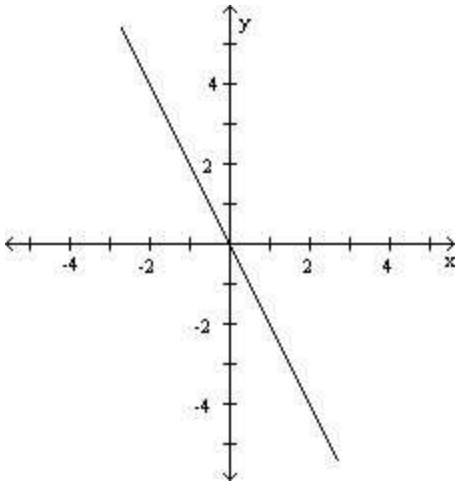
Name_____

AP Calculus AB
Worksheet - Differentiability

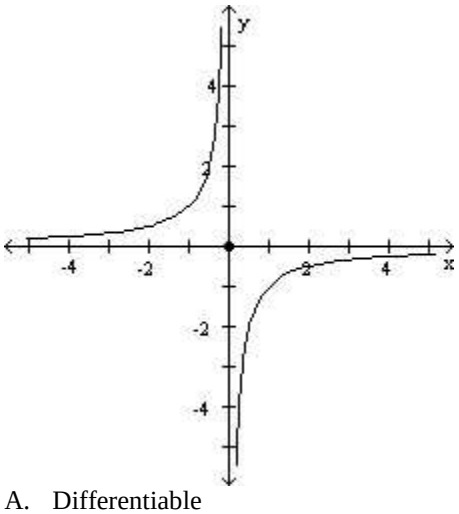
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.
The figure shows the graph of a function. At the given value of *x*, does the function appear to be differentiable, continuous but not differentiable, or neither continuous nor differentiable?

1. *x=0*



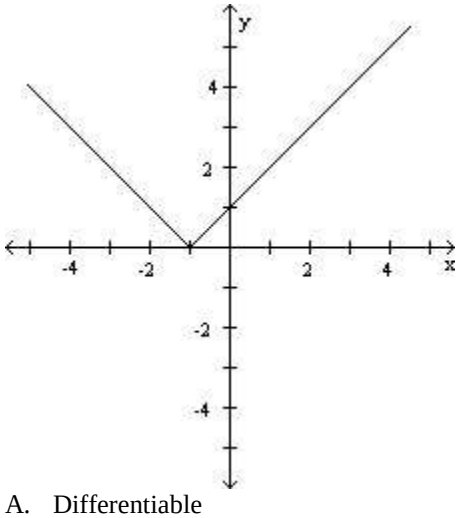
A. Differentiable
B. Continuous but not differentiable
C. Neither continuous nor differentiable

2. *x=0*



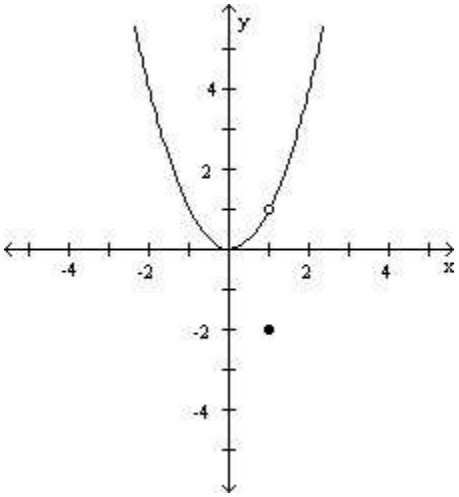
A. Differentiable
B. Continuous but not differentiable
C. Neither continuous nor differentiable

3. *x=-1*



A. Differentiable
B. Continuous but not differentiable
C. Neither continuous nor differentiable

4. *x=1*



A. Differentiable
B. Continuous but not differentiable
C. Neither continuous nor differentiable

If the function is not differentiable at the given value of *x*, tell whether the problem is a corner, cusp, vertical tangent, or a discontinuity.

5. y = -3|x| - 9, at x = 0

A. vertical tangent
B. cusp
C. corner
D. function is differentiable at *x=0*

6. $y = \frac{8}{x+2}$, at *x* = -2

A. vertical tangent
B. cusp
C. corner
D. function is differentiable at *x=2*

7. $y = 3 - \sqrt[3]{x}$, at *x* = 0

A. vertical tangent
B. cusp
C. corner
D. function is differentiable at *x=0*

8. $y = \sqrt[3]{|x+12|}$, at *x* = -12

A. vertical tangent
B. cusp
C. corner
D. function is differentiable at *x=-12*

Determine the values of *x* for which the function is differentiable.

9) _____

9) $y = 6x - 1$
A) All reals except 0.16666667
B) All reals
C) All reals except -1
D) All reals except 6

10) _____

10) $y = \frac{1}{x-7}$
A) All reals except 1
B) All reals
C) All reals except 7
D) All reals except -7

11) _____

11) $y = \frac{1}{x^2 - 121}$
A) All reals
B) All reals except 11
C) All reals except 121
D) All reals except -11 and 11

12) _____

12) $y = x^2 - 49$
A) All reals except 7
B) All reals except 49
C) All reals
D) All reals except -7 and 7

13) _____

13) $y = \frac{1}{x^2 + 64}$
A) All reals except 64
B) All reals
C) All reals except -8 and 8
D) All reals except 8

14) _____

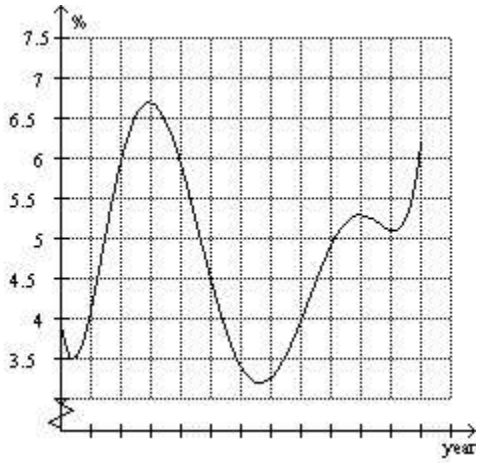
14) $y = \sqrt{x-5}$
A) All reals greater than -5
B) All reals greater than or equal to 5
C) All reals except 5
D) All reals greater than 5

15) _____

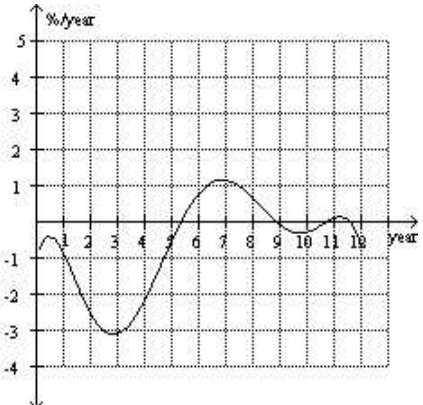
15) $y = \sqrt{x^2 + 9}$
A) All reals except 3
B) All reals except 9
C) All reals
D) All reals except -3 and 3

Solve the problem.

16) The graph shows the yearly average interest rates for 30-year mortgages for years since 1988 (Year 0 corresponds to 1988). Sketch a graph of the rate of change of interest rates with respect to time. 16) _____



A)



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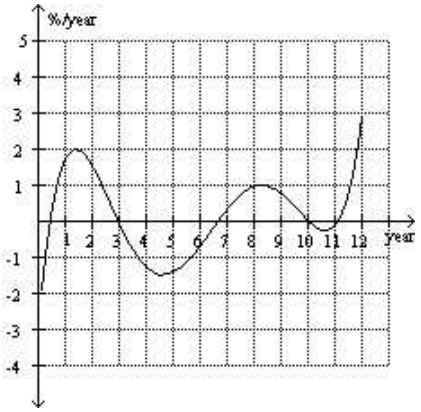
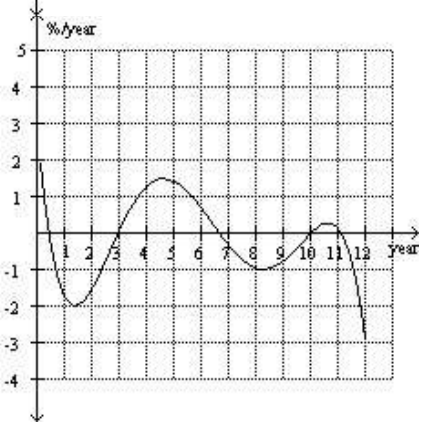
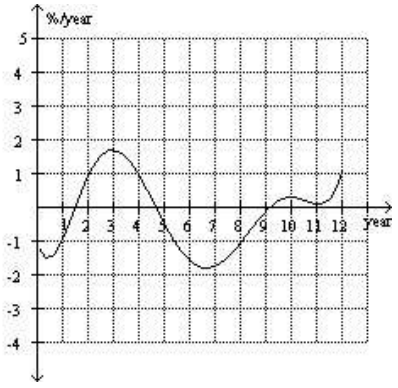


C)

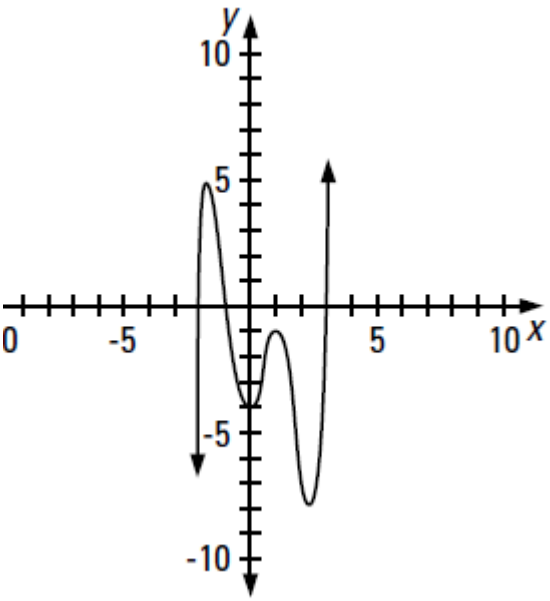


D)

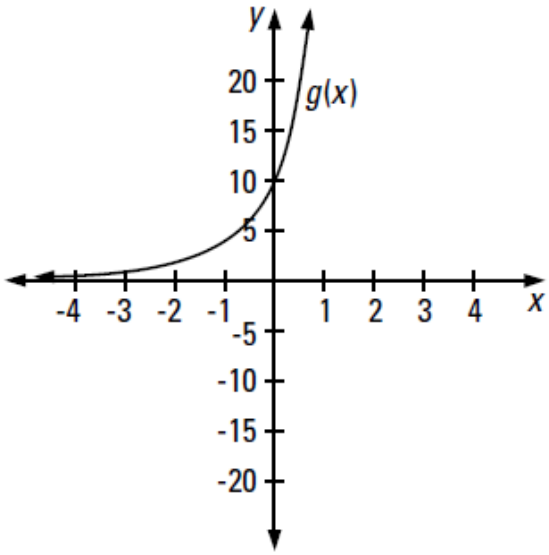


Sketch a graph of the derivative of the given function.

17.



18.



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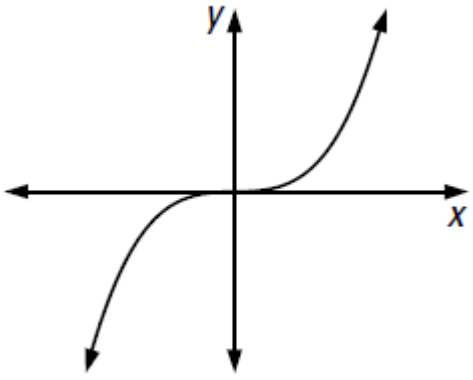
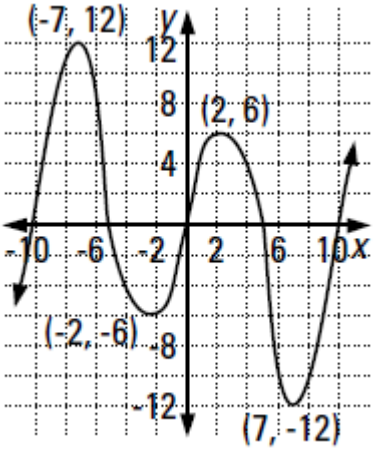
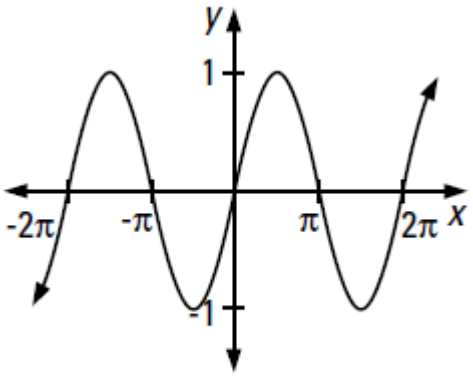


20.



21.



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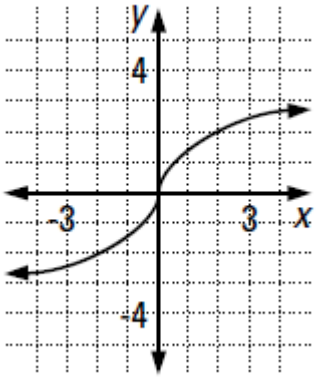
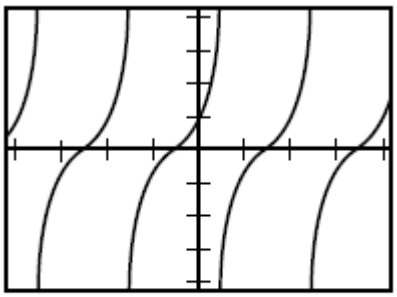


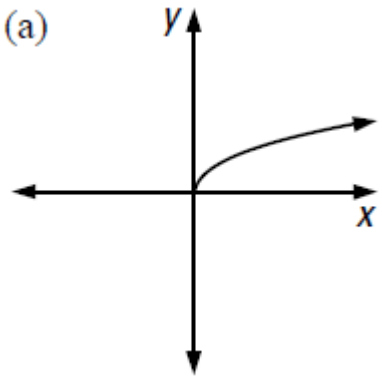
23.



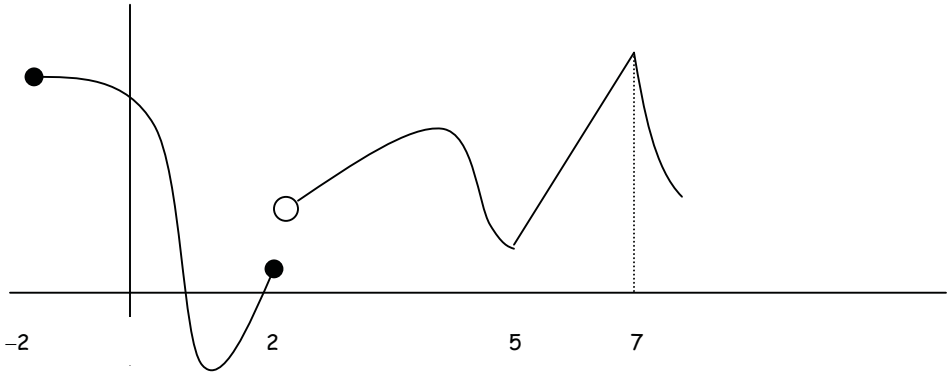
$-2\pi \le x \le 2\pi$ $\quad x$-scale $= \frac{\pi}{2}$

$-4 \le y \le 4$ $\quad y$-scale $= 1$

24.



1. The graph of f is given below. State the numbers at which f is not differentiable and why. Your reason should be based on the definition of differentiability at a number.



f is not differentiable at x = −2 because $f'_-(-2)$ does not exist.

f is not differentiable at x = 2 because $f'_-(2) \neq f'_+(2) \therefore f'(2)$ does not exist.
f is not differentiable at x = 5 because $f'_-(5) \neq f'_+(5) \therefore f'(5)$ does not exist.
f is not differentiable at x = 7 because $f'_-(7) \neq f'_+(7) \therefore f'(7)$ does not exist.

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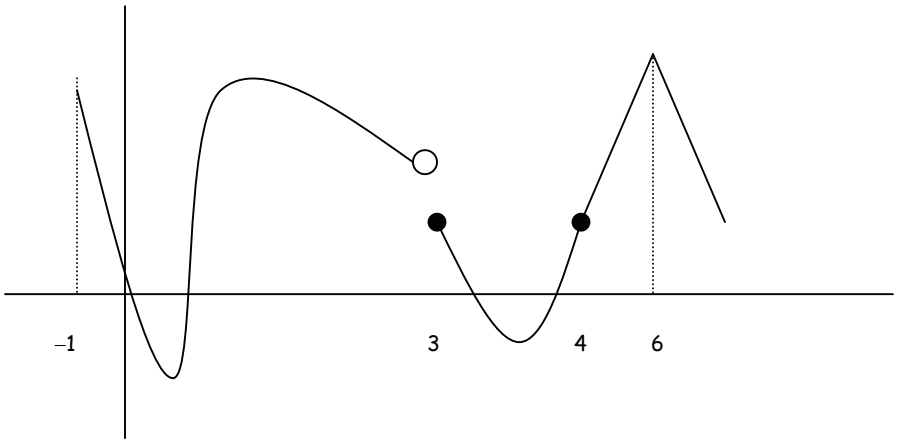
2. The graph of f is given below. State the numbers at which f is not differentiable and why. Your reason should be based on the definition of differentiability at a number. Also state where f is not continuous and tell why. Your reason should be based on the definition of continuity at a number.



f is not differentiable at x = 3 because f is not continuous at x = 3.
f is not differentiable at x = 4 because $f'_-(4) \neq f'_+(4) \therefore f'(4)$ does not exist.
f is not differentiable at x = 6 because $f'_-(6) \neq f'_+(6) \therefore f'(6)$ does not exist.

3. Show that $f(x) = |x - 6|$ is not differentiable at $x = 6$.

$$f(x) = \begin{cases} x - 6 & \text{if } x \geq 6 \\ 6 - x & \text{if } x < 6 \end{cases}$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 6 \\ -1 & \text{if } x < 6 \end{cases}$$

Now, since $f'_+(6) = 1$ and $f'_-(6) = -1$, then $f'_+(x) \neq f'_-(x)$ $\therefore$ $f'(6)$ does not exist and f is not differentiable at $x = 6$.

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4. Where is the greatest integer function $f(x) = [[x]]$ not differentiable?

f is not differentiable at any integer because f is not continuous at any integer.

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5. Where and why is the following function not continuous? Where and why is it not differentiable?

$$f(x) = \begin{cases} \dfrac{x^3 - x}{x^2 + x} & \text{if } x < 1 \text{ but } x \neq 0 \\ 0 & \text{if } x = 0 \\ 1 - x & \text{if } x \geq 1 \end{cases}$$

Continuity test at $x = 0$

i) $f(0) = 0$

ii) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} (x - 1) = -1$

Since $f(0) \neq \lim\limits_{x \to 0} f(x)$, f is not continuous at $x = 0$ and therefore f is not differentiable at $x = 0$

Continuity test at $x = 1$

i) $f(1) = 0$

ii) $\lim\limits_{x \to 1^-} f(x) = 0$

$\lim\limits_{x \to 1^+} f(x) = 0$

$\therefore \lim\limits_{x \to 1} f(x) = 0$

Since $f(1) = \lim\limits_{x \to 1} f(x)$, f is continuous at $x = 1$.

Differentiability test at $x = 1$

$$f'(x) = \begin{cases} 1 & x < 1 \text{ and } x \neq 0 \\ 0 & x = 0 \\ -1 & x > 1 \end{cases}$$

Since $f'_+(1) = -1$ amd $f'_-(1) = 1$, $f'_+(1) \neq f'_-(1)$ and $f'(1)$ does not exist. $\therefore$ f is not differentiable at $x = 1$

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6. If $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x - 4 & \text{if } x > 0 \end{cases}$, find $f'(x)$ and tell where (if anywhere) the derivative does not exist.

$$f'(x) = \begin{cases} 2x & x < 0 \\ 1 & x > 0 \end{cases}$$

Since $f'_+(0) = 1$ and $f'_-(0) = 0$, then $f'_+(0) \neq f'_-(0)$ $\therefore$ $f'(0)$ does not exist and f is not differentiable at $x = 0$.