1-7 Inverse Relations and Functions

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write yes or no.

1. \( f(x) = x^2 + 6x + 9 \)

**SOLUTION:**

The graph of \( f(x) = x^2 + 6x + 9 \) below shows that it is possible to find a horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

\[ \text{[Graph]} \]

\([-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}\]

3. \( f(x) = x^2 - 10x + 25 \)

**SOLUTION:**

The graph of \( f(x) = x^2 - 10x + 25 \) below shows that it is possible to find a horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

\[ \text{[Graph]} \]

\([-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}\]

5. \( f(x) = \sqrt{2x} \)

**SOLUTION:**

It appears from the portion of the graph of \( f(x) = \sqrt{2x} \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

\[ \text{[Graph]} \]

\([-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}\]

7. \( f(x) = \sqrt{x + 4} \)

**SOLUTION:**

It appears from the portion of the graph of \( f(x) = \sqrt{x + 4} \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

\[ \text{[Graph]} \]

\([-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}\]
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9. \( f(x) = \frac{5}{x - 6} \)

**SOLUTION:**
It appears from the portion of the graph of \( f(x) = \frac{5}{x - 6} \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does exist.

15. \( h(x) = x^7 + 2x^3 - 10x^2 \)

**SOLUTION:**
The graph of \( h(x) = x^7 + 2x^3 - 10x^2 \) below shows that it is possible to find a horizontal line that intersects the graph of \( h(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

11. \( f(x) = x^3 - 9 \)

**SOLUTION:**
It appears from the portion of the graph of \( f(x) = x^3 - 9 \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does exist.

13. \( g(x) = -3x^4 + 6x^2 - x \)

**SOLUTION:**
The graph of \( g(x) = -3x^4 + 6x^2 - x \) below shows that it is possible to find a horizontal line that intersects the graph of \( g(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

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17. \( f(x) = \sqrt{6 - x^2} \)

**SOLUTION:**
The graph of \( f(x) = \sqrt{6 - x^2} \) below shows that it is possible to find a horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does not exist.

\[
\begin{align*}
[-10, 10] \text{ scl: 1 by [-3, 3] scl: 1}
\end{align*}
\]

19. \( f(x) = \frac{4 - x}{x} \)

**SOLUTION:**
It appears from the portion of the graph of \( f(x) = \frac{4 - x}{x} \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does exist.

\[
\begin{align*}
[-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}
\end{align*}
\]

The function \( f \) has domain \((-\infty, 0) \cup (0, \infty)\) and range \((-\infty, -1) \cup (-1, \infty)\).

\[
\begin{align*}
&y = \frac{4 - x}{x} \\
x &= \frac{4 - y}{y} \\
x = 4 - y \\
x + y &= 4 \\
y(x + 1) &= 4 \\
y &= \frac{4}{x + 1}
\end{align*}
\]

17. \( f(x) = \sqrt{6 - x^2} \)

\[
\begin{align*}
f^{-1}(x) &= \frac{4}{x + 1}
\end{align*}
\]

From the graph \( y = \frac{4}{x + 1} \) below, you can see that the inverse relation has domain \((-\infty, -1) \cup (-1, \infty)\) and range \((-\infty, 0) \cup (0, \infty)\).

\[
\begin{align*}
[-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}
\end{align*}
\]

The domain and range of \( f \) are equal to the range and domain of \( f^{-1} \), respectively. Therefore, no further restrictions are necessary. \( f^{-1}(x) = \frac{4}{x + 1} \) for \( x \neq -1 \).

21. \( f(x) = \frac{6}{\sqrt{8 - x}} \)

**SOLUTION:**
It appears from the portion of the graph of \( f(x) = \frac{6}{\sqrt{8 - x}} \) shown below that there is no horizontal line that intersects the graph of \( f(x) \) more than once. Therefore, you can conclude that an inverse function does exist.

\[
\begin{align*}
[-10, 10] \text{ scl: 1 by [-10, 10] scl: 1}
\end{align*}
\]

The function \( f \) has domain \((-\infty, 8)\) and range \((0, \infty)\).
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Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse exists. Then reflect the points and connect the points with a smooth curve that resembles the original graph.

\[ y = \frac{6}{\sqrt{8-x}} \]
\[ x = \frac{6}{\sqrt{8-y}} \]
\[ \sqrt{8-y} = \frac{6}{x} \]
\[ 8-y = \left( \frac{6}{x} \right)^2 \]
\[ 8-y = \frac{36}{x^2} \]
\[ 8 - \frac{36}{x^2} = y \]
\[ f^{-1}(x) = 8 - \frac{36}{x^2} \]

From the graph \( y = 8 - \frac{36}{x^2} \) below, you can see that the inverse relation has domain \([-\infty, 0) \cup (0, \infty)\) and range \((-\infty, 8)\).

By restricting the domain of the inverse relation to \((0, \infty)\), the domain and range of \(f^{-1}\) are equal to the range and domain of \(f^{-1}\), respectively. Therefore, \(f^{-1}(x) = 8 - \frac{36}{x^2}\) for \(x > 0\).

23. \(f(x) = \frac{6x+3}{x-8}\)

**SOLUTION:**

It appears from the portion of the graph of \(f(x) = \frac{6x+3}{x-8}\) shown below that there is no horizontal line that intersects the graph of \(f(x)\) more than once. Therefore, you can conclude that an inverse function does exist.

The function \(f\) has domain \((-\infty, 8) \cup (8, \infty)\) and range \((-\infty, 6) \cup (6, \infty)\).

\[ y = \frac{6x+3}{x-8} \]
\[ x = \frac{6y+3}{y-8} \]
\[ xy - 8x = 6y + 3 \]
\[ xy - 6y = 8x + 3 \]
\[ y(x-6) = 8x + 3 \]
\[ y = \frac{8x+3}{x-6} \]
\[ f^{-1}(x) = \frac{8x+3}{x-6} \]

From the graph \(y = \frac{8x+3}{x-6}\) below, you can see that the inverse relation has domain \((-\infty, 6) \cup (6, \infty)\) and range \((-\infty, 8) \cup (8, \infty)\).

The domain and range of \(f\) are equal to the range and domain of \(f^{-1}\), respectively. Therefore, no further restrictions are necessary. \(f^{-1}(x) = \frac{8x+3}{x-6}\) for \(x \neq 6\).
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Use the graph of each function to graph its inverse function.

39.

**SOLUTION:**
Graph the line $y = x$ and reflect the points. Then connect the points with a smooth curve that resembles the original graph.

41.

**SOLUTION:**
Graph the line $y = x$ and reflect the points. Then connect the points with a smooth curve that resembles the original graph.