaluate F:

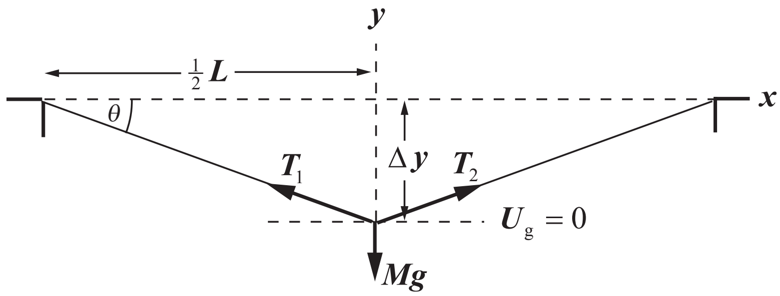
$$\boldsymbol{F} = \frac{245.2\,\text{kJ}}{357\,\text{m}} = \boxed{687\,\text{N}}$$

(*c*) Assuming a 3 min ride to the top, the average power delivered to the elevator is:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{U}}{\Delta \boldsymbol{t}} = \frac{245.2\,\text{kJ}}{(3\,\text{min})(60\,\text{s/min})} = \boxed{1.36\,\text{kW}}$$

13 • A tightrope walker whose mass is 50 kg walks across a tightrope held between two supports 10 m apart; the tension in the rope is 5000 N when she stands at the exact center of the rope. Estimate: (*a*) the sag in the tightrope when the acrobat stands in the exact center, and (*b*) the change in her gravitational potential energy from when she steps onto the tightrope to when she stands at its exact center.

Picture the Problem The diagram depicts the situation when the tightrope walker is at the center of rope and shows a coordinate system in which the +*x* direction is to the right and +*y* direction is upward. *M* represents her mass and the vertical components of tensions $\vec{\boldsymbol{T}}_1$ and $\vec{\boldsymbol{T}}_2$, which are equal in magnitude, support her weight. We can apply a condition for static equilibrium in the vertical direction to relate the tension in the rope to the angle θ and use trigonometry to find Δy as a function of θ.



(*a*) Use trigonometry to relate the sag Δy in the rope to its length L and θ:

$$\tan\theta = \frac{\Delta y}{\frac{1}{2}L} \Rightarrow \Delta y = \tfrac{1}{2}L\tan\theta \quad (1)$$

Apply $\sum F_y = 0$ to the tightrope walker when she is at the center of the rope to obtain:

$$2T\sin\theta - Mg = 0$$

where T is the magnitude of $\vec{T}_1$ and $\vec{T}_2$.

Solve for θ to obtain:

$$\theta = \sin^{-1}\left(\frac{Mg}{2T}\right)$$

Substituting for θ in equation (1) yields:

$$\Delta y = \tfrac{1}{2}L\tan\left[\sin^{-1}\left(\frac{Mg}{2T}\right)\right]$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \tfrac{1}{2}(10\text{ m})\tan\left[\sin^{-1}\left[\frac{(50\text{ kg})(9.81\text{ m/s}^2)}{2(5000\text{ N})}\right]\right] = 0.2455\text{ m} = \boxed{25\text{ cm}}$$

(*b*) Express the change in the tightrope walker's gravitational potential energy as the rope sags:

$$\Delta U = U_{\text{at center}} - U_{\text{end}} = 0 + Mg\Delta y$$
$$= Mg\Delta y$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (50\text{ kg})(9.81\text{ m/s}^2)(-0.2455\text{ m})$$
$$= \boxed{-0.12\text{ kJ}}$$

14 •• *The metabolic rate* is defined as the rate at which the body uses chemical energy to sustain its life functions. The average metabolic rate experimentally has been found to be proportional to the total skin surface area of the body. The surface area for a 5-ft, 10-in. male weighing 175 lb is about 2.0 m^2, and for a 5-ft, 4-in. female weighing 110 lb it is approximately 1.5 m^2. There is about a 1 percent change in surface area for every three pounds above or below the weights quoted here and a 1 percent change for every inch above or below the heights quoted. (*a*) Estimate your average metabolic rate over the course of a day using the following guide for metabolic rates (per square meter of skin area) for various physical activities: sleeping, 40 W/m^2; sitting, 60 W/m^2; walking, 160 W/m^2; moderate physical activity, 175 W/m^2; and moderate aerobic exercise, 300 W/m^2. How do your results compare to the power of a 100-W light bulb? (*b*) Express your average metabolic rate in terms of kcal/day (1 kcal = 4.19 kJ). (A kcal is the ″food calorie″ used by nutritionists.) (*c*) An estimate used by nutritionists is that each day the ″average person″ must eat roughly 12–15 kcal of food for each pound of body weight to maintain his or her weight. From the calculations in Part (*b*), are these estimates plausible?

Picture the Problem We'll use the data for the "typical male" described above and assume that he spends 8 hours per day sleeping, 2 hours walking, 8 hours sitting, 1 hour in aerobic exercise, and 5 hours doing moderate physical activity. We can approximate his energy utilization using $E_{\text{activity}} = AP_{\text{activity}}\Delta t_{\text{activity}}$, where A is the surface area of his body, P_{activity} is the rate of energy consumption in a given activity, and $\Delta t_{\text{activity}}$ is the time spent in the given activity. His total energy consumption will be the sum of the five terms corresponding to his daily activities.

(*a*) Express the energy consumption of the hypothetical male:

$$\boldsymbol{E} = \boldsymbol{E}_{\text{sleeping}} + \boldsymbol{E}_{\text{walking}} + \boldsymbol{E}_{\text{sitting}} + \boldsymbol{E}_{\text{mod. act.}} + \boldsymbol{E}_{\text{aerobic act.}} \qquad (1)$$

Evaluate E_{sleeping}:

$$E_{\text{sleeping}} = AP_{\text{sleeping}}\Delta t_{\text{sleeping}} = \left(2.0\,\text{m}^2\right)\left(40\,\text{W/m}^2\right)\left(8.0\,\text{h}\right)\left(3600\,\text{s/h}\right) = 2.30\times10^6\ \text{J}$$

Evaluate E_{walking}:

$$E_{\text{walking}} = AP_{\text{walking}}\Delta t_{\text{walking}} = \left(2.0\,\text{m}^2\right)\left(160\,\text{W/m}^2\right)\left(2.0\,\text{h}\right)\left(3600\,\text{s/h}\right) = 2.30\times10^6\ \text{J}$$

Evaluate E_{sitting}

$$E_{\text{sitting}} = AP_{\text{sitting}}\Delta t_{\text{sitting}} = \left(2.0\,\text{m}^2\right)\left(60\,\text{W/m}^2\right)\left(8.0\,\text{h}\right)\left(3600\,\text{s/h}\right) = 3.46\times10^6\ \text{J}$$

Evaluate $E_{\text{mod. act.}}$:

$$E_{\text{mod. act.}} = AP_{\text{mod. act.}}\Delta t_{\text{mod. act.}} = \left(2.0\,\text{m}^2\right)\left(175\,\text{W/m}^2\right)\left(5.0\,\text{h}\right)\left(3600\,\text{s/h}\right) = 6.30\times10^6\ \text{J}$$

Evaluate $E_{\text{aerobic act.}}$:

$$E_{\text{aerobic act.}} = AP_{\text{aerobic act.}}\Delta t_{\text{aerobic act.}} = \left(2.0\,\text{m}^2\right)\left(300\,\text{W/m}^2\right)\left(1.0\,\text{h}\right)\left(3600\,\text{s/h}\right) = 2.16\times10^6\ \text{J}$$

Substitute numerical values in equation (1) and evaluate E:

$$\begin{aligned} E &= 2.30\times10^6\ \text{J} + 2.30\times10^6\ \text{J} + 3.46\times10^6\ \text{J} + 6.30\times10^6\ \text{J} + 2.16\times10^6\ \text{J} \\ &= 16.5\times10^6\ \text{J} = \boxed{17\ \text{MJ}} \end{aligned}$$

Express the average metabolic rate represented by this energy consumption:

$$P_{\text{av}} = \frac{E}{\Delta t}$$

Substitute numerical values and evaluate P_{av}:

$$\boldsymbol{P}_{\text{av}} = \frac{16.5\times10^6\ \text{J}}{\left(24\,\text{h}\right)\left(3600\,\text{s/h}\right)} = \boxed{0.19\,\text{kW}}$$

or about twice that of a 100 W light bulb.

(*b*) Express his average energy consumption in terms of kcal/day:

$$\boldsymbol{E} = \frac{16.5\times10^6\ \text{J/d}}{4190\,\text{J/kcal}} = 3938\,\text{kcal/d} \approx \boxed{3.9\,\text{Mcal/d}}$$

(*c*) $\dfrac{3940\,\text{kcal}}{175\,\text{lb}} \approx 23\dfrac{\text{kcal}}{\text{lb}}$ is higher than the estimate given in the statement of the problem. However, by adjusting the day's activities, the metabolic rate can vary by more than a factor of 2.

15 •• **[SSM]** Assume that your maximum metabolic rate (the maximum rate at which your body uses its chemical energy) is 1500 W (about 2.7 hp). Assuming a 40 percent efficiency for the conversion of chemical energy into mechanical energy, estimate the following: (*a*) the shortest time you could run up four flights of stairs if each flight is 3.5 m high, (*b*) the shortest time you could climb the Empire State Building (102 stories high) using your Part (*a*) result. Comment on the feasibility of you actually achieving Part (*b*) result.

Picture the Problem The rate at which you expend energy, that is do work, is defined as *power* and is the ratio of the work done to the time required to do the work.

(*a*) Relate the rate at which you can expend energy to the work done in running up the four flights of stairs:

$$\varepsilon P = \frac{\Delta W}{\Delta t} \Rightarrow \Delta t = \frac{\Delta W}{\varepsilon P}$$

where *e* is the efficiency for the conversion of chemical energy into mechanical energy.

The work you do in climbing the stairs increases your gravitational potential energy:

$$\Delta W = mgh$$

Substitute for ΔW to obtain:

$$\Delta t = \frac{mgh}{\varepsilon P} \qquad (1)$$

Assuming that your mass is 70 kg, substitute numerical values in equation (1) and evaluate Δt:

$$\Delta t = \frac{(70\,\text{kg})(9.81\,\text{m/s}^2)(4\times 3.5\,\text{m})}{(0.40)(1500\,\text{W})} \approx \boxed{16\,\text{s}}$$

(*b*) Substituting numerical values in equation (1) yields:

$$\Delta t = \frac{(70\,\text{kg})(9.81\,\text{m/s}^2)(102\times 3.5\,\text{m})}{(0.40)(1500\,\text{W})} = 409\,\text{s} \approx \boxed{6.8\,\text{min}}$$

The time of about 6.8 min is clearly not reasonable. The fallacy is that you cannot do work at the given rate of 250 W for more than very short intervals of time.

16 •• You are in charge of determining when the uranium fuel rods in a local nuclear power plant are to be replaced with fresh ones. To make this determination you decide to estimate how much the mass of a core of a nuclear-fueled electric-generating plant is reduced per unit of electric energy produced. (*Note:* In such a generating plant the reactor core generates thermal energy, which is then transformed to electric energy by a steam turbine. It requires 3.0 J of thermal energy for each 1.0 J of electric energy produced.) What are your results for the production of (*a*) 1.0 J of thermal energy? (*b*) enough electric energy to keep a 100-W light bulb burning for 10.0 y? (*c*) electric energy at a constant rate of 1.0 GW for a year? (This is typical of modern plants.)

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) Relate the rest mass consumed to the energy produced and solve for *m*:

$$E_0 = mc^2 \Rightarrow m = \frac{E_0}{c^2} \qquad (1)$$

Substitute numerical values and evaluate m:

$$m = \frac{1.0\,\text{J}}{\left(2.998\times10^{8}\,\text{m/s}\right)^2} = \boxed{1.1\times10^{-17}\,\text{kg}}$$

(b) Because the reactor core must produce 3 J of thermal energy for each joule of electrical energy produced:

$$\boldsymbol{E} = 3\boldsymbol{P}\Delta\boldsymbol{t}$$

Substitute for E_0 in equation (1) to obtain:

$$\boldsymbol{m} = \frac{3\boldsymbol{P}\Delta\boldsymbol{t}}{\boldsymbol{c}^2} \qquad (2)$$

Substitute numerical values in equation (2) and evaluate m:

$$\boldsymbol{m} = \frac{3(100\,\text{W})(10\,\text{y})\left(\frac{365.24\,\text{d}}{\text{y}}\right)\left(\frac{24\,\text{h}}{\text{d}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right)}{\left(2.998\times10^{8}\,\text{m/s}\right)^2} = \boxed{1.1\,\mu\text{g}}$$

(c) Substitute numerical values in equation (2) and evaluate m:

$$\boldsymbol{m} = \frac{3(1.0\,\text{GW})(1.0\,\text{y})\left(\frac{365.24\,\text{d}}{\text{y}}\right)\left(\frac{24\,\text{h}}{\text{d}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right)}{\left(2.998\times10^{8}\,\text{m/s}\right)^2} = \boxed{1.1\,\text{kg}}$$

17 •• [SSM] The chemical energy released by burning a gallon of gasoline is approximately 1.3×10^5 kJ. Estimate the total energy used by all of the cars in the United States during the course of one year. What fraction does this represent of the total energy use by the United States in one year (currently about 5×10^{20} J)?

Picture the Problem There are about 3×10^8 people in the United States. On the assumption that the average family has 4 people in it and that they own two cars, we have a total of 1.5×10^8 automobiles on the road (excluding those used for industry). We'll assume that each car uses about 15 gal of fuel per week.

Calculate, based on the assumptions identified above, the total annual consumption of energy derived from gasoline:

$$\left(1.5\times10^{8}\,\text{auto}\right)\left(15\frac{\text{gal}}{\text{auto}\cdot\text{week}}\right)\left(52\frac{\text{weeks}}{\boldsymbol{y}}\right)\left(1.3\times10^{5}\,\frac{\text{kJ}}{\text{gal}}\right) = \boxed{1.5\times10^{19}\,\text{J/y}}$$

Express this rate of energy use as a fraction of the total annual energy use by the United States:

$$\frac{1.5\times10^{19}\ \text{J/y}}{5\times10^{20}\ \text{J/y}}\approx\boxed{3\%}$$

18 •• The maximum efficiency of a solar-energy panel in converting solar energy into useful electrical energy is currently about 12 percent. In a region such as the southwestern United States the solar intensity reaching Earth's surface is about 1.0 kW/m^2 on average during the day. Estimate the area that would have to be covered by solar panels in order to supply the energy requirements of the United States (approximately 5 × 10^{20} J/y) and compare it to the area of Arizona. Assume cloudless skies.

Picture the Problem The energy consumption of the U.S. works out to an average power consumption of about 1.6×10^{13} watt. The solar constant is roughly 10^3 W/m^2 (reaching the ground), or about 120 W/m^2 of useful power with a 12% conversion efficiency. Letting P represent the daily rate of energy consumption, we can relate the power available at the surface of the earth to the required area of the solar panels using $P = IA$. Using the internet, one finds that the area of Arizona is about 114,000 mi^2 or 3.0 × 10^{11} m^2.

Relate the required area to the electrical energy to be generated by the solar panels:

$$\boldsymbol{A}=\frac{\boldsymbol{P}}{\boldsymbol{I}}$$

where I is the solar intensity that reaches the surface of the Earth.

Substitute numerical values and evaluate A:

$$\boldsymbol{A}=\frac{2\left(1.6\times10^{13}\ \text{W}\right)}{120\ \text{W/m}^2}=\boxed{2.7\times10^{11}\ \text{m}^2}$$

where the factor of 2 comes from the fact that the sun is only "up" for roughly half the day.

Express the ratio of A to the area of Arizona to obtain:

$$\frac{\boldsymbol{A}}{\boldsymbol{A}_{\text{Arizona}}}=\frac{2.7\times10^{11}\ \text{m}^2}{3.0\times10^{11}\ \text{m}^2}\approx0.90$$

That is, the required area is about 90% of the area of Arizona.

Remarks: A more realistic estimate that would include the variation of sunlight over the day and account for latitude and weather variations might very well increase the area required by an order of magnitude.

19 •• Hydroelectric power plants convert gravitational potential energy into more useful forms by flowing water downhill through a turbine system to generate electric energy. The Hoover Dam on the Colorado River is 211 m high and generates 4 × 10^{-9} kW·h/y. At what rate (in L/s) must water be flowing through the turbines to generate this power? The density of water is 1.00 kg/L.

Assume a total efficiency of 90.0 percent in converting the water's potential energy into electrical energy.

Picture the Problem We can relate the energy available from the water in terms of its mass, the vertical distance it has fallen, and the efficiency of the process. Differentiation of this expression with respect to time will yield the rate at which water must pass through its turbines to generate Hoover Dam's annual energy output.

Assuming a total efficiency ε, use the expression for the gravitational potential energy near the earth's surface to express the energy available from the water when it has fallen a distance h:

$$\boldsymbol{E = \varepsilon mgh}$$

Differentiate this expression with respect to time to obtain:

$$P = \frac{d}{dt}\left[\varepsilon mgh\right] = \varepsilon gh\frac{dm}{dt} = \varepsilon\rho gh\frac{dV}{dt}$$

Solving for dV/dt yields:

$$\frac{dV}{dt} = \frac{P}{\varepsilon\rho gh} \qquad (1)$$

Using its definition, relate the dam's annual power output to the energy produced:

$$P = \frac{\Delta E}{\Delta t}$$

Substituting for P in equation (1) yields:

$$\frac{dV}{dt} = \frac{\Delta E}{\varepsilon\rho gh\Delta t}$$

Substitute numerical values and evaluate dV/dt:

$$\frac{dV}{dt} = \frac{4.00\times10^{9}\ \text{kW}\cdot\text{h}}{(0.90)(1.00\,\text{kg/L})(9.81\,\text{m/s}^2)(211\,\text{m})\left(365.24\,\text{d}\times\frac{24\ \text{h}}{\text{d}}\right)} = \boxed{2.4\times10^{5}\ \text{L/s}}$$

Force, Potential Energy, and Equilibrium

20 • Water flows over Victoria Falls, which is 128 m high, at an average rate of 1.4×10^{6} kg/s. If half the potential energy of this water were converted into electric energy, how much electric power would be produced by these falls?

Picture the Problem The water going over the falls has gravitational potential energy relative to the base of the falls. As the water falls, the falling water acquires kinetic energy until, at the base of the falls; its energy is entirely kinetic.

The rate at which energy is delivered to the base of the falls is given by $P = dW/dt = -dU/dt$.

Express the rate at which energy is being delivered to the base of the falls; remembering that half the potential energy of the water is converted to electric energy:

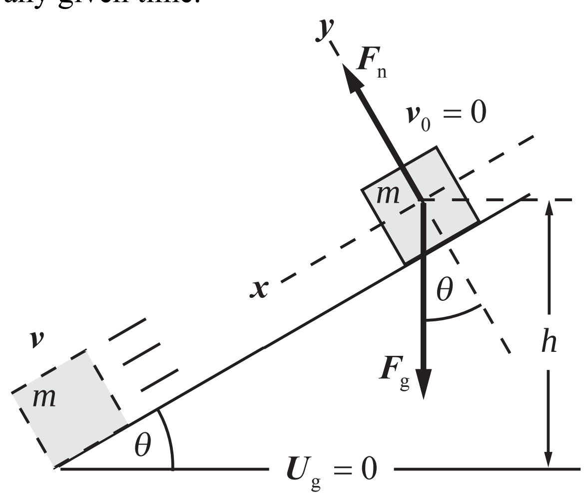
$$P = \frac{dW}{dt} = -\frac{dU}{dt} = -\tfrac{1}{2}\frac{d}{dt}(mgh)$$
$$= -\tfrac{1}{2}gh\frac{dm}{dt}$$

Substitute numerical values and evaluate P:

$$P = -\tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(-128\,\text{m}\right)\left(1.4\times10^{6}\,\text{kg/s}\right) = \boxed{0.88\,\text{GW}}$$

21 • A 2.0-kg box slides down a long, frictionless incline of angle 30º. It starts from rest at time $t = 0$ at the top of the incline at a height of 20 m above the ground. (*a*) What is the potential energy of the box relative to the ground at $t = 0$? (*b*) Use Newton's laws to find the distance the box travels during the interval $0.0\text{ s} < t < 1.0\text{ s}$ and its speed at $t = 1.0$ s. (*c*) Find the potential energy and the kinetic energy of the box at $t = 1.0$ s. (*d*) Find the kinetic energy and the speed of the box just as it reaches the ground at the bottom of the incline.

Picture the Problem In the absence of friction, the sum of the potential and kinetic energies of the box remains constant as it slides down the incline. We can use the conservation of the mechanical energy of the system to calculate where the box will be and how fast it will be moving at any given time. We can also use Newton's 2nd law to show that the acceleration of the box is constant and constant-acceleration equations to calculate where the box will be and how fast it will be moving at any given time.



(*a*) Express the gravitational potential energy of the box, relative to the ground, at the top of the incline:

$$U_i = mgh$$

Substitute numerical values and evaluate U_i:

$$U_i = (2.0\text{ kg})(9.81\text{ m/s}^2)(20\text{ m}) = 392\text{ J} = \boxed{0.39\text{ kJ}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the box to its initial speed, acceleration and time-of-travel:

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}at^2 \qquad (1)$$

Apply $\sum F_x = ma_x$ to the box as it slides down the incline and solve for its acceleration:

$$F_g \sin\theta = ma$$

or, because $F_g = mg$,

$mg\sin\theta = ma$ and

$$a = g\sin\theta$$

Substitute for a in equation (1) to obtain:

$$\Delta x = \tfrac{1}{2}(g\sin\theta)t^2$$

Substitute numerical values and evaluate $\Delta x(t = 1.0\text{ s})$:

$$\Delta x(1.0\text{ s}) = \tfrac{1}{2}(9.81\text{ m/s}^2)(\sin 30°)(1.0\text{ s})^2 = 2.45\text{ m} = \boxed{2.5\text{ m}}$$

Using a constant-acceleration equation, relate the speed of the box at any time to its initial speed and acceleration:

$$v = v_0 + at$$

or, because $v_0 = 0$,

$$v = at = g(\sin\theta)t$$

Substitute numerical values and evaluate $v(1.0\text{ s})$:

$$v(1.0\text{ s}) = (9.81\text{ m/s}^2)(\sin 30°)(1.0\text{ s}) = 4.91\text{ m/s} = \boxed{4.9\text{ m/s}}$$

(*c*) The kinetic energy of the box is given by:

$$K = \tfrac{1}{2}mv^2$$

or, because $v = g(\sin\theta)t$,

$$K(t) = \tfrac{1}{2}mg^2(\sin^2\theta)t^2$$

Substitute numerical values and evaluate $K(1.0\text{ s})$:

$$\boldsymbol{K}(1.0\text{ s})=\tfrac{1}{2}(2.0\text{ kg})(9.81\text{ m/s}^2)^2(\sin^2 30)(1.0\text{ s})^2=24.1\text{ J}=\boxed{24\text{ J}}$$

Express the potential energy of the box after it has traveled for 1.0 s in terms of its initial potential energy and its kinetic energy:

$$\boldsymbol{U}=\boldsymbol{U}_{\text{i}}-\boldsymbol{K}=392\text{ J}-24\text{ J}$$
$$=\boxed{0.37\text{ kJ}}$$

(*d*) Apply conservation of mechanical energy to the box-earth system as the box as it slides down the incline:

$$\boldsymbol{W}_{\text{ext}}=\Delta\boldsymbol{K}+\Delta\boldsymbol{U}=0$$

or, because $K_{\text{i}}=U_{\text{f}}=0$,

$$\boldsymbol{K}_{\text{f}}-\boldsymbol{U}_{\text{i}}=0 \qquad (2)$$

Solving for K_{f} yields:

$$K_{\text{f}}=U_{\text{i}}=\boxed{0.39\text{ kJ}}$$

From equation (2) we have:

$$\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_{\text{f}}^2=\boldsymbol{U}_{\text{i}}\Rightarrow\boldsymbol{v}_{\text{f}}=\sqrt{\frac{2\boldsymbol{U}_{\text{i}}}{\boldsymbol{m}}}$$

Substitute numerical values and evaluate v_{f}:

$$v_{\text{f}}=\sqrt{\frac{2(392\text{ J})}{2.0\text{ kg}}}=\boxed{20\text{ m/s}}$$

22 • A constant force F_x = 6.0 N is in the +x direction. (*a*) Find the potential-energy function $U(x)$ associated with this force if $U(x_0)=0$. (*b*) Find a function $U(x)$ such that $U(4\ .0\text{ m})=0$. (*c*) Find a function $U(x)$ such that $U(6\ .0\text{ m})=14$ J.

Picture the Problem The potential energy function $U(x)$ is defined by the equation $U(x)-U(x_0)=-\int_{x_0}^{x}F\,dx.$ We can use the given force function to determine $U(x)$ and then the conditions on $U(x)$ to determine the potential functions that satisfy the given conditions.

(*a*) Use the definition of the potential energy function to find the potential energy function associated with F_x:

$$\boldsymbol{U}(\boldsymbol{x})=\boldsymbol{U}(\boldsymbol{x}_0)-\int_{x_0}^{x}\boldsymbol{F}_x\boldsymbol{dx}$$
$$=\boldsymbol{U}(\boldsymbol{x}_0)-\int_{x_0}^{x}(6\text{ N})\boldsymbol{dx}$$
$$=\boldsymbol{U}(\boldsymbol{x}_0)-(6.0\text{ N})(\boldsymbol{x}-\boldsymbol{x}_0)$$

Because $U(x_0) = 0$:

$$U(x) = \boxed{-(6.0\,\mathrm{N})(x - x_0)}$$

(*b*) Use the result obtained in (*a*) to find $U(x)$ that satisfies the condition that $U(4.0\text{ m}) = 0$:

$$U(4.0\,\mathrm{m}) = -(6.0\,\mathrm{N})(4.0\,\mathrm{m} - x_0) = 0 \Rightarrow x_0 = 4.0\,\mathrm{m}$$

and

$$U(x) = -(6.0\,\mathrm{N})(x - 4.0\,\mathrm{m}) = \boxed{24\,\mathrm{J} - (6.0\,\mathrm{N})x}$$

(*c*) Use the result obtained in (*a*) to find $U(x)$ that satisfies the condition that $U(6.0\text{ m}) = 14$ J:

$$U(6.0\,\mathrm{m}) = -(6.0\,\mathrm{N})(6.0\,\mathrm{m} - x_0) = 14\,\mathrm{J} \Rightarrow x_0 = \frac{25}{3}\,\mathrm{m}$$

and

$$U(x) = -(6.0\,\mathrm{N})\left(x - \frac{25}{3.0}\,\mathrm{m}\right) = \boxed{50\,\mathrm{J} - (6.0\,\mathrm{N})x}$$

23 • A spring has a force constant of 1.0×10^4 N/m . How far must the spring be stretched for its potential energy to equal (*a*) 50 J, and (*b*) 100 J?

Picture the Problem The potential energy of a stretched or compressed ideal spring U_s is related to its force (stiffness) constant k and stretch or compression Δx by $U_s = \frac{1}{2}kx^2$.

(*a*) Relate the potential energy stored in the spring to the distance it has been stretched:

$$U_s = \tfrac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2U_s}{k}}$$

Substitute numerical values and evaluate x:

$$x = \sqrt{\frac{2(50\,\mathrm{J})}{1.0\times10^4\,\mathrm{N/m}}} = \boxed{10\,\mathrm{cm}}$$

(*b*) Proceed as in (*a*) with $U_s = 100$ J:

$$x = \sqrt{\frac{2(100\,\mathrm{J})}{1.0\times10^4\,\mathrm{N/m}}} = \boxed{14\,\mathrm{cm}}$$

24 • (*a*) Find the force F_x associated with the potential-energy function $U = Ax^4$, where A is a constant. (*b*) At what value(s) of x does the force equal zero?

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(Ax^4\right) = \boxed{-4Ax^3}$$

(*b*) Set $F_x = 0$ and solve for x to obtain:

$$F_x = 0 \Rightarrow \boxed{x = 0}$$

25 •• **[SSM]** The force F_x is associated with the potential-energy function $U = C/x$, where C is a positive constant. (*a*) Find the force F_x as a function of x. (*b*) Is this force directed toward the origin or away from it in the region $x > 0$? Repeat the question for the region $x < 0$. (*c*) Does the potential energy U increase or decrease as x increases in the region $x > 0$? (*d*) Answer Parts (*b*) and (*c*) where C is a negative constant.

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(\frac{C}{x}\right) = \boxed{\frac{C}{x^2}}$$

(*b*) Because $C > 0$, if $x > 0$, F_x is positive and $\vec{F}$ points away from the origin. If $x < 0$, F_x is still positive and $\vec{F}$ points toward the origin.

(*c*) Because U is inversely proportional to x and $C > 0$, $U(x)$ decreases with increasing x.

(*d*) When $C < 0$, if $x > 0$, F_x is negative and $\vec{F}$ points toward the origin. If $x < 0$, F_x is negative and $\vec{F}$ points away from the origin.

Because U is inversely proportional to x and $C < 0$, $U(x)$ becomes less negative as x increases and $U(x)$ increases with increasing x.

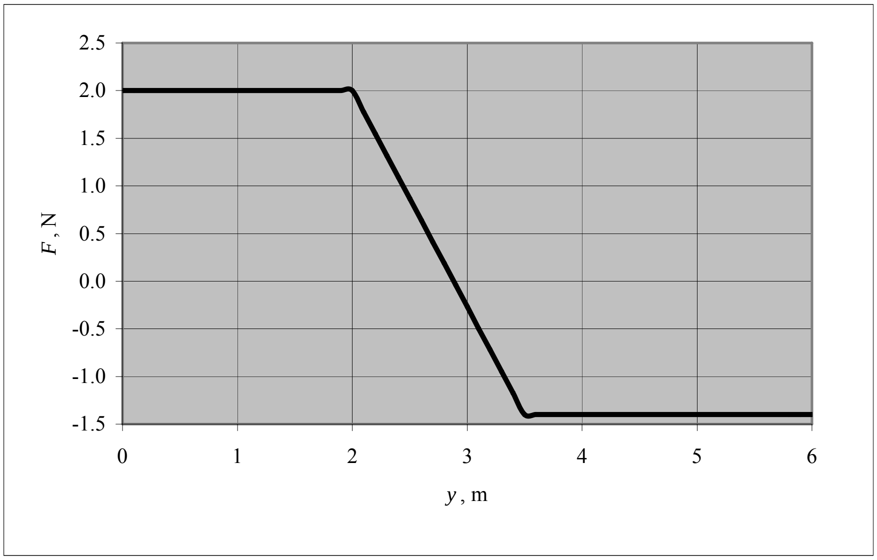
26 •• The force F_y is associated with the potential-energy function $U(y)$. On the potential-energy curve for U versus y, shown in Figure 7-37, the segments AB and CD are straight lines. Plot F_y versus y. Include numerical values, with units, on both axes. These values can be obtained from the U versus y plot.

Picture the Problem F_y is defined to be the negative of the derivative of the potential-energy function with respect to y; that is, $F_y = -dU/dy$. Consequently, we can obtain F_y by examining the slopes of the graph of U as a function of y.

The table to the right summarizes the information we can obtain from Figure 7-37:

Interval	Slope (N)	F_y (N)
A→B	−2	2
B→C	transitional	2 → −1.4
C→D	1.4	−1.4

The following graph shows *F* as a function of *y*:



27 •• The force acting on an object is given by $F_x = a/x^2$. At $x = 5.0$ m, the force is known to point in the $-x$ direction and have a magnitude of 25.0 N. Determine the potential energy associated with this force as a function of *x*, assuming we assign a reference value of –10.0 J at $x = 2.0$ m for the potential energy.

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to *x*, i.e. $F_x = -dU/dx$. Consequently, given *F* as a function of *x*, we can find *U* by integrating F_x with respect to *x*. Applying the condition on $F_x x$ will allow us to determine the value of *a* and using the fact that the potential energy is –10.0 J at $x = 2.00$ m will give us the value of U_0.

Evaluate the integral of F_x with respect to *x*:

$$U(x) = -\int F_x dx = -\int \frac{a}{x^2} dx = \frac{a}{x} + U_0 \qquad (1)$$

Because $F_x(5.0\text{ m}) = -25.0\text{ N}$:

$$\frac{a}{(5.00\text{ m})^2} = -25.0\text{ N}$$

Solving for a yields: $a = -625\ \text{N}\cdot\text{m}^2$

Substitute for a in equation (1) to obtain:

$$U(x) = \frac{-625\ \text{N}\cdot\text{m}^2}{x} + U_0 \qquad (2)$$

Applying the condition $U(2.00\ \text{m}) = -10.0\ \text{J}$ yields:

$$-10.0\ \text{J} = \frac{-625\ \text{N}\cdot\text{m}^2}{2.00\ \text{m}} + U_0$$

Solve for U_0 to obtain: $U_0 = 303\ \text{J}$

Substituting for U_0 in equation (2) yields:

$$U(x) = \boxed{\frac{-625\ \text{N}\cdot\text{m}^2}{x} + 303\ \text{J}}$$

28 •• The potential energy of an object constrained to the x axis is given by $U(x) = 3x^2 - 2x^3$, where U is in joules and x is in meters. (*a*) Determine the force F_x associated with this potential energy function. (*b*) Assuming no other forces act on the object, at what positions is this object in equilibrium? (*c*) Which of these equilibrium positions are stable and which are unstable?

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate $F_x = -\dfrac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(3x^2 - 2x^3\right) = \boxed{6x(x-1)}$$

(*b*) We know that, at equilibrium, $F_x = 0$:

When $F_x = 0$, $6x(x-1) = 0$. Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 1\,\text{m.}}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{dU}{dx} = \frac{d}{dx}\left(3x^2 - 2x^3\right) = 6x - 6x^2$$

and

$$\frac{d^2U}{dx^2} = 6 - 12x$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 6 > 0$$

$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 1$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=1\,\text{m}} = 6 - 12 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 1\,\text{m}}$$

29 •• [SSM] The potential energy of an object constrained to the x axis is given by $U(x) = 8x^2 - x^4$, where U is in joules and x is in meters. (*a*) Determine the force F_x associated with this potential energy function. (*b*) Assuming no other forces act on the object, at what positions is this object in equilibrium? (*c*) Which of these equilibrium positions are stable and which are unstable?

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate the negative of the derivative of U with respect to x:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(8x^2 - x^4\right)$$
$$= 4x^3 - 16x = \boxed{4x(x+2)(x-2)}$$

(*b*) The object is in equilibrium wherever $F_{\text{net}} = F_x = 0$:

$4x(x+2)(x-2) = 0 \Rightarrow$ the equilibrium points are $\boxed{x = -2\,\text{m}, 0, \text{and } 2\,\text{m.}}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{d^2U}{dx^2} = \frac{d}{dx}\left(16x - 4x^3\right) = 16 - 12x^2$$

Evaluating $\boldsymbol{d^2U/dx^2}$ at $x = -2$ m, 0 and $x = 2$ m yields the following results:

x, m	d^2U/dx^2	Equilibrium
−2	−32	Unstable
0	16	Stable
2	−32	Unstable

Remarks: You could also decide whether the equilibrium positions are stable or unstable by plotting *F*(*x*) and examining the curve at the equilibrium positions.

30 •• The net force acting on an object constrained to the x axis is given by $F_x(x) = x^3 - 4x$. (The force is in newtons and x in meters.) Locate the positions of unstable and stable equilibrium. Show that each of these positions is either stable or unstable by calculating the force one millimeter on either side of the locations.

Picture the Problem The equilibrium positions are those values of x for which $F(x) = 0$. Whether the equilibrium positions are stable or unstable depends on whether the signs of the force either side of the equilibrium position are the same (unstable equilibrium) of opposite (stable equilibrium).

Determine the equilibrium locations by setting $F_{\text{net}} = F(x) = 0$:

$F(x) = x^3 - 4x = x(x^2 - 4) = 0$
and the positions of stable and unstable equilibrium are at

$$\boxed{\boldsymbol{x} = -2\text{ m}, 0 \text{ and } 2\text{ m}}\,.$$

Noting that we need only determine whether each value of $F(x)$ is positive or negative, evaluate $F(x)$ at $x = -201$ mm and $x = -199$ mm to determine the stability at $x = -200$ mm … and repeat these calculations at $x = -1$ mm, 1 mm and $x = 199$ mm, 201 mm to complete the following table:

x, mm	$\boldsymbol{F_{x-1\,\text{mm}}}$	$\boldsymbol{F_{x+1\,\text{mm}}}$	Equilibrium
−200	< 0	< 0	Unstable
0	> 0	< 0	Stable
200	> 0	> 0	Unstable

Remarks: You can very easily confirm these results by using your graphing calculator to plot *F*(*x*). You could also, of course, find *U*(*x*) from *F*(*x*) and examine it at the equilibrium positions.

31 •• The potential energy of a 4.0-kg object constrained to the x axis is given by $U = 3x^2 - x^3$ for $x \le 3.0$ m and $U = 0$ for $x \ge 3.0$ m, where U is in joules and x is in meters, and the only force acting on this object is the force associated with this potential energy function. (*a*) At what positions is this object in equilibrium? (*b*) Sketch a plot of U versus x. (*c*) Discuss the stability of the equilibrium for the values of x found in Part (*a*). (*d*) If the total mechanical energy of the particle is 12 J, what is its speed at $x = 2.0$ m?

Picture the Problem $F_x x$ is defined to be the negative of the derivative of the potential-energy function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we can examine the graph of U.

(*a*) Evaluate $F_x = -\dfrac{dU}{dx}$ for $x \le 3.0$ m:

$$F_x = -\frac{d}{dx}\left(3x^2 - x^3\right) = 3x(2-x)$$

Set $F_x = 0$ and solve for those values of x for which the 4.0-kg object is in equilibrium:

$3x(2 - x) = 0$

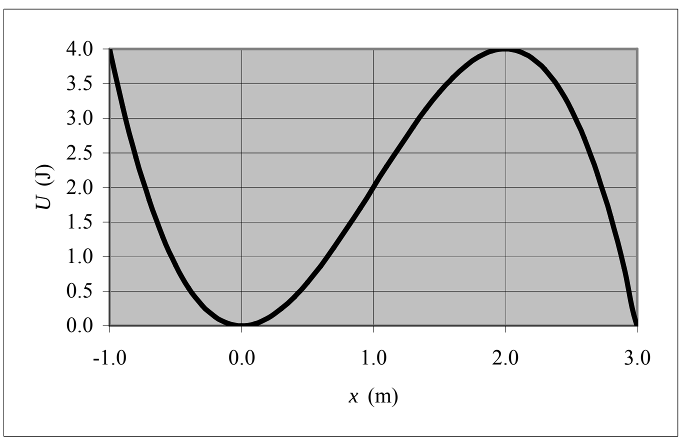
Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 2.0\,\text{m.}}$

Because $U = 0$:

$$\boldsymbol{F_x}(\boldsymbol{x} \ge 3\,\text{m}) = -\frac{\boldsymbol{dU}}{\boldsymbol{dx}} = 0$$

Therefore, the object is in neutral equilibrium for $x \ge 3.0$ m.

(*b*) A graph of $U(x)$ in the interval -1.0 m $\le x \le 3.0$ m follows:



(*c*) From the graph, $U(x)$ is a minimum at $x = 0$ and so the equilibrium is stable at this point

From the graph, $U(x)$ is a maximum at $x = 2.0$ m and so the equilibrium is unstable at this point.

(*d*) Relate the kinetic energy of the object K to its total energy E and its potential energy U:

$$\boldsymbol{K} = \tfrac{1}{2}\boldsymbol{mv}^2 = \boldsymbol{E} - \boldsymbol{U} \Rightarrow v = \sqrt{\frac{2(E-U)}{m}}$$

Substitute numerical values and evaluate $v(2.0\text{ m})$:

$$\boldsymbol{v}(2.0\,\text{m}) = \sqrt{\frac{2\left(12\,\text{J} - \left(\left(3.0\,\text{J/m}^2\right)(2.0\,\text{m})^2 - \left(1.0\,\text{J/m}^3\right)(2.0\,\text{m})^3\right)\right)}{4.0\,\text{kg}}} = \boxed{2.0\,\text{m/s}}$$

32 •• A force is given by $F_x = Ax^{-3}$, where $A = 8.0\ \text{N}\cdot\text{m}^3$. (*a*) For positive values of *x*, does the potential energy associated with this force increase or decrease with increasing *x*? (You can determine the answer to this question by imagining what happens to a particle that is placed at rest at some point *x* and is then released.) (*b*) Find the potential-energy function *U* associated with this force such that *U* approaches zero as *x* approaches infinity. (*c*) Sketch *U* versus *x*.

Picture the Problem F_x is defined to be the negative of the derivative of the potential-energy function with respect to *x*, that is $F_x = -dU/dx$. Consequently, given *F* as a function of *x*, we can find *U* by integrating F_x with respect to *x*.

(*a*) Evaluate the negative of the integral of $F(x)$ with respect to *x*:

$$U(x) = -\int F(x) = -\int Ax^{-3}dx$$

$$= \frac{1}{2}\frac{A}{x^2} + U_0$$

where U_0 is a constant whose value is determined by conditions on $U(x)$.

For $x > 0$:

$$\boxed{U \text{ decreases as } x \text{ increases}}$$

(*b*) As $x \to \infty$, $\frac{1}{2}\frac{A}{x^2} \to 0$. Hence:

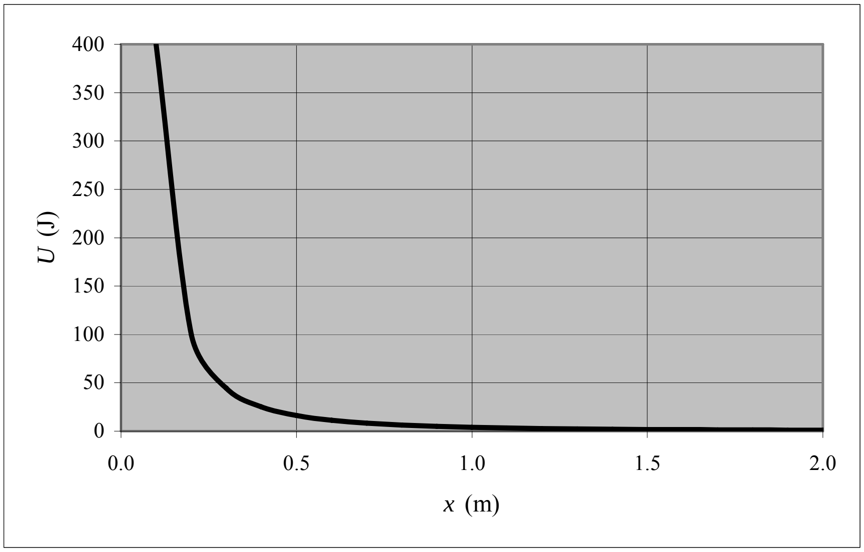
$U_0 = 0$

and

$$\boldsymbol{U}(\boldsymbol{x}) = \frac{1}{2}\frac{\boldsymbol{A}}{\boldsymbol{x}^2} = \frac{1}{2}\left(\frac{8.0\,\text{N}\cdot\text{m}^3}{\boldsymbol{x}^2}\right)$$

$$= \boxed{\frac{4.0}{\boldsymbol{x}^2}\text{N}\cdot\text{m}^3}$$

(c) The graph of $U(x)$ follows:



33 •• **[SSM]** A straight rod of negligible mass is mounted on a frictionless pivot, as shown in Figure 7-38. Blocks have masses m_1 and m_2 are attached to the rod at distances ℓ_1 and ℓ_2. (a) Write an expression for the gravitational potential energy of the blocks-Earth system as a function of the angle θ made by the rod and the horizontal. (b) For what angle θ is this potential energy a minimum? Is the statement ″systems tend to move toward a configuration of minimum potential energy″ consistent with your result? (c) Show that if $m_1\ell_1 = m_2\ell_2$, the potential energy is the same for all values of θ. (When this holds, the system will balance at any angle θ. This result is known as *Archimedes' law of the lever*.)

Picture the Problem The gravitational potential energy of this system of two objects is the sum of their individual potential energies and is dependent on an arbitrary choice of where, or under what condition(s), the gravitational potential energy is zero. The best choice is one that simplifies the mathematical details of the expression for U. In this problem let's choose $U = 0$ where $\theta = 0$.

(a) Express U for the 2-object system as the sum of their gravitational potential energies; noting that because the object whose mass is m_2 is above the position we have chosen for $U = 0$, its potential energy is positive while that of the object whose mass is m_1 is negative:

$$\begin{aligned} U(\theta) &= U_1 + U_2 \\ &= m_2 g\ell_2 \sin\theta - m_1 g\ell_1 \sin\theta \\ &= \boxed{(m_2\ell_2 - m_1\ell_1)g\sin\theta} \end{aligned}$$

(*b*) Differentiate *U* with respect to θ and set this derivative equal to zero to identify extreme values:

$$\frac{dU}{d\theta}=(m_2\ell_2-m_1\ell_1)g\cos\theta=0$$

from which we can conclude that $\cos\theta = 0$ and $\theta = \cos^{-1}0$.

To be physically meaningful, $-\pi/2\le\theta\le\pi/2$. Hence:

$$\theta=\pm\pi/2$$

Express the 2nd derivative of *U* with respect to θ and evaluate this derivative at $\theta=\pm\pi/2$:

$$\frac{d^2U}{d\theta^2}=-(m_2\ell_2-m_1\ell_1)g\sin\theta$$

If we assume, in the expression for *U* that we derived in (*a*), that $m_2\ell_2 - m_1\ell_1 > 0$, then $U(\theta)$ is a sine function and, in the interval of interest,

$$-\pi/2\le\theta\le\pi/2,$$

takes on its minimum value when $\theta = -\pi/2$:

$$\left.\frac{d^2U}{d\theta^2}\right|_{-\pi/2}>0 \text{ and}$$

$$\boxed{U \text{ is a minimum at } \theta=-\pi/2}$$

$$\left.\frac{d^2U}{d\theta^2}\right|_{\pi/2}<0 \text{ and}$$

$$\boxed{U \text{ is a maximum at } \theta=\pi/2}$$

(c) If m2ℓ2 = m1ℓ1, then:

$$m_1\ell_1-m_2\ell_2=0$$

and

$$\boxed{U=0 \text{ independent of } \theta.}$$

Remarks: An alternative approach to establishing that *U* is a maximum at $\theta = \pi/2$ is to plot its graph and note that, in the interval of interest, *U* is concave downward with its maximum value at $\theta = \pi/2$. Similarly, it can be shown that *U* is a minimum at $\theta = -\pi/2$ (Part (*b*)).

34 •• An Atwood's machine (Figure 7-39) consists of masses m_1 and m_2, and a pulley of negligible mass and friction. Starting from rest, the speed of the two masses is 4.0 m/s at the end of 3.0 s. At that time, the kinetic energy of the system is 80 J and each mass has moved a distance of 6.0 m. Determine the values of m_1 and m_2.

Picture the Problem In a simple Atwood's machine, the only effect of the pulley is to connect the motions of the two objects on either side of it; that is, it could be replaced by a piece of polished pipe. We can relate the kinetic energy of the rising and falling objects to the mass of the system and to their common speed and relate their accelerations to the sum and difference of their masses … leading to simultaneous equations in m_1 and m_2.

Relate the kinetic energy of the system to the total mass being accelerated:

$$K = \tfrac{1}{2}(m_1 + m_2)v^2 \Rightarrow m_1 + m_2 = \frac{2K}{v^2}$$

Substitute numerical values and evaluate $m_1 + m_2$:

$$m_1 + m_2 = \frac{2(80\,\text{J})}{(4.0\,\text{m/s})^2} = 10.0\,\text{kg} \qquad (1)$$

In Chapter 4, the acceleration of the masses was shown to be:

$$a = \frac{m_1 - m_2}{m_1 + m_2}g$$

Because $v(t) = at$, we can eliminate a in the previous equation to obtain:

$$v(t) = \frac{m_1 - m_2}{m_1 + m_2}gt$$

Solving for $m_1 - m_2$ yields:

$$m_1 - m_2 = \frac{(m_1 + m_2)v(t)}{gt}$$

Substitute numerical values and evaluate $m_1 - m_2$:

$$m_1 - m_2 = \frac{(10\,\text{kg})(4.0\,\text{m/s})}{(9.81\,\text{m/s}^2)(3.0\,\text{s})} = 1.36\,\text{kg} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\boldsymbol{m_1} = \boxed{5.7\,\text{kg}} \text{ and } \boldsymbol{m_2} = \boxed{4.3\,\text{kg}}$$

35 ••• You have designed a novelty desk clock, as shown in Figure 7-40. You are worried that it is not ready for market because the clock itself might be in an unstable equilibrium configuration. You decide to apply your knowledge of potential energies and equilibrium conditions and analyze the situation. The clock (mass m) is supported by two light cables running over the two frictionless pulleys of negligible diameter, which are attached to counterweights that each have mass M. (*a*) Find the potential energy of the system as a function of the distance y. (*b*) Find the value of y for which the potential energy of the system is a minimum. (*c*) If the potential energy is a minimum, then the system is in equilibrium. Apply Newton's second law to the clock and show that it is in equilibrium (the forces on it sum to zero) for the value of y obtained for Part (*b*).

(d) Finally, determine whether you are going to be able to market this gadget: is this a point of stable or unstable equilibrium?

Picture the Problem Let L be the total length of one cable and the zero of gravitational potential energy be at the top of the pulleys. We can find the value of y for which the potential energy of the system is an extremum by differentiating $U(y)$ with respect to y and setting this derivative equal to zero. We can establish that this value corresponds to a minimum by evaluating the second derivative of $U(y)$ at the point identified by the first derivative. We can apply Newton's 2nd law to the clock to confirm the result we obtain by examining the derivatives of $U(y)$.

(*a*) Express the potential energy of the system as the sum of the potential energies of the clock and counterweights:

$$U(y)=U_{\text{clock}}(y)+U_{\text{weights}}(y)$$

Substitute for $U_{\text{clock}}(y)$ and $U_{\text{weights}}(y)$ to obtain:

$$U(y)=\boxed{-mgy-2Mg\left(L-\sqrt{y^2+d^2}\right)}$$

(*b*) Differentiate $U(y)$ with respect to y:

$$\frac{dU(y)}{dy}=-\frac{d}{dy}\left[mgy+2Mg\left(L-\sqrt{y^2+d^2}\right)\right]=-\left[mg-2Mg\frac{y}{\sqrt{y^2+d^2}}\right]$$

For extreme values (relative maxima and minima):

$$mg-2Mg\frac{y'}{\sqrt{y'^2+d^2}}=0$$

Solve for y' to obtain:

$$y'=d\sqrt{\frac{m^2}{4M^2-m^2}}$$

Find $\dfrac{d^2U(y)}{dy^2}$:

$$\frac{d^2U(y)}{dy^2}=-\frac{d}{dy}\left[mg-2Mg\frac{y}{\sqrt{y^2+d^2}}\right]$$

$$=\frac{2Mgd^2}{\left(y^2+d^2\right)^{3/2}}$$

Evaluate $\frac{d^2U(y)}{dy^2}$ at $y = y'$:

$$\left.\frac{d^2U(y)}{dy^2}\right|_{y'} = \left.\frac{2Mgd^2}{\left(y^2+d^2\right)^{3/2}}\right|_{y'}$$

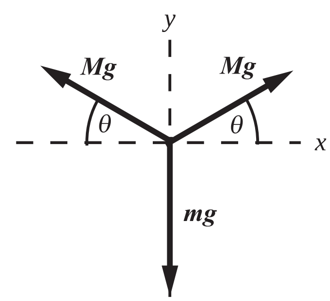
$$= \frac{2Mgd}{\left(\frac{m^2}{4M^2-m^2}+1\right)^{3/2}}$$

$$> 0$$

and the potential energy is a minimum at

$$y = \boxed{d\sqrt{\frac{m^2}{4M^2-m^2}}}$$

(*c*) The free-body diagram, showing the magnitudes of the forces acting on the support point just above the clock, is shown to the right:



Apply $\sum F_y = 0$ to this point to obtain:

$$2Mg\sin\theta - mg = 0 \Rightarrow \sin\theta = \frac{m}{2M}$$

Express $\sin\theta$ in terms of y and d:

$$\sin\theta = \frac{y}{\sqrt{y^2+d^2}}$$

Equate the two expressions for $\sin\theta$ to obtain:

$$\frac{m}{2M} = \boxed{\frac{y}{\sqrt{y^2+d^2}}}$$

which is equivalent to the first equation in Part (*b*).

(*d*) This is a point of stable equilibrium. If the clock is displaced downward, θ increases, leading to a larger upward force on the clock. Similarly, if the clock is displaced upward, the net force from the cables decreases. Because of this, the clock will be pulled back toward the equilibrium point if it is displaced away from it.

Remarks: Because we've shown that the potential energy of the system is a minimum at $y = y'$ (i.e., $U(y)$ is concave upward at that point), we can conclude that this point is one of stable equilibrium.

The Conservation of Mechanical Energy

36 • A block of mass *m* on a horizontal frictionless tabletop is pushed against a horizontal spring, compressing it a distance *x*, and the block is then released. The spring propels the block along the tabletop, giving a speed *v*. The same spring is then used to propel a second block of mass 4*m*, giving it a speed 3*v*. What distance was the spring compressed in the second case? Express your answer in terms of *x*.

Picture the Problem The work done in compressing the spring is stored in the spring as potential energy. When the block is released, the energy stored in the spring is transformed into the kinetic energy of the block. Equating these energies will give us a relationship between the compressions of the spring and the speeds of the blocks.

Let the numeral 1 refer to the first case and the numeral 2 to the second case. Relate the compression of the spring in the second case to its potential energy, which equals its initial kinetic energy when released:

$$\begin{aligned}\tfrac{1}{2}kx_2^2 &= \tfrac{1}{2}m_2v_2^2\\ &= \tfrac{1}{2}(4m_1)(3v_1)^2\\ &= 18m_1v_1^2\end{aligned}$$

Relate the compression of the spring in the first case to its potential energy, which equals its initial kinetic energy when released:

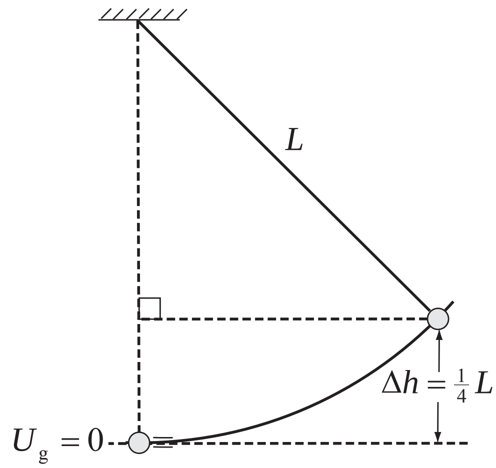
$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m_1v_1^2 \Rightarrow \boldsymbol{m_1v_1^2} = \boldsymbol{kx_1^2}$$

Substitute for $\boldsymbol{m_1v_1^2}$ to obtain:

$$\tfrac{1}{2}kx_2^2 = 18kx_1^2 \Rightarrow x_2 = \boxed{6x_1}$$

37 • A simple pendulum of length *L* with a bob of mass *m* is pulled aside until the bob is at a height *L*/4 above its equilibrium position. The bob is then released. Find the speed of the bob as it passes through the equilibrium position. Neglect any effects due to air resistance.

Picture the Problem The diagram shows the pendulum bob in its initial position. Let the zero of gravitational potential energy be at the low point of the pendulum's swing, the equilibrium position, and let the system include the pendulum and the earth. We can find the speed of the bob at it passes through the equilibrium position by applying conservation of mechanical energy to the system.



Apply conservation of mechanical energy to the system to obtain:

$$\boldsymbol{W}_{\text{ext}} = \Delta \boldsymbol{K} + \Delta \boldsymbol{U}$$

or, because $W_{\text{ext}} = 0$,

$$\Delta \boldsymbol{K} + \Delta \boldsymbol{U} = 0$$

Because $K_i - U_f = 0$:

$$K_f - U_i = 0$$

Substituting for K_f and U_i yields:

$$\tfrac{1}{2}mv_f^2 - mg\Delta h = 0 \Rightarrow v_f = \sqrt{2g\Delta h}$$

Express Δh in terms of the length L of the pendulum:

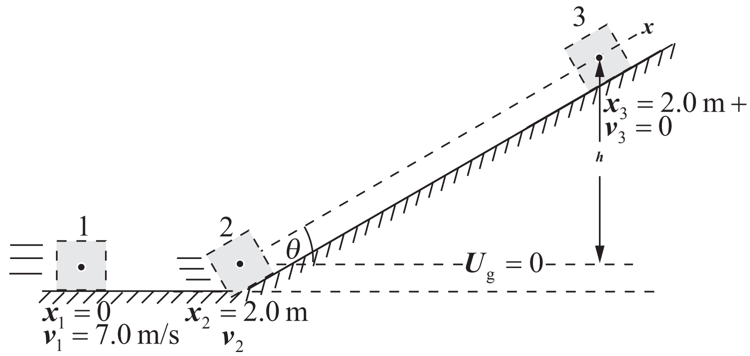
$$\Delta h = \frac{L}{4}$$

Substitute for Δh in the expression for v_f and simplify to obtain:

$$v_f = \boxed{\sqrt{\frac{gL}{2}}}$$

38 • A 3.0-kg block slides along a frictionless horizontal surface with a speed of 7.0 m/s (Figure 7-41). After sliding a distance of 2.0 m, the block makes a smooth transition to a frictionless ramp inclined at an angle of 40° to the horizontal. What distance along the ramp does the block slide before coming momentarily to rest?

Picture the Problem The pictorial representation shows the block in its initial, intermediate, and final states. It also shows a choice for $U_g = 0$. Let the system consist of the block, ramp, and the earth. Because the surfaces are frictionless, the initial kinetic energy of the system is equal to its final gravitational potential energy when the block has come to rest on the incline.



Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U$$

or, because $W_{ext} = 0$,

$$\Delta K + \Delta U = 0$$

Because $K_3 = U_1 = 0$:

$$-K_1 + U_3 = 0$$

Substituting for K_1 and U_3 yields:

$$-\tfrac{1}{2}mv_1^2 + mgh = 0 \Rightarrow h = \frac{v_1^2}{2g}$$

where h is the change in elevation of the block as it slides to a momentary stop on the ramp.

Relate the height h to the displacement ℓ of the block along the ramp and the angle the ramp makes with the horizontal:

$$h = \ell \sin\theta$$

Equate the two expressions for h and solve for ℓ to obtain:

$$\ell \sin\theta = \frac{v_1^2}{2g} \Rightarrow \ell = \frac{v_1^2}{2g\sin\theta}$$

Substitute numerical values and evaluate ℓ:

$$\ell = \frac{(7.0\text{ m/s})^2}{2(9.81\text{ m/s}^2)\sin 40°} = \boxed{3.9\text{ m}}$$

39 • The 3.00-kg object in Figure 7-42 is released from rest at a height of 5.00 m on a curved frictionless ramp. At the foot of the ramp is a spring of force constant 400 N/m. The object slides down the ramp and into the spring, compressing it a distance *x* before coming momentarily to rest. (*a*) Find *x*. (*b*) Describe the motion object (if any) after the block momentarily comes to rest?

Picture the Problem Let the system consist of the earth, the block, and the spring. With this choice there are no external forces doing work to change the energy of the system. Let $U_g = 0$ at the elevation of the spring. Then the initial gravitational potential energy of the 3.00-kg object is transformed into kinetic energy as it slides down the ramp and then, as it compresses the spring, into potential energy stored in the spring.

(*a*) Apply conservation of mechanical energy to the system to relate the distance the spring is compressed to the initial potential energy of the block:

$$W_{ext} = \Delta K + \Delta U = 0$$

and, because $\Delta K = 0$,

$$-mgh + \tfrac{1}{2}kx^2 = 0 \Rightarrow x = \sqrt{\frac{2mgh}{k}}$$

Substitute numerical values and evaluate x:

$$\boldsymbol{x} = \sqrt{\frac{2(3.00\,\text{kg})(9.81\,\text{m/s}^2)(5.00\,\text{m})}{400\,\text{N/m}}} = \boxed{0.858\,\text{m}}$$

(*b*) The energy stored in the compressed spring will accelerate the block, launching it back up the incline and the block will retrace its path, rising to a height of 5.00 m.

40 • You are designing a game for small children and want to see if the ball's maximum speed is sufficient to require the use of goggles. In your game, a 15.0-g ball is to be shot from a spring gun whose spring has a force constant of 600 N/m. The spring will be compressed 5.00 cm when in use. How fast will the ball be moving as it leaves the gun and how high will the ball go if the gun is aimed vertically upward? What would be your recommendation on the use of goggles?

Picture the Problem With U_g chosen to be zero at the uncompressed level of the spring, the ball's initial gravitational potential energy is negative. Let the system consist of the ball, the spring and gun, and the earth. The difference between the initial potential energy of the spring and the gravitational potential energy of the ball is first converted into the kinetic energy of the ball and then into gravitational potential energy as the ball rises and slows … eventually coming momentarily to rest.

Apply conservation of mechanical energy to the system as the ball leaves the gun to obtain:

$$W_{ext} = \Delta K + \Delta U_g + \Delta U_s = 0$$

or, because $K_i = U_{s,f} = U_{g,f} = 0$,

$$\tfrac{1}{2}mv_f^2 + mgx - \tfrac{1}{2}kx^2 = 0$$

Solving for v_f yields:

$$v_f = \sqrt{\left(\frac{k}{m}x - 2g\right)x}$$

Substitute numerical values and evaluate v_f:

$$\boldsymbol{v}_{\mathrm{f}}=\sqrt{\left[\left(\frac{600\ \mathrm{N/m}}{0.0150\ \mathrm{kg}}\right)(0.0500\ \mathrm{m})-2\left(9.81\ \mathrm{m/s^2}\right)\right](0.0500\ \mathrm{m})}=\boxed{9.95\ \mathrm{m/s}}$$

This initial speed of the ball is fast enough to warrant the use of goggles.

Apply conservation of mechanical energy to the system as the ball rises to its maximum height to obtain:

$$W_{\mathrm{ext}}=\Delta K+\Delta U_{\mathrm{g}}+\Delta U_{\mathrm{s}}=0$$

or, because $\Delta K = U_{s,f} = 0$,

$$mgh+mgx-\tfrac{1}{2}kx^2=0$$

where h is the maximum height of the ball.

Solving for h yields:

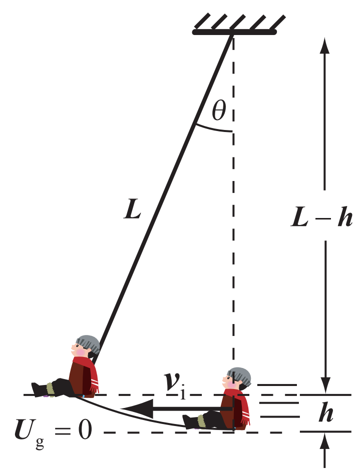
$$h=\frac{kx^2}{2mg}-x$$

Substitute numerical values and evaluate h:

$$\boldsymbol{h}=\frac{(600\,\mathrm{N/m})(0.0500\,\mathrm{m})^2}{2(0.0150\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)}-0.0500\,\mathrm{m}=\boxed{5.05\,\mathrm{m}}$$

41 • **[SSM]** A 16-kg child on a 6.0-m-long playground swing moves with a speed of 3.4 m/s when the swing seat passes through its lowest point. What is the angle that the swing makes with the vertical when the swing is at its highest point? Assume that the effects due to air resistance are negligible, and assume that the child is not pumping the swing.

Picture the Problem Let the system consist of the earth and the child. Then W_{ext} = 0. Choose U_g = 0 at the child's lowest point as shown in the diagram to the right. Then the child's initial energy is entirely kinetic and its energy when it is at its highest point is entirely gravitational potential. We can determine h from conservation of mechanical energy and then use trigonometry to determine θ.



Using the diagram, relate θ to h and L:

$$\theta=\cos^{-1}\left(\frac{L-h}{L}\right)=\cos^{-1}\left(1-\frac{h}{L}\right)\quad(1)$$

Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U = 0$$

or, because $K_f = U_{g,i} = 0$,

$$-K_i + U_{g,f} = 0$$

Substituting for K_i and $U_{g,f}$ yields:

$$-\tfrac{1}{2}mv_i^2 + mgh = 0 \Rightarrow h = \frac{v_i^2}{2g}$$

Substitute for h in equation (1) to obtain:

$$\theta = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left(1 - \frac{(3.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(6.0\,\text{m})}\right) = \boxed{26°}$$

42 •• The system shown in Figure 7-44 is initially at rest when the lower string is cut. Find the speed of the objects when they are momentarily at the same height. The frictionless pulley has negligible mass.

Picture the Problem Let the system include the two objects and the earth. Then $W_{ext} = 0$. Choose $U_g = 0$ at the elevation at which the two objects meet. With this choice, the initial potential energy of the 3.0-kg object is positive and that of the 2.0-kg object is negative. Their sum, however, is positive. Given our choice for $U_g = 0$, this initial potential energy is transformed entirely into kinetic energy.

Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U_g = 0$$

or, because $W_{ext} = 0$,

$$\Delta K = -\Delta U_g$$

Noting that m represents the sum of the masses of the objects as they are both moving in the final state, substitute for ΔK:

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -\Delta U_g$$

or, because $v_i = 0$,

$$\tfrac{1}{2}mv_f^2 = -\Delta U_g \Rightarrow v_f = \sqrt{\frac{-2\Delta U_g}{m}}$$

ΔU_g is given by:

$$\Delta U_g = U_{g,f} - U_{g,i} = 0 - (m_3 - m_2)gh$$

Substitute for ΔU_g to obtain:

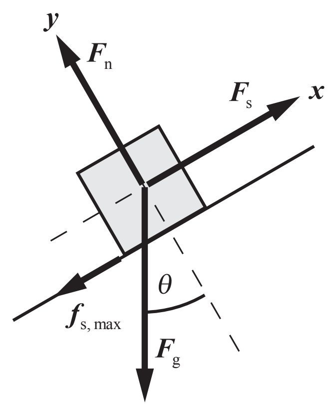
$$v_f = \sqrt{\frac{2(m_3 - m_2)gh}{m}}$$

Substitute numerical values and evaluate v_f:

$$\boldsymbol{v}_f = \sqrt{\frac{2(3.0\,\text{kg} - 2.0\,\text{kg})(0.50\,\text{m})(9.81\,\text{m/s}^2)}{3.0\,\text{kg} + 2.0\,\text{kg}}} = \boxed{1.4\,\text{m/s}}$$

43 •• A block of mass *m* rests on an inclined plane (Figure 7-44). The coefficient of static friction between the block and the plane is μ_s. A gradually increasing force is pulling down on the spring (force constant *k*). Find the potential energy *U* of the spring (in terms of the given symbols) at the moment the block begins to move.

Picture the Problem F_s is the force exerted by the spring and, because the block is on the verge of sliding, $f_s = f_{s,max}$. We can use Newton's 2nd law, under equilibrium conditions, to express the elongation of the spring as a function of *m, k* and θ and then substitute in the expression for the potential energy stored in a stretched or compressed spring.



Express the potential energy of the spring when the block is about to move:

$$U = \tfrac{1}{2}kx^2$$

Apply $\sum \vec{F} = m\vec{a}$, under equilibrium conditions, to the block:

$$\sum \boldsymbol{F}_x = \boldsymbol{F}_s - \boldsymbol{f}_{s,max} - \boldsymbol{mg}\sin\boldsymbol{\theta} = 0$$

and

$$\sum \boldsymbol{F}_y = \boldsymbol{F}_n - \boldsymbol{mg}\cos\boldsymbol{\theta} = 0$$

Using $f_{s,\,max} = \mu_s F_n$ and $F_s = kx$, eliminate $f_{s,\,max}$ and F_s from the *x* equation and solve for *x*:

$$x = \frac{mg(\sin\theta + \mu_s\cos\theta)}{k}$$

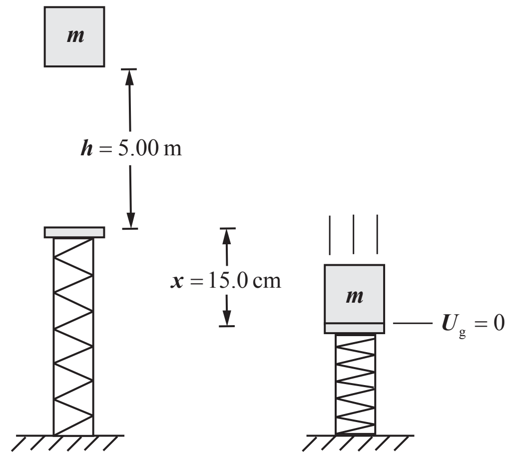
Substitute for *x* in the expression for *U* and simplify to obtain:

$$U = \tfrac{1}{2}k\left[\frac{mg(\sin\theta + \mu_s\cos\theta)}{k}\right]^2$$

$$= \boxed{\frac{[mg(\sin\theta + \mu_s\cos\theta)]^2}{2k}}$$

44 •• A 2.40-kg block is dropped onto a spring (Figure 7-45) with a force constant of 3.96×10^3 N/m from a height of 5.00 m. When the block is momentarily at rest, the spring is compressed by 25.0 cm. Find the speed of the block when the compression of the spring is 15.0 cm.

Picture the Problem Let the system include the block, the spring, and the earth. Let $U_g = 0$ where the spring is compressed 15.0 cm. Then the mechanical energy when the compression of the spring is 15.0 cm will be partially kinetic and partially stored in the spring. We can use conservation of mechanical energy to relate the initial potential energy of the system to the energy stored in the spring and the kinetic energy of block when it has compressed the spring 15.0 cm.



Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta U + \Delta K = 0$$

or

$$U_{g,f} - U_{g,i} + U_{s,f} - U_{s,i} + K_f - K_i = 0$$

Because $U_{g,f} = U_{s,i} = K_i = 0$:

$$-U_{g,i} + U_{s,f} + K_f = 0$$

Substitute to obtain:

$$-mg(h+x) + \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = 0$$

Solving for v yields:

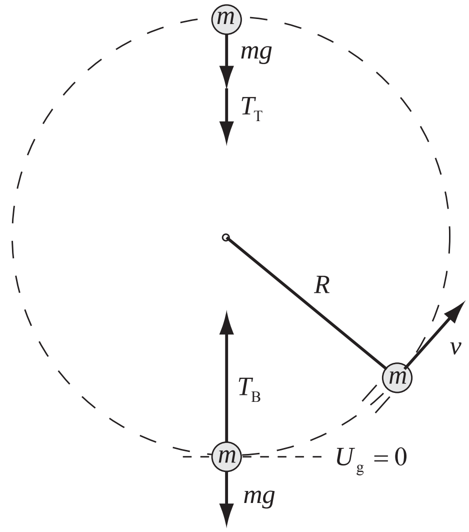
$$v = \sqrt{2g(h+x) - \frac{kx^2}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(5.00\,\text{m} + 0.150\,\text{m}) - \frac{(3.96\times10^3\,\text{N/m})(0.150\,\text{m})^2}{2.40\,\text{kg}}} = \boxed{8.00\,\text{m/s}}$$

45 •• **[SSM]** A ball at the end of a string moves in a vertical circle with constant mechanical energy E. What is the difference between the tension at the bottom of the circle and the tension at the top?

Picture the Problem The diagram represents the ball traveling in a circular path with constant energy. U_g has been chosen to be zero at the lowest point on the circle and the superimposed free-body diagrams show the forces acting on the ball at the top (T) and bottom (B) of the circular path. We'll apply Newton's 2nd law to the ball at the top and bottom of its path to obtain a relationship between T_T and T_B and conservation of mechanical energy to relate the speeds of the ball at these two locations.



Apply $\sum F_{radial} = ma_{radial}$ to the ball at the bottom of the circle and solve for T_B:

$$T_B - mg = m\frac{v_B^2}{R}$$

and

$$T_B = mg + m\frac{v_B^2}{R} \qquad (1)$$

Apply $\sum F_{radial} = ma_{radial}$ to the ball at the top of the circle and solve for T_T:

$$T_T + mg = m\frac{v_T^2}{R}$$

and

$$T_T = -mg + m\frac{v_T^2}{R} \qquad (2)$$

Subtract equation (2) from equation (1) to obtain:

$$T_B - T_T = mg + m\frac{v_B^2}{R} - \left(-mg + m\frac{v_T^2}{R}\right) = m\frac{v_B^2}{R} - m\frac{v_T^2}{R} + 2mg \qquad (3)$$

Using conservation of mechanical energy, relate the energy of the ball at the bottom of its path to its mechanical energy at the top of the circle:

$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_T^2 + mg(2R)$$

or

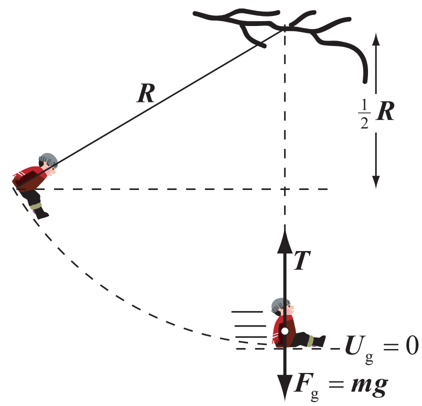
$$m\frac{v_B^2}{R} - m\frac{v_T^2}{R} = 4mg$$

Substituting in equation (3) yields:

$$T_B - T_T = \boxed{6mg}$$

46 •• A girl of mass m is taking a picnic lunch to her grandmother. She ties a rope of length R to a tree branch over a creek and starts to swing from rest at a point that is a distance $R/2$ lower than the branch. What is the minimum breaking tension for the rope if it is not to break and drop the girl into the creek?

Picture the Problem Let the system consist of the girl and the earth and let $U_g = 0$ at the lowest point in the girl's swing. We can apply conservation of mechanical energy to the system to relate the girl's speed v to R. The force diagram shows the forces acting on the girl at the low point of her swing. Applying Newton's 2nd law to her will allow us to establish the relationship between the tension T and her speed.



Apply $\sum F_{radial} = ma_{radial}$ to the girl at her lowest point and solve for T:

$$T - mg = m\frac{v^2}{R}$$

and

$$T = mg + m\frac{v^2}{R} \quad (1)$$

Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

Substituting for K_f and U_i yields:

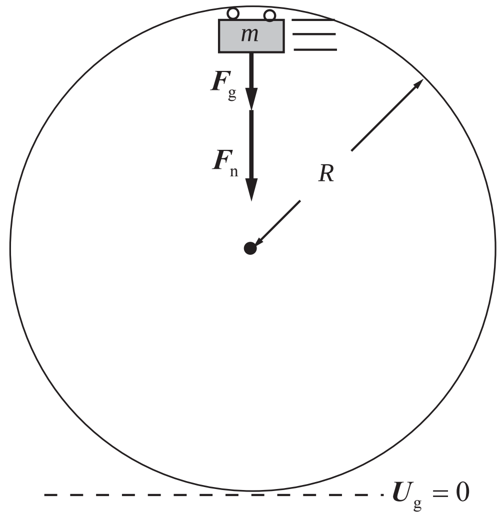
$$\tfrac{1}{2}mv^2 - mg\frac{R}{2} = 0 \Rightarrow \frac{v^2}{R} = g$$

Substitute for v^2/R in equation (1) and simplify to obtain:

$$T = mg + mg = \boxed{2mg}$$

47 •• A 1500-kg roller coaster car starts from rest a height $H = 23.0$ m (Figure 7-46) above the bottom of a 15.0-m-diameter loop. If friction is negligible, determine the downward force of the rails on the car when the upside-down car is at the top of the loop.

Picture the Problem Let the system include the car, the track, and the earth. The pictorial representation shows the forces acting on the car when it is upside down at the top of the loop. Choose $U_g = 0$ at the bottom of the loop. We can express F_n in terms of v and R by apply Newton's 2nd law to the car and then obtain a second expression in these same variables by applying conservation of mechanical energy to the system. The simultaneous solution of these equations will yield an expression for F_n in terms of known quantities.



Apply $\sum F_{radial} = ma_{radial}$ to the car at the top of the circle and solve for F_n:

$$F_n + mg = m\frac{v^2}{R}$$

and

$$F_n = m\frac{v^2}{R} - mg \qquad (1)$$

Using conservation of mechanical energy, relate the energy of the car at the beginning of its motion to its energy when it is at the top of the loop:

$$W_{ext} = \Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K_f + U_f - U_i = 0$$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}mv^2 + mg(2R) - mgH = 0$$

Solving for $m\dfrac{v^2}{R}$ yields:

$$m\frac{v^2}{R} = 2mg\left(\frac{H}{R} - 2\right) \qquad (2)$$

Substitute equation (2) in equation (1) to obtain:

$$F_n = 2mg\left(\frac{H}{R} - 2\right) - mg$$

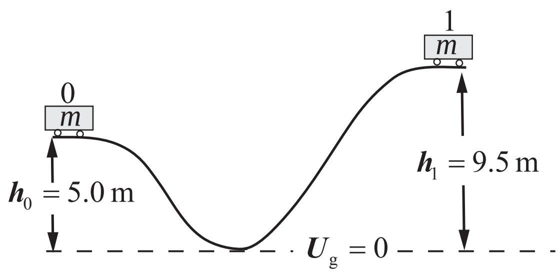
$$= mg\left(\frac{2H}{R} - 5\right)$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_{\rm n} = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left[\frac{2(23.0\,\text{m})}{7.50\,\text{m}} - 5\right] = \boxed{16.7\,\text{kN}}$$

48 •• A single roller-coaster car is moving with speed v_0 on the first section of track when it descends a 5.0-m-deep valley, then climbs to the top of a hill that is 4.5 m above the first section of track. Assume any effects of friction or of air resistance are negligible. (*a*) What is the minimum speed v_0 required if the car is to travel beyond the top of the hill? (*b*) Can we affect this speed by changing the depth of the valley to make the coaster pick up more speed at the bottom? Explain.

Picture the Problem Let the system include the roller coaster, the track, and the earth and denote the starting position with the numeral 0 and the top of the second hill with the numeral 1. We can use the work-energy theorem to relate the energies of the coaster at its initial and final positions. Let m be the mass of the roller coaster.



(*a*) Use conservation of mechanical energy to relate the work done by external forces to the change in the energy of the system:

$$W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta K + \Delta U$$

Because the track is frictionless, $W_{\text{ext}} = 0$:

$$\Delta K + \Delta U = 0$$

and

$$K_1 - K_0 + U_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh_1 - mgh_0 = 0$$

Solving for v_0 yields:

$$v_0 = \sqrt{v_1^2 + 2g(h_1 - h_0)}$$

If the coaster just makes it to the top of the second hill, $v_1 = 0$ and:

$$v_0 = \sqrt{2g(h_1 - h_0)}$$

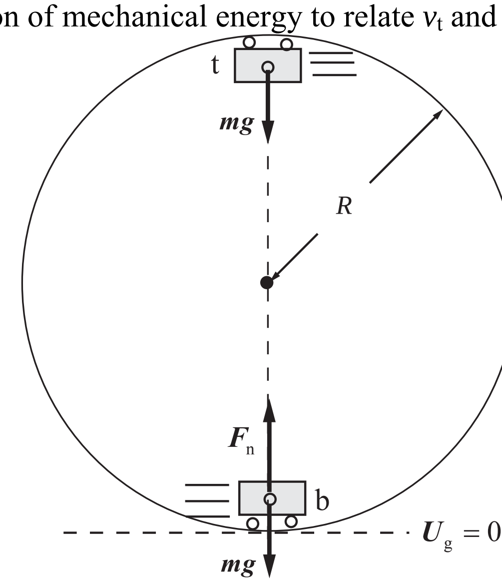
Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{2(9.81\,\text{m/s}^2)(9.5\,\text{m} - 5.0\,\text{m})} = \boxed{9.4\,\text{m/s}}$$

(*b*) No. Note that the required speed depends only on the difference in the heights of the two hills.

49 •• The Gravitron single-car roller coaster consists of a single loop-the-loop. The car is initially pushed, giving it just the right mechanical energy so the riders on the coaster will feel "weightless" when they pass through the top of the circular arc. How heavy will they feel when they pass through the bottom of the arc (that is, what is the normal force pressing up on them when they are at the bottom of the loop)? Express the answer as a multiple of *mg* (their actual weight). Assume any effects of friction or of air resistance are negligible.

Picture the Problem Let the radius of the loop be *R* and the mass of one of the riders be *m*. At the top of the loop, the centripetal force on her is her weight (the force of gravity). The two forces acting on her at the bottom of the loop are the normal force exerted by the seat of the car, pushing up, and the force of gravity, pulling down. We can apply Newton's 2nd law to her at both the top and bottom of the loop to relate the speeds at those locations to *m* and *R* and, at *b*, to *F*, and then use conservation of mechanical energy to relate v_t and v_b.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the bottom of the circular arc:

$$F - mg = m\frac{v_b^2}{R} \Rightarrow F = mg + m\frac{v_b^2}{R} \quad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the top of the circular arc:

$$mg = m\frac{v_t^2}{R} \Rightarrow v_t^2 = gR$$

Apply conservation of mechanical energy to the system to obtain:

$K_b - K_t + U_b - U_t = 0$
or, because $U_b = 0$,
$K_b - K_t - U_t = 0$

Substitute for K_b, K_t, and U_t to obtain:

$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_t^2 - 2mgR = 0$

Solving for v_b^2 yields:

$v_b^2 = 5gR$

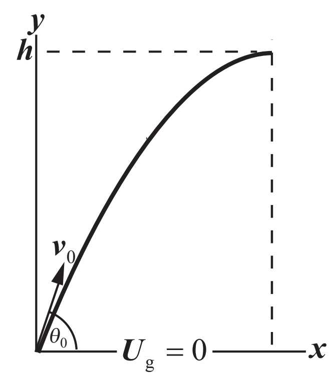
Substitute for v_b^2 in equation (1) and simplify to obtain:

$$F = mg + m\frac{5gR}{R} = \boxed{6mg}$$

That is, the rider will feel six times heavier than her normal weight.

50 •• A stone is thrown upward at an angle of 53° above the horizontal. Its maximum height above the release point is 24 m. What was the stone's initial speed? Assume any effects of air resistance are negligible.

Picture the Problem Let the system consist of the stone and the earth and ignore the influence of air resistance. Then $W_{ext} = 0$. Choose $U_g = 0$ as shown in the figure. Apply conservation of mechanical energy to describe the energy transformations as the stone rises to the highest point of its trajectory.



Apply conservation of mechanical energy to the system:

$W_{ext} = \Delta K + \Delta U = 0$
and
$K_1 - K_0 + U_1 - U_0 = 0$

Because $U_0 = 0$:

$K_1 - K_0 + U_1 = 0$

Substitute for the kinetic and potential energies yields:

$\frac{1}{2}mv_x^2 - \frac{1}{2}mv^2 + mgh = 0$

In the absence of air resistance, the horizontal component of $\vec{v}$ is constant and equal to $v_x = v\cos\theta$:

$\frac{1}{2}m(v\cos\theta)^2 - \frac{1}{2}mv^2 + mgh = 0$

Solving for v yields:

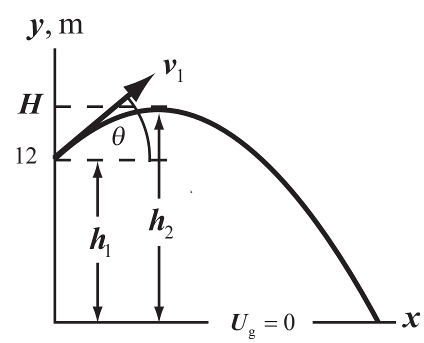
$$v=\sqrt{\frac{2gh}{1-\cos^2\theta}}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v}=\sqrt{\frac{2(9.81\,\text{m/s}^2)(24\,\text{m})}{1-\cos^2 53°}}=\boxed{27\,\text{m/s}}$$

51 •• A 0.17-kg baseball is launched from the roof of a building 12 m above the ground. Its initial velocity is 30 m/s at 40° above the horizontal. Assume any effects of air resistance are negligible. (*a*) What is the maximum height above the ground the ball reaches? (*b*) What is the speed of the ball as it strikes the ground?

Picture the Problem The figure shows the ball being thrown from the roof of the building. Let the system consist of the ball and the earth. Then $W_{ext} = 0$. Choose $U_g = 0$ at ground level. We can use conservation of mechanical energy to determine the maximum height of the ball and its speed at impact with the ground.



(*a*) Apply conservation of mechanical energy to obtain:

$$W_{ext}=\Delta K+\Delta U=0$$

or

$$K_2-K_1+U_2-U_1=0$$

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_2^2-\tfrac{1}{2}mv_1^2+mgh_2-mgh_1=0$$

Note that, at point 2, the ball is moving horizontally and:

$$v_2=v_1\cos\theta$$

Substitute for v_2 and h_2 to obtain:

$$\tfrac{1}{2}m(v_1\cos\theta)^2-\tfrac{1}{2}mv_1^2+mgH-mgh_1=0$$

Solving for H yields:

$$H=h_1-\frac{v^2}{2g}\left(\cos^2\theta-1\right)$$

Substitute numerical values and evaluate H:

$$H=12\,\text{m}-\frac{(30\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\left(\cos^2 40°-1\right)=\boxed{31\,\text{m}}$$

(*b*) Apply conservation of mechanical energy to the system to relate the initial mechanical energy of the ball to its just-before-impact energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

or, because $U_f = 0$,

$$K_f - K_1 - U_1 = 0$$

Substituting for K_f, K_1, and U_1 yields:

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 - mgh_i = 0$$

Solve for v_f to obtain:

$$v_f = \sqrt{v_i^2 + 2gh_i}$$

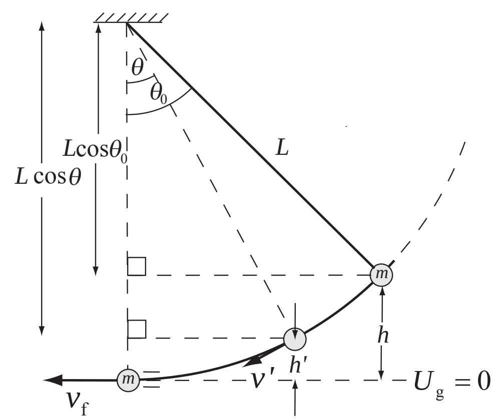
Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{(30\,\text{m/s})^2 + 2(9.81\,\text{m/s}^2)(12\,\text{m})}$$

$$= \boxed{34\,\text{m/s}}$$

52 •• An 80-cm-long pendulum with a 0.60-kg bob is released from rest at an initial angle of θ_0 with the vertical. At the bottom of the swing, the speed of the bob is 2.8 m/s. (*a*) What is θ_0? (*b*) What angle does the pendulum make with the vertical when the speed of the bob is 1.4 m/s? Is this angle equal to $\frac{1}{2}\theta_0$? Explain why or why not.

Picture the Problem The figure shows the pendulum bob in its release position and in the two positions in which it is in motion with the given speeds. Let the system consist of the pendulum and the earth and choose $U_g = 0$ at the low point of the swing. We can apply conservation of mechanical energy to relate the two angles of interest to the speeds of the bob at the intermediate and low points of its trajectory.



(*a*) Apply conservation of mechanical energy to the system to obtain:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$\boldsymbol{K}_f - \boldsymbol{K}_i + \boldsymbol{U}_f - \boldsymbol{U}_i = 0.$$

Because $K_i = U_f = 0$:

$$\boldsymbol{K}_f - \boldsymbol{U}_i = 0 \qquad (1)$$

Refer to the pictorial representation to see that U_i is given by:

$$U_i = mgh = mgL(1 - \cos\theta_0)$$

Substitute for K_f and U_i in equation (1) to obtain:

$$\tfrac{1}{2}mv_f^2 - mgL(1-\cos\theta_0) = 0$$

Solving for θ_0 yields:

$$\theta_0 = \cos^{-1}\left(1-\frac{v^2}{2gL}\right)$$

Substitute numerical values and evaluate θ_0:

$$\boldsymbol{\theta}_0 = \cos^{-1}\left[1-\frac{(2.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.80\,\text{m})}\right]$$
$$= \boxed{60°}$$

(*b*) Letting primed quantities describe the indicated location, use conservation of mechanical energy to obtain :

$$\boldsymbol{K}_f' - \boldsymbol{K}_i + \boldsymbol{U}_f' - \boldsymbol{U}_i = 0$$

Because $K_i = 0$:

$$K_f' + U_f' - U_i = 0$$

Refer to the pictorial representation to see that U_f' is given by:

$$U_f' = mgh' = mgL(1-\cos\theta)$$

Substitute for K_f', U_f' and U_i :

$$\tfrac{1}{2}m(v_f')^2 + mgL(1-\cos\theta) - mgL(1-\cos\theta_0) = 0$$

Solving for θ yields :

$$\theta = \cos^{-1}\left[\frac{(v_f')^2}{2gL}+\cos\theta_0\right]$$

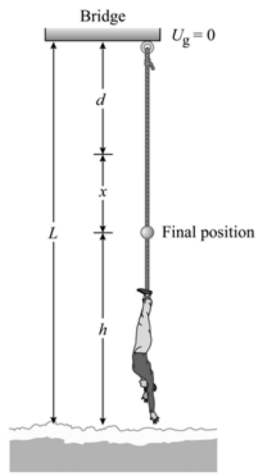
Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \cos^{-1}\left[\frac{(1.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.80\,\text{m})}+\cos 60°\right] = \boxed{51°}$$

No. The change in gravitational potential energy is linearly dependent on the cosine of the angle rather than on the angle itself.

53 •• The Royal Gorge bridge over the Arkansas River is 310 m above the river. A 60-kg bungee jumper has an elastic cord with an unstressed length of 50 m attached to her feet. Assume that, like an ideal spring, the cord is massless and provides a linear restoring force when stretched. The jumper leaps, and at her lowest point she barely touches the water. After numerous ascents and descents, she comes to rest at a height *h* above the water. Model the jumper as a point particle and assume that any effects of air resistance are negligible. (*a*) Find *h*. (*b*) Find the maximum speed of the jumper.

Picture the Problem Choose $U_g = 0$ at the bridge and let the system be the earth, the jumper and the bungee cord. Then $W_{ext} = 0$. We can use conservation of mechanical energy to relate to relate her initial and final gravitational potential energies to the energy stored in the stretched bungee cord U_s. In Part (*b*), we'll use a similar strategy but include a kinetic energy term because we are interested in finding her maximum speed.



(*a*) Express her final height *h* above the water in terms of *L*, *d* and the distance *x* the bungee cord has stretched:

$$h = L - d - x \qquad (1)$$

Use conservation of mechanical energy to relate her gravitational potential energy as she just touches the water to the energy stored in the stretched bungee cord:

$$W_{ext} = \Delta K + \Delta U = 0$$

Because $\Delta K = 0$ and $\Delta U = \Delta U_g + \Delta U_s$:

$$-mgL + \tfrac{1}{2}ks^2 = 0 \Rightarrow k = \frac{2mgL}{s^2}$$

where *s* is the maximum distance the bungee cord has stretched.

Find the maximum distance the bungee cord stretches:

$s = 310\text{ m} - 50\text{ m} = 260\text{ m}.$

Substitute numerical values and evaluate *k*:

$$k = \frac{2(60\,\text{kg})(9.81\,\text{m/s}^2)(310\,\text{m})}{(260\,\text{m})^2} = 5.40\,\text{N/m}$$

Express the relationship between the forces acting on her when she has finally come to rest *x*:

$$F_{net} = kx - mg = 0 \Rightarrow x = \frac{mg}{k}$$

Substitute numerical values and evaluate x:

$$x=\frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}}=109\,\text{m}$$

Substitute in equation (1) and evaluate h:

$$\boldsymbol{h}=310\,\text{m}-50\,\text{m}-109\,\text{m}=151\,\text{m}=\boxed{0.15\,\text{km}}$$

(b) Using conservation of mechanical energy, express her total energy E:

$$E=K+U_g+U_s=E_i=0$$

Because v is a maximum when K is a maximum, solve for K to obtain:

$$K=-U_g-U_s=mg(d+x)-\tfrac{1}{2}kx^2 \qquad (2)$$

Use the condition for an extreme value to obtain:

$$\frac{dK}{dx}=mg-kx=0\Rightarrow x=\frac{mg}{k}$$

Substitute numerical values and evaluate x:

$$\boldsymbol{x}=\frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}}=109\,\text{m}$$

From equation (2) we have:

$$\tfrac{1}{2}mv^2=mg(d+x)-\tfrac{1}{2}kx^2$$

Solve for v to obtain:

$$v=\sqrt{2g(d+x)-\frac{kx^2}{m}}$$

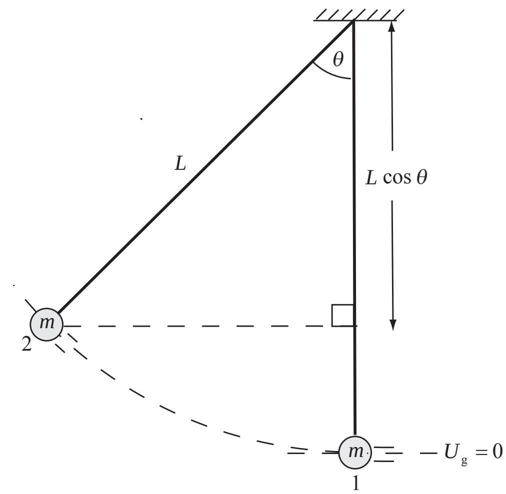
Substitute numerical values and evaluate v for x = 109 m:

$$\boldsymbol{v}=\sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}+109\,\text{m})-\frac{(5.4\,\text{N/m})(109\,\text{m})^2}{60\,\text{kg}}}=\boxed{45\,\text{m/s}}$$

Because $\dfrac{d^2K}{dx^2}=-k<0$, x = 109 m corresponds to K_{max} and so v is a maximum.

54 •• A pendulum consists of a 2.0-kg bob attached to a light 3.0-m-long string. While hanging at rest with the string vertical, the bob is struck a sharp horizontal blow, giving it a horizontal velocity of 4.5 m/s. At the instant the string makes an angle of 30° with the vertical, what is (a) the speed, (b) the gravitational potential energy (relative to its value is at the lowest point), and (c) the tension in the string? (d) What is the angle of the string with the vertical when the bob reaches its greatest height?

Picture the Problem Let the system be the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the low point of the bob's swing and apply conservation of mechanical energy to the system. When the bob reaches the 30° position its energy will be partially kinetic and partially potential. When it reaches its maximum height, its energy will be entirely potential. Applying Newton's 2nd law will allow us to express the tension in the string as a function of the bob's speed and its angular position.



(*a*) Apply conservation of mechanical energy to relate the energies of the bob at points 1 and 2:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Because $U_1 = 0$:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + U_2 = 0 \qquad (1)$$

The potential energy of the system when the bob is at point 2 is given by:

$$U_2 = mgL(1 - \cos\theta)$$

Substitute for U_2 in equation (1) to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgL(1 - \cos\theta) = 0$$

Solving for v_2 yields:

$$v_2 = \sqrt{v_1^2 - 2gL(1 - \cos\theta)}$$

Substitute numerical values and evaluate v_2:

$$\boldsymbol{v}_2 = \sqrt{(4.5\,\text{m/s})^2 - 2(9.81\,\text{m/s}^2)(3.0\,\text{m})(1 - \cos 30°)} = 3.52\,\text{m/s} = \boxed{3.5\,\text{m/s}}$$

(*b*) From (*a*) we have:

$$U_2 = mgL(1 - \cos\theta)$$

Substitute numerical values and evaluate U_2:

$$\boldsymbol{U}_2 = (2.0\,\text{kg})(9.81\,\text{m/s}^2)(3.0\,\text{m})(1 - \cos 30°) = \boxed{7.9\,\text{J}}$$

(*c*) Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the bob to obtain:

$$T - mg\cos\theta = m\frac{v_2^2}{L}$$

Solving for *T* yields:

$$T = m\left(g\cos\theta + \frac{v_2^2}{L}\right)$$

Substitute numerical values and evaluate *T*:

$$\boldsymbol{T} = (2.0\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 30° + \frac{(3.52\,\text{m/s})^2}{3.0\,\text{m}}\right] = \boxed{25\,\text{N}}$$

(*d*) When the bob reaches its greatest height:

$$U = U_{\max} = mgL(1 - \cos\theta_{\max})$$

and

$$-K_1 + U_{\max} = 0$$

Substitute for K_1 and $U_{\max}$:

$$-\tfrac{1}{2}mv_1^2 + mgL(1 - \cos\theta_{\max}) = 0$$

Solve for $\theta_{\max}$ to obtain:

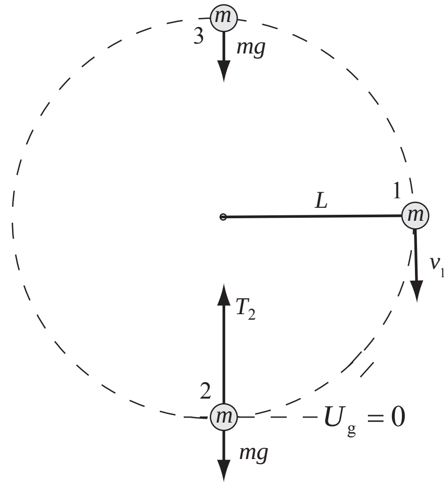
$$\theta_{\max} = \cos^{-1}\left(1 - \frac{v_1^2}{2gL}\right)$$

Substitute numerical values and evaluate $\theta_{\max}$:

$$\boldsymbol{\theta}_{\max} = \cos^{-1}\left[1 - \frac{(4.5\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(3.0\,\text{m})}\right] = \boxed{49°}$$

55 •• **[SSM]** A pendulum consists of a string of length *L* and a bob of mass *m*. The bob is rotated until the string is horizontal. The bob is then projected downward with the minimum initial speed needed to enable the bob to make a full revolution in the vertical plane. (*a*) What is the maximum kinetic energy of the bob? (*b*) What is the tension in the string when the kinetic energy is maximum?

Picture the Problem Let the system consist of the earth and pendulum bob. Then $W_{\text{ext}} = 0$. Choose $U_{\text{g}} = 0$ at the bottom of the circle and let points 1, 2 and 3 represent the bob's initial point, lowest point and highest point, respectively. The bob will gain speed and kinetic energy until it reaches point 2 and slow down until it reaches point 3; so it has its maximum kinetic energy when it is at point 2. We can use Newton's 2nd law at points 2 and 3 in conjunction with conservation of mechanical energy to find the maximum kinetic energy of the bob and the tension in the string when the bob has its maximum kinetic energy.



(*a*) Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the bob at the top of the circle and solve for v_3^2:

$$mg = m\frac{v_3^2}{L} \Rightarrow v_3^2 = gL$$

Apply conservation of mechanical energy to the system to express the relationship between K_2, K_3 and U_3:

$$\boldsymbol{K}_3 - \boldsymbol{K}_2 + \boldsymbol{U}_3 - \boldsymbol{U}_2 = 0$$

or, because $U_2 = 0$,

$$\boldsymbol{K}_3 - \boldsymbol{K}_2 + \boldsymbol{U}_3 = 0$$

Solving for K_2 yields:

$$\boldsymbol{K}_2 = \boldsymbol{K}_{\text{max}} = \boldsymbol{K}_3 + \boldsymbol{U}_3$$

Substituting for K_3 and U_3 yields:

$$\boldsymbol{K}_{\text{max}} = \tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_3^2 + \boldsymbol{mg}(2\boldsymbol{L})$$

Substitute for v_3^2 and simplify to obtain:

$$\boldsymbol{K}_{\text{max}} = \tfrac{1}{2}\boldsymbol{m}(\boldsymbol{gL}) + 2\boldsymbol{mgL} = \boxed{\tfrac{5}{2}\boldsymbol{mgL}}$$

(*b*) Apply $\sum \boldsymbol{F}_{\text{radial}} = \boldsymbol{ma}_{\text{c}}$ to the bob at the bottom of the circle and solve for T_2:

$$F_{\text{net}} = T_2 - mg = m\frac{v_2^2}{L}$$

and

$$T_2 = mg + m\frac{v_2^2}{L} \qquad (1)$$

Use conservation of mechanical energy to relate the energies of the bob at points 2 and 3 and solve for K_2:

$$K_3 - K_2 + U_3 - U_2 = 0 \text{ where } U_2 = 0$$

and

$$K_2 = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mg(2L)$$

Substitute for v_3^2 and K_2 to obtain:

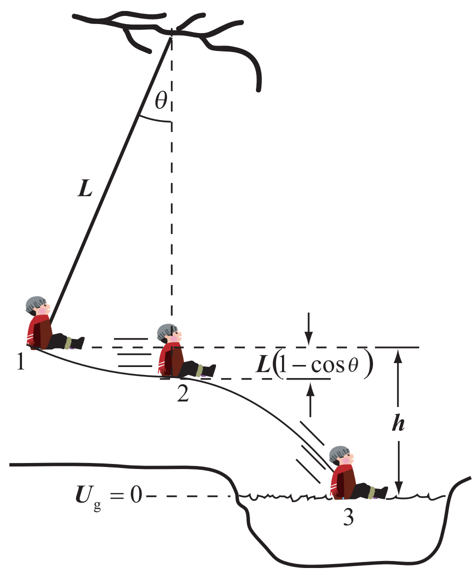
$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}m(gL) + mg(2L) \Rightarrow v_2^2 = 5gL$$

Substitute for v_2^2 in equation (1) and simplify to obtain:

$$T_2 = mg + m\frac{5gL}{L} = \boxed{6mg}$$

56 •• A child whose weight is 360 N swings out over a pool of water using a rope attached to the branch of a tree at the edge of the pool. The branch is 12 m above ground level and the surface of the water is 1.8 m below ground level. The child holds onto the rope at a point 10.6 m from the branch and moves back until the angle between the rope and the vertical is 23°. When the rope is in the vertical position, the child lets go and drops into the pool. Find the speed of the child just as he impacts the surface of the water. (Model the child as a point particle attached to the rope 10.6 m from the branch.)

Picture the Problem Let the system consist of the earth and child. Then $W_{ext} = 0$. In the figure, the child's initial position is designated with the numeral 1; the point at which the child releases the rope and begins to fall with a 2, and its point of impact with the water is identified with a 3. Choose $U_g = 0$ at the water level. While one could apply conservation of mechanical energy between points 1 and 2 and then between points 2 and 3, it is more direct to consider the energy transformations between points 1 and 3.



Apply conservation of mechanical energy between points 1 and 3:

$$W_{ext} = \Delta K + \Delta U = 0$$
$$K_3 - K_1 + U_3 - U_1 = 0$$

where U_3 and K_1 are zero.

Substitute for K_3 and U_1;

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1-\cos\theta)] = 0$$

Solving for v_3 yields:

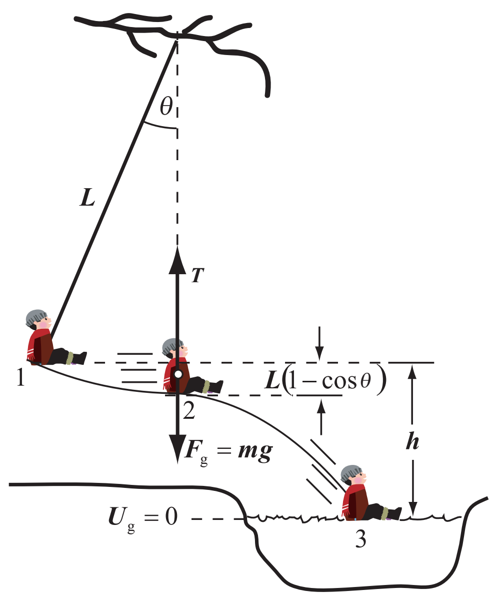
$$v_3 = \sqrt{2g[h + L(1-\cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\,\text{m/s}^2)[3.2\,\text{m} + (10.6\,\text{m})(1-\cos 23°)]} = \boxed{8.9\,\text{m/s}}$$

57 •• Walking by a pond, you find a rope attached to a stout tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your weight. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.) (*a*) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing? (*b*) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?

Picture the Problem Let the system consist of you and the earth. Then there are no external forces to do work on the system and $W_{\text{ext}} = 0$. In the figure, your initial position is designated with the numeral 1, the point at which you release the rope and begin to fall with a 2, and your point of impact with the water is identified with a 3. Choose $U_g = 0$ at the water level. We can apply Newton's 2nd law to the forces acting on you at point 2 and apply conservation of mechanical energy between points 1 and 2 to determine the maximum angle at which you can begin your swing and then between points 1 and 3 to determine the speed with which you will hit the water.



(*a*) Use conservation of mechanical energy to relate your speed at point 2 to your potential energy there and at point 1:

$$W_{\text{ext}} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Because $K_1 = 0$:

$$\tfrac{1}{2}mv_2^2 + mgh - [mgL(1-\cos\theta) + mgh] = 0$$

Solve this equation for θ to obtain:

$$\theta = \cos^{-1}\left[1 - \frac{v_2^2}{2gL}\right] \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to yourself at point 2 and solve for T:

$$T - mg = m\frac{v_2^2}{L} \text{ and } T = mg + m\frac{v_2^2}{L}$$

Because you've estimated that the rope might break if the tension in it exceeds your weight by 80 N, it must be that:

$$m\frac{v_2^2}{L} = 80\,\text{N} \Rightarrow v_2^2 = \frac{(80\,\text{N})L}{m}$$

Let's assume that your mass is 70 kg. Then:

$$\boldsymbol{v}_2^2 = \frac{(80\,\text{N})(4.6\,\text{m})}{70\,\text{kg}} = 5.26\,\text{m}^2/\text{s}^2$$

Substitute numerical values in equation (1) to obtain:

$$\theta = \cos^{-1}\left[1 - \frac{5.26\,\text{m}^2/\text{s}^2}{2(9.81\,\text{m/s}^2)(4.6\,\text{m})}\right] = 19.65° = \boxed{20°}$$

(*b*) Apply conservation of mechanical energy between points 1 and 3:

$$W_{\text{ext}} = \Delta K + \Delta U = 0$$

or, because $U_3 = K_1 = 0$,

$$\boldsymbol{K}_3 - \boldsymbol{U}_1 = 0$$

Substitute for K_3 and U_1 to obtain:

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1-\cos\theta)] = 0$$

Solving for v_3 yields:

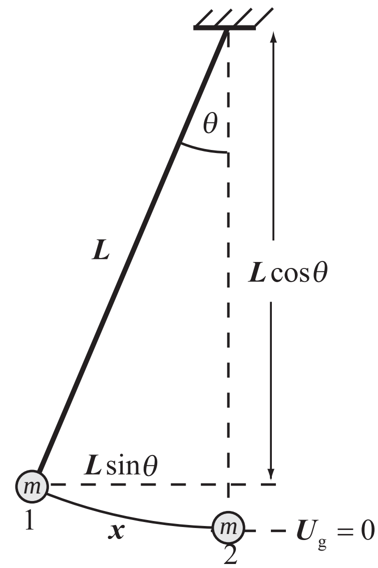
$$v_3 = \sqrt{2g[h + L(1-\cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$\boldsymbol{v}_3 = \sqrt{2(9.81\,\text{m/s}^2)[1.8\,\text{m} + (4.6\,\text{m})(1-\cos 19.65°)]} = \boxed{6.4\,\text{m/s}}$$

58 ••• A pendulum bob of mass m is attached to a light string of length L and is also attached to a spring of force constant k. With the pendulum in the position shown in Figure 7-47, the spring is at its unstressed length. If the bob is now pulled aside so that the string makes a *small* angle θ with the vertical and released, what is the speed of the bob as it passes through the equilibrium position? *Hint: Recall the small-angle approximations: if θ is expressed in radians, and if $|\boldsymbol{\theta}| < 1$, then* $\sin\theta \approx \tan\theta \approx \theta$ *and* $\cos\theta \approx 1 - \frac{1}{2}\theta^2$.

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the bob and the earth. Given this choice, there are no external forces doing work on the system. Because $\theta << 1$, we can use the trigonometric series for the sine and cosine functions to approximate these functions. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of mechanical energy to the system as the pendulum bob swings from point 1 to point 2:

$$W_{ext} = \Delta K + \Delta U = 0$$

or, because $K_1 = U_2 = 0$,

$$K_2 - U_1 = 0$$

Substituting for K_2 and U_1 yields:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}kx^2 - mgL(1-\cos\theta) = 0$$

Note, from the figure, that when $\theta << 1$, $x \approx L\sin\theta$:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}k(L\sin\theta)^2 - mgL(1-\cos\theta) = 0$$

Also, when $\theta << 1$:

$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

Substitute for $\sin\theta$ and $\cos\theta$ to obtain:

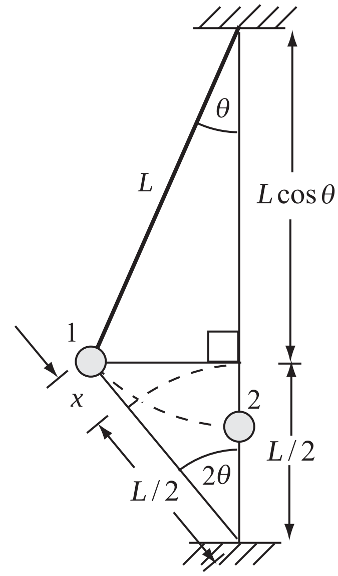
$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}k(L\theta)^2 - mgL\left[1 - \left(1 - \tfrac{1}{2}\theta^2\right)\right]$$

Solving for v_2 yields:

$$v_2 = \boxed{L\theta\sqrt{\frac{k}{m} + \frac{g}{L}}}$$

59 ••• **[SSM]** A pendulum is suspended from the ceiling and attached to a spring fixed to the floor directly below the pendulum support (Figure 7-48). The mass of the pendulum bob is m, the length of the pendulum is L, and the force constant is k. The unstressed length of the spring is $L/2$ and the distance between the floor and ceiling is $1.5L$. The pendulum is pulled aside so that it makes an angle θ with the vertical and is then released from rest. Obtain an expression for the speed of the pendulum bob as the bob passes through a point directly below the pendulum support.

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the earth, ceiling, spring, and pendulum bob. Given this choice, there are no external forces doing work to change the energy of the system. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of mechanical energy to the system as the pendulum bob swings from point 1 to point 2:

$$W_{ext} = \Delta K + \Delta U_g + \Delta U_s = 0$$

or, because $K_1 = U_{g,2} = U_{s,2} = 0$,

$$K_2 - U_{g,1} - U_{s,1} = 0$$

Substituting for K_2, $U_{g,1}$, and $U_{s,2}$ yields:

$$\tfrac{1}{2}mv_2^2 - mgL(1-\cos\theta) - \tfrac{1}{2}kx^2 = 0 \quad (1)$$

Apply the Pythagorean theorem to the lower triangle in the diagram to obtain:

$$\left(x + \tfrac{1}{2}L\right)^2 = L^2\left[\sin^2\theta + \left(\tfrac{3}{2} - \cos\theta\right)^2\right] = L^2\left[\sin^2\theta + \tfrac{9}{4} - 3\cos\theta + \cos^2\theta\right]$$
$$= L^2\left(\tfrac{13}{4} - 3\cos\theta\right)$$

Take the square root of both sides of the equation to obtain:

$$x + \tfrac{1}{2}L = L\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)}$$

Solving for x yields:

$$x = L\left[\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)} - \tfrac{1}{2}\right]$$

Substitute for x in equation (1) to obtain:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kL^2\left[\sqrt{\left(\tfrac{13}{4} - 3\cos\theta\right)} - \tfrac{1}{2}\right]^2 + mgL(1-\cos\theta)$$

Solving for v_2 yields:

$$v_2 = \boxed{L\sqrt{2\frac{g}{L}(1-\cos\theta) + \frac{k}{m}\left(\sqrt{\tfrac{13}{4} - 3\cos\theta} - \tfrac{1}{2}\right)^2}}$$

Total Energy and Non-conservative Forces

60 • In a volcanic eruption, 4.00 km^3 of mountain with an average density of 1600 kg/m^3 was raised an average height of 500 m. (*a*) What is the minimum amount of energy, in joules, that was released during this eruption? (*b*) The energy released by thermonuclear bombs is measured in megatons of TNT, where 1 megaton of TNT = 4.2×10^{15} J. Convert your answer for Part (*a*) to megatons of TNT.

Picture the Problem The energy of the eruption is initially in the form of the kinetic energy of the material it thrusts into the air. This energy is then transformed into gravitational potential energy as the material rises.

(*a*) Express the energy of the eruption in terms of the height Δh to which the debris rises:

$$E = mg\Delta h$$

Relate the mass of the material to its density and volume:

$$m = \rho V$$

Substitute for m to obtain:

$$E = \rho V g\Delta h$$

Substitute numerical values and evaluate E:

$$\boldsymbol{E} = \left(1600\,\text{kg/m}^3\right)\left(4.00\,\text{km}^3\right)\left(9.81\,\text{m/s}^2\right)\left(500\,\text{m}\right) = \boxed{3.14\times10^{16}\ \text{J}}$$

(*b*) Convert 3.14×10^{16} J to megatons of TNT:

$$3.14\times10^{16}\ \text{J} = 3.14\times10^{16}\ \text{J}\times\frac{1\,\text{Mton TNT}}{4.2\times10^{15}\ \text{J}} = \boxed{7.5\,\text{Mton TNT}}$$

61 • To work off a large pepperoni pizza you ate on Friday night, on Saturday morning you climb a 120-m-high hill. (*a*) Assuming a reasonable value for your mass, determine your increase in gravitational potential energy. (*b*) Where does this energy come from? (*c*) The human body is typically 20 percent efficient. How much energy was converted into thermal energy? (*d*) How much chemical energy is expended by you during the climb? Given that oxidation (burning) of a single slice of pepperoni pizza releases about 1.0 MJ (250 food calories) of energy, do you think one climb up the hill is enough?

Picture the Problem The work you did equals the change in your gravitational potential energy and is enabled by the transformation of metabolic energy in your muscles. Let the system consist of you and the earth and apply the conservation of mechanical energy to this system.

(*a*) Your increase in gravitational potential energy is:

$$\Delta U_g = mg\Delta h$$

Assuming that your mass is 70 kg, your increase in gravitational potential energy is:

$$\Delta U_g = (70\,\text{kg})(9.81\,\text{m/s}^2)(120\,\text{m}) = 82.4\,\text{kJ} = \boxed{82\,\text{kJ}}$$

(*b*) The energy required to do this work comes from the conversion of stored internal chemical energy into gravitational potential energy and thermal energy.

(*c*) Because 20% of the energy you expend is converted into gravitational potential energy, five times this amount is converted into thermal energy:

$$\Delta E_{\text{therm}} = -5\Delta U_g$$

Substitute the numerical value of ΔU_g and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = -5(82.4\,\text{kJ}) = -412\,\text{kJ} = \boxed{-410\,\text{kJ}}$$

(*d*) Apply the conservation of mechanical energy to the system to obtain:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} + \Delta E_{\text{chem}} = 0$$

or, because you begin and end your ascent at rest, $\Delta K = 0$ and,

$$\Delta U_g + \Delta E_{\text{therm}} + \Delta E_{\text{chem}} = 0$$

Solving for ΔE_{chem} yields:

$$\Delta E_{\text{chem}} = -\Delta U_g - \Delta E_{\text{therm}}$$

Substitute numerical values and evaluate ΔE_{chem}:

$$\Delta E_{\text{chem}} = -(82.4\text{ kJ}) - (-412\text{ kJ}) = \boxed{330\text{ kJ}}$$

Given this small decrease in your mass, one climb of the hill is certainly not enough to rid yourself of the caloric intake of even one slice of pizza.

62 • A 2000-kg car moving at an initial speed of 25 m/s along a horizontal road skids to a stop in 60 m. (*a*) Find the energy dissipated by friction. (*b*) Find the coefficient of kinetic friction between the tires and the road. (*Note*: When stopping without skidding and using conventional brakes, 100 percent of the kinetic energy is dissipated by friction within the brakes. With regenerative braking, such as that used in hybrid vehicles, only 70 percent of the kinetic energy is dissipated.)

Picture the Problem Let the car and the earth constitute the system. As the car skids to a stop on a horizontal road, its kinetic energy is transformed into internal (thermal) energy. Knowing that energy is transformed into heat by friction, we can use the definition of the coefficient of kinetic friction to calculate its value.

(*a*) The energy dissipated by friction is given by:

$$f\Delta s = \Delta E_{\text{therm}}$$

Apply the work-energy theorem for problems with kinetic friction:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta E_{\text{mech}} + f\Delta s$$

or, because $\Delta E_{\text{mech}} = \Delta K = -K_{\text{i}}$ and $W_{\text{ext}} = 0$,

$$0 = -\tfrac{1}{2}mv_{\text{i}}^2 + f\Delta s \Rightarrow f\Delta s = \tfrac{1}{2}mv_{\text{i}}^2$$

Substitute numerical values and evaluate $f\Delta s$:

$$f\Delta s = \tfrac{1}{2}(2000\text{ kg})(25\text{ m/s})^2 = 6.25\times10^5\text{ J} = \boxed{6.3\times10^5\text{ J}}$$

(*b*) Relate the kinetic friction force to the coefficient of kinetic friction and the weight of the car:

$$\boldsymbol{f}_{\text{k}} = \boldsymbol{\mu}_{\text{k}}\boldsymbol{mg} \Rightarrow \boldsymbol{\mu}_{\text{k}} = \frac{\boldsymbol{f}_{\text{k}}}{\boldsymbol{mg}} \quad (1)$$

Express the relationship between the energy dissipated by friction and the kinetic friction force:

$$\Delta\boldsymbol{E}_{\text{therm}} = \boldsymbol{f}_{\text{k}}\Delta\boldsymbol{s} \Rightarrow \boldsymbol{f}_{\text{k}} = \frac{\Delta\boldsymbol{E}_{\text{therm}}}{\Delta\boldsymbol{s}}$$

Substitute for f_{k} in equation (1) to obtain:

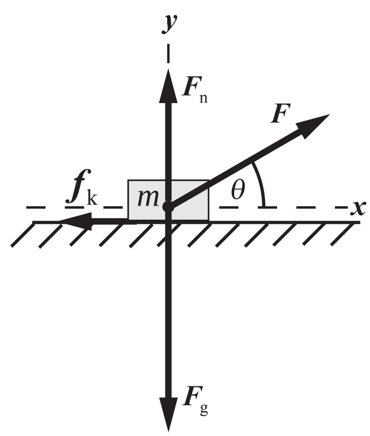
$$\mu_{\text{k}} = \frac{\Delta E_{\text{therm}}}{mg\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{6.25\times10^5\ \text{J}}{(2000\,\text{kg})(9.81\,\text{m/s}^2)(60\,\text{m})} = \boxed{0.53}$$

63 • An 8.0-kg sled is initially at rest on a horizontal road. The coefficient of kinetic friction between the sled and the road is 0.40. The sled is pulled a distance of 3.0 m by a force of 40 N applied to the sled at an angle of 30° above the horizontal. (*a*) Find the work done by the applied force. (*b*) Find the energy dissipated by friction. (*c*) Find the change in the kinetic energy of the sled. (*d*) Find the speed of the sled after it has traveled 3.0 m.

Picture the Problem Let the system consist of the sled and the earth. Then the 40-N force is external to the system. The free-body diagram shows the forces acting on the sled as it is pulled along a horizontal road. The work done by the applied force can be found using the definition of work. To find the energy dissipated by friction, we'll use Newton's 2nd law to determine f_k and then use it in the definition of work. The change in the kinetic energy of the sled is equal to the net work done on it. Finally, knowing the kinetic energy of the sled after it has traveled 3.0 m will allow us to solve for its speed at that location.



(*a*) The work done by the applied force is given by:

$$\boldsymbol{W}_{\text{ext}} = \vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{s}} = \boldsymbol{Fs}\cos\boldsymbol{\theta}$$

Substitute numerical values and evaluate W_{ext}:

$$W_{\text{ext}} = (40\,\text{N})(3.0\,\text{m})\cos30° = 103.9\,\text{J} = \boxed{0.10\,\text{kJ}}$$

(*b*) The energy dissipated by friction as the sled is dragged along the surface is given by:

$$\Delta E_{\text{therm}} = f\Delta x = \mu_k F_n\Delta x \qquad (1)$$

Apply $\sum F_y = ma_y$ to the sled:

$$F_n + F\sin\theta - mg = 0$$

Solving for F_n yields:

$$F_n = mg - F\sin\theta$$

Substitute for F_n in equation (1) to obtain:

$$\Delta E_{\text{therm}} = \mu_k \Delta x(mg - F\sin\theta)$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (0.40)(3.0\,\text{m})\left[(8.0\,\text{kg})(9.81\,\text{m/s}^2) - (40\,\text{N})\sin 30°\right] = 70.2\,\text{J} = \boxed{70\,\text{J}}$$

(*c*) Apply the work-energy theorem for systems with kinetic friction:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta E_{\text{mech}} + f\Delta s$$

or, because $\Delta E_{\text{mech}} = \Delta K + \Delta U$ and $\Delta U = 0$,

$$W_{\text{ext}} = \Delta K + \Delta E_{\text{therm}}$$

Solving for ΔK yields:

$$\Delta \boldsymbol{K} = \boldsymbol{W}_{\text{ext}} - \Delta \boldsymbol{E}_{\text{therm}}$$

Substitute numerical values and evaluate ΔK:

$$\Delta \boldsymbol{K} = 103.9\,\text{J} - 70.2\,\text{J} = 33.7\,\text{J} = \boxed{34\,\text{J}}$$

(*d*) Because $K_i = 0$:

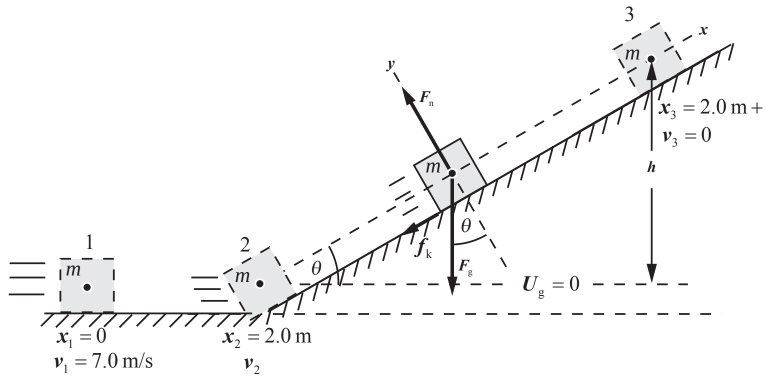
$$K_f = \Delta K = \tfrac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2\Delta K}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2(33.7\,\text{J})}{8.0\,\text{kg}}} = \boxed{2.9\,\text{m/s}}$$

64 •• Using Figure 7-41, suppose that the surfaces described are not frictionless and that the coefficient of kinetic friction between the block and the surfaces is 0.30. Find (*a*) the speed of the block when it reaches the ramp, and (*b*) the distance that the block slides along the inclined surface before coming momentarily to rest. (Neglect any energy dissipated along the transition curve.)

Picture the Problem The pictorial representation shows the block in its initial, intermediate, and final states. It also shows a choice for $U_g = 0$. Let the system consist of the block, ramp, and the earth. Then the kinetic energy of the block at the foot of the ramp is equal to its initial kinetic energy less the energy dissipated by friction. The block's kinetic energy at the foot of the incline is partially converted to gravitational potential energy and partially converted to thermal energy (dissipated by friction) as the block slides up the incline. The free-body diagram shows the forces acting on the block as it slides up the incline. Applying Newton's 2$^{\text{nd}}$ law to the block will allow us to determine f_k and express the energy dissipated by friction.



(*a*) Apply conservation of energy to the system while the block is moving horizontally:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U + f\Delta s$$

or, because $\Delta U = W_{ext} = 0$,

$$0 = \Delta K + f\Delta s = K_2 - K_1 + f\Delta s$$

Solving for K_2 yields:

$$K_2 = K_1 - f\Delta s$$

Substitute for K_2, K_1, and $f\Delta s$ to obtain:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv_1^2 - \mu_k mg\Delta x$$

Solving for v_2 yields:

$$v_2 = \sqrt{v_1^2 - 2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{(7.0\text{ m/s})^2 - 2(0.30)(9.81\text{ m/s}^2)(2.0\text{ m})} = 6.10\text{ m/s} = \boxed{6.1\text{ m/s}}$$

(*b*) Apply conservation of energy to the system while the block is on the incline:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U + f\Delta s$$

or, because $K_3 = U_2 = W_{ext} = 0$,

$$0 = -K_2 + U_3 + f\Delta s \qquad (1)$$

Apply $\sum F_y = ma_y$ to the block when it is on the incline:

$$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$$

Express $f\Delta s$:

$$f\Delta s = f_k\ell = \mu_k F_n\ell = \mu_k mg\ell\cos\theta$$

The final potential energy of the block is:

$$U_3 = mg\ell\sin\theta$$

Substitute for U_3, K_2, and $f\Delta s$ in equation (1) to obtain:

$$0 = -\tfrac{1}{2}mv_2^2 + mg\ell\sin\theta + \mu_k mg\ell\cos\theta$$

Solving for ℓ yields:

$$\ell = \frac{\frac{1}{2}\boldsymbol{v}_2^2}{\boldsymbol{g}(\sin\boldsymbol{\theta} + \boldsymbol{\mu}_\mathrm{k}\cos\boldsymbol{\theta})}$$

Substitute numerical values and evaluate ℓ:

$$\ell = \frac{\frac{1}{2}(6.10\,\mathrm{m/s})^2}{(9.81\,\mathrm{m/s^2})(\sin 40° + (0.30)\cos 40°)} = \boxed{2.2\,\mathrm{m}}$$

65 •• **[SSM]** The 2.0-kg block in Figure 7-49 slides down a frictionless curved ramp, starting from rest at a height of 3.0 m. The block then slides 9.0 m on a rough horizontal surface before coming to rest. (*a*) What is the speed of the block at the bottom of the ramp? (*b*) What is the energy dissipated by friction? (*c*) What is the coefficient of kinetic friction between the block and the horizontal surface?

Picture the Problem Let the system include the block, the ramp and horizontal surface, and the earth. Given this choice, there are no external forces acting that will change the energy of the system. Because the curved ramp is frictionless, mechanical energy is conserved as the block slides down it. We can calculate its speed at the bottom of the ramp by using conservation of energy. The potential energy of the block at the top of the ramp or, equivalently, its kinetic energy at the bottom of the ramp is converted into thermal energy during its slide along the horizontal surface.

(*a*) Let the numeral 1 designate the initial position of the block and the numeral 2 its position at the foot of the ramp. Choose $U_\mathrm{g} = 0$ at point 2 and use conservation of energy to relate the block's potential energy at the top of the ramp to its kinetic energy at the bottom:

$$W_\mathrm{ext} = \Delta E_\mathrm{mech} + \Delta E_\mathrm{therm}$$

or, because $W_\mathrm{ext} = K_\mathrm{i} = U_\mathrm{f} = \Delta E_\mathrm{therm} = 0$,

$$0 = \tfrac{1}{2}mv_2^2 - mg\Delta h = 0 \Rightarrow v_2 = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_2:

$$\boldsymbol{v}_2 = \sqrt{2(9.81\,\mathrm{m/s^2})(3.0\,\mathrm{m})} = 7.67\,\mathrm{m/s} = \boxed{7.7\,\mathrm{m/s}}$$

(*b*) The energy dissipated by friction is responsible for changing the thermal energy of the system:

$$W_f + \Delta K + \Delta U = \Delta E_{therm} + \Delta K + \Delta U = 0$$

Because $\Delta K = 0$ for the slide:

$$W_f = -\Delta U = -(U_2 - U_1) = U_1$$

Substituting for U_1 yields:

$$W_f = mg\Delta h$$

Substitute numerical values and evaluate U_1:

$$W_f = (2.0\,\text{kg})(9.81\,\text{m/s}^2)(3.0\,\text{m}) = 58.9\,\text{J} = \boxed{59\,\text{J}}$$

(*c*) The energy dissipated by friction is given by:

$$\Delta E_{therm} = f\Delta s = \mu_k mg\Delta x$$

Solving for μ_k yields:

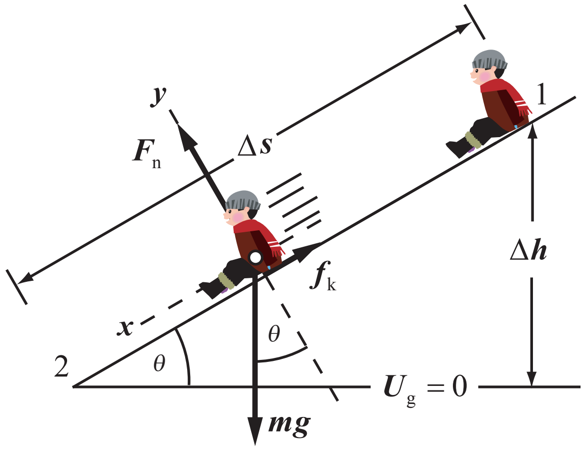
$$\mu_k = \frac{\Delta E_{therm}}{mg\Delta x}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{58.9\,\text{J}}{(2.0\,\text{kg})(9.81\,\text{m/s}^2)(9.0\,\text{m})} = \boxed{0.33}$$

66 •• A 20-kg girl slides down a playground slide with a vertical drop of 3.2 m. When she reaches the bottom of the slide, her speed is 1.3 m/s. (*a*) How much energy was dissipated by friction? (*b*) If the slide is inclined at 20° with the horizontal, what is the coefficient of kinetic friction between the girl and the slide?

Picture the Problem Let the system consist of the earth, the girl, and the slide. Given this choice, there are no external forces doing work to change the energy of the system**.** By the time she reaches the bottom of the slide, her potential energy at the top of the slide has been converted into kinetic and thermal energy. Choose $U_g = 0$ at the bottom of the slide and denote the top and bottom of the slide as shown in the figure. We'll use the work-energy theorem with friction to relate these quantities and the forces acting on her during her slide to determine the friction force that transforms some of her initial potential energy into thermal energy.



(*a*) Express the work-energy theorem:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

Because $U_2 = K_1 = W_{\text{ext}} = 0$:

$$0 = K_2 - U_1 + \Delta E_{\text{therm}} = 0$$

or

$$\Delta E_{\text{therm}} = U_1 - K_2 = mg\Delta h - \tfrac{1}{2}mv_2^2$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta \boldsymbol{E}_{\text{therm}} = (20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m}) - \tfrac{1}{2}(20\,\text{kg})(1.3\,\text{m/s})^2 = 611\,\text{J} = \boxed{0.61\,\text{kJ}}$$

(*b*) Relate the energy dissipated by friction to the kinetic friction force and the distance over which this force acts:

$$\Delta E_{\text{therm}} = f\Delta s = \mu_k F_n \Delta s$$

Solve for μ_k to obtain:

$$\mu_k = \frac{\Delta E_{\text{therm}}}{F_n \Delta s} \qquad (1)$$

Apply $\sum F_y = ma_y$ to the girl and solve for F_n:

$$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$$

Referring to the figure, relate Δh to Δs and θ:

$$\Delta s = \frac{\Delta h}{\sin\theta}$$

Substitute for Δs and F_n in equation (1) and simplify to obtain:

$$\mu_k = \frac{\Delta E_{\text{therm}}}{mg\dfrac{\Delta h}{\sin\theta}\cos\theta} = \frac{\Delta E_{\text{therm}}\tan\theta}{mg\Delta h}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(611\,\text{J})\tan 20°}{(20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m})} = \boxed{0.35}$$

67 •• In Figure 7-50, the coefficient of kinetic friction between the 4.0-kg block and the shelf is 0.35. (*a*) Find the energy dissipated by friction when the 2.0-kg block falls a distance *y*. (*b*) Find the change in mechanical energy E_{mech} of the two-block–Earth system during the time it takes the 2.0-kg block to fall a distance *y*. (*c*) Use your result for Part (*b*) to find the speed of either block after the 2.0-kg block falls 2.0 m.

Picture the Problem Let the system consist of the two blocks, the shelf, and the earth. Given this choice, there are no external forces doing work to change the energy of the system. Due to the friction between the 4.0-kg block and the surface on which it slides, not all of the energy transformed during the fall of the 2.0-kg block is realized in the form of kinetic energy. We can find the energy dissipated by friction and then use the work-energy theorem with kinetic friction to find the speed of either block when they have moved the given distance.

(*a*) The energy dissipated by friction when the 2.0-kg block falls a distance *y* is given by:

$$\Delta E_{\text{therm}} = f\Delta s = \mu_k m_1 g y$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (0.35)(4.0\,\text{kg})(9.81\,\text{m/s}^2)y = (13.7\,\text{N})y = \boxed{(14\,\text{N})y}$$

(*b*) From the work-energy theorem with kinetic friction we have:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$$

or, because $W_{\text{ext}} = 0$ and $E_{\text{mech,i}} = 0$,

$$\boldsymbol{E}_{\text{mech}} = -\Delta\boldsymbol{E}_{\text{therm}} = \boxed{-(14\,\text{N})\boldsymbol{y}}$$

(*c*) Express the total mechanical energy of the system:

$$\tfrac{1}{2}(m_1 + m_2)v^2 - m_2 g y + \Delta E_{\text{therm}} = 0$$

Solving for *v* yields:

$$v = \sqrt{\frac{2(m_2 g y - \Delta E_{\text{therm}})}{m_1 + m_2}}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{\frac{2\left[(2.0\,\text{kg})(9.81\,\text{m/s}^2)(2.0\,\text{m}) - (13.7\,\text{N})(2.0\,\text{m})\right]}{4.0\,\text{kg} + 2.0\,\text{kg}}} = \boxed{2.0\,\text{m/s}}$$

68 •• A small object of mass m moves in a horizontal circle of radius r on a rough table. It is attached to a horizontal string fixed at the center of the circle. The speed of the object is initially v_0. After completing one full trip around the circle, the speed of the object is $0.5v_0$. (*a*) Find the energy dissipated by friction during that one revolution in terms of m, v_0, and r. (*b*) What is the coefficient of kinetic friction? (*c*) How many more revolutions will the object make before coming to rest?

Picture the Problem Let the system consist of the particle, the table, and the earth. Then $W_{ext} = 0$ and the energy dissipated by friction during one revolution is the change in the thermal energy of the system.

(*a*) Apply the work-energy theorem with kinetic friction to obtain:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm}$$

or, because $\Delta U = W_{ext} = 0$,

$$0 = \Delta K + \Delta E_{therm}$$

Substitute for ΔK_f and simplify to obtain:

$$\begin{aligned}\Delta E_{therm} &= -\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right)\\ &= -\left[\tfrac{1}{2}m\left(\tfrac{1}{2}v_0\right)^2 - \tfrac{1}{2}m(v_0)^2\right]\\ &= \boxed{\tfrac{3}{8}mv_0^2}\end{aligned}$$

(*b*) Relate the energy dissipated by friction to the distance traveled and the coefficient of kinetic friction:

$$\Delta E_{therm} = f\Delta s = \mu_k mg\Delta s = \mu_k mg(2\pi r)$$

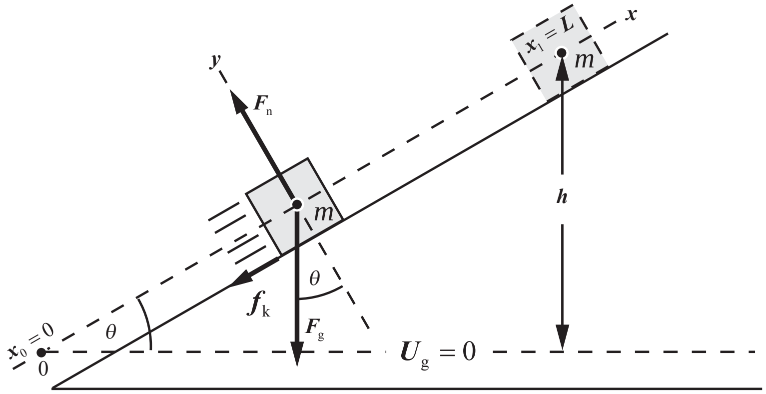
Substitute for ΔE and solve for μ_k to obtain:

$$\mu_k = \frac{\Delta E_{therm}}{2\pi mgr} = \frac{\tfrac{3}{8}mv_0^2}{2\pi mgr} = \boxed{\frac{3v_0^2}{16\pi gr}}$$

(*c*) Because the object lost $\tfrac{3}{4}\boldsymbol{K}_i$ in one revolution, it will only require another 1/3 revolution to lose the remaining $\tfrac{1}{4}\boldsymbol{K}_i$

69 •• **[SSM]** The initial speed of a 2.4-kg box traveling up a plane inclined 37° to the horizontal is 3.8 m/s. The coefficient of kinetic friction between the box and the plane is 0.30. (*a*) How far along the incline does the box travel before coming to a stop? (*b*) What is its speed when it has traveled half the distance found in Part (*a*)?

Picture the Problem The box will slow down and stop due to the dissipation of thermal energy. Let the system be the earth, the box, and the inclined plane and apply the work-energy theorem with friction. With this choice of the system, there are no external forces doing work to change the energy of the system. The pictorial representation shows the forces acting on the box when it is moving up the incline.



(*a*) Apply the work-energy theorem with friction to the system:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U + \Delta E_{therm}$$

Substitute for ΔK, ΔU, and ΔE_{therm} to obtain:

$$0 = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh + \mu_k F_n L \quad (1)$$

Referring to the free-body diagram, relate the normal force to the weight of the box and the angle of the incline:

$$F_n = mg\cos\theta$$

Relate h to the distance L along the incline:

$$h = L\sin\theta$$

Substitute in equation (1) to obtain:

$$\mu_k mgL\cos\theta + \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgL\sin\theta = 0 \qquad (2)$$

Solving equation (2) for L yields:

$$L = \frac{v_0^2}{2g(\mu_k\cos\theta + \sin\theta)}$$

Substitute numerical values and evaluate L:

$$\boldsymbol{L} = \frac{(3.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[(0.30)\cos 37° + \sin 37°]} = 0.8747\,\text{m} = \boxed{0.87\,\text{m}}$$

(b) Let $\boldsymbol{v}_{\frac{1}{2}\boldsymbol{L}}$ represent the box's speed when it is halfway up the incline. Then equation (2) becomes:

$$\mu_k mg\left(\tfrac{1}{2}L\right)\cos\theta + \tfrac{1}{2}mv_{\frac{1}{2}L}^2 - \tfrac{1}{2}mv_0^2 + mg\left(\tfrac{1}{2}L\right)\sin\theta = 0$$

Solving for $\boldsymbol{v}_{\frac{1}{2}\boldsymbol{L}}$ yields :

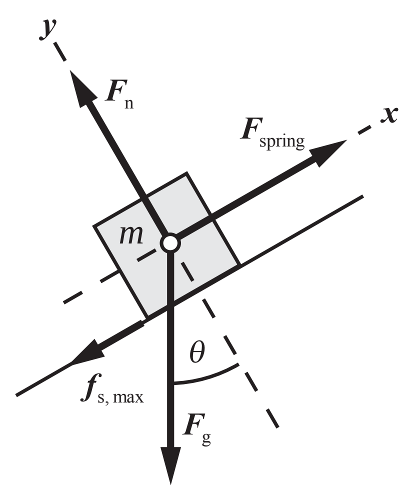
$$v_{\frac{1}{2}L} = \sqrt{v_0^2 - gL(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate $\boldsymbol{v}_{\frac{1}{2}\boldsymbol{L}}$:

$$\boldsymbol{v}_f = \sqrt{(3.8\,\text{m/s})^2 - (9.81\,\text{m/s}^2)(0.8747\,\text{m})[\sin 37° + (0.30)\cos 37°]} = \boxed{2.7\,\text{m/s}}$$

70 ••• A block of mass *m* rests on a plane inclined an angle θ with the horizontal (Figure 7-51). A spring with force constant *k* is attached to the block. The coefficient of static friction between the block and plane is μ_s. The spring is pulled upward along the plane very slowly. (a) What is the extension of the spring the instant the block begins to move? (b) The block stops moving just as the extension of the contracting spring reaches zero. Express μ_k (the kinetic coefficient of friction) in terms of μ_s and θ.

Picture the Problem Let the system consist of the earth, the block, the incline, and the spring. With this choice of the system, there are no external forces doing work to change the energy of the system. The free-body diagram shows the forces acting on the block just before it begins to move. We can apply Newton's 2nd law to the block to obtain an expression for the extension of the spring at this instant. We'll apply the work-energy theorem with friction to the second part of the problem.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the block when it is on the verge of sliding:

$$\sum F_x = F_{\text{spring}} - f_{\text{s,max}} - mg\sin\theta = 0$$

and

$$\sum F_y = F_{\text{n}} - mg\cos\theta = 0$$

Eliminate F_{n}, $f_{\text{s,max}}$, and F_{spring} between the two equations to obtain:

$$kd - \mu_{\text{s}} mg\cos\theta - mg\sin\theta = 0$$

Solving for d yields:

$$d = \boxed{\frac{mg}{k}\left(\sin\theta + \mu_{\text{s}}\cos\theta\right)}$$

(b) Begin with the work-energy theorem with friction and no work being done by an external force:

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{\text{therm}}$$

Because the block is at rest in both its initial and final states, $\Delta K = 0$ and:

$$\Delta U_g + \Delta U_s + \Delta E_{\text{therm}} = 0 \qquad (1)$$

Let $U_{\text{g}} = 0$ at the initial position of the block. Then:

$$\Delta U_{\text{g}} = U_{\text{g,final}} - U_{\text{g,initial}} = mgh - 0 = mgd\sin\theta$$

Express the change in the energy stored in the spring as it relaxes to its unstretched length:

$$\Delta U_{\text{s}} = U_{\text{s,final}} - U_{\text{s,initial}} = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}kd^2$$

The energy dissipated by friction is:

$$\Delta E_{\text{therm}} = f\Delta s = f_{\text{k}} d = \mu_{\text{k}} F_{\text{n}} d = \mu_{\text{k}} mgd\cos\theta$$

Substitute in equation (1) to obtain:

$$mgd\sin\theta - \tfrac{1}{2}kd^2 + \mu_{\text{k}} mgd\cos\theta = 0$$

or

$$mg\sin\theta - \tfrac{1}{2}kd + \mu_{\text{k}} mg\cos\theta = 0$$

Substituting for d (from Part (a)) yields:

$$mg\sin\theta - \tfrac{1}{2}k\left[\frac{mg}{k}\left(\sin\theta + \mu_{\text{s}}\cos\theta\right)\right] + \mu_{\text{k}} mg\cos\theta = 0$$

Finally, solve for μ_{k} to obtain:

$$\boldsymbol{\mu}_{\text{k}} = \boxed{\tfrac{1}{2}\left(\boldsymbol{\mu}_{\text{s}} - \tan\boldsymbol{\theta}\right)}$$

Mass and Energy

71 • (*a*) Calculate the rest energy of 1.0 g of dirt. (*b*) If you could convert this energy completely into electrical energy and sell it for \$0.10/kW·h, how much money would you take in? (*c*) If you could power a 100-W light bulb with this energy, for how long could you keep the bulb lit?

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) The rest energy of the dirt is given by:

$$E_0 = mc^2$$

Substitute numerical values and evaluate E_0:

$$E_0 = \left(1.0\times10^{-3}\ \text{kg}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2$$
$$= 8.988\times10^{13}\ \text{J} = \boxed{9.0\times10^{13}\ \text{J}}$$

(*b*) Express kW·h in joules:

$$1\,\text{kW}\cdot\text{h} = \left(1\times10^{3}\ \text{J/s}\right)\left(1\,\text{h}\times\frac{3600\ \text{s}}{\text{h}}\right)$$
$$= 3.60\times10^{6}\ \text{J}$$

Convert 8.988×10^{13} J to kW·h:

$$8.988\times10^{13}\ \text{J} = \left(8.988\times10^{13}\ \text{J}\right)\times\left(\frac{1\,\text{kW}\cdot\text{h}}{3.60\times10^{6}\ \text{J}}\right)$$
$$= 2.50\times10^{7}\ \text{kW}\cdot\text{h}$$

Determine the price of the electrical energy:

$$\text{Price} = \left(2.50\times10^{7}\ \text{kW}\cdot\text{h}\right)\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right)$$
$$= \boxed{\$2.5\times10^{6}}$$

(*c*) Relate the energy consumed to its rate of consumption and the time:

$$\Delta E = P\Delta t \Rightarrow \Delta t = \frac{\Delta E}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{8.988\times10^{13}\ \text{J}}{100\ \text{W}} = 8.988\times10^{11}\ \text{s}$$
$$= \boxed{9.0\times10^{11}\ \text{s}}$$
$$= 8.988\times10^{11}\ \text{s}\times\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}$$
$$= \boxed{2.8\times10^{4}\ \text{y}}$$

72 • One kiloton of TNT, when detonated, yields an explosive energy of roughly 4×10^{12} J. How much less is the total mass of the bomb remnants after the explosion than before? If you could find and reassemble the pieces, would this loss of mass be noticeable?

Picture the Problem We can use the equation expressing the equivalence of energy and matter, $E = \Delta mc^2$, to find the reduction in the mass of the bomb due to the explosion.

Solve $E = \Delta mc^2$ for Δm:

$$\Delta m = \frac{E}{c^2}$$

Substitute numerical values and evaluate Δm:

$$\Delta m = \frac{4\times10^{12}\ \text{J}}{\left(2.998\times10^{8}\ \text{m/s}\right)^2} \approx \boxed{4\times10^{-5}\ \text{kg}}$$

Express the ratio of Δm to the mass of the bomb before its explosion:

$$\frac{\Delta m}{m_{\text{bomb}}} = \frac{4\times10^{-5}\ \text{kg}}{1\ \text{kton}\times\dfrac{2000\ \text{lb}}{\text{ton}}\times\dfrac{1\ \text{kg}}{2.2046\ \text{lb}}}$$
$$\approx 5\times10^{-11}$$

No, not noticeable! The mass change, compared to the mass of the bomb, is negligible.

73 • Calculate your rest energy in both mega electron-volts and joules. If that energy could be converted completely to the kinetic energy of your car, estimate its speed. Use the nonrelativistic expression for kinetic energy and comment on whether or not your answer justifies using the nonrelativistic expression for kinetic energy.

Picture the Problem Your rest energy is given by $E_0 = mc^2$.

Your rest energy if given by:

$$E_0 = mc^2$$

Assuming that your mass is 70 kg, substitute numerical values and evaluate E_0:

$$E_0 = (70\text{ kg})(2.998\times10^8\text{ m/s})^2 = 6.292\times10^{18}\text{ J} = \boxed{6.3\times10^{18}\text{ J}}$$

Convert E_0 to MeV to obtain:

$$E_0 = 6.292\times10^{18}\text{ J}\times\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}} = \boxed{3.9\times10^{31}\text{ MeV}}$$

The nonrelativistic expression for the kinetic energy of your car is:

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Assuming the mass of your car to be 1.4×10^3 kg (approximately 3000 lb), substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(6.292\times10^{18}\text{ J})}{1.4\times10^3\text{ kg}}} \approx \boxed{9.5\times10^7\text{ m/s}}$$

As expected, this result is close enough to the speed of light (and thus incorrect) because the non-relativistic expressions do not apply if the car's energy is of the order of the magnitude of its rest energy. In this case we assumed they were equal.

74 • If a black hole and a "normal" star orbit each other, gases from the normal star falling into the black hole can have their temperature increased by millions of degrees due to frictional heating. When the gases are heated that much, they begin to radiate light in the X-ray region of the electromagnetic spectrum (high-energy light photons). Cygnus X-1, the second strongest known X-ray source in the sky, is thought to be one such binary system; it radiates at an estimated power of 4×10^{31} W. If we assume that 1.0 percent of the in-falling mass escapes as X ray energy, at what rate is the black hole gaining mass?

Picture the Problem We can differentiate the mass-energy equation to obtain an expression for the rate at which the black hole gains energy.

Using the mass-energy relationship, express the energy radiated by the black hole:

$$E = (0.010)mc^2$$

Differentiate this expression to obtain an expression for the rate at which the black hole is radiating energy:

$$\frac{dE}{dt} = \frac{d}{dt}\left[(0.010)mc^2\right] = (0.010)c^2\frac{dm}{dt}$$

Solving for dm/dt yields:

$$\frac{dm}{dt} = \frac{dE/dt}{(0.010)c^2}$$

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt}=\frac{4\times10^{31}\text{ watt}}{(0.010)(2.998\times10^{8}\text{ m/s})^{2}}$$
$$\approx \boxed{4\times10^{16}\text{ kg/s}}$$

75 • **[SSM]** You are designing the fuel requirements for a small fusion electric-generating plant. Assume 33% conversion to electric energy. For the deuterium–tritium (D–T) fusion reaction in Example 7-18, calculate the number of reactions per second that are necessary to generate 1.00 kW of electric power.

Picture the Problem The number of reactions per second is given by the ratio of the power generated to the energy released per reaction. The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per second.

In Example 7-18 it is shown that the energy per reaction is 17.59 MeV. Convert this energy to joules:

$$17.59\text{ MeV}=(17.59\text{ MeV})(1.602\times10^{-19}\text{ J/eV})=28.18\times10^{-13}\text{ J}$$

Assuming 33% conversion to electric energy, the number of reactions per second is:

$$\frac{1000\text{ J/s}}{(0.33)(28.18\times10^{-13}\text{ J/reaction})}\approx\boxed{1.1\times10^{15}\text{ reactions/s}}$$

76 • Use Table 7-1 to calculate the energy needed to remove one neutron from a stationary alpha particle, leaving a stationary helion plus a neutron with a kinetic energy of 1.5 MeV.

Picture the Problem The energy required for this reaction is the sum of 1.5 MeV and the difference between the rest energy of ^{4}He and the sum of the rest energies of a helion (^{3}He) and a neutron.

The required energy is given by: $\quad E_{\text{total}}=E+K_{\text{n}} \qquad (1)$

Express the reaction: $\quad {}^{4}\text{He}\rightarrow{}^{3}\text{He}+\text{n}$

The rest energy of a neutron (Table 7-1) is: $\quad$ 939.573 MeV

The rest energy of ^{4}He (Table 7-1) is: $\quad$ 3727.409 MeV

The rest energy of ^{3}He is: $\quad$ 2808.432 MeV

Substitute numerical values to find the difference in the rest energy of ^{4}He and the sum of the rest energies of ^{3}He and n:

$$\boldsymbol{E} = [3727.409 - (2808.432 + 939.573)]\ \text{MeV} = 20.596\,\text{MeV}$$

Substitute numerical values in equation (1) and evaluate E_{total}:

$$E_{\text{total}} = 20.596\,\text{MeV} + 1.5\,\text{MeV} = \boxed{22.1\,\text{MeV}}$$

77 • A free neutron can decay into a proton plus an electron and an electron antineutrino [an electron antineutrino (symbol $\bar{\nu}_{\text{e}}$) is a nearly massless elementary particle]: $\text{n} \to \text{p} + \text{e}^- + \bar{\boldsymbol{\nu}}_{\text{e}}$. Use Table 7-1 to calculate the energy released during this reaction.

Picture the Problem The energy released during this reaction is the difference between the rest energy of a neutron and the sum of the rest energies of a proton and an electron.

The rest energy of a proton (Table 7-1) is: 938.280 MeV

The rest energy of an electron (Table 7-1) is: 0.511 MeV

The rest energy of a neutron (Table 7-1) is: 939.573 MeV

Substitute numerical values to find the difference in the rest energy of a neutron and the sum of the rest energies of a positron and an electron:

$$E = [939.573 - (938.280 + 0.511)]\ \text{MeV} = \boxed{0.782\,\text{MeV}}$$

78 •• During one type of nuclear fusion reaction, two deuterons combine to produce an alpha particle. (*a*) How much energy is released during this reaction? (*b*) How many such reactions must take place per second to produce 1 kW of power?

Picture the Problem The reaction is ${}^{2}\text{H}+{}^{2}\text{H}\to{}^{4}\text{He}+E$. The energy released in this reaction is the difference between twice the rest energy of ^{2}H and the rest energy of ^{4}He. The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per reaction.

(*a*) The rest energy of ^{4}He (Table 7-1) is:

$$3727.409\text{ MeV}$$

The rest energy of a deuteron, ^{2}H, (Table 7-1) is:

$$1875.628\text{ MeV}$$

The energy released in the reaction is:

$$\begin{aligned} E &= [2(1875.628)-3727.409]\text{ MeV} \\ &= 23.847\text{ MeV}\times\frac{1.602\times10^{-19}\text{ J}}{\text{eV}} \\ &= 3.820\times10^{-12}\text{ J} = \boxed{3.82\times10^{-12}\text{ J}} \end{aligned}$$

(*b*) The number of reactions per second is:

$$\frac{1.00\times10^{3}\text{ J/s}}{3.820\times10^{-12}\text{ J/reaction}} = \boxed{2.62\times10^{14}\text{ reactions/s}}$$

79 •• A large nuclear power plant produces 1000 MW of electrical power by nuclear fission. (*a*) By how many kilograms does the mass of the nuclear fuel decrease by in one year? (Assume an efficiency of 33 percent for a nuclear power plant.) (*b*) In a coal-burning power plant, each kilogram of coal releases 31 MJ of thermal energy when burned. How many kilograms of coal are needed each year for a 1000-MW coal-burning power plant? (Assume an efficiency of 38 percent for a coal-burning power plant.)

Picture the Problem The annual consumption of matter by the fission plant is the ratio of its annual energy output to the square of the speed of light. The annual consumption of coal in a coal-burning power plant is the ratio of its annual energy output to energy per unit mass of the coal.

(*a*) The yearly consumption of matter is given by:

$$\Delta\boldsymbol{m} = \frac{\boldsymbol{E}}{\boldsymbol{\varepsilon}\boldsymbol{c}^{2}}$$

where E is the energy to be generated and ε is the efficiency of the plant.

Because the energy to be generated is the product of the power output of the plant and the elapsed time:

$$\Delta m = \frac{P\Delta t}{\varepsilon\, c^2} \qquad (1)$$

Substitute numerical values and evaluate Δm:

$$\Delta m = \frac{(1000\text{ MW})\left(1\text{ y}\times\dfrac{3.156\times10^{7}\text{ s}}{\text{y}}\right)}{(0.33)\left(2.998\times10^{8}\text{ m/s}\right)^2} = \boxed{1.1\text{ kg}}$$

(*b*) For a coal-burning power plant, equation (1) becomes:

$$\Delta m_{\text{coal}} = \frac{P\Delta t}{\varepsilon\left(\dfrac{\text{energy released}}{\text{unit mass}}\right)}$$

Substitute numerical values and evaluate Δm_{coal}:

$$\Delta m_{\text{coal}} = \frac{(1000\text{ MW})\left(1\text{ y}\times\dfrac{3.156\times10^{7}\text{ s}}{\text{y}}\right)}{(0.38)(31\text{ MJ/kg})} = \boxed{2.7\times10^{9}\text{ kg}}$$

Remarks: 2.7 × 10^9 kg is approximately 3 million tons!

Quantization of Energy

80 •• A mass on the end of a spring with a force constant of 1000 N/kg oscillates at a frequency of 2.5 oscillations per second. (*a*) Determine the quantum number, *n*, of the state it is in if it has a total energy of 10 J. (*b*) What is its ground state energy?

Picture the Problem The energy number *n* of a state whose energy is *E* is given by $E = \left(n+\frac{1}{2}\right)hf$ where *h* is Planck's constant and *f* is the frequency of the state.

(*a*) The energy of the vibrational state is given by:

$$E = \left(n+\tfrac{1}{2}\right)hf$$

or, because we expect *n* to be very large,

$$E \approx nhf \Rightarrow n = \frac{E}{hf}$$

Substitute numerical values and evaluate n:

$$\boldsymbol{n} = \frac{10\text{ J}}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.5\text{ s}^{-1}\right)} = \boxed{6.0\times10^{33}}$$

(*b*) The ground state energy of the oscillator is the energy of the system when $n = 0$:

$$\boldsymbol{E}_0 = \tfrac{1}{2}\boldsymbol{hf}$$

Substitute numerical values and evaluate E_0:

$$\boldsymbol{E}_0 = \tfrac{1}{2}\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.5\text{ s}^{-1}\right) = \boxed{8.3\times10^{-34}\text{ J}}$$

81 •• Repeat Problem 80, but consider instead an atom in a solid vibrating at a frequency of 1.00×10^{14} oscillations per second and having a total energy of 2.7 eV.

Picture the Problem The energy number n of a state whose energy is E is given by $\boldsymbol{E} = \left(\boldsymbol{n}+\frac{1}{2}\right)\boldsymbol{hf}$ where h is Planck's constant and f is the frequency of the state.

(*a*) The energy of the vibrational state is given by:

$$E = \left(n+\tfrac{1}{2}\right)hf \Rightarrow n = \frac{E}{hf} - \frac{1}{2}$$

Substitute numerical values and evaluate n:

$$\boldsymbol{n} = \frac{2.7\text{ eV}\times\dfrac{1.602\times10^{-19}\text{ J}}{\text{eV}}}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(10^{14}\text{ s}^{-1}\right)} - \frac{1}{2} = \boxed{6}$$

(*b*) The ground state energy of the oscillator is the energy of the system when $n = 0$:

$$\boldsymbol{E}_0 = \tfrac{1}{2}\boldsymbol{hf}$$

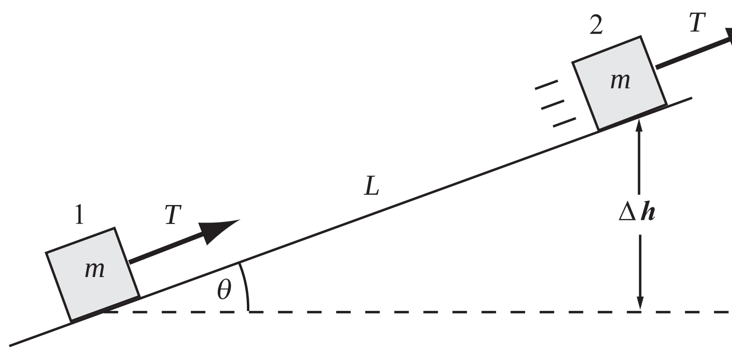
Substitute numerical values and evaluate E_0:

$$\boldsymbol{E}_0 = \tfrac{1}{2}\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(10^{14}\text{ s}^{-1}\right) = 3.315\times10^{-20}\text{ J}\times\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}} = \boxed{0.21\text{ eV}}$$

General Problems

82 • A block of mass m, starting from rest, is pulled up a frictionless inclined plane that makes an angle θ with the horizontal by a string parallel to the plane. The tension in the string is T. After traveling a distance L, the speed of the block is v_f. Derive an expression for work done by the tension force.

Picture the Problem Let the system consist of the block, the earth, and the incline. Then the tension in the string is an external force that will do work to change the energy of the system. Because the incline is frictionless; the work done by the tension in the string as it displaces the block on the incline is equal to the sum of the changes in the kinetic and gravitational potential energies.



Relate the work done on the block by the tension force to the changes in the kinetic and gravitational potential energies of the block:

$$W_{\text{tension force}} = W_{\text{ext}} = \Delta U + \Delta K \quad (1)$$

Referring to the figure, express the change in the potential energy of the block as it moves from position 1 to position 2:

$$\Delta U = mg\Delta h = mgL\sin\theta$$

Because the block starts from rest:

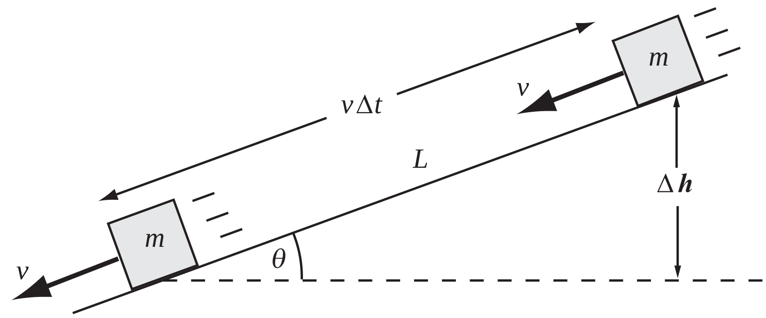
$$\Delta K = K_2 = \tfrac{1}{2}mv_f^2$$

Substitute for ΔU and ΔK in equation (1) to obtain:

$$W_{\text{tension force}} = \boxed{mgL\sin\theta + \tfrac{1}{2}mv_f^2}$$

83 • A block of mass m slides with constant speed v down a plane inclined at angle θ with the horizontal. Derive an expression for the energy dissipated by friction during the time interval Δt.

Picture the Problem Let the system include the earth, the block, and the inclined plane. Then there are no external forces to do work on the system and $W_{\text{ext}} = 0$. Apply the work-energy theorem with friction to find an expression for the energy dissipated by friction.



Apply the work-energy theorem with friction to the block:

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

Because the velocity of the block is constant, $\Delta K = 0$ and:

$$\Delta E_{\text{therm}} = -\Delta U = -mg\Delta h$$

In time Δt the block slides a distance $v\Delta t$. From the figure:

$$\Delta h = -v\Delta t \sin\theta$$

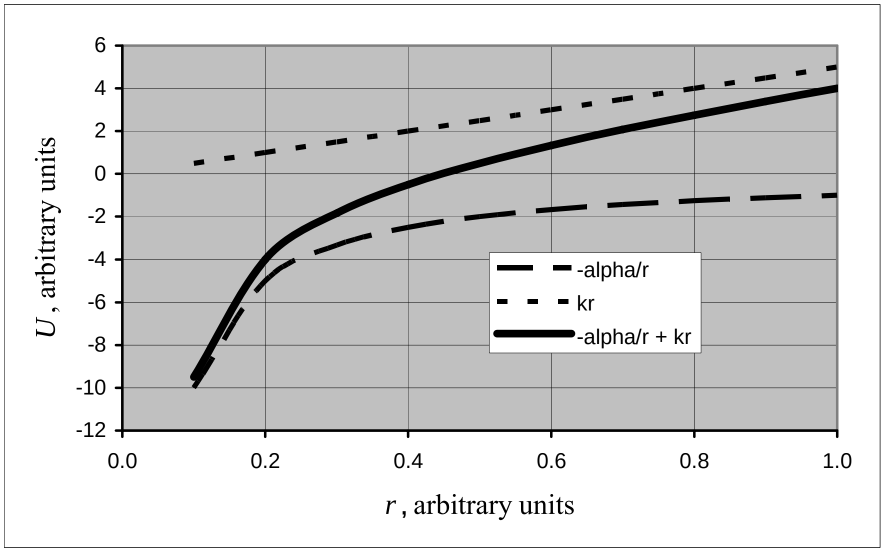
Substitute for Δh to obtain:

$$\Delta E_{\text{therm}} = \boxed{mgv\Delta t \sin\theta}$$

84 • In particle physics, the potential energy associated with a pair of quarks bound together by the strong nuclear force is in one particular theoretical model written as the following function: $U(r) = -(\alpha/r) + kr$, where k and α are positive constants, and r is the distance of separation between the two quarks. (a) Sketch the general shape of the potential-energy function. (b) What is a general form for the force each quark exerts on the other? (c) At the two extremes of very small and very large values of r, what does the force simplify to?

Picture the Problem The force between the two quarks is given by $F = -\dfrac{dU(r)}{dr}$.

(*a*) The following graph was plotted using a spreadsheet program. α was set to 1 and *k* was set to 5.



(*b*) *F* is given by:

$$F = -\frac{dU(r)}{dr}$$

Substitute for *U*(*r*) and evaluate *F* to obtain:

$$F = -\frac{d}{dr}\left(-\frac{\alpha}{r} + kr\right) = \boxed{-\frac{\alpha}{r^2} + k}$$

(*c*) For *r* >> 1:

$$F_{r>>1} \to \boxed{k}$$

For *r* << 1:

$$F_{r<<1} \to \boxed{-\frac{\alpha}{r^2}}$$

85 • **[SSM]** You are in charge of "solar-energizing" your grandfather's farm. At the farm's location, an average of 1.0 kW/m^2 reaches the surface during the daylight hours on a clear day. If this could be converted at 25% efficiency to electric energy, how large a collection area would you need to run a 4.0-hp irrigation water pump during the daylight hours?

Picture the Problem The solar constant is the average energy per unit area and per unit time reaching the upper atmosphere. This physical quantity can be thought of as the power per unit area and is known as *intensity*.

Letting I_{surface} represent the intensity of the solar radiation at the surface of the earth, express I_{surface} as a function of power and the area on which this energy is incident:

$$\varepsilon I_{\text{surface}} = \frac{P}{A} \Rightarrow \boldsymbol{A} = \frac{\boldsymbol{P}}{\boldsymbol{\varepsilon I}_{\text{surface}}}$$

where ε is the efficiency of conversion to electric energy.

Substitute numerical values and evaluate A:

$$\boldsymbol{A} = \frac{4.0\ \text{hp} \times \frac{746\ \text{W}}{\text{hp}}}{(0.25)(1.0\ \text{kW/m}^2)} = \boxed{12\ \text{m}^2}$$

86 •• The radiant energy from the Sun that reaches Earth's orbit is 1.35 kW/m^2. (*a*) Even when the Sun is directly overhead and under dry desert conditions, 25% of this energy is absorbed and/or reflected by the atmosphere before it reaches Earth's surface. If the average frequency of the electromagnetic radiation from the Sun is 5.5×10^{14} Hz, how many photons per second would be incident upon a 1.0-m^2 solar panel? (*b*) Suppose the efficiency of the panels for converting the radiant energy to electrical energy and delivering it is a highly efficient 10.0%. How large a solar panel is needed to supply the needs of a 5.0-hp solar-powered car (assuming the car runs directly off the solar panel and not batteries) during a race in Cairo at noon on March 21? (*c*) Assuming a more-realistic efficiency of 3.3% and panels capable of rotating to be always perpendicular to the sunlight, how large an array of solar panels is needed to supply the power needs of the International Space Station (ISS)? The ISS requires about 110 kW of continuous electric power.

Picture the Problem The number of photons *n* incident on a solar panel is related to the energy *E* of the incident radiation ($\boldsymbol{E = nhf}$) and the intensity of the solar radiation is the rate at which it delivers energy per unit area.

(*a*) The number of photons *n* incident on the solar panel is related to the energy *E* of the radiation:

$$\boldsymbol{E = nhf \Rightarrow n = \frac{E}{hf}} \qquad (1)$$

The intensity *I* of the radiation is given by:

$$\boldsymbol{I = \frac{P}{A} = \frac{E}{A\Delta t} \Rightarrow E = IA\Delta t}$$

Substituting for *E* in equation (1) yields:

$$\boldsymbol{n = \frac{IA\Delta t}{hf}}$$

The number of photons arriving per unit time is given by:

$$\frac{n}{\Delta t}=\frac{IA}{hf}\text{ or }\frac{n}{\Delta t}=\frac{\varepsilon I'A}{hf}$$

where I' is the unreduced solar constant and ε is the percentage of the energy absorbed.

Substitute numerical values and evaluate the number of photons arriving per unit time:

$$\frac{n}{\Delta t}=\frac{(0.75)(1.35\text{ kW/m}^2)(1.0\text{ m}^2)}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(5.5\times10^{14}\text{ s}^{-1})}=\boxed{2.8\times10^{21}\text{ s}^{-1}}$$

(*b*) The effective intensity of the radiation is given by:

$$I=\frac{P}{\varepsilon A}\Rightarrow A=\frac{P}{\varepsilon I}\qquad(2)$$

where ε is the efficiency of energy conversion.

Substitute numerical values and evaluate A:

$$A=\frac{5.0\text{ hp}\times\frac{746\text{ W}}{\text{hp}}}{(0.10)(1.35\text{ kW/m}^2)}=\boxed{28\text{ m}^2}$$

(*c*) Substitute numerical values in equation (2) to obtain:

$$A=\frac{5.0\text{ hp}\times\frac{746\text{ W}}{\text{hp}}}{(0.033)(1.35\text{ kW/m}^2)}=\boxed{84\text{ m}^2}$$

87 •• In 1964, after the 1250-kg jet-powered car *Spirit of America* lost its parachute and went out of control during a run at Bonneville Salt Flats, Utah, it left skid marks about 8.00 km long. (This earned a place in the Guinness Book of World Records for longest skid marks.) (*a*) If the car was moving initially at a speed of about 800 km/h, and was still going at about 300 km/h when it crashed into a brine pond, estimate the coefficient of kinetic friction μ_k. (*b*) What was the kinetic energy of the car 60 s after the skid began?

Picture the Problem Let the system include the earth and the *Spirit of America*. Then there are no external forces to do work on the car and $W_{ext}=0$. We can use the work-energy theorem for problems with kinetic friction to relate the coefficient of kinetic friction to the given information. A constant-acceleration equation will yield the car's velocity when 60 s have elapsed.

(*a*) Apply the work-energy theorem with friction to relate the coefficient of kinetic friction μ_k to the initial and final kinetic energies of the car:

$$W_{ext} = \Delta K + \Delta U + \Delta E_{therm} = 0$$

or

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 + \mu_k mg\Delta s = 0$$

Solving for μ_k yields:

$$\mu_k = \frac{v_i^2 - v_f^2}{2g\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{\left[\left(800\,\frac{\text{km}}{\text{h}}\right)^2 - \left(300\,\frac{\text{km}}{\text{h}}\right)^2\right]\times\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{2\left(9.81\,\text{m/s}^2\right)\left(8.00\,\text{km}\right)} = \boxed{0.270}$$

(*b*) The kinetic energy of the car as a function of its speed is:

$$K = \tfrac{1}{2}mv^2 \qquad (1)$$

Using a constant-acceleration equation, relate the speed of the car to its acceleration, initial speed, and the elapsed time:

$$v = v_0 + at \qquad (2)$$

Express the braking force acting on the car:

$$F_{net} = -f_k = -\mu_k mg = ma$$

Solving for *a* yields:

$$a = -\mu_k g$$

Substitute for *a* in equation (2) to obtain:

$$v = v_0 - \mu_k gt$$

Substituting for *v* in equation (1) yields an expression for the kinetic energy of car as a function of the time it has been skidding:

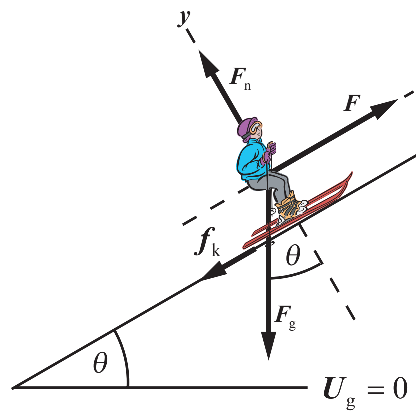
$$K(t) = \tfrac{1}{2}m\left(v_0 - \mu_k gt\right)^2$$

Substitute numerical values and evaluate $K(60\text{ s})$:

$$K(60\,\text{s}) = \tfrac{1}{2}(1250\,\text{kg})\left[800\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}} - (0.270)\left(9.81\,\text{m/s}^2\right)(60\,\text{s})\right]^2 = \boxed{54\,\text{GJ}}$$

88 •• A T-bar tow is planned in a new ski area. At any one time, it will be required, to pull a maximum of 80 skiers up a 600-m slope inclined at 15° above the horizontal at a speed of 2.50 m/s. The coefficient of kinetic friction between the skiers skis and the snow is typically 0.060. As the manager of the facility, what motor power should you request of the construction contractor if the mass of the average skier is 75.0 kg. Assume you want to be ready for any emergency and will order a motor whose power rating is 50% larger than the bare minimum.

Picture the Problem The free-body diagram shows the forces acting on a skier as he/she is towed up the slope at constant speed. We can apply the work-energy theorem to find the minimum rate at which the motor will have to supply energy to tow the skiers up an incline whose length is ℓ.



Apply the work-energy theorem to the skiers:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} = \Delta K + \Delta U_g + \Delta E_{therm}$$

Because $\Delta K = 0, \Delta E_{therm} = f_k \ell$, and $\Delta U_g = m_{tot} g \ell \sin\theta$:

$$W_{ext} = m_{tot} g \ell \sin\theta + f_k \ell \qquad (1)$$

The external work done by the electric motor is given by:

$$W_{ext} = P_{min}\Delta t = P_{min}\frac{\ell}{v}$$

where v is the speed with which the skiers are towed up the incline.

The kinetic friction force is given by:

$$f_k = \mu_k F_n = \mu_k m_{tot} g \cos\theta$$

Substituting for W_{ext} and f_k in equation (1) yields:

$$P_{min}\frac{\ell}{v} = m_{tot} g \ell \sin\theta + \mu_k m_{tot} g \cos\theta$$

Solve for P_{min} to obtain:

$$P_{min} = m_{tot} g v (\sin\theta + \mu_k \cos\theta)$$

Because you want a safety factor of 50%, the power output of the motor you should order should be 150% of P_{min}:

$$P = 1.5 m_{tot} g v (\sin\theta + \mu_k \cos\theta)$$

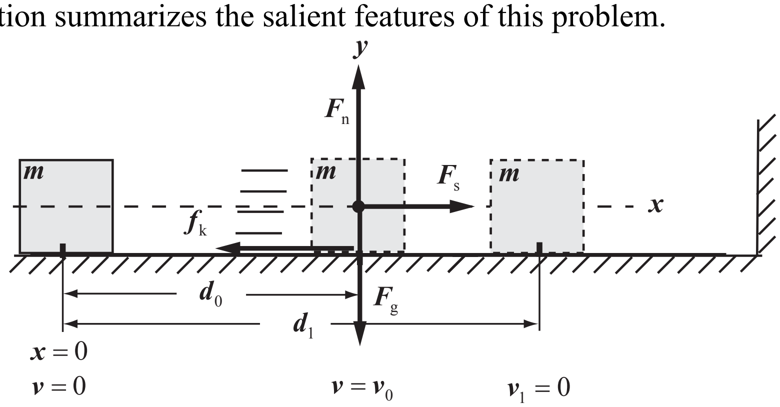
Substitute numerical values and evaluate P:

$$\boldsymbol{P} = (1.5)(80)(75.0\,\text{kg})(9.81\,\text{m/s}^2)(2.50\,\text{m/s})[\sin 15.0° + (0.060)\cos 15.0°]$$
$$= \boxed{70\,\text{kW}}$$

Remarks: We could have solved this problem using Newton's 2nd law.

89 •• A box of mass m on the floor is connected to a horizontal spring of force constant k (Figure 7-52). The coefficient of kinetic friction between the box and the floor is μ_k. The other end of the spring is connected to a wall. The spring is initially unstressed. If the box is pulled away from the wall a distance d_0 and released, the box slides toward the wall. Assume the box does not slide so far that the coils of the spring touch. (*a*) Obtain an expression for the distance d_1 the box slides before it first comes to a stop, (*b*) Assuming $d_1 > d_0$, obtain an expression for the speed of the box when has slid a distance d_0 following the release.
(*c*) Obtain the special value of μ_k such that $d_1 = d_0$.

Picture the Problem Let the system include the Earth, the box, and the surface on which the box slides and apply the work-energy theorem for problems with kinetic friction to the box to derive the expressions for distance the box slides and the speed of the box when it first reaches its equilibrium position. The pictorial representation summarizes the salient features of this problem.



(*a*) Apply the work-energy theorem for problems with kinetic friction to the box as it moves from $x = 0$ to $x = d_1$ to obtain:

$$\boldsymbol{W}_{\text{ext}} = \Delta \boldsymbol{E}_{\text{sys}} = \Delta \boldsymbol{E}_{\text{mech}} + \Delta \boldsymbol{E}_{\text{therm}}$$

or, because $W_{\text{ext}} = \Delta K = \Delta U_g = 0$, $\Delta E_{\text{therm}} = f_k \Delta x$, and $\Delta \boldsymbol{E}_{\text{mech}} = \Delta \boldsymbol{U}_s$,

$$\Delta \boldsymbol{U}_s + \boldsymbol{f}_k \Delta \boldsymbol{x} = 0$$

Substitute for ΔU_s and Δx to obtain:

$$\tfrac{1}{2}k(d_1 - d_0)^2 - \tfrac{1}{2}kd_0^2 + f_k d_1 = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the box to obtain: $\boldsymbol{F}_{\text{n}} - \boldsymbol{F}_{\text{g}} = 0 \Rightarrow \boldsymbol{F}_{\text{n}} = \boldsymbol{F}_{\text{g}} = \boldsymbol{mg}$

f_{k} is given by: $f_{\text{k}} = \mu_{\text{k}} F_{\text{n}} = \mu_{\text{k}} mg$

Substituting for f_{k} in equation (1) yields: $\frac{1}{2}k(d_1 - d_0)^2 - \frac{1}{2}kd_0^2 + \mu_{\text{k}} mgd_1 = 0$

Solve for d_1 to obtain:

$$d_1 = \boxed{2d_0 - \frac{2\mu_{\text{k}} mg}{k}}$$

(*b*) Apply the work-energy theorem to the box as it moves from $x = 0$ to $x = d_0$ to obtain:

$W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$

or, because $W_{\text{ext}} = \Delta U_{\text{g}} = 0$,
$\Delta \boldsymbol{K} + \Delta \boldsymbol{U}_{\text{s}} + \Delta \boldsymbol{E}_{\text{therm}} = 0$

Noting that $\boldsymbol{K}_0 = \boldsymbol{U}_{\text{s,f}} = 0$, substitute for ΔK, ΔU_{s}, and ΔE_{therm} to obtain:

$\frac{1}{2}mv_0^2 - \frac{1}{2}kd_0^2 + f_{\text{k}} d_0 = 0$

Substituting the expression for f_{k} obtained in (*a*) yields:

$\frac{1}{2}mv_0^2 - \frac{1}{2}kd_0^2 + \mu_{\text{k}} mgd_0 = 0$

Solving for $\boldsymbol{v}_0$ yields:

$$v_0 = \boxed{\sqrt{\frac{k}{m}d_0^2 - 2\mu_{\text{k}} gd_0}}$$

(*c*) Let $\boldsymbol{d}_1 = \boldsymbol{d}_0$ in the expression for $\boldsymbol{d}_1$ derived in (*a*) to obtain:

$$d_0 = 2d_0 - \frac{2\mu_{\text{k}} mg}{k} \Rightarrow \mu_{\text{k}} = \boxed{\frac{kd_0}{2mg}}$$

Remarks: You can obtain the same Part (*c*) result by setting $\boldsymbol{v}_0 = 0$ in the expression derived in Part (*b*).

90 •• You operate a small grain elevator near Champaign, Illinois. One of your silos uses a bucket elevator that carries a full load of 800 kg through a vertical distance of 40 m. (A bucket elevator works with a continuous belt, like a conveyor belt.) (*a*) What is the power provided by the electric motor powering the bucket elevator when the bucket elevator ascends with a full load at a speed of 2.3 m/s? (*b*) Assuming the motor is 85% efficient, how much does it cost you to run this elevator, per day, assuming it runs 60 percent of the time between 7:00 A.M. and 7:00 P.M. with and average load of 85 percent of a full load? Assume the cost of electric energy in your location is 15 cents per kilowatt hour.

Picture the Problem The power provided by a motor that is delivering sufficient energy to exert a force $\vec{F}$ on a load which it is moving at a speed $\vec{v}$ is $\vec{F} \cdot \vec{v}$.

(*a*) The power provided by the motor is given by:

$$\boldsymbol{P}=\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{v}}=\boldsymbol{Fv}\cos\boldsymbol{\theta}$$

or, because $\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{v}}$ are in the same direction, $\boldsymbol{P}=\boldsymbol{Fv}$ (1)

Because the elevator is ascending with constant speed, the required force is:

$$\boldsymbol{F}=\boldsymbol{m}_{\text{load}}\boldsymbol{g}$$

Substitute for F in equation (1) to obtain:

$$\boldsymbol{P}=\boldsymbol{m}_{\text{load}}\boldsymbol{gv}$$

Substitute numerical values and evaluate P:

$$P=(800\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s})=18.05\,\text{kW}=\boxed{18\,\text{kW}}$$

(*b*) The daily cost of operating the elevator is given by:

$$\boldsymbol{C}_{\text{daily}}=\boldsymbol{E}_{\text{used}}\boldsymbol{c} \quad (2)$$

where c is the per unit cost of the energy.

The energy used by the motor is:

$$E_{\text{used}}=\frac{P_{\text{motor}}}{\varepsilon}\Delta t$$

where ε is the efficiency of the motor and Δt is the number of hours the elevator operates daily.

Substituting for E_{used} in equation (2) yields:

$$C_{\text{daily}}=\frac{P_{\text{motor}}\Delta tc}{\varepsilon}$$

Substitute numerical values and evaluate C_{daily}:

$$\boldsymbol{C}_{\text{daily}}=\frac{(18.05\,\text{kW})(12\,\text{h}\times 0.60)\left(\dfrac{\$0.15}{\text{kWh}}\right)}{0.85}=\boxed{\$22.93}$$

91 •• To reduce the power requirement of elevator motors, elevators are counterbalanced with weights connected to the elevator by a cable that runs over a pulley at the top of the elevator shaft. Neglect any effects of friction in the pulley. If a 1200-kg elevator that carries a maximum load of 800 kg is counterbalanced with a mass of 1500 kg, (*a*) what is the power provided by the motor when the elevator ascends fully loaded at a speed of 2.3 m/s? (*b*) How much power is provided by the motor when the elevator ascends at 2.3 m/s without a load?

Picture the Problem The power provided by a motor that is delivering sufficient energy to exert a force $\vec{F}$ on a load which it is moving at a speed $\vec{v}$ is $\vec{F}\cdot\vec{v}$ .The counterweight does negative work and the power of the motor is reduced from that required with no counterbalance.

(*a*) The power provided by the motor is given by:

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

or, because $\vec{F}$ and $\vec{v}$ are in the same direction,

$$P = Fv \qquad (1)$$

Because the elevator is counterbalanced and ascending with constant speed, the tension in the support cable(s) is:

$$F = \left(m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}}\right)g$$

Substitute for F in equation (1) to obtain:

$$P = \left(m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}}\right)gv$$

Substitute numerical values and evaluate P:

$$P = (1200\,\text{kg} + 800\,\text{kg} - 1500\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = 11.28\,\text{kW} = \boxed{11\,\text{kW}}$$

(*b*) Without a load:

$$F = \left(m_{\text{elev}} - m_{\text{cw}}\right)g$$

and

$$P = Fv = \left(m_{\text{elev}} - m_{\text{cw}}\right)gv$$

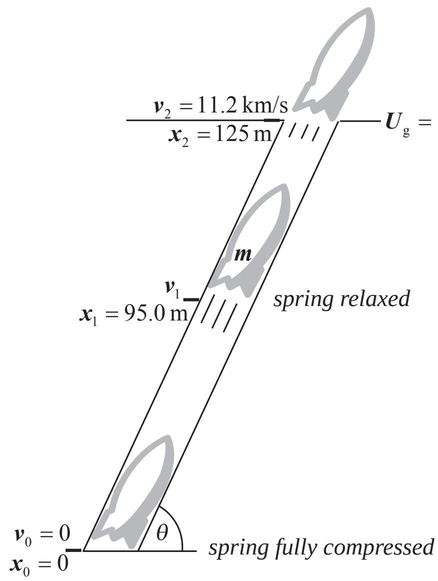
Substitute numerical values and evaluate P:

$$P = (1200\,\text{kg} - 1500\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = -6.77\,\text{kW} = \boxed{-6.8\,\text{kW}}$$

92 •• In old science fiction movies, writers attempted to come up with novel ways of launching spacecraft toward the moon. In one hypothetical case, a screenwriter envisioned launching a moon probe from a deep, smooth tunnel, inclined at 65.0° above the horizontal. At the bottom of the tunnel a very stiff spring designed to launch the craft was anchored. The top of the spring, when the spring is unstressed, is 30.0 m from the upper end of the table. The screenwriter knew from his research that to reach the moon, the 318-kg probe should have a speed of at least 11.2 km/s when it exits the tunnel. If the spring is compressed by

95.0 m just before launch, what is the minimum value for its force constant to achieve a successful launch? Neglect friction with the tunnel walls and floor.

Picture the Problem Let the system consist of the earth, spring, tunnel, and the spacecraft and the zero of gravitational potential energy be at the surface of the earth. Then there are no external forces to do work on the system and W_{ext} = 0. We can use conservation of mechanical energy to find the minimum value of the force constant that will result in a successful launch. The pictorial representation summarizes the details of the launch. Note that the spacecraft slows somewhat over the last 30 m of its launch.



(*a*) Apply conservation of mechanical energy to the spacecraft as it moves from $x = x_0$ to $x = x_2$ to obtain:

$$W_{ext} = \Delta E_{mech}$$

or, because $W_{ext} = 0$,

$$\Delta E_{mech} = 0 \qquad (1)$$

The change in the mechanical energy of the system is:

$$\begin{aligned}\Delta E_{mech} &= \Delta K + \Delta U_g + \Delta U_s \\ &= K_2 - K_0 + U_{g,2} - U_{g,0} \\ &\quad + U_{s,2} - U_{s,0}\end{aligned}$$

Because $K_0 = U_{g,2} = U_{s,2} = 0$:

$$\Delta E_{mech} = K_2 - U_{g,0} - U_{s,0}$$

Substituting for K_2, $U_{g,0}$, and $U_{s,0}$ yields:

$$\begin{aligned}\Delta E_{mech} &= \tfrac{1}{2}mv_2^2 - (-mgx_2\sin\theta) - \tfrac{1}{2}kx_1^2 \\ &= \tfrac{1}{2}mv_2^2 + mgx_2\sin\theta - \tfrac{1}{2}kx_1^2\end{aligned}$$

Substituting for ΔE_{mech} in equation (1) yields:

$$\tfrac{1}{2}mv_2^2 + mgx_2\sin\theta - \tfrac{1}{2}kx_1^2 = 0$$

Solving for k yields:

$$k = \frac{mv_2^2 + 2mgx_2\sin\theta}{x_1^2}$$

Substitute numerical values and evaluate k:

$$\boldsymbol{k} = \frac{(318\,\text{kg})(11.2\,\text{km/s})^2 + 2(318\,\text{kg})(9.81\,\text{m/s}^2)(125\,\text{m})\sin 65.0°}{(95.0\,\text{m})^2}$$

$$= \boxed{4.42\times10^3\ \text{kN/m}}$$

93 •• **[SSM]** In a volcanic eruption, a 2-kg piece of porous volcanic rock is thrown straight upward with an initial speed of 40 m/s. It travels upward a distance of 50 m before it begins to fall back to Earth. (*a*) What is the initial kinetic energy of the rock? (*b*) What is the increase in thermal energy due to air resistance during ascent? (*c*) If the increase in thermal energy due to air resistance on the way down is 70% of that on the way up, what is the speed of the rock when it returns to its initial position?

Picture the Problem Let the system consist of the earth, rock and air. Given this choice, there are no external forces to do work on the system and $W_{\text{ext}} = 0$. Choose $U_g = 0$ to be where the rock begins its upward motion. The initial kinetic energy of the rock is partially transformed into potential energy and partially dissipated by air resistance as the rock ascends. During its descent, its potential energy is partially transformed into kinetic energy and partially dissipated by air resistance.

(*a*) The initial kinetic energy of the rock is given by:

$$\boldsymbol{K}_{\text{i}} = \tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_{\text{i}}^2$$

Substitute numerical values and evaluate K_i:

$$\boldsymbol{K}_{\text{i}} = \tfrac{1}{2}(2.0\,\text{kg})(40\,\text{m/s})^2 = \boxed{1.6\,\text{kJ}}$$

(*b*) Apply the work-energy theorem with friction to relate the energies of the system as the rock ascends:

$$\Delta K + \Delta U + \Delta E_{\text{therm}} = 0$$

or, because $K_f = 0$,

$$-K_{\text{i}} + \Delta U + \Delta E_{\text{therm}} = 0$$

Solving for ΔE_{therm} yields:

$$\Delta E_{\text{therm}} = K_{\text{i}} - \Delta U$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta \boldsymbol{E}_{\text{therm}} = 1.6\,\text{kJ} - (2.0\,\text{kg})(9.81\,\text{m/s}^2)(50\,\text{m}) = 0.619\,\text{kJ} = \boxed{0.6\,\text{kJ}}$$

(*c*) Apply the work-energy theorem with friction to relate the energies of the system as the rock descends:

$$\Delta \boldsymbol{K} + \Delta \boldsymbol{U} + 0.70\Delta \boldsymbol{E}_{\text{therm}} = 0$$

Because $K_i = U_f = 0$: $$K_f - U_i + 0.70\Delta E_{therm} = 0$$

Substitute for the energies to obtain: $$\tfrac{1}{2}mv_f^2 - mgh + 0.70\Delta E_{therm} = 0$$

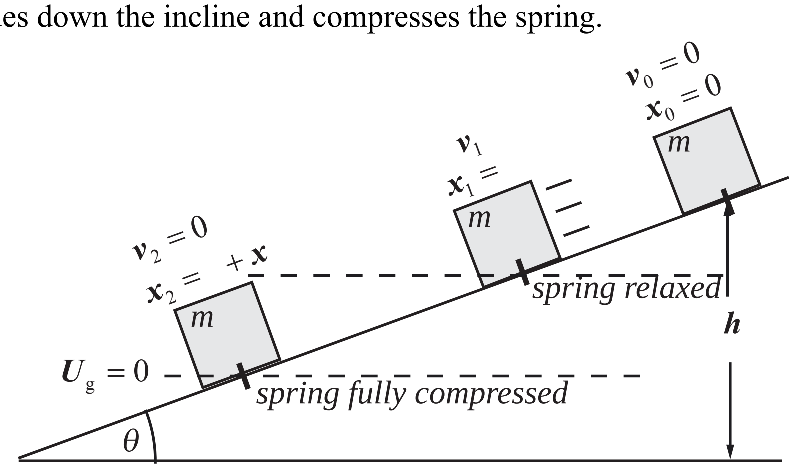
Solve for v_f to obtain: $$v_f = \sqrt{2gh - \frac{1.40\Delta E_{therm}}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}) - \frac{1.40(0.619\,\text{kJ})}{2.0\,\text{kg}}} = \boxed{23\,\text{m/s}}$$

94 •• A block of mass *m* starts from rest at a height *h* and slides down a frictionless plane inclined at angle θ with the horizontal, as shown in Figure 7-53. The block strikes a spring of force constant *k*. Find the distance the spring is compressed when the block momentarily stops.

Picture the Problem Let the distance the block slides before striking the spring be ℓ. The pictorial representation shows the block at the top of the incline ($x_0 = 0$), just as it strikes the spring ($x_1 = \ell$), and the block against the fully compressed spring ($x_2 = \ell + x$). Let the block, spring, and the earth comprise the system. Then $W_{ext} = 0$. Let $U_g = 0$ where the spring is at maximum compression. We can apply the work-energy theorem to the block to relate the energies of the system as the block slides down the incline and compresses the spring.



Apply the work-energy theorem to the block from x_0 to x_2:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$\Delta K + U_{g,2} - U_{g,0} + U_{s,2} - U_{s,0} = 0$$

Because $\Delta K = U_{g,2} = U_{s,0} = 0$: $$-U_{g,0} + U_{s,2} = 0$$

Substitute for each of these energy terms to obtain: $$-mgh_0 + \tfrac{1}{2}kx^2 = 0 \quad (1)$$
where x is the distance the spring compresses.

h_0 is given by: $$h_0 = x_2 \sin\theta = (\ell + x)\sin\theta$$

Substitute for h_0 in equation (1) to obtain: $$-mg(\ell + x)\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Rewrite this equation explicitly as a quadratic equation to obtain: $$x^2 - \frac{2mg\sin\theta}{k}x - \frac{2mg\ell\sin\theta}{k} = 0$$

Solving for x yields:

$$x = \boxed{\frac{mg}{k}\sin\theta + \sqrt{\left(\frac{mg}{k}\right)^2 \sin^2\theta + \frac{2mg\ell}{k}\sin\theta}}$$

Note that the negative sign between the two terms leads to a non-physical solution and has been ignored.

95 •• **[SSM]** A block of mass m is suspended from a wall bracket by a spring and is free to move vertically (Figure 7-54). The $+y$ direction is downward and the origin is at the position of the block when the spring is unstressed. (*a*) Show that the potential energy as a function of position may be expressed as $U = \frac{1}{2}ky^2 - mgy$, (*b*) Using a **spreadsheet** program or graphing calculator, make a graph of U as a function of y with $k = 2$ N/m and $mg = 1$ N. (*c*) Explain how this graph shows that there is a position of stable equilibrium for a positive value of y. Using the Part (*a*) expression for U, determine (symbolically) the value of y when the block is at its equilibrium position. (*d*) From the expression for U, find the net force acting on m at any position y. (*e*) The block is released from rest with the spring unstressed; if there is no friction, what is the maximum value of y that will be reached by the mass? Indicate y_{max} on your graph/spreadsheet.

Picture the Problem Given the potential energy function as a function of y, we can find the net force acting on a given system from $F = -dU/dy$. The maximum extension of the spring; that is, the lowest position of the mass on its end, can be found by applying the work-energy theorem. The equilibrium position of the system can be found by applying the work-energy theorem with friction … as can the amount of thermal energy produced as the system oscillates to its equilibrium position. In Part (*c*), setting dU/dy equal to zero and solving the resulting equation for y will yield the value of y when the block is in its equilibrium position (*a*) The potential energy of the oscillator is the sum of the gravitational potential energy of block and the energy stored in the stretched spring:

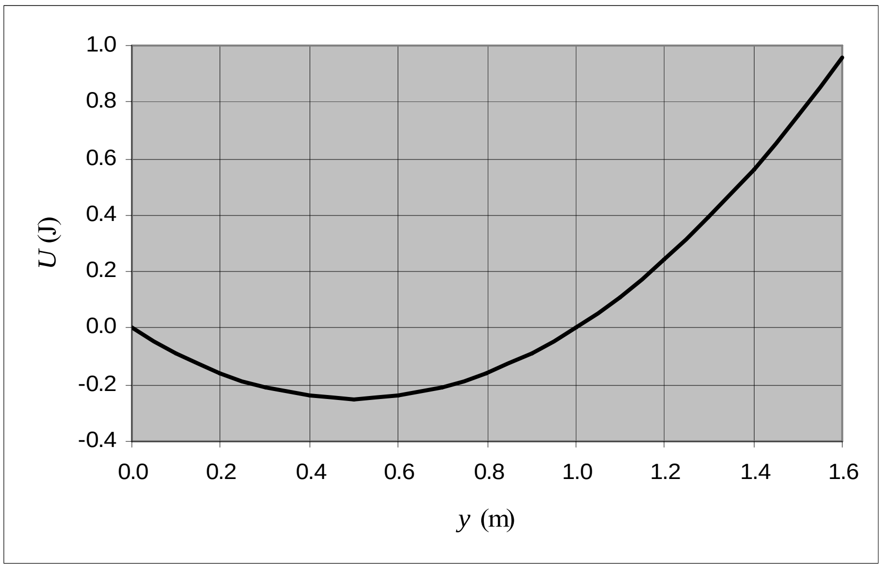
$$\boldsymbol{U} = \boldsymbol{U}_{\mathrm{g}} + \boldsymbol{U}_{\mathrm{s}}$$

Letting the zero of gravitational potential energy be at the oscillator's equilibrium position yields:

$$\boldsymbol{U} = \boxed{\tfrac{1}{2}\boldsymbol{k}\boldsymbol{y}^2 - \boldsymbol{mgy}}$$

where *y* is the distance the spring is stretched.

(*b*) A graph of *U* as a function of *y* follows. Because *k* and *m* are not specified, *k* has been set equal to 2 and *mg* to 1.



(*c*) The fact that *U* is a minimum near *y* = 0.5 m tells us that this is a position of stable equilibrium.

Differentiate *U* with respect to *y* to obtain:

$$\frac{dU}{dy} = \frac{d}{dy}\left(\tfrac{1}{2}ky^2 - mgy\right) = ky - mg$$

Setting this expression equal to zero for extrema yields:

$$ky - mg = 0 \Rightarrow y = \boxed{\frac{mg}{k}}$$

(*d*) Evaluate the negative of the derivative of *U* with respect to *y*:

$$F = -\frac{dU}{dy} = -\frac{d}{dy}\left(\tfrac{1}{2}ky^2 - mgy\right)$$

$$= \boxed{-ky + mg}$$

(*e*) Apply conservation of energy to the movement of the mass from $y = 0$ to $y = y_{max}$:

$$\Delta K + \Delta U + \Delta E_{therm} = 0$$

Because $\Delta K = 0$ (the object starts from rest and is momentarily at rest at $\boldsymbol{y} = \boldsymbol{y}_{max}$) and (no friction), it follows that:

$$\Delta \boldsymbol{U} = \boldsymbol{U}(\boldsymbol{y}_{max}) - \boldsymbol{U}(0) = 0$$

Because $U(0) = 0$:

$$\boldsymbol{U}(\boldsymbol{y}_{max}) = 0 \Rightarrow \tfrac{1}{2}\boldsymbol{k}\boldsymbol{y}_{max}^2 - \boldsymbol{mgy}_{max} = 0$$

Solve for y_{max} to obtain:

$$y_{max} = \boxed{\frac{2mg}{k}}$$

On the graph, y_{max} is at (1.0, 0.0).

96 •• A spring-loaded gun is cocked by compressing a short, strong spring by a distance *d*. It fires a signal flare of mass *m* directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height *h* above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of *m*, v_0, *d*, *h*, and *g*.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant *k*? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated into thermal energy?

Picture the Problem The energy stored in the compressed spring is initially transformed into the kinetic energy of the signal flare and then into gravitational potential energy and thermal energy as the flare climbs to its maximum height. Let the system contain the earth, the air, and the flare so that $W_{ext} = 0$. We can use the work-energy theorem with friction in the analysis of the energy transformations during the motion of the flare.

(*a*) The work done on the spring in compressing it is equal to the kinetic energy of the flare at launch:

$$\boldsymbol{W}_s = \boldsymbol{K}_{i,\,flare} = \boxed{\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_0^2}$$

(*b*) Ignoring changes in gravitational potential energy (that is, assume that the compression of the spring is small compared to the maximum elevation of the flare), apply the conservation of mechanical energy to the transformation that takes place as the spring decompresses and gives the flare its launch speed:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{s,f} - U_{s,i} = 0$$

Because $K_i = U_{s,f} = 0$:

$$K_f - U_{s,i} = 0$$

Substitute for K_f and $U_{s,i}$ to obtain:

$$\tfrac{1}{2}mv_0^2 - \tfrac{1}{2}kd^2 = 0 \Rightarrow k = \boxed{\frac{mv_0^2}{d^2}}$$

(c) Apply the work-energy theorem with friction to the upward trajectory of the flare:

$$\Delta K + \Delta U_g + \Delta E_{therm} = 0$$

Solve for ΔE_{therm}:

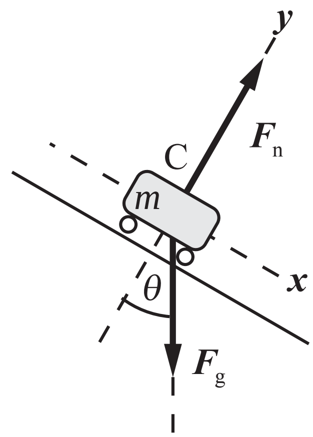
$$\Delta E_{therm} = -\Delta K - \Delta U_g$$
$$= K_i - K_f + U_i - U_f$$

Because $K_f = U_i = 0$:

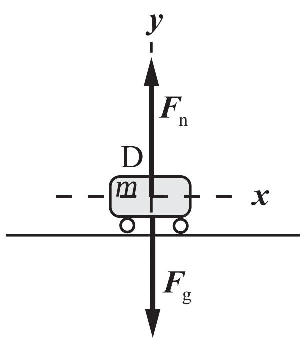
$$\Delta E_{therm} = \boxed{\tfrac{1}{2}mv_0^2 - mgh}$$

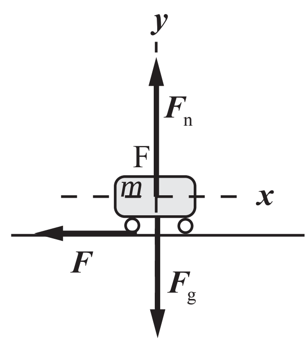
97 •• Your firm is designing a new roller-coaster ride. The permit process requires the calculation of forces and accelerations at various important locations on the ride. Each roller-coaster car will have a total mass (including passengers) of 500 kg and will travel freely along the winding frictionless track shown in Figure 7-55. Points A, E, and G are on horizontal straight sections, all at the same height of 10 m above the ground. Point C is at a height of 10 m above the ground on an inclined section of slope angle 30°. Point B is at the crest of a hill, while point D is at ground level at the bottom of a valley; the radius of curvature at both of these points is 20 m. Point F is at the middle of a banked horizontal curve with a radius of curvature of 30 m, and at the same height as points A, E, and G. At point A the speed of the car is 12 m/s. (*a*) If the car is just barely to make it over the hill at point B, what must be the height of point B above the ground? (*b*) If the car is to just barely make it over the hill at point B, what should be the magnitude of the force exerted by the track on the car at that point? (*c*) What will be the acceleration of the car at point C? (*d*) What will be the magnitude and direction of the force exerted by the track on the car at point D? (*e*) What will be the magnitude and direction of the force exerted by the track on the car at point F? (*f*) At point G a constant braking force is to be applied to the car, bringing it to a halt in a distance of 25 m. What is the magnitude of this required braking force?

Picture the Problem Let $U_D = 0$. Choose the system to include the earth, the track, and the car. Then there are no external forces to do work on the system and change its energy and we can use Newton's 2nd law and the work-energy theorem to describe the system's energy transformations to point G … and then the work-energy theorem with friction to determine the braking force that brings the car to a stop. The free-body diagram for point C is shown above.



The free-body diagrams for the rollercoaster cars at points D and F are shown below.





(*a*) Apply the work-energy theorem to the system's energy transformations between A and B:

$$\Delta K + \Delta U = 0$$

or

$$K_B - K_A + U_B - U_A = 0$$

If we assume that the car arrives at point B with $v_B = 0$, then:

$$-\tfrac{1}{2}mv_A^2 + mg\Delta h = 0 \Rightarrow \Delta \boldsymbol{h} = \frac{\boldsymbol{v}_A^2}{2\boldsymbol{g}}$$

where Δh is the difference in elevation between A and B.

The height above the ground is given by:

$$h + \Delta h = h + \frac{v_A^2}{2g}$$

Substitute numerical values and evaluate $h + \Delta h$:

$$\boldsymbol{h}+\Delta\boldsymbol{h}=10\,\text{m}+\frac{(12\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}=17.3\,\text{m}$$
$$=\boxed{17\,\text{m}}$$

(*b*) If the car just makes it to point B; i.e., if it gets there with $v_B = 0$, then the force exerted by the track on the car will be the normal force:

$$\boldsymbol{F}_{\text{track on car}}=\boldsymbol{F}_{\text{n}}=\boldsymbol{mg}$$

Substitute numerical values and evaluate $F_{\text{track on car}}$:

$$F_{\text{track on car}}=(500\,\text{kg})(9.81\,\text{m/s}^2)$$
$$=\boxed{4.91\,\text{kN}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car at point C (see the FBD) and solve for a:

$$mg\sin\theta=ma\Rightarrow a=g\sin\theta$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a}=(9.81\,\text{m/s}^2)\sin30^\circ=\boxed{4.9\,\text{m/s}^2}$$

(*d*) Apply $\sum F_y = ma_y$ to the car at point D (see the FBD) and solve for F_n:

$$F_{\text{n}}-mg=m\frac{v_{\text{D}}^2}{R}\Rightarrow F_{\text{n}}=mg+m\frac{v_{\text{D}}^2}{\text{R}}$$

Apply the work-energy theorem to the system's energy transformations between B and D:

$$\Delta K+\Delta U=0$$
or
$$K_{\text{D}}-K_{\text{B}}+U_{\text{D}}-U_{\text{B}}=0$$

Because $K_B = U_D = 0$:

$$K_{\text{D}}-U_{\text{B}}=0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_{\text{D}}^2-mg(h+\Delta h)=0$$

Solving for v_{D}^2 yields:

$$v_{\text{D}}^2=2g(h+\Delta h)$$

Substitute for v_{D}^2 in the expression for F_n and simplify to obtain:

$$F_{\text{n}}=mg+m\frac{v_{\text{D}}^2}{R}=mg+m\frac{2g(h+\Delta h)}{R}$$
$$=mg\left[1+\frac{2(h+\Delta h)}{R}\right]$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_n = (500\,\text{kg})(9.81\,\text{m/s}^2)\left[1 + \frac{2(17.3\,\text{m})}{20\,\text{m}}\right]$$
$$= \boxed{13\,\text{kN, directed upward.}}$$

(*e*) $\vec{\boldsymbol{F}}$ has two components at point F; one horizontal (the inward force that the track exerts) and the other vertical (the normal force). Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the car at point F:

$$\sum F_y = F_n - mg = 0 \Rightarrow F_n = mg$$

and

$$\sum F_x = F_c = m\frac{v_F^2}{R}$$

Express the resultant of these two forces:

$$F = \sqrt{F_c^2 + F_n^2} = \sqrt{\left(m\frac{v_F^2}{R}\right)^2 + (mg)^2}$$
$$= m\sqrt{\frac{v_F^4}{R^2} + g^2}$$

Substitute numerical values and evaluate F:

$$\boldsymbol{F} = (500\,\text{kg})\sqrt{\frac{(12\,\text{m/s})^4}{(30\,\text{m})^2} + (9.81\,\text{m/s}^2)^2}$$
$$= \boxed{5.5\,\text{kN}}$$

The angle the resultant makes with the x axis is given by:

$$\theta = \tan^{-1}\left(\frac{F_n}{F_c}\right) = \tan^{-1}\left(\frac{gR}{v_F^2}\right)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{(9.81\,\text{m/s}^2)(30\,\text{m})}{(12\,\text{m/s})^2}\right] = 63.9°$$
$$= \boxed{64°}$$

(*f*) Apply the work-energy theorem with friction to the system's energy transformations between F and the car's stopping position:

$$-K_G + \Delta E_{\text{therm}} = 0$$

and

$$\Delta E_{\text{therm}} = K_G = \tfrac{1}{2}mv_G^2$$

The work done by friction is also given by:

$$\Delta E_{\text{therm}} = f\Delta s = F_{\text{brake}}d$$

where d is the stopping distance.

Equate the two expressions for ΔE_{therm} and solve for F_{brake}:

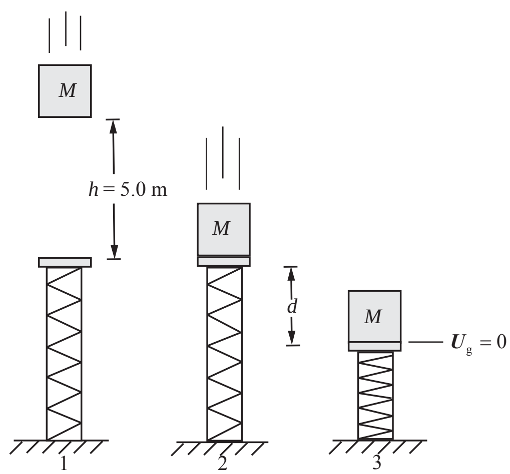
$$F_{\text{brake}} = \frac{mv_F^2}{2d}$$

Substitute numerical values and evaluate F_{brake}:

$$\boldsymbol{F}_{\text{brake}} = \frac{(500\,\text{kg})(12\,\text{m/s})^2}{2(25\,\text{m})} = \boxed{1.4\,\text{kN}}$$

98 •• The cable of a 2000-kg elevator has broken, and the elevator is moving downward at a steady speed of 1.5 m/s. A safety braking system that works on friction prevents the downward speed from increasing. (*a*) At what rate is the braking system converting mechanical energy to thermal energy? (*b*) While the elevator is moving downward at 1.5 m/s, the braking system fails and the elevator is in free-fall for a distance of 5.0 m before hitting the top of a large safety spring with force constant of 1.5×10^4 N/m. After the elevator hits the top of the spring, find the distance *d* that the spring is compressed before the elevator is brought to rest.

Picture the Problem The rate of conversion of mechanical energy can be determined from $P = \vec{F} \cdot \vec{v}$. The pictorial representation shows the elevator moving downward just as it goes into freefall as state 1. In state 2 the elevator is moving faster and is about to strike the relaxed spring. The momentarily at rest elevator on the compressed spring is shown as state 3. Let U_g = 0 where the spring has its maximum compression and the system consist of the earth, the elevator, and the spring. Then W_{ext} = 0 and we can apply conservation of mechanical energy to the analysis of the falling elevator and compressing spring.



(*a*) Express the rate of conversion of mechanical energy to thermal energy as a function of the speed of the elevator and braking force acting on it:

$$P = F_{\text{braking}} v_0$$

Because the elevator is moving with constant speed, the net force acting on it is zero and:

$$F_{\text{braking}} = Mg$$

Substitute for F_{braking} to obtain:

$$P = Mgv_0$$

Substitute numerical values and evaluate P:

$$P = (2000\,\text{kg})(9.81\,\text{m/s}^2)(1.5\,\text{m/s}) = \boxed{29\,\text{kW}}$$

(*b*) Apply the conservation of mechanical energy to the falling elevator and compressing spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = U_{g,3} = U_{s,1} = 0$:

$$-\tfrac{1}{2}Mv_0^2 - Mg(h+d) + \tfrac{1}{2}kd^2 = 0$$

Rewrite this equation as a quadratic equation in d, the maximum compression of the spring:

$$d^2 - \left(\frac{2Mg}{k}\right)d - \frac{M}{k}\left(2gh + v_0^2\right) = 0$$

Solve for d to obtain:

$$d = \frac{Mg}{k} \pm \sqrt{\frac{M^2g^2}{k^2} + \frac{M}{k}\left(2gh + v_0^2\right)}$$

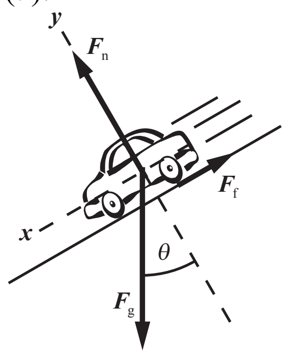
Substitute numerical values and evaluate d:

$$d = \frac{(2000\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\times10^4\ \text{N/m}} + \sqrt{\frac{(2000\,\text{kg})^2(9.81\,\text{m/s}^2)^2}{(1.5\times10^4\ \text{N/m})^2} + \frac{2000\,\text{kg}}{1.5\times10^4\ \text{N/m}}\left[2(9.81\,\text{m/s}^2)(5.0\,\text{m}) + (1.5\,\text{m/s})^2\right]}$$
$$= \boxed{5.2\,\text{m}}$$

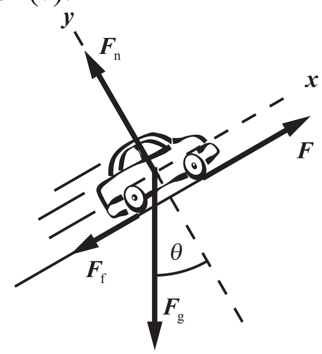
99 ••• **[SSM]** To measure the combined force of friction (rolling friction plus air drag) on a moving car, an automotive engineering team you are on turns off the engine and allows the car to coast down hills of known steepness. The team collects the following data: (1) On a 2.87° hill, the car can coast at a steady 20 m/s. (2) On a 5.74° hill, the steady coasting speed is 30 m/s. The total mass of the car is 1000 kg. (*a*) What is the magnitude of the combined force of friction at 20 m/s (F_{20}) and at 30 m/s (F_{30})? (*b*) How much power must the engine deliver to drive the car on a level road at steady speeds of 20 m/s (P_{20}) and 30 m/s (P_{30})? (*c*) The maximum power the engine can deliver is 40 kW. What is the angle of the steepest incline up which the car can maintain a steady 20 m/s? (*d*) Assume that the engine delivers the same total useful work from each liter of gas, no matter what the speed. At 20 m/s on a level road, the car gets 12.7 km/L. How many kilometers per liter does it get if it goes 30 m/s instead?

Picture the Problem We can use Newton's 2nd law to determine the force of friction as a function of the angle of the hill for a given constant speed. The power output of the engine is given by $P = \vec{F}_f \cdot \vec{v}$.

FBD for (*a*):



FBD for (*c*):



(*a*) Apply $\sum F_x = ma_x$ to the car:

$$\boldsymbol{mg}\sin\boldsymbol{\theta} - \boldsymbol{F} = 0 \Rightarrow \boldsymbol{F} = \boldsymbol{mg}\sin\boldsymbol{\theta}$$

Evaluate F for the two speeds:

$$\boldsymbol{F}_{20} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin(2.87°) = \boxed{491\,\text{N}}$$

and

$$\boldsymbol{F}_{30} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin(5.74°) = \boxed{981\,\text{N}}$$

(*b*) The power an engine must deliver on a level road in order to overcome friction loss is given by:

$$\boldsymbol{P} = \boldsymbol{F}_f\boldsymbol{v}$$

Evaluate this expression for v = 20 m/s and 30 m/s:

$$\boldsymbol{P}_{20} = (491\,\text{N})(20\,\text{m/s}) = \boxed{9.8\,\text{kW}}$$

and

$$\boldsymbol{P}_{30} = (981\,\text{N})(30\,\text{m/s}) = \boxed{29\,\text{kW}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car:

$$\sum \boldsymbol{F}_x = \boldsymbol{F} - \boldsymbol{mg}\sin\boldsymbol{\theta} - \boldsymbol{F}_f = 0$$

Solving for F yields:

$$\boldsymbol{F} = \boldsymbol{mg}\sin\boldsymbol{\theta} + \boldsymbol{F}_f$$

Relate F to the power output of the engine and the speed of the car:

$$\boldsymbol{F} = \frac{\boldsymbol{P}}{\boldsymbol{v}}$$

Equate these expressions for F to obtain:

$$\frac{\boldsymbol{P}}{\boldsymbol{v}} = \boldsymbol{mg}\sin\boldsymbol{\theta} + \boldsymbol{F}_{\mathrm{f}}$$

Solving for θ yields:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{\frac{\boldsymbol{P}}{\boldsymbol{v}} - \boldsymbol{F}_{\mathrm{f}}}{\boldsymbol{mg}}\right]$$

Substitute numerical values and evaluate θ for $\boldsymbol{F}_{\mathrm{f}} = \boldsymbol{F}_{20}$:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{\frac{40\,\mathrm{kW}}{20\,\mathrm{m/s}} - 491\,\mathrm{N}}{(1000\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}\right] = \boxed{8.8°}$$

(*d*) Express the equivalence of the work done by the engine in driving the car at the two speeds:

$$W_{\mathrm{engine}} = F_{20}(\Delta s)_{20} = F_{30}(\Delta s)_{30}$$

Let ΔV represent the volume of fuel consumed by the engine driving the car on a level road and divide both sides of the work equation by ΔV to obtain:

$$F_{20}\frac{(\Delta s)_{20}}{\Delta V} = F_{30}\frac{(\Delta s)_{30}}{\Delta V}$$

Solve for $\frac{(\Delta s)_{30}}{\Delta V}$:

$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{F_{20}}{F_{30}}\frac{(\Delta s)_{20}}{\Delta V}$$

Substitute numerical values and evaluate $\frac{(\Delta s)_{30}}{\Delta V}$:

$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{491\,\mathrm{N}}{981\,\mathrm{N}}(12.7\,\mathrm{km/L}) = \boxed{6.36\,\mathrm{km/L}}$$

100 •• (*a*) Calculate the kinetic energy of a 1200-kg car moving at 50 km/h. (*b*) If friction (rolling friction and air drag) results in a retarding force of 300 N at a speed of 50 km/h, what is the minimum energy needed to move the car a distance of 300 m at a constant speed of 50 km/h?

Picture the Problem While on a horizontal surface, the work done by an automobile engine changes the kinetic energy of the car and does work against friction. These energy transformations are described by the work-energy theorem with friction. Let the system include the earth, the roadway, and the car *but not the car's engine*.

(*a*) The kinetic energy of the car is:

$$K = \tfrac{1}{2}(1200\,\text{kg})\left(50\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2$$
$$= \boxed{0.12\,\text{MJ}}$$

(*b*) The required energy equals the energy dissipated by friction:

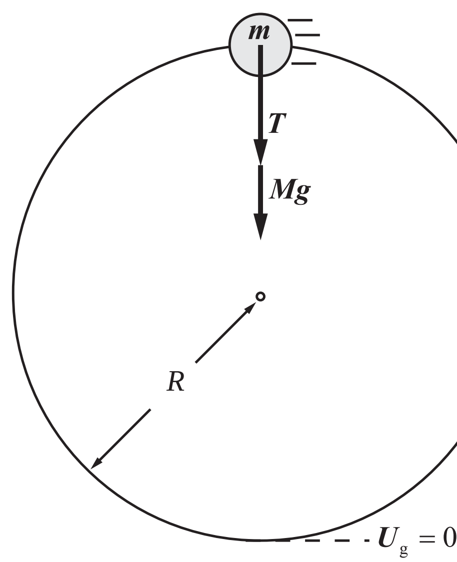
$$\Delta E_{\text{therm}} = f\Delta s$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\Delta E_{\text{therm}} = (300\,\text{N})(300\,\text{m}) = \boxed{90.0\,\text{kJ}}$$

101 ••• A pendulum consists of a string of length L with a small bob of mass m. The bob is held to the side with the string horizontal (see Figure 7-56). Then the bob is released from rest. At the lowest point of the swing, the string catches on a thin peg a distance R above the lowest point. Show that R must be smaller than $2L/5$ if the string is to remain taut as the bob swings around the peg in a full circle.

Picture the Problem Assume that the bob is moving with speed v as it passes the top vertical point when looping around the peg. There are two forces acting on the bob: the tension in the string (if any) and the force of gravity, Mg; both point downward when the ball is in the topmost position. The minimum possible speed for the bob to pass the vertical occurs when the tension is 0; from this, gravity must supply the centripetal force required to keep the ball moving in a circle. We can use conservation of mechanical energy to relate v to L and R.



Express the condition that the bob swings around the peg in a full circle:

$$M\frac{v^2}{R} > Mg \Rightarrow \frac{v^2}{R} > g \qquad (1)$$

Use conservation of mechanical energy to relate the kinetic energy of the bob at the bottom of the loop to its potential energy at the top of its swing:

$$\tfrac{1}{2}Mv^2 = Mg(L - 2R)$$

Solving for v^2 yields:

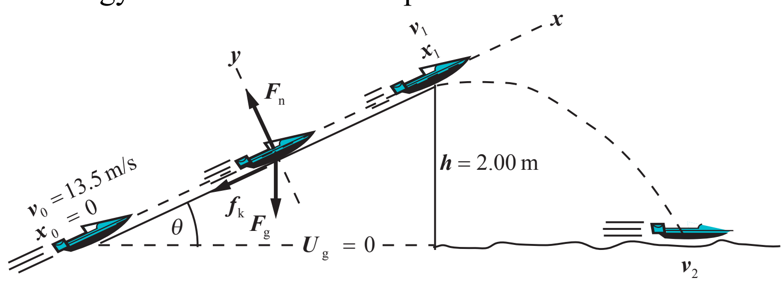
$$v^2 = 2g(L - 2R)$$

Substitute for v^2 in equation (1) to obtain:

$$\frac{2g(L-2R)}{R} > g \Rightarrow R < \boxed{\frac{2}{5}L}$$

102 •• A 285-kg stunt boat is driven on the surface of a lake at a constant speed of 13.5 m/s toward a ramp, which is angled at 25.0° above the horizontal. The coefficient of friction between the boat bottom and the ramp's surface is 0.150, and the raised end of the ramp is 2.00 m above the water surface. (*a*) Assuming the engines are cut off when the boat hits the ramp, what is the speed of the boat as it leaves the ramp? (*b*) What is the speed of the boat when it strikes the water again? Neglect any effects due to air resistance.

Picture the Problem The pictorial representation summarizes the details of the problem. Let the system consist of the earth, the boat, and the ramp. Then no external forces do work on the system. We can use the work-energy theorem for problems with kinetic friction to find the speed of the boat at the top of the ramp and the work-energy theorem to find the speed of the boat when it hits the water.



(*a*) Apply the work-energy theorem to the boat as it slides up the ramp to obtain:

$$\boldsymbol{W}_{\text{ext}} = \Delta\boldsymbol{E}_{\text{mech}} + \Delta\boldsymbol{E}_{\text{therm}}$$

or, because $W_{\text{ext}} = 0$,

$$\Delta E_{\text{mech}} + \Delta E_{\text{therm}} = 0 \qquad (1)$$

ΔE_{mech} is given by:

$$\Delta\boldsymbol{E}_{\text{mech}} = \Delta\boldsymbol{K} + \Delta\boldsymbol{U}_{\text{g}}$$

$$= \boldsymbol{K}_1 - \boldsymbol{K}_0 + \boldsymbol{U}_{\text{g},1} - \boldsymbol{U}_{\text{g},0}$$

or, because $U_{\text{g},0} = 0$,

$$\Delta\boldsymbol{E}_{\text{mech}} = \boldsymbol{K}_1 - \boldsymbol{K}_0 + \boldsymbol{U}_{\text{g},1}$$

Substituting for K_1, K_0, and $U_{g,1}$ yields:

$$\Delta E_{mech} = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh$$

ΔE_{therm} is given by:

$$\Delta E_{therm} = f_k x_1 = \mu_k F_n x_1$$

Because $F_n = mg\cos\theta$:

$$\Delta E_{therm} = \mu_k mgx_1\cos\theta$$

Substituting for ΔE_{mech} and ΔE_{therm} in equation (1) yields:

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh + \mu_k mgx_1\cos\theta = 0$$

Referring to the pictorial representation, express x_1 in terms of h to obtain:

$$x_1 = \frac{h}{\sin\theta}$$

Substituting for x_1 yields:

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh + \frac{\mu_k mgh\cos\theta}{\sin\theta} = 0$$

Solve for v_1 to obtain:

$$v_1\sqrt{v_0^2 - 2gh(1+\mu_k\cot\theta)}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{(13.5\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(2.00\text{ m})[1+(0.150)\cot(25.0°)]} = 11.42\text{ m/s}$$
$$= \boxed{11.4\text{ m/s}}$$

(*b*) Apply the work-kinetic energy theorem to the boat while it is airborne:

$$W_{ext} = \Delta K + \Delta U_g$$

or, because $W_{ext} = 0$,

$$\Delta K + \Delta U_g = 0$$

Substitute for ΔK and ΔU_g to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - mgh = 0$$

Solving for v_2 yields:

$$v_2 = \sqrt{v_1^2 + 2gh}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{(11.42\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(2.00\text{ m})} = \boxed{13.0\text{ m/s}}$$

103 •• A standard introductory-physics lab-experiment to examine the conservation of energy and Newton’s laws is shown in Figure 7-57. A glider is set up on a linear air track and is attached by a string over a massless-frictionless pulley to a hanging weight. The mass of the glider is *M*, while the mass of the hanging weight is *m*. When the air supply to the air track is turned on, the track becomes essentially frictionless. You then release the hanging weight and measure the speed of the glider after the weight has fallen a given distance (y). To show that the measured speed is the speed predicted by theory; (*a*) apply conservation of mechanical energy and calculate the speed as a function of y. To verify this calculation; (*b*) apply Newton's second and third laws directly by sketching a free-body diagram for each of the two masses and applying Newton's laws to find their accelerations. Then use kinematics to calculate the speed of the glider as a function of y.

Picture the Problem For Part (*a*), we'll let the system include the glider, track, weight, and the earth. The speeds of the glider and the falling weight will be the same while they are in motion. Let their common speed when they have moved a distance Y be v and let the zero of potential energy be at the elevation of the weight when it has fallen the distance Y. We can use conservation of mechanical energy to relate the speed of the glider (and the weight) to the distance the weight has fallen. In Part (*b*), we'll let the direction of motion be the x direction, the tension in the connecting string be T, and apply Newton's 2nd law to the glider and the weight to find their common acceleration. Because this acceleration is constant, we can use a constant-acceleration equation to find their common speed when they have moved a distance Y.

(*a*) Apply the work-energy theorem to the system to obtain:

$$\boldsymbol{W}_{\text{ext}} = \Delta \boldsymbol{K} + \Delta \boldsymbol{U} = 0$$

or, because $W_{\text{ext}} = 0$,

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

Because the system starts from rest and $U_{\text{f}} = 0$:

$$K_{\text{f}} - U_{\text{i}} = 0$$

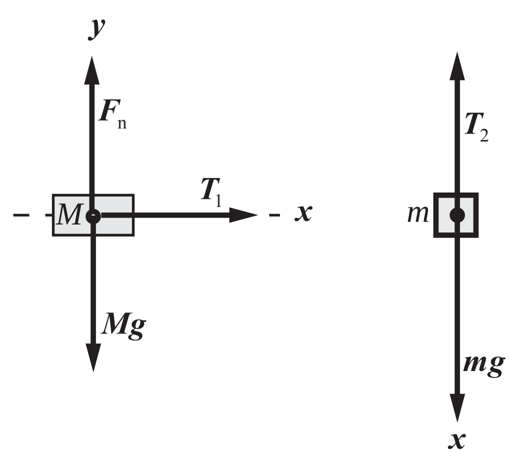
Substitute for K_{f} and U_{i} to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2 - mgY = 0$$

Solving for v yields:

$$v = \boxed{\sqrt{\frac{2mgY}{M+m}}}$$

(*b*) The free-body diagrams for the glider and the weight are shown to the right:



Apply Newton's 3rd law to obtain: $\left|\vec{T}_1\right| = \left|\vec{T}_2\right| = T$

Apply $\sum F_x = ma$ to the glider: $T = Ma$

Apply $\sum F_x = ma$ to the hanging weight: $mg - T = ma$

Add these equations to eliminate T and obtain: $mg = Ma + ma \Rightarrow a = g\dfrac{m}{m+M}$

Using a constant-acceleration equation, relate the speed of the glider to its initial speed and to the distance that the weight has fallen: $v^2 = v_0^2 + 2aY$
or, because $v_0 = 0$,
$v^2 = 2aY$

Substitute for a and solve for v to obtain: $v = \boxed{\sqrt{\dfrac{2mgY}{M+m}}}$, the same result we obtained in Part (a).

104 •• In one model of a person jogging, the energy expended is assumed to go into accelerating and decelerating the feet and the lower portions of the legs. If the jogging speed is v then the maximum speed of the foot and lower leg is about $2v$. (From the moment a foot leaves the ground, to the moment it next contacts the ground, the foot travels nearly twice as far as the torso, so it must be going, on average, nearly twice as fast as the torso.) If the mass of the foot and lower portion of a leg is m, the energy needed to accelerate the foot and lower portion of a leg from rest to speed $2v$ is $\frac{1}{2}m(2v)^2 = 2mv^2$, and the same energy is needed to decelerate this mass back to rest for the next stride. Assume that the mass of the foot and lower portion of a man's leg is 5.0 kg and that he jogs at a speed of 3.0 m/s with 1.0 m between one footfall and the next. The energy he must provide to each leg in each 2.0 m of travel is $2mv^2$, so the energy he must provide to both legs during each second of jogging is $6mv^2$. Calculate the rate of the man's energy expenditure using this model, assuming that his muscles have an efficiency of 20 percent.

Picture the Problem We're given $P = dW/dt$ and are asked to evaluate it under the assumed conditions.

We're given that the rate of energy expenditure by the man is: $\boldsymbol{P} = 6\boldsymbol{mv}^2$

Substitute numerical values and evaluate P: $\boldsymbol{P} = 6(10\,\text{kg})(3.0\,\text{m/s})^2 = 540\,\text{W}$

Express the rate of energy expenditure P' assuming that his muscles have an efficiency of 20%:

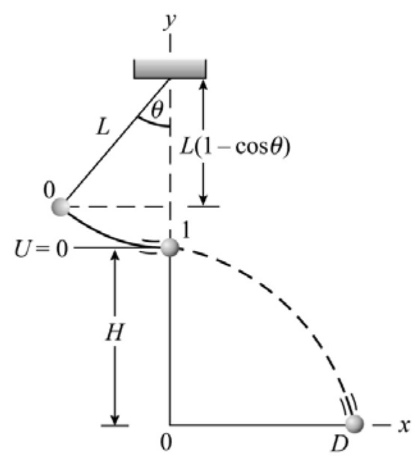
$$P = \tfrac{1}{5}P' \Rightarrow \boldsymbol{P' = 5P}$$

Substitute numerical values and evaluate P':

$$\boldsymbol{P'} = 5(540\,\text{W}) = \boxed{2.7\,\text{kW}}$$

105 •• **[SSM]** A high school teacher once suggested measuring the magnitude of free-fall acceleration by the following method: Hang a mass on a very fine thread (length L) to make a pendulum with the mass a height H above the floor when at its lowest point P. Pull the pendulum back so that the thread makes an angle θ_0 with the vertical. Just above point P, place a razor blade that is positioned to cut through the thread as the mass swings through point P. Once the thread is cut, the mass is projected horizontally, and hits the floor a horizontal distance D from point P. The idea was that the measurement of D as a function of θ_0 should somehow determine g. Apart from some obvious experimental difficulties, the experiment had one fatal flaw: D does not depend on g! Show that this is true, and that D depends only on the angle θ_0.

Picture the Problem The pictorial representation shows the bob swinging through an angle θ before the thread is cut and the ball is launched horizontally. Let its speed at position 1 be v. We can use conservation of mechanical energy to relate v to the change in the potential energy of the bob as it swings through the angle θ. We can find its flight time Δt from a constant-acceleration equation and then express D as the product of v and Δt.



Relate the distance D traveled horizontally by the bob to its launch speed v and time of flight Δt:

$$D = v\Delta t \qquad (1)$$

Use conservation of mechanical energy to relate its launch speed v to the length of the pendulum L and the angle θ:

$$K_1 - K_0 + U_1 - U_0 = 0$$
or, because $U_1 = K_0 = 0$,
$$K_1 - U_0 = 0$$

Substitute for K_1 and U_0 to obtain:

$$\tfrac{1}{2}mv^2 - mgL(1-\cos\theta) = 0$$

Solving for v yields:

$$v = \sqrt{2gL(1-\cos\theta)}$$

In the absence of air resistance, the horizontal and vertical motions of the bob are independent of each other and we can use a constant-acceleration equation to express the time of flight (the time to fall a distance H):

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $\Delta y = -H$, $a_y = -g$, and $v_{0y} = 0$,

$$-H = -\tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{2H/g}$$

Substitute in equation (1) and simplify to obtain:

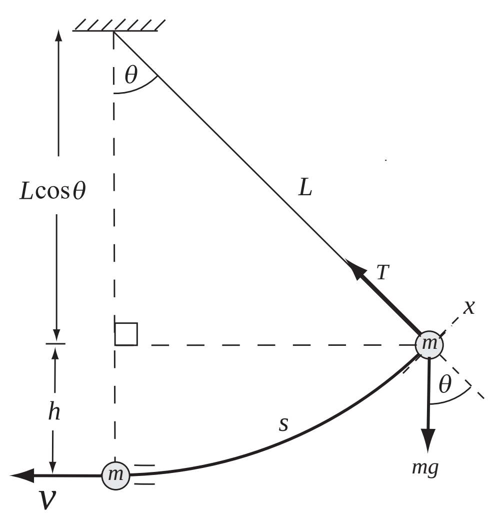
$$D = \sqrt{2gL(1-\cos\theta)}\sqrt{\frac{2H}{g}}$$

$$= \boxed{2\sqrt{HL(1-\cos\theta)}}$$

which shows that, while D depends on θ, it is independent of g.

106 ••• The bob of a pendulum of length L is pulled aside so that the string makes an angle θ_0 with the vertical, and the bob is then released. In Example 7-5, the conservation of energy was used to obtain the speed of the bob at the bottom of its swing. In this problem, you are to obtain the same result using Newton's second law. (*a*) Show that the tangential component of Newton's second law gives $dv/dt = -g\sin\theta$, where v is the speed and θ is the angle between the string and the vertical. (*b*) Show that v can be written $v = L\,d\theta/dt$. (*c*) Use this result and the chain rule for derivatives to obtain $\frac{dv}{dt} = \frac{dv}{d\theta}\frac{v}{L}$. (*d*) Combine the results of Parts (*a*) and (*c*) to obtain $v\,dv = -gL\sin\theta\,d\theta$. (*e*) Integrate the left side of the equation in Part (*d*) from $v = 0$ to the final speed v and the right side from $\theta = \theta_0$ to $\theta = 0$, and show that the result is equivalent to $v = \sqrt{2gh}$, where h is the original height of the bob above the bottom of its swing.

Picture the Problem The free-body diagram shows the forces acting on the pendulum bob. The application of Newton's 2nd law leads directly to the required expression for the tangential acceleration. Recall that, provided θ is in radian measure, $s = L\theta$. Differentiation with respect to time produces the result called for in Part (*b*). The remaining parts of the problem simply require following the directions for each part.



(*a*) Apply $\sum F_x = ma_x$ to the bob: $F_{\tan} = -mg\sin\theta = ma_{\tan}$

Solving for $a_{\tan}$ yields: $\boldsymbol{a}_{\tan} = \boldsymbol{dv/dt} = \boxed{-\boldsymbol{g}\sin\boldsymbol{\theta}}$

(*b*) Relate the arc distance s to the length of the pendulum L and the angle θ: $s = L\theta$

Differentiate s with respect to time: $ds/dt = \boxed{v = Ld\theta/dt}$

(*c*) Multiply $\frac{dv}{dt}$ by $\frac{d\theta}{d\theta}$ and substitute for $\frac{d\theta}{dt}$ from Part (*b*):

$$\frac{dv}{dt} = \frac{dv}{dt}\frac{d\theta}{d\theta} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \boxed{\frac{dv}{d\theta}\left(\frac{v}{L}\right)}$$

(*d*) Equate the expressions for $\boldsymbol{dv/dt}$ from (*a*) and (*c*) to obtain:

$$\frac{dv}{d\theta}\left(\frac{v}{L}\right) = -g\sin\theta$$

Separating the variables yields: $\boldsymbol{vdv} = \boxed{-\boldsymbol{gL}\sin\boldsymbol{\theta}\,\boldsymbol{d\theta}}$

(e) Integrate the left side of the equation in Part (*d*) from $v = 0$ to the final speed v and the right side from $\boldsymbol{\theta} = \boldsymbol{\theta}_0$ to $\boldsymbol{\theta} = 0$:

$$\int_0^v vdv = \int_{\theta_0}^0 -gL\sin\theta\,d\theta$$

Integrate both sides of the equation to obtain:

$$\tfrac{1}{2}v^2 = gL(1-\cos\theta_0)$$

Note, from the figure, that $\boldsymbol{h} = \boldsymbol{L}(1-\cos\boldsymbol{\theta}_0)$. Substitute to obtain:

$$\tfrac{1}{2}\boldsymbol{v}^2 = \boldsymbol{gh} \Rightarrow \boldsymbol{v} = \boxed{\sqrt{2\boldsymbol{gh}}}$$

107 ••• A rock climber is rappelling down the face of a cliff when his hold slips and he slides down over the rock face, supported only by the bungee cord he attached to the top of the cliff. The cliff face is in the form of a smooth quarter-cylinder with height (and radius) $H = 300$ m (Figure 7-58). Treat the bungee cord as a spring with force constant $k = 5.00$ N/m and unstressed length $L = 60.0$ m. The climber's mass is 85.0 kg. (*a*) Using a **spreadsheet** program, make a graph of the rock climber's potential energy as a function of *s*, his distance from the top of the cliff *measured along the curved surface*. Use values of *s* between 60.0 m and 200 m. (*b*) His fall began when he was a distance $s_i = 60.0$ m from the top of the cliff, and ended when he was a distance $s_f = 110$ m from the top. Determine how much energy is dissipated by friction between the time he initially slipped and the time when he came to a stop.

Picture the Problem The potential energy of the climber is the sum of his gravitational potential energy and the potential energy stored in the spring-like bungee cord. Let θ be the angle which the position of the rock climber on the cliff face makes with a vertical axis and choose the zero of gravitational potential energy to be at the bottom of the cliff. We can use the definitions of U_g and U_{spring} to express the climber's total potential energy and the work-energy theorem for problems with friction to determine how energy is dissipated by friction between the time he initially slipped and finally came to a stop.

(*a*) The total potential energy of the climber is the sum of $U_{\text{bungee cord}}$ and U_g:

$$U(s) = U_{\text{bungee cord}} + U_g \qquad (1)$$

$U_{\text{bungee cord}}$ is given by:

$$U_{\text{bungee cord}} = \tfrac{1}{2}k(s-L)^2$$

U_g is given by:

$$U_g = Mgy = MgH\cos\theta = MgH\cos\left(\frac{s}{H}\right)$$

Substitute for $U_{\text{bunge cord}}$ and U_g in equation (1) to obtain:

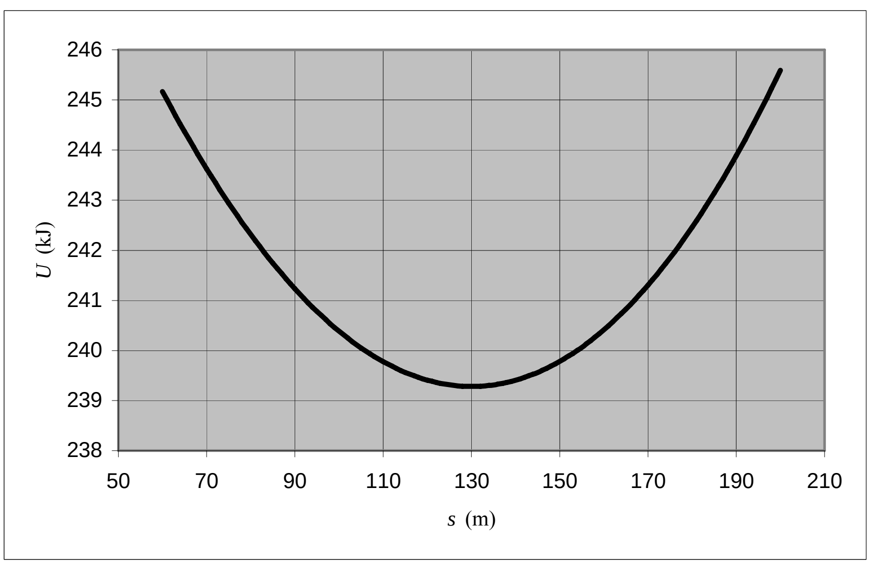
$$U(s) = \tfrac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right)$$

A spreadsheet solution is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B3	300	H
B4	5.00	k
B5	60.0	L
B6	85.0	M
B7	9.81	g
D11	60.0	s
D12	D11+1	$s+1$
E11	0.5*$B$4*(D11−$B$5)^2 +$B$6*$B$7*$B$3*(cos(D11/$B$3))	$\frac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right)$

	A	B	C	D	E
1					
2					
3	H =	300	m		
4	k =	5.00	N/m		
5	L =	60.0	m		
6	m =	85.0	kg		
7	g =	9.81	m/s^2		
8					
9				s	$U(s)$
10				(m)	(J)
11				60	2.452E+05
12				61	2.450E+05
13				62	2.448E+05
14				63	2.447E+05
15				64	2.445E+05
59				108	2.399E+05
60				109	2.398E+05
61				110	2.398E+05
62				111	2.397E+05
63				112	2.397E+05

The following graph was plotted using the data from columns D (s) and E ($U(s)$).



(b) Apply the work-kinetic energy theorem for problems with friction to the climber to obtain:

$$W_{ext} = \Delta K + \Delta U(s) + \Delta E_{therm}$$

or, because $W_{ext} = \Delta K = 0$,

$$\Delta U(s) + \Delta E_{therm} = 0$$

Solve for the energy dissipated by friction to obtain:

$$\Delta \boldsymbol{E}_{\text{therm}} = -\Delta \boldsymbol{U}(\boldsymbol{s}) = -(\boldsymbol{U}(110\text{ m}) - \boldsymbol{U}(60.0\text{ m})) = -\boldsymbol{U}(110\text{ m}) + \boldsymbol{U}(60.0\text{ m})$$

Substituting for $U(110\text{ m})$ and $U(60.0\text{ m})$ and simplifying yields:

$$\begin{aligned}\Delta E_{\text{therm}} &= -\left[\tfrac{1}{2}k(110\text{ m} - L)^2 + MgH\cos\left(\frac{110\text{ m}}{H}\right)\right] \\ &\quad + \left[\tfrac{1}{2}k(60.0\text{ m} - L)^2 + MgH\cos\left(\frac{60.0\text{ m}}{H}\right)\right] \\ &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\cos\left(\frac{110\text{ m}}{H}\right) + \tfrac{1}{2}k(60.0\text{ m} - L)^2 \\ &\quad + MgH\cos\left(\frac{60.0\text{ m}}{H}\right)\end{aligned}$$

Because $L = 60.0$ m, the third term is zero. Simplifying yields:

$$\begin{aligned}\Delta E_{\text{therm}} &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\cos\left(\frac{110\text{ m}}{H}\right) + MgH\cos\left(\frac{60.0\text{ m}}{H}\right) \\ &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\left[\cos\left(\frac{110\text{ m}}{H}\right) - \cos\left(\frac{60.0\text{ m}}{H}\right)\right]\end{aligned}$$

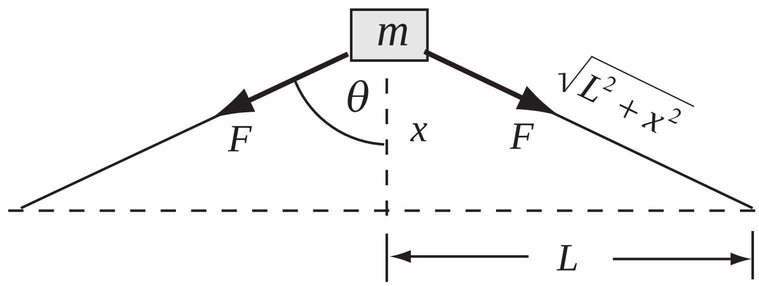
Substitute numerical values and evaluate $\Delta \boldsymbol{E}_{\text{therm}}$:

$$\begin{aligned}\Delta \boldsymbol{E}_{\text{therm}} &= -\tfrac{1}{2}(5.00\text{ N/m})(110\text{ m} - 60.0\text{ m})^2 \\ &\quad - (85.0\text{ kg})(9.81\text{ m/s}^2)(300\text{ m})\left[\cos\left(\frac{110\text{ m}}{(300\text{ m})}\right) - \cos\left(\frac{60.0\text{ m}}{(300\text{ m})}\right)\right] \\ &\approx \boxed{5.4\text{ kJ}}\end{aligned}$$

Remarks: You can obtain this same result by examining the partial spreadsheet printout or the graph shown above.

108 ••• A block of wood (mass *m*) is connected to two massless springs, as shown in Figure 7-59. Each spring has unstressed length *L* and force constant *k*. (*a*) If the block is displaced a distance *x*, as shown, what is the change in the potential energy stored in the springs? (*b*) What is the magnitude of the force pulling the block back toward the equilibrium position? (*c*) Using a **spreadsheet** program or graphing calculator, make a graph of the potential energy *U* as a function of *x* for $0 \le x \le 0.20$ m. Assume $k = 1.0$ N/m, $L = 0.10$ m, and $m = 1.0$ kg. (*d*) If the block is displaced a distance $x = 0.10$ m and released, what is its speed as it passes through the equilibrium point? Assume that the block is resting on a frictionless surface.

Picture the Problem The diagram shows the forces the springs exert on the block. Because the block is resting on a horizontal surface and they have no role in the motion of the block, the gravitational force and the normal force are not shown. The change in the potential energy stored in the springs is due to the elongation of both springs when the block is displaced a distance x from its equilibrium position and we can find ΔU using $\frac{1}{2}k(\Delta L)^2$. We can find the magnitude of the force pulling the block back toward its equilibrium position by finding the sum of the magnitudes of the y components of the forces exerted by the springs. In Part (d) we can use conservation of mechanical energy to find the speed of the block as it passes through its equilibrium position.



(a) Express the change in the potential energy stored in the springs when the block is displaced a distance x:

$$\Delta U = 2\left[\tfrac{1}{2}k(\Delta L)^2\right] = k(\Delta L)^2$$

where ΔL is the change in length of either spring.

Use the diagram to express ΔL:

$$\Delta L = \sqrt{L^2 + x^2} - L$$

Substitute for ΔL to obtain:

$$\Delta U = \boxed{k\left(\sqrt{L^2 + x^2} - L\right)^2}$$

(b) Sum the forces acting on the block to express $F_{\text{restoring}}$:

$$F_{\text{restoring}} = 2F\cos\theta = 2k\Delta L\cos\theta$$

$$= 2k\Delta L\frac{x}{\sqrt{L^2 + x^2}}$$

Substitute for ΔL to obtain:

$$F_{\text{restoring}} = 2k\left(\sqrt{L^2 + x^2} - L\right)\frac{x}{\sqrt{L^2 + x^2}}$$

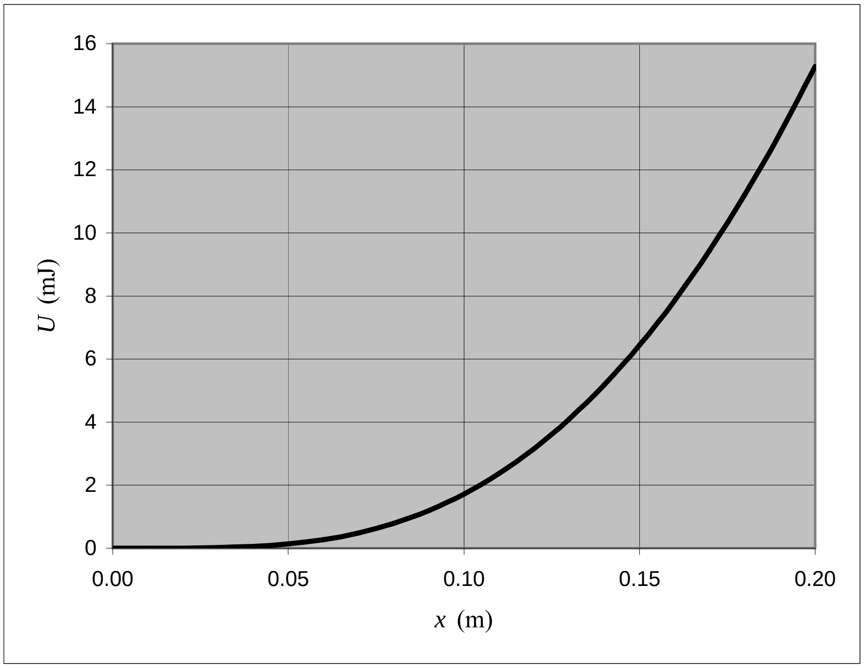
$$= \boxed{2kx\left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right)}$$

(c) A spreadsheet program to calculate $U(x)$ is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B1	0.1	L
B2	1.0	k
B3	1.0	M
C8	C7+0.01	x
D7	$B$2*((C7^2+$B$1^2)^0.5−$B$1)^2	$U(x)$

	A	B	C	D
1	L =	0.1	m	
2	k =	1.0	N/m	
3	m =	1.0	kg	
4				
5			x	$U(x)$
6			(m)	(J)
7			0	0
8			0.01	2.49E−07
9			0.02	3.92E−06
10			0.03	1.94E−05
11			0.04	5.93E−05
12			0.05	1.39E−04
23			0.16	7.86E−03
24			0.17	9.45E−03
25			0.18	1.12E−02
26			0.19	1.32E−02
27			0.20	1.53E−02

The following graph was plotted using the data from columns C (x) and D ($U(x)$).



(d) Use conservation of mechanical energy to relate the kinetic energy of the block as it passes through the equilibrium position to the change in its potential energy as it returns to its equilibrium position:

$$K_{\text{equilibrium}} = \Delta U$$

or

$$\tfrac{1}{2}mv^2 = \Delta U \Rightarrow v = \sqrt{\frac{2\Delta U}{m}}$$

Substitute for ΔU and simplify to obtain:

$$v = \sqrt{\frac{2k\left(\sqrt{L^2+x^2}-L\right)^2}{m}} = \left(\sqrt{L^2+x^2}-L\right)\sqrt{\frac{2k}{m}}$$

Substitute numerical values and evaluate v:

$$v = \left(\sqrt{(0.10\,\text{m})^2+(0.10\,\text{m})^2}-0.10\,\text{m}\right)\sqrt{\frac{2(1.0\,\text{N/m})}{1.0\,\text{kg}}} = \boxed{5.9\,\text{cm/s}}$$