CollegeBoard

Answer Explanations

SAT® Practice Test #3

5LSA08

Choices B, C, and D are incorrect because each provides a statement that does not logically connect to the examples that follow.

QUESTION 43.

Choice D is the best answer. It accurately states that the information in the proposed additional sentence is not related to formal portraits of cats, the main topic of the paragraph.

Choices A, B, and C are incorrect because each fails to recognize that the proposed sentence interrupts the logical development of the paragraph.

QUESTION 44.

Choice D is the best answer. The tone corresponds with that established in the passage, and the phrasing appropriately focuses on the cats' contribution to protecting artwork rather than on simply killing rodents.

Choices A, B, and C are incorrect because none makes explicit the link between the cats' hunting activities and the service to the museum.

Section 3: Math Test — No Calculator

QUESTION 1.

Choice C is correct. The painter's fee is given by $nK\ell h$, where n is the number of walls, K is a constant with units of dollars per square foot, ℓ is the length of each wall in feet, and h is the height of each wall in feet. Examining this equation shows that ℓ and h will be used to determine the area of each wall. The variable n is the number of walls, so n times the area of the walls will give the amount of area that will need to be painted. The only remaining variable is K, which represents the cost per square foot and is determined by the painter's time and the price of paint. Therefore, K is the only factor that will change if the customer asks for a more expensive brand of paint.

Choice A is incorrect because a more expensive brand of paint would not cause the height of each wall to change. Choice B is incorrect because a more expensive brand of paint would not cause the length of each wall to change. Choice D is incorrect because a more expensive brand of paint would not cause the number of walls to change.

QUESTION 2.

Choice D is correct. Dividing each side of the equation $3r = 18$ by 3 gives $r = 6$. Substituting 6 for r in the expression $6r + 3$ gives $6(6) + 3 = 39$.

Alternatively, the expression $6r + 3$ can be rewritten as $2(3r) + 3$. Substituting 18 for $3r$ in the expression $2(3r) + 3$ yields $2(18) + 3 = 36 + 3 = 39$.

Choice A is incorrect because 6 is the value of r; however, the question asks for the value of the expression $6r + 3$. Choices B and C are incorrect because if $6r + 3$ were equal to either of these values, then it would not be possible for $3r$ to be equal to 18, as stated in the question.

QUESTION 3.

Choice D is correct. By definition, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ for any positive integers m and n. It follows, therefore, that $a^{\frac{2}{3}} = \sqrt[3]{a^2}$.

Choice A is incorrect. By definition, $a^{\frac{1}{n}} = \sqrt[n]{a}$ for any positive integer n. Applying this definition as well as the power property of exponents to the expression $\sqrt{a^{\frac{1}{3}}}$ yields $\sqrt{a^{\frac{1}{3}}} = \left(a^{\frac{1}{3}}\right)^{\frac{1}{2}} = a^{\frac{1}{6}}$. Because $a^{\frac{1}{6}} \neq a^{\frac{2}{3}}$, $\sqrt{a^{\frac{1}{3}}}$ is not the correct answer. Choice B is incorrect. By definition, $a^{\frac{1}{n}} = \sqrt[n]{a}$ for any positive integer n. Applying this definition as well as the power property of exponents to the expression $\sqrt{a^3}$ yields $\sqrt{a^3} = (a^3)^{\frac{1}{2}} = a^{\frac{3}{2}}$. Because $a^{\frac{3}{2}} \neq a^{\frac{2}{3}}$, $\sqrt{a^3}$ is not the correct answer. Choice C is incorrect. By definition, $a^{\frac{1}{n}} = \sqrt[n]{a}$ for any positive integer n. Applying this definition as well as the power property of exponents to the expression $\sqrt[3]{a^{\frac{1}{2}}}$ yields $\sqrt[3]{a^{\frac{1}{2}}} = \left(a^{\frac{1}{2}}\right)^{\frac{1}{3}} = a^{\frac{1}{6}}$. Because $a^{\frac{1}{6}} \neq a^{\frac{2}{3}}$, $\sqrt[3]{a^{\frac{1}{2}}}$ is not the correct answer.

QUESTION 4.

Choice B is correct. To fit the scenario described, 30 must be twice as large as x. This can be written as $2x = 30$.

Choices A, C, and D are incorrect. These equations do not correctly relate the numbers and variables described in the stem. For example, the expression in choice C states that 30 is half as large as x, not twice as large as x.

QUESTION 5.

Choice C is correct. Multiplying each side of $\frac{5}{x} = \frac{15}{x + 20}$ by $x(x + 20)$ gives $15x = 5(x + 20)$. Distributing the 5 over the values within the parentheses yields $15x = 5x + 100$, and then subtracting $5x$ from each side gives $10x = 100$. Finally, dividing both sides by 10 gives $x = 10$. Therefore, the value of $\frac{x}{5}$ is $\frac{10}{5} = 2$.

Choice A is incorrect because it is the value of x, not $\frac{x}{5}$. Choices B and D are incorrect and may be the result of errors in arithmetic operations on the given equation.

QUESTION 6.

Choice C is correct. Multiplying each side of the equation $2x - 3y = -14$ by 3 gives $6x - 9y = -42$. Multiplying each side of the equation $3x - 2y = -6$ by 2 gives $6x - 4y = -12$. Then, subtracting the sides of $6x - 4y = -12$ from the corresponding sides of $6x - 9y = -42$ gives $-5y = -30$. Dividing each side of the equation $-5y = -30$ by -5 gives $y = 6$. Finally, substituting 6 for y in $2x - 3y = -14$ gives $2x - 3(6) = -14$, or $x = 2$. Therefore, the value of $x - y$ is $2 - 6 = -4$.

Alternatively, adding the corresponding sides of $2x - 3y = -14$ and $3x - 2y = -6$ gives $5x - 5y = -20$, from which it follows that $x - y = -4$.

Choices A, B, and D are incorrect and may be the result of an arithmetic error when solving the system of equations.

QUESTION 7.

Choice C is correct. If $x - b$ is a factor of $f(x)$, then $f(b)$ must equal 0. Based on the table, $f(4) = 0$. Therefore, $x - 4$ must be a factor of $f(x)$.

Choice A is incorrect because $f(2) \neq 0$; choice B is incorrect because no information is given about the value of $f(3)$, so $x - 3$ may or may not be a factor of $f(x)$; and choice D is incorrect because $f(5) \neq 0$.

QUESTION 8.

Choice A is correct. The linear equation $y = kx + 4$ is in slope-intercept form, and so the slope of the line is k. Since the line contains the point (c, d), the coordinates of this point satisfy the equation $y = kx + 4$: $d = kc + 4$. Solving this equation for the slope, k, gives $k = \frac{d - 4}{c}$.

Choices B, C, and D are incorrect and may be the result of errors in substituting the coordinates of (c, d) in $y = kx + 4$ or of errors in solving for k in the resulting equation.

QUESTION 9.

Choice A is correct. If a system of two linear equations has no solution, then the lines represented by the equations in the coordinate plane are parallel. The equation $kx - 3y = 4$ can be rewritten as $y = \frac{k}{3}x - \frac{4}{3}$, where $\frac{k}{3}$ is the slope of the line, and the equation $4x - 5y = 7$ can be rewritten as $y = \frac{4}{5}x - \frac{7}{5}$, where $\frac{4}{5}$ is the slope of the line. If two lines are parallel, then the slopes of the line are equal. Therefore, $\frac{4}{5} = \frac{k}{3}$, or $k = \frac{12}{5}$. (Since the y-intercepts of the lines represented by the equations are $-\frac{4}{3}$ and $-\frac{7}{5}$, the lines are parallel, not identical.)

Choices B, C, and D are incorrect and may be the result of a computational error when rewriting the equations or solving the equation representing the equality of the slopes for k.

QUESTION 10.

Choice A is correct. Substituting 25 for y in the equation $y = (x - 11)^2$ gives $25 = (x - 11)^2$. It follows that $x - 11 = 5$ or $x - 11 = -5$, so the x-coordinates of the two points of intersection are $x = 16$ and $x = 6$, respectively. Since both points of intersection have a y-coordinate of 25, it follows that the two points are (16, 25) and (6, 25). Since these points lie on the horizontal line $y = 25$, the distance between these points is the positive difference of the x-coordinates: $16 - 6 = 10$.

Choices B, C, and D are incorrect and may be the result of an error in solving the quadratic equation that results when substituting 25 for y in the given quadratic equation.

QUESTION 11.

Choice B is correct. Since the angles marked $y°$ and $u°$ are vertical angles, $y = u$. Subtracting the sides of $y = u$ from the corresponding sides of $x + y = u + w$ gives $x = w$. Since the angles marked $w°$ and $z°$ are vertical angles, $w = z$. Therefore, $x = z$, and so I must be true.

The equation in II need not be true. For example, if $x = w = z = t = 70$ and $y = u = 40$, then all three pairs of vertical angles in the figure have equal measure and the given condition $x + y = u + w$ holds. But it is not true in this case that y is equal to w. Therefore, II need not be true.

Since the top three angles in the figure form a straight angle, it follows that $x + y + z = 180$. Similarly, $w + u + t = 180$, and so $x + y + z = w + u + t$. Subtracting the sides of the given equation $x + y = u + w$ from the corresponding sides of $x + y + z = w + u + t$ gives $z = t$. Therefore, III must be true. Since only I and III must be true, the correct answer is choice B.

Choices A, C, and D are incorrect because each of these choices includes II, which need not be true.

QUESTION 12.

Choice A is correct. The parabola with equation $y = a(x - 2)(x + 4)$ crosses the x-axis at the points $(-4, 0)$ and $(2, 0)$. The x-coordinate of the vertex of the parabola is halfway between the x-coordinates of $(-4, 0)$ and $(2, 0)$. Thus, the x-coordinate of the vertex is $\frac{-4 + 2}{2} = -1$. This is the value of c. To find the y-coordinate of the vertex, substitute -1 for x in $y = a(x - 2)(x + 4)$: $y = a(x - 2)(x + 4) = a(-1 - 2)(-1 + 4) = a(-3)(3) = -9a$.

Therefore, the value of d is $-9a$.

Choice B is incorrect because the value of the constant term in the equation is not the y-coordinate of the vertex, unless there were no linear terms in the quadratic. Choice C is incorrect and may be the result of a sign error in finding the x-coordinate of the vertex. Choice D is incorrect because the negative of the coefficient of the linear term in the quadratic is not the y-coordinate of the vertex.

QUESTION 13.

Choice B is correct. Since $24x^2 + 25x - 47$ divided by $ax - 2$ is equal to $-8x - 3$ with remainder -53, it is true that $(-8x - 3)(ax - 2) - 53 = 24x^2 + 25x - 47$. (This can be seen by multiplying each side of the given equation by $ax - 2$). This can be rewritten as $-8ax^2 + 16x - 3ax = 24x^2 + 25x - 47$. Since the coefficients of the x^2-term have to be equal on both sides of the equation, $-8a = 24$, or $a = -3$.

Choices A, C, and D are incorrect and may be the result of either a conceptual misunderstanding or a computational error when trying to solve for the value of a.

QUESTION 14.

Choice A is correct. Dividing each side of the given equation by 3 gives the equivalent equation $x^2 + 4x + 2 = 0$. Then using the quadratic formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, with $a = 1$, $b = 4$, and $c = 2$, gives the solutions $x = -2 \pm \sqrt{2}$.

Choices B, C, and D are incorrect and may be the result of errors when applying the quadratic formula.

QUESTION 15.

Choice D is correct. If C is graphed against F, the slope of the graph is equal to $\frac{5}{9}$ degrees Celsius/degrees Fahrenheit, which means that for an increase of 1 degree Fahrenheit, the increase is $\frac{5}{9}$ of 1 degree Celsius. Thus, statement I is true. This is the equivalent to saying that an increase of 1 degree Celsius is equal to an increase of $\frac{9}{5}$ degrees Fahrenheit. Since $\frac{9}{5} = 1.8$, statement II is true. On the other hand, statement III is not true, since a temperature increase of $\frac{9}{5}$ degrees Fahrenheit, not $\frac{5}{9}$ degree Fahrenheit, is equal to a temperature increase of 1 degree Celsius.

Choices A, B, and C are incorrect because each of these choices omits a true statement or includes a false statement.

QUESTION 16.

The correct answer is either 1 or 2. The given equation can be rewritten as $x^5 - 5x^3 + 4x = 0$. Since the polynomial expression on the left has no constant term, it has x as a factor: $x(x^4 - 5x^2 + 4) = 0$. The expression in parentheses is a quadratic equation in x^2 that can be factored, giving $x(x^2 - 1)(x^2 - 4) = 0$. This further factors as $x(x - 1)(x + 1)(x - 2)(x + 2) = 0$. The solutions for x are $x = 0$, $x = 1$, $x = -1$, $x = 2$, and $x = -2$. Since it is given that $x > 0$, the possible values of x are $x = 1$ and $x = 2$. Either 1 or 2 may be gridded as the correct answer.

QUESTION 17.

The correct answer is 2. First, clear the fractions from the given equation by multiplying each side of the equation by 36 (the least common multiple of 4, 9, and 12). The equation becomes $28x - 16x = 9 + 15$. Combining like terms on each side of the equation yields $12x = 24$. Finally, dividing both sides of the equation by 12 yields $x = 2$.

Alternatively, since $\frac{7}{9}x - \frac{4}{9}x = \frac{3}{9}x = \frac{1}{3}x$ and $\frac{1}{4} + \frac{5}{12} = \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$, the given equation simplifies to $\frac{1}{3}x = \frac{2}{3}$. Multiplying each side of $\frac{1}{3}x = \frac{2}{3}$ by 3 yields $x = 2$.

QUESTION 18.

The correct answer is 105. Since $180 - z = 2y$ and $y = 75$, it follows that $180 - z = 150$, and so $z = 30$. Thus, each of the base angles of the isosceles triangle on the right has measure $\frac{180° - 30°}{2} = 75°$. Therefore, the measure of the angle marked $x°$ is $180° - 75° = 105°$, and so the value of x is 105.

QUESTION 19.

The correct answer is 370. A system of equations can be used where h represents the number of calories in a hamburger and f represents the number of calories in an order of fries. The equation $2h + 3f = 1700$ represents the fact that 2 hamburgers and 3 orders of fries contain a total of 1700 calories, and the equation $h = f + 50$ represents the fact that one hamburger contains 50 more calories than an order of fries. Substituting $f + 50$ for h in $2h + 3f = 1700$ gives $2(f + 50) + 3f = 1700$. This equation can be solved as follows:

$$2f + 100 + 3f = 1700$$
$$5f + 100 = 1700$$
$$5f = 1600$$
$$f = 320$$

The number of calories in an order of fries is 320, so the number of calories in a hamburger is 50 more than 320, or 370.

QUESTION 20.

The correct answer is $\frac{3}{5}$ or .6. Triangle ABC is a right triangle with its right angle at B. Thus, $\overline{AC}$ is the hypotenuse of right triangle ABC, and $\overline{AB}$ and $\overline{BC}$ are the legs of right triangle ABC. By the Pythagorean theorem, $AB = \sqrt{20^2 - 16^2} = \sqrt{400 - 256} = \sqrt{144} = 12$. Since triangle DEF is similar to triangle ABC, with vertex F corresponding to vertex C, the measure of angle F equals the measure of angle C. Thus, $\sin F = \sin C$. From the side lengths of triangle ABC, $\sin C = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12}{20} = \frac{3}{5}$. Therefore, $\sin F = \frac{3}{5}$. Either $\frac{3}{5}$ or its decimal equivalent, .6, may be gridded as the correct answer.

Section 4: Math Test – Calculator

QUESTION 1.

Choice C is correct. Marilyn's distance from her campsite remained the same during the time she ate lunch. This is represented by a horizontal segment in the graph. The only horizontal segment in the graph starts at a time of about 1:10 P.M. and ends at about 1:40 P.M. Therefore, Marilyn finished her lunch and continued her hike at about 1:40 P.M.

Choices A, B, and D are incorrect and may be the result of a misinterpretation of the graph. For example, choice B is the time Marilyn started her lunch, and choice D is the time Marilyn was at the maximum distance from her campsite.

QUESTION 2.

Choice B is correct. Of the 25 people who entered the contest, there are 8 females under age 40 and 2 males age 40 or older. Therefore, the probability that the contest winner will be either a female under age 40 or a male age 40 or older is $\frac{8}{25} + \frac{2}{25} = \frac{10}{25}$.

Choice A is incorrect and may be the result of dividing 8 by 2, instead of adding 8 to 2, to find the probability. Choice C is incorrect; it is the probability that the contest winner will be either a female under age 40 or a female age 40 or older. Choice D is incorrect and may be the result of multiplying 8 and 2, instead of adding 8 and 2, to find the probability.

QUESTION 3.

Choice C is correct. Based on the graph, sales increased in the first 3 years since 1997, which is until year 2000, and then generally decreased thereafter.

Choices A, B, and D are incorrect; each of these choices contains inaccuracies in describing the general trend of music album sales from 1997 to 2000.

QUESTION 4.

Choice C is correct. The graph of $y = f(n)$ in the coordinate plane is a line that passes through each of the points given in the table. From the table, one can see that an increase of 1 unit in n results in an increase of 3 units in $f(n)$; for example, $f(2) - f(1) = 1 - (-2) = 3$. Therefore, the graph of $y = f(n)$ in the coordinate plane is a line with slope 3. Only choice C is a line with slope 3. The y-intercept of the line is the value of $f(0)$. Since an increase of 1 unit in n results in an increase of 3 units in $f(n)$, it follows that $f(1) - f(0) = 3$. Since $f(1) = -2$, it follows that $f(0) = f(1) - 3 = -5$. Therefore, the y-intercept of the graph of $f(n)$ is -5, and the slope-intercept equation for $f(n)$ is $f(n) = 3n - 5$.

Choices A, B, and D are incorrect because each equation has the incorrect slope of the line (the y-intercept in each equation is also incorrect).

QUESTION 5.

Choice B is correct. Since 7 percent of the 562 juniors is 0.07(562) and 5 percent of the 602 seniors is 0.05(602), the expression 0.07(562) + 0.05(602) can be evaluated to determine the total number of juniors and seniors inducted into the honor society. Of the given choices, 69 is closest to the value of the expression.

Choice A is incorrect and may be the result of adding the number of juniors and seniors and the percentages given and then using the expression (0.07 + 0.05)(562 + 602). Choices C and D are incorrect and may be the result of finding either only the number of juniors inducted or only the number of seniors inducted.

QUESTION 6.

Choice A is correct. The sum of the two polynomials is $(3x^2 - 5x + 2) + (5x^2 - 2x - 6)$. This can be rewritten by combining like terms:

$$(3x^2 - 5x + 2) + (5x^2 - 2x - 6) = (3x^2 + 5x^2) + (-5x - 2x) + (2 - 6) = 8x^2 - 7x - 4.$$

Choice B is incorrect and may be the result of a sign error when combining the coefficients of the x-term. Choice C is incorrect and may be the result of adding the exponents, as well as the coefficients, of like terms. Choice D is incorrect and may be the result of a combination of the errors described in B and C.

QUESTION 7.

Choice D is correct. To solve the equation for w, multiply both sides of the equation by the reciprocal of $\frac{3}{5}$, which is $\frac{5}{3}$. This gives $\left(\frac{5}{3}\right) \cdot \frac{3}{5}w = \frac{4}{5} \cdot \left(\frac{5}{3}\right)$, which simplifies to $w = \frac{20}{9}$.

Choices A, B, and C are incorrect and may be the result of errors in arithmetic when simplifying the given equation.

QUESTION 8.

Choice C is correct. In the equation $y = 0.56x + 27.2$, the value of x increases by 1 for each year that passes. Each time x increases by 1, y increases by 0.56 since 0.56 is the slope of the graph of this equation. Since y represents the average number of students per classroom in the year represented by x, it follows that, according to the model, the estimated increase each year in the average number of students per classroom at Central High School is 0.56.

Choice A is incorrect because the total number of students in the school in 2000 is the product of the average number of students per classroom and the total number of classrooms, which would appropriately be approximated by the y-intercept (27.2) times the total number of classrooms, which is not given. Choice B is incorrect because the average number of students per classroom in 2000 is given by the y-intercept of the graph of the equation, but the question is asking for the meaning of the number 0.56, which is the slope. Choice D is incorrect because 0.56 represents the estimated yearly change in the average number of students per classroom. The estimated difference between the average number of students per classroom in 2010 and 2000 is 0.56 times the number of years that have passed between 2000 and 2010, that is, $0.56 \times 10 = 5.6$.

QUESTION 9.

Choice B is correct. Because Nate walks 25 meters in 13.7 seconds, and 4 minutes is equal to 240 seconds, the proportion $\frac{25 \text{ meters}}{13.7 \text{ sec}} = \frac{x \text{ meters}}{240 \text{ sec}}$ can be used to find out how many meters, x, Nate walks in 4 minutes. The proportion can be simplified to $\frac{25}{13.7} = \frac{x}{240}$, because the units of meters per second cancel, and then each side of the equation can be multiplied by 240, giving $\frac{(240)(25)}{13.7} = x \approx 438$. Therefore, of the given options, 450 meters is closest to the distance Nate will walk in 4 minutes.

Choice A is incorrect and may be the result of setting up the proportion as $\frac{13.7 \text{ sec}}{25 \text{ meters}} = \frac{x \text{ meters}}{240 \text{ sec}}$ and finding that $x \approx 132$, which is close to 150. Choices C and D are incorrect and may be the result of errors in calculation.

QUESTION 10.

Choice D is correct. On Mercury, the acceleration due to gravity is 3.6 m/sec^2. Substituting 3.6 for g and 90 for m in the formula $W = mg$ gives $W = 90(3.6) = 324$ newtons.

Choice A is incorrect and may be the result of dividing 90 by 3.6. Choice B is incorrect and may be the result of subtracting 3.6 from 90 and rounding to the nearest whole number. Choice C is incorrect because an object with a weight of 101 newtons on Mercury would have a mass of about 28 kilograms, not 90 kilograms.

QUESTION 11.

Choice B is correct. On Earth, the acceleration due to gravity is 9.8 m/sec^2. Thus, for an object with a weight of 150 newtons, the formula $W = mg$ becomes $150 = m(9.8)$, which shows that the mass of an object with a weight of 150 newtons on Earth is about 15.3 kilograms. Substituting this mass into the formula $W = mg$ and now using the weight of 170 newtons gives $170 = 15.3g$, which shows that the second planet's acceleration due to gravity is about 11.1 m/sec^2. According to the table, this value for the acceleration due to gravity holds on Saturn.

Choices A, C, and D are incorrect. Using the formula $W = mg$ and the values for g in the table shows that an object with a weight of 170 newtons on these planets would not have the same mass as an object with a weight of 150 newtons on Earth.

QUESTION 12.

Choice D is correct. A zero of a function corresponds to an x-intercept of the graph of the function in the xy-plane. Therefore, the complete graph of the function f, which has five distinct zeros, must have five x-intercepts. Only the graph in choice D has five x-intercepts, and therefore, this is the only one of the given graphs that could be the complete graph of f in the xy-plane.

Choices A, B, and C are incorrect. The number of x-intercepts of each of these graphs is not equal to five; therefore, none of these graphs could be the complete graph of f, which has five distinct zeros.

QUESTION 13.

Choice D is correct. Starting with the original equation, $h = -16t^2 + vt + k$, in order to get v in terms of the other variables, $-16t^2$ and k need to be subtracted from each side. This yields $vt = h + 16t^2 - k$, which when divided by t will give v in terms of the other variables. However, the equation $v = \frac{h + 16t^2 - k}{t}$ is not one of the options, so the right side needs to be further simplified. Another way to write the previous equation is $v = \frac{h - k}{t} + \frac{16t^2}{t}$, which can be simplified to $v = \frac{h - k}{t} + 16t$.

Choices A, B, and C are incorrect and may be the result of arithmetic errors when rewriting the original equation to express v in terms of h, t, and k.

QUESTION 14.

Choice A is correct. The hotel charges \$0.20 per minute to use the meeting-room phone. This per-minute rate can be converted to the hourly rate using the conversion 1 hour = 60 minutes, as shown below.

$$\frac{\$0.20}{\text{minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{\$(0.20 \times 60)}{\text{hour}}$$

Thus, the hotel charges $\$(0.20 \times 60)$ per hour to use the meeting-room phone. Therefore, the cost c, in dollars, for h hours of use is $c = (0.20 \times 60)h$, which is equivalent to $c = 0.20(60h)$.

Choice B is incorrect because in this expression the per-minute rate is multiplied by h, the number of <u>hours</u> of phone use. Furthermore, the equation indicates that there is a flat fee of \$60 in addition to the per-minute or per-hour rate. This is not the case. Choice C is incorrect because the expression indicates that the hotel charges $\$\left(\frac{60}{0.20}\right)$ per hour for use of the meeting-room phone, not \$0.20(60) per hour. Choice D is incorrect because the expression indicates that the hourly rate is $\frac{1}{60}$ times the per-minute rate, not 60 times the per-minute rate.

QUESTION 15.

Choice A is the correct answer. Experimental research is a method used to study a small group of people and generalize the results to a larger population. However, in order to make a generalization involving cause and effect:

- The population must be well defined.
- The participants must be selected at random.
- The participants must be randomly assigned to treatment groups.

When these conditions are met, the results of the study can be generalized to the population with a conclusion about cause and effect. In this study, all conditions are met and the population from which the participants were selected are people with poor eyesight. Therefore, a general conclusion can be drawn about the effect of Treatment X on the population of people with poor eyesight.

Choice B is incorrect. The study did not include all available treatments, so no conclusion can be made about the relative effectiveness of all available treatments. Choice C is incorrect. The participants were selected at random from a large population of people with poor eyesight. Therefore, the results can be generalized only to that population and not to anyone in general. Also, the conclusion is too strong: an experimental study might show that people are likely to be helped by a treatment, but it cannot show that <u>anyone</u> who takes the treatment will be helped. Choice D is incorrect.

This conclusion is too strong. The study shows that Treatment X is likely to improve the eyesight of people with poor eyesight, but it cannot show that the treatment definitely will cause improvement in eyesight for every person. Furthermore, since the people undergoing the treatment in the study were selected from people with poor eyesight, the results can be generalized only to this population, not to all people.

QUESTION 16.

Choice B is correct. For any value of x, say $x = x_0$, the point $(x_0, f(x_0))$ lies on the graph of f and the point $(x_0, g(x_0))$ lies on the graph of g. Thus, for any value of x, say $x = x_0$, the value of $f(x_0) + g(x_0)$ is equal to the sum of the y-coordinates of the points on the graphs of f and g with x-coordinate equal to x_0. Therefore, the value of x for which $f(x) + g(x)$ is equal to 0 will occur when the y-coordinates of the points representing $f(x)$ and $g(x)$ at the same value of x are equidistant from the x-axis and are on opposite sides of the x-axis. Looking at the graphs, one can see that this occurs at $x = -2$: the point $(-2, -2)$ lies on the graph of f, and the point $(-2, 2)$ lies on the graph of g. Thus, at $x = -2$, the value of $f(x) + g(x)$ is $-2 + 2 = 0$.

Choices A, C, and D are incorrect because none of these x-values satisfy the given equation, $f(x) + g(x) = 0$.

QUESTION 17.

Choice B is correct. The quantity of the product supplied to the market is given by the function $S(P) = \frac{1}{2}P + 40$. If the price P of the product increases by \$10, the effect on the quantity of the product supplied can be determined by substituting $P + 10$ for P as the argument in the function. This gives $S(P + 10) = \frac{1}{2}(P + 10) + 40 = \frac{1}{2}P + 45$, which shows that $S(P + 10) = S(P) + 5$. Therefore, the quantity supplied to the market will increase by 5 units when the price of the product is increased by \$10.

Alternatively, look at the coefficient of P in the linear function S. This is the slope of the graph of the function, where P is on the horizontal axis and $S(P)$ is on the vertical axis. Since the slope is $\frac{1}{2}$, for every increase of 1 in P, there will be an increase of $\frac{1}{2}$ in $S(P)$, and therefore, an increase of 10 in P will yield an increase of 5 in $S(P)$.

Choice A is incorrect. If the quantity supplied decreases as the price of the product increases, the function $S(P)$ would be decreasing, but $S(P) = \frac{1}{2}P + 40$ is an increasing function. Choice C is incorrect and may be the result of assuming the slope of the graph of $S(P)$ is equal to 1. Choice D is incorrect and may be the result of confusing the y-intercept of the graph of $S(P)$ with the slope, and then adding 10 to the y-intercept.

QUESTION 18.

Choice B is correct. The quantity of the product supplied to the market will equal the quantity of the product demanded by the market if $S(P)$ is equal to $D(P)$, that is, if $\frac{1}{2}P + 40 = 220 - P$. Solving this equation gives $P = 120$, and so \$120 is the price at which the quantity of the product supplied will equal the quantity of the product demanded.

Choices A, C, and D are incorrect. At these dollar amounts, the quantities given by $S(P)$ and $D(P)$ are not equal.

QUESTION 19.

Choice C is correct. It is given that 1 ounce of graphene covers 7 football fields. Therefore, 48 ounces can cover $7 \times 48 = 336$ football fields. If each football field has an area of $1\frac{1}{3}$ acres, than 336 football fields have a total area of $336 \times 1\frac{1}{3} = 448$ acres. Therefore, of the choices given, 450 acres is closest to the number of acres 48 ounces of graphene could cover.

Choice A is incorrect and may be the result of dividing, instead of multiplying, the number of football fields by $1\frac{1}{3}$. Choice B is incorrect and may be the result of finding the number of football fields, not the number of acres, that can be covered by 48 ounces of graphene. Choice D is incorrect and may be the result of setting up the expression $\frac{7 \times 48 \times 4}{3}$ and then finding only the numerator of the fraction.

QUESTION 20.

Choice B is correct. To answer this question, find the point in the graph that represents Michael's 34-minute swim and then compare the actual heart rate for that swim with the expected heart rate as defined by the line of best fit. To find the point that represents Michael's swim that took 34 minutes, look along the vertical line of the graph that is marked "34" on the horizontal axis. That vertical line intersects only one point in the scatterplot, at 148 beats per minute. On the other hand, the line of best fit intersects the vertical line representing 34 minutes at 150 beats per minute. Therefore, for the swim that took 34 minutes, Michael's actual heart rate was $150 - 148 = 2$ beats per minute less than predicted by the line of best fit.

Choices A, C, and D are incorrect and may be the result of misreading the scale of the graph.

QUESTION 21.

Choice C is correct. Let I be the initial savings. If each successive year, 1% of the current value is added to the value of the account, then after 1 year, the amount in the account will be $I + 0.01I = I(1 + 0.01)$; after 2 years, the amount in the account will be $I(1 + 0.01) + 0.01I(1 + 0.01) = (1 + 0.01)I(1 + 0.01) = I(1 + 0.01)^2$; and after t years, the amount in the account will be $I(1 + 0.01)^t$. This is exponential growth of the money in the account.

Choice A is incorrect. If each successive year, 2% of the initial savings, I, is added to the value of the account, then after t years, the amount in the account will be $I + 0.02It$, which is linear growth. Choice B is incorrect. If each successive year, 1.5% of the initial savings, I, and \$100 is added to the value of the the account, then after t years the amount in the account will be $I + (0.015I + 100)t$, which is linear growth. Choice D is incorrect. If each successive year, \$100 is added to the value of the account, then after t years the amount in the account will be $I + 100t$, which is linear growth.

QUESTION 22.

Choice B is correct. One of the three numbers is x; let the other two numbers be y and z. Since the sum of three numbers is 855, the equation $x + y + z = 855$ is true. The statement that x is 50% more than the sum of the other two numbers can be represented as $x = 1.5(y + z)$, or $\frac{x}{1.5} = y + z$. Substituting $\frac{x}{1.5}$ for $y + z$ in $x + y + z = 855$ gives $x + \frac{x}{1.5} = 855$. This last equation can be rewritten as $x + \frac{2x}{3} = 855$, or $\frac{5x}{3} = 855$. Therefore, x equals $\frac{3}{5} \times 855 = 513$.

Choices A, C, and D are incorrect and may be the result of calculation errors.

QUESTION 23.

Choice C is correct. Since the angles are acute and $\sin(a°) = \cos(b°)$, it follows from the complementary angle property of sines and cosines that $a + b = 90$. Substituting $4k - 22$ for a and $6k - 13$ for b gives $(4k - 22) + (6k - 13) = 90$, which simplifies to $10k - 35 = 90$. Therefore, $10k = 125$, and $k = 12.5$.

Choice A is incorrect and may be the result of mistakenly assuming that $a + b$ and making a sign error. Choices B and D are incorrect because they result in values for a and b such that $\sin(a°) \neq \cos(b°)$.

QUESTION 24.

Choice D is correct. Let c be the number of students in Mr. Kohl's class. The conditions described in the question can be represented by the equations $n = 3c + 5$ and $n + 21 = 4c$. Substituting $3c + 5$ for n in the second equation gives $3c + 5 + 21 = 4c$, which can be solved to find $c = 26$.

Choices A, B, and C are incorrect because the values given for the number of students in the class cannot fulfill both conditions given in the question. For example, if there were 16 students in the class, then the first condition would imply that there are $3(16) + 5 = 53$ milliliters of solution in the beaker, but the second condition would imply that there are $4(16) - 21 = 43$ milliliters of solution in the beaker. This contradiction shows that there cannot be 16 students in the class.

QUESTION 25.

Choice D is correct. The volume of the grain silo can be found by adding the volumes of all the solids of which it is composed. The silo is made up of a cylinder with height 10 feet (ft) and base radius 5 feet and two cones, each having height 5 ft and base radius 5 ft. The formulas $V_{\text{cylinder}} = \pi r^2 h$ and $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$ can be used to determine the total volume of the silo. Since the two cones have identical dimensions, the total volume, in cubic feet, of the silo is given by $V_{\text{silo}} = \pi(5)^2(10) + (2)\left(\frac{1}{3}\right)\pi(5)^2(5) = \left(\frac{4}{3}\right)(250)\pi$, which is approximately equal to 1,047.2 cubic feet.

Choice A is incorrect because this is the volume of only the two cones. Choice B is incorrect because this is the volume of only the cylinder. Choice C is incorrect because this is the volume of only one of the cones plus the cylinder.

QUESTION 26.

Choice C is correct. The line passes through the origin, $(2, k)$, and $(k, 32)$. Any two of these points can be used to find the slope of the line. Since the line passes through $(0, 0)$ and $(2, k)$, the slope of the line is equal to $\frac{k-0}{2-0} = \frac{k}{2}$. Similarly, since the line passes through $(0, 0)$ and $(k, 32)$, the slope of the line is equal to $\frac{32-0}{k-0} = \frac{32}{k}$. Since each expression gives the slope of the same line, it must be true that $\frac{k}{2} = \frac{32}{k}$. Multiplying each side of $\frac{k}{2} = \frac{32}{k}$ by $2k$ gives $k^2 = 64$, from which it follows that $k = 8$ or $k = -8$. Therefore, of the given choices, only 8 could be the value of k.

Choices A, B, and D are incorrect and may be the result of calculation errors.

QUESTION 27.

Choice C is correct. Let ℓ and w be the length and width, respectively, of the original rectangle. The area of the original rectangle is $A = \ell w$. The rectangle is altered by increasing its length by 10 percent and decreasing its width by p percent; thus, the length of the altered rectangle is 1.1ℓ, and the width of the altered rectangle is $\left(1 - \frac{p}{100}\right)w$. The alterations decrease the area by 12 percent, so the area of the altered rectangle is $(1 - 0.12)A = 0.88A$.

The altered rectangle is the product of its length and width, so $0.88A = (1.1\ell)\left(1 - \frac{p}{100}\right)w$. Since $A = \ell w$, this last equation can be rewritten as $0.88A = (1.1)\left(1 - \frac{P}{100}\right)\ell w = (1.1)\left(1 - \frac{P}{100}\right)A$, from which it follows that $0.88 = (1.1)\left(1 - \frac{P}{100}\right)$, or $0.8 = \left(1 - \frac{P}{100}\right)$. Therefore, $\frac{P}{100} = 0.2$, and so the value of p is 20.

Choice A is incorrect and may be the result of confusing the 12 percent decrease in area with the percent decrease in width. Choice B is incorrect because decreasing the width by 15 percent results in a 6.5 percent decrease in area, not a 12 percent decrease. Choice D is incorrect and may be the result of adding the percents given in the question (10 + 12).

QUESTION 28.

Choice D is correct. For the present population to decrease by 10 percent, it must be multiplied by the factor 0.9. Since the engineer estimates that the population will decrease by 10 percent every 20 years, the present population, 50,000, must be multiplied by $(0.9)^n$, where n is the number of 20-year periods that will have elapsed t years from now. After t years, the number of 20-year periods that have elapsed is $\frac{t}{20}$. Therefore, $50{,}000(0.9)^{\frac{t}{20}}$ represents the engineer's estimate of the population of the city t years from now.

Choices A, B, and C are incorrect because each of these choices either confuses the percent decrease with the multiplicative factor that represents the percent decrease or mistakenly multiplies t by 20 to find the number of 20-year periods that will have elapsed in t years.

QUESTION 29.

Choice A is correct. Let x be the number of left-handed female students and let y be the number of left-handed male students. Then the number of right-handed female students will be $5x$ and the number of right-handed male students will be $9y$. Since the total number of left-handed students is 18 and the total number of right-handed students is 122, the system of equations below must be satisfied.

$$\begin{cases} x + y = 18 \\ 5x + 9y = 122 \end{cases}$$

Solving this system gives $x = 10$ and $y = 8$. Thus, 50 of the 122 right-handed students are female. Therefore, the probability that a right-handed student selected at random is female is $\frac{50}{122}$, which to the nearest thousandth is 0.410.

Choices B, C, and D are incorrect and may be the result of incorrect calculation of the missing values in the table.

QUESTION 30.

Choice A is correct. Subtracting the sides of $3y + c = 5y - 7$ from the corresponding sides of $3x + b = 5x - 7$ gives $(3x - 3y) + (b - c) = (5x - 5y)$. Since $b = c - \frac{1}{2}$, or $b - c = -\frac{1}{2}$, it follows that $(3x - 3y) + \left(-\frac{1}{2}\right) = (5x - 5y)$. Solving this equation for x in terms of y gives $x = y - \frac{1}{4}$. Therefore, x is y minus $\frac{1}{4}$.

Choices B, C, and D are incorrect and may be the result of making a computational error when solving the equations for x in terms of y.

QUESTION 31.

The correct answer is either 4 or 5. Because each student ticket costs \$2 and each adult ticket costs \$3, the total amount, in dollars, that Chris spends on x student tickets and 1 adult ticket is $2(x) + 3(1)$. Because Chris spends at least \$11 but no more than \$14 on the tickets, one can write the compound inequality $2x + 3 \geq 11$ and $2x + 3 \leq 14$. Subtracting 3 from each side of both inequalities and then dividing each side of both inequalities by 2 yields $x \geq 4$ and $x \leq 5.5$. Thus, the value of x must be an integer that is both greater than or equal to 4 and less than or equal to 5.5. Therefore, $x = 4$ or $x = 5$. Either 4 or 5 may be gridded as the correct answer.

QUESTION 32.

The correct answer is 58.6. The mean of a data set is determined by calculating the sum of the values and dividing by the number of values in the data set. The sum of the ages, in years, in the data set is 703, and the number of values in the data set is 12. Thus, the mean of the ages, in years, of the first 12 United States presidents at the beginning of their terms is $\frac{703}{12}$. The fraction $\frac{703}{12}$ cannot be entered into the grid, so the decimal equivalent, rounded to the nearest tenth, is the correct answer. This rounded decimal equivalent is 58.6.

QUESTION 33.

The correct answer is 9. To rewrite the difference $(-3x^2 + 5x - 2) - 2(x^2 - 2x - 1)$ in the form $ax^2 + bx + c$, the expression can be simplified by using the distributive property and combining like terms as follows:

$(-3x^2 + 5x - 2) - (2x^2 - 4x - 2)$

$(-3x^2 - 2x^2) + (5x - (-4x)) + (-2 -(-2))$

$-5x^2 + 9x + 0$

The coefficient of x is the value of b, which is 9.

Alternatively, since b is the coefficient of x in the difference $(-3x^2 + 5x - 2) - 2(x^2 - 2x - 1)$, one need only compute the x-term in the difference. The x-term is $5x - 2(-2x) = 5x + 4x = 9x$, so the value of b is 9.

QUESTION 34.

The correct answer is $\frac{5}{8}$ or .625. A complete rotation around a point is 360° or 2π radians. Since the central angle AOB has measure $\frac{5\pi}{4}$ radians, it represents $\frac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}$ of a complete rotation around point O. Therefore, the sector formed by central angle AOB has area equal to $\frac{5}{8}$ the area of the entire circle. Either the fraction $\frac{5}{8}$ or its decimal equivalent, .625, may be gridded as the correct answer.

QUESTION 35.

The correct answer is 50. The mean of a data set is the sum of the values in the data set divided by the number of values in the data set. The mean of 75 is obtained by finding the sum of the first 10 ratings and dividing by 10. Thus, the sum of the first 10 ratings was 750. In order for the mean of the first 20 ratings to be at least 85, the sum of the first 20 ratings must be at least $(85)(20) = 1700$. Therefore, the sum of the next 10 ratings must be at least $1700 - 750 = 950$. The maximum rating is 100, so the maximum possible value of the sum of the 12th through 20th ratings is $9 \times 100 = 900$. Therefore, for the store to be able to have an average of at least 85 for the first 20 ratings, the least possible value for the 11th rating is $950 - 900 = 50$.

QUESTION 36.

The correct answer is 750. The inequalities $y \leq -15x + 3000$ and $y \leq 5x$ can be graphed in the xy-plane. They are represented by the half-planes below and include the boundary lines $y = -15x + 3000$ and $y = 5x$, respectively. The solution set of the system of inequalities will be the intersection of these half-planes, including the boundary lines, and the solution (a, b) with the greatest possible value of b will be the point of intersection of the boundary lines. The intersection of boundary lines of these inequalities can be found by setting them equal to each other: $5x = -15x + 3000$, which has solution $x = 150$. Thus, the x-coordinate of the point of intersection is 150. Therefore, the y-coordinate of the point of intersection of the boundary lines is $5(150) = -15(150) + 3000 = 750$. This is the maximum possible value of b for a point (a, b) that is in the solution set of the system of inequalities.

QUESTION 37.

The correct answer is 7. The average number of shoppers, N, in the checkout line at any time is $N = rt$, where r is the number of shoppers entering the checkout line per minute and T is the average number of minutes each shopper spends in the checkout line. Since 84 shoppers per hour make a purchase, 84 shoppers per hour enter the checkout line. This needs to be converted to the number of shoppers per minute. Since there are 60 minutes in one hour, the rate is $\frac{84 \text{ shoppers}}{60 \text{ minutes}} = 1.4$ shoppers per minute. Using the given formula with $r = 1.4$ and $t = 5$ yields $N = rt = (1.4)(5) = 7$. Therefore, the average number of shoppers, N, in the checkout line at any time during business hours is 7.

QUESTION 38.

The correct answer is 60. The estimated average number of shoppers in the original store at any time is 45. In the new store, the manager estimates that an average of 90 shoppers per <u>hour</u> enter the store, which is equivalent to 1.5 shoppers per minute. The manager also estimates that each shopper stays in the store for an average of 12 minutes. Thus, by Little's law, there are, on average, $N = rt = (1.5)(12) = 18$ shoppers in the new store at any time. This is $\frac{45 - 18}{45} \times 100 = 60$ percent less than the average number of shoppers in the original store at any time.