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# **Algebra Based Physics**

## **Newton's Law of Universal Gravitation**

2016-01-19

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# Newton's Law of Universal Gravitation

*Click on the topic to go to that section*

- **Gravitational Force**
- **Gravitational Field**
- **Surface Gravity**
- **Gravitational Field in Space**
- **Orbital Motion**
- **Kepler's Third Law of Motion**

# Gravitational Force



[https://www.njctl.org/video/?v=IP\\_u0xQvP04](https://www.njctl.org/video/?v=IP_u0xQvP04)



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# Newton's Law of Universal Gravitation

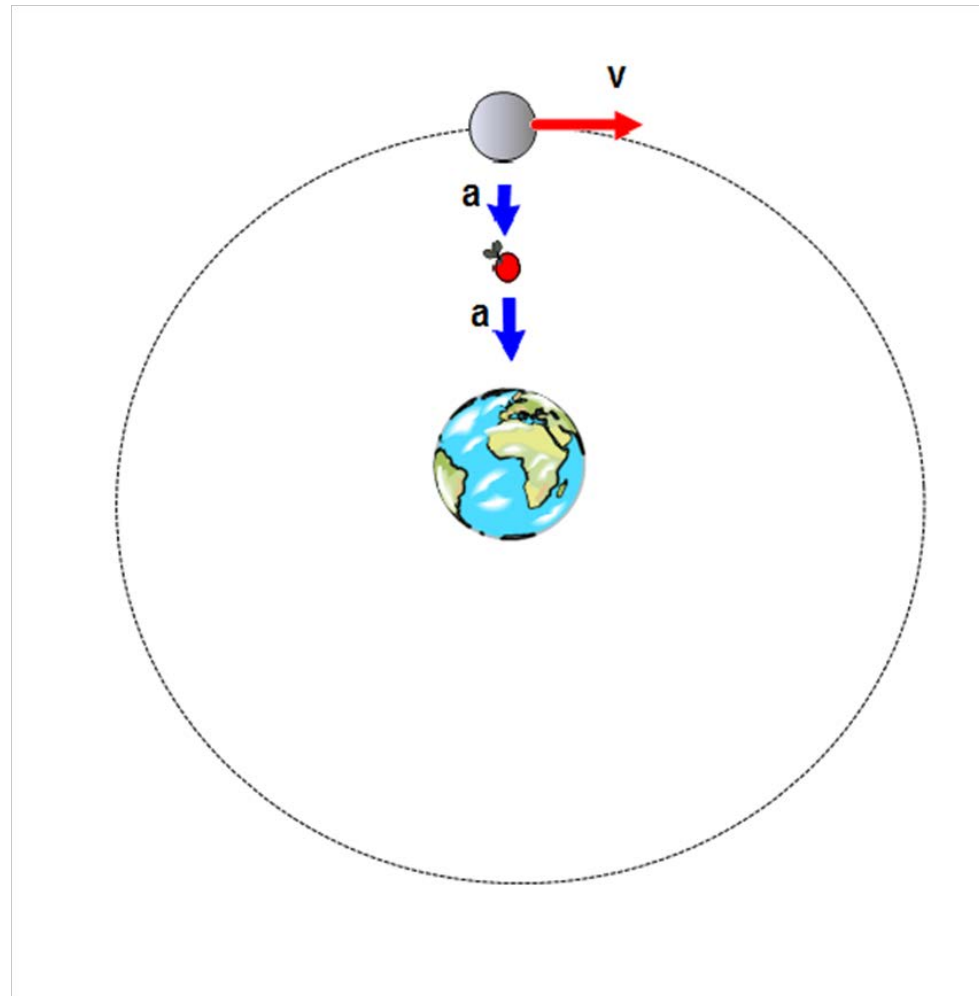
It has been well known since ancient times that Earth is a sphere and objects that are near the surface tend fall down.



# Newton's Law of Universal Gravitation

Newton connected the idea that objects, like apples, fall towards the center of Earth with the idea that the moon orbits around Earth...it's also falling towards the center of Earth.

The moon just stays in circular motion since it has a velocity perpendicular to its acceleration.

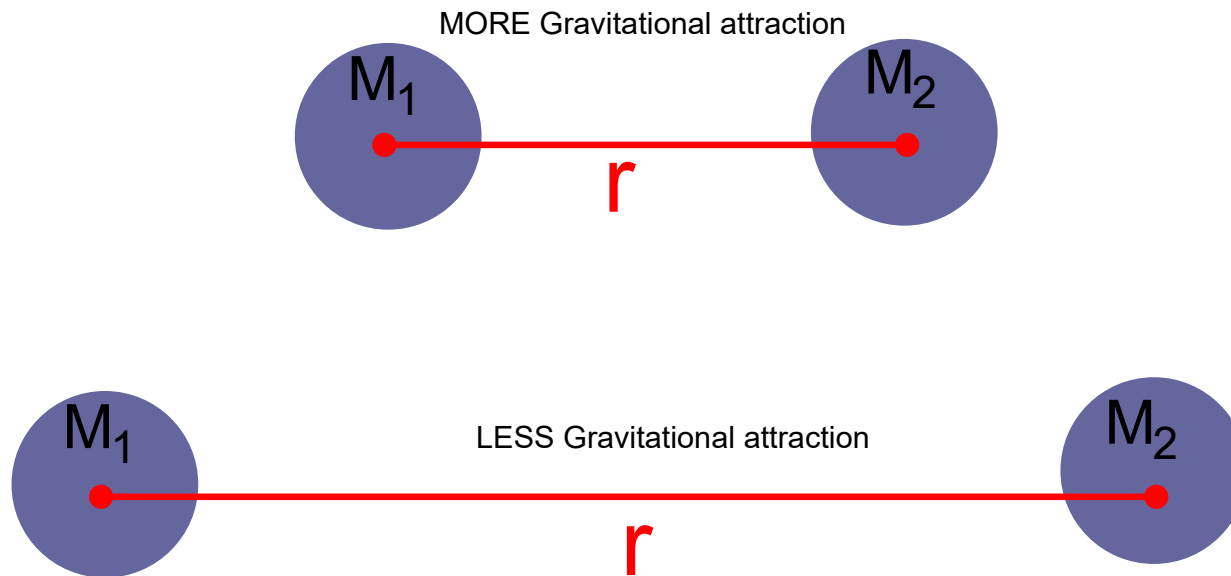


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# Newton's Law of Universal Gravitation

Newton concluded that all objects attract one another with a "gravitational force". The magnitude of the gravitational force decreases as the centers of the masses increases in distance.



# Gravitational Constant

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

In 1798, Henry Cavendish measured  $G$  using a torsion beam balance. He did not initially set out to measure  $G$ , he was instead trying to measure the density of the Earth.



*H. Cavendish*



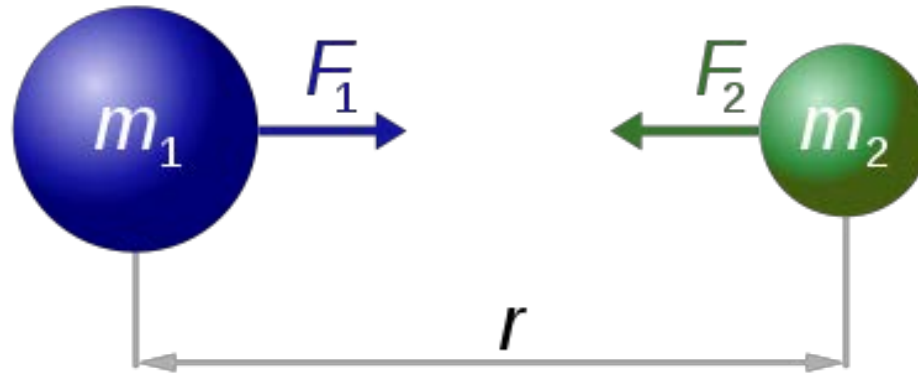
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# Newton's Law of Universal Gravitation

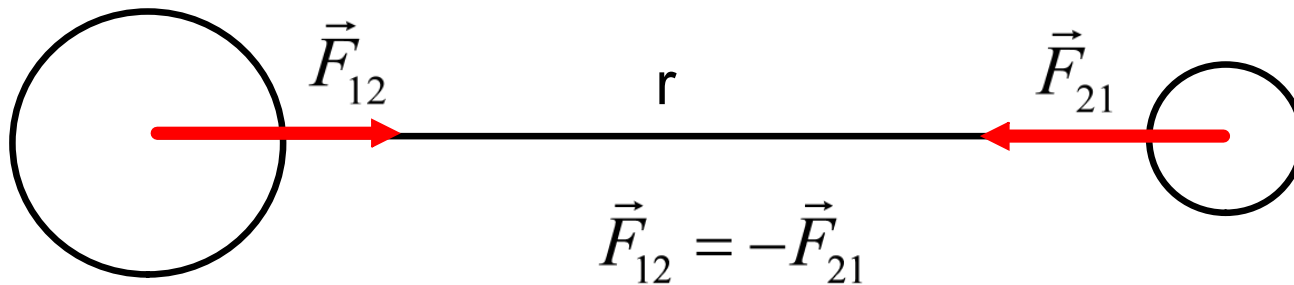
Mathematically, the magnitude of the gravitational force decreases with the inverse of the square of the distance between the centers of the masses and in proportion to the product of the masses.



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

# Newton's Law of Universal Gravitation

The direction of the force is along the line connecting the centers of the two masses. Each mass feels a force of attraction towards the other mass...along that line.



# Newton's Law of Universal Gravitation

Newton's third law tells us that the force on each mass is equal.

That means that if I drop a pen, the force of Earth pulling the pen down is equal to the force of the pen pulling Earth up.

However, since the mass of Earth is so much larger, that force causes the pen to accelerate down, while the movement of Earth up is completely unmeasurable.

1 What is the magnitude of the gravitational force between two 1 kg objects which are located 1.0 m apart?

A  $3.3 \times 10^{-11}$  N

B  $1.7 \times 10^{-11}$  N

C  $2.7 \times 10^{-10}$  N

D  $6.7 \times 10^{-11}$  N

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$



[https://www.njctl.org/video/?v=IP\\_u0xQvP04](https://www.njctl.org/video/?v=IP_u0xQvP04)



Answer

2 What is the magnitude of the gravitational force acting on a 4.0 kg object which is 1.0 m from a 1.0 kg object?

A  $3.3 \times 10^{-11} \text{ N}$

B  $1.7 \times 10^{-11} \text{ N}$

C  $2.7 \times 10^{-10} \text{ N}$

D  $6.7 \times 10^{-11} \text{ N}$

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$



<https://www.njctl.org/video/?v=dPFsPm5UYhg>



Answer

3 What is the magnitude of the gravitational force acting on a 1.0 kg object which is 1.0 m from a 4.0 kg object?

A  $3.3 \times 10^{-11} \text{ N}$

B  $1.7 \times 10^{-11} \text{ N}$

C  $2.7 \times 10^{-10} \text{ N}$

D  $6.7 \times 10^{-11} \text{ N}$

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$



<https://www.njctl.org/video/?v=iOovJt118lc>



Answer

4 What is the magnitude of the gravitational force acting on a 1.0 kg object which is 2.0 m from a 4.0 kg object?

- A  $3.3 \times 10^{-11}$  N
- B  $1.7 \times 10^{-11}$  N
- C  $2.7 \times 10^{-10}$  N
- D  $6.7 \times 10^{-11}$  N

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$



<https://www.njctl.org/video/?v=tjcf5sqwLT0>



5 What is the magnitude of the gravitational force between Earth and its moon?

$$r = 3.8 \times 10^8 \text{ m}$$

$$m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$$

$$m_{\text{moon}} = 7.3 \times 10^{22} \text{ kg}$$

A  $2.0 \times 10^{18} \text{ N}$

B  $2.0 \times 10^{19} \text{ N}$

C  $2.0 \times 10^{20} \text{ N}$

D  $2.0 \times 10^{21} \text{ N}$

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$



<https://www.njctl.org/video/?v=4MieN1BT4Yc>



Answer



6 What is the magnitude of the gravitational force between Earth and its sun?

$$r = 1.5 \times 10^{11} \text{ m}$$

$$m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$$

$$m_{\text{sun}} = 2.0 \times 10^{30} \text{ kg}$$

A  $3.6 \times 10^{18} \text{ N}$

B  $3.6 \times 10^{19} \text{ N}$

C  $3.6 \times 10^{21} \text{ N}$

D  $3.6 \times 10^{22} \text{ N}$

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$



<https://www.njctl.org/video/?v=mAC5GoXJGE>



Answer

# \* Gravitational Field



[https://www.njctl.org/video/?v=p\\_OteaRhSsk](https://www.njctl.org/video/?v=p_OteaRhSsk)



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## \*Gravitational Field

While the force between two objects can always be computed by using the formula for  $F_G$ ; it's sometimes convenient to consider one mass as creating a gravitational field and the other mass responding to that field.

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$F_G = \left( \frac{Gm_1}{r^2} \right) m_2$$

$$F_G = \left( \frac{GM}{r^2} \right) m$$

$$F_G = \text{weight} = mg$$

$$g = \frac{GM}{r^2}$$

# \*Gravitational Field

The magnitude of the gravitational field created by an object varies from location to location in space; it depends on the distance from the object and the object's mass.

Gravitational field,  $g$ , is a vector. Its direction is always towards the object creating the field.

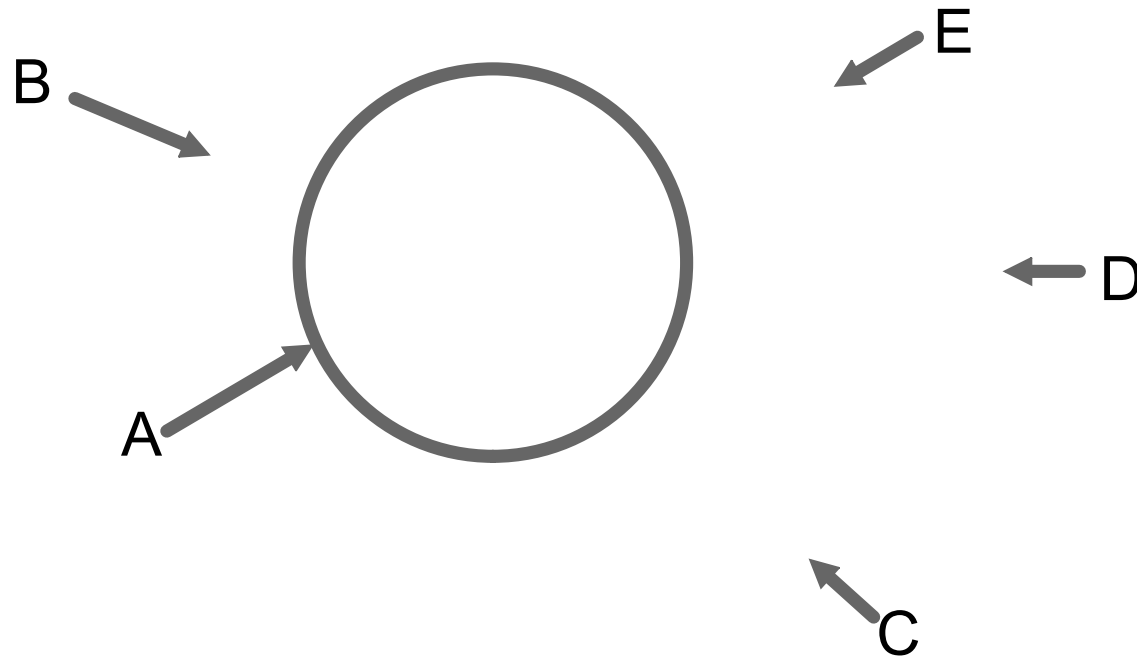
That's the direction of the force that a test mass would experience if placed at that location. In fact,  $g$  is the acceleration that a mass would experience if placed at that location in space.



$$g = \frac{GM}{r^2}$$

# \*Gravitational Field

7 Where is the gravitational field the strongest?



Answer

8\*What happens to the gravitational field if the distance from the center of an object doubles?

- A It doubles
- B It quadruples
- C It is cut to one half
- D It is cut to one fourth

Answer

9\*What happens to the gravitational field if the mass of an object doubles?

- A It doubles
- B It quadruples
- C It is cut to one half
- D It is cut to one fourth

Answer



[https://www.njctl.org/video/?v=E1KR\\_75YCIA](https://www.njctl.org/video/?v=E1KR_75YCIA)



# **\* Surface Gravity**

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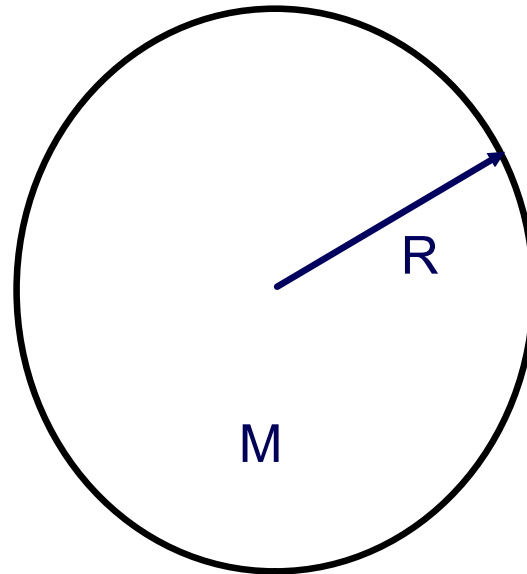


## \*Surface Gravity

Planets, stars, moons, all have a gravitational field...since they all have mass.

That field is largest at the object's surface, where the distance from the center of the object is the smallest...when "r" is the radius of the object.

By the way, only the mass of the planet that's closer to the center of the planet than you are contributes to its gravitational field. So the field actually gets smaller if you tunnel down below the surface.



$$g = \frac{GM}{R^2}$$

- \* 10 Determine the surface gravity of Earth. Its mass is  $6.0 \times 10^{24}$  kg and its radius is  $6.4 \times 10^6$  m.

Answer



<https://www.njctl.org/video/?v=oc8zZ7MNFtE>



\*11 Determine the surface gravity of Earth's moon. Its mass is  $7.4 \times 10^{22}$  kg and its radius is  $1.7 \times 10^6$  m.

Answer



<https://www.njctl.org/video/?v=R63PGhOwbV8>



- \* 12 Determine the surface gravity of Earth's sun. Its mass is  $2.0 \times 10^{30}$  kg and its radius is  $7.0 \times 10^8$  m.

Answer



<https://www.njctl.org/video/?v=d64u-4vMrHo>



\* 13 Compute  $g$  for the surface of a planet whose radius is double that of the Earth and whose mass is triple that of Earth.

Answer



<https://www.njctl.org/video/?v=iUWtYR2gxsQ>



# **Gravitational Field in Space**

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## \*Gravitational field in space

While gravity gets weaker as you get farther from a planet, it never becomes zero.

There is always some gravitational field present due to every planet, star and moon in the universe.



## \*Gravitational field in space

The local gravitational field is usually dominated by nearby masses since gravity gets weaker as the inverse square of the distance.

The contribution of a planet to the local gravitational field can be calculated using the same equation we've been using. You just have to be careful about "r".

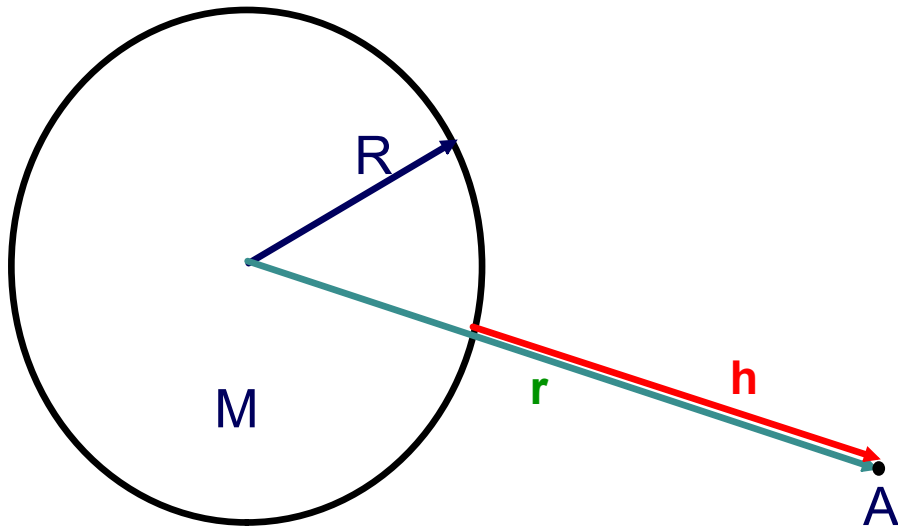




## \*Gravitational field in space

The contribution of a planet to the local gravitational field can be calculated using the same equation we've been using. You just have to be careful about "r".

If a location, "A", is a height "h" above a planet of radius "R", it is a distance "r" from the planet's center, where  $r = R + h$ .



$$g_A = \frac{GM}{r^2}$$

$$g_A = \frac{GM}{(R + h)^2}$$

\*14 Determine the gravitational field of Earth at a height of  $6.4 \times 10^6$  m (1 Earth radius). Earth's mass is  $6.0 \times 10^{24}$  kg and its radius is  $6.4 \times 10^6$  m.

Answer



[https://www.njctl.org/video/?v=7Z\\_AZ2L53vs](https://www.njctl.org/video/?v=7Z_AZ2L53vs)



- \* 15 Determine the gravitational field of Earth at a height  $2.88 \times 10^8$  m above its surface (the height of the moon above Earth).

Earth's mass is  $6.0 \times 10^{24}$  kg and its radius is  $6.4 \times 10^6$  m.

Answer



[https://www.njcti.org/video/?v=q\\_Esi6tz1yk](https://www.njcti.org/video/?v=q_Esi6tz1yk)



# The International Space Station (ISS)

The International Space Station (ISS) is a research facility, the on-orbit construction of which began in 1998. The space station is in a Low Earth Orbit and can be seen from Earth with the naked eye!

It orbits at an altitude of approximately 350 km (190 mi) above the surface of the Earth, and travels at an average speed of 27,700 kilometers (17,210 mi) per hour. This means the astronauts see 15 sunrises everyday!



[https://www.njctl.org/video/?v=4Kysw9\\_Xhi0](https://www.njctl.org/video/?v=4Kysw9_Xhi0)



- \* 16 The occupants of the International Space Station (ISS) float and appear to be weightless. Determine the strength of Earth's gravitational field acting on astronauts in the ISS.

Earth's mass is  $6.0 \times 10^{24}$  kg and its radius is  $6.4 \times 10^6$  m. The ISS is 350km ( $3.5 \times 10^5$ m) above the surface of Earth.

Answer



[https://www.njctl.org/video/?v=4Kysw9\\_Xhi0](https://www.njctl.org/video/?v=4Kysw9_Xhi0)



\*17 How does the gravitational field acting on the occupants in the space station compare to the gravitational field acting on you now?

- A It's the same
- B It's slightly less
- C It's about half as strong
- D There is no gravity acting on them

Answer



<https://www.njctl.org/video/?v=DaDGZF-rPTY>



# **\*\* Orbital Motion**

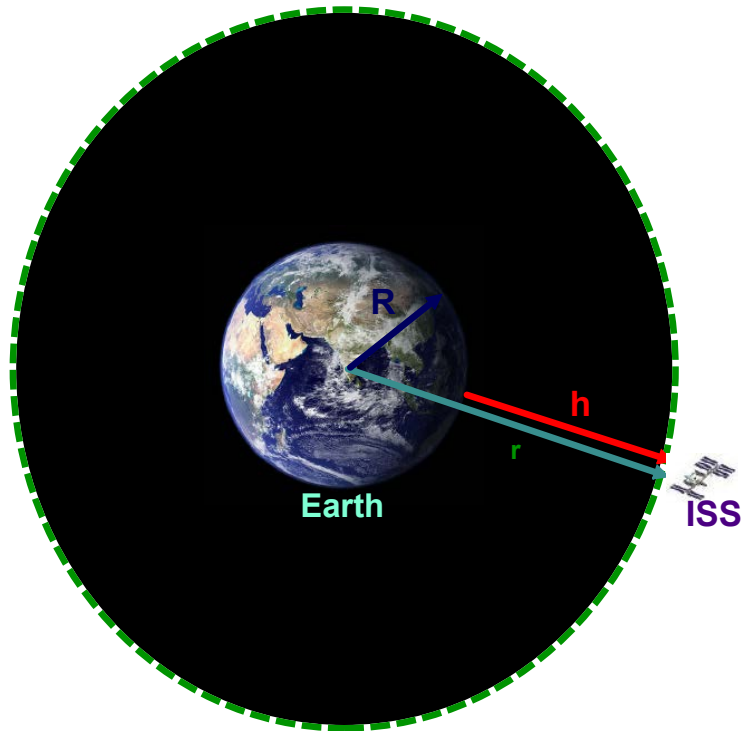


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## \*\* Orbital Motion



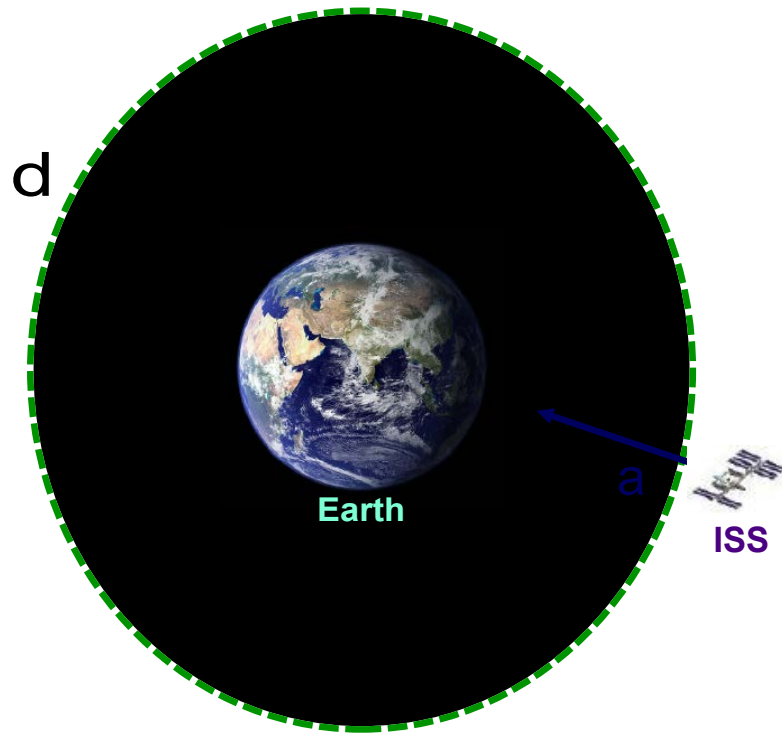
We've already determined that the gravitational field acting on the occupants of the space station, and on the space station itself, is not very different than the force acting on us.

How come they don't fall to Earth?

This diagram should look really familiar....



## \*\* Orbital Motion

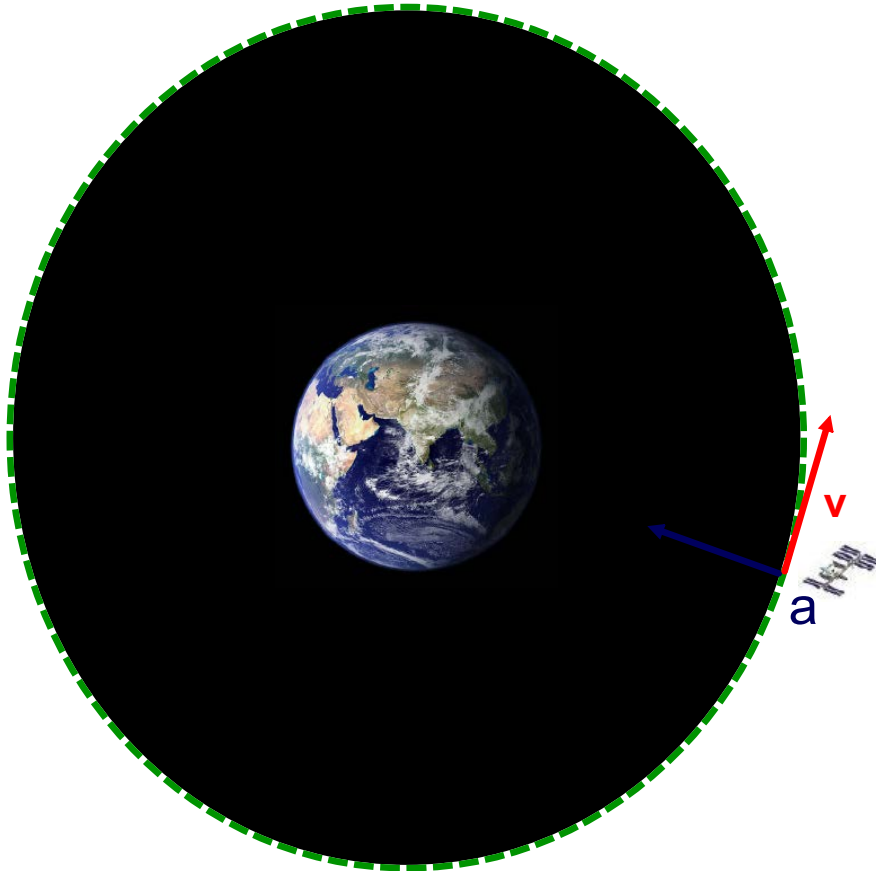


The gravitational field will be pointed towards the center of Earth and represents the acceleration that a mass would experience at that location (regardless of the mass).

In this case any object would simply fall to Earth.

How could that be avoided?

## \*\* Orbital Motion



If the object has a tangential velocity perpendicular to its acceleration, it will go in a circle.

It will keep falling to Earth, but never strike Earth.

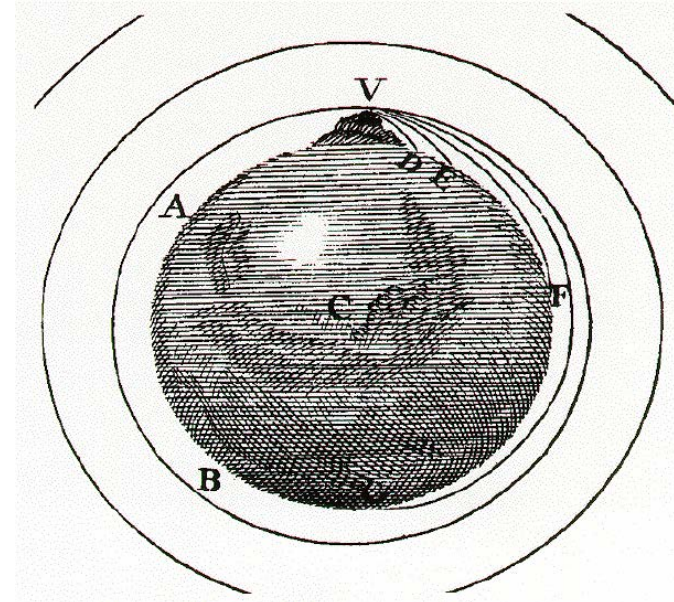


<https://www.njctl.org/video/?v=iQOHRKKNLQ>



## \*\* Orbital Motion

Here is Newton's own drawing of a thought experiment where a cannon on a very high mountain (above the atmosphere) shoots a shell with increasing speed, shown by trajectories for the shell of D, E, F, and G and finally so fast that it never falls to earth, but goes into orbit.

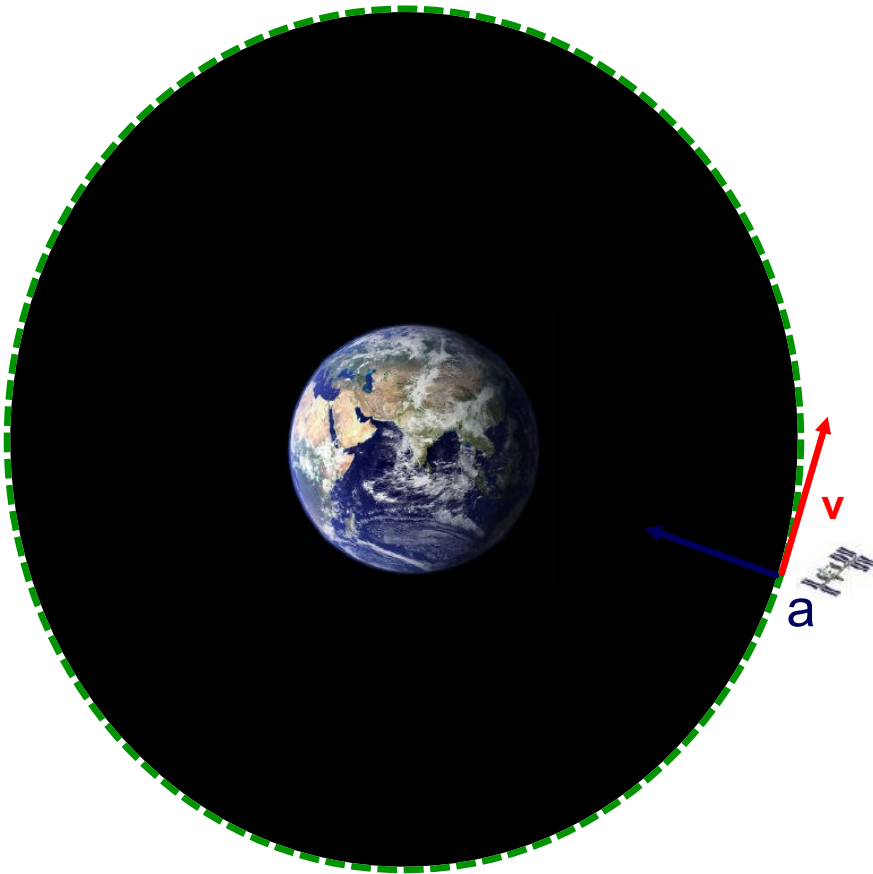


<https://www.njctl.org/video/?v=olZV-ixRTcY>



## \*\* Orbital Motion

We can calculate the velocity necessary to maintain a stable orbit at a distance "r" from the center of a planet of mass "M".



$$\Sigma F = ma$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r} = v^2$$

$$v = \sqrt{\frac{GM}{r}}$$

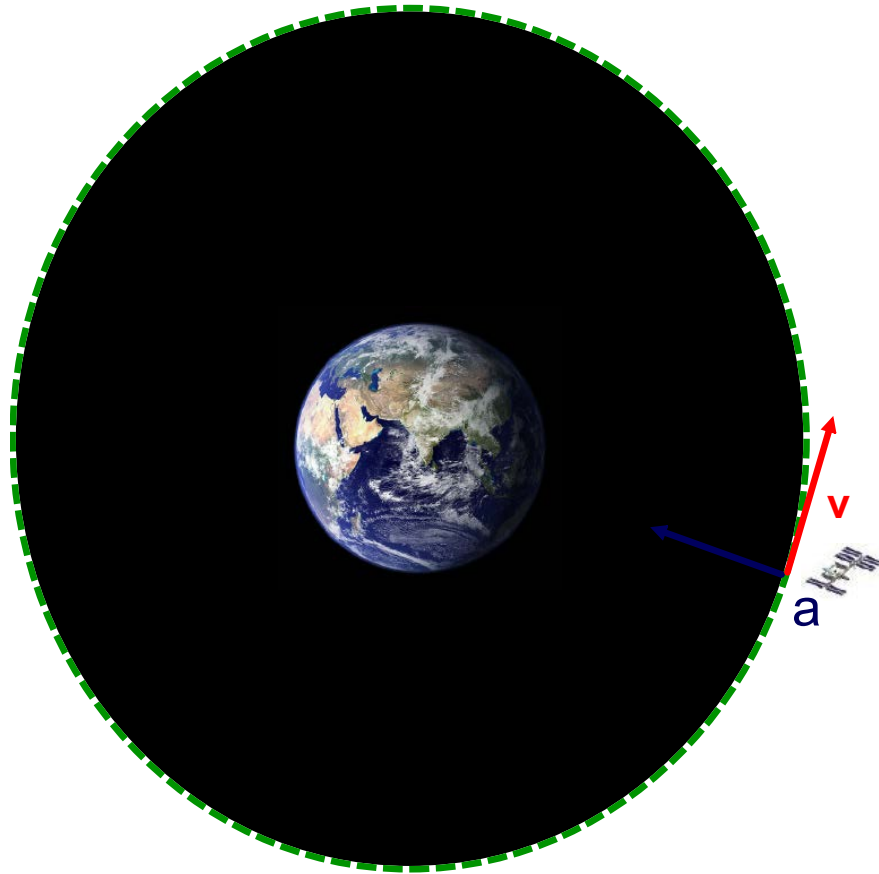


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## \*\* Orbital Motion

From that, we can calculate the period,  $T$ , of any object's orbit.



$$v = \sqrt{\frac{GM}{r}} \quad \text{or} \quad v = \sqrt{gr}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v}$$

**\*\* 18** Compute  $g$  at a distance of  $7.3 \times 10^8$  m from the center of a spherical object whose mass is  $3.0 \times 10^{27}$  kg.

Answer



[https://www.njctl.org/video/?v=mRIBe\\_tA\\_eg](https://www.njctl.org/video/?v=mRIBe_tA_eg)



**\*\* 19** Use your previous answer to determine the velocity, both magnitude and direction, for an object orbiting at a distance of  $7.3 \times 10^8$  m from the center of a spherical object whose mass is  $3.0 \times 10^{27}$  kg.

Answer



<https://www.njctl.org/video/?v=kCWb0U-Y4Jg>



**\*\* 20** Use your previous answer to determine the orbital period of for an object orbiting at a distance of  $7.3 \times 10^8$  m from the center of a spherical object whose mass is  $3.0 \times 10^{27}$  kg.

Answer



<https://www.njctl.org/video/?v=5m1BuMWL5LI>





- \* 21 Compute  $g$  at a height of 59 earth radii above the surface of Earth.

**Answer**



View solution

<https://youtu.be/CDdqUjsO8TM>

**\*\* 22** Use your previous answer to determine the velocity, both magnitude and direction, for an object orbiting at height of  $59 R_E$  above the surface of Earth.

**Answer**

**\*\* 23** Use your previous answer to determine the orbital period of an object orbiting at height of  $59 R_E$  above the surface of Earth.

Answer

# **\*\* Kepler's Third Law of Motion**



<https://www.njctl.org/video/?v=5vr5aiqDYyY>



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## \*\* Orbital Motion

Now, we can find the relationship between the period,  $T$ , and the orbital radius,  $r$ , for any orbit.

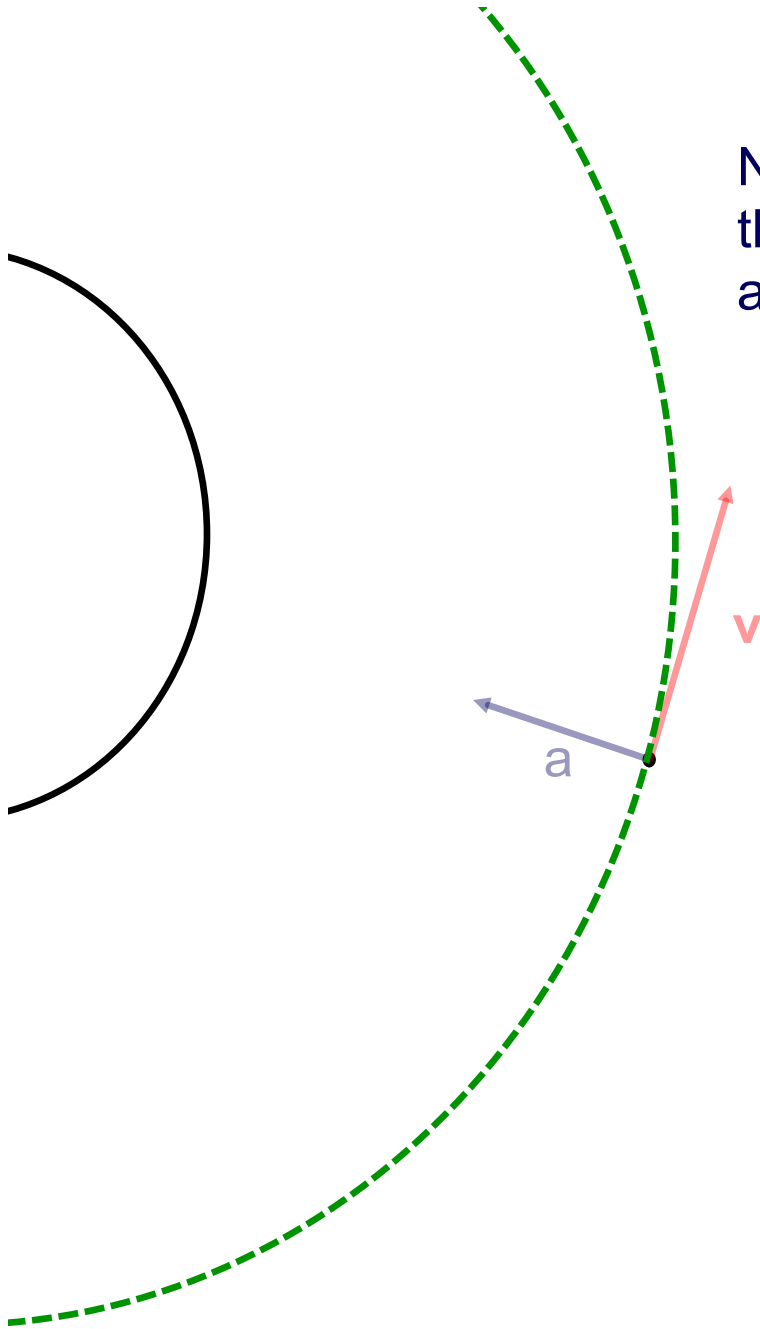
$$v = \frac{2\pi r}{T}$$

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$GMT^2 = 4\pi^2 r^3$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$



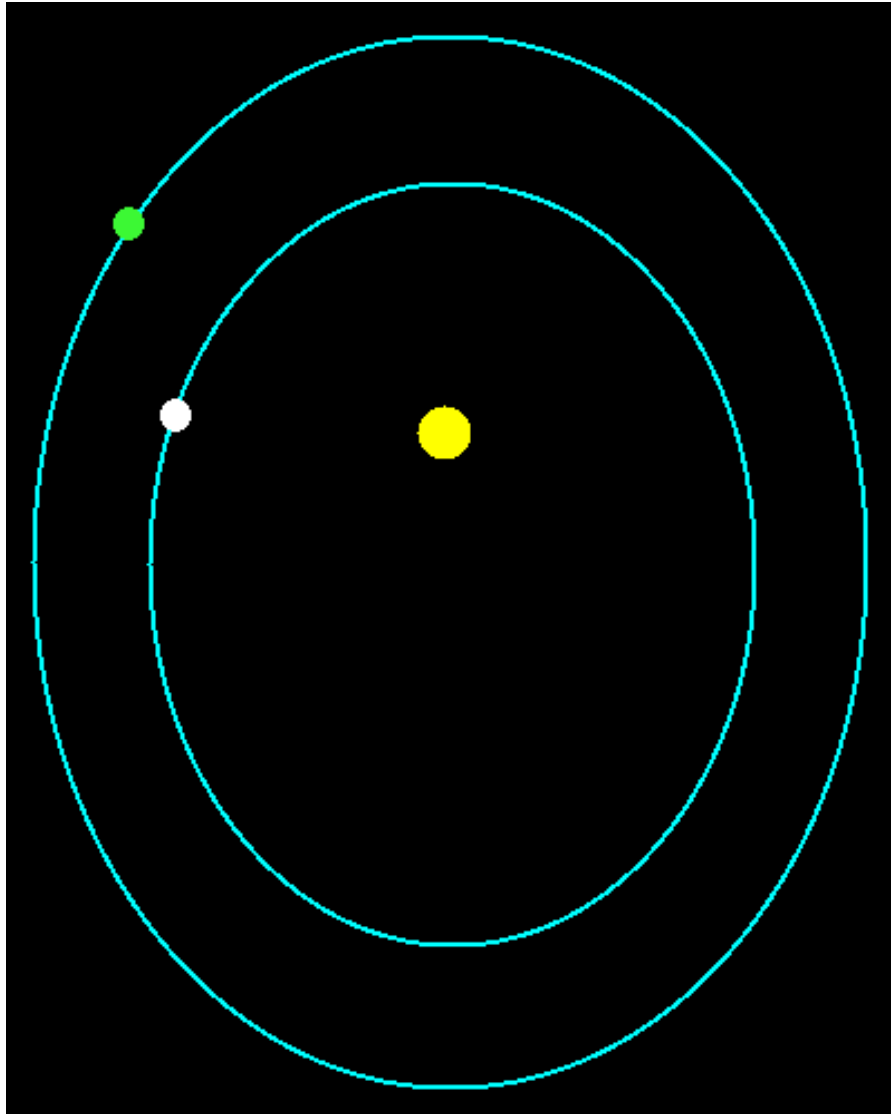
## **\*\*Kepler's Third Law**

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Kepler had noted that the ratio of  $T^2 / r^3$  yields the same result for all the planets. That is, the square of the period of any planet's orbit divided by the cube of its distance from the sun always yields the same number.

We have now shown why:  $(4\pi^2) / (GM)$  is a constant; its the same for all orbiting objects, where  $M$  is the mass of the object being orbited; it is independent of the object that is orbiting.

## \*\* Kepler's Third Law



If you know the period ( $T$ ) of a planet's orbit, you can determine its distance ( $r$ ) from the sun.

Since all planets orbiting the sun have the same period to distance ratio, the following is true:

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$\frac{T_{\text{(white)}}^2}{r_{\text{(white)}}^3} = \frac{T_{\text{(green)}}^2}{r_{\text{(green)}}^3}$$

**\*\*24** The period of the Moon is 27.3 days and its orbital radius is  $3.8 \times 10^8$  m. What would be the orbital radius of an object orbiting Earth with a period of 20 days?

**Answer**



<https://www.njctl.org/video/?v=EqzMY3p5cyc>





**\*\* 25** What is the orbital period (in days) of an unknown object orbiting the sun with an orbital radius twice the radius of Earth?

Answer



<https://www.njctl.org/video/?v=itQif01ytNQ>

