# Honors Pre-Calculus Summer Assignment

1) Your summer assignment will include a review of Algebra 2 topics. The following pages contain a copy of Chapter P (Prerequisites) of your textbook for next year.

2) You should be capable of doing all of these problems without the use of a calculator.

3) This assignment will be collected.

4) You should do these problems showing all work in a neat, organized manner. Graphs should be done accurately (axes and points labeled).

**Section P.1** *#6, 8, 11, 14, 20, 24, 25, 29, 32, 38, 40, 48, 50, 52*

**Section P.2** *#2, 5, 9, 12, 16, 18, 21, 26, 27, 31, 43, 46*

**Section P.3** *#3, 8, 18, 20, 23, 25, 27, 33, 35, 41, 50*

**Section P.4** *#4, 9, 12, 18, 23, 26, 42, 48*

**Section P.5** *#3, 9, 13, 18, 21, 26, 42, 59*

**Section P.6** *#2, 7, 11, 26, 30, 35, 38, 42*

**Section P.7** *#1, 7, 14, 24, 25, 33*

If you have any questions, please see me. I will occasionally check my e-mail over the summer if you have any questions during that time.

Good luck and have a wonderful summer!

Mr. Franz

## Chapter P Overview

Historically, algebra was used to represent problems with symbols (algebraic models) and solve them by reducing the solution to algebraic manipulation of symbols. This technique is still important today. In addition, graphing calculators are now used to represent problems with graphs (graphical models) and solve them with the numerical and graphical techniques of technology.

We begin with basic properties of real numbers and introduce absolute value, distance formulas, midpoint formulas, and equations of circles. Slope of a line is used to write standard equations for lines, and applications involving linear equations are discussed. The basics of complex numbers are explained. Equations and inequalities are solved using both algebraic and graphical techniques.

# P.1 Real Numbers

### What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

### ... and why

These topics are fundamental in the study of mathematics and science.

## Representing Real Numbers

A **real number** is any number that can be written as a decimal. Real numbers are represented by symbols such as $-8, 0, 1.75, 2.333..., 0.\overline{36}, 8/5, \sqrt{3}, \sqrt[3]{16}, e,$ and $\pi$.

The set of real numbers contains several important subsets:

| | |
|---|---|
| The **natural (or counting) numbers**: | $\{1, 2, 3, \dots\}$ |
| The **whole numbers**: | $\{0, 1, 2, 3, \dots\}$ |
| The **integers**: | $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ |

Braces { } are used to enclose the **elements**, or **objects**, of a set. The rational numbers are another important subset of the real numbers. A **rational number** is any number that can be written as a ratio $a/b$ of two integers, where $b \neq 0$. We can use **set-builder notation** to define the rational numbers:

$$\left\{ \frac{a}{b} \middle| a, b \text{ are integers, and } b \neq 0 \right\}$$

The vertical bar that follows $a/b$ is read "such that."

The decimal form of a rational number either **terminates** like $7/4 = 1.75$, or is **infinitely repeating** like $4/11 = 0.363636... = 0.\overline{36}$. The bar over the 36 indicates the block of digits that repeats. A real number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating. For example, $\sqrt{3} = 1.7320508...$ and $\pi = 3.14159265....$

A real number can be approximated by giving a few of its digits. Sometimes we can find the decimal form of rational numbers with calculators, but not very often.



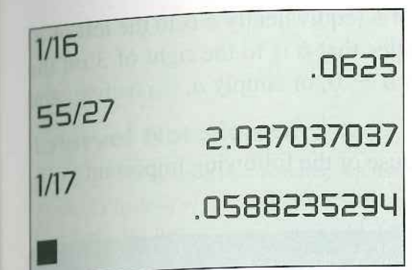



**FIGURE P.1** Calculator decimal representations of 1/16, 55/27, and 1/17 with the calculator set in floating decimal mode. (Example 1)

### EXAMPLE 1 Examining Decimal Forms of Rational Numbers

Determine the decimal form of 1/16, 55/27, and 1/17.

**SOLUTION** Figure P.1 suggests that the decimal form of 1/16 terminates and that of 55/27 repeats in blocks of 037.

$$\frac{1}{16} = 0.0625 \quad \text{and} \quad \frac{55}{27} = 2.\overline{037}$$

We cannot predict the *exact* decimal form of 1/17 from Figure P.1; however, we can say that $1/17 \approx 0.0588235294$. The symbol $\approx$ is read "*is approximately equal to*." We can use long division (see Exercise 66) to prove that

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

Now try Exercise 3.

The real numbers and the points of a line can be matched *one-to-one* to form a **real number line**. We start with a horizontal line and match the real number zero with a point $O$, the **origin**. **Positive numbers** are assigned to the right of the origin, and **negative numbers** to the left, as shown in Figure P.2.

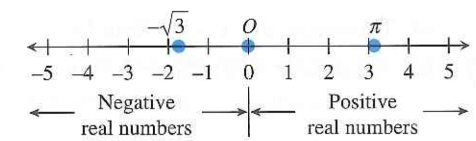



**FIGURE P.2** The real number line.

Every real number corresponds to one and only one point on the real number line, and every point on the real number line corresponds to one and only one real number. Between every pair of real numbers on the number line there are infinitely many more real numbers.

The number associated with a point is **the coordinate of the point**. As long as the context is clear, we will follow the standard convention of using the real number for both the name of the point and its coordinate.

## Order and Interval Notation

The set of real numbers is **ordered**. This means that we can use inequalities to compare any two real numbers that are not equal and say that one is "less than" or "greater than" the other.

### Unordered Systems

Not all number systems are ordered. For example, the complex number system, to be introduced in Section P.6, has no natural ordering.

### Order of Real Numbers

Let $a$ and $b$ be any two real numbers.

| Symbol | Definition | Read |
|---|---|---|
| $a > b$ | $a - b$ is positive | $a$ is greater than $b$ |
| $a < b$ | $a - b$ is negative | $a$ is less than $b$ |
| $a \geq b$ | $a - b$ is positive or zero | $a$ is greater than or equal to $b$ |
| $a \leq b$ | $a - b$ is negative or zero | $a$ is less than or equal to $b$ |

The symbols $>$, $<$, $\geq$, and $\leq$ are **inequality symbols**.

### Opposites and Number Line

$a < 0 \Leftrightarrow -a > 0$

If $a < 0$, then $a$ is to the left of 0 on the real number line, and its opposite, $-a$, is to the right of 0. Thus, $-a > 0$. This logic can be reversed: If $-a > 0$, then $a < 0$.

Geometrically, $a > b$ means that $a$ is to the right of $b$ (equivalently $b$ is to the left of $a$) on the real number line. For example, $6 > 3$ implies that 6 is to the right of 3 on the real number line. Note also that $a > 0$ means that $a - 0$, or simply $a$, is positive, and $a < 0$ means that $a$ is negative.

We are able to compare any two real numbers because of the following important property of the real numbers.

### Trichotomy Property

Let $a$ and $b$ be any two real numbers. Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b$$

Inequalities can be used to describe **intervals** of real numbers, as illustrated in Example 2.

### EXAMPLE 2 Interpreting Inequalities

Describe and graph the interval of real numbers for the inequality.

**(a)** $x < 3$ **(b)** $-1 < x \le 4$

**SOLUTION**

**(a)** The inequality $x < 3$ describes all real numbers less than 3 (Figure P.3a).

**(b)** The *double inequality* $-1 < x \le 4$ represents all real numbers between $-1$ and 4, excluding $-1$ and including 4 (Figure P.3b). Now try Exercise 5.

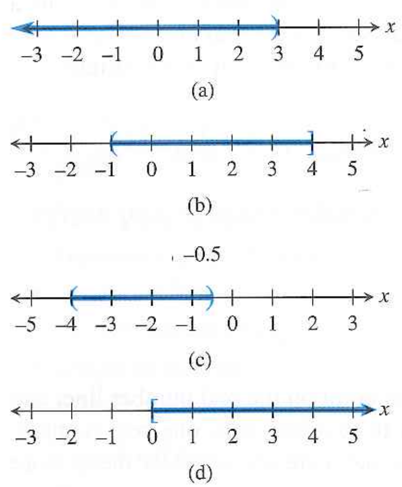


**FIGURE P.3** In graphs of inequalities, parentheses correspond to $<$ and $>$ and brackets to $\le$ and $\ge$. (Examples 2 and 3)

### EXAMPLE 3 Writing Inequalities

Write an interval of real numbers using an inequality and draw its graph.

**(a)** The real numbers between $-4$ and $-0.5$

**(b)** The real numbers greater than or equal to zero

**SOLUTION**

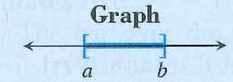
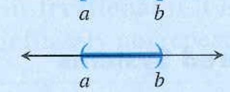
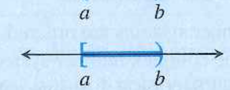
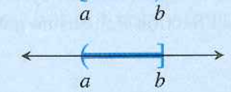
**(a)** $-4 < x < -0.5$ (Figure P.3c)

**(b)** $x \ge 0$ (Figure P.3d) Now try Exercise 15.

As shown in Example 2, inequalities define *intervals* on the real number line. We often use $[2, 5]$ to describe the *bounded interval* determined by $2 \le x \le 5$. This interval is **closed** because it contains its *endpoints* 2 and 5. There are four types of **bounded intervals**.

### Bounded Intervals of Real Numbers

Let $a$ and $b$ be real numbers with $a < b$.

| Interval Notation | Interval Type | Inequality Notation | Graph |
|---|---|---|---|
| $[a, b]$ | Closed | $a \le x \le b$ |  |
| $(a, b)$ | Open | $a < x < b$ |  |
| $[a, b)$ | Half-open | $a \le x < b$ |  |
| $(a, b]$ | Half-open | $a < x \le b$ |  |

The numbers $a$ and $b$ are the **endpoints** of each interval.



### Interval Notation at $\pm\infty$

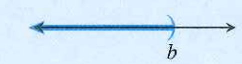
Because $-\infty$ is *not* a real number, we use $(-\infty, 2)$ instead of $[-\infty, 2)$ to describe $x < 2$. Similarly, we use $[-1, \infty)$ instead of $[-1, \infty]$ to describe $x \ge -1$.

We use the interval notation $(-\infty, \infty)$ to represent the entire set of real numbers. The symbols $-\infty$ (*negative infinity*) and $\infty$ (*positive infinity*) allow us to use interval notation for unbounded intervals and are *not* real numbers.

The interval of real numbers determined by the inequality $x < 2$ can be described by the *unbounded interval* $(-\infty, 2)$. This interval is **open** because it does *not* contain its endpoint 2. In addition to $(-\infty, \infty)$, there are four types of **unbounded intervals**.

### Unbounded Intervals of Real Numbers

Let $a$ and $b$ be real numbers.

| Interval Notation | Interval Type | Inequality Notation | Graph |
|---|---|---|---|
| $[a, \infty)$ | Closed | $x \ge a$ |  |
| $(a, \infty)$ | Open | $x > a$ |  |
| $(-\infty, b]$ | Closed | $x \le b$ |  |
| $(-\infty, b)$ | Open | $x < b$ |  |

Each of these intervals has exactly one endpoint, namely $a$ or $b$.

### EXAMPLE 4 Converting Between Intervals and Inequalities

Convert interval notation to inequality notation or vice versa. Find the endpoints and state whether the interval is bounded, its type, and graph the interval.

**(a)** $[-6, 3)$ **(b)** $(-\infty, -1)$ **(c)** $-2 \le x \le 3$

**SOLUTION**

**(a)** The interval $[-6, 3)$ corresponds to $-6 \le x < 3$ and is bounded and half-open (Figure P.4a). The endpoints are $-6$ and 3.

**(b)** The interval $(-\infty, -1)$ corresponds to $x < -1$ and is unbounded and open (Figure P.4b). The only endpoint is $-1$.

**(c)** The inequality $-2 \le x \le 3$ corresponds to the closed, bounded interval $[-2, 3]$ (Figure P.4c). The endpoints are $-2$ and 3. Now try Exercise 29.

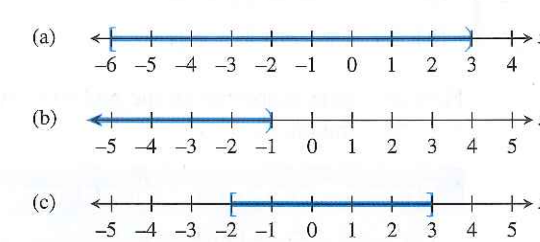


**FIGURE P.4** Graphs of the intervals of real numbers in Example 4.

## Basic Properties of Algebra

Algebra involves the use of letters and other symbols to represent real numbers. A **variable** is a letter or symbol (for example, $x$, $y$, $t$, $\theta$) that represents an unspecified real number. A **constant** is a letter or symbol (for example, $-2$, 0, $\sqrt{3}$, $\pi$) that represents a specific real number. An **algebraic expression** is a combination of variables and constants involving addition, subtraction, multiplication, division, powers, and roots.

We state some of the properties of the arithmetic operations of addition, subtraction, multiplication, and division, represented by the symbols $+, -, \times$ (or $\cdot$) and $\div$ (or /), respectively. Addition and multiplication are the primary operations. Subtraction and division are defined in terms of addition and multiplication.

**Subtraction**: $a - b = a + (-b)$

**Division**: $\frac{a}{b} = a\left(\frac{1}{b}\right), b \neq 0$

### Subtraction vs. Negative Numbers

On many calculators, there are two "−" keys, one for subtraction and one for negative numbers or opposites. Be sure you know how to use both keys correctly. Misuse can lead to incorrect results.

In the above definitions, $-b$ is the **additive inverse** or **opposite** of $b$, and $1/b$ is the **multiplicative inverse** or **reciprocal** of $b$. Perhaps surprisingly, additive inverses are not always negative numbers. The additive inverse of 5 is the negative number $-5$. However, the additive inverse of $-3$ is the positive number 3.

The following properties hold for real numbers, variables, and algebraic expressions.

### Properties of Algebra

Let $u$, $v$, and $w$ be real numbers, variables, or algebraic expressions.

**1. Commutative properties**
Addition: $u + v = v + u$
Multiplication: $uv = vu$

**2. Associative properties**
Addition:
$(u + v) + w = u + (v + w)$
Multiplication: $(uv)w = u(vw)$

**3. Identity properties**
Addition: $u + 0 = u$
Multiplication: $u \cdot 1 = u$

**4. Inverse properties**
Addition: $u + (-u) = 0$
Multiplication: $u \cdot \frac{1}{u} = 1, u \neq 0$

**5. Distributive properties**
Multiplication over addition:
$u(v + w) = uv + uw$
$(u + v)w = uw + vw$
Multiplication over subtraction:
$u(v - w) = uv - uw$
$(u - v)w = uw - vw$

The left-hand sides of the equations for the distributive property show the **factored form** of the algebraic expressions, and the right-hand sides show the **expanded form**.

### EXAMPLE 5 Using the Distributive Property

**(a)** Write the expanded form of $(a + 2)x$.

**(b)** Write the factored form of $3y - by$.

**SOLUTION**

**(a)** $(a + 2)x = ax + 2x$

**(b)** $3y - by = (3 - b)y$

Now try Exercise 37.

Here are some properties of the additive inverse together with examples that help illustrate their meanings.

### Properties of the Additive Inverse

Let $u$ and $v$ be real numbers, variables, or algebraic expressions.

| Property | Example |
|---|---|
| 1. $-(-u) = u$ | $-(-3) = 3$ |
| 2. $(-u)v = u(-v) = -(uv)$ | $(-4)3 = 4(-3) = -(4 \cdot 3) = -12$ |
| 3. $(-u)(-v) = uv$ | $(-6)(-7) = 6 \cdot 7 = 42$ |
| 4. $(-1)u = -u$ | $(-1)5 = -5$ |
| 5. $-(u + v) = (-u) + (-v)$ | $-(7 + 9) = (-7) + (-9) = -16$ |



## Integer Exponents

Exponential notation is used to shorten products of factors that repeat. For example,

$$(-3)(-3)(-3)(-3) = (-3)^4 \quad \text{and} \quad (2x + 1)(2x + 1) = (2x + 1)^2.$$

### Exponential Notation

Let $a$ be a real number, variable, or algebraic expression and $n$ a positive integer. Then

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}},$$

where $n$ is the **exponent**, $a$ is the **base**, and $a^n$ is the **nth power of a**, read as "$a$ to the $n$th power."

The two exponential expressions in Example 6 have the same value but have different bases. Be sure you understand the difference.

### Understanding Notation

$(-3)^2 = 9$
$-3^2 = -9$
Be careful!

### EXAMPLE 6 Identifying the Base

**(a)** In $(-3)^5$, the base is $-3$.

**(b)** In $-3^5$, the base is 3.

Now try Exercise 43.

Here are the basic properties of exponents together with examples that help illustrate their meanings.

### Properties of Exponents

Let $u$ and $v$ be real numbers, variables, or algebraic expressions and $m$ and $n$ be integers. All bases are assumed to be nonzero.

| Property | Example |
|---|---|
| 1. $u^m u^n = u^{m+n}$ | $5^3 \cdot 5^4 = 5^{3+4} = 5^7$ |
| 2. $\frac{u^m}{u^n} = u^{m-n}$ | $\frac{x^9}{x^4} = x^{9-4} = x^5$ |
| 3. $u^0 = 1$ | $8^0 = 1$ |
| 4. $u^{-n} = \frac{1}{u^n}$ | $y^{-3} = \frac{1}{y^3}$ |
| 5. $(uv)^m = u^m v^m$ | $(2z)^5 = 2^5 z^5 = 32z^5$ |
| 6. $(u^m)^n = u^{mn}$ | $(x^2)^3 = x^{2 \cdot 3} = x^6$ |
| 7. $\left(\frac{u}{v}\right)^m = \frac{u^m}{v^m}$ | $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$ |

To simplify an expression involving powers means to rewrite it so that each factor appears only once, all exponents are positive, and exponents and constants are combined as much as possible.

### Moving Factors

Be sure you understand how exponent property 4 permits us to move factors from the numerator to the denominator and vice versa:

$$\frac{v^{-m}}{u^{-n}} = \frac{u^n}{v^m}$$

## EXAMPLE 7 Simplifying Expressions Involving Powers

(a) $(2ab^3)(5a^2b^5) = 10(aa^2)(b^3b^5) = 10a^3b^8$

(b) $\frac{u^2v^{-2}}{u^{-1}v^3} = \frac{u^2u^1}{v^2v^3} = \frac{u^3}{v^5}$

(c) $\left(\frac{x^2}{2}\right)^{-3} = \left(\frac{2}{x^2}\right)^3 = \frac{2^3}{(x^2)^3} = \frac{8}{x^6}$

Now try Exercise 47.

## Scientific Notation

Any positive number can be written in **scientific notation**,

$$c \times 10^m, \text{ where } 1 \le c < 10 \text{ and } m \text{ is an integer.}$$

This notation provides a way to work with very large and very small numbers. For example, the distance between the Earth and the Sun is about 93,000,000 miles. In scientific notation,

$$93{,}000{,}000 \text{ mi} = 9.3 \times 10^7 \text{ mi.}$$

The *positive exponent* 7 indicates that moving the decimal point in 9.3 to the right 7 places produces the decimal form of the number.

The mass of an oxygen molecule is about

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 054 \text{ g.}$$

In scientific notation,

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 054 \text{ g} = 5.4 \times 10^{-23} \text{ g.}$$

The *negative exponent* −23 indicates that moving the decimal point in 5.4 to the left 23 places produces the decimal form of the number.

## EXAMPLE 8 Converting to and from Scientific Notation

(a) $2.375 \times 10^8 = 237{,}500{,}000$

(b) $0.000000349 = 3.49 \times 10^{-7}$

Now try Exercises 57 and 59.

## EXAMPLE 9 Using Scientific Notation

Simplify $\frac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000}$.

**SOLUTION**

$$\begin{aligned}\frac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000} &= \frac{(3.6 \times 10^5)(4.5 \times 10^9)}{1.8 \times 10^4}\\ &= \frac{(3.6)(4.5)}{1.8} \times 10^{5+9-4}\\ &= 9 \times 10^{10}\\ &= 90{,}000{,}000{,}000\end{aligned}$$

Now try Exercise 63.

Using a Calculator Figure P.5 shows two ways to perform the computation. In the first, the numbers are entered in decimal form. In the second, the numbers are entered in scientific notation. The calculator uses "9E10" to stand for $9 \times 10^{10}$.



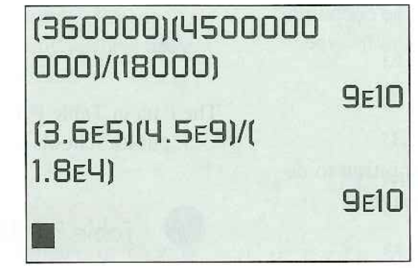



**FIGURE P.5** Be sure you understand how your calculator displays scientific notation. (Example 9)

## QUICK REVIEW P.1

1. List the positive integers between −3 and 7.
2. List the integers between −3 and 7.
3. List all negative integers greater than −4.
4. List all positive integers less than 5.

In Exercises 5 and 6, use a calculator to evaluate the expression. Round the value to two decimal places.

5. (a) $4(-3.1)^3 - (-4.2)^5$ (b) $\frac{2(-5.5) - 6}{7.4 - 3.8}$
6. (a) $5[3(-1.1)^2 - 4(-0.5)^3]$ (b) $5^{-2} + 2^{-4}$

In Exercises 7 and 8, evaluate the algebraic expression for the given values of the variables.

7. $x^3 - 2x + 1, x = -2, 1.5$
8. $a^2 + ab + b^2, a = -3, b = 2$

In Exercises 9 and 10, list the possible remainders.

9. When the positive integer $n$ is divided by 7
10. When the positive integer $n$ is divided by 13

## SECTION P.1 Exercises

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

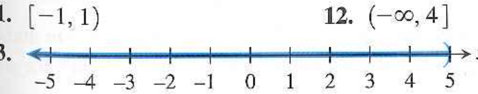
In Exercises 1–4, find the decimal form for the rational number. State whether it repeats or terminates.

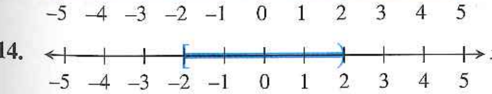
1. −37/8
2. 15/99
3. −13/6
4. 5/37

In Exercises 5–10, describe and graph the interval of real numbers.

5. $x \le 2$
6. $-2 \le x < 5$
7. $(-\infty, 7)$
8. $[-3, 3]$
9. $x$ is negative.
10. $x$ is greater than or equal to 2 and less than or equal to 6.

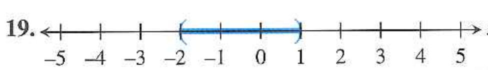
In Exercises 11–16, use an inequality to describe the interval of real numbers.

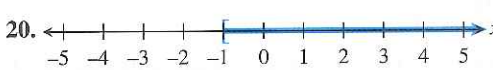
11. $[-1, 1)$
12. $(-\infty, 4]$
13. 

14. 

15. $x$ is between −1 and 2.
16. $x$ is greater than or equal to 5.

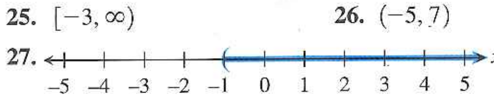
In Exercises 17–22, use interval notation to describe the interval of real numbers.

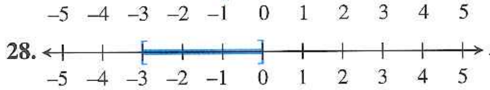
17. $x > -3$
18. $-7 < x < -2$
19. 

20. 

21. $x$ is greater than −3 and less than or equal to 4.
22. $x$ is positive.

In Exercises 23–28, use words to describe the interval of real numbers.

23. $4 < x \le 9$
24. $x \ge -1$
25. $[-3, \infty)$
26. $(-5, 7)$
27. 

28. 

In Exercises 29–32, convert to inequality notation. Find the endpoints and state whether the interval is bounded or unbounded and its type.

**29.** $(-3, 4]$ **30.** $(-3, -1)$

**31.** $(-\infty, 5)$ **32.** $[-6, \infty)$

In Exercises 33–36, use both inequality and interval notation to describe the set of numbers. State the meaning of any variables you use.

**33. Writing to Learn** Bill is at least 29 years old.

**34. Writing to Learn** No item at Sarah's Variety Store costs more than \$2.00.

**35. Writing to Learn** The price of a gallon of gasoline varies from \$3.099 to \$4.399.

**36. Writing to Learn** Salary raises at California State University at Chico will be between 2% and 6.5% this year.

In Exercises 37–40, use the distributive property to write the factored form or the expanded form of the given expression.

**37.** $a(x^2 + b)$ **38.** $(y - z^3)c$

**39.** $ax^2 + dx^2$ **40.** $a^3z + a^3w$

In Exercises 41 and 42, find the additive inverse of the number.

**41.** $6 - \pi$ **42.** $-7$

In Exercises 43 and 44, identify the base of the exponential expression.

**43.** $-5^2$ **44.** $(-2)^7$

**45. Group Activity** Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(3x)y = 3(xy)$ (b) $a^2b = ba^2$

(c) $a^2b + (-a^2b) = 0$ (d) $(x + 3)^2 + 0 = (x + 3)^2$

(e) $a(x + y) = ax + ay$

**46. Group Activity** Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(x + 2)\dfrac{1}{x + 2} = 1$ (b) $1 \cdot (x + y) = x + y$

(c) $2(x - y) = 2x - 2y$

(d) $2x + (y - z) = 2x + (y + (-z))$
$= (2x + y) + (-z) =$
$(2x + y) - z$

(e) $\dfrac{1}{a}(ab) = \left(\dfrac{1}{a}a\right)b = 1 \cdot b = b$

In Exercises 47–52, simplify the expression. Assume that the variables in the denominators are nonzero.

**47.** $\dfrac{x^4y^3}{x^2y^5}$ **48.** $\dfrac{(3x^2)^2y^4}{3y^2}$

**49.** $\left(\dfrac{4}{x^2}\right)^2$ **50.** $\left(\dfrac{2}{xy}\right)^{-3}$

**51.** $\dfrac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$ **52.** $\left(\dfrac{4a^3b}{a^2b^3}\right)\left(\dfrac{3b^2}{2a^2b^4}\right)$

The data in Table P.1 give the expenditures in millions of dollars for U.S. public schools for the 2008–2009 school year.

**Table P.1 U.S. Public School Expenditures**

| Category | Amount (millions of \$) |
|---|---|
| Current expenditures | 518,997 |
| Capital outlay | 65,882 |
| Interest on school debt | 16,691 |
| Total | 610,110 |

*Source: National Center for Education Statistics, U.S. Department of Education, as reported in The World Almanac and Book of Facts 2012.*

In Exercises 53–56, write the amount of expenditures in dollars obtained from the category in scientific notation.

**53.** Current expenditures

**54.** Capital outlay

**55.** Interest on school debt

**56.** Total

In Exercises 57 and 58, write the number in scientific notation.

**57.** The mean distance from Jupiter to the Sun is about 483,900,000 miles.

**58.** The electric charge, in coulombs, of an electron is about −0.000 000 000 000 000 000 16.

In Exercises 59–62, write the number in decimal form.

**59.** $3.33 \times 10^{-8}$ **60.** $6.73 \times 10^{11}$

**61.** The distance that light travels in 1 year (*one light year*) is about $5.87 \times 10^{12}$ mi.

**62.** The mass of a neutron is about $1.6747 \times 10^{-24}$ g.

In Exercises 63 and 64, use scientific notation to simplify.

**63.** $\dfrac{(1.3 \times 10^{-7})(2.4 \times 10^8)}{1.3 \times 10^9}$ without using a calculator

**64.** $\dfrac{(3.7 \times 10^{-7})(4.3 \times 10^6)}{2.5 \times 10^7}$

## Explorations

**65. Investigating Exponents** For positive integers $m$ and $n$, we can use the definition to show that $a^m a^n = a^{m+n}$.

(a) Examine the equation $a^m a^n = a^{m+n}$ for $n = 0$ and explain why it is reasonable to define $a^0 = 1$ for $a \neq 0$.

(b) Examine the equation $a^m a^n = a^{m+n}$ for $n = -m$ and explain why it is reasonable to define $a^{-m} = 1/a^m$ for $a \neq 0$.



**66. Decimal Forms of Rational Numbers** Here is the third step when we divide 1 by 17. (The first two steps are not shown, because the quotient is 0 in both cases.)

$$\begin{array}{r} 0.05 \\ 17\overline{)1.00} \\ \underline{85} \\ 15 \end{array}$$

By convention we say that 1 is the first remainder in the long division process, 10 is the second, and 15 is the third remainder.

(a) Continue this long division process until a remainder is repeated, and complete the following table:

| Step | Quotient | Remainder |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 0 | 10 |
| 3 | 5 | 15 |
| ⋮ | ⋮ | ⋮ |

(b) Explain why the digits that occur in the quotient between the pair of repeating remainders determine the infinitely repeating portion of the decimal representation. In this case

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

(c) Explain why this procedure will always determine the infinitely repeating portion of a rational number whose decimal representation does not terminate.

## Standardized Test Questions

**67. True or False** The additive inverse of a real number must be negative. Justify your answer.

**68. True or False** The reciprocal of a positive real number must be less than 1. Justify your answer.

In Exercises 69–72, solve these problems without using a calculator.

**69. Multiple Choice** Which of the following inequalities corresponds to the interval $[-2, 1)$?

(A) $x \leq -2$ (B) $-2 \leq x \leq 1$

(C) $-2 < x < 1$ (D) $-2 < x \leq 1$

(E) $-2 \leq x < 1$

**70. Multiple Choice** What is the value of $(-2)^4$?

(A) 16 (B) 8

(C) 6 (D) −8

(E) −16

**71. Multiple Choice** What is the base of the exponential expression $-7^2$?

(A) −7 (B) 7

(C) −2 (D) 2

(E) 1

**72. Multiple Choice** Which of the following is the simplified form of $\dfrac{x^6}{x^2}$, $x \neq 0$?

(A) $x^{-4}$ (B) $x^2$

(C) $x^3$ (D) $x^4$

(E) $x^8$

## Extending the Ideas

The **magnitude** of a real number is its distance from the origin.

**73.** List the whole numbers whose magnitudes are less than 7.

**74.** List the natural numbers whose magnitudes are less than 7.

**75.** List the integers whose magnitudes are less than 7.

**76. Writing to Learn Combining Rational and Irrational Numbers** In each case, write an explanation to justify your answer.

(a) When two rational numbers are added, is the sum a rational number?

(b) When two rational numbers are multiplied, is the product a rational number?

(c) When a rational number and an irrational number are added, is the sum a rational number?

(d) When a *nonzero* rational number and an irrational number are multiplied, is the product a rational number?

# P.2 Cartesian Coordinate System

### What you'll learn about

- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications

### ... and why

These topics provide the foundation for the material that will be addressed in this textbook.

## Cartesian Plane

The points in a plane correspond to ordered pairs of real numbers, just as the points on a line are associated with individual real numbers. This correspondence creates the **Cartesian plane**, or the **rectangular coordinate system**, in the plane.

To construct a rectangular coordinate system, or a Cartesian plane, draw a pair of perpendicular real number lines, one horizontal and the other vertical, with the lines intersecting at their respective 0-points (Figure P.6). The horizontal line is usually the ***x*-axis** and the vertical line is usually the ***y*-axis**. The positive direction on the $x$-axis is to the right, and the positive direction on the $y$-axis is up. Their point of intersection, $O$, is the **origin of the Cartesian plane**.

Each point $P$ of the plane is associated with an **ordered pair $(x, y)$** of real numbers, the **(Cartesian) coordinates of the point**. The ***x*-coordinate** represents the intersection of the $x$-axis with the vertical line from $P$, and the ***y*-coordinate** represents the intersection of the $y$-axis with the horizontal line from $P$. Figure P.6 shows the points $P$ and $Q$ with coordinates $(4, 2)$ and $(-6, -4)$, respectively. We use the ordered pair $(a, b)$ for both the name of the point and its coordinates.

The coordinate axes divide the Cartesian plane into four **quadrants**, as shown in Figure P.7.

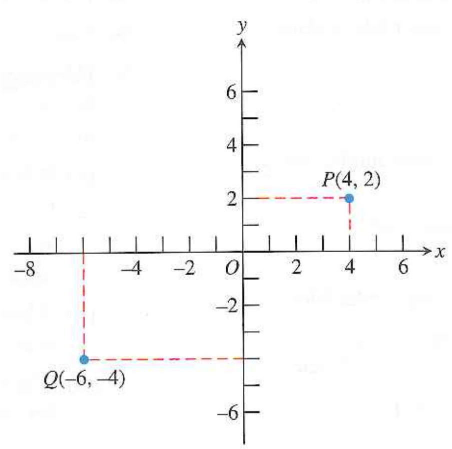



**FIGURE P.6** The Cartesian coordinate plane.

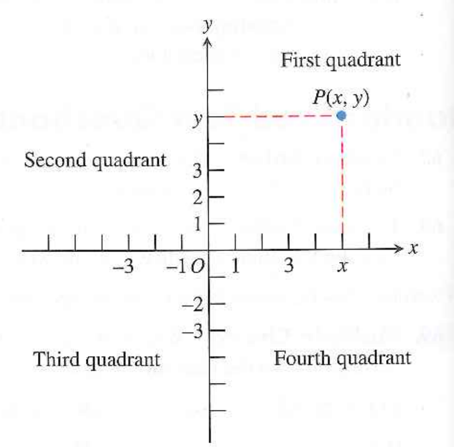



**FIGURE P.7** The four quadrants. Points on the $x$- or $y$-axis are not in any quadrant.

**Table P.2 U.S. Exports to Mexico**

| Year | U.S. Exports (billions of dollars) |
|---|---|
| 2003 | 97.4 |
| 2004 | 110.8 |
| 2005 | 120.4 |
| 2006 | 134.0 |
| 2007 | 136.0 |
| 2008 | 151.2 |
| 2009 | 128.9 |
| 2010 | 163.5 |

*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*

### EXAMPLE 1 Plotting Data on U.S. Exports to Mexico

The values in billions of dollars of U.S. exports to Mexico from 2003 to 2010 are given in Table P.2. Plot the (year, export value) ordered pairs on a rectangular coordinate system.

**SOLUTION** The points are plotted in Figure P.8 on page 13. **Now try Exercise 31.**

A **scatter plot** is a graph of the $(x, y)$ data pairs on a Cartesian plane. Figure P.8 is a scatter plot of the data from Table P.2.

## Absolute Value of a Real Number

The *absolute value of a real number* suggests its **magnitude** (size). For example, the absolute value of 3 is 3, and the absolute value of $-5$ is 5.



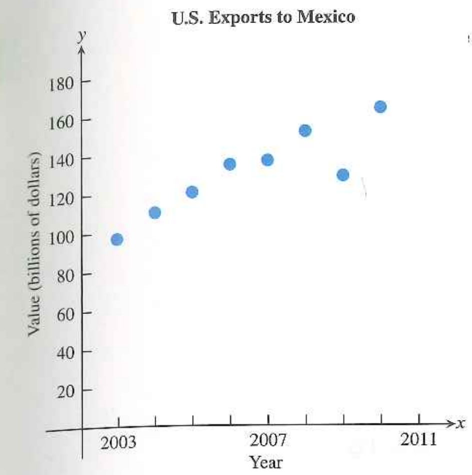



**FIGURE P.8** The graph for Example 1.

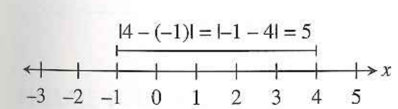



**FIGURE P.9** Finding the distance between $-1$ and 4.

### Absolute Value and Distance

If we let $b = 0$ in the distance formula, we see that the distance between $a$ and 0 is $|a|$. Thus, the absolute value of a number is its distance from zero.

**DEFINITION Absolute Value of a Real Number**

The **absolute value of a real number $a$** is

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \\ 0, & \text{if } a = 0. \end{cases}$$

### EXAMPLE 2 Using the Definition of Absolute Value

Evaluate:

**(a)** $|-4|$ **(b)** $|\pi - 6|$

**SOLUTION**

**(a)** Because $-4 < 0$, $|-4| = -(-4) = 4$.

**(b)** Because $\pi \approx 3.14$, $\pi - 6$ is negative, so $\pi - 6 < 0$. Thus, $|\pi - 6| = -(\pi - 6) = 6 - \pi \approx 2.858$. **Now try Exercise 9.**

Here is a summary of some important properties of absolute value.

**Properties of Absolute Value**

Let $a$ and $b$ be real numbers.

**1.** $|a| \ge 0$ **2.** $|-a| = |a|$

**3.** $|ab| = |a||b|$ **4.** $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \ne 0$

## Distance Formulas

The *distance* between $-1$ and 4 on the number line is 5 (Figure P.9). This distance may be found by subtracting the smaller number from the larger: $4 - (-1) = 5$. If we use absolute value, the order of subtraction does not matter:

$$|4 - (-1)| = |-1 - 4| = 5$$

**Distance Formula (Number Line)**

Let $a$ and $b$ be real numbers. The **distance between $a$ and $b$** is

$$|a - b|.$$

Note that $|a - b| = |b - a|$.

To find the *distance* between two points that lie on the same horizontal or vertical line in the Cartesian plane, we use the distance formula for points on a number line. For example, the distance between points $x_1$ and $x_2$ on the $x$-axis is $|x_1 - x_2| = |x_2 - x_1|$ and the distance between points $y_1$ and $y_2$ on the $y$-axis is $|y_1 - y_2| = |y_2 - y_1|$.

To find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ that do not lie on the same horizontal or vertical line, we form the right triangle determined by $P$, $Q$, and $R(x_2, y_1)$ (Figure P.10).

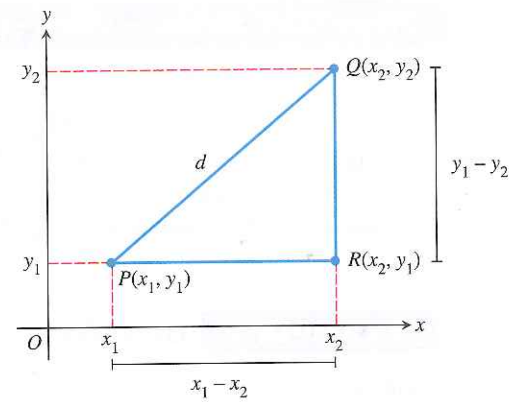



**FIGURE P.10** Forming a right triangle with hypotenuse $\overline{PQ}$.

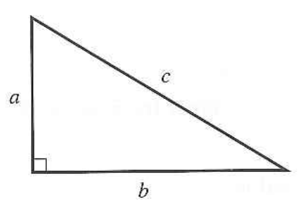



**FIGURE P.11** The Pythagorean Theorem: $c^2 = a^2 + b^2$.

The distance from $P$ to $R$ is $|x_1 - x_2|$, and the distance from $R$ to $Q$ is $|y_1 - y_2|$. By the **Pythagorean Theorem** (Figure P.11), the distance $d$ between $P$ and $Q$ is

$$d = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}.$$

Because $|x_1 - x_2|^2 = (x_1 - x_2)^2$ and $|y_1 - y_2|^2 = (y_1 - y_2)^2$, we obtain the following formula.

**Distance Formula (Coordinate Plane)**

The **distance $d$ between points $P(x_1, y_1)$ and $Q(x_2, y_2)$** in the coordinate plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

**EXAMPLE 3** Finding the Distance Between Two Points

Find the distance $d$ between the points $(1, 5)$ and $(6, 2)$.

**SOLUTION**

$$\begin{aligned} d &= \sqrt{(1-6)^2 + (5-2)^2} && \text{The distance formula} \\ &= \sqrt{(-5)^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \approx 5.831 && \text{Using a calculator} \end{aligned}$$

Now try Exercise 11.

## Midpoint Formulas

When the endpoints of a segment on a number line are known, we take the average of their coordinates to find the midpoint of the segment.

**Midpoint Formula (Number Line)**

The **midpoint of the line segment with endpoints $a$ and $b$** is

$$\frac{a + b}{2}.$$



**EXAMPLE 4** Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints $-9$ and $3$ on a number line is

$$\frac{(-9) + 3}{2} = \frac{-6}{2} = -3.$$

See Figure P.12. Now try Exercise 23.

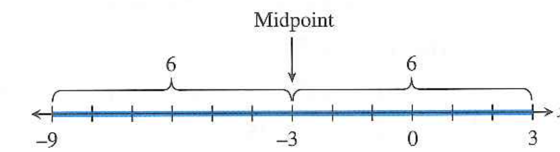



**FIGURE P.12** Notice that the distance from the midpoint, $-3$, to 3 or to $-9$ is 6. (Example 4)

Just as with number lines, the midpoint of a line segment in the coordinate plane is determined by its endpoints. Each coordinate of the midpoint is the average of the corresponding coordinates of its endpoints.

**Midpoint Formula (Coordinate Plane)**

The **midpoint of the line segment with endpoints $(a, b)$ and $(c, d)$** is

$$\left(\frac{a + c}{2}, \frac{b + d}{2}\right).$$

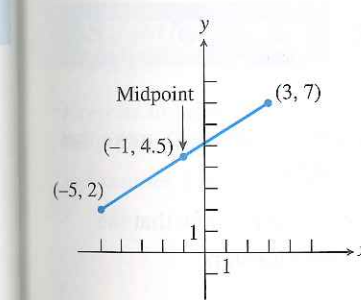



**FIGURE P.13** Midpoint of a line segment (Example 5).

**EXAMPLE 5** Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints $(-5, 2)$ and $(3, 7)$ is

$$(x, y) = \left(\frac{-5 + 3}{2}, \frac{2 + 7}{2}\right) = (-1, 4.5).$$

See Figure P.13. Now try Exercise 25.

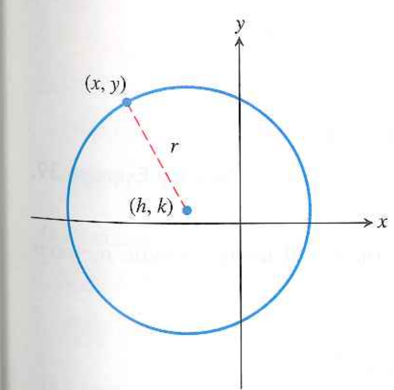



**FIGURE P.14** The circle with center $(h, k)$ and radius $r$.

## Equations of Circles

A **circle** is the set of points in a plane at a fixed distance (**radius**) from a fixed point (**center**). Figure P.14 shows the circle with center $(h, k)$ and radius $r$. If $(x, y)$ is any point on the circle, the distance formula gives

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

Squaring both sides, we obtain the following equation for a circle.

**DEFINITION Standard Form Equation of a Circle**

The **standard form equation of a circle** with center $(h, k)$ and radius $r$ is

$$(x - h)^2 + (y - k)^2 = r^2.$$

**EXAMPLE 6** Finding Standard Form Equations of Circles

Find the standard form equation of the circle.

**(a)** Center $(-4, 1)$, radius 8 **(b)** Center $(0, 0)$, radius 5

**SOLUTION**

**(a)** $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation

$(x - (-4))^2 + (y - 1)^2 = 8^2$ Substitute $h = -4, k = 1, r = 8$.

$(x + 4)^2 + (y - 1)^2 = 64$

**(b)** $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation

$(x - 0)^2 + (y - 0)^2 = 5^2$ Substitute $h = 0, k = 0, r = 5$.

$x^2 + y^2 = 25$

Now try Exercise 41.

## Applications

**EXAMPLE 7** Using an Inequality to Express Distance

We can state that "the distance between $x$ and $-3$ is less than 9" using the inequality

$$|x - (-3)| < 9 \quad \text{or} \quad |x + 3| < 9.$$

Now try Exercise 51.

The converse of the Pythagorean Theorem is true. That is, if the sum of squares of the lengths of the two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

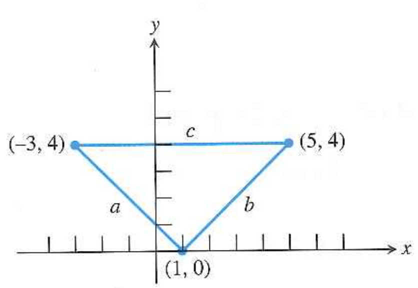



**FIGURE P.15** The triangle in Example 8.

**EXAMPLE 8** Verifying Right Triangles

Use the converse of the Pythagorean Theorem and the distance formula to prove that the points $(-3, 4)$, $(1, 0)$, and $(5, 4)$ determine a right triangle.

**SOLUTION** The three points are plotted in Figure P.15. We need to show that the lengths of the sides of the triangle satisfy the Pythagorean relationship $a^2 + b^2 = c^2$. Applying the distance formula we find that

$$a = \sqrt{(-3 - 1)^2 + (4 - 0)^2} = \sqrt{32}$$

$$b = \sqrt{(1 - 5)^2 + (0 - 4)^2} = \sqrt{32}$$

$$c = \sqrt{(-3 - 5)^2 + (4 - 4)^2} = \sqrt{64}$$

The triangle is a right triangle because

$$a^2 + b^2 = (\sqrt{32})^2 + (\sqrt{32})^2 = 32 + 32 = 64 = c^2.$$

Now try Exercise 39.

Properties of geometric figures can sometimes be confirmed using analytic methods such as the midpoint formulas.

**EXAMPLE 9** Using the Midpoint Formula

It is a fact from geometry that the diagonals of a parallelogram bisect each other. Prove this with a midpoint formula.



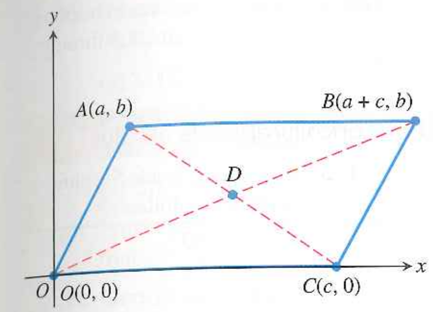



**FIGURE P.16** The coordinates of $B$ must be $(a + c, b)$ in order for $CB$ to be parallel to $OA$. (Example 9)

**SOLUTION** We can position a parallelogram in the rectangular coordinate plane as shown in Figure P.16. Applying the midpoint formula for the coordinate plane to segments $OB$ and $AC$, we find that

$$\text{midpoint of segment } OB = \left(\frac{0 + a + c}{2}, \frac{0 + b}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$$

$$\text{midpoint of segment } AC = \left(\frac{a + c}{2}, \frac{b + 0}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$$

The midpoints of segments $OA$ and $AC$ are the same, so the diagonals of the parallelogram $OABC$ meet at their midpoints and thus bisect each other.

Now try Exercise 37.

## QUICK REVIEW P.2

In Exercises 1 and 2, plot the two numbers on a number line. Then find the distance between them.

**1.** $\sqrt{7}, \sqrt{2}$ **2.** $-\frac{5}{3}, -\frac{9}{5}$

In Exercises 3 and 4, plot the real numbers on a number line.

**3.** $-3, 4, 2.5, 0, -1.5$ **4.** $-\frac{5}{2}, -\frac{1}{2}, \frac{2}{3}, 0, -1$

In Exercises 5 and 6, plot the points.

**5.** $A(3, 5), B(-2, 4), C(3, 0), D(0, -3)$

**6.** $A(-3, -5), B(2, -4), C(0, 5), D(-4, 0)$

In Exercises 7–10, use a calculator to evaluate the expression. Round your answer to two decimal places.

**7.** $\frac{-17 + 28}{2}$ **8.** $\sqrt{13^2 + 17^2}$

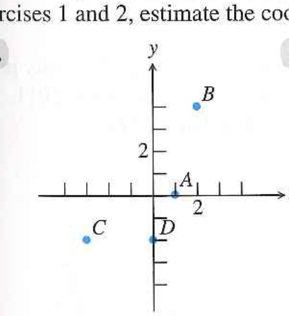
**9.** $\sqrt{6^2 + 8^2}$ **10.** $\sqrt{(17 - 3)^2 + (-4 - 8)^2}$

## SECTION P.2 Exercises

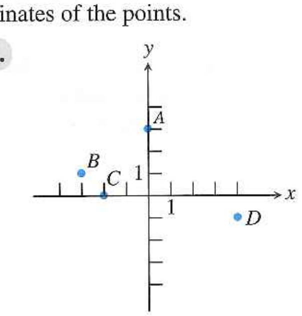
**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1 and 2, estimate the coordinates of the points.

**1.**




**2.**




In Exercises 3 and 4, name the quadrants containing the points.

**3.** **(a)** $(2, 4)$ **(b)** $(0, 3)$ **(c)** $(-2, 3)$ **(d)** $(-1, -4)$

**4.** **(a)** $\left(\frac{1}{2}, \frac{3}{2}\right)$ **(b)** $(-2, 0)$ **(c)** $(-1, -2)$ **(d)** $\left(-\frac{3}{2}, -\frac{7}{3}\right)$

In Exercises 5–8, evaluate the expression.

**5.** $3 + |-3|$ **6.** $2 - |-2|$

**7.** $|(-2)3|$ **8.** $\frac{-2}{|-2|}$

In Exercises 9 and 10, rewrite the expression without using absolute value symbols.

**9.** $|\pi - 4|$ **10.** $|\sqrt{5} - 5/2|$

In Exercises 11–18, find the distance between the points.

**11.** $-9.3, 10.6$ **12.** $-5, -17$

**13.** $(-3, -1), (5, -1)$ **14.** $(-4, -3), (1, 1)$

**15.** $(0, 0), (3, 4)$ **16.** $(-1, 2), (2, -3)$

**17.** $(-2, 0), (5, 0)$ **18.** $(0, -8), (0, -1)$

In Exercises 19–22, find the perimeter and area of the figure determined by the points.

**19.** $(-5, 3), (0, -1), (4, 4)$

**20.** $(-2, -2), (-2, 2), (2, 2), (2, -2)$

**21.** $(-3, -1), (-1, 3), (7, 3), (5, -1)$

**22.** $(-2, 1), (-2, 6), (4, 6), (4, 1)$

In Exercises 23–28, find the midpoint of the line segment with the given endpoints.

**23.** $-9.3, 10.6$ **24.** $-5, -17$

**25.** $(-1, 3), (5, 9)$

**26.** $(3, \sqrt{2}), (6, 2)$

**27.** $(-7/3, 3/4), (5/3, -9/4)$

**28.** $(5, -2), (-1, -4)$

In Exercises 29–34, draw a scatter plot of the data given in the table.

**29. U.S. Motor Vehicle Production** The total number of motor vehicles in thousands $(y)$ produced by the United States each year from 2004 through 2010 is given in the table. *(Source: Automotive News Data Center and R. L. Polk Marketing Systems as reported in The World Almanac and Book of Facts 2012.)*

| $x$ | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|
| $y$ | 12,021 | 12,018 | 11,351 | 10,611 | 8,503 | 5,591 | 7,632 |

**30. World Motor Vehicle Production** The total number of motor vehicles in thousands $(y)$ produced in the world each year from 2004 through 2010 is given in the table. *(Source: American Automobile Manufacturers Association as reported in The World Almanac and Book of Facts 2012.)*

| $x$ | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|
| $y$ | 65,654 | 67,892 | 70,992 | 74,647 | 67,602 | 59,096 | 73,311 |

**31. U.S. Imports from Mexico** The total in billions of dollars of U.S. imports from Mexico from 2003 through 2010 is given in Table P.3.

**Table P.3 U.S. Imports from Mexico**

| Year | U.S. Imports (billions of dollars) |
|---|---|
| 2003 | 138.1 |
| 2004 | 155.9 |
| 2005 | 170.1 |
| 2006 | 198.3 |
| 2007 | 210.7 |
| 2008 | 215.9 |
| 2009 | 176.7 |
| 2010 | 229.9 |

*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*

**32. U.S. Agricultural Exports** The total in billions of dollars of U.S. agricultural exports from 2003 through 2010 is given in Table P.4.

**Table P.4 U.S. Agricultural Exports**

| Year | U.S. Agricultural Exports (billions of dollars) |
|---|---|
| 2003 | 59.4 |
| 2004 | 61.4 |
| 2005 | 63.2 |
| 2006 | 70.9 |
| 2007 | 90.0 |
| 2008 | 114.8 |
| 2009 | 98.5 |
| 2010 | 115.8 |

*Source: U.S. Department of Agriculture, The World Almanac and Book of Facts 2012.*

**33. U.S. Agricultural Trade Surplus** The total in billions of dollars of U.S. agricultural trade surplus from 2003 through 2010 is given in Table P.5.

**Table P.5 U.S. Agricultural Trade Surplus**

| Year | U.S. Agricultural Trade Surplus (billions of dollars) |
|---|---|
| 2003 | 12.0 |
| 2004 | 7.4 |
| 2005 | 3.9 |
| 2006 | 5.6 |
| 2007 | 18.1 |
| 2008 | 34.3 |
| 2009 | 26.8 |
| 2010 | 34.0 |

*Source: U.S. Department of Agriculture, The World Almanac and Book of Facts 2012.*

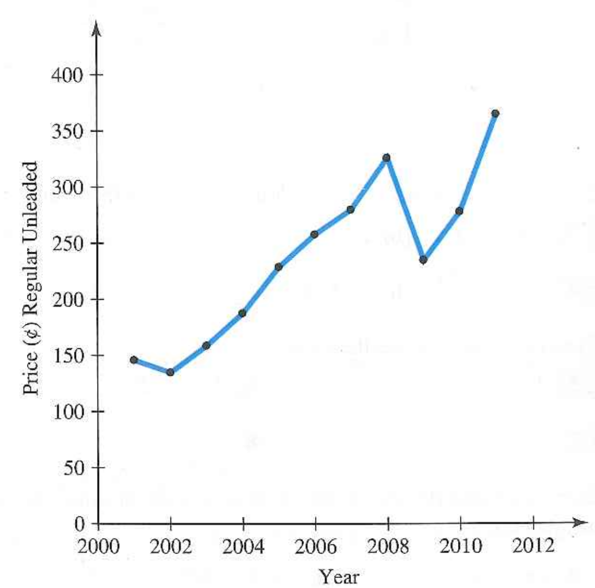
**34. U.S. Exports to Canada** The total in billions of dollars of U.S. exports to Canada from 2003 through 2010 is given in Table P.6.

**Table P.6 U.S. Exports to Canada**

| Year | U.S. Exports (billions of dollars) |
|---|---|
| 2003 | 169.9 |
| 2004 | 189.9 |
| 2005 | 211.9 |
| 2006 | 230.7 |
| 2007 | 248.9 |
| 2008 | 261.2 |
| 2009 | 204.7 |
| 2010 | 249.1 |

*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*

In Exercises 35 and 36, use the graph for the average U.S. city prices (in cents) of regular unleaded gasoline for the years 2001–2011. *(Source: The World Almanac and Book of Facts 2012.)*






**35. Reading from Graphs** Estimate the price of gasoline (in dollars) for

(a) 2002 (b) 2007 (c) 2010

**36. Percent Increase** Estimate the percent increase in the price of gasoline from

(a) 2001 to 2006 (b) 2009 to 2011

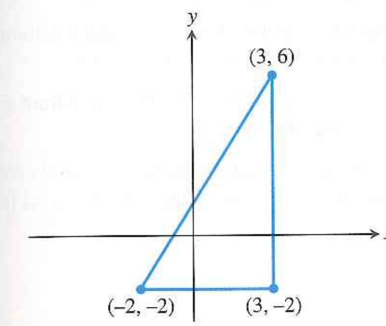
**37.** Prove that the figure determined by the points is an isosceles triangle: $(1, 3)$, $(4, 7)$, $(8, 4)$

**38. Group Activity** Prove that the diagonals of the figure determined by the points bisect each other.

(a) Square $(-7, -1)$, $(-2, 4)$, $(3, -1)$, $(-2, -6)$

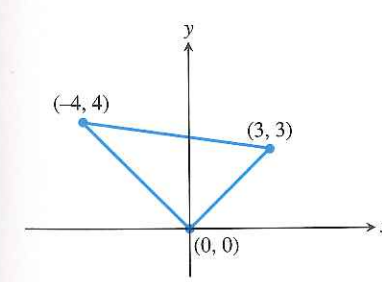
(b) Parallelogram $(-2, -3)$, $(0, 1)$, $(6, 7)$, $(4, 3)$

**39.** (a) Find the lengths of the sides of the triangle in the figure.




(b) **Writing to Learn** Prove that the triangle is a right triangle.

**40.** (a) Find the lengths of the sides of the triangle in the figure.




(b) **Writing to Learn** Prove that the triangle is a right triangle.

In Exercises 41–44, find the standard form equation for the circle.

**41.** Center $(1, 2)$, radius 5

**42.** Center $(-3, 2)$, radius 1

**43.** Center $(-1, -4)$, radius 3

**44.** Center $(0, 0)$, radius $\sqrt{3}$

In Exercises 45–48, find the center and radius of the circle.

**45.** $(x - 3)^2 + (y - 1)^2 = 36$

**46.** $(x + 4)^2 + (y - 2)^2 = 121$

**47.** $x^2 + y^2 = 5$

**48.** $(x - 2)^2 + (y + 6)^2 = 25$

In Exercises 49–52, write the statement using absolute value notation.

**49.** The distance between $x$ and 4 is 3.

**50.** The distance between $y$ and $-2$ is greater than or equal to 4.

**51.** The distance between $x$ and $c$ is less than $d$ units.

**52.** $y$ is more than $d$ units from $c$.

**53.** Let $(4, 4)$ be the midpoint of the line segment determined by the points $(1, 2)$ and $(a, b)$. Determine $a$ and $b$.

**54. Writing to Learn Isosceles but Not Equilateral** Prove that the triangle determined by the points $(3, 0)$, $(-1, 2)$, and $(5, 4)$ is isosceles but not equilateral.

**55. Writing to Learn Equidistant Point** Prove that the midpoint of the hypotenuse of the right triangle with vertices $(0, 0)$, $(5, 0)$, and $(0, 7)$ is equidistant from the three vertices.

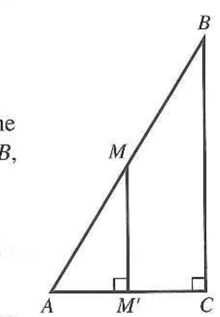
**56. Writing to Learn** Describe the set of real numbers that satisfy $|x - 2| < 3$.

**57. Writing to Learn** Describe the set of real numbers that satisfy $|x + 3| \geq 5$.

## Standardized Test Questions

**58. True or False** If $a$ is a real number, then $|a| \geq 0$. Justify your answer.

**59. True or False** Let $\Delta ABC$ and $\Delta AMM'$ be right triangles as shown in the figure. If $M$ is the midpoint of segment $AB$, then $M'$ is the midpoint of segment $AC$. Justify your answer.




In Exercises 60–63, solve these problems without using a calculator.

**60. Multiple Choice** Which of the following is equal to $|1 - \sqrt{3}|$?

(A) $1 - \sqrt{3}$ (B) $\sqrt{3} - 1$

(C) $(1 - \sqrt{3})^2$ (D) $\sqrt{2}$

(E) $\sqrt{1/3}$

**61. Multiple Choice** Which of the following is the midpoint of the line segment with endpoints $-3$ and 2?

(A) 5/2 (B) 1

(C) $-1/2$ (D) $-1$

(E) $-5/2$

62. **Multiple Choice** Which of the following is the center of the circle $(x - 3)^2 + (y + 4)^2 = 2$?
(A) $(3, -4)$ (B) $(-3, 4)$
(C) $(4, -3)$ (D) $(-4, 3)$
(E) $(3/2, -2)$

63. **Multiple Choice** Which of the following points is in the third quadrant?
(A) $(0, -3)$ (B) $(-1, 0)$
(C) $(2, -1)$ (D) $(-1, 2)$
(E) $(-2, -3)$

## Explorations

64. **Dividing a Line Segment into Thirds**
(a) Find the coordinates of the points one-third and two-thirds of the way from $a = 2$ to $b = 8$ on a number line.
(b) Repeat (a) for $a = -3$ and $b = 7$.
(c) Find the coordinates of the points one-third and two-thirds of the way from $a$ to $b$ on a number line.
(d) Find the coordinates of the points one-third and two-thirds of the way from the point $(1, 2)$ to the point $(7, 11)$ in the coordinate plane.
(e) Find the coordinates of the points one-third and two-thirds of the way from the point $(a, b)$ to the point $(c, d)$ in the coordinate plane.

## Extending the Ideas

65. **Writing to Learn Equidistant Point from Vertices of a Right Triangle** Prove that the midpoint of the hypotenuse of any right triangle is equidistant from the three vertices.

66. **Comparing Areas** Consider the four points $A(0, 0)$, $B(0, a)$, $C(a, a)$, and $D(a, 0)$. Let $P$ be the midpoint of the line segment $CD$ and $Q$ the point one-fourth of the way from $A$ to $D$ on segment $AD$.
(a) Find the area of triangle $BPQ$.
(b) Compare the area of triangle $BPQ$ with the area of square $ABCD$.

In Exercises 67–69, let $P(a, b)$ be a point in the first quadrant.

67. Find the coordinates of the point $Q$ in the fourth quadrant so that the $x$-axis is the perpendicular bisector of $PQ$.

68. Find the coordinates of the point $Q$ in the second quadrant so that the $y$-axis is the perpendicular bisector of $PQ$.

69. Find the coordinates of the point $Q$ in the third quadrant so that the origin is the midpoint of the segment $PQ$.

70. **Writing to Learn** Prove that the distance formula for the number line is a special case of the distance formula for the Cartesian plane.



# P.3 Linear Equations and Inequalities

**What you'll learn about**
- Equations
- Solving Equations
- Linear Equations in One Variable
- Linear Inequalities in One Variable

**... and why**
These topics provide the foundation for algebraic techniques needed throughout this textbook.

## Equations

An **equation** is a statement of equality between two expressions. Here are some properties of equality that we use to solve equations algebraically.

**Properties of Equality**

Let $u$, $v$, $w$, and $z$ be real numbers, variables, or algebraic expressions.

| | |
|---|---|
| 1. **Reflexive** | $u = u$ |
| 2. **Symmetric** | If $u = v$, then $v = u$. |
| 3. **Transitive** | If $u = v$, and $v = w$, then $u = w$. |
| 4. **Addition** | If $u = v$ and $w = z$, then $u + w = v + z$. |
| 5. **Multiplication** | If $u = v$ and $w = z$, then $uw = vz$. |

## Solving Equations

A **solution of an equation in $x$** is a value of $x$ for which the equation is true. To **solve an equation in $x$** means to find all values of $x$ for which the equation is true, that is, to find all solutions of the equation.

**EXAMPLE 1** Confirming a Solution

Prove that $x = -2$ is a solution of the equation $x^3 - x + 6 = 0$.

**SOLUTION**

$$(-2)^3 - (-2) + 6 \stackrel{?}{=} 0$$
$$-8 + 2 + 6 \stackrel{?}{=} 0$$
$$0 = 0$$

Now try Exercise 1.

## Linear Equations in One Variable

The most basic equation in algebra is a *linear equation*.

**DEFINITION Linear Equation in x**

A **linear equation in $x$** is one that can be written in the form

$$ax + b = 0,$$

where $a$ and $b$ are real numbers and $a \neq 0$.

The equation $2z - 4 = 0$ is linear in the variable $z$. The equation $3u^2 - 12 = 0$ is *not* linear in the variable $u$. A linear equation in one variable has exactly one solution. We solve such an equation by transforming it into an *equivalent equation* whose solution is obvious. Two or more equations are **equivalent** if they have the same solutions. For example, the equations $2z - 4 = 0$, $2z = 4$, and $z = 2$ are all equivalent. Here are operations that produce equivalent equations.

### Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

| Operation | Given Equation | Equivalent Equation |
|---|---|---|
| **1.** Combine like terms, reduce fractions, and remove grouping symbols. | $2x + x = \frac{3}{9}$ | $3x = \frac{1}{3}$ |
| **2.** Perform the same operation on both sides. | | |
| (a) Add $(-3)$. | $x + 3 = 7$ | $x = 4$ |
| (b) Subtract $(2x)$. | $5x = 2x + 4$ | $3x = 4$ |
| (c) Multiply by a nonzero constant (1/3). | $3x = 12$ | $x = 4$ |
| (d) Divide by a nonzero constant (3). | $3x = 12$ | $x = 4$ |

The next two examples illustrate how to use equivalent equations to solve linear equations.

### EXAMPLE 2 Solving a Linear Equation

Solve $2(2x - 3) + 3(x + 1) = 5x + 2$. Support the result with a calculator.

**SOLUTION**

$$2(2x - 3) + 3(x + 1) = 5x + 2$$
$$4x - 6 + 3x + 3 = 5x + 2 \quad \text{Distributive property}$$
$$7x - 3 = 5x + 2 \quad \text{Combine like terms.}$$
$$2x = 5 \quad \text{Add 3, and subtract } 5x.$$
$$x = 2.5 \quad \text{Divide by 2.}$$

To support our algebraic work we can evaluate the expressions in the original equation for $x = 2.5$. Figure P.17 shows that each side of the original equation is equal to 14.5 if $x = 2.5$.

Now try Exercise 23.

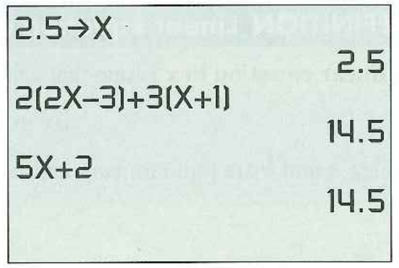



**FIGURE P.17** The top line stores the number 2.5 into the variable $x$. (Example 2)

If an equation involves fractions, find the least common denominator (LCD) of the fractions and multiply both sides by the LCD. This is sometimes referred to as *clearing the equation of fractions*. Example 3 illustrates.



#### Integers and Fractions

Notice in Example 3 that $2 = \frac{2}{1}$.

### EXAMPLE 3 Solving a Linear Equation Involving Fractions

Solve

$$\frac{5y - 2}{8} = 2 + \frac{y}{4}.$$

**SOLUTION** The denominators are 8, 1, and 4. The LCD of the fractions is 8. (See Appendix A.3 if necessary.)

$$\frac{5y - 2}{8} = 2 + \frac{y}{4}$$
$$8\left(\frac{5y - 2}{8}\right) = 8\left(2 + \frac{y}{4}\right) \quad \text{Multiply by the LCD 8.}$$
$$8 \cdot \frac{5y - 2}{8} = 8 \cdot 2 + 8 \cdot \frac{y}{4} \quad \text{Distributive property}$$
$$5y - 2 = 16 + 2y \quad \text{Simplify.}$$
$$5y = 18 + 2y \quad \text{Add 2.}$$
$$3y = 18 \quad \text{Subtract } 2y.$$
$$y = 6 \quad \text{Divide by 3.}$$

We leave it to you to check the solution using either paper and pencil or a calculator.

Now try Exercise 25.

## Linear Inequalities in One Variable

We used inequalities to describe order on the number line in Section P.1. For example, if $x$ is to the left of 2 on the number line, or if $x$ is any real number less than 2, we write $x < 2$. The most basic inequality in algebra is a *linear inequality*.

### DEFINITION Linear Inequality in x

A **linear inequality in $x$** is one that can be written in the form

$$ax + b < 0, \quad ax + b \le 0, \quad ax + b > 0, \quad \text{or} \quad ax + b \ge 0,$$

where $a$ and $b$ are real numbers and $a \neq 0$.

To **solve an inequality in $x$** means to find all values of $x$ for which the inequality is true. A **solution of an inequality in $x$** is a value of $x$ for which the inequality is true. The set of all solutions of an inequality is the **solution set** of the inequality. We **solve an inequality** by finding its solution set. Here is a list of properties we use to solve inequalities.

#### Direction of an Inequality

Multiplying (or dividing) an inequality by a positive number preserves the direction of the inequality. Multiplying (or dividing) an inequality by a negative number reverses the direction.

### Properties of Inequalities

Let $u$, $v$, $w$, and $z$ be real numbers, variables, or algebraic expressions, and $c$ a real number.

| | |
|---|---|
| **1. Transitive** | If $u < v$ and $v < w$, then $u < w$. |
| **2. Addition** | If $u < v$, then $u + w < v + w$. |
| | If $u < v$ and $w < z$, then $u + w < v + z$. |
| **3. Multiplication** | If $u < v$ and $c > 0$, then $uc < vc$. |
| | If $u < v$ and $c < 0$, then $uc > vc$. |

The above properties are true if $<$ is replaced by $\le$. There are similar properties for $>$ and $\ge$.

The set of solutions of a linear inequality in one variable forms an interval of real numbers. Just as with linear equations, we solve a linear inequality by transforming it into an *equivalent inequality* whose solutions are obvious. Two or more inequalities are **equivalent** if they have the same solution set. The properties of inequalities listed above describe operations that transform an inequality into an equivalent one.

### EXAMPLE 4 Solving a Linear Inequality

Solve $3(x - 1) + 2 \le 5x + 6$.

**SOLUTION**

$$3(x - 1) + 2 \le 5x + 6$$

$$3x - 3 + 2 \le 5x + 6 \quad \text{Distributive property}$$

$$3x - 1 \le 5x + 6 \quad \text{Combine like terms.}$$

$$3x \le 5x + 7 \quad \text{Add 1.}$$

$$-2x \le 7 \quad \text{Subtract } 5x.$$

$$\left(-\frac{1}{2}\right) \cdot -2x \ge \left(-\frac{1}{2}\right) \cdot 7 \quad \text{Multiply by } -1/2. \text{ (The inequality reverses.)}$$

$$x \ge -3.5$$

The solution set of the inequality is the set of all real numbers greater than or equal to $-3.5$. In interval notation, the solution set is $[-3.5, \infty)$.

**Now try Exercise 41.**

Because the solution set of a linear inequality is an interval of real numbers, we can display the solution set with a number line graph as illustrated in Example 5.

### EXAMPLE 5 Solving a Linear Inequality Involving Fractions

Solve the inequality, and graph its solution set.

$$\frac{x}{3} + \frac{1}{2} > \frac{x}{4} + \frac{1}{3}$$

**SOLUTION** The LCD of the fractions is 12.

$$\frac{x}{3} + \frac{1}{2} > \frac{x}{4} + \frac{1}{3}$$

$$12 \cdot \left(\frac{x}{3} + \frac{1}{2}\right) > 12 \cdot \left(\frac{x}{4} + \frac{1}{3}\right) \quad \text{Multiply by the LCD 12.}$$

$$4x + 6 > 3x + 4 \quad \text{Simplify.}$$

$$x + 6 > 4 \quad \text{Subtract } 3x.$$

$$x > -2 \quad \text{Subtract 6.}$$

The solution set is the interval $(-2, \infty)$. Its graph is shown in Figure P.18.

**Now try Exercise 43.**

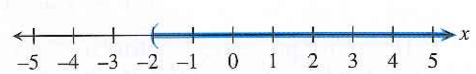



**FIGURE P.18** The graph of the solution set of the inequality in Example 5.



Sometimes two inequalities are combined in a *double inequality*, which is solved by isolating $x$ as the middle expression. Example 6 illustrates this.

### EXAMPLE 6 Solving a Double Inequality

Solve the inequality, and graph its solution set.

$$-3 < \frac{2x + 5}{3} \le 5$$

**SOLUTION**

$$-3 < \frac{2x + 5}{3} \le 5$$

$$-9 < 2x + 5 \le 15 \quad \text{Multiply by 3.}$$

$$-14 < 2x \le 10 \quad \text{Subtract 5.}$$

$$-7 < x \le 5 \quad \text{Divide by 2.}$$

The solution set is the set of all real numbers greater than $-7$ and less than or equal to 5. In interval notation, the solution is set $(-7, 5]$. Its graph is shown in Figure P.19.

**Now try Exercise 47.**

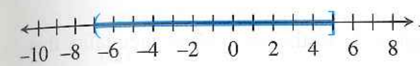



**FIGURE P.19** The graph of the solution set of the double inequality in Example 6.

## QUICK REVIEW P.3

In Exercises 1 and 2, simplify the expression by combining like terms.

1. $2x + 5x + 7 + y - 3x + 4y + 2$
2. $4 + 2x - 3z + 5y - x + 2y - z - 2$

In Exercises 3 and 4, use the distributive property to expand the products. Simplify the resulting expression by combining like terms.

3. $3(2x - y) + 4(y - x) + x + y$
4. $5(2x + y - 1) + 4(y - 3x + 2) + 1$

In Exercises 5–10, use the LCD to combine the fractions. Simplify the resulting fraction.

5. $\frac{2}{y} + \frac{3}{y}$
6. $\frac{1}{y - 1} + \frac{3}{y - 2}$
7. $2 + \frac{1}{x}$
8. $\frac{1}{x} + \frac{1}{y} - x$
9. $\frac{x + 4}{2} + \frac{3x - 1}{5}$
10. $\frac{x}{3} + \frac{x}{4}$

## SECTION P.3 Exercises

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1–4, which values of $x$ are solutions of the equation?

1. $2x^2 + 5x = 3$

   (a) $x = -3$ (b) $x = -\frac{1}{2}$ (c) $x = \frac{1}{2}$

2. $\frac{x}{2} + \frac{1}{6} = \frac{x}{3}$

   (a) $x = -1$ (b) $x = 0$ (c) $x = 1$

3. $\sqrt{1 - x^2} + 2 = 3$

   (a) $x = -2$ (b) $x = 0$ (c) $x = 2$

4. $(x - 2)^{1/3} = 2$

   (a) $x = -6$ (b) $x = 8$ (c) $x = 10$

In Exercises 5–10, determine whether the equation is linear in $x$.

5. $5 - 3x = 0$
6. $5 = 10/2$
7. $x + 3 = x - 5$
8. $x - 3 = x^2$
9. $2\sqrt{x} + 5 = 10$
10. $x + \frac{1}{x} = 1$

In Exercises 11–24, solve the equation without using a calculator.

11. $3x = 24$
12. $4x = -16$
13. $3t - 4 = 8$
14. $2t - 9 = 3$
15. $2x - 3 = 4x - 5$
16. $4 - 2x = 3x - 6$
17. $4 - 3y = 2(y + 4)$
18. $4(y - 2) = 5y$
19. $\frac{1}{2}x = \frac{7}{8}$
20. $\frac{2}{3}x = \frac{4}{5}$

**21.** $\frac{1}{2}x + \frac{1}{3} = 1$ **22.** $\frac{1}{3}x + \frac{1}{4} = 1$

**23.** $2(3 - 4z) - 5(2z + 3) = z - 17$

**24.** $3(5z - 3) - 4(2z + 1) = 5z - 2$

In Exercises 25–28, solve the equation. Support your answer with a calculator.

**25.** $\frac{2x - 3}{4} + 5 = 3x$ **26.** $2x - 4 = \frac{4x - 5}{3}$

**27.** $\frac{t + 5}{8} - \frac{t - 2}{2} = \frac{1}{3}$ **28.** $\frac{t - 1}{3} + \frac{t + 5}{4} = \frac{1}{2}$

**29. Writing to Learn** Write a statement about a solution of an equation suggested by the computations in the figure.

(a)
```
-2→X
                -2
2X²+X-6
                 0
```
(b)
```
3/2→X
               1.5
2X²+X-6
                 0
```

**30. Writing to Learn** Write a statement about a solution of an equation suggested by the computations in the figure.

(a)
```
2→X
                 2
7X+5
                19
4X-7
                 1
```
(b)
```
-4→X
                -4
7X+5
               -23
4X-7
               -23
```

In Exercises 31–34, which values of $x$ are solutions of the inequality?

**31.** $2x - 3 < 7$

(a) $x = 0$ (b) $x = 5$ (c) $x = 6$

**32.** $3x - 4 \geq 5$

(a) $x = 0$ (b) $x = 3$ (c) $x = 4$

**33.** $-1 < 4x - 1 \leq 11$

(a) $x = 0$ (b) $x = 2$ (c) $x = 3$

**34.** $-3 \leq 1 - 2x \leq 3$

(a) $x = -1$ (b) $x = 0$ (c) $x = 2$

In Exercises 35–42, solve the inequality, and draw a number line graph of the solution set.

**35.** $x - 4 < 2$ **36.** $x + 3 > 5$

**37.** $2x - 1 \leq 4x + 3$ **38.** $3x - 1 \geq 6x + 8$

**39.** $2 \leq x + 6 < 9$ **40.** $-1 \leq 3x - 2 < 7$

**41.** $2(5 - 3x) + 3(2x - 1) \leq 2x + 1$

**42.** $4(1 - x) + 5(1 + x) > 3x - 1$

In Exercises 43–54, solve the inequality.

**43.** $\frac{5x + 7}{4} \leq -3$ **44.** $\frac{3x - 2}{5} > -1$

**45.** $4 \geq \frac{2y - 5}{3} \geq -2$ **46.** $1 > \frac{3y - 1}{4} > -1$

**47.** $0 \leq 2z + 5 < 8$ **48.** $-6 < 5t - 1 < 0$

**49.** $\frac{x - 5}{4} + \frac{3 - 2x}{3} < -2$ **50.** $\frac{3 - x}{2} + \frac{5x - 2}{3} < -1$

**51.** $\frac{2y - 3}{2} + \frac{3y - 1}{5} < y - 1$

**52.** $\frac{3 - 4y}{6} - \frac{2y - 3}{8} \geq 2 - y$

**53.** $\frac{1}{2}(x - 4) - 2x \leq 5(3 - x)$

**54.** $\frac{1}{2}(x + 3) + 2(x - 4) < \frac{1}{3}(x - 3)$

In Exercises 55–58, find the solutions of the equation or inequality that are displayed in Figure P.20.

**55.** $x^2 - 2x < 0$ **56.** $x^2 - 2x = 0$

**57.** $x^2 - 2x > 0$ **58.** $x^2 - 2x \leq 0$

| X | Y1 | |
|---|---|---|
| 0 | 0 | |
| 1 | -1 | |
| 2 | 0 | |
| 3 | 3 | |
| 4 | 8 | |
| 5 | 15 | |
| 6 | 24 | |
| Y1 = X²–2X | | |

**FIGURE P.20** The second column gives values of $y_1 = x^2 - 2x$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.

**59. Writing to Learn** Explain how the second equation was obtained from the first.

$$x - 3 = 2x + 3, \quad 2x - 6 = 4x + 6$$

**60. Writing to Learn** Explain how the second equation was obtained from the first.

$$2x - 1 = 2x - 4, \quad x - \frac{1}{2} = x - 2$$

**61. Group Activity** Determine whether the two equations are equivalent.

(a) $3x = 6x + 9, \quad x = 2x + 9$

(b) $6x + 2 = 4x + 10, \quad 3x + 1 = 2x + 5$

**62. Group Activity** Determine whether the two equations are equivalent.

(a) $3x + 2 = 5x - 7, \quad -2x + 2 = -7$

(b) $2x + 5 = x - 7, \quad 2x = x - 7$



## Standardized Test Questions

**63. True or False** $-6 > -2$. Justify your answer.

**64. True or False** $2 \leq \frac{6}{3}$. Justify your answer.

In Exercises 65–68, you may use a graphing calculator to solve these problems.

**65. Multiple Choice** Which of the following equations is equivalent to the equation $3x + 5 = 2x + 1$?

(A) $3x = 2x$ (B) $3x = 2x + 4$

(C) $\frac{3}{2}x + \frac{5}{2} = x + 1$ (D) $3x + 6 = 2x$

(E) $3x = 2x - 4$

**66. Multiple Choice** Which of the following inequalities is equivalent to the inequality $-3x < 6$?

(A) $3x < -6$ (B) $x < 10$

(C) $x > -2$ (D) $x > 2$

(E) $x > 3$

**67. Multiple Choice** Which of the following is the solution to the equation $x(x + 1) = 0$?

(A) $x = 0$ or $x = -1$ (B) $x = 0$ or $x = 1$

(C) Only $x = -1$ (D) Only $x = 0$

(E) Only $x = 1$

**68. Multiple Choice** Which of the following represents an equation equivalent to the equation

$$\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{1}{3}$$

that is cleared of fractions?

(A) $2x + 1 = x - 1$ (B) $8x + 6 = 3x - 4$

(C) $4x + 3 = \frac{3}{2}x - 2$ (D) $4x + 3 = 3x - 4$

(E) $4x + 6 = 3x - 4$

## Explorations

**69. Testing Inequalities on a Calculator**

(a) The calculator we use indicates that the statement $2 < 3$ is true by returning the value 1 (for true) when $2 < 3$ is entered. Try it with your calculator.

(b) The calculator we use indicates that the statement $2 < 1$ is false by returning the value 0 (for false) when $2 < 1$ is entered. Try it with your calculator.

(c) Use your calculator to test which of these two numbers is larger: 799/800, 800/801.

(d) Use your calculator to test which of these two numbers is larger: −102/101, −103/102.

(e) If your calculator returns 0 when you enter $2x + 1 < 4$, what can you conclude about the value stored in $x$?

## Extending the Ideas

**70. Perimeter of a Rectangle** The formula for the perimeter $P$ of a rectangle is

$$P = 2(L + W).$$

Solve this equation for $W$.

**71. Area of a Trapezoid** The formula for the area $A$ of a trapezoid is

$$A = \frac{1}{2}h(b_1 + b_2).$$

Solve this equation for $b_1$.

**72. Volume of a Sphere** The formula for the volume $V$ of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

Solve this equation for $r$.

**73. Celsius and Fahrenheit** The formula for Celsius temperature in terms of Fahrenheit temperature is

$$C = \frac{5}{9}(F - 32).$$

Solve the equation for $F$.

# P.4 Lines in the Plane

**What you'll learn about**
- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables

**... and why**

Linear equations are used extensively in applications involving business and behavioral science.

## Slope of a Line

The slope of a nonvertical line is the ratio of the amount of vertical change to the amount of horizontal change between any two points on the line. For the points $(x_1, y_1)$ and $(x_2, y_2)$, the vertical change is $\Delta y = y_2 - y_1$ and the horizontal change is $\Delta x = x_2 - x_1$. $\Delta y$ is read "delta" $y$. See Figure P.21.

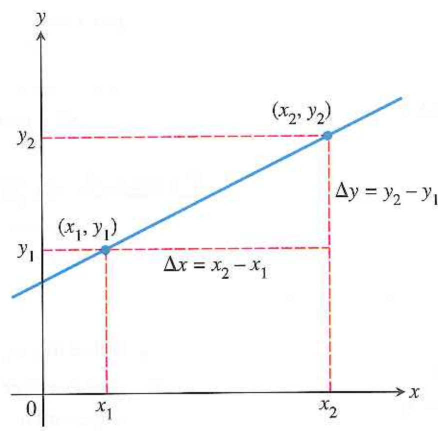



**FIGURE P.21** The slope of a nonvertical line can be found from the coordinates of any two points of the line.

**DEFINITION Slope of a Line**

The **slope** of a nonvertical line through the points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

If the line is vertical, then $x_1 = x_2$ and the slope is undefined.

**EXAMPLE 1** Finding the Slope of a Line

Find the slope of the line through the two points. Sketch a graph of the line.

**(a)** $(-1, 2)$ and $(4, -2)$ **(b)** $(1, 1)$ and $(3, 4)$

**SOLUTION**

**(a)** The two points are $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (4, -2)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 2}{4 - (-1)} = -\frac{4}{5}.$$

**(b)** The two points are $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (3, 4)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 1} = \frac{3}{2}.$$

The graphs of these two lines are shown in Figure P.22.

Now try Exercise 3.

**Slope Formula**

The slope does not depend on the order of the points. We could use $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 2)$ in Example 1a. Check it out.



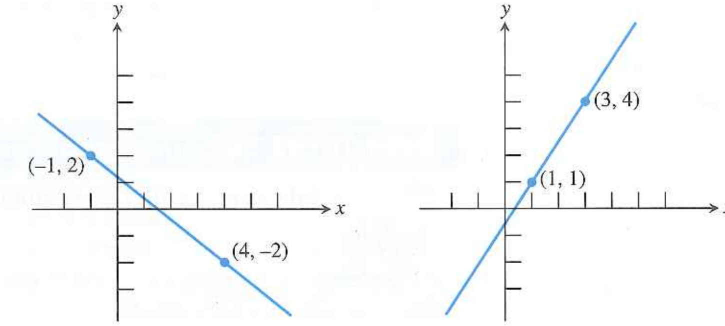



**FIGURE P.22** The graphs of the two lines in Example 1.

Figure P.23 shows a vertical line through the points $(3, 2)$ and $(3, 7)$. If we try to calculate its slope using the slope formula $(y_2 - y_1)/(x_2 - x_1)$, we get zero in the denominator. So, it makes sense to say that a vertical line does not have a slope, or that its slope is undefined.

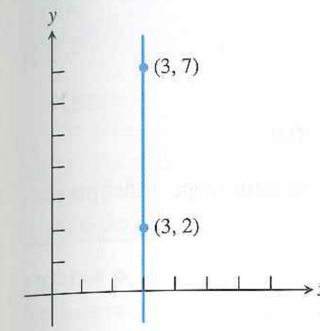



**FIGURE P.23** Applying the slope formula to this vertical line gives $m = 5/0$, which is not defined. Thus, the slope of a vertical line is undefined.

## Point-Slope Form Equation of a Line

If we know the coordinates of one point on a line and the slope of the line, then we can find an equation for that line. For example, the line in Figure P.24 passes through the point $(x_1, y_1)$ and has slope $m$. If $(x, y)$ is any other point on this line, the definition of the slope yields the equation

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1).$$

An equation written this way is in the *point-slope form*.

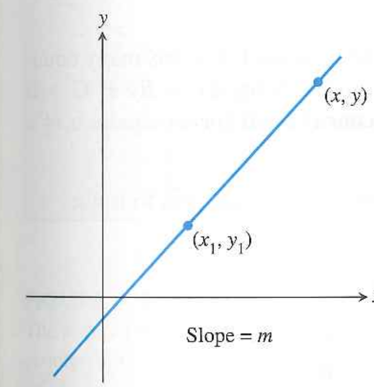



**FIGURE P.24** The line through $(x_1, y_1)$ with slope $m$.

**DEFINITION Point-Slope Form of an Equation of a Line**

The **point-slope form** of an equation of a line that passes through the point $(x_1, y_1)$ and has slope $m$ is

$$y - y_1 = m(x - x_1).$$

**EXAMPLE 2** Using the Point-Slope Form

Use the point-slope form to find an equation of the line that passes through the point $(-3, -4)$ and has slope 2.

**SOLUTION** Substitute $x_1 = -3$, $y_1 = -4$, and $m = 2$ into the point-slope form, and simplify the resulting equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-4) = 2(x - (-3)) \qquad x_1 = -3,\ y_1 = -4,\ m = 2$$
$$y + 4 = 2(x + 3) \qquad \text{Simplify.}$$

For graphing purposes, this equation can be written as $y = 2(x + 3) - 4$ or as $y = 2x + 2$.

Now try Exercise 11.

**y-Intercept**

The $b$ in $y = mx + b$ is often referred to as "the $y$-intercept" instead of "the $y$-coordinate of the $y$-intercept."

## Slope-Intercept Form Equation of a Line

The **y-intercept** of a nonvertical line is the point where the line intersects the $y$-axis. If we know the $y$-intercept and the slope of the line, we can apply the point-slope form to find an equation of the line.

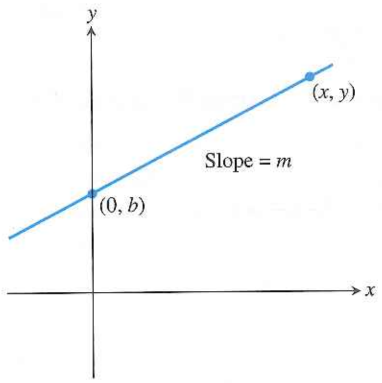



**FIGURE P.25** The line with slope $m$ and $y$-intercept $b$.

### Alternative Solution

You could solve Example 3 using the point-slope form:

$$y - 6 = 3(x - (-1))$$
$$y = 3(x + 1) + 6$$
$$y = 3x + 3 + 6$$
$$y = 3x + 9$$

Figure P.25 shows a line with slope $m$ and $y$-intercept $(0, b)$, or $b$ for short. A point-slope form equation for this line is $y - b = m(x - 0)$. By rewriting this equation we obtain the form known as the *slope-intercept form*.

**DEFINITION Slope-Intercept Form of an Equation of a Line**

The **slope-intercept form** of an equation of a line with slope $m$ and $y$-intercept $(0, b)$ is

$$y = mx + b.$$

**EXAMPLE 3** Using the Slope-Intercept Form

Using the slope-intercept form, write an equation of the line with slope 3 that passes through the point $(-1, 6)$.

**SOLUTION**

$y = mx + b$ — Slope-intercept form

$y = 3x + b$ — $m = 3$

$6 = 3(-1) + b$ — $y = 6$ when $x = -1$

$b = 9$

The slope-intercept form of the equation is $y = 3x + 9$. **Now try Exercise 21.**

We should not use the phrase "*the* equation of a line" because each line has many equations. Every line has an equation that can be written in the form $Ax + By + C = 0$ where $A$ and $B$ are not both zero. This form is the **general form** for an equation of a line.

If $B \neq 0$, the general form can be changed to the slope-intercept form as follows:

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$y = \underbrace{-\frac{A}{B}}_{\text{slope}}x + \underbrace{\left(-\frac{C}{B}\right)}_{y\text{-intercept}}$$

**Forms of Equations of Lines**

| | |
|---|---|
| **General form:** | $Ax + By + C = 0$, $A$ and $B$ not both zero |
| **Slope-intercept form:** | $y = mx + b$ |
| **Point-slope form:** | $y - y_1 = m(x - x_1)$ |
| **Vertical line:** | $x = a$ |
| **Horizontal line:** | $y = b$ |

## Graphing Linear Equations in Two Variables

A **linear equation in $x$ and $y$** is one that can be written in the form

$$Ax + By = C,$$

where $A$ and $B$ are not both zero. Rewriting the equation as $Ax + By - C = 0$ we see that it is closely related to the general form. If $B = 0$, the line is vertical, and if $A = 0$, the line is horizontal.



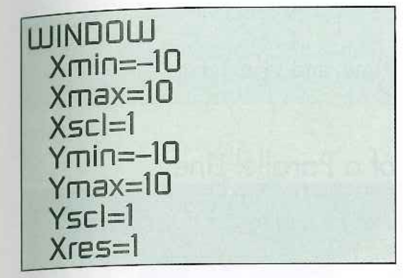
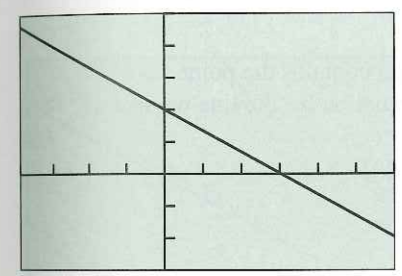



**FIGURE P.26** The window dimensions for the *standard window*. The notation "$[-10, 10]$ by $[-10, 10]$" is used to represent window dimensions like these.



[–4, 6] by [–3, 5]

**FIGURE P.27** The graph of $2x + 3y = 6$. The points $(0, 2)$ ($y$-intercept) and $(3, 0)$ ($x$-intercept) appear to lie on the graph and, as pairs, are solutions of the equation, providing visual support that the graph is correct. (Example 4)

### Viewing Window

The **viewing window** $[-4, 6]$ by $[-3, 5]$ in Figure P.27 means $-4 \leq x \leq 6$ and $-3 \leq y \leq 5$.

### Square Viewing Window

A **square viewing window** on a grapher is one in which angles appear to be true. For example, the line $y = x$ will appear to make a 45° angle with the positive $x$-axis. Furthermore, a distance of 1 on the $x$- and $y$-axes will appear to be the same. That is, if Xscl = Yscl, the distance between consecutive tick marks on the $x$- and $y$-axes will appear to be the same.

The **graph** of an equation in $x$ and $y$ consists of all pairs $(x, y)$ that are solutions of the equation. For example, $(1, 2)$ is a **solution** of the equation $2x + 3y = 8$ because substituting $x = 1$ and $y = 2$ into the equation leads to the true statement $8 = 8$. The pairs $(-2, 4)$ and $(2, 4/3)$ are also solutions.

Because the graph of a linear equation in $x$ and $y$ is a straight line, find two solutions and then connect them with a straight line to draw the graph. If a line is neither horizontal nor vertical, then two easy points to find are its $x$-intercept and the $y$-intercept. The **$x$-intercept** is the point $(x', 0)$ where the graph intersects the $x$-axis. Set $y = 0$ and solve for $x$ to find the $x$-intercept. The coordinates of the $y$-intercept are $(0, y')$. Set $x = 0$ and solve for $y$ to find the $y$-intercept.

**Graphing with a Graphing Utility**

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form $y =$ (an expression in $x$).
2. Enter the expression into the grapher.
3. Select an appropriate **viewing window** (see Figure P.26).
4. Press the "graph" key.

A graphing utility, or *grapher*, computes $y$-values for a select set of $x$-values between Xmin and Xmax and plots the corresponding $(x, y)$ points.

**EXAMPLE 4** Using a Graphing Utility

Draw the graph of $2x + 3y = 6$.

**SOLUTION** First we solve for $y$.

$2x + 3y = 6$

$3y = -2x + 6$ — Solve for $y$.

$y = -\frac{2}{3}x + 2$ — Divide by 3.

Figure P.27 shows the graph of $y = -(2/3)x + 2$, or equivalently, the graph of the linear equation $2x + 3y = 6$ in the $[-4, 6]$ by $[-3, 5]$ viewing window. **Now try Exercise 27.**

## Parallel and Perpendicular Lines

**EXPLORATION 1 Investigating Graphs of Linear Equations**

1. What do the graphs of $y = mx + b$ and $y = mx + c$, $b \neq c$, have in common? How are they different?
2. Graph $y = 2x$ and $y = -(1/2)x$ in a *square viewing window* (see margin note). On the calculator we use, the "decimal window" is square. Estimate the angle between the two lines.
3. Repeat part 2 for $y = mx$ and $y = -(1/m)x$ with $m = 1, 3, 4$, and 5.

Parallel lines and perpendicular lines were involved in Exploration 1. Using a grapher to decide whether lines are parallel or perpendicular is risky. Here is an algebraic test to determine whether two lines are parallel or perpendicular.

### Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if their slopes are equal. Any two distinct vertical lines are parallel.
2. Two nonvertical lines are perpendicular if and only if their slopes $m_1$ and $m_2$ are opposite reciprocals, that is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

A vertical line is perpendicular to a horizontal line, and vice versa.

### EXAMPLE 5 Finding an Equation of a Parallel Line

Find an equation of the line through $P(1, -2)$ that is parallel to the line $L$ with equation $3x - 2y = 1$.

**SOLUTION** We find the slope of $L$ by writing its equation in slope-intercept form.

$$3x - 2y = 1 \quad \text{Equation for } L$$
$$-2y = -3x + 1 \quad \text{Subtract } 3x.$$
$$y = \frac{3}{2}x - \frac{1}{2} \quad \text{Divide by } -2.$$

The slope of $L$ is 3/2.

The line whose equation we seek has slope 3/2 and contains the point $(x_1, y_1) = (1, -2)$. Thus, the point-slope form equation for the line we seek is

$$y + 2 = \frac{3}{2}(x - 1),$$

which can be written in slope-intercept form as

$$y = \frac{3}{2}x - \frac{7}{2}.$$

Now try Exercise 41(a).

### EXAMPLE 6 Finding an Equation of a Perpendicular Line

Find an equation of the line through $P(2, -3)$ that is perpendicular to the line $L$ with equation $4x + y = 3$. Support the result with a grapher.

**SOLUTION** We find the slope of $L$ by writing its equation in slope-intercept form.

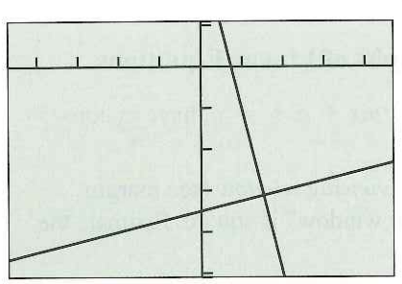
$$4x + y = 3 \quad \text{Equation for } L$$
$$y = -4x + 3 \quad \text{Subtract } 4x.$$

The slope of $L$ is $-4$.

The line whose equation we seek has slope $-1/(-4) = 1/4$ and passes through the point $(x_1, y_1) = (2, -3)$. Thus, the point-slope form equation for the line we seek is

$$y - (-3) = \frac{1}{4}(x - 2)$$
$$y + 3 = \frac{1}{4}x - \frac{1}{2} \quad \text{Distributive property}$$
$$y = \frac{1}{4}x - \frac{7}{2}$$

Figure P.28 shows the graphs of the two equations in a square viewing window and suggests that the graphs are indeed perpendicular. Now try Exercise 43(b).



[−4.7, 4.7] by [−5.1, 1.1]

**FIGURE P.28** The graphs of $y = -4x + 3$ and $y = (1/4)x - 7/2$ in this square viewing window appear to intersect at a right angle. (Example 6)



## Applying Linear Equations in Two Variables

Linear equations and their graphs occur frequently in applications. Algebraic solutions to these application problems often require finding an equation of a line and solving a linear equation in one variable. Grapher techniques complement algebraic ones.

### EXAMPLE 7 Finding the Depreciation of Real Estate

Camelot Apartments purchased a \$50,000 building. For tax purposes, its value depreciates \$2000 per year over a 25-year period.

(a) Write a linear equation giving the value $y$ of the building in terms of the years $x$ after the purchase.

(b) In how many years will the value of the building be \$24,500?

**SOLUTION**

(a) We need to determine the value of $m$ and $b$ so that $y = mx + b$, where $0 \le x \le 25$. We know that $y = 50{,}000$ when $x = 0$, so the line has $y$-intercept $(0, 50{,}000)$ and $b = 50{,}000$. One year after purchase $(x = 1)$, the value of the building is $50{,}000 - 2000 = 48{,}000$. So when $x = 1$, $y = 48{,}000$. Using algebra, we find

$$y = mx + b$$
$$48{,}000 = m \cdot 1 + 50{,}000 \quad y = 48{,}000 \text{ when } x = 1$$
$$-2000 = m \quad \text{Subtract } 50{,}000.$$

The value $y$ of the building $x$ years after its purchase is

$$y = -2000x + 50{,}000.$$

(b) We need to find the value of $x$ when $y = 24{,}500$.

$$y = -2000x + 50{,}000$$

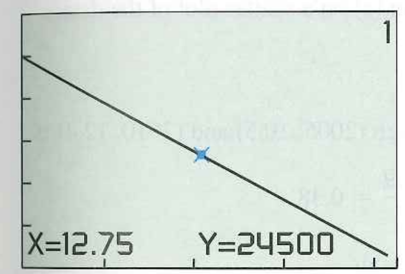
Using algebra again, we find

$$24{,}500 = -2000x + 50{,}000 \quad \text{Set } y = 24{,}500.$$
$$-25{,}500 = -2000x \quad \text{Subtract } 50{,}000.$$
$$12.75 = x \quad \text{Divide by } -2000.$$

The depreciated value of the building will be \$24,500 exactly 12.75 years, or 12 years 9 months, after the building was purchased by Camelot Apartments. We can support our algebraic work both graphically and numerically. The trace coordinates in Figure P.29a show graphically that $(12.75, 24{,}500)$ is a solution of $y = -2000x + 50{,}000$. This means that $y = 24{,}500$ when $x = 12.75$.

Figure P.29b is a table of values for $y = -2000x + 50{,}000$ for a few values of $x$. The fourth line of the table shows numerically that $y = 24{,}500$ when $x = 12.75$.

Now try Exercise 45.




[0, 23.5] by [0, 60000]

(a)

| X | $Y_1$ | |
|---|---|---|
| 12 | 26000 | |
| 12.25 | 25500 | |
| 12.5 | 25000 | |
| 12.75 | 24500 | |
| 13 | 24000 | |
| 13.25 | 23500 | |
| 13.5 | 23000 | |

$Y_1$ = −2000X+50000

(b)

**FIGURE P.29** A (a) graph and (b) table of values for $y = -2000x + 50{,}000$. (Example 7)

Figure P.30 on page 34 shows Americans' income from 2005 to 2010 in trillions of dollars and a corresponding scatter plot of the data. In Example 8, we model the data in Figure P.30 with a linear equation.

| Year | Amount (trillions of dollars) |
|---|---|
| 2005 | 10.5 |
| 2006 | 11.3 |
| 2007 | 11.9 |
| 2008 | 12.5 |
| 2009 | 11.9 |
| 2010 | 12.4 |

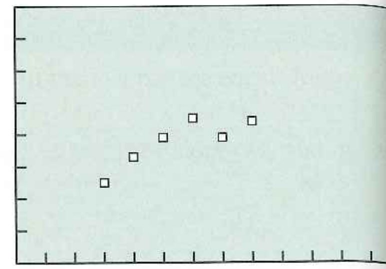



**FIGURE P.30** Americans' personal income. (Example 8)
*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*

### EXAMPLE 8 Finding a Linear Model for Americans' Personal Income over Time

Americans' total personal income in trillions of dollars is given in Figure P.30.

**(a)** Write a linear equation for Americans' income $y$ in terms of the year $x$ using the points (2005, 10.5) and (2010, 12.4).

**(b)** Use the equation in (a) to estimate Americans' income in 2008.

**(c)** Use the equation in (a) to predict Americans' income in 2015.

**(d)** Superimpose a graph of the linear equation in (a) on a scatter plot of the data like the one shown in Figure P.30.

**SOLUTION**

**(a)** Let $y = mx + b$. The slope of the line through (2005, 10.5) and (2010, 12.4) is

$$m = \frac{12.4 - 10.5}{2010 - 2005} = \frac{1.9}{5} = 0.38.$$

Using this slope and the point (2005, 10.5) yields

$$y = 0.38(x - 2005) + 10.5.$$

**(b)** To estimate Americans' total personal income for the year 2008, we let $x = 2008$ in the equation found in part (a):

$$\begin{aligned} y &= 0.38(2008 - 2005) + 10.5 \\ &= 0.38 \cdot 3 + 10.5 \\ &= 1.14 + 10.5 \\ &= 11.64 \end{aligned}$$

This value of roughly \$11.6 trillion underestimates the actual total income of \$12.5 trillion.

**(c)** To predict Americans' total personal income for 2015, we let $x = 2015$:

$$\begin{aligned} y &= 0.38(2015 - 2005) + 10.5 \\ &= 0.38 \cdot 10 + 10.5 \\ &= 3.8 + 10.5 \\ &= 14.3 \end{aligned}$$

Our linear model projects that the total personal income of all Americans in 2015 should be about \$14.3 trillion.

**(d)** Figure P.31 shows the scatter plot as well as the graph of the line we used to make our estimation and prediction. **Now try Exercise 51.**

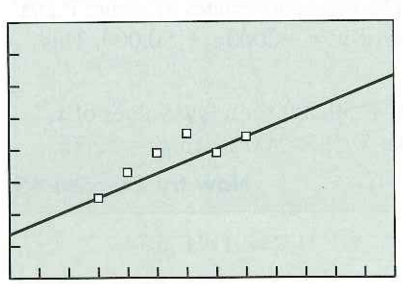



**FIGURE P.31** Linear model for Americans' personal income. (Example 8)



### The Speed of Light

Whether light traveled instantaneously or actually took some time was an open question for thousands of years. Galileo Galilei (1564–1642) was one of the first to approximate the speed of light. Jean Bernard Léon Foucault (1819–1868) established a modern estimate for light's speed. The speed of light played an important role in Einstein's development of special relativity and continues to be important in physics and astronomy. In 2013, whether the speed of light in a vacuum is truly a universal constant was called into question.

## CHAPTER OPENER Problem (from page 1)

**Problem:** Assume that the speed of light is about 299,800 km/sec (186,300 mi/sec).

**(a)** If light travels from the Sun to Earth in 8.32 min, approximate the distance between Earth and the Sun.

**(b)** If the distance from the Moon to Earth is roughly 384,400 km, approximate the time required for light to travel from the Moon to Earth.

**(c)** If light travels on average from the Sun to Pluto in about 5 hr 28 min, approximate the average distance between the Sun and Pluto.

**Solution:** We use the linear equation $d = r \cdot t$ (distance = rate × time) and the given rate $r = 299{,}800$ km/sec.

**(a)** Because $t = 8.32$ min $= 499.2$ sec,

$$d = r \cdot t = 299{,}800 \text{ km/sec} \times 499.2 \text{ sec} \approx 150{,}000{,}000 \text{ km}.$$

The distance from the Sun to Earth is about 150 million kilometers (93 million miles).

**(b)** Because $d = 384{,}400$ km,

$$t = \frac{d}{r} = \frac{384{,}400 \text{ km}}{299{,}800 \text{ km/sec}} \approx 1.282 \text{ sec}.$$

The time it takes light to travel from the Moon to Earth is about 1.282 sec.

**(c)** Because $t = 5$ hr 28 min $= 328$ min $= 19{,}680$ sec,

$$d = r \cdot t = 299{,}800 \text{ km/sec} \times 19{,}680 \text{ sec} = 5{,}900{,}064{,}000 \text{ km}.$$

The average distance from the Sun to Pluto is about $5.9 \times 10^9$ km.

## QUICK REVIEW P.4

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1–4, solve for $x$.

**1.** $-75x + 25 = 200$

**2.** $400 - 50x = 150$

**3.** $3(1 - 2x) + 4(2x - 5) = 7$

**4.** $2(7x + 1) = 5(1 - 3x)$

In Exercises 5–8, solve for $y$.

**5.** $2x - 5y = 21$

**6.** $\frac{1}{3}x + \frac{1}{4}y = 2$

**7.** $2x + y = 17 + 2(x - 2y)$

**8.** $x^2 + y = 3x - 2y$

In Exercises 9 and 10, simplify the fraction.

**9.** $\dfrac{9 - 5}{-2 - (-8)}$

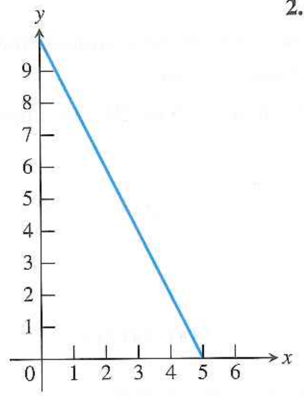
**10.** $\dfrac{-4 - 6}{-14 - (-2)}$

## SECTION P.4 Exercises

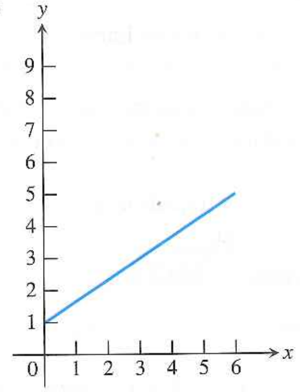
**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1 and 2, estimate the slope of the line.

**1.**



**2.**



In Exercises 3–6, find the slope of the line through the pair of points.

**3.** $(-3, 5)$ and $(4, 9)$ **4.** $(-2, 1)$ and $(5, -3)$

**5.** $(-2, -5)$ and $(-1, 3)$ **6.** $(5, -3)$ and $(-4, 12)$

In Exercises 7–10, find the value of $x$ or $y$ so that the line through the pair of points has the given slope.

| | Points | Slope |
|---|---|---|
| **7.** | $(x, 3)$ and $(5, 9)$ | $m = 2$ |
| **8.** | $(-2, 3)$ and $(4, y)$ | $m = -3$ |
| **9.** | $(-3, -5)$ and $(4, y)$ | $m = 3$ |
| **10.** | $(-8, -2)$ and $(x, 2)$ | $m = 1/2$ |

In Exercises 11–14, find a point-slope form equation for the line through the point with given slope.

| | Point | Slope | | Point | Slope |
|---|---|---|---|---|---|
| **11.** | $(1, 4)$ | $m = 2$ | **12.** | $(-4, 3)$ | $m = -2/3$ |
| **13.** | $(5, -4)$ | $m = -2$ | **14.** | $(-3, 4)$ | $m = 3$ |

In Exercises 15–20, find a general form equation for the line through the pair of points.

**15.** $(-7, -2)$ and $(1, 6)$ **16.** $(-3, -8)$ and $(4, -1)$

**17.** $(1, -3)$ and $(5, -3)$ **18.** $(-1, -5)$ and $(-4, -2)$

**19.** $(-1, 2)$ and $(2, 5)$ **20.** $(4, -1)$ and $(4, 5)$

In Exercises 21–26, find a slope-intercept form equation for the line.

**21.** The line through $(0, 5)$ with slope $m = -3$

**22.** The line through $(1, 2)$ with slope $m = 1/2$

**23.** The line through the points $(-4, 5)$ and $(4, 3)$

**24.** The line through the points $(4, 2)$ and $(-3, 1)$

**25.** The line $2x + 5y = 12$

**26.** The line $7x - 12y = 96$

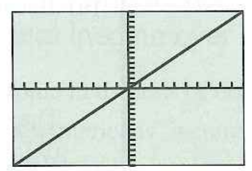
In Exercises 27–30, graph the linear equation on a grapher. Choose a viewing window that shows the line intersecting both the $x$- and $y$-axes.

**27.** $8x + y = 49$ **28.** $2x + y = 35$

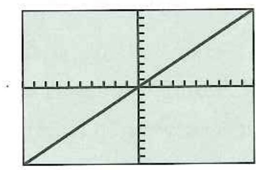
**29.** $123x + 7y = 429$ **30.** $2100x + 12y = 3540$

In Exercises 31 and 32, the line contains the origin and the point in the upper right corner of the grapher screen.

**31. Writing to Learn** Which line shown here has the greater slope? Explain.

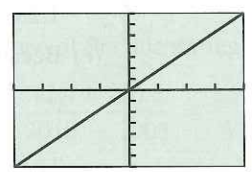


[–10, 10] by [–15, 15]
(a)

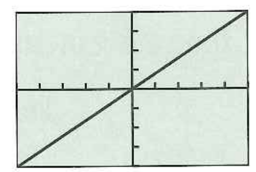


[–10, 10] by [–10, 10]
(b)

**32. Writing to Learn** Which line shown here has the greater slope? Explain.



[–20, 20] by [–35, 35]
(a)



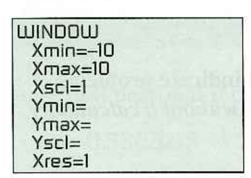
[–5, 5] by [–20, 20]
(b)

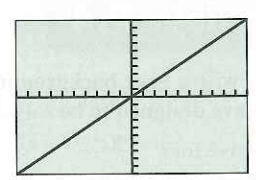
In Exercises 33–36, find the value of $x$ and the value of $y$ for which $(x, 14)$ and $(18, y)$ are points on the graph.

**33.** $y = 0.5x + 12$ **34.** $y = -2x + 18$

**35.** $3x + 4y = 26$ **36.** $3x - 2y = 14$

In Exercises 37–40, find the values for Ymin, Ymax, and Yscl that will make the graph of the line appear in the viewing window as shown here.



**37.** $y = 3x$ **38.** $y = 5x$

**39.** $y = \frac{2}{3}x$ **40.** $y = \frac{5}{4}x$



In Exercises 41–44, (a) find an equation for the line passing through the point and parallel to the given line, and (b) find an equation for the line passing through the point and perpendicular to the given line. Support your work graphically.

| | Point | Line |
|---|---|---|
| **41.** | $(1, 2)$ | $y = 3x - 2$ |
| **42.** | $(-2, 3)$ | $y = -2x + 4$ |
| **43.** | $(3, 1)$ | $2x + 3y = 12$ |
| **44.** | $(6, 1)$ | $3x - 5y = 15$ |

**45. Real Estate Appreciation** Bob Michaels purchased a house 8 years ago for \$42,000. This year it was appraised at \$67,500.

**(a)** A linear equation $V = mt + b$, $0 \le t \le 15$, represents the value $V$ of the house for 15 years after it was purchased. Determine $m$ and $b$.

**(b)** Graph the equation and trace to estimate in how many years after purchase this house will be worth \$72,500.

**(c)** Write and solve an equation algebraically to determine how many years after purchase this house will be worth \$74,000.

**(d)** Determine how many years after purchase this house will be worth \$80,250.

**46. Investment Planning** Mary Ellen plans to invest \$18,000, putting part of the money $x$ into a savings account that pays 5% annually and the rest into an account that pays 8% annually.

**(a)** What are the possible values of $x$ in this situation?

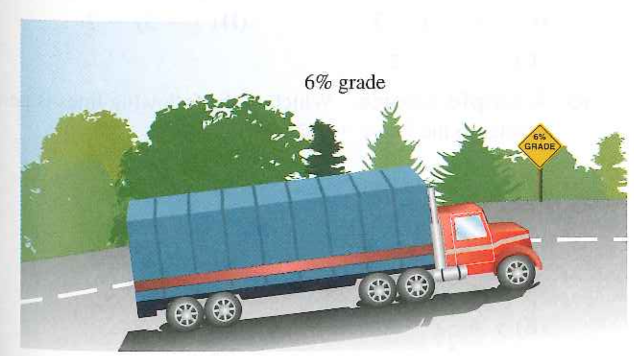
**(b)** If Mary Ellen invests $x$ dollars at 5%, write an equation that describes the total interest $I$ received from both accounts at the end of one year.

**(c)** Graph and trace to estimate how much Mary Ellen invested at 5% if she earned \$1020 in total interest at the end of the first year.

**(d)** Use your grapher to generate a table of values for $I$ to find out how much Mary Ellen should invest at 5% to earn \$1185 in total interest in one year.

**47. Navigation** A commercial jet airplane climbs at takeoff with slope $m = 3/8$. How far in the horizontal direction will the airplane fly to reach an altitude of 12,000 ft above the takeoff point?

**48. Grade of a Highway** Interstate 70 west of Denver, Colorado, has a section posted as a 6% grade. This means that for a horizontal change of 100 ft there is a 6-ft vertical change.




**(a)** Find the slope of this section of the highway.

**(b)** On a highway with a 6% grade what is the horizontal distance required to climb 250 ft?

**(c)** A sign along the highway says 6% grade for the next 7 mi. Estimate how many feet of vertical change there are along those next 7 mi. (There are 5280 ft in 1 mile.)

**49. Writing to Learn Building Specifications** Asphalt shingles do not meet code specifications on a roof that has less than a 4-12 pitch. A 4-12 pitch means there are 4 ft of vertical change in 12 ft of horizontal change. A certain roof has slope $m = 3/8$. Could asphalt shingles be used on that roof? Explain.

**50. Revisiting Example 8** Use the linear equation found in Example 8 to estimate Americans' income in 2004, 2006, 2007 displayed in Figure P.30.

**51. Americans' Spending** The $(x, y)$ table shows total personal consumption expenditures $(y)$ in the United States in trillions of dollars for selected years $(x)$. (*Source: U.S. Bureau of Economic Analysis, The World Almanac and Book of Facts 2012.*)

| $x$ | 1990 | 1995 | 2000 | 2005 | 2010 |
|---|---|---|---|---|---|
| $y$ | 3.8 | 5.0 | 6.8 | 8.8 | 10.2 |

**(a)** Write a linear equation for Americans' spending $(y)$ in terms of the year $(x)$, using the points (1990, 3.8) and (2010, 10.2).

**(b)** Use the equation in (a) to estimate Americans' expenditures in 2005.

**(c)** Use the equation in (a) to predict Americans' expenditures in 2015.

**(d)** Superimpose a graph of the linear equation in (a) on a scatter plot of the data.

**52. U.S. Imports from Mexico** The $(x, y)$ table shows total U.S. imports from Mexico $(y)$ in billions of dollars for selected years $(x)$. (*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*)

| $x$ | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 138.1 | 155.9 | 170.1 | 198.3 | 210.7 | 215.9 | 176.7 | 229.9 |

**(a)** Use the pairs (2003, 138.1) and (2010, 229.9) to write a linear equation for $x$ and $y$.

**(b)** Superimpose the graph of the linear equation in (a) on a scatter plot of the data.

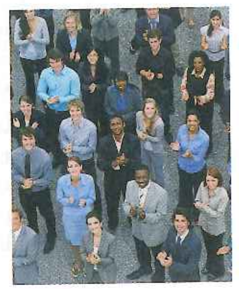
**(c)** Use the equation in (a) to predict the total U.S. imports from Mexico in 2015.

53. **World Population** Table P.7 shows the midyear worldwide human population for selected years.

**Table P.7 World Population**

| Year | Population (millions) |
|---|---|
| 1980 | 4453 |
| 1990 | 5282 |
| 2000 | 6085 |
| 2010 | 6972 |
| 2013 | 7130 |

*Source: U.S. Census Bureau.*



(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Draw a scatter plot of the data.

(b) Use the 1980 and 2010 data to write a linear equation for the population $y$ (in millions) in terms of the year $x$. Superimpose the graph of the linear equation on the scatter plot of the data.

(c) Use the graph in (b) to predict the midyear world population in 2020.

54. **U.S. Exports to Canada** Table P.8 shows the total exports from the United States to Canada in billions of dollars for selected years.

**Table P.8 U.S. Exports to Canada**

| Year | U.S. Exports (billions of dollars) |
|---|---|
| 2003 | 169.9 |
| 2004 | 189.9 |
| 2005 | 211.9 |
| 2006 | 230.7 |
| 2007 | 248.9 |
| 2008 | 261.2 |
| 2009 | 204.7 |
| 2010 | 249.1 |

*Source: U.S. Census Bureau, The World Almanac and Book of Facts 2012.*

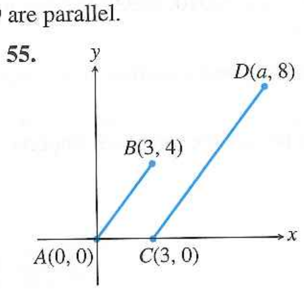
(a) Draw a scatter plot of the data.

(b) Use the 2003 and 2010 data to write a linear equation for the U.S. exports $y$ in terms of the year $x$. Superimpose the graph of the equation on the scatter plot.

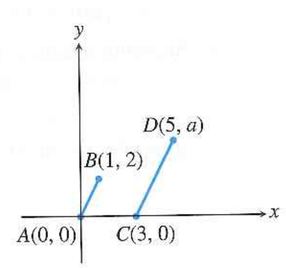
(c) Use the equation in (b) to predict the U.S. exports to Canada for 2015.

In Exercises 55 and 56, determine $a$ so that the line segments $AB$ and $CD$ are parallel.

55.

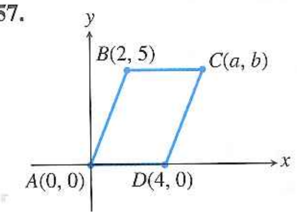



56.

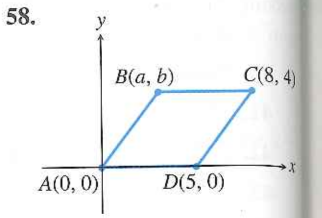



In Exercises 57 and 58, determine $a$ and $b$ so that figure $ABCD$ is a parallelogram.

57.




58.




59. **Writing to Learn Perpendicular Lines**

(a) Is it possible for two lines with positive slopes to be perpendicular? Explain.

(b) Is it possible for two lines with negative slopes to be perpendicular? Explain.

60. **Group Activity Parallel and Perpendicular Lines**

(a) Assume that $c \neq d$ and $a$ and $b$ are not both zero. Show that $ax + by = c$ and $ax + by = d$ are parallel lines. Explain why the restrictions on $a$, $b$, $c$, and $d$ are necessary.

(b) Assume that $a$ and $b$ are not both zero. Show that $ax + by = c$ and $bx - ay = d$ are perpendicular lines. Explain why the restrictions on $a$ and $b$ are necessary.

## Standardized Test Questions

61. **True or False** The slope of a vertical line is zero. Justify your answer.

62. **True or False** The graph of any equation of the form $ax + by = c$, where $a$ and $b$ are not both zero, is always a line. Justify your answer.

In Exercises 63–66, you may use a graphing calculator to solve these problems.

63. **Multiple Choice** Which of the following is an equation of the line through the point $(-2, 3)$ with slope 4?

(A) $y - 3 = 4(x + 2)$ (B) $y + 3 = 4(x - 2)$
(C) $x - 3 = 4(y + 2)$ (D) $x + 3 = 4(y - 2)$
(E) $y + 2 = 4(x - 3)$

64. **Multiple Choice** Which of the following is an equation of the line with slope 3 and $y$-intercept $-2$?

(A) $y = 3x + 2$ (B) $y = 3x - 2$
(C) $y = -2x + 3$ (D) $x = 3y - 2$
(E) $x = 3y + 2$

65. **Multiple Choice** Which of the following lines is perpendicular to the line $y = -2x + 5$?

(A) $y = 2x + 1$ (B) $y = -2x - \frac{1}{5}$
(C) $y = -\frac{1}{2}x + \frac{1}{3}$ (D) $y = -\frac{1}{2}x + 3$
(E) $y = \frac{1}{2}x - 3$



66. **Multiple Choice** Which of the following is the slope of the line through the two points $(-2, 1)$ and $(1, -4)$?

(A) $-\frac{3}{5}$ (B) $\frac{3}{5}$
(C) $-\frac{5}{3}$ (D) $\frac{5}{3}$
(E) $-3$

## Explorations

67. **Exploring the Graph of $\frac{x}{a} + \frac{y}{b} = c$, $a \neq 0$, $b \neq 0$**

Let $c = 1$.

(a) Draw the graph for $a = 3$, $b = -2$.

(b) Draw the graph for $a = -2$, $b = -3$.

(c) Draw the graph for $a = 5$, $b = 3$.

(d) Use your graphs in (a), (b), (c) to conjecture what $a$ and $b$ represent when $c = 1$. Prove your conjecture.

(e) Repeat (a)–(d) for $c = 2$. What do $a$ and $b$ represent when $c = 2$?

(f) If $c = -1$, what do $a$ and $b$ represent?

68. **Investigating Graphs of Linear Equations**

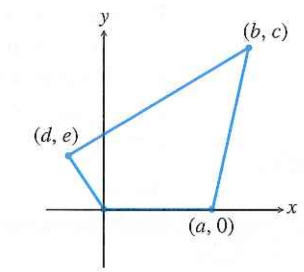
(a) Graph $y = mx$ for $m = -3, -2, -1, 1, 2, 3$ in the window $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?

(b) If $m > 0$, what do the graphs of $y = mx$ and $y = -mx$ have in common? How are they different?

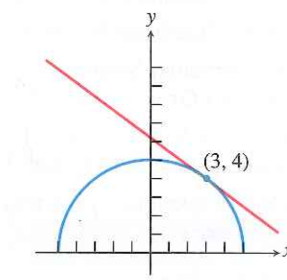
(c) Graph $y = 0.3x + b$ for $b = -3, -2, -1, 0, 1, 2, 3$ in $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?

## Extending the Ideas

69. **Connecting Algebra and Geometry** Show that if the midpoints of consecutive sides of any quadrilateral (see figure) are connected, the result is a parallelogram.



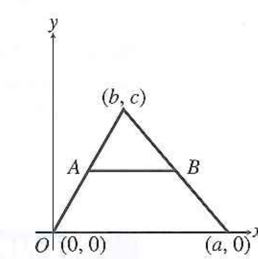

Art for Exercise 69




Art for Exercise 70

70. **Connecting Algebra and Geometry** Consider the semicircle of radius 5 centered at $(0, 0)$ as shown in the figure. Find an equation of the line tangent to the semicircle at the point $(3, 4)$. (*Hint:* A line tangent to a circle is perpendicular to the radius at the point of tangency.)

71. **Connecting Algebra and Geometry** Show that in any triangle (see figure), the line segment joining the midpoints of two sides is parallel to the third side and is half as long.



# P.5 Solving Equations Graphically, Numerically, and Algebraically

### What you'll learn about

- Solving Equations Graphically
- Solving Quadratic Equations
- Approximating Solutions of Equations Graphically
- Approximating Solutions of Equations Numerically with Tables
- Solving Equations by Finding Intersections

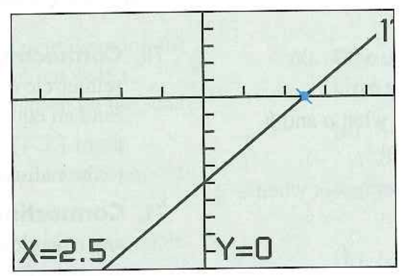
### ... and why

These basic techniques are involved in using a graphing utility to solve equations in this textbook.

## Solving Equations Graphically

The graph of the equation $y = 2x - 5$ (in $x$ and $y$) can be used to solve the equation $2x - 5 = 0$ (in $x$). Using the techniques of Section P.3, we can show algebraically that $x = 5/2$ is a solution of $2x - 5 = 0$. Therefore, the ordered pair $(5/2, 0)$ is a solution of $y = 2x - 5$. Figure P.32 suggests that the $x$-intercept of the graph of the line $y = 2x - 5$ is the point $(5/2, 0)$ as it should be.

One way to solve an equation graphically is to find all its $x$-intercepts. There are many graphical techniques that can be used to find $x$-intercepts.




$[-4.7, 4.7]$ by $[-10, 5]$

**FIGURE P.32** Using the TRACE feature of a grapher, we see that $(2.5, 0)$ is an $x$-intercept of the graph of $y = 2x - 5$ and, therefore, $x = 2.5$ is a solution of the equation $2x - 5 = 0$.

### EXAMPLE 1 Solving by Finding $x$-Intercepts

Solve the equation $2x^2 - 3x - 2 = 0$ graphically. Confirm algebraically.

**SOLUTION**

**Solve Graphically** Find the $x$-intercepts of the graph of $y = 2x^2 - 3x - 2$ (Figure P.33). We use TRACE to see that $(-0.5, 0)$ and $(2, 0)$ are $x$-intercepts of this graph. Thus, the solutions of this equation are $x = -0.5$ and $x = 2$. Answers obtained graphically are really approximations, although in general they are very good approximations.

**Confirm Algebraically** In this case, we can use factoring to find exact values.

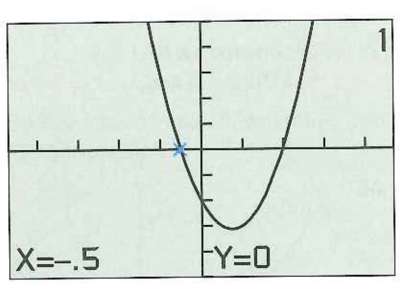
$$2x^2 - 3x - 2 = 0$$
$$(2x + 1)(x - 2) = 0 \quad \text{Factor.}$$

We can conclude that

$$2x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -1/2 \quad \text{or} \quad x = 2$$

So, $x = -1/2$ and $x = 2$ are the exact solutions of the original equation.

Now try Exercise 1.




$[-4.7, 4.7]$ by $[-5, 5]$

**FIGURE P.33** It appears that $(-0.5, 0)$ and $(2, 0)$ are $x$-intercepts of the graph of $y = 2x^2 - 3x - 2$. (Example 1)

The algebraic solution procedure used in Example 1 is a special case of the following important property.



**Zero Factor Property**

Let $a$ and $b$ be real numbers.

If $ab = 0$, then $a = 0$ or $b = 0$.

## Solving Quadratic Equations

Linear equations $(ax + b = 0)$ and *quadratic equations* are two members of the family of *polynomial equations*, which will be studied in detail in Chapter 2.

**DEFINITION Quadratic Equation in x**

A **quadratic equation in $x$** is one that can be written in the form

$$ax^2 + bx + c = 0,$$

where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

We review some of the basic algebraic techniques for solving quadratic equations. One algebraic technique that we have already used in Example 1 is *factoring*.

Quadratic equations of the form $(ax + b)^2 = c$ are fairly easy to solve as illustrated in Example 2.

**Finding Square Roots**

If $t^2 = k > 0$, then $t = \sqrt{k}$ or $t = -\sqrt{k}$.

### EXAMPLE 2 Solving by Extracting Square Roots

Solve $(2x - 1)^2 = 9$ algebraically.

**SOLUTION**

$$(2x - 1)^2 = 9$$
$$2x - 1 = \pm 3 \quad \text{Extract square roots.}$$
$$2x = 4 \quad \text{or} \quad 2x = -2 \quad \text{Add 1.}$$
$$x = 2 \quad \text{or} \quad x = -1 \quad \text{Divide by 2.}$$

Now try Exercise 9.

The technique of Example 2 is more general than you might think because every quadratic equation can be written in the form $(x + b)^2 = c$. The procedure we need to accomplish this is *completing the square*.

**Completing the Square**

To solve $x^2 + bx = c$ by **completing the square**, add $(b/2)^2$ to both sides of the equation and factor the left side of the new equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}$$

In general, to solve a quadratic equation by completing the square, first we divide both sides of the equation by the coefficient of $x^2$, and then we complete the square as illustrated in Example 3.

## EXAMPLE 3 Solving by Completing the Square

Solve $4x^2 - 20x + 17 = 0$ by completing the square.

**SOLUTION**

$$4x^2 - 20x + 17 = 0$$

$$x^2 - 5x + \frac{17}{4} = 0 \qquad \text{Divide by 4.}$$

$$x^2 - 5x = -\frac{17}{4} \qquad \text{Subtract } \frac{17}{4}.$$

Completing the square for the equation above we obtain

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = -\frac{17}{4} + \left(-\frac{5}{2}\right)^2 \qquad \text{Add } \left(-\frac{5}{2}\right)^2.$$

$$\left(x - \frac{5}{2}\right)^2 = 2 \qquad \text{Factor and simplify.}$$

$$x - \frac{5}{2} = \pm\sqrt{2} \qquad \text{Extract square roots.}$$

$$x = \frac{5}{2} \pm \sqrt{2}$$

$$x = \frac{5}{2} + \sqrt{2} \approx 3.91 \text{ or } x = \frac{5}{2} - \sqrt{2} \approx 1.09$$

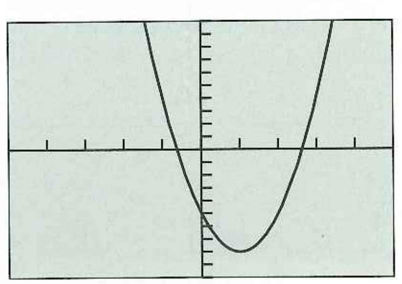
**Now try Exercise 13**

The procedure of Example 3 can be used to solve the general quadratic equation $ax^2 + bx + c = 0$, producing the following formula. (See Exercise 68.)

### Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$



[-5, 5] by [-10, 10]

**FIGURE P.34** The graph of $y = 3x^2 - 6x - 5$. (Example 4)

## EXAMPLE 4 Using the Quadratic Formula

Solve the equation $3x^2 - 6x = 5$.

**SOLUTION** First we subtract 5 from both sides of the equation to put it in the form $ax^2 + bx + c = 0$: $3x^2 - 6x - 5 = 0$. We can see that $a = 3$, $b = -6$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-5)}}{2(3)} \qquad a = 3, b = -6, c = -5$$

$$x = \frac{6 \pm \sqrt{96}}{6} \qquad \text{Simplify.}$$

$$x = \frac{6 + \sqrt{96}}{6} \approx 2.63 \quad \text{or} \quad x = \frac{6 - \sqrt{96}}{6} \approx -0.63$$

The graph of $y = 3x^2 - 6x - 5$ in Figure P.34 supports that the $x$-intercepts are approximately $-0.63$ and $2.63$.

**Now try Exercise 19**



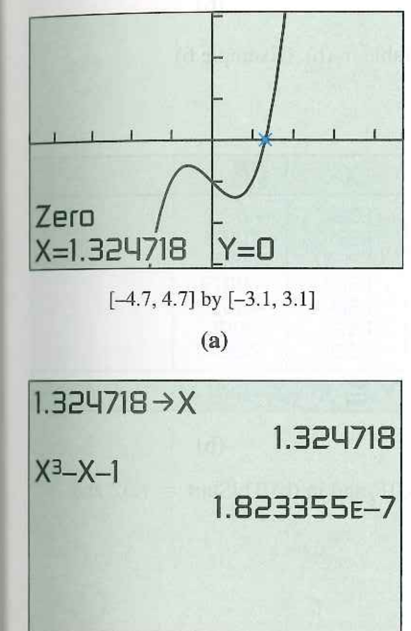
### Solving Quadratic Equations Algebraically

There are four basic ways to solve quadratic equations algebraically.

1. **Factoring** (see Example 1)
2. **Extracting Square Roots** (see Example 2)
3. **Completing the Square** (see Example 3)
4. **Using the Quadratic Formula** (see Example 4)

## Approximating Solutions of Equations Graphically

A solution of the equation $x^3 - x - 1 = 0$ is a value of $x$ that makes the value of $y = x^3 - x - 1$ equal to zero. Example 5 illustrates a built-in procedure on graphing calculators to find such values of $x$.




[-4.7, 4.7] by [-3.1, 3.1]

(a)

(b)

**FIGURE P.35** The graph of $y = x^3 - x - 1$. (a) shows that $(1.324718, 0)$ is an approximation to the $x$-intercept of the graph. (b) supports this conclusion. (Example 5)

## EXAMPLE 5 Solving Graphically

Solve the equation $x^3 - x - 1 = 0$ graphically.

**SOLUTION** Figure P.35a suggests that $x \approx 1.324718$ is the solution we seek. Figure P.35b provides numerical support that $x = 1.324718$ is a close approximation to the solution because, when $x = 1.324718$, $x^3 - x - 1 \approx 1.82 \times 10^{-7}$, which is nearly zero.

**Now try Exercise 31.**

When solving equations graphically, we usually get approximate solutions and not exact solutions. We will use the following agreement about accuracy in this book.

### Agreement About Approximate Solutions

For applications, round to a value that is reasonable for the context of the problem. For all other problems, round to two decimal places unless directed otherwise.

With this accuracy agreement, we would report the approximate solution found in Example 5 as 1.32.

## Approximating Solutions of Equations Numerically with Tables

The table feature on graphing calculators provides a numerical *zoom-in procedure* that we can use to find accurate solutions of equations. We illustrate this procedure in Example 6 with the same equation used in Example 5.

## EXAMPLE 6 Solving Using Tables

Solve the equation $x^3 - x - 1 = 0$ using grapher tables.

**SOLUTION** From Figure P.35a, we know that the solution we seek is between $x = 1$ and $x = 2$. Figure P.36a sets the starting point of the table (TblStart = 1) at $x = 1$ and the input increments in the table ($\Delta$Tbl = 0.1) at 0.1. Figure P.36b shows that the zero of $x^3 - x - 1$ lies between $x = 1.3$ and $x = 1.4$.

The next two steps in this process are shown in Figure P.37.

From Figure P.37a, we can read that the zero is between $x = 1.32$ and $x = 1.33$; from Figure P.37b, we can read that the zero is between $x = 1.324$ and $x = 1.325$. Thus, we conclude that the zero is approximately 1.32, according to our accuracy agreement.

**Now try Exercise 37**

TABLE SETUP
TblStart=1
ΔTbl=.1
Indpnt: Auto Ask
Depend: Auto Ask

(a)

| X | Y1 |
|---|---|
| 1 | -1 |
| 1.1 | -.769 |
| 1.2 | -.472 |
| 1.3 | -.103 |
| 1.4 | .344 |
| 1.5 | .875 |
| 1.6 | 1.496 |

Y1 = X3–X–1

(b)

**FIGURE P.36** (a) gives the setup that produces the table in (b). (Example 6)

| X | Y1 |
|---|---|
| 1.3 | -.103 |
| 1.31 | -.0619 |
| 1.32 | -.02 |
| 1.33 | .02264 |
| 1.34 | .0661 |
| 1.35 | .11038 |
| 1.36 | .15546 |

Y1 = X3–X–1

(a)

| X | Y1 |
|---|---|
| 1.32 | -.02 |
| 1.321 | -.0158 |
| 1.322 | -.0116 |
| 1.323 | -.0073 |
| 1.324 | -.0031 |
| 1.325 | .0012 |
| 1.326 | .00547 |

Y1 = X3–X–1

(b)

**FIGURE P.37** In (a) TblStart = 1.3 and ΔTbl = 0.01, and in (b) TblStart = 1.32 and ΔTbl = 0.001. (Example 6)

## EXPLORATION 1 Finding Real Zeros of Equations

Consider the equation $4x^2 - 12x + 7 = 0$.

1. Use a graph to show that this equation has two real solutions, one between 0 and 1 and the other between 2 and 3.
2. Use the numerical zoom-in procedure illustrated in Example 6 to find each zero accurate to two decimal places.
3. Use the built-in zero finder (see Example 5) to find the two solutions. Then round them to two decimal places.
4. If you are familiar with the graphical zoom-in process, use it to find each solution accurate to two decimal places.
5. Compare the numbers obtained in parts 2, 3, and 4.
6. Support the results obtained in parts 2, 3, and 4 numerically.
7. Use the numerical zoom-in procedure illustrated in Example 6 to find each zero accurate to six decimal places. Compare with the answer found in part 3 with the zero finder.



## Solving Equations by Finding Intersections

Sometimes we can rewrite an equation and solve it graphically by finding the *points of intersection* of two graphs. A point $(a, b)$ is a **point of intersection** of two graphs if it lies on both graphs.

We illustrate this procedure with the absolute value equation in Example 7.

### EXAMPLE 7 Solving by Finding Intersections

Solve the equation $|2x - 1| = 6$.

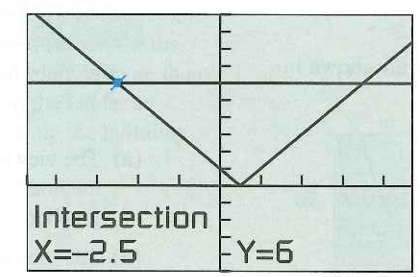
**SOLUTION** Figure P.38 suggests that the V-shaped graph of $y = |2x - 1|$ intersects the graph of the horizontal line $y = 6$ twice. We can use TRACE or the intersection feature of our grapher to see that the two points of intersection have coordinates $(-2.5, 6)$ and $(3.5, 6)$. This means that the original equation has two solutions: $-2.5$ and $3.5$.

We can use algebra to find the exact solutions. The only two real numbers with absolute value 6 are 6 itself and $-6$. So, if $|2x - 1| = 6$, then

$$2x - 1 = 6 \quad \text{or} \quad 2x - 1 = -6$$

$$x = \frac{7}{2} = 3.5 \quad \text{or} \quad x = -\frac{5}{2} = -2.5$$

**Now try Exercise 39.**




[-4.7, 4.7] by [-5, 10]

**FIGURE P.38** The graphs of $y = |2x - 1|$ and $y = 6$ intersect at $(-2.5, 6)$ and $(3.5, 6)$. (Example 7)

## QUICK REVIEW P.5

In Exercises 1–4, expand the product.

1. $(3x - 4)^2$
2. $(2x + 3)^2$
3. $(2x + 1)(3x - 5)$
4. $(3y - 1)(5y + 4)$

In Exercises 5–8, factor the expression.

5. $25x^2 - 20x + 4$
6. $15x^3 - 22x^2 + 8x$
7. $3x^3 + x^2 - 15x - 5$
8. $y^4 - 13y^2 + 36$

In Exercises 9 and 10, combine the fractions and reduce the resulting fraction to lowest terms.

9. $\frac{x}{2x + 1} - \frac{2}{x + 3}$
10. $\frac{x + 1}{x^2 - 5x + 6} - \frac{3x + 11}{x^2 - x - 6}$

## SECTION P.5 Exercises

In Exercises 1–6, solve the equation graphically by finding x-intercepts. Confirm by using factoring to solve the equation.

1. $x^2 - x - 20 = 0$
2. $2x^2 + 5x - 3 = 0$
3. $4x^2 - 8x + 3 = 0$
4. $x^2 - 8x = -15$
5. $x(3x - 7) = 6$
6. $x(3x + 11) = 20$

In Exercises 7–12, solve the equation by extracting square roots.

7. $4x^2 = 25$
8. $2(x - 5)^2 = 17$
9. $3(x + 4)^2 = 8$
10. $4(u + 1)^2 = 18$
11. $2y^2 - 8 = 6 - 2y^2$
12. $(2x + 3)^2 = 169$

In Exercises 13–18, solve the equation by completing the square.

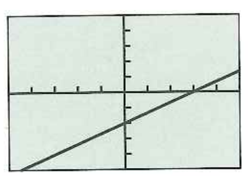
13. $x^2 + 6x = 7$
14. $x^2 + 5x - 9 = 0$
15. $x^2 - 7x + \frac{5}{4} = 0$
16. $4 - 6x = x^2$
17. $2x^2 - 7x + 9 = (x - 3)(x + 1) + 3x$
18. $3x^2 - 6x - 7 = x^2 + 3x - x(x + 1) + 3$

In Exercises 19–24, solve the equation using the quadratic formula.

19. $x^2 + 8x - 2 = 0$
20. $2x^2 - 3x + 1 = 0$
21. $3x + 4 = x^2$
22. $x^2 - 5 = \sqrt{3}x$
23. $x(x + 5) = 12$
24. $x^2 - 2x + 6 = 2x^2 - 6x - 26$

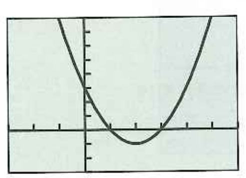
In Exercises 25–28, estimate any x- and y-intercepts that are shown in the graph.

25.



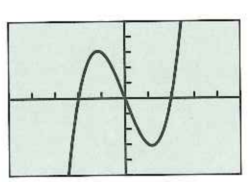
[–5, 5] by [–5, 5]

26.



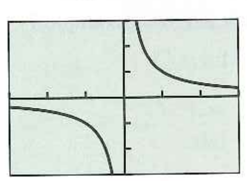
[–3, 6] by [–3, 8]

27



[–5, 5] by [–5, 5]

28



[–3, 3] by [–3, 3]

In Exercises 29–34, solve the equation graphically by finding x-intercepts.

29. $x^2 + x - 1 = 0$
30. $4x^2 + 20x + 23 = 0$
31. $x^3 + x^2 + 2x - 3 = 0$
32. $x^3 - 4x + 2 = 0$
33. $x^2 + 4 = 4x$
34. $x^2 + 2x = -2$

In Exercises 35 and 36, the table permits you to estimate a zero of an expression. State the expression and give the zero as accurately as can be read from the table.

35.

| X | $Y_1$ | |
|---|---|---|
| .4 | –.04 | |
| .41 | –.0119 | |
| .42 | .0164 | |
| .43 | .0449 | |
| .44 | .0736 | |
| .45 | .1025 | |
| .46 | .1316 | |

$Y_1 = X^2+2X-1$

36.

| X | $Y_1$ | |
|---|---|---|
| –1.735 | –.0177 | |
| –1.734 | –.0117 | |
| –1.733 | –.0057 | |
| –1.732 | 3E–4 | |
| –1.731 | .0063 | |
| –1.73 | .01228 | |
| –1.729 | .01826 | |

$Y_1 = X^3-3X$

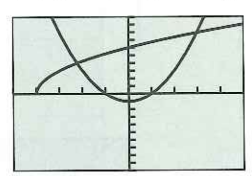
In Exercises 37 and 38, use tables to find the indicated number of solutions of the equation accurate to two decimal places.

37. Two solutions of $x^2 - x - 1 = 0$
38. One solution of $-x^3 + x + 1 = 0$

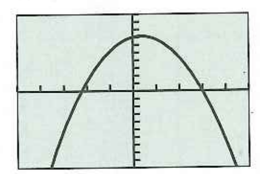
In Exercises 39–44, solve the equation graphically by finding intersections. Confirm your answer algebraically.

39. $|t - 8| = 2$
40. $|x + 1| = 4$
41. $|2x + 5| = 7$
42. $|3 - 5x| = 4$
43. $|2x - 3| = x^2$
44. $|x + 1| = 2x - 3$

45. **Interpreting Graphs** The graphs in the two viewing windows shown here can be used to solve the equation $3\sqrt{x + 4} = x^2 - 1$ graphically.



[–5, 5] by [–10, 10]
(a)



[–5, 5] by [–10, 10]
(b)

(a) The viewing window in (a) illustrates the intersection method for solving. Identify the two equations that are graphed.

(b) The viewing window in (b) illustrates the x-intercept method for solving. Identify the equation that is graphed.

(c) **Writing to Learn** How are the intersection points in (a) related to the x-intercepts in (b)?

46. **Writing to Learn Revisiting Example 6** Explain why all real numbers x that satisfy $1.324 < x < 1.325$ round to 1.32.

In Exercises 47–56, use a method of your choice to solve the equation.

47. $x^2 + x - 2 = 0$
48. $x^2 - 3x = 12 - 3(x - 2)$
49. $|2x - 1| = 5$
50. $x + 2 - 2\sqrt{x + 3} = 0$
51. $x^3 + 4x^2 - 3x - 2 = 0$
52. $x^3 - 4x + 2 = 0$
53. $|x^2 + 4x - 1| = 7$
54. $|x + 5| = |x - 3|$
55. $|0.5x + 3| = x^2 - 4$
56. $\sqrt{x + 7} = -x^2 + 5$



57. **Group Activity Discriminant of a Quadratic** The radicand $b^2 - 4ac$ in the quadratic formula is called the **discriminant** of the quadratic polynomial $ax^2 + bx + c$ because it can be used to describe the nature of its zeros.

(a) **Writing to Learn** If $b^2 - 4ac > 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.

(b) **Writing to Learn** If $b^2 - 4ac = 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.

(c) **Writing to Learn** If $b^2 - 4ac < 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.

58. **Group Activity Discriminant of a Quadratic** Use the information learned in Exercise 57 to create a quadratic polynomial with the following numbers of real zeros. Support your answers graphically.

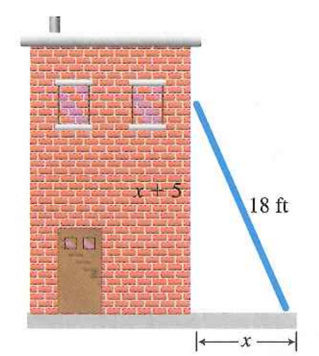
(a) Two real zeros?

(b) Exactly one real zero

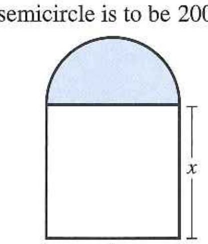
(c) No real zeros

59. **Size of a Soccer Field** Several of the World Cup '94 soccer matches were played in Stanford University's stadium in Menlo Park, California. The field is 30 yd longer than it is wide, and the area of the field is 8800 yd². What are the dimensions of this soccer field?

60. **Height of a Ladder** John's paint crew knows from experience that its 18-ft ladder is particularly stable when the distance from the ground to the top of the ladder is 5 ft more than the distance from the building to the base of the ladder as shown in the figure. In this position, how far up the building does the ladder reach?




61. **Finding the Dimensions of a Norman Window** A Norman window has the shape of a square with a semicircle mounted on it. Find the width of the window if the total area of the square and the semicircle is to be 200 ft².




### Standardized Test Questions

62. **True or False** If 2 is an x-intercept of the graph of $y = ax^2 + bx + c$, then 2 is a solution of the equation $ax^2 + bx + c = 0$. Justify your answer.

63. **True or False** If $2x^2 = 18$, then x must be equal to 3. Justify your answer.

In Exercises 64–67, you may use a graphing calculator to solve these problems.

64. **Multiple Choice** Which of the following are the solutions of the equation $x(x - 3) = 0$?
(A) Only $x = 3$
(B) Only $x = -3$
(C) $x = 0$ and $x = -3$
(D) $x = 0$ and $x = 3$
(E) There are no solutions.

65. **Multiple Choice** Which of the following replacements for ? make $x^2 - 5x + ?$ a perfect square?
(A) $-\frac{5}{2}$
(B) $\left(-\frac{5}{2}\right)^2$
(C) $(-5)^2$
(D) $\left(-\frac{2}{5}\right)^2$
(E) $-6$

66. **Multiple Choice** Which of the following are the solutions of the equation $2x^2 - 3x - 1 = 0$?
(A) $\frac{3}{4} \pm \sqrt{17}$
(B) $\frac{3 \pm \sqrt{17}}{4}$
(C) $\frac{3 \pm \sqrt{17}}{2}$
(D) $\frac{-3 \pm \sqrt{17}}{4}$
(E) $\frac{3 \pm 1}{4}$

67. **Multiple Choice** Which of the following are the solutions of the equation $|x - 1| = -3$?
(A) Only $x = 4$
(B) Only $x = -2$
(C) Only $x = 2$
(D) $x = 4$ and $x = -2$
(E) There are no solutions.

### Explorations

68. **Deriving the Quadratic Formula** Follow these steps to use completing the square to solve $ax^2 + bx + c = 0$, $a \neq 0$.

(a) Subtract c from both sides of the original equation and divide both sides of the resulting equation by a to obtain

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

**(b)** Add the square of one-half of the coefficient of $x$ in (a) to both sides and simplify to obtain

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

**(c)** Extract square roots in (b) and solve for $x$ to obtain the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

## Extending the Ideas

**69. Finding Number of Solutions** Consider the equation $|x^2 - 4| = c$.

**(a)** Find a value of $c$ for which this equation has four solutions. (There are many such values.)

**(b)** Find a value of $c$ for which this equation has three solutions. (There is only one such value.)

**(c)** Find a value of $c$ for which this equation has two solutions. (There are many such values.)

**(d)** Find a value of $c$ for which this equation has no solutions. (There are many such values.)

**(e) Writing to Learn** Are there any other possible numbers of solutions of this equation? Explain.

**70. Sums and Products of Solutions of $ax^2 + bx + c = 0, a \neq 0$** Suppose that $b^2 - 4ac > 0$.

**(a)** Show that the sum of the two solutions of this equation is $-b/a$.

**(b)** Show that the product of the two solutions of this equation is $c/a$.

**71. Exercise 70 Continued** The equation $2x^2 + bx + c = 0$ has two solutions $x_1$ and $x_2$. If $x_1 + x_2 = 5$ and $x_1 \cdot x_2 = 3$, find the two solutions.



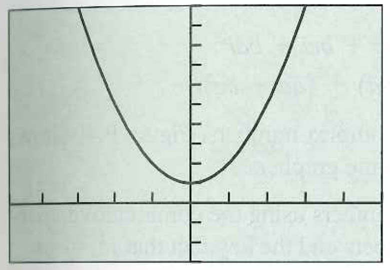
# P.6 Complex Numbers

### What you'll learn about

- Complex Numbers
- Operations with Complex Numbers
- Complex Conjugates and Division
- Complex Solutions of Quadratic Equations

### ... and why

The zeros of polynomials are complex numbers.



[–5, 5] by [–3, 10]

**FIGURE P.39** The graph of $f(x) = x^2 + 1$ has no $x$-intercepts.

### Historical Note

René Descartes (1596–1650) coined the term imaginary in a time when negative solutions to equations were considered *false*. Carl Friedrich Gauss (1777–1855) gave us the term complex number and the symbol $i$ for $\sqrt{-1}$. Today practical applications of complex numbers abound.

## Complex Numbers

Figure P.39 shows that the function $f(x) = x^2 + 1$ has no real zeros, so $x^2 + 1 = 0$ has no real-number solutions. To remedy this situation, mathematicians in the 17th century extended the definition of $\sqrt{a}$ to include negative real numbers $a$. First the number $i = \sqrt{-1}$ is defined as a solution of the equation $i^2 + 1 = 0$ and is the **imaginary unit**. Then for any negative real number $\sqrt{a} = \sqrt{|a|} \cdot i$.

The extended system of numbers, called the *complex numbers*, consists of all real numbers and sums of real numbers and real number multiples of $i$. The following are all examples of complex numbers:

$$-6,\quad 5i,\quad \sqrt{5},\quad -7i,\quad \frac{5}{2}i + \frac{2}{3},\quad -2 + 3i,\quad 5 - 3i,\quad \frac{1}{3} + \frac{4}{5}i$$

**DEFINITION Complex Number**

A **complex number** is any number that can be written in the form

$$a + bi,$$

where $a$ and $b$ are real numbers. The real number $a$ is the **real part**, the real number $b$ is the **imaginary part**, and $a + bi$ is the **standard form**.

A real number $a$ is the complex number $a + 0i$, so *all real numbers are also complex numbers*. If $a = 0$ and $b \neq 0$, then $a + bi$ becomes $bi$, and is an **imaginary number**. For instance, $5i$ and $-7i$ are imaginary numbers.

Two complex numbers are **equal** if and only if their real and imaginary parts are equal. For example,

$$x + yi = 2 + 5i \quad \text{if and only if} \quad x = 2 \text{ and } y = 5.$$

## Operations with Complex Numbers

Adding complex numbers is done by adding their real and imaginary parts separately. Subtracting complex numbers is also done using the same parts.

**DEFINITION Addition and Subtraction of Complex Numbers**

If $a + bi$ and $c + di$ are two complex numbers, then

**Sum:** $(a + bi) + (c + di) = (a + c) + (b + d)i,$

**Difference:** $(a + bi) - (c + di) = (a - c) + (b - d)i.$

**EXAMPLE 1** Adding and Subtracting Complex Numbers

**(a)** $(7 - 3i) + (4 + 5i) = (7 + 4) + (-3 + 5)i = 11 + 2i$

**(b)** $(2 - i) - (8 + 3i) = (2 - 8) + (-1 - 3)i = -6 - 4i$

Now try Exercise 3.

The **additive identity** for the complex numbers is $0 = 0 + 0i$. The **additive invers** of $a + bi$ is $-(a + bi) = -a - bi$ because

$$(a + bi) + (-a - bi) = 0 + 0i = 0.$$

Many of the properties of real numbers also hold for complex numbers. These include

- *Commutative* properties of addition and multiplication,
- *Associative* properties of addition and multiplication, and
- *Distributive* properties of multiplication over addition and subtraction.

Using these properties and the fact that $i^2 = -1$, complex numbers can be multiplie by treating them as algebraic expressions.

### EXAMPLE 2 Multiplying Complex Numbers

$$\begin{aligned}(2 + 3i)\cdot(5 - i) &= 2(5 - i) + 3i(5 - i) && \text{Distributive property}\\ &= 10 - 2i + 15i - 3i^2 && \text{Distributive property}\\ &= 10 + 13i - 3(-1) && i^2 = -1\\ &= 13 + 13i\end{aligned}$$

**Now try Exercise 9**

We can generalize Example 2 as follows:

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2\\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

Many graphers can perform basic calculations on complex numbers. Figure P.40 show how the operations of Examples 1 and 2 look on some graphers.

We compute positive integer powers of complex numbers using the commutative, asso ciative, and distributive properties of complex numbers and the key fact that $i^2 = -1$.

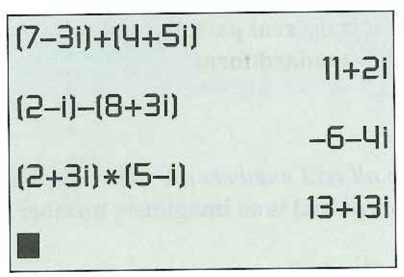



**FIGURE P.40** Complex number operations on a grapher. (Examples 1 and 2)

### EXAMPLE 3 Raising a Complex Number to a Power

If $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, find $z^2$ and $z^3$.

**SOLUTION**

$$\begin{aligned}z^2 &= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\\ &= \frac{1}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2\\ &= \frac{1}{4} + \frac{2\sqrt{3}}{4}i + \frac{3}{4}(-1)\\ &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i\end{aligned}$$

$$\begin{aligned}z^3 = z^2\cdot z &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\\ &= -\frac{1}{4} - \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2\\ &= -\frac{1}{4} + 0i + \frac{3}{4}(-1)\\ &= -1\end{aligned}$$

Figure P.41 supports these results numerically. **Now try Exercise 2**

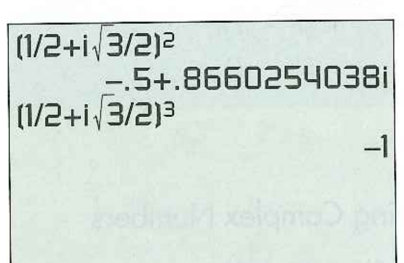



**FIGURE P.41** The square and cube of a complex number. (Example 3)



Example 3 demonstrates that $1/2 + (\sqrt{3}/2)i$ is a cube root of $-1$ and a solution of $x^3 + 1 = 0$. In Section 2.5, we will explore complex zeros of polynomial functions in depth. Section 6.6 delves into the geometry of complex numbers.

## Complex Conjugates and Division

The product of the complex numbers $a + bi$ and $a - bi$ is a positive real number:

$$(a + bi)\cdot(a - bi) = a^2 - (bi)^2 = a^2 + b^2$$

We introduce the following definition to describe this special relationship.

**DEFINITION Complex Conjugate**

The **complex conjugate** of the complex number $z = a + bi$ is

$$\bar{z} = \overline{a + bi} = a - bi.$$

The **multiplicative identity** for the complex numbers is $1 = 1 + 0i$. The **multiplicative inverse**, or **reciprocal**, of $z = a + bi$ is

$$z^{-1} = \frac{1}{z} = \frac{1}{a + bi} = \frac{1}{a + bi}\cdot\frac{a - bi}{a - bi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}i.$$

In general, a quotient of two complex numbers, written in fraction form, can be simplified as we just simplified $1/z$—by multiplying the numerator and denominator of the fraction by the complex conjugate of the denominator.

### EXAMPLE 4 Dividing Complex Numbers

Write the complex number in standard form.

(a) $\frac{2}{3 - i}$  (b) $\frac{5 + i}{2 - 3i}$

**SOLUTION** Multiply the numerator and denominator by the complex conjugate of the denominator.

(a)
$$\begin{aligned}\frac{2}{3 - i} &= \frac{2}{3 - i}\cdot\frac{3 + i}{3 + i}\\ &= \frac{6 + 2i}{3^2 + 1^2}\\ &= \frac{6}{10} + \frac{2}{10}i\\ &= \frac{3}{5} + \frac{1}{5}i\end{aligned}$$

(b)
$$\begin{aligned}\frac{5 + i}{2 - 3i} &= \frac{5 + i}{2 - 3i}\cdot\frac{2 + 3i}{2 + 3i}\\ &= \frac{10 + 15i + 2i + 3i^2}{2^2 + 3^2}\\ &= \frac{7 + 17i}{13}\\ &= \frac{7}{13} + \frac{17}{13}i\end{aligned}$$

**Now try Exercise 33.**

## Complex Solutions of Quadratic Equations

Recall that the solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$, are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The radicand $b^2 - 4ac$ is the **discriminant**, and tells us whether the solutions are real numbers. In particular, if $b^2 - 4ac < 0$, the solutions involve the square root of a negative number and thus lead to complex-number solutions. In all, there are thre cases, which we now summarize:

**Discriminant of a Quadratic Equation**

For a quadratic equation $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$,

- If $b^2 - 4ac > 0$, there are two distinct real solutions.
- If $b^2 - 4ac = 0$, there is one (repeated) real solution.
- If $b^2 - 4ac < 0$, there is a complex conjugate pair of solutions.

### EXAMPLE 5 Solving a Quadratic Equation

Solve $x^2 + x + 1 = 0$.

**SOLUTION**

**Solve Algebraically** Using the quadratic formula with $a = b = c = 1$, we obtain

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-(1) \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i.$$

So the solutions are $-1/2 + (\sqrt{3}/2)i$ and $-1/2 - (\sqrt{3}/2)i$, a complex conjugate pair.

**Confirm Numerically** Substituting $-1/2 + (\sqrt{3}/2)i$ into the original equation, we obtain

$$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + 1$$
$$= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + 1 = 0.$$

By a similar computation we can confirm the second solution.

Now try Exercise 41

## QUICK REVIEW P.6

In Exercises 1–4, add or subtract, and simplify.

1. $(2x + 3) + (-x + 6)$
2. $(3y - x) + (2x - y)$
3. $(2a + 4d) - (a + 2d)$
4. $(6z - 1) - (z + 3)$

In Exercises 5–10, multiply and simplify.

5. $(x - 3)(x + 2)$
6. $(2x - 1)(x + 3)$
7. $(x - \sqrt{2})(x + \sqrt{2})$
8. $(x + 2\sqrt{3})(x - 2\sqrt{3})$
9. $[x - (1 + \sqrt{2})][x - (1 - \sqrt{2})]$
10. $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$

## SECTION P.6 Exercises

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1–8, write the sum or difference in the standard form $a + bi$ without using a calculator.

1. $(2 - 3i) + (6 + 5i)$
2. $(2 - 3i) + (3 - 4i)$
3. $(7 - 3i) + (6 - i)$
4. $(2 + i) - (9i - 3)$
5. $(2 - i) + (3 - \sqrt{-3})$
6. $(\sqrt{5} - 3i) + (-2 + \sqrt{-9})$
7. $(i^2 + 3) - (7 + i^3)$
8. $(\sqrt{7} + i^2) - (6 - \sqrt{-81})$



In Exercises 9–16, write the product in standard form without using a calculator.

9. $(2 + 3i)(2 - i)$
10. $(2 - i)(1 + 3i)$
11. $(1 - 4i)(3 - 2i)$
12. $(5i - 3)(2i + 1)$
13. $(7i - 3)(2 + 6i)$
14. $(\sqrt{-4} + i)(6 - 5i)$
15. $(-3 - 4i)(1 + 2i)$
16. $(\sqrt{-2} + 2i)(6 + 5i)$

In Exercises 17–20, write the expression in the form $bi$, where $b$ is a real number.

17. $\sqrt{-16}$
18. $\sqrt{-25}$
19. $\sqrt{-3}$
20. $\sqrt{-5}$

In Exercises 21–24, find the real numbers $x$ and $y$ that make the equation true.

21. $2 + 3i = x + yi$
22. $3 + yi = x - 7i$
23. $(5 - 2i) - 7 = x - (3 + yi)$
24. $(x + 6i) = (3 - i) + (4 - 2yi)$

In Exercises 25–28, write the complex number in standard form.

25. $(3 + 2i)^2$
26. $(1 - i)^3$
27. $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4$
28. $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$

In Exercises 29–32, find the product of the complex number and its conjugate.

29. $2 - 3i$
30. $5 - 6i$
31. $-3 + 4i$
32. $-1 - \sqrt{2}i$

In Exercises 33–40, write the expression in standard form without using a calculator.

33. $\frac{1}{2 + i}$
34. $\frac{i}{2 - i}$
35. $\frac{2 + i}{2 - i}$
36. $\frac{2 + i}{3i}$
37. $\frac{(2 + i)^2(-i)}{1 + i}$
38. $\frac{(2 - i)(1 + 2i)}{5 + 2i}$
39. $\frac{(1 - i)(2 - i)}{1 - 2i}$
40. $\frac{(1 - \sqrt{2}i)(1 + i)}{(1 + \sqrt{2}i)}$

In Exercises 41–44, solve the equation.

41. $x^2 + 2x + 5 = 0$
42. $3x^2 + x + 2 = 0$
43. $4x^2 - 6x + 5 = x + 1$
44. $x^2 + x + 11 = 5x - 8$

## Standardized Test Questions

45. **True or False** There are no complex numbers $z$ satisfying $z = -\bar{z}$. Justify your answer.

46. **True or False** For the complex number $i$, $i + i^2 + i^3 + i^4 = 0$. Justify your answer.

In Exercises 47–50, solve the problem without using a calculator.

47. **Multiple Choice** Which of the following is the standard form for the product $(2 + 3i)(2 - 3i)$?
(A) $-5 + 12i$ (B) $4 - 9i$ (C) $13 - 3i$
(D) $-5$ (E) $13 + 0i$

48. **Multiple Choice** Which of the following is the standard form for the quotient $\frac{1}{i}$?
(A) 1 (B) $-1$ (C) $i$ (D) $-1/i$ (E) $0 - i$

49. **Multiple Choice** Assume that $2 - 3i$ is a solution of $ax^2 + bx + c = 0$, where $a, b, c$ are real numbers. Which of the following is also a solution of the equation?
(A) $2 + 3i$ (B) $-2 - 3i$ (C) $-2 + 3i$
(D) $3 + 2i$ (E) $\frac{1}{2 - 3i}$

50. **Multiple Choice** Which of the following is the standard form for the power $(1 - i)^3$?
(A) $-4i$ (B) $-2 + 2i$ (C) $-2 - 2i$ (D) $2 + 2i$ (E) $2 - 2i$

## Explorations

51. **Group Activity The Powers of *i***
(a) Simplify the complex numbers $i, i^2, \ldots, i^8$ by evaluating each one.
(b) Simplify the complex numbers $i^{-1}, i^{-2}, \ldots, i^{-8}$ by evaluating each one.
(c) Evaluate $i^0$.
(d) **Writing to Learn** Discuss your results from (a)–(c) with the members of your group, and write a summary statement about the integer powers of $i$.

52. **Writing to Learn** Describe the nature of the graph of $f(x) = ax^2 + bx + c$ when $a$, $b$, and $c$ are real numbers and the equation $ax^2 + bx + c = 0$ has nonreal complex solutions.

## Extending the Ideas

53. Prove that the difference between a complex number and its conjugate is a complex number whose real part is 0.
54. Prove that the product of a complex number and its complex conjugate is a complex number whose imaginary part is zero.
55. Prove that the complex conjugate of a product of two complex numbers is the product of their complex conjugates.
56. Prove that the complex conjugate of a sum of two complex numbers is the sum of their complex conjugates.
57. **Writing to Learn** Explain why $-i$ is a solution of $x^2 - ix + 2 = 0$ but $i$ is not.

# P.7 Solving Inequalities Algebraically and Graphically

### What you'll learn about

- Solving Absolute Value Inequalities
- Solving Quadratic Inequalities
- Approximating Solutions to Inequalities
- Projectile Motion

### ... and why

These techniques are involved in using a graphing utility to solve inequalities in this textbook.

## Solving Absolute Value Inequalities

We now extend the methods for solving inequalities introduced in Section P.3. We star with two basic rules for solving absolute value inequalities.

**Solving Absolute Value Inequalities**

Let $u$ be an algebraic expression in $x$, and let $a$ be a real number with $a \geq 0$.

**1.** If $|u| < a$, then $u$ is in the interval $(-a, a)$. That is,

$$|u| < a \quad \text{if and only if} \quad -a < u < a.$$

**2.** If $|u| > a$, then $u$ is in the interval $(-\infty, -a)$ or $(a, \infty)$, that is,

$$|u| > a \quad \text{if and only if} \quad u < -a \text{ or } u > a.$$

The inequalities $<$ and $>$ can be replaced with $\leq$ and $\geq$, respectively. See Figure P.42.

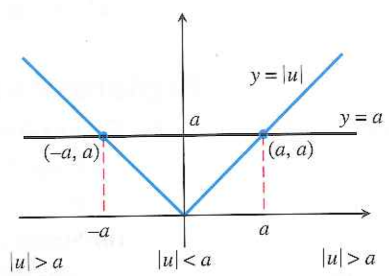



**FIGURE P.42** The solution of $|u| < a$ is represented by the portion of the number line for which the graph of $y = |u|$ is below the graph of $y = a$. The solution of $|u| > a$ is represented by the portion of the number line where the graph of $y = |u|$ is above the graph of $y = a$.

**EXAMPLE 1** Solving an Absolute Value Inequality

Solve $|x - 4| < 8$.

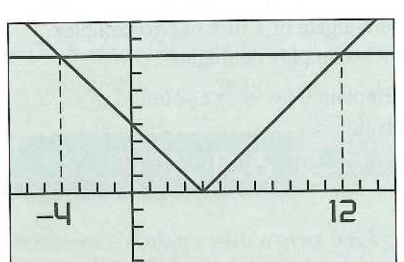
**SOLUTION**

$$\begin{aligned} |x - 4| &< 8 && \text{Original inequality} \\ -8 < x - 4 &< 8 && \text{Equivalent double inequality} \\ -4 < x &< 12 && \text{Add 4.} \end{aligned}$$

As an interval the solution is $(-4, 12)$.

Figure P.43 shows that points on the graph of $y = |x - 4|$ are below the points on the graph of $y = 8$ for values of $x$ between $-4$ and 12.

*Now try Exercise 3.*




[−7, 15] by [−5, 10]

**FIGURE P.43** The graphs of $y = |x - 4|$ and $y = 8$. (Example 1)

**EXAMPLE 2** Solving Another Absolute Value Inequality

Solve $|3x - 2| \geq 5$.

**SOLUTION** The solution of this absolute value inequality consists of the solutions of both of these inequalities.

$$\begin{aligned} 3x - 2 \leq -5 \quad &\text{or} \quad 3x - 2 \geq 5 \\ 3x \leq -3 \quad &\text{or} \quad 3x \geq 7 && \text{Add 2.} \\ x \leq -1 \quad &\text{or} \quad x \geq \frac{7}{3} && \text{Divide by 3.} \end{aligned}$$



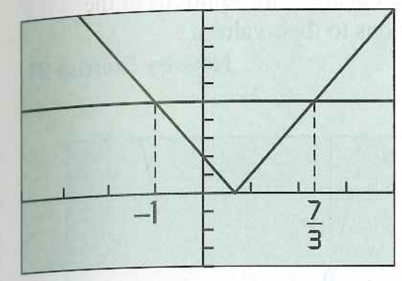
The solution consists of all numbers that are in either one of the two intervals $(-\infty, -1]$ or $[7/3, \infty)$, which may be written as $(-\infty, -1] \cup [7/3, \infty)$. The notation "$\cup$" is read as "union." (See the margin note.)

Figure P.44 shows that points on the graph of $y = |3x - 2|$ are above or on the points on the graph of $y = 5$ for values of $x$ to the left of and including $-1$ and to the right of and including 7/3.

*Now try Exercise 7.*

**Union of Two Sets**

The **union of two sets $A$ and $B$**, denoted by $A \cup B$, is the set of all objects that belong to $A$ or $B$ or both.




[−4, 4] by [−4, 10]

**FIGURE P.44** The graphs of $y = |3x - 2|$ and $y = 5$. (Example 2)

## Solving Quadratic Inequalities

To solve a quadratic inequality such as $x^2 - x - 12 > 0$ we begin by solving the corresponding quadratic equation $x^2 - x - 12 = 0$. Then we determine the values of $x$ for which the graph of $y = x^2 - x - 12$ lies above the $x$-axis.

**EXAMPLE 3** Solving a Quadratic Inequality

Solve $x^2 - x - 12 > 0$.

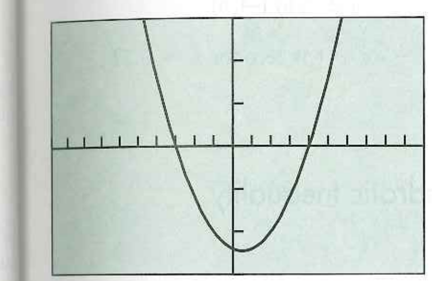
**SOLUTION** First we solve the corresponding equation $x^2 - x - 12 = 0$.

$$\begin{aligned} x^2 - x - 12 &= 0 \\ (x - 4)(x + 3) &= 0 && \text{Factor.} \\ x - 4 = 0 \quad \text{or} \quad x + 3 &= 0 && ab = 0 \Rightarrow a = 0 \text{ or } b = 0 \\ x = 4 \quad \text{or} \quad x &= -3 && \text{Solve for } x. \end{aligned}$$

The solutions of the corresponding quadratic equation are $-3$ and 4, and they are not solutions of the original inequality because $0 > 0$ is false. Figure P.45 shows that the points on the graph of $y = x^2 - x - 12$ are above the $x$-axis for values of $x$ to the left of $-3$ and to the right of 4.

The solution of the original inequality is $(-\infty, -3) \cup (4, \infty)$.

*Now try Exercise 11.*



[−10, 10] by [−15, 15]

**FIGURE P.45** The graph of $y = x^2 - x - 12$ appears to cross the $x$-axis at $x = -3$ and $x = 4$. (Example 3)

In Example 4, the quadratic inequality involves the symbol $\leq$. In this case, the solutions of the corresponding quadratic equation are also solutions of the inequality.

**EXAMPLE 4** Solving Another Quadratic Inequality

Solve $2x^2 + 3x \leq 20$.

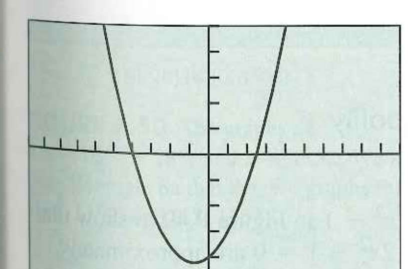
**SOLUTION** First we subtract 20 from both sides of the inequality to obtain $2x^2 + 3x - 20 \leq 0$. Next, we solve the corresponding quadratic equation $2x^2 + 3x - 20 = 0$.

$$\begin{aligned} 2x^2 + 3x - 20 &= 0 \\ (x + 4)(2x - 5) &= 0 && \text{Factor.} \\ x + 4 = 0 \quad \text{or} \quad 2x - 5 &= 0 && ab = 0 \Rightarrow a = 0 \text{ or } b = 0 \\ x = -4 \quad \text{or} \quad x &= \frac{5}{2} && \text{Solve for } x. \end{aligned}$$

The solutions of the corresponding quadratic equation are $-4$ and $5/2 = 2.5$. You can check that they are also solutions of the inequality.

Figure P.46 shows that the points on the graph of $y = 2x^2 + 3x - 20$ are below the $x$-axis for values of $x$ between $-4$ and 2.5. The solution of the original inequality is $[-4, 2.5]$. We use square brackets because the endpoints $-4$ and 2.5 are solutions of the inequality.

*Now try Exercise 9.*



[−10, 10] by [−25, 25]

**FIGURE P.46** The graph of $y = 2x^2 + 3x - 20$ appears to be below the $x$-axis for $-4 < x < 2.5$. (Example 4)

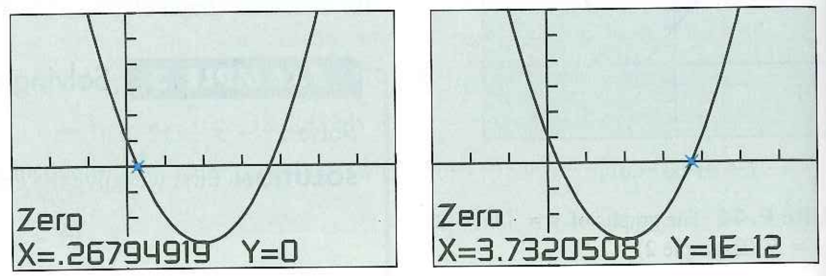
In Examples 3 and 4 the corresponding quadratic equation factored. If this doesn't happen we will need to approximate the zeros of the quadratic equation if it has any. Then we use our accuracy agreement from Section P.5 and write the endpoints of any intervals accurate to two decimal places as illustrated in Example 5.

## EXAMPLE 5 Solving a Quadratic Inequality Graphically

Solve $x^2 - 4x + 1 \geq 0$ graphically.

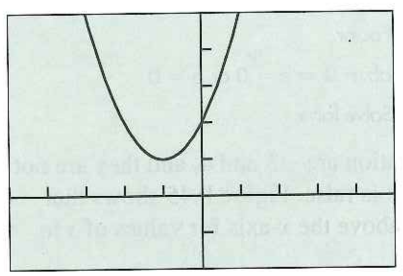
**SOLUTION** We can use the graph of $y = x^2 - 4x + 1$ in Figure P.47 to determine that the solutions of the equation $x^2 - 4x + 1 = 0$ are about 0.27 and 3.73. Thus, the solution of the original inequality is roughly $(-\infty, 0.27] \cup [3.73, \infty)$. We use square brackets because the zeros of the quadratic equation are solutions of the inequality even though we only have approximations to their values.

Now try Exercise 21




[−3, 7] by [−4, 6]    [−3, 7] by [−4, 6]

**FIGURE P.47** This figure suggests that $y = x^2 - 4x + 1$ is zero for $x \approx 0.27$ and $x \approx 3.73$. (Example 5)



[−5, 5] by [−2, 5]

**FIGURE P.48** The values of $y = x^2 + 2x + 2$ are never negative. (Example 6)

## EXAMPLE 6 Showing That a Quadratic Inequality Has No Solution

Solve $x^2 + 2x + 2 < 0$.

**SOLUTION** Figure P.48 shows that the graph of $y = x^2 + 2x + 2$ lies above the $x$-axis for all values for $x$. Thus, the inequality $x^2 + 2x + 2 < 0$ has *no* solution.

Now try Exercise 25

Figure P.48 also shows that the solution of the inequality $x^2 + 2x + 2 > 0$ is the set of all real numbers or, in interval notation, $(-\infty, \infty)$. A quadratic inequality can also have exactly one solution (see Exercise 31).

## Approximating Solutions to Inequalities

To solve the inequality in Example 7 we approximate the zeros of the corresponding graph. Then we determine the values of $x$ for which the corresponding graph is above or on the $x$-axis.

## EXAMPLE 7 Solving a Cubic Inequality

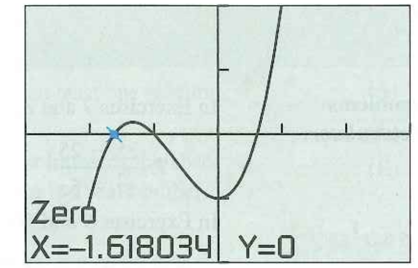
Solve $x^3 + 2x^2 - 1 \geq 0$ graphically.

**SOLUTION** We can use the graph of $y = x^3 + 2x^2 - 1$ in Figure P.49 to show that the solutions of the corresponding equation $x^3 + 2x^2 - 1 = 0$ are approximately $-1.62, -1$, and 0.62. The points on the graph of $y = x^3 + 2x^2 - 1$ are above the $x$-axis for values of $x$ between $-1.62$ and $-1$, and for values of $x$ to the right of 0.62.

The solution of the inequality is $[-1.62, -1] \cup [0.62, \infty)$. We use square brackets because the zeros of $y = x^3 + 2x^2 - 1$ are solutions of the inequality.

Now try Exercise 27






[−3, 3] by [−2, 2]

**FIGURE P.49** The graph of $y = x^3 + 2x^2 - 1$ appears to be above the $x$-axis between the two negative $x$-intercepts and to the right of the positive $x$-intercept. (Example 7)

## Projectile Motion

The movement of an object that is propelled vertically, but then subject only to the force of gravity, is an example of **projectile motion**.

### Projectile Motion

Suppose an object is launched vertically from a point $s_0$ feet above the ground with an initial velocity of $v_0$ feet per second. The vertical position $s$ (in feet) of the object $t$ seconds after it is launched is

$$s = -16t^2 + v_0t + s_0.$$

## EXAMPLE 8 Finding the Height of a Projectile

A projectile is launched straight up from ground level with an initial velocity of 288 ft/sec.

(a) When will the projectile's height above ground be 1152 ft?

(b) When will the projectile's height above ground be at least 1152 ft?

**SOLUTION** Here $s_0 = 0$ and $v_0 = 288$. So, the projectile's height is $s = -16t^2 + 288t$.

(a) We need to determine when $s = 1152$.

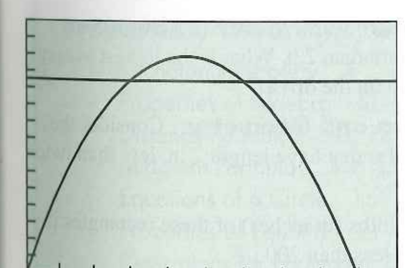
$$\begin{aligned} s &= -16t^2 + 288t \\ 1152 &= -16t^2 + 288t && \text{Substitute } s = 1152. \\ 16t^2 - 288t + 1152 &= 0 && \text{Add } 16t^2 - 288t. \\ t^2 - 18t + 72 &= 0 && \text{Divide by 16.} \\ (t - 6)(t - 12) &= 0 && \text{Factor.} \\ t = 6 \quad \text{or} \quad t &= 12 && \text{Solve for } t. \end{aligned}$$

The projectile is 1152 ft above ground twice; the first time at $t = 6$ sec on the way up, and the second time at $t = 12$ sec on the way down (Figure P.50).

(b) The projectile will be at least 1152 ft above ground when $s \geq 1152$. We can see from Figure P.50 together with the algebraic work in (a) that the solution is $[6, 12]$. This means that the projectile is at least 1152 ft above ground for times between $t = 6$ sec and $t = 12$ sec, including 6 and 12 sec.

In Exercise 32 we ask you to use algebra to solve the inequality $s = -16t^2 + 288t \geq 1152$.

Now try Exercise 33.



[0, 20] by [0, 1500]

**FIGURE P.50** The graphs of $s = -16t^2 + 288t$ and $s = 1152$. We know from Example 8a that the two graphs intersect at (6, 1152) and (12, 1152).

## QUICK REVIEW P.7

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.**

In Exercises 1–3, solve for $x$.

1. $-7 < 2x - 3 < 7$
2. $5x - 2 \ge 7x + 4$
3. $|x + 2| = 3$

In Exercises 4–6, factor the expression completely.

4. $4x^2 - 9$
5. $x^3 - 4x$
6. $9x^2 - 16y^2$

In Exercises 7 and 8, reduce the fraction to lowest terms.

7. $\dfrac{z^2 - 25}{z^2 - 5z}$
8. $\dfrac{x^2 + 2x - 35}{x^2 - 10x + 25}$

In Exercises 9 and 10, add the fractions and simplify.

9. $\dfrac{x}{x - 1} + \dfrac{x + 1}{3x - 4}$
10. $\dfrac{2x - 1}{x^2 - x - 2} + \dfrac{x - 3}{x^2 - 3x + 2}$

## SECTION P.7 Exercises

In Exercises 1–8, solve the inequality algebraically. Write the solution in interval notation and draw its number line graph.

1. $|x + 4| \ge 5$
2. $|2x - 1| > 3.6$
3. $|x - 3| < 2$
4. $|x + 3| \le 5$
5. $|4 - 3x| - 2 < 4$
6. $|3 - 2x| + 2 > 5$
7. $\left|\dfrac{x + 2}{3}\right| \ge 3$
8. $\left|\dfrac{x - 5}{4}\right| \le 6$

In Exercises 9–16, solve the inequality. Use algebra to solve the corresponding equation.

9. $2x^2 + 17x + 21 \le 0$
10. $6x^2 - 13x + 6 \ge 0$
11. $2x^2 + 7x > 15$
12. $4x^2 + 2 < 9x$
13. $2 - 5x - 3x^2 < 0$
14. $21 + 4x - x^2 > 0$
15. $x^3 - x \ge 0$
16. $x^3 - x^2 - 30x \le 0$

In Exercises 17–26, solve the inequality graphically.

17. $x^2 - 4x < 1$
18. $12x^2 - 25x + 12 \ge 0$
19. $6x^2 - 5x - 4 > 0$
20. $4x^2 - 1 \le 0$
21. $9x^2 + 12x - 1 \ge 0$
22. $4x^2 - 12x + 7 < 0$
23. $4x^2 + 1 > 4x$
24. $x^2 + 9 \le 6x$
25. $x^2 - 8x + 16 < 0$
26. $9x^2 + 12x + 4 \ge 0$

In Exercises 27–30, solve the cubic inequality graphically.

27. $3x^3 - 12x + 2 \ge 0$
28. $8x - 2x^3 - 1 < 0$
29. $2x^3 + 2x > 5$
30. $4 \le 2x^3 + 8x$

31. **Group Activity** Give an example of a quadratic inequality with the indicated solution.
    (a) All real numbers
    (b) No solution
    (c) Exactly one solution
    (d) $[-2, 5]$
    (e) $(-\infty, -1) \cup (4, \infty)$
    (f) $(-\infty, 0] \cup [4, \infty)$

32. **Revisiting Example 8** Solve the inequality $-16t^2 + 288t \ge 1152$ algebraically and compare your answer with the result obtained in Example 10.

33. **Projectile Motion** A projectile is launched straight up from ground level with an initial velocity of 256 ft/sec.
    (a) When will the projectile's height above ground be 768 ft?
    (b) When will the projectile's height above ground be at least 768 ft?
    (c) When will the projectile's height above ground be less than or equal to 768 ft?

34. **Projectile Motion** A projectile is launched straight up from ground level with an initial velocity of 272 ft/sec.
    (a) When will the projectile's height above ground be 960 ft?
    (b) When will the projectile's height above ground be more than 960 ft?
    (c) When will the projectile's height above ground be less than or equal to 960 ft?

35. **Writing to Learn** Explain the role of equation solving in the process of solving an inequality. Give an example.

36. **Travel Planning** Barb wants to drive to a city 105 mi from her home in no more than 2 h. What is the lowest average speed she must maintain on the drive?

37. **Connecting Algebra and Geometry** Consider the collection of all rectangles that have length 2 in. less than twice their width.
    (a) Find the possible widths (in inches) of these rectangles if their perimeters are less than 200 in.
    (b) Find the possible widths (in inches) of these rectangles if their areas are less than or equal to 1200 in.$^2$.

38. **Boyle's Law** For a certain gas, $P = 400/V$, where $P$ is pressure and $V$ is volume. If $20 \le V \le 40$, what is the corresponding range for $P$?

39. **Cash-Flow Planning** A company has current assets (cash, property, inventory, and accounts receivable) of \$200,000 and current liabilities (taxes, loans, and accounts payable) of \$50,000. How much can it borrow if it wants its ratio of assets to liabilities to be no less than 2? Assume the amount borrowed is added to both current assets and current liabilities.



## Standardized Test Questions

40. **True or False** The absolute value inequality $|x - a| < b$, where $a$ and $b$ are real numbers, always has at least one solution. Justify your answer.

41. **True or False** Every real number is a solution of the absolute value inequality $|x - a| \ge 0$, where $a$ is a real number. Justify your answer.

In Exercises 42–45, solve these problems without using a calculator.

42. **Multiple Choice** Which of the following is the solution to $|x - 2| < 3$?
    (A) $x = -1$ or $x = 5$
    (B) $[-1, 5)$
    (C) $[-1, 5]$
    (D) $(-\infty, -1) \cup (5, \infty)$
    (E) $(-1, 5)$

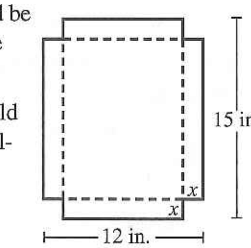
43. **Multiple Choice** Which of the following is the solution to $x^2 - 2x + 2 \ge 0$?
    (A) $[0, 2]$
    (B) $(-\infty, 0) \cup (2, \infty)$
    (C) $(-\infty, 0] \cup [2, \infty)$
    (D) All real numbers
    (E) There is no solution.

44. **Multiple Choice** Which of the following is the solution to $x^2 > x$?
    (A) $(-\infty, 0) \cup (1, \infty)$
    (B) $(-\infty, 0] \cup [1, \infty))$
    (C) $(1, \infty)$
    (D) $(0, \infty)$
    (E) There is no solution.

45. **Multiple Choice** Which of the following is the solution to $x^2 \le 1$?
    (A) $(-\infty, 1]$
    (B) $(-1, 1)$
    (C) $[1, \infty)$
    (D) $[-1, 1]$
    (E) There is no solution.

## Explorations

46. **Constructing a Box with No Top** An open box is formed by cutting squares from the corners of a regular piece of cardboard (see figure) and folding up the flaps.
    (a) What size corner squares should be cut to yield a box with a volume of 125 in.$^3$?
    (b) What size corner squares should be cut to yield a box with a volume more than 125 in.$^3$?
    (c) What size corner squares should be cut to yield a box with a volume of at most 125 in.$^3$?




## Extending the Ideas

In Exercises 47 and 48, use a combination of algebraic and graphical techniques to solve the inequalities.

47. $|2x^2 + 7x - 15| < 10$
48. $|2x^2 + 3x - 20| \ge 10$

## CHAPTER P Key Ideas

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