

INDEPENDENCE OF HORIZONTAL AND VERTICAL VELOCITIES

The velocity of any object that is traveling along a curved path is constantly changing. Even if the object is moving at a constant speed, the direction is always changing as the path of the object bends. Thus, the object is accelerating.

Any object that is hurled through the air and continues its motion without outside assistance is called a PROJECTILE. Some examples of projectiles are missiles, bullets, and balls rolling off the end of tables. The path that a projectile takes is called a TRAJECTORY. Most trajectories are curved.

Projectiles move in two directions as they fly through the air. Part of their motion is in the horizontal direction. Yet they also move vertically. Thus, a ball that is rolled off the end of a table will move away from the edge both downward (due to gravity) and outward (due to its speed as it rolled across the table). The two separate velocities, one horizontal and the other vertical, have no effect on one another.

An object that is fired horizontally from a particular height will hit the ground at the same time as an object that is dropped from that height (provided that they are dropped at the same time). Both objects accelerate towards the ground as they fall at -9.81 m/s^2 due to gravity. The horizontal velocity of the object that is fired has no effect on this acceleration. The horizontal velocity causes the fired object to hit the ground further away horizontally from the firing point than the object that is dropped.

If a car is traveling along at 30 m/s, the horizontal velocity of the ball is 30 m/s. If you toss the ball upward with a velocity of 10 m/s, the horizontal velocity of the ball is still 30 m/s. If you toss the ball upward at 60 m/s, the horizontal velocity of the ball is still 30 m/s. Once the ball leaves your hand, its vertical velocity begins to change. The ball slows down until it reaches its apex and then accelerates downward as it falls back into your hand. Throughout its flight the vertical velocity of the ball is changing because of gravity. Yet, the horizontal velocity of the ball remains the same.

Since these velocities are independent, they can be dealt with as separate parts of one problem.

In the horizontal direction, the velocity is constant. Therefore, the horizontal part of the problem uses the standard equation for constant (or average) velocity...

$$v_h = \frac{d_h}{t}$$

where:

v_h = horizontal velocity

d_h = horizontal distance

t = time that the object is in the air

Using this equation you can solve for horizontal distance, horizontal velocity, and time in the air, depending on what information you have.

The horizontal distance traveled also depends on the time that the object is in the air. For horizontal projectiles, the time that the object is in the air depends on the height that the object is fired from. The time that the object is in the air is the same whether the object is dropped or fired horizontally. Therefore, you can treat the vertical portion of the problem as if the object were in free fall. You may use the standard free fall equations to help you calculate the time...

$$d_v = \frac{1}{2}gt^2$$

where:

d_v = vertical distance

g = acceleration due to gravity (-9.81 m/s²)

t = time that the object is in the air

Using this equation you can find both the vertical distance (which will be negative because the object is falling) and the time that the object is in the air. This equation is most useful in finding the latter since we often know the height from which we fire the projectile (which is the vertical distance).

Once we know the time that the object is in the air, we can use the horizontal velocity equations to predict the distance that the projectile will travel or the horizontal velocity that we need to reach a given horizontal distance.

STEPS FOR CALCULATING THE HORIZONTAL DISTANCE COVERED BY A PROJECTILE FIRED HORIZONTALLY FROM A GIVEN HEIGHT

- 1.) Since the vertical distance is usually given, use the equation below to calculate the time that the projectile is in the air. Use $g = -9.81 \text{ m/s}^2$.

$$t = \sqrt{\frac{2d_v}{g}}$$

- 2.) Use the time calculate in Step #1 and the given horizontal velocity in the equation below to calculate the horizontal distance that the projectile will travel.

$$d_h = v_h t$$

NAME: _____

DATE: _____

1. How far from the bottom of a 45 meter tall cliff will an arrow strike the ground if it is shot horizontally with a speed of 150 m/s?

DATA. List the given information

_____ = 45 m

_____ = 150 m/s

STEP ONE. Use the equation $t = \sqrt{\frac{2d_v}{g}}$ to find the time that the arrow is in the air.

_____ seconds

STEP TWO. Use the equation $d_h = v_h t$ to calculate the range (horizontal distance) of the projectile.

_____ m

2. How far will a rock travel from the base of a cliff if the cliff is 25 meters tall and the rock is thrown horizontally with a velocity of 67 m/s?

DATA. List the given information

_____ = 25 m

_____ = 67 m/s

STEP ONE. Use the equation $t = \sqrt{\frac{2d_v}{g}}$ to find the time that the arrow is in the air.

_____ seconds

STEP TWO. Use the equation $d_h = v_h t$ to calculate the range (horizontal distance) of the projectile.

_____ m

NAME: _____

DATE: _____

FOR EACH OF THE FOLLOWING, IGNORE THE EFFECTS OF AIR RESISTANCE
SHOW ALL WORK!

1. A cannon is shot horizontally at a speed of 360 m/s off of a cliff 135 meters high. How far away from the bottom of the cliff will the cannonball strike the cat which is sleeping on the ground?

Data	Equations	Math	Answer

2. A cat is standing motionless on a long straight sidewalk. Alvin the Accurate is the bombardier in a plane flying westward over this same road, and directly toward the cat with a speed of 315 m/s at an altitude of 2500 meters. How far before the cat must the bomb be released to get a hit?

Data	Equations	Math	Answer

3. Billy the Rooftop Bowler is at it again but he slips and his ball rolls off the edge of his 75 meter tall building at a speed of 12.5 m/s. How far from the building will the ball hit the cat walking across the street below?

Data	Equations	Math	Answer