

# **Concept-Development Practice Page** **Chapter 18: Solids**

**18-1**

## Scaling Squares

1.

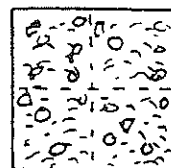

 If you have a square 1 cm on each side, what is its area? 1 cm<sup>2</sup>

 A SQUARE OF SIDE 1 HAS AN AREA 1<sup>2</sup>


2.


 If you have a square 2 cm on each side, what is its area? 4 cm<sup>2</sup>

3. Consider a square piece of pizza, 10 cm x 10 cm. Another piece of pizza measures 20 cm x 20 cm. How does the area of the larger pizza compare?

100 cm<sup>2</sup> vs. 400 cm<sup>2</sup> 4x


4. If you have a square 4 cm on each side, what is its area? 16 cm<sup>2</sup>

5. If you have a square 10 cm on each side, what is its area? 100 cm<sup>2</sup>

6. If you double each side of a square, how many times as large does its area become?

4x

7. If you triple each side of a square, how many times as large does its area become?

9x

8. *True or false:* If the side of a square is increased by a certain factor, say 5, then the area increases by the *square* of the factor, in this case 5<sup>2</sup> (or 25). True

So, if you scale up the side of a square by a factor of 10, its area will increase by a factor of 10<sup>2</sup>, or 100.

 WHEN YOU DOUBLE THE SIDE OF A SQUARE, YOU GET 4x TIMES THE AREA
 
 AND IF YOU TRIPLE THE SIDE OF A SQUARE YOU GET 9x TIMES THE AREA
 
 IN GENERAL, IF YOU X THE SIDE OF A SQUARE, YOU X<sup>2</sup> THE AREA!
 

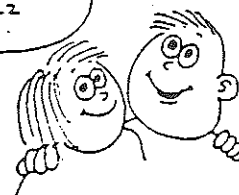
## Scaling Circles

1. Complete the table.

CIRCLES		
RADIUS	CIRCUMFERENCE	AREA
1 cm	$2\pi(1\text{ cm}) = 2\pi\text{ cm}$	$\pi(1\text{ cm})^2 = \pi\text{ cm}^2$
2 cm	$2\pi(2\text{ cm}) = 4\pi\text{ cm}$	$\pi(2\text{ cm})^2 = 4\pi$
3 cm	$2\pi(3\text{ cm}) = 6\pi\text{ cm}$	$\pi(3\text{ cm})^2 = 9\pi$
10 cm	$2\pi(10\text{ cm}) = 20\pi\text{ cm}$	$\pi(10\text{ cm})^2 = 100\pi$

FOR THE CIRCUMFERENCE OF A CIRCLE,  $C = 2\pi r$

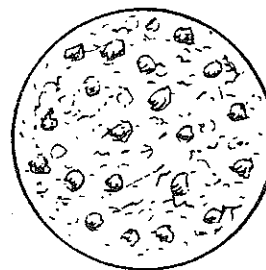
AND FOR THE AREA OF A CIRCLE,  $A = \pi r^2$



2. From your completed table, when the radius of a circle is doubled, its area increases by a factor of  $2^2 = 4$ . When the radius is increased by a factor of 10, the area increases by a factor of  $10^2 = 100$ .

3. Consider a round pizza that costs \$2.00. Another pizza of the same thickness has twice the diameter. How much should the larger pizza cost?

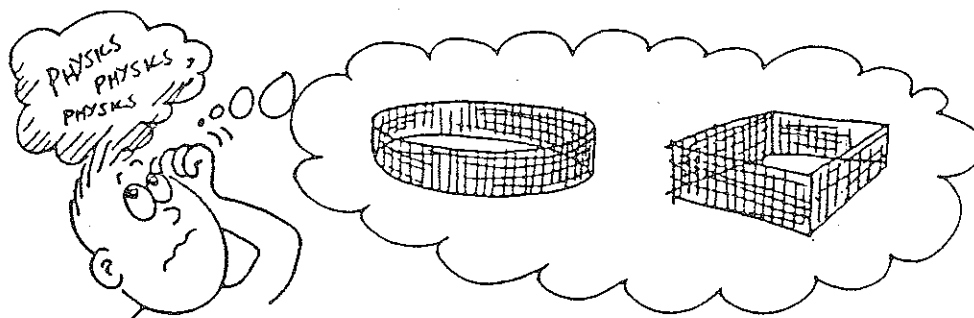
$2 \times r = 4 \times \text{Area} = \$8.00$



4. True or false: If the radius of a circle is increased by a certain factor, say 5, then the area increases by the *square* of the factor, in this case  $5^2$  or 25. True

So if you scale up the radius of a circle by a factor of 10, its area will increase by a factor of 100.

5. (Application:) Suppose you raise chickens and spend \$50 to buy wire for a chicken pen. To hold the most chickens inside, you should make the shape of the pen  
(square) (circular) (either, for both provide the same area)



# **Concept-Development Practice Page** **Chapter 18: Solids**

**18-2**

## **Scaling Cubes**

1. Consider a cube, say 1 cm x 1 cm x 1 cm (about the size of a sugar cube). Its volume is 1 cm<sup>3</sup>. The surface area of one of its faces is 1 cm<sup>2</sup>. This is also the area of any cross-section (a slice through the cube that is parallel to any of its faces). The total surface area of the cube is 6 cm<sup>2</sup>, because it has 6 faces (4 sides and top and bottom; count them).

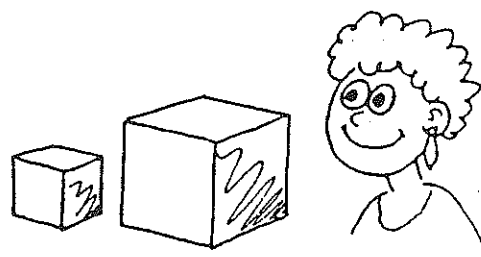
Now consider a second cube, scaled up by a factor of 2 so it is 2 cm x 2 cm x 2 cm.

- a. What is the total surface areas of each cube?

1st cube 6 cm<sup>2</sup>; 2nd cube 24 cm<sup>2</sup>

- b. How many times more is the total surface area of the second cube compared to that of the first?

4x



- c. What are the volumes of the cubes?

1st cube 1 cm<sup>3</sup>; 2nd cube 8 cm<sup>3</sup>

- d. How many times more is the volume of the second cube compared to that of the first?

8x

- e. Compare the surface-area-to-volume ratio for both cubes.

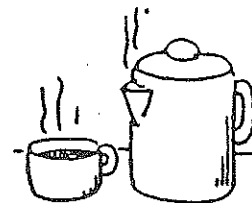
1st cube:  $\frac{\text{surface area}}{\text{volume}} = \frac{6}{1}$

2nd cube:  $\frac{\text{surface area}}{\text{volume}} = \frac{24}{8} = \frac{3}{1}$

2. As the size of a cube (or object of any shape) increases, the ratio of surface area to volume decreases. This means that the gain in surface area is proportionally less than the gain in volume (area gains only as the *square* of the increase, while volume gains as the *cube* of the increase). Apply this relationship and good thinking to explain the following:

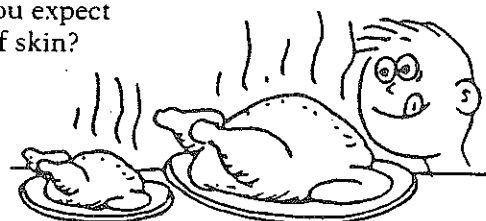
- a. Why does a cup of coffee cool faster than the pot from which it is poured? Or, if you want your coffee to stay warm, should you pour it into a cup or leave it in the pot? Explain.

Cup has more surface area per unit  
Volume so rate of cooling is higher.



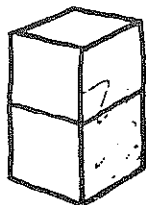
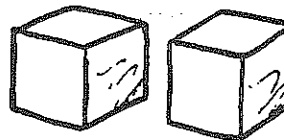
- b. Chickens, Cornish game hens, and turkeys have approximately the same body shape. If you had two birds, one with a body twice as wide, twice as thick, and twice as long, how would you expect their weights to compare? How about the amounts of skin?

8x weight (proportional to volume)  
4x skin (proportional to Area)



## Scaling Solids

1. a. Consider a pair of cubes, each 1 cm on a side. The volume of each cube is 1  $\text{cm}^3$ , so the total volume of both cubes is 2  $\text{cm}^3$ . The total surface area of each cube is 6  $\text{cm}^2$ , so the total surface area of both cubes is 12  $\text{cm}^2$ .

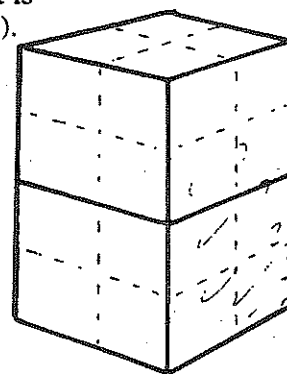


- b. If the two cubes are glued together, to make a double cube, the total volume is 2  $\text{cm}^3$  and the total exposed surface area is 10  $\text{cm}^2$ .

2. The sketch to the right is a scaled-up version of the small double cube. It is scaled up by a factor of 2 (twice as wide, twice as thick, and twice as tall).

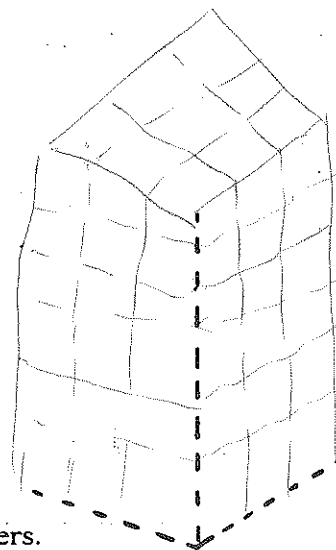
- a. Its volume is 16  $\text{cm}^3$ .  
 b. Its total surface area is 40  $\text{cm}^2$ .  
 c. Which has the greater surface-area-to-volume ratio, the small or the scaled-up double cube?

small cube



3. Scale up the double cube of Question 2 by a factor of 3 (three times as tall and three times as wide both ways). Make a sketch of it in the space at the right.

- a. Its volume is 54  $\text{cm}^3$ .  
 b. Its total surface area is 90  $\text{cm}^2$ .  
 c. Which of the three versions of the double cube has the most surface area? #3  
 d. Which has the most surface area compared to its volume? #1



4. True or false: As the volume of an object increases, its surface area also increases, but the ratio of surface area to volume decreases. True

5. The effects of scaling are good for some creatures and bad for others. Write either good (G) or bad (B) for each of the following:

- a. An insect falling from a tall tree G b. An elephant falling from the same tree B  
 c. A small fish trying to flee a big fish G d. A big fish chasing a small fish B  
 e. An insect that falls in the water G f. A hungry sparrow B