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Algebra Based Physics

Momentum

2016-01-20

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Momentum

Click on the topic to go to that section

https://www.njctl.org/video/?v=vCZuOPzzPb0

Momentum

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https://www.njctl.org/video/?v=ImQuL1LkzQs

Momentum Defined

Newton's First Law tells us that – objects remain in motion with a constant velocity unless acted upon by a force.

In our experience:

When two objects of different masses travel with the same velocity, the one with more mass is harder to stop.

When objects of the equal masses travel with different speeds, the faster one is harder to stop.

A new quantity, *momentum* (p), that takes these observations into account:

$$momentum = mass \times velocity$$

$$p = mv$$

View a video about momentum, from Bill Nye the Science Guy!

https://www.njctl.org/video/?v=y2Gb4NIv0Xg

Momentum is a Vector Quantity

The momentum (p) of a single object is the product of its mass and its velocity.

Momentum - like force, acceleration and velocity - is a vector.

$$\vec{p} = m\vec{v}$$

The unit for momentum is the product of the units in the formula:

kg·m/s.

1 Which has more momentum?

A A large truck moving at 30 m/s

B A small car moving at 30 m/s

C Both have the same momentum.

Answer

https://www.njctl.org/video/?v=TEgJZPNKrF8

2 What is the momentum of a 20 kg object with a velocity of +5.0 m/s?

Answer

https://www.njctl.org/video/?v=5K5szGWdTS8

3 What is the momentum of a 20 kg object with a velocity of −5.0 m/s?

Answer

https://www.njctl.org/video/?v=tJJNPix769s

4 What is the velocity of a 5.0 kg object whose momentum is −15.0 kg· m/s?

Answer

https://www.njctl.org/video/?v=zSNVIfYxlKY

5 What is the mass of an object whose momentum is 35 kg·m/s when its velocity is 7.0 m/s?

Answer

https://www.njctl.org/video/?v=y1143dHjhl8

Momentum Change & Impulse

https://www.njctl.org/video/?v=pCAvyCCA3L0

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Change in Momentum

Suppose that there is an event that changes an object's momentum.

from p_0 - the initial momentum (just before the event)

by Δp - the change in momentum

to p_f - the final momentum (just after the event)

The equation for momentum change is:

$$p_0 + \Delta p = p_f$$

Momentum Change = Impulse

Momentum change equation: $\Delta p = p_f - p_0$

Newton's First Law tells us that the velocity (and so the momentum) of an object won't change unless the object is affected by an external force.

When an outside force F acts on the object for a time Δt, it delivers an *impulse* I to the object that *changes its momentum*: $I = p_f - p_0$

Where the *impulse* is: $I = F\Delta t$

SI Unit for Impulse

There no specially named unit for impulse.
We just use the product of the units of force and time...

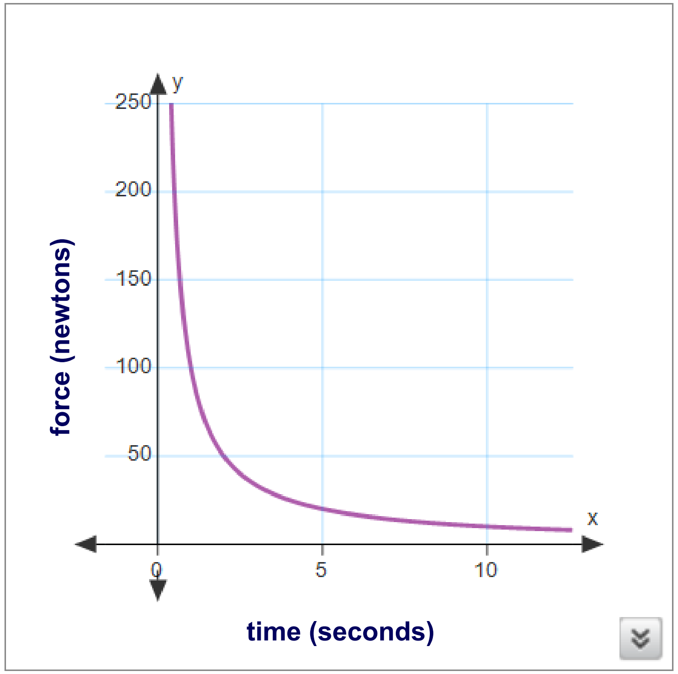
N·s

or

kg·m/s

Effect of Collision Time on Force

Impulse = $F\Delta t$ = change in momentum



Changing the duration (t) of an impulse by a small amount can greatly reduce the force on an object

Real World Applications

Impulse = $F\Delta t$ = change in momentum

Car Design

Air bags

Collisions

Crush zones

Jumping/Landing

Boxing/Martial Arts

Baseball, Golf...

6 An external force of 25 N acts on a system for 10 s. How big is the impulse delivered to the system?

Answer

https://www.njctl.org/video/?v=_819N8wiu_o

7 In the previous problem, an external force of 25 N acted on a system for 10 s. We found that the impulse delivered was 250 N-s. What is the magnitude of the change in momentum of the system?

Answer

https://www.njctl.org/video/?v=3rkbZoAdxVk

8 The momentum change of an object is equal to the ______.

A force acting on it

B impulse acting on it

C velocity change of the object

D object's mass times the force acting on it

Answer

https://www.njctl.org/video/?v=b_ZkShN9sjU

9 Air bags are used in cars because they:

A increase the force with which you hit the dashboard

B increase the duration (time) of impact in a collision

C decrease the momentum of a collision

D decrease the impulse in a collision

Answer

https://www.njctl.org/video/?v=sihCqAZaMMQ

10 One car crashes into a concrete barrier. Another car crashes into a collapsible barrier at the same speed. What is the difference between the 2 crashes?

A change in momentum

B force on the car

C impact time

D both B & C are true

Answer

https://www.njctl.org/video/?v=ZZoM42bQb-I

11 In order to increase the final momentum of a golf ball, we could:

A not change the speed of the golf club after the collision

B increase the force acting on it

C increase the time of contact between the club and ball

D all of the above

Answer

https://www.njctl.org/video/?v=7hgSkFwqQPo

12 An external force acts on an object for 0.0020 s. During that time the object's momentum increases by 400 kg-m/s. What was the magnitude of the force?

Answer

https://www.njctl.org/video/?v=c5OVsX3cpcg

13 * A 50,000 N force acts for 0.030 s on a 2.5 kg object that was initially at rest. What is its final velocity?

Answer

https://www.njctl.org/video/?v=cWBa7xdDYDg

The Momentum of a System of Objects

https://www.njctl.org/video/?v=Lux_N6ovevM

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The Momentum of a System of Objects

If a system contains more than one object, the total momentum is the *vector sum* of the momenta of those objects.

$$p_{system} = \sum p$$

$$p_{system} = p_1 + p_2 + p_3 + \ldots$$

$$p_{system} = m_1 v_1 + m_2 v_2 + m_3 v_3 + \ldots$$

The Momentum of a System of Objects

$$p_{system} = m_1v_1 + m_2v_2 + m_3v_3 + ...$$

To determine total momentum of a system:

Choose a direction considered to be positive

Assign positive values to momenta in that direction

Assign negative values to momenta in the opposite direction

Add the momenta to get total momentum.

Example

Determine the momentum of a system of two objects: $m_{1,}$ has a mass of 6 kg and a velocity of 13 m/s towards the east and m_2, has a mass of 14 kg and a velocity of 7 m/s towards the west.

(Let east be positive)

$m_1 = 6$ kg
$v_1 = 13$ m/s

$m_2 = 14$ kg
$v_2 = -7$ m/s

$p_{system} = p_1 + p_2$

$p_{system} = m_1v_1 + m_2v_2$

$p_{system} = 6kg(13m/s) + 14kg(-7m/s)$

$p_{system} = 78kg\text{-}m/s + -98kg\text{-}m/s$

$p_{system} = -20$ kg-m/s

14 Determine the magnitude of the momentum of a system of two objects: m_1, has a mass of 6.0 kg and a velocity of 20 m/s north and m_2, has a mass of 3 kg and a velocity 20 m/s south.

Answer

https://www.njctl.org/video/?v=xaB6gNQWseQ

15 Determine the momentum of a system of two objects: the first has a mass of 8 kg and a velocity of 8 m/s to the east while the second has a mass of 5 kg and a velocity of 15 m/s to the west.

Answer

https://www.njctl.org/video/?v=vwAotKOd2S4

16 Determine the momentum of a system of 3 objects: The first has a mass of 7.0 kg and a velocity of 23 m/s north; the second has a mass of 9.0 kg and a velocity of 7 m/s north; and the third has a mass of 5.0 kg and a velocity of 42 m/s south.

Answer

https://www.njctl.org/video/?v=ZQuJE6HSk6M

Conservation of Momentum

https://www.njctl.org/video/?v=IKC9_xNa0Fw

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Conservation Laws

Some of the most powerful concepts in science are called Conservation Laws:

apply to closed systems - where the objects only interact with each other and nothing else.

enable us to solve problems without worrying about the details of an event.

Momentum is Conserved

In the last unit we learned that energy is conserved.

Like energy, momentum is a conserved property of nature. It is not created or destroyed;

So in a closed system we will always have the same amount of momentum.

The only way the momentum of a system can change is if momentum is added or taken away by an outside force.

Conservation of Momentum

To apply Conservation of Momentum,
Take snapshots of a system just before and after an event.
By comparing these two snapshots we can learn a lot.

Conservation of Momentum and Impulse

Recall from our discussion of change of momentum and impulse:

$$p_{\mathbf{0}} + \mathbf{I} = p_f$$

When a net external force acts on an object, it imparts an impulse *I* to the object, changing its momentum.

This is exactly the same for a system of objects.

$$p_{\mathbf{0}(system)} + \mathbf{I} = p_{\mathbf{f}(system)}$$

If there is no net external force on the system, the momentum of the system is conserved.

$$p_{\mathbf{0}(system)} = p_{\mathbf{f}(system)}$$

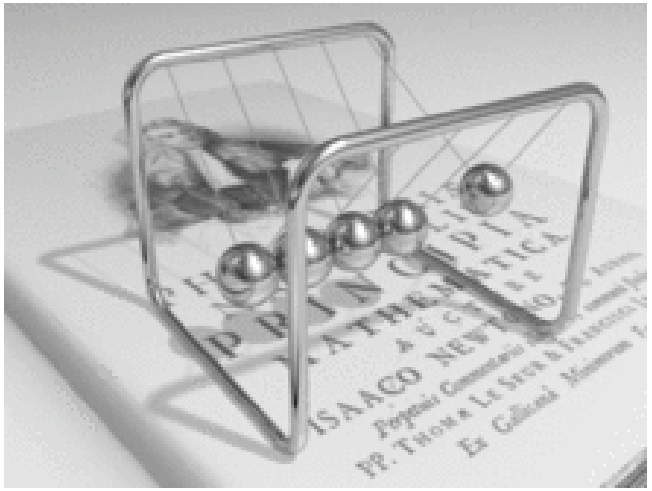
*Conservation of Momentum & Impulse Proof

Both the Conservation of Momentum and the concept of Impulse follow directly from Newton's Second Law:

$F = ma$	where F is the net external force
$F = m(\Delta v/\Delta t)$	since $a = \Delta v/\Delta t$
$F\Delta t = m\Delta v$	after multiplying both sides by Δt
$F\Delta t = \Delta(mv)$	since m is constant
$F\Delta t = \Delta p$	substituting Δp for Δmv.
$F\Delta t = I$	definition of impulse, I
$I = \Delta p$	substituting I for $F\Delta t$
$I = p_f - p_0$	since $\Delta p = p_f - p_0$
$p_0 + I = p_f$	
$p_0 = p_f$	when there is no net external force (F=0), I=0 so momentum is conserved

Conservation of Momentum and Impulse

A Newton's cradle demonstrates conservation of momentum.



$$p_{\mathbf{0}(system)} = p_{\mathbf{f}(system)}$$

Inelastic Collisions & Explosions

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Conservation Laws, Collisions and Explosions

Objects in an isolated system can interact with each other in a number of ways...

They can collide

If they are stuck together, they can explode (push apart)

In an isolated system measurements show both momentum and total energy are conserved, but the energy can change from one form to another.

Conservation of momentum and change in kinetic energy help us predict what happened or what will happen in one of these events.

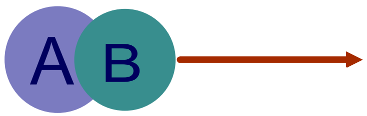
Collisions and Explosions

We differentiate between collisions and explosions by the way the energy changes or does not change form.

Explosions: an object or objects break apart because potential energy stored in one or more of the objects is transformed into kinetic energy

Before (moving together)

$p_A + p_B = (m_A + m_B)v$



After (moving apart)

$p_A' = m_A v_A'$

$p_B' = m_B v_B'$

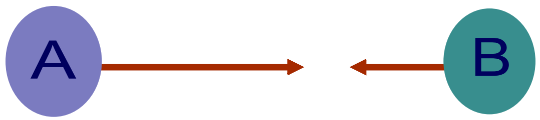
Collisions and Explosions

We differentiate between collisions and explosions by the way the energy changes or does not change form.

Inelastic collisions: two objects collide and stick together converting some kinetic energy into bonding energy, heat, sound

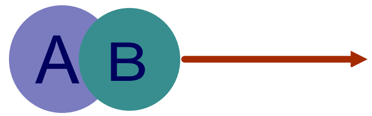
Before (moving towards the other)

$p_A = m_A v_A$ $p_B = m_B v_B$



After (moving together)

$p_A' + p_B' = (m_A + m_B)v'$



Collisions and Explosions

We differentiate between collisions and explosions by the way the energy changes or does not change form.

Elastic collisions: two objects collide and bounce off each other while conserving kinetic energy

Before (moving towards)

$p_A=m_Av_A$ $p_B=m_Bv_B$

After (moving apart)

$p_A'=m_Av_A'$ $p_B'=m_Bv_B'$

Collisions and Explosions - Summarized

Event	Description	Momentum Conserved?	Kinetic Energy Conserved?
Inelastic Collision	General collision: Objects bounce off each other	Yes	No. Some kinetic energy is converted to heat, sound... energy
Inelastic Collision	Objects stick together	Yes	No. Kinetic energy is converted to potential energy, bonding, or heat, sound...energy
Elastic Collision	Objects bounce off each other	Yes	Yes
Explosion	Object or objects break apart	Yes	No. Release of potential energy increases kintetic energy

17 In ________ collisions momentum is conserved.

A Elastic

B Inelastic

C All

Answer

https://www.njctl.org/video/?v=4yaQ5ehM71M

18 * In _______ collisions kinetic energy is conserved.

A Elastic

B Inelastic

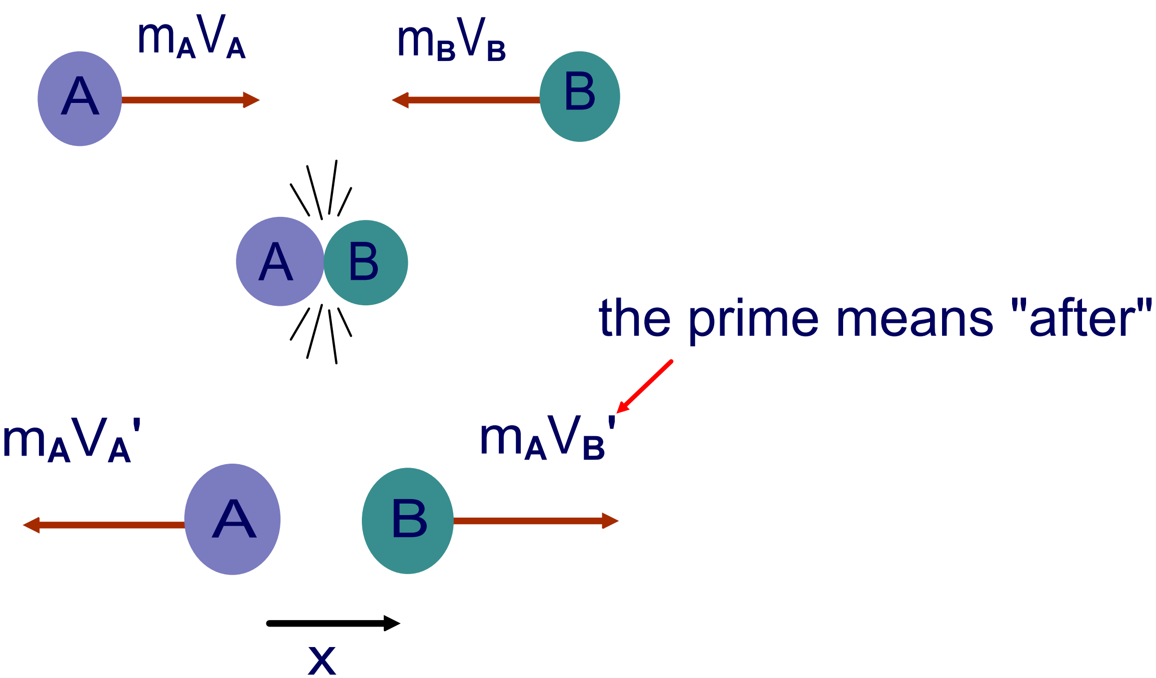
C All

Answer

https://www.njctl.org/video/?v=itOeKBO4duE

Conservation of Momentum

During a collision or an explosion, measurements show that the total momentum does not change:



$$m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B'$$

https://www.njctl.org/video/?v=OBC0jVu2VmY

Explosions

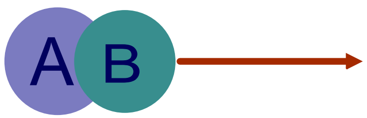
In an explosion, one object (or coupled objects) breaks apart into two or more pieces moving afterwards as separate objects.

We will assume:

- the object (or a coupled pair of objects) breaks into two pieces
- explosion is along the same line as the initial velocity

Before (moving together)

$p_A + p_B = (m_A + m_B)v$



After (moving apart)

$p_A' = m_A v_A'$ $\quad$ $p_B' = m_B v_B'$

$$p_A + p_B = p_A' + p_B'$$

$$(m_A + m_B)v = m_A v_A' + m_B v_B'$$

19 A 5 kg cannon ball is loaded into a 300 kg cannon. When the cannon is fired, it recoils at 5 m/s. What is the cannon balls's velocity after the explosion?

Answer

https://www.njctl.org/video/?v=CU7x4YwVXyE

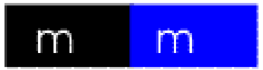
20 Two railcars, one with a mass of 4000 kg and the other with a mass of 6000 kg, are at rest and stuck together. To separate them a small explosive is set off between them. The 4000 kg car is measured travelling 6 m/s. How fast is the 6000 kg car going?

Answer

https://www.njctl.org/video/?v=i3ZiGT23rqI

Perfectly Inelastic Collisions

In perfectly inelastic collisions, two objects collide and stick together, moving afterwards as one object.



$$p_A + p_B = p_A' + p_B'$$

$$m_Av_A + m_Bv_B = (m_A + m_B)v'$$

https://www.njctl.org/video/?v=ifJv2G-7F5I

21 A 13,500 kg railroad freight car travels on a level track at a speed of 4.5 m/s. It collides and couples with a 25,000 kg second car, initially at rest and with brakes released. No external force acts on the system. What is the speed of the two cars after colliding?

Answer

https://www.njctl.org/video/?v=GK8cTCtDXqk

22 A cannon ball with a mass of 100 kg flies in horizontal direction with a speed of 800 m/s and strikes a ship initially at rest. The mass of the ship is 15,000 kg. Find the speed of the ship after the ball becomes embedded in it.

Answer

https://www.njctl.org/video/?v=xY6l-yS2Q0w

23 A 40 kg girl skates at 5.5 m/s on ice toward her 70 kg friend who is standing still, with open arms. As they collide and hold each other, what is the speed of the couple?

Answer

https://www.njctl.org/video/?v=atB4vWqfiws

Elastic Collisions

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* Elastic Collisions

In an elastic collision, two objects collide and bounce off each other and both momentum and kinetic energy are conserved.

Before (moving towards)

$p_A = m_A v_A$ $\qquad$ $p_B = m_B v_B$

After (moving apart)

$p_A' = m_A v_A'$ $\qquad$ $p_B' = m_B v_B'$

If we know the masses and any two of the velocities,

these two conservation equations

$$p_A + p_B = p_A' + p_B' \quad \& \quad KE_A + KE_B = KE_A' + KE_B'$$

enable us to calculate the other two velocities.

**Derivation of Elastic Collision Condition

Conservation of Momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

Conservation of Kinetic Energy

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2$$

$$m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$$

$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$$

$$\frac{\cancel{m_1}(v_1 + v_1')\cancel{(v_1 - v_1')} = \cancel{m_2}(v_2' + v_2)\cancel{(v_2' - v_2)}}{\cancel{m_1}\cancel{(v_1 - v_1')} = \cancel{m_2}\cancel{(v_2' - v_2)}}$$

$$(v_1 + v_1') = (v_2' + v_2)$$

$$(v_1 - v_2) = (v_2' - v_1')$$

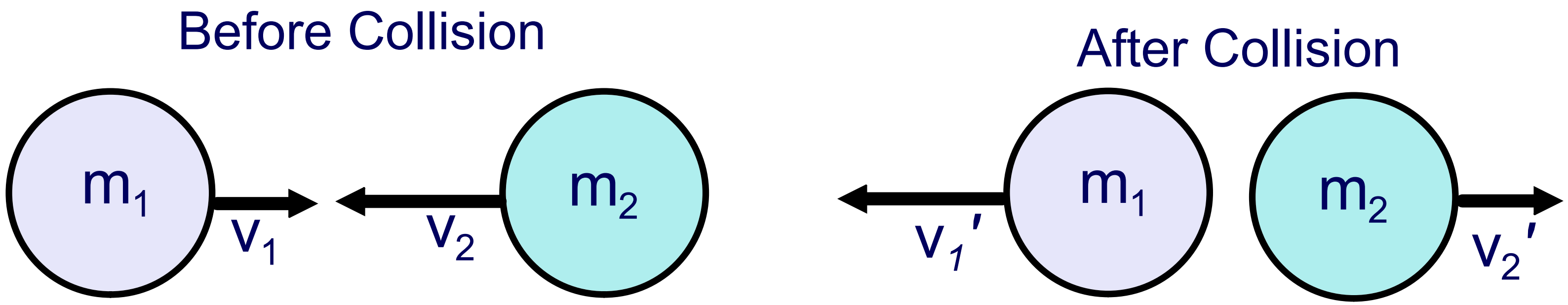
* Properties of Elastic Collisions

For all elastic collisions, regardless of the masses of the objects, the objects separate after the collision with the same relative speed that they collided with.

$$(v_1 - v_2) = (v_2' - v_1')$$

In an elastic collision between two objects of identical masses, the two objects exchange velocities.

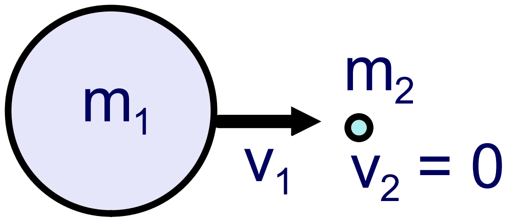
$$v_1' = v_2 \text{ and } v_2' = v_1$$



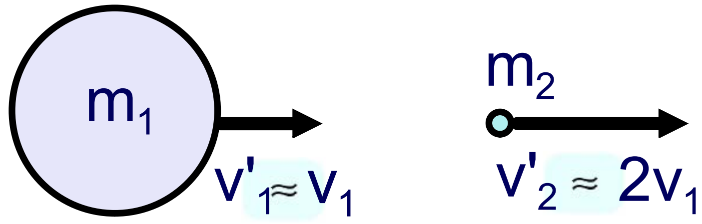
* Properties of Elastic Collisions

In an elastic collision where one object is much more massive than the other, the velocity of the smaller mass after the collision will be about twice that of the projectile while the more massive object's velocity will be almost unchanged.

Before Collision



After Collision



24 * Two objects have an elastic collision. Before they collide they are approaching each other with a velocity of 4 m/s relative to each other. With what velocity do they go apart from one another?

Answer

https://www.njctl.org/video/?v=XyNeHYB7NMs

25 * Two objects have an elastic collision. One object, m_1, has an initial velocity of +4.0 m/s and m_2 has a velocity of -3.0 m/s. After the collision, m_1 has a velocity of 1.0 m/s. What is the velocity of m_2?

Answer

https://www.njctl.org/video/?v=rPWM1MOYkwc

26 * A bowling ball has a velocity of +v when it collides with a ping pong ball that is at rest. The velocity of the bowling ball is virtually unaffected by the collision. What will be the speed of the ping pong ball?

Answer

https://www.njctl.org/video/?v=dWt7xci6YAc

27 * A baseball bat has a velocity of +v when it collides with a baseball that has a velocity of -2v. The bat barely changes velocity during the collision. How fast is the baseball going after it's hit?

Answer

https://www.njctl.org/video/?v=fuYkX4LSjoA

28 * Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +6.0 m/s and m_2 is -3.0 m/s. What is the velocity of m_1 after the collision?

Answer

https://www.njctl.org/video/?v=wGtYWS0Rmdw

29 * Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +6.0m/s and m_2 is -3.0m/s. What is the velocity of m_2 after the collision?

Answer

https://www.njctl.org/video/?v=gCk9pPRmfN4

30 * Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +3.0m/s and m_2 is +2.0m/s. What is the velocity of m_1 after the collision?

Answer

https://www.njctl.org/video/?v=3cfD9uIMGZg

31 * Two objects with identical masses have an elastic collision: the initial velocity of m_1 is +3.0 m/s and m_2 is +2.0 m/s. What is the velocity of m_2 after the collision?

Answer

https://www.njctl.org/video/?v=O16Nh9CCxE0

Attachments

Eqn1.pict