Chapter 4

4-4 Study Guide and Intervention

Graphing Sine and Cosine Functions

Transformations of Sine and Cosine Functions  A sinusoid is a transformation of the graph of the sine function. The general form of the sinusoidal functions sine and cosine are $y = a \sin (bx + c) + d$ or $y = a \cos (bx + c) + d$. The graph of $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$ have the following characteristics.

- **Amplitude** $= |a|$
- **Period** $= \frac{2\pi}{|b|}$
- **Frequency** $= \frac{|b|}{2\pi}$
- **Phase Shift** $= -\frac{c}{b}$
- **Vertical Shift** $= d$
- **Midline** $= d$

**Example**

State the amplitude, period, frequency, phase shift, and vertical shift of $y = -2 \cos \left(\frac{x}{4} - \frac{1}{2}\right) + 2$. Then graph two periods of the function.

- **Amplitude** $= |a| = 2$
- **Period** $= \frac{2\pi}{|b|} = \frac{2\pi}{\frac{1}{4}} = 8\pi$
- **Frequency** $= |b| = \frac{1}{4}$
- **Phase Shift** $= -\frac{c}{b} = 2$
- **Vertical Shift** $= d = 2$

**Exercises**

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

1. $y = 3 \sin \left(2x + \frac{\pi}{2}\right)$
2. $y = \cos \left(x + \frac{\pi}{3}\right) + 2$
3. $y = 3 \sin \left(2x + \frac{\pi}{4}\right) + 1$
4. $y = \cos \left(3x\right) - 2$

State the frequency and midline of each function.

1. $y = 3 \sin \left(\frac{\pi x}{2} + \frac{\pi}{3}\right)$
2. $y = \cos \left(x - \frac{\pi}{2}\right) + 1$

Write a sine function with the given characteristics.

1. amplitude $= 2$, period $= 4$, phase shift $= \frac{1}{4}$, vertical shift $= 4$
2. amplitude $= 1.2$, phase shift $= \frac{3\pi}{4}$, vertical shift $= 1$

**Applications of Sinusoidal Functions**

The data can be modeled by a sinusoidal function of the form $y = a \sin (bx + c) + d$. Find the maximum $M$ and minimum $m$ values of the data, and use these values to find $a$, $b$, $c$, and $d$.

- **Amplitude formula** $a = \frac{M - m}{2}$
- **Period formula** $b = \frac{2\pi}{p}$
- **Phase shift formula** $c = -\frac{b d}{a}$
- **Vertical shift formula** $d = \frac{M + m}{2}$

**Example**

The table shows the average monthly temperatures for Ann Arbor, Michigan, in degrees Fahrenheit ($^\circ$F). Write a sinusoidal function that models the average monthly temperatures as a function of time $x$, where $x = 1$ represents January.

- **Month**
  - Jan: $30^\circ$
  - Feb: $24^\circ$
  - Mar: $40^\circ$
  - Apr: $50^\circ$
  - May: $71^\circ$
  - June: $80^\circ$
  - July: $84^\circ$
  - Aug: $81^\circ$
  - Sept: $74^\circ$
  - Oct: $62^\circ$
  - Nov: $48^\circ$
  - Dec: $30^\circ$

The data can be modeled by a sinusoidal function of the form $y = a \sin (bx + c) + d$. Find the maximum $M$ and minimum $m$ values of the data, and use these values to find $a$, $b$, $c$, and $d$.

1. **Amplitude formula** $a = \frac{M - m}{2}$
2. **Period formula** $b = \frac{2\pi}{p}$
3. **Phase shift formula** $c = -\frac{b d}{a}$
4. **Vertical shift formula** $d = \frac{M + m}{2}$

**Exercise**

1. **MUSEUM ATTENDANCE** The table gives the number of visitors in thousands to a museum for each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visitors</td>
<td>10</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>24</td>
<td>30</td>
<td>52</td>
<td>39</td>
</tr>
</tbody>
</table>

- **a.** Write a trigonometric function that models the monthly attendance at the museum using $x = 1$ to represent January.
  - $y = 12 \sin \left(\frac{\pi x - \frac{\pi}{2}}{2}\right) + 20$

- **b.** According to your model, how many people should the museum expect to visit during October? 16,292 people
4-4 Practice

Graphing Sine and Cosine Functions

Describe how the graphs of \( f(x) \) and \( g(x) \) are related. Then find the amplitude of \( g(x) \) and sketch two periods of both functions on the same coordinate axes.

1. \( f(x) = \sin x \)
   \[ g(x) = \frac{1}{3} \sin x \]
   The graph of \( g(x) \) is the graph of \( f(x) \) reflected vertically. The amplitude of \( g(x) \) is \( \frac{1}{3} \).

2. \( f(x) = \cos x \)
   \[ g(x) = -\frac{1}{3} \cos x \]
   The graph of \( g(x) \) is the graph of \( f(x) \) compressed vertically and reflected in the x-axis. The amplitude of \( g(x) \) is \( \frac{1}{3} \).

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

3. \( y = 2 \sin \left( x + \frac{\pi}{2} \right) - 3 \)
   \[ \text{amplitude} = 2; \quad \text{period} = 2\pi; \quad \text{frequency} = \frac{1}{2}; \quad \text{phase shift} = -\frac{\pi}{2}; \quad \text{vertical shift} = -3 \]

4. \( y = \frac{1}{2} \cos (2x - \pi) + 2 \)
   \[ \text{amplitude} = \frac{1}{2}; \quad \text{period} = \pi; \quad \text{frequency} = \frac{1}{4}; \quad \text{phase shift} = \pi; \quad \text{vertical shift} = 2 \]

Write a sinusoidal function with the given amplitude, period, phase shift, and vertical shift.

5. Sine function: amplitude = 15, period = 4\( \pi \), phase shift = \( \pi/2 \), vertical shift = -10
   \[ y = \pm 15 \sin \left( \frac{x}{4} - \frac{\pi}{8} \right) - 10 \]

6. Cosine function: amplitude = 3, period = \( \frac{\pi}{2} \), phase shift = -\( \frac{\pi}{2} \), vertical shift = 5
   \[ y = \pm \frac{3}{2} \cos \left( 6x + \frac{\pi}{2} \right) + 5 \]

7. Music: A piano tuner strikes a tuning fork note A above middle C and sets in motion vibrations that can be modeled by \( y = 0.001 \sin 880t \). Find the amplitude and period of the function.
   \[ \text{Amplitude} = 0.001; \quad \text{Period} = \frac{\pi}{440} \]

Chapter 4 Practice

NAME ___________________________ DATE __________ PERIOD ________

4-4 Word Problem Practice

Graphing Sine and Cosine Functions

1. METEOROLOGY The average monthly temperatures for Baltimore, Maryland, are shown below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>32</td>
</tr>
<tr>
<td>Feb</td>
<td>35</td>
</tr>
<tr>
<td>Mar</td>
<td>44</td>
</tr>
<tr>
<td>Apr</td>
<td>53</td>
</tr>
<tr>
<td>May</td>
<td>63</td>
</tr>
<tr>
<td>June</td>
<td>73</td>
</tr>
<tr>
<td>July</td>
<td>82</td>
</tr>
<tr>
<td>Aug</td>
<td>74</td>
</tr>
<tr>
<td>Sept</td>
<td>69</td>
</tr>
<tr>
<td>Oct</td>
<td>57</td>
</tr>
<tr>
<td>Nov</td>
<td>47</td>
</tr>
<tr>
<td>Dec</td>
<td>37</td>
</tr>
</tbody>
</table>

a. Determine the amplitude, period, phase shift, and vertical shift of a sinusoidal function that models the monthly temperatures using \( x = 1 \) to represent January.

   \[ \text{amplitude} = 22.5; \quad \text{period} = 12 \text{ months}; \quad \text{phase shift} = -5 \text{ months}; \quad \text{vertical shift} = 54.5 \]

   Sample answer:
   \[ y = 22.5 \cos \left( \frac{\pi}{6} x + \frac{5\pi}{6} \right) + 54.5 \]

b. Write an equation of a sinusoidal function that models the monthly temperatures.

   \[ y = -9 \cos \left( \frac{\pi}{2} x + 2 \right) \]

   Sample answer: \( 77^\circ; \quad 35^\circ \)

2. BOATING A buoy, bobbing up and down in the water as waves pass it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 2 feet.

   a. Determine the amplitude and period of a sinusoidal function that models the bobbing buoy.

      \[ 1.5; \quad 10 \text{ s} \]

   b. Write an equation of a sinusoidal function that models the bobbing buoy, using \( x = 0 \) as its highest point.

      Sample answer:
      \[ y = 1.5 \cos \left( \frac{\pi}{5} x \right) \]

3. A student graphed a periodic function with a period of \( n \). The student then translated the graph \( c \) units to the right and obtained the original graph. Describe the relationship between \( c \) and \( n \).

   Sample answer: \( c \) is a positive multiple of \( n \).

4. SWING Marsha is pushing her brother Bobby on a rope swing over a creek. When she starts the swing, he is 7 feet above the water. After 2 seconds, Bobby is 11 feet above the water past the edge of the creek. Assume that the distance from the edge of the creek varies sinusoidally with time and that the distance \( y \) is positive when Bobby is over the water and negative when he is over land. Write a trigonometric function that models the distance Bobby is from the edge of the creek at time \( t \) seconds.

   \[ y = -9 \cos \left( \frac{\pi}{2} t + 2 \right) \]

   Sample answer: \( 77^\circ; \quad 35^\circ \)

5. ROLLER COASTER Part of a roller coaster track is a sinusoidal function.

   The high and low points are separated by 150 feet horizontally and 82 feet vertically as shown. The low point is 6 feet above the ground.

   a. Write a sinusoidal function that models the distance the roller coaster track is above the ground at a given horizontal distance \( x \).

      \[ y = 41 \cos \left( \frac{\pi}{150} x + 47 \right) \]

   b. Point A is 40 feet to the right of the \( y \)-axis. How far above the ground is the track at point A?

      Point A is 74.4 feet above the ground.