



Answer Explanations

SAT[®] Practice Test #4

QUESTION 40.

Choice B is the best answer because it provides the singular nouns “writer” and “speaker” to agree with the singular pronoun “anyone.”

Choices A, C, and D are incorrect because none creates pronoun-referent agreement.

QUESTION 41.

Choice D is the best answer because it expresses in the clearest, simplest way the idea that many game designers start out as programmers.

Choices A, B, and C are incorrect because each is unnecessarily wordy and obscures meaning.

QUESTION 42.

Choice D is the best answer because it logically and appropriately modifies the phrase “collaboration skills.”

Choices A, B, and C are incorrect because none appropriately describes the value of collaboration skills.

QUESTION 43.

Choice A is the best answer because it provides a logical subject for the modifying phrase “demanding and deadline driven.”

Choices B, C, and D are incorrect because each creates a dangling modifier.

QUESTION 44.

Choice B is the best answer because sentence 5 expresses the main point upon which the paragraph elaborates.

Choices A, C, and D are incorrect because none places sentence 5 in the appropriate position to set up the details contained in the paragraph.

Section 3: Math Test — No Calculator

QUESTION 1.

Choice A is correct. The expression $|x - 1| - 1$ will equal 0 if $|x - 1| = 1$. This is true for $x = 2$ and for $x = 0$. For example, substituting $x = 2$ into the expression $|x - 1| - 1$ and simplifying the result yields $|2 - 1| - 1 = |1| - 1 = 1 - 1 = 0$. Therefore, there is a value of x for which $|x - 1| - 1$ is equal to 0.

Choice B is incorrect. By definition, the absolute value of any expression is a nonnegative number. Substituting any value for x into the expression

$|x + 1|$ will yield a nonnegative number as the result. Because the sum of a nonnegative number and a positive number is positive, $|x + 1| + 1$ will be a positive number for any value of x . Therefore, $|x + 1| + 1 \neq 0$ for any value of x . Choice C is incorrect. By definition, the absolute value of any expression is a nonnegative number. Substituting any value for x into the expression $|1 - x|$ will yield a nonnegative number as the result. Because the sum of a nonnegative number and a positive number is positive, $|1 - x| + 1$ will be a positive number for any value of x . Therefore, $|1 - x| + 1 \neq 0$ for any value of x . Choice D is incorrect. By definition, the absolute value of any expression is a nonnegative number. Substituting any value for x into the expression $|x - 1|$ will yield a nonnegative number as the result. Because the sum of a nonnegative number and a positive number is positive, $|x - 1| + 1$ will be a positive number for any value of x . Therefore, $|x - 1| + 1 \neq 0$ for any value of x .

QUESTION 2.

Choice A is correct. Since $f(x) = \frac{3}{2}x + b$ and $f(6) = 7$, substituting 6 for x in $f(x) = \frac{3}{2}x + b$ gives $f(6) = \frac{3}{2}(6) + b = 7$. Then, solving the equation $\frac{3}{2}(6) + b = 7$ for b gives $\frac{18}{2} + b = 7$, or $9 + b = 7$. Thus, $b = 7 - 9 = -2$. Substituting this value back into the original function gives $f(x) = \frac{3}{2}x - 2$; therefore, one can evaluate $f(-2)$ by substituting -2 for x : $\frac{3}{2}(-2) - 2 = -\frac{6}{2} - 2 = -3 - 2 = -5$.

Choice B is incorrect as it is the value of b , not of $f(-2)$. Choice C is incorrect as it is the value of $f(2)$, not of $f(-2)$. Choice D is incorrect as it is the value of $f(6)$, not of $f(-2)$.

QUESTION 3.

Choice A is correct. The first equation can be rewritten as $x = 6y$. Substituting $6y$ for x in the second equation gives $4(y + 1) = 6y$. The left-hand side can be rewritten as $4y + 4$, giving $4y + 4 = 6y$. Subtracting $4y$ from both sides of the equation gives $4 = 2y$, or $y = 2$.

Choices B, C, and D are incorrect and may be the result of a computational or conceptual error when solving the system of equations.

QUESTION 4.

Choice B is correct. If $f(x) = -2x + 5$, then one can evaluate $f(-3x)$ by substituting $-3x$ for every instance of x . This yields $f(-3x) = -2(-3x) + 5$, which simplifies to $6x + 5$.

Choices A, C, and D are incorrect and may be the result of miscalculations in the substitution or of misunderstandings of how to evaluate $f(-3x)$.

QUESTION 5.

Choice C is correct. The expression $3(2x + 1)(4x + 1)$ can be simplified by first distributing the 3 to yield $(6x + 3)(4x + 1)$, and then expanding to obtain $24x^2 + 12x + 6x + 3$. Combining like terms gives $24x^2 + 18x + 3$.

Choice A is incorrect and may be the result of performing the term-by-term multiplication of $3(2x + 1)(4x + 1)$ and treating every term as an x -term. Choice B is incorrect and may be the result of correctly finding $(6x + 3)(4x + 1)$, but then multiplying only the first terms, $(6x)(4x)$, and the last terms, $(3)(1)$, but not the outer or inner terms. Choice D is incorrect and may be the result of incorrectly distributing the 3 to both terms to obtain $(6x + 3)(12x + 3)$, and then adding $3 + 3$ and $6x + 12x$ and incorrectly adding the exponents of x .

QUESTION 6.

Choice B is correct. The equation $\frac{a - b}{b} = \frac{3}{7}$ can be rewritten as $\frac{a}{b} - \frac{b}{b} = \frac{3}{7}$, from which it follows that $\frac{a}{b} - 1 = \frac{3}{7}$, or $\frac{a}{b} = \frac{3}{7} + 1 = \frac{10}{7}$.

Choices A, C, and D are incorrect and may be the result of calculation errors in rewriting $\frac{a - b}{b} = \frac{3}{7}$. For example, choice A may be the result of a sign error in rewriting $\frac{a - b}{b}$ as $\frac{a}{b} + \frac{b}{b} = \frac{a}{b} + 1$.

QUESTION 7.

Choice D is correct. In Amelia's training schedule, her longest run in week 16 will be 26 miles and her longest run in week 4 will be 8 miles. Thus, Amelia increases the distance of her longest run by 18 miles over the course of 12 weeks. Since Amelia increases the distance of her longest run each week by a constant amount, the amount she increases the distance of her longest run each week is $\frac{26 - 8}{16 - 4} = \frac{18}{12} = \frac{3}{2} = 1.5$ miles.

Choices A, B, and C are incorrect because none of these training schedules would result in increasing Amelia's longest run from 8 miles in week 4 to 26 miles in week 16. For example, choice A is incorrect because if Amelia increases the distance of her longest run by 0.5 miles each week and has her longest run of 8 miles in week 4, her longest run in week 16 would be $8 + 0.5 \cdot 12 = 14$ miles, not 26 miles.

QUESTION 8.

Choice A is correct. For an equation of a line in the form $y = mx + b$, the constant m is the slope of the line. Thus, the line represented by $y = -3x + 4$ has slope -3 . Lines that are parallel have the same slope. To find out which of the given equations represents a line with the same slope as the line represented by $y = -3x + 4$, one can rewrite each equation in the form $y = mx + b$, that is, solve each equation for y . Choice A, $6x + 2y = 15$, can

be rewritten as $2y = -6x + 15$ by subtracting $6x$ from each side of the equation. Then, dividing each side of $2y = -6x + 15$ by 2 gives $y = -\frac{6}{2}x + \frac{15}{2} = -3x + \frac{15}{2}$. Therefore, this line has slope -3 and is parallel to the line represented by $y = -3x + 4$. (The lines are parallel, not coincident, because they have different y -intercepts.)

Choices B, C, and D are incorrect and may be the result of common misunderstandings about which value in the equation of a line represents the slope of the line.

QUESTION 9.

Choice D is correct. The question states that $\sqrt{x-a} = x-4$ and that $a = 2$, so substituting 2 for a in the equation yields $\sqrt{x-2} = x-4$. To solve for x , square each side of the equation, which gives $(\sqrt{x-2})^2 = (x-4)^2$, or $x-2 = (x-4)^2$. Then, expanding $(x-4)^2$ yields $x-2 = x^2-8x+16$, or $0 = x^2-9x+18$. Factoring the right-hand side gives $0 = (x-3)(x-6)$, and so $x = 3$ or $x = 6$. However, for $x = 3$, the original equation becomes $\sqrt{3-2} = 3-4$, which yields $1 = -1$, which is not true. Hence, $x = 3$ is an extraneous solution that arose from squaring each side of the equation. For $x = 6$, the original equation becomes $\sqrt{6-2} = 6-4$, which yields $\sqrt{4} = 2$, or $2 = 2$. Since this is true, the solution set of $\sqrt{x-2} = x-4$ is $\{6\}$.

Choice A is incorrect because it includes the extraneous solution in the solution set. Choice B is incorrect and may be the result of a calculation or factoring error. Choice C is incorrect because it includes only the extraneous solution, and not the correct solution, in the solution set.

QUESTION 10.

Choice D is correct. Multiplying each side of $\frac{t+5}{t-5} = 10$ by $t-5$ gives $t+5 = 10(t-5)$. Distributing the 10 over the values in the parentheses yields $t+5 = 10t-50$. Subtracting t from each side of the equation gives $5 = 9t-50$, and then adding 50 to each side gives $55 = 9t$. Finally, dividing each side by 9 yields $t = \frac{55}{9}$.

Choices A, B, and C are incorrect and may be the result of calculation errors or using the distribution property improperly.

QUESTION 11.

Choice C is correct. Since $y = (2x-3)(x+9)$ and $x = 2y+5$, it follows that $x = 2((2x-3)(x+9)) + 5 = 4x^2 + 30x - 54$. This can be rewritten as $4x^2 + 29x - 54 = 0$. Because the discriminant of this quadratic equation, $29^2 - (4)(-54) = 29^2 + 4(54)$, is positive, this equation has 2 distinct roots. Using each of the roots as the value of x and finding y from the equation $x = 2y + 5$ gives 2 ordered pairs (x, y) that satisfy the given system of

equations. Since no other value of x satisfies $4x^2 + 29x - 54 = 0$, there are no other ordered pairs that satisfy the given system. Therefore, there are 2 ordered pairs (x, y) that satisfy the given system of equations.

Choices A and B are incorrect and may be the result of either a miscalculation or a conceptual error. Choice D is incorrect because a system of one quadratic equation and one linear equation cannot have infinitely many solutions.

QUESTION 12.

Choice C is correct. Since the price of Ken's sandwich was x dollars, and Paul's sandwich was \$1 more, the price of Paul's sandwich was $x + 1$ dollars. Thus, the total cost of the sandwiches was $2x + 1$ dollars. Since this cost was split evenly, Ken and Paul each paid $\frac{2x + 1}{2} = x + 0.5$ dollars plus a 20% tip. After adding the 20% tip, each of them paid $(x + 0.5) + 0.2(x + 0.5) = 1.2(x + 0.5) = 1.2x + 0.6$ dollars.

Choices A, B, and D are incorrect. These expressions do not model the given context. They may be the result of errors in setting up the expression or of calculation errors.

QUESTION 13.

Choice B is correct. One can find the intersection points of the two graphs by setting the functions $f(x)$ and $g(x)$ equal to one another and then solving for x . This yields $8x^2 - 2 = -8x^2 + 2$. Adding $8x^2$ and 2 to each side of the equation gives $16x^2 = 4$. Then dividing each side by 16 gives $x^2 = \frac{1}{4}$, and then taking the square root of each side gives $x = \pm \frac{1}{2}$. From the graph, the value of k is the x -coordinate of the point of intersection on the positive x -axis. Therefore, $k = \frac{1}{2}$.

Alternatively, since $(k, 0)$ lies on the graph of both f and g , it follows that $f(k) = g(k) = 0$. Thus, evaluating $f(x) = 8x^2 - 2$ at $x = k$ gives $0 = 8k^2 - 2$. Adding 2 to each side yields $2 = 8k^2$ and then dividing each side by 8 gives $\frac{1}{4} = k^2$. Taking the square root of each side then gives $k = \pm \frac{1}{2}$. From the graph, k is positive, so $k = \frac{1}{2}$.

Choices A, C, and D are incorrect and may be the result of calculation errors in solving for x or k .

QUESTION 14.

Choice A is correct. To rewrite $\frac{8 - i}{3 - 2i}$ in the standard form $a + bi$, multiply the numerator and denominator of $\frac{8 - i}{3 - 2i}$ by the conjugate, $3 + 2i$. This gives $\left(\frac{8 - i}{3 - 2i}\right)\left(\frac{3 + 2i}{3 + 2i}\right) = \frac{24 + 16i - 3i + (-i)(2i)}{3^2 - (2i)^2}$. Since $i^2 = -1$, this last fraction

can be rewritten as $\frac{24 + 16i - 3i + 2}{9 - (-4)} = \frac{26 + 13i}{13}$, which simplifies to $2 + i$.

Therefore, when $\frac{8 - i}{3 - 2i}$ is rewritten in the standard form $a + bi$, the value of a is 2.

Choices B, C, and D are incorrect and may be the result of errors in symbolic manipulation. For example, choice B could be the result of mistakenly rewriting $\frac{8 - i}{3 - 2i}$ as $\frac{8}{3} + \frac{1}{2}i$.

QUESTION 15.

Choice B is correct. The given quadratic equation can be rewritten as $2x^2 - kx - 4p = 0$. Applying the quadratic formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to this equation with $a = 2$, $b = -k$, and $c = -4p$ gives the solutions $\frac{k}{4} \pm \frac{\sqrt{k^2 + 32p}}{4}$.

Choices A, C, and D are incorrect and may be the result of errors in applying the quadratic formula.

QUESTION 16.

The correct answer is 9. Since the three shelves of the triangular shelf system are parallel, the three triangles in the figure are similar. Since the shelves divide the left side of the largest triangle in the ratio 2 to 3 to 1, the similarity ratios of the triangles are as follows.

- ▶ Smallest to middle: 2 to 5
- ▶ Smallest to largest: 2 to 6, or 1 to 3
- ▶ Middle to largest: 5 to 6

The height of the largest shampoo bottle that can stand upright on the middle shelf is equal to the height of the middle shelf. The height of the entire triangular shelf system is 18 inches. This is the height of the largest triangle. The height of the middle shelf is the height of the middle triangle minus the height of the smallest triangle. Since the similarity ratio of the middle triangle to the largest triangle is 5 to 6, the height of the middle shelf is $\frac{5}{6}(18) = 15$ inches. Since the similarity ratio of the smallest triangle to the largest triangle is 1 to 3, the height of the middle shelf is $\frac{1}{3}(18) = 6$ inches. Therefore, the height of the middle shelf is 9 inches.

QUESTION 17.

The correct answer is .6 or $\frac{3}{5}$. The angles marked x° and y° are acute angles in a right triangle. Thus, they are complementary angles. By the complementary angle relationship between sine and cosine, it follows that $\sin(x^\circ) = \cos(y^\circ)$. Therefore, the cosine of y° is .6. Either .6 or the equivalent fraction $\frac{3}{5}$ may be gridded as the correct answer.

Alternatively, since the sine of x° is .6, the ratio of the side opposite the x° angle to the hypotenuse is .6. The side opposite the x° angle is the side adjacent to the y° angle. Thus, the ratio of the side adjacent to the y° angle to the hypotenuse, which is equal to the cosine of y° , is equal to .6.

QUESTION 18.

The correct answer is 5. The four-term polynomial expression can be factored completely, by grouping, as follows:

$$(x^3 - 5x^2) + (2x - 10) = 0$$

$$x^2(x - 5) + 2(x - 5) = 0$$

$$(x - 5)(x^2 + 2) = 0$$

By the zero product property, set each factor of the polynomial equal to 0 and solve each resulting equation for x . This gives $x = 5$ or $x = \pm i\sqrt{2}$, respectively. Because the question asks for the real value of x that satisfies the equation, the correct answer is 5.

QUESTION 19.

The correct answer is 0. Multiplying each side of $-3x + 4y = 20$ by 2 gives $-6x + 8y = 40$. Adding each side of $-6x + 8y = 40$ to the corresponding side of $6x + 3y = 15$ gives $11y = 55$, or $y = 5$. Finally, substituting 5 for y in $6x + 3y = 15$ gives $6x + 3(5) = 15$, or $x = 0$.

QUESTION 20.

The correct answer is 25. In the mesosphere, an increase of 10 kilometers in the distance above Earth results in a decrease in the temperature by k° Celsius where k is a constant. Thus, the temperature in the mesosphere is linearly dependent on the distance above Earth. Using the values provided and the slope formula, one can calculate the unit rate of change for the temperature in the mesosphere to be $\frac{-80 - (-5)}{80 - 50} = \frac{-75}{30} = \frac{-2.5}{1}$. The slope indicates that, within the mesosphere, if the distance above Earth increases by 1 kilometer, the temperature decreases by 2.5° Celsius. Therefore, if the distance above Earth increases by $(1 \times 10) = 10$ kilometers, the temperature will decrease by $(2.5 \times 10) = 25^\circ$ Celsius. Thus, the value of k is 25.

Section 4: Math Test — Calculator

QUESTION 1.

Choice B is correct. Let m be the number of movies Jill rented online during the month. Since the monthly membership fee is \$9.80 and there is an additional fee of \$1.50 to rent each movie online, the total of the membership fee and the movie rental fees, in dollars, can be written as $9.80 + 1.50m$. Since

the total of these fees for the month was \$12.80, the equation $9.80 + 1.50m = 12.80$ must be true. Subtracting 9.80 from each side and then dividing each side by 1.50 yields $m = 2$.

Choices A, C, and D are incorrect and may be the result of errors in setting up or solving the equation that represents the context.

QUESTION 2.

Choice C is correct. Donald believes he can increase his typing speed by 5 words per minute each month. Therefore, in m months, he believes he can increase his typing speed by $5m$ words per minute. Because he is currently able to type at a speed of 180 words per minute, he believes that in m months, he will be able to increase his typing speed to $180 + 5m$ words per minute.

Choice A is incorrect because the expression indicates that Donald currently types 5 words per minute and will increase his typing speed by 180 words per minute each month. Choice B is incorrect because the expression indicates that Donald currently types 225 words per minute, not 180 words per minute. Choice D is incorrect because the expression indicates that Donald will decrease, not increase, his typing speed by 5 words per minute each month.

QUESTION 3.

Choice C is correct. Because there are 16 ounces in 1 pound, a 3-pound pizza weighs $3 \times 16 = 48$ ounces. One half of the pizza weighs $\frac{1}{2} \times 48 = 24$ ounces, and one-third of the half weighs $\frac{1}{3} \times 24 = 8$ ounces.

Alternatively, since $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, cutting the pizza into halves and then into thirds results in a pizza that is cut into sixths. Therefore, each slice of the 48-ounce pizza weighs $\frac{1}{6} \times 48 = 8$ ounces.

Choice A is incorrect and is the result of cutting each half into sixths rather than thirds. Choice B is incorrect and is the result of cutting each half into fourths rather than thirds. Choice D is incorrect and is the result of cutting the whole pizza into thirds.

QUESTION 4.

Choice B is correct. Because Nick surveyed a random sample of the freshman class, his sample was representative of the entire freshman class. Thus, the percent of students in the entire freshman class expected to prefer the Fall Festival in October is appropriately estimated by the percent of students who preferred it in the sample, 25.6%. Thus, of the 225 students in the freshman class, approximately $225 \times 0.256 = 57.6$ students would be expected to prefer having the Fall Festival in October. Of the choices given, this is closest to 60.

Choices A, C, and D are incorrect. These choices may be the result of misapplying the concept of percent or of calculation errors.

QUESTION 5.

Choice B is correct. The density of an object is equal to the mass of the object divided by the volume of the object, which can be expressed as $\text{density} = \frac{\text{mass}}{\text{volume}}$. Thus, if an object has a density of 3 grams per milliliter and a mass of 24 grams, the equation becomes $3 \text{ grams/milliliter} = \frac{24 \text{ grams}}{\text{volume}}$. This can be rewritten as $\text{volume} = \frac{24 \text{ grams}}{3 \text{ grams/milliliter}} = 8 \text{ milliliters}$.

Choice A is incorrect and may be the result of confusing the density and the volume and setting up the density equation as $24 = \frac{3}{\text{volume}}$. Choice C is incorrect and may be the result of a conceptual error that leads to subtracting 3 from 24. Choice D is incorrect and may be the result of confusing the mass and the volume and setting up the density equation as $24 = \frac{\text{volume}}{3}$.

QUESTION 6.

Choice A is correct. Let a be the number of hours Angelica worked last week. Since Raul worked 11 more hours than Angelica, Raul worked $a + 11$ hours last week. Since they worked a combined total of 59 hours, the equation $a + (a + 11) = 59$ must hold. This equation can be simplified to $2a + 11 = 59$, or $2a = 48$. Therefore, $a = 24$, and Angelica worked 24 hours last week.

Choice B is incorrect because it is the number of hours Raul worked last week. Choice C is incorrect. If Angelica worked 40 hours and Raul worked 11 hours more, Raul would have worked 51 hours, and the combined total number of hours they worked would be 91, not 59. Choice D is incorrect and may be the result of solving the equation $a + 11 = 59$ rather than $a + (a + 11) = 59$.

QUESTION 7.

Choice A is correct. According to the table, of the 50 movies with the greatest ticket sales in 2012, 4 are comedy movies with a PG-13 rating. Therefore, the proportion of the 50 movies with the greatest ticket sales in 2012 that are comedy movies with a PG-13 rating is $\frac{4}{50}$, or equivalently, $\frac{2}{25}$.

Choice B is incorrect; $\frac{9}{50}$ is the proportion of the 50 movies with the greatest ticket sales in 2012 that are comedy movies, regardless of rating. Choice C is incorrect; $\frac{2}{11} = \frac{4}{22}$ is the proportion of movies with a PG-13 rating that are comedy movies. Choice D is incorrect; $\frac{11}{25} = \frac{22}{50}$ is the proportion of the 50 movies with the greatest ticket sales in 2012 that have a rating of PG-13.

QUESTION 8.

Choice D is correct. The quadrants of the xy -plane are defined as follows: Quadrant I is above the x -axis and to the right of the y -axis; Quadrant II is above the x -axis and to the left of the y -axis; Quadrant III is below the x -axis and to the left of the y -axis; and Quadrant IV is below the x -axis and to the right of the y -axis. It is possible for line ℓ to pass through Quadrants II, III, and IV, but not Quadrant I, only if line ℓ has negative x - and y -intercepts. This implies that line ℓ has a negative slope, since between the negative x -intercept and the negative y -intercept the value of x increases (from negative to zero) and the value of y decreases (from zero to negative); so the quotient of the change in y over the change in x , that is, the slope of line ℓ , must be negative.

Choice A is incorrect because a line with an undefined slope is a vertical line, and if a vertical line passes through Quadrant IV, it must pass through Quadrant I as well. Choice B is incorrect because a line with a slope of zero is a horizontal line and, if a horizontal line passes through Quadrant II, it must pass through Quadrant I as well. Choice C is incorrect because if a line with a positive slope passes through Quadrant IV, it must pass through Quadrant I as well.

QUESTION 9.

Choice B is correct. According to the table, in 2012 there was a total of $14,766 + 47,896 = 62,662$ registered voters between 18 and 44 years old, and $3,453 + 11,237 = 14,690$ of them were from the Midwest region. Therefore, the probability that a randomly chosen registered voter who was between 18 and 44 years old in 2012 was from Midwest region is $\frac{14,690}{62,662} \approx 0.234$. Of the given choices, 0.25 is closest to this value.

Choices A, C, and D are incorrect and may be the result of errors in selecting the correct proportion or in calculating the correct value.

QUESTION 10.

Choice A is correct. According to the graph, the animal with the longest gestation period (60 days) has a life expectancy of 3 years.

Choices B, C, and D are incorrect. All the animals that have a life expectancy of 4, 8, or 10 years have a gestation period that is shorter than 60 days, which is the longest gestation period.

QUESTION 11.

Choice A is correct. The ratio of life expectancy to gestation period for the animal represented by point A is approximately $\frac{7 \text{ years}}{23 \text{ days}}$, or about

0.3 years/day, which is greater than the ratio for the animals represented by the other labeled points (the ratios for points B , C , and D , in units of years of life expectancy per day of gestation, are approximately $\frac{8}{44}$, $\frac{8}{51}$, and $\frac{10}{51}$ respectively, each of which is less than 0.2 years/day).

Choices B , C , and D are incorrect and may be the result of errors in calculating the ratio or in reading the graph.

QUESTION 12.

Choice C is correct. All of the given choices are polynomials. If the graph of a polynomial function f in the xy -plane has an x -intercept at b , then $(x - b)$ must be a factor of $f(x)$. Since -3 , -1 , and 1 are each x -intercepts of the graph of f , it follows that $(x + 3)$, $(x + 1)$, and $(x - 1)$ must each be a factor of $f(x)$. The factored polynomial function in choice C is the only polynomial given with these 3 factors.

Choices A , B , and D are incorrect because they do not contain all three factors that must exist if the graph of the polynomial function f has x -intercepts at -3 , -1 , and 1 .

QUESTION 13.

Choice C is correct. The mosquito population starts at 100 in week 0 and then is multiplied by a factor of 10 every 5 weeks. Thus, if $P(t)$ is the mosquito population after t weeks, then based on the table, $P(t) = 100(10)^{\frac{t}{5}}$, which indicates an exponential growth relationship.

Choices A , B , and D are incorrect and may be the result of an incorrect interpretation of the relationship or errors in modeling the relationship.

QUESTION 14.

Choice D is correct. According to the given formula, the amount of money generated for a year at 5% interest, compounded monthly, is $1,000\left(1 + \frac{5}{1,200}\right)^{12}$, whereas the amount of money generated at 3% interest, compounded monthly, is $1,000\left(1 + \frac{3}{1,200}\right)^{12}$. Therefore, the difference between these two amounts, $1,000\left(1 + \frac{5}{1,200}\right)^{12} - 1,000\left(1 + \frac{3}{1,200}\right)^{12}$, shows how much additional money is generated at an interest rate of 5% than at an interest rate of 3%.

Choices A , B , and C are incorrect and may be the result of misinterpreting the given formula. For example, the expression in choice C gives how many times as much money, not how much additional money, is generated at an interest rate of 5% than at an interest rate of 3%.

QUESTION 15.

Choice B is correct. The graph of $y = ax^b$, where a is positive and b is negative, has a positive y -intercept and rapidly decreases (in particular, decreases at a faster rate than a linear function) toward the x -axis as x increases. Of the scatterplots shown, only the one in choice B would be appropriately modeled by such a function.

Choice A is incorrect, as this scatterplot is appropriately modeled by a linear function. Choice C is incorrect, as this scatterplot is appropriately modeled by an increasing function. Choice D is incorrect, as this scatterplot shows no clear relationship between x and y .

QUESTION 16.

Choice A is correct. The total cost y , in dollars, of buying the materials and renting the tools for x days from Store A and Store B is found by substituting the respective values for these stores from the table into the given equation, $y = M + (W + K)x$, as shown below.

$$\text{Store A: } y = 750 + (15 + 65)x = 750 + 80x$$

$$\text{Store B: } y = 600 + (25 + 80)x = 600 + 105x$$

Thus, the number of days, x , for which the total cost of buying the materials and renting the tools from Store B is less than or equal to the total cost of buying the materials and renting the tools from Store A can be found by solving the inequality $600 + 105x \leq 750 + 70x$. Subtracting $80x$ and 600 from each side of $600 + 105x \leq 750 + 70x$ and combining like terms yields $25x \leq 150$. Dividing each side of $25x \leq 150$ by 25 yields $x \leq 6$.

Choice B is incorrect. The inequality $x \geq 6$ is the number of days for which the total cost of buying the materials and renting the tools from Store B is greater than or equal to the total cost of buying the materials and renting the tools from Store A. Choices C and D are incorrect and may be the result of an error in setting up or simplifying the inequality.

QUESTION 17.

Choice D is correct. The total cost, y , of buying the materials and renting the tools in terms of the number of days, x , is given as $y = M + (W + K)x$. If this relationship is graphed in the xy -plane, the slope of the graph is equal to $W + K$, which is the daily rental cost of the wheelbarrow plus the daily rental cost of the concrete mixer, that is, the total daily rental costs of the tools.

Choice A is incorrect because the total cost of the project is y . Choice B is incorrect because the total cost of the materials is M , which is the y -intercept of the graph of $y = M + (W + K)x$. Choice C is incorrect because the total daily cost of the project is the total cost of the project divided by the total number of days the project took and, since materials cost more than 0 dollars, this is not the same as the total daily rental costs.

QUESTION 18.

Choice C is correct. The volume V of a right circular cylinder is given by the formula $V = \pi r^2 h$, where r is the base radius of the cylinder and h is the height of the cylinder. Since each glass has an internal diameter of 3 inches, each glass has a base radius of $\frac{3}{2}$ inches. Since the height of the milk in each glass is 6 inches, the volume of milk in each glass is $V = \pi \left(\frac{3}{2}\right)^2 (6) \approx 42.41$ cubic inches. The total number of glasses Jim can pour from 1 gallon is equal to $\frac{\text{number of cubic inches in 1 gallon}}{\text{number of cubic inches in 1 glass}} = \frac{231}{42.41}$, which is approximately 5.45 glasses. Since the question asks for the largest number of full glasses Jim can pour, the number of glasses needs to be rounded down to 5.

Choices A, B, and D are incorrect and may be the result of conceptual errors or calculation errors. For example, choice D is incorrect because even though Jim can pour more than 5 full glasses, he will not have enough milk to pour a full 6th glass.

QUESTION 19.

Choice A is correct. Adding 4 to each side of the inequality $3p - 2 \geq 1$ yields the inequality $3p + 2 \geq 5$. Therefore, the least possible value of $3p + 2$ is 5.

Choice B is incorrect because it gives the least possible value of $3p$, not of $3p + 2$. Choice C is incorrect. If the least possible value of $3p + 2$ were 2, then it would follow that $3p + 2 \geq 2$. Subtracting 4 from each side of this inequality would yield $3p - 2 \geq -2$. This contradicts the given inequality, $3p - 2 \geq 1$. Therefore, the least possible value of $3p + 2$ cannot be 2. Choice D is incorrect because it gives the least possible value of p , not of $3p + 2$.

QUESTION 20.

Choice C is correct. Since the biomass of the lake doubles each year, the biomass starts at a positive value and then increases exponentially over time. Of the graphs shown, only the graph in choice C is of an increasing exponential function.

Choice A is incorrect because the biomass of the lake must start at a positive value, not zero. Furthermore, this graph shows linear growth, not exponential growth. Choice B is incorrect because the biomass of the lake must start at a positive value, not zero. Furthermore, this graph has vertical segments and is not a function. Choice D is incorrect because the biomass of the lake does not remain the same over time.

QUESTION 21.

Choice C is correct. The exact coordinates of the scatterplot in the xy -plane cannot be read from the bar graph provided. However, for a data point to be

above the line $y = x$, the value of y must be greater than the value of x . That is, the consumption in 2010 must be greater than the consumption in 2000. This occurs for 3 types of energy sources shown in the bar graph: biofuels, geothermal, and wind.

Choices A, B, and D are incorrect and may be the result of a conceptual error in presenting the data shown in a scatterplot. For example, choice B is incorrect because there are 2 data points in the scatterplot that lie below the line $y = x$.

QUESTION 22.

Choice B is correct. Reading the graph, the amount of wood power used in 2000 was 2.25 quadrillion BTUs and the amount used in 2010 was 2.00 quadrillion BTUs. To find the percent decrease, find the difference between the two numbers, divide by the original value, and then multiply by 100: $\frac{2.25 - 2.00}{2.25} \times 100 = \frac{0.25}{2.25} \times 100 \approx 11.1$ percent. Of the choices given, 11% is closest to the percent decrease in the consumption of wood power from 2000 to 2010.

Choices A, C, and D are incorrect and may be the result of errors in reading the bar graph or in calculating the percent decrease.

QUESTION 23.

Choice B is correct. The standard deviation is a measure of how far the data set values are from the mean. In the data set for City A, the large majority of the data are in three of the five possible values, which are the three values closest to the mean. In the data set for City B, the data are more spread out, with many values at the minimum and maximum values. Therefore, by observation, the data for City B have a larger standard deviation.

Alternatively, one can calculate the mean and visually inspect the difference between the data values and the mean. For City A the mean is $\frac{1,655}{21} \approx 78.8$, and for City B the mean is $\frac{1,637}{21} \approx 78.0$. The data for City A are closely clustered near 79, which indicates a small standard deviation. The data for City B are spread out away from 78, which indicates a larger standard deviation.

Choices A, C, and D are incorrect and may be the result of misconceptions about the standard deviation.

QUESTION 24.

Choice C is correct. Since segment AB is a diameter of the circle, it follows that arc \widehat{ADB} is a semicircle. Thus, the circumference of the circle is twice the length of arc \widehat{ADB} which is $2(8\pi) = 16\pi$. Since the circumference of a circle is 2π times the radius of the circle, the radius of this circle is 16π divided by 2π , which is equal to 8.

Choices A, B, and D are incorrect and may be the result of losing track of factors of 2 or of solving for the diameter of the circle instead of the radius. For example, choice D is the diameter of the circle.

QUESTION 25.

Choice B is correct. In $f(x)$, factoring out the greatest common factor, $2x$, yields $f(x) = 2x(x^2 + 3x + 2)$. It is given that $g(x) = x^2 + 3x + 2$, so using substitution, $f(x)$ can be rewritten as $f(x) = 2x \cdot g(x)$. In the equation $p(x) = f(x) + 3g(x)$, substituting $2x \cdot g(x)$ for $f(x)$ yields $p(x) = 2x \cdot g(x) + 3 \cdot g(x)$. In $p(x)$, factoring out the greatest common factor, $g(x)$, yields $p(x) = (g(x))(2x + 3)$. Because $2x + 3$ is a factor of $p(x)$, it follows that $p(x)$ is divisible by $2x + 3$.

Choices A, C, and D are incorrect because $2x + 3$ is not a factor of the polynomials $h(x)$, $r(x)$, or $s(x)$. Using the substitution $f(x) = 2x \cdot g(x)$, and factoring further, $h(x)$, $r(x)$, and $s(x)$ can be rewritten as follows:

$$h(x) = (x + 1)(x + 2)(2x + 1)$$

$$r(x) = (x + 1)(x + 2)(4x + 3)$$

$$s(x) = 2(x + 1)(x + 2)(3x + 1)$$

Because $2x + 3$ is not a factor of $h(x)$, $r(x)$, or $s(x)$, it follows that $h(x)$, $r(x)$, and $s(x)$ are not divisible by $2x + 3$.

QUESTION 26.

Choice C is correct. If $-y < x < y$, the value of x is either between $-y$ and 0 or between 0 and y , so statement I, $|x| < y$ is true. It is possible that the value of x is greater than zero, but x could be negative. For example, a counterexample to statement II, $x > 0$, is $x = -2$ and $y = 3$, yielding $-3 < -2 < 3$, so the given condition is satisfied. Statement III must be true since $-y < x < y$ implies that $-y < y$, so y must be greater than 0. Therefore, statements I and III are the only statements that must be true.

Choices A, B, and D are incorrect because each of these choices either omits a statement that must be true or includes a statement that could be false.

QUESTION 27.

Choice D is correct. To interpret what the number 61 in the equation of the line of best fit represents, one must first understand what the data in the scatterplot represent. Each of the points in the scatterplot represents a large US city, graphed according to its population density (along the horizontal axis) and its relative housing cost (along the vertical axis). The line of best fit for this data represents the expected relative housing cost for a certain population density, based on the data points in the graph. Thus, one might say, on average, a city of population density x is expected to have a relative

housing cost of $y\%$, where $y = 0.0125x + 61$. The number 61 in the equation represents the y -intercept of the line of best fit, in that when the population density, x , is 0, there is an expected relative housing cost of 61%. This might not make the best sense within the context of the problem, in that when the population density is 0, the population is 0, so there probably wouldn't be any housing costs. However, it could be interpreted that for cities with low population densities, housing costs were likely around or above 61% (since below 61% would be for cities with negative population densities, which is impossible).

Choice A is incorrect because it interprets the values of the vertical axis as dollars and not percentages. Choice B is incorrect because the lowest housing cost is about 61% of the national average, not 61% of the highest housing cost. Choice C is incorrect because one cannot absolutely assert that no city with a low population density had housing costs below 61% of the national average, as the model shows that it is unlikely, but not impossible.

QUESTION 28.

Choice D is correct. The minimum value of a quadratic function appears as a constant in the vertex form of its equation, which can be found from the standard form by completing the square. Rewriting $f(x) = (x + 6)(x - 4)$ in standard form gives $f(x) = x^2 + 2x - 24$. Since the coefficient of the linear term is 2, the equation for $f(x)$ can be rewritten in terms of $(x + 1)^2$ as follows:

$$f(x) = x^2 + 2x - 24 = (x^2 + 2x + 1) - 1 - 24 = (x + 1)^2 - 25$$

Since the square of a real number is always nonnegative, the vertex form $f(x) = (x + 1)^2 - 25$ shows that the minimum value of f is -25 (and occurs at $x = -1$). Therefore, this equivalent form of f shows the minimum value of f as a constant.

Choices A and C are incorrect because they are not equivalent to the given equation for f . Choice B is incorrect because the minimum value of f , which is -25 , does not appear as a constant or a coefficient.

QUESTION 29.

Choice B is correct. Since the average of 2 numbers is the sum of the 2 numbers divided by 2, the equations $x = \frac{m+9}{2}$, $y = \frac{2m+15}{2}$ and $z = \frac{3m+18}{2}$ are true. The average of x , y , and z is given by $\frac{x+y+z}{3}$. Substituting the preceding expressions in m for each variable gives $\frac{\frac{m+9}{2} + \frac{2m+15}{2} + \frac{3m+18}{2}}{3}$. This fraction can be simplified to $\frac{6m+42}{6}$, or $m+7$.

Choices A, C, and D are incorrect and may be the result of conceptual errors or calculation errors. For example, choice D is the sum of x , y , and z , not the average.

QUESTION 30.

Choice D is correct. The equation $f(x) = k$ gives the solutions to the system of equations $y = f(x) = x^3 - x^2 - x - \frac{11}{4}$ and $y = k$. A real solution of a system of two equations corresponds to a point of intersection of the graphs of the two equations in the xy -plane. The graph of $y = k$ is a horizontal line that contains the point $(0, k)$. Thus, the line with equation $y = -3$ is a horizontal line that intersects the graph of the cubic equation three times, and it follows that the equation $f(x) = -3$ has three real solutions.

Choices A, B, and C are incorrect because the graphs of the corresponding equations are horizontal lines that do not intersect the graph of the cubic equation three times.

QUESTION 31.

The correct answer is 1160. The pool contains 600 gallons of water before the hose is turned on, and water flows from the hose into the pool at a rate of 8 gallons per minute. Thus, the number of gallons of water in the pool m minutes after the hose is turned on is given by the expression $600 + 8m$. Therefore, after 70 minutes, there will be $600 + 8(70) = 1160$ gallons of water in the pool.

QUESTION 32.

The correct answer is $\frac{1}{2}$ or .5. The equation that models the normal systolic blood pressure, in millimeters of mercury, for a male x years old, $P = \frac{x + 220}{2}$, can be rewritten as $P = \frac{1}{2}x + 110$. For each increase of 1 year in age, the value of x increases by 1; hence, P becomes $\frac{1}{2}(x + 1) + 110 = \left(\frac{1}{2}x + 110\right) + \frac{1}{2}$. That is, P increases by $\frac{1}{2}$ millimeter of mercury. Either the fraction $\frac{1}{2}$ or its decimal equivalent, .5, may be gridded as the correct answer.

QUESTION 33.

The correct answer is 4.55. Since there are 16 Roman digits in a Roman pes, 75 digits is equal to $\frac{75}{16}$ pes. Since 1 pes is equal to 11.65 inches, $\frac{75}{16}$ pes is equal to $\frac{75}{16}(11.65)$ inches. Since 12 inches is equal to 1 foot, $\frac{75}{16}(11.65)$ inches is equal to $\frac{75}{16}(11.65)\left(\frac{1}{12}\right) 4.55078125$ feet. Therefore, 75 digits is equal to $\frac{75}{16}(11.65)\left(\frac{1}{12}\right) = 4.55078125$ feet. Rounded to the nearest hundredth of a foot, 75 Roman digits is equal to 4.55 feet.

QUESTION 34.

The correct answer is 150. In the study, 240 male and 160 plus another 100 female bats have been tagged, so that 500 bats have been tagged altogether. If x more male bats must be tagged for $\frac{3}{5}$ of the total number of bats to be male, the proportion $\frac{\text{male bats}}{\text{total bats}} = \frac{240 + x}{500 + x} = \frac{3}{5}$ must be true. Multiplying each side of $\frac{240 + x}{500 + x} = \frac{3}{5}$ by $5(500 + x)$ gives $5(240 + x) = 3(500 + x)$, which simplifies to $1200 + 5x = 1500 + 3x$. Therefore, $x = 150$, and 150 more male bats must be tagged; this will bring the total to 390 male bats out of 650 bats, which is equal to $\frac{3}{5}$.

QUESTION 35.

The correct answer is 2.25 or $\frac{9}{4}$. Let q_s be the dynamic pressure of the slower fluid moving with velocity v_s , and let q_f be the dynamic pressure of the faster fluid moving with velocity v_f . Then $v_f = 1.5v_s$.

Given the equation $q = \frac{1}{2}nv^2$, substituting the dynamic pressure and velocity of the faster fluid gives $q_f = \frac{1}{2}nv_f^2$. Since $v_f = 1.5v_s$, the expression $1.5v_s$ can be substituted for v_f in this equation, giving $q_f = \frac{1}{2}n(1.5v_s)^2$. This can be rewritten as $q_f = (2.25)\frac{1}{2}nv_s^2 = (2.25)q_s$. Therefore, the ratio of the dynamic pressure of the faster fluid is $\frac{q_f}{q_s} = \frac{2.25q_s}{q_s} = 2.25$. Either 2.25 or the equivalent improper fraction $\frac{9}{4}$ may be gridded as the correct answer.

QUESTION 36.

The correct answer is 29, 30, 31, 32, 33, or 34. Since the radius of the circle is 10, its circumference is 20π . The full circumference of a circle is 360° . Thus, an arc of length s on the circle corresponds to a central angle of x° , where $\frac{x}{360} = \frac{s}{20\pi}$, or $x = \frac{360}{20\pi}s$. Since $5 < s < 6$, it follows that $\frac{360}{20\pi}(5) < x < \frac{360}{20\pi}(6)$, which becomes, to the nearest tenth, $28.6 < x < 34.4$. Therefore, the possible integer values of x are 29, 30, 31, 32, 33, and 34. Any one of these numbers may be gridded as the correct answer.

QUESTION 37.

The correct answer is .72. According to the analyst's estimate, the value V , in dollars, of the stock will decrease by 28% each week for t weeks, where $t = 1, 2,$ or 3 , with its value being given by the formula $V = 360(r)^t$. This equation is an example of exponential decay. A stock losing 28% of its value each week is the same as the stock's value decreasing to 72% of its value from the previous week, since $V - (.28)V = (.72)V$. Using this information, after 1 week the value, in dollars, of the stock will be $V = 360(.72)$; after 2 weeks the value of the stock will be $V = 360(.72)(.72) = 360(.72)^2$; and after 3 weeks the value of the stock will be $V = 360(.72)(.72)(.72) = 360(.72)^3$. For all of the values of t in question, namely $t = 1, 2,$ and 3 , the equation $V = 360(.72)^t$ is true. Therefore, the analyst should use .72 as the value of r .

QUESTION 38.

The correct answer is 134. The analyst's prediction is that the stock will lose 28 percent of its value for each of the next three weeks. Thus, the predicted value of the stock after 1 week is $\$360 - (0.28)\$360 = \$259.20$; after 2 weeks, $\$259.20 - (0.28)\$259.20 \approx \$186.62$; and after 3 weeks, $\$186.62 - (0.28)\$186.62 \approx \$134.37$. Therefore, to the nearest dollar, the stock analyst believes the stock will be worth 134 dollars after three weeks.