Algebra 2
Chapter 7 - Chapter Test Solutions

1. $y = 3(0.25)^x$
exponential decay
y-intercept: $(0, 3)$

2. $y = 2(6)^{-x}$
$y = 2\left(\frac{1}{6}\right)^x$
exponential decay
y-intercept: $(0, 2)$

3. $y = 0.1(10)^x$
exponential growth
y-intercept: $(0, 0.1)$

4. $y = 3e^x$
exponential growth
y-intercept: $(0, 3)$

5. $y = 3^x + 2$
The parent graph $y = 3^x$ is translated 2 units up.
domain: all real numbers
range: $y > 2$
asymptote: $y = 2$

6. $y = \left(\frac{1}{2}\right)^{x+1}$
The parent graph $y = \left(\frac{1}{2}\right)^x$ is translated 1 unit to the left.
domain: all real numbers
range: $y > 0$
asymptote: $y = 0$

7. $y = -(2)^{x+2}$
The parent graph $y = 2^x$ is translated 2 units to the left and reflected across the x-axis.
domain: all real numbers
range: $y < 0$
asymptote: $y = 0$

8. $5^4 = 625$
$\log_5 625 = 4$

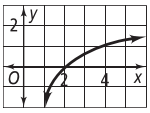
9. $e^0 = 1$
$\log_e 1 = 0$

10. $\log_2 8 = x$
$2^x = 8$
$2^x = 2^3$
$x = 3$

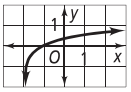
11. $\log_7 7 = x$
$7^x = 7$
$x = 1$

12. $y = \log_5 \frac{1}{125}$
$5^y = \frac{1}{125}$
$y = -3$

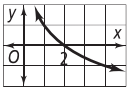
13. $\log_{11} 1 = x$
$11^x = 1$
$11^x = 11^0$
$x = 0$

14. $y = \log_3(x - 1)$


The parent graph $y = \log_3 x$ is translated 1 unit to the right.
domain: $x > 1$
range: all real numbers
no y-intercept
asymptote: $x = 1$

15. $y = \frac{1}{2}\log_3(x + 2)$


The parent graph $y = \log_3 x$ is translated 2 units to the left and compressed by a factor of $\frac{1}{2}$.
domain: $x > -2$
range: all real numbers
y-intercept: $\left(0, \frac{1}{2}\log_3 2\right) \approx (0, 0.03155)$
asymptote: $x = -2$

16. $y = 1 - \log_2 x$


The parent graph $y = \log_2 x$ is reflected across the x-axis and translated 1 unit up.
domain: $x > 0$
range: all real numbers
no y-intercept
asymptote: $x = 0$

17. $\log_2 4 + 3\log_2 9 = \log_2 4 + \log_2 9^3$
$= \log_2 (4 \cdot 9^3)$
$= \log_2 2916$

18. $3 \log a - 2 \log b = \log a^3 - \log b^2$
$= \log \frac{a^3}{b^2}$

19. $\log_7 \frac{a}{b} = \log_7 a - \log_7 b$

20. $\log 3x^3y^2 = \log 3 + \log x^3 + \log y^2$
$= \log 3 + 3 \log x + 2 \log y$

21. $\log_9 27 - \log_9 9 = \log_9 \frac{27}{9}$
$x = \log_9 3$
$9^x = 3$
$x = \frac{1}{2}$

22. $2 \log 5 + \log 40 = \log 5^2 + \log 40$
$= \log \left(5^2 \cdot 40\right)$
$x = \log 1000$
$10^x = 1000$
$10^x = 10^3$
$x = 3$

23. $(27)^{3x} = 81$
$\left(3^3\right)^{3x} = 3^4$
$3^{9x} = 3^4$
$9x = 4$
$x = \frac{4}{9} = 0.\overline{44}$

24. $3^{x-1} = 24$
$\log 3^{x-1} = \log 24$
$(x-1)\log 3 = \log 24$
$x - 1 = \frac{\log 24}{\log 3}$
$x = \frac{\log 24}{\log 3} + 1$
$x \approx 3.89$

25. $2e^{3x} = 16$
$e^{3x} = 8$
$\ln e^{3x} = \ln 8$
$3x = \ln 8$
$x = \frac{\ln 8}{3}$
$x \approx 0.69$

26. $2 \log x = -4$
$\log x = -2$
$x = 10^{-2}$
$x = 0.01$

27. $\log_3 16 = \frac{\log 16}{\log 3}$

28. $\log_2 10 = \frac{\log 10}{\log 2} = \frac{1}{\log 2}$

29. $\log_7 8 = \frac{\log 8}{\log 7}$

30. $\log_4 9 = \frac{\log 9}{\log 4}$

31. $\ln 2 + \ln x = 1$
$\ln x = 1 - \ln 2$
$x = e^{1-\ln 2}$
$x \approx 1.359$

32. $\ln (x+1) + \ln (x-1) = 4$
$\ln (x+1)(x-1) = 4$
$\ln \left(x^2 - 1\right) = 4$
$x^2 - 1 = e^4$
$x^2 = e^4 + 1$
$x = \sqrt{e^4 + 1}$
$x \approx 7.456$

33. $\ln (2x-1)^2 = 7$
$(2x-1)^2 = e^7$
$(2x-1)^2 \approx 1096.63$
$2x - 1 \approx \pm\sqrt{1096.63}$
$2x - 1 \approx \pm 33.115$
$2x - 1 \approx 33.115$ or $2x - 1 \approx -33.115$
$2x \approx 34.115$ or $2x \approx -32.115$
$x \approx 17.058$ or $x \approx -16.058$

34. $3 \ln x - \ln 2 = 4$
$3 \ln x = 4 + \ln 2$
$\ln x = \frac{1}{3}(4 + \ln 2)$
$x = e^{\frac{1}{3}(4+\ln 2)}$
$x \approx 4.780$

35. Taking common logarithms of both sides gives $2x = \dfrac{\log\ 4}{\log\ 3}$.

Taking logarithms to the base 3 of both sides gives $2x = \log_3\ 4$, which by the Change of Base Formula is equivalent to $2x = \dfrac{\log\ 4}{\log\ 3}$.

$$\begin{aligned} 3^{2x} &= 4 \\ \log\ 3^{2x} &= \log\ 4 \\ 2x\ \log\ 3 &= \log\ 4 \\ x &= \frac{\log\ 4}{2\ \log\ 3} \\ x &\approx 0.6309 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 3^{2x} &= 4 \\ \log_3\ 3^{2x} &= \log_3\ 4 \\ 2x &= \log_3\ 4 \\ x &= \frac{1}{2}\log_3\ 4 \\ x &\approx 0.6309 \end{aligned}$$

36. Answers may vary. Sample:

exponential growth: $y = \dfrac{1}{3}(2)^x$

exponential decay: $y = 7\left(\dfrac{1}{5}\right)^x$

Both answers should be of the form $y = ab^x$.

For the growth model, $b > 1$. For the decay model, $0 < b < 1$.

37.
$$\begin{aligned} A &= Pe^{rt} \\ 2000 &= 1500e^{0.07t} \\ \frac{4}{3} &= e^{0.07t} \\ \ln\ \frac{4}{3} &= \ln\ e^{0.07t} \\ 0.07t &= \ln\ \frac{4}{3} \\ t &= \frac{1}{0.07} \cdot \ln\ \frac{4}{3} \\ t &\approx 4.11 \end{aligned}$$

It will be about 4.11 years before there is \$2000 in the account.