# Algebra 2
## Chapter 6 - Chapter Review Solutions

1. The number under a radical sign is called the radicand.

3. A radical expression can always be written using a rational exponent.

5. $\sqrt{25} = \sqrt{5^2}$
$= \pm 5$

7. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3}$
$= -2$

9. $\sqrt{81x^2} = \sqrt{9^2x^2}$
$= \sqrt{(9x)^2}$
$= 9|x|$

11. $\sqrt[4]{16x^{12}} = \sqrt[4]{2^4(x^3)^4}$
$= \sqrt[4]{(2x^3)^4}$
$= 2|x^3|$

13. $\sqrt{\dfrac{9x^4}{36}} = \sqrt{\dfrac{1}{4}x^4}$
$= \sqrt{\left(\dfrac{1}{2}x^2\right)^2}$
$= \dfrac{x^2}{2}$

15. $\sqrt[3]{9} \cdot \sqrt[3]{3} = \sqrt[3]{9 \cdot 3}$
$= \sqrt[3]{27}$
$= \sqrt[3]{3^3}$
$= 3$

17. $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$
$= \sqrt{16}$
$= \sqrt{4^2}$
$= 4$

19. $5\sqrt[3]{9y^2} \cdot \sqrt[3]{24y} = 5\sqrt[3]{9 \cdot 24 \cdot y^2 \cdot y}$
$= 5\sqrt[3]{216y^3}$
$= 5\sqrt[3]{(6y)^3}$
$= 5 \cdot 6y = 30y$

21. $\dfrac{\sqrt[3]{81x^5y^3}}{\sqrt[3]{3x^2}} = \sqrt[3]{\dfrac{81x^5y^3}{3x^2}}$
$= \sqrt[3]{27x^3y^3}$
$= \sqrt[3]{(3xy)^3}$
$= 3xy$

23. $\dfrac{\sqrt{8}}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{48}}{6}$
$= \dfrac{4\sqrt{3}}{6}$
$= \dfrac{2\sqrt{3}}{3}$

25. $\dfrac{\sqrt[3]{6x^2y^4}}{2\sqrt[3]{5x^7y}} = \dfrac{1}{2}\sqrt[3]{\dfrac{6y^3}{5x^5}}$
$= \dfrac{y\sqrt[3]{6}}{2x\sqrt[3]{5x^2}} \cdot \dfrac{\sqrt[3]{25x}}{\sqrt[3]{25x}}$
$= \dfrac{y\sqrt[3]{150x}}{10x^2}$

27. $3\sqrt{20x} + 8\sqrt{45x} - 4\sqrt{5x} = 3\sqrt{4 \cdot 5x} + 8\sqrt{9 \cdot 5x} - 4\sqrt{5x}$
$= 3\sqrt{2^2}\sqrt{5x} + 8\sqrt{3^2}\sqrt{5x} - 4\sqrt{5x}$
$= 3 \cdot 2\sqrt{5x} + 8 \cdot 3\sqrt{5x} - 4\sqrt{5x}$
$= 6\sqrt{5x} + 24\sqrt{5x} - 4\sqrt{5x}$
$= (6 + 24 - 4)\sqrt{5x}$
$= 26\sqrt{5x}$

29. $\left(3+\sqrt{2}\right)\left(4+\sqrt{2}\right) = 3 \cdot 4 + 3\sqrt{2} + 4\sqrt{2} + \left(\sqrt{2}\right)^2$
$= 12 + (3+4)\sqrt{2} + 2$
$= 14 + 7\sqrt{2}$

31. $\left(10+\sqrt{6}\right)\left(10-\sqrt{3}\right) = 10^2 + 10\sqrt{6} - 10\sqrt{3} - \sqrt{6}\sqrt{3}$
$= 100 + 10\sqrt{6} - 10\sqrt{3} - \sqrt{18}$
$= 100 + 10\sqrt{6} - 10\sqrt{3} - \sqrt{9}\sqrt{2}$
$= 100 + 10\sqrt{6} - 10\sqrt{3} - 3\sqrt{2}$

33.

$$\frac{3+\sqrt{18}}{1+\sqrt{8}} = \frac{\left(3+\sqrt{18}\right)}{\left(1+\sqrt{8}\right)}\cdot\frac{\left(1-\sqrt{8}\right)}{\left(1-\sqrt{8}\right)}$$
$$= \frac{3-3\sqrt{8}+\sqrt{18}-\sqrt{144}}{1-\sqrt{64}}$$
$$= \frac{3-6\sqrt{2}+3\sqrt{2}-12}{1-8}$$
$$= \frac{-9-3\sqrt{2}}{-7}$$
$$= \frac{-\left(9+3\sqrt{2}\right)}{-7}$$
$$= \frac{9+3\sqrt{2}}{7}$$

35.

$$81^{\frac{1}{4}} = \sqrt[4]{81}$$
$$= \sqrt[4]{3^4}$$
$$= 3$$

37.

$$5^{\frac{3}{2}}\cdot 5^{\frac{1}{2}} = 5^{\frac{3}{2}+\frac{1}{2}}$$
$$= 5^{\frac{4}{2}}$$
$$= 5^2$$
$$= 25$$

39.

$$\left(-8y^9\right)^{\frac{1}{3}} = (-8)^{\frac{1}{3}}y^{9\cdot\frac{1}{3}}$$
$$= \sqrt[3]{-8}y^3$$
$$= \sqrt[3]{(-2)^3}y^3$$
$$= -2y^3$$

41.

$$\left(x^{\frac{1}{6}}y^{\frac{1}{3}}\right)^{-18} = x^{\frac{1}{6}(-18)}y^{\frac{1}{3}(-18)}$$
$$= x^{-3}y^{-6}$$
$$= \frac{1}{x^3y^6}$$

43.

$$\left(\frac{x^{\frac{1}{3}}}{y^{-\frac{2}{3}}}\right)^9 = \frac{x^{\frac{1}{3}(9)}}{y^{-\frac{2}{3}(9)}}$$
$$= \frac{x^3}{y^{-6}}$$
$$= x^3y^6$$

45.

$$3\sqrt{2x+6} = 18$$
$$\sqrt{2x+6} = 6$$
$$\left(\sqrt{2x+6}\right)^2 = 6^2$$
$$2x+6 = 36$$
$$2x = 30$$
$$x = 15$$

Check:

$$3\sqrt{2x+6} \stackrel{?}{=} 18$$
$$3\sqrt{2(15)+6} \stackrel{?}{=} 18$$
$$3\sqrt{36} \stackrel{?}{=} 18$$
$$3\cdot 6 \stackrel{?}{=} 18$$
$$18 = 18$$

The solution is $x = 15$.

47.

$$4(3x-3)^{\frac{2}{3}} = 36$$
$$(3x-3)^{\frac{2}{3}} = 9$$

$$\left[(3x-3)^{\frac{2}{3}}\right]^{\frac{3}{2}} = 9^{\frac{3}{2}} \quad \text{or} \quad \left[(3x-3)^{\frac{2}{3}}\right]^{\frac{3}{2}} = -9^{\frac{3}{2}}$$

$$3x-3 = \left(\sqrt{9}\right)^3 \qquad 3x-3 = -\left(\sqrt{9}\right)^3$$
$$3x-3 = 3^3 \qquad 3x-3 = -3^3$$
$$3x-3 = 27 \qquad 3x-3 = -27$$
$$3x = 30 \qquad 3x = -24$$
$$x = 10 \qquad x = -8$$

Check:

$$4(3x-3)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(3\cdot 10-3)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(27)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(\sqrt[3]{27})^2 \stackrel{?}{=} 36$$
$$4\cdot 9 \stackrel{?}{=} 36$$
$$36 = 36$$

$$4(3x-3)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(3\cdot\left(-8\right)-3)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(-27)^{\frac{2}{3}} \stackrel{?}{=} 36$$
$$4(\sqrt[3]{27})^2 \stackrel{?}{=} 36$$
$$4\left(-3\right)^2 \stackrel{?}{=} 36$$
$$4\cdot 9 \stackrel{?}{=} 36$$
$$36 = 36$$

The solutions are $x = 10$ and $x = -8$.

49. $\sqrt{x+6}+2=x+6$

$$\begin{aligned}\sqrt{x+6}&=x+4\\ \left(\sqrt{x+6}\right)^2&=\left(x+4\right)^2\\ x+6&=x^2+8x+16\\ 0&=x^2+7x+10\\ 0&=(x+2)(x+5)\\ x&=-2 \text{ or } x=-5\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{x+6}+2&\stackrel{?}{=}x+6\\ \sqrt{-2+6}+2&\stackrel{?}{=}-2+6\\ \sqrt{4}+2&\stackrel{?}{=}4\\ 2+2&\stackrel{?}{=}4\\ 4&=4\end{aligned}$$

$$\begin{aligned}\sqrt{x+6}+2&\stackrel{?}{=}x+6\\ \sqrt{-5+6}+2&\stackrel{?}{=}-5+6\\ \sqrt{1}+2&\stackrel{?}{=}1\\ 1+2&\stackrel{?}{=}1\\ 3&\neq 1\end{aligned}$$

The only solution is $x=-2$.

51. $\sqrt{2x+9}-\sqrt{x}=3$

$$\begin{aligned}\sqrt{2x+9}&=3+\sqrt{x}\\ \left(\sqrt{2x+9}\right)^2&=\left(3+\sqrt{x}\right)^2\\ 2x+9&=9+6\sqrt{x}+x\\ x&=6\sqrt{x}\\ x^2&=\left(6\sqrt{x}\right)^2\\ x^2&=36x\\ x^2-36x&=0\\ x(x-36)&=0\\ x&=0 \text{ or } x=36\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{2x+9}-\sqrt{x}&\stackrel{?}{=}3\\ \sqrt{2(0)+9}-\sqrt{0}&\stackrel{?}{=}3\\ \sqrt{9}&\stackrel{?}{=}3\\ 3&=3\end{aligned}$$

$$\begin{aligned}\sqrt{2x+9}-\sqrt{x}&\stackrel{?}{=}3\\ \sqrt{2(36)+9}-\sqrt{36}&\stackrel{?}{=}3\\ \sqrt{81}-6&\stackrel{?}{=}3\\ 9-6&\stackrel{?}{=}3\\ 3&=3\end{aligned}$$

The solutions are $x=0,\ 36$.

53. $f(x)=x-4$
$g(x)=x^2-16$

$$\begin{aligned}f(x)+g(x)&=(x-4)+(x^2-16)\\ &=x^2+x-20\end{aligned}$$

domain: all real numbers

55. $f(x)=x-4$
$g(x)=x^2-16$

$$\begin{aligned}f(x)\cdot g(x)&=(x-4)(x^2-16)\\ &=x^3-4x^2-16x+64\end{aligned}$$

domain: all real numbers

57. $g(x)=5x-2$
$h(x)=x^2+1$

$$\begin{aligned}(h\circ g)(x)&=h(g(x))\\ &=\left(5x-2\right)^2+1\\ &=25x^2-20x+4+1\\ &=25x^2-20x+5\end{aligned}$$

$$\begin{aligned}(h\circ g)(-1)&=25(-1)^2-20(-1)+5\\ &=25+20+5\\ &=50\end{aligned}$$

59. $g(x)=5x-2$
$h(x)=x^2+1$

$$\begin{aligned}(g\circ h)(x)&=g(h(x))\\ &=5(x^2+1)-2\\ &=5x^2+5-2\\ &=5x^2+3\end{aligned}$$

$$\begin{aligned}(g\circ h)(2)&=5(2)^2+3\\ &=5\cdot 4+3\\ &=23\end{aligned}$$

61. $C(x)=x-1$
$D(x)=0.5x$

50% decrease first, then the $1 off :
$C(D(x))=0.5x-1$

$$\begin{aligned}C(D(4))&=0.5(4)-1\\ &=2-1\\ &=1\end{aligned}$$

The cost of the cereal is $1.00.

$1 off first then the 50% decrease
$$\begin{aligned}D(C(x))&=0.5(x-1)\\ &=0.5x-0.5\end{aligned}$$

$$\begin{aligned}D(C(4))&=0.5(4)-0.5\\ &=2-0.5\\ &=1.50\end{aligned}$$

The cost of the cereal is $1.50.

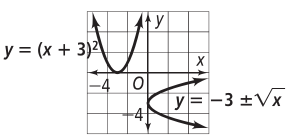
Use the coupon after the store discount.

63.
$$f(x) = 15 - 3x$$
$$y = 15 - 3x$$
$$x = 15 - 3y$$
$$x - 15 = -3y$$
$$\frac{x-15}{-3} = y$$
$$y = 5 - \frac{1}{3}x$$
$$f^{-1}(x) = 5 - \frac{1}{3}x$$

For each $x$ in the domain of $f^{-1}$, there is only one value of $y$ in the range. The inverse is a function.

65.
$$f(x) = \left(2x-3\right)^2$$
$$y = \left(2x-3\right)^2$$
$$x = \left(2y-3\right)^2$$
$$\pm\sqrt{x} = \sqrt{\left(2y-3\right)^2}$$
$$\pm\sqrt{x} = 2y-3$$
$$\frac{3 \pm \sqrt{x}}{2} = y$$
$$f^{-1}(x) = \frac{3 \pm \sqrt{x}}{2}$$

For each $x$ in the domain of $f^{-1}$, there are two values of $y$ in the range. The inverse is not a function.

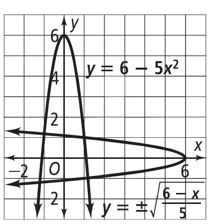
67.



domain of $f$: all real numbers
range of $f$: $y \geq 0$

domain of $f^{-1}$: $x \geq 0$

range of $f^{-1}$: all real numbers

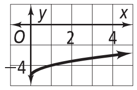
69.



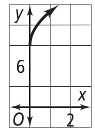

domain of $f$: all real numbers
range of $f$: $y \leq 6$

domain of $f^{-1}$: $x \leq 6$

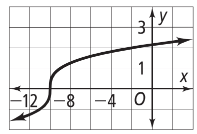
range of $f^{-1}$: all real numbers

71.


domain: $x \geq 0$
range: $y \geq -5$

73.


domain: $x \geq 0$
range: $y \geq 9$

75.


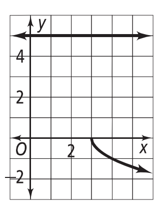
domain: all real numbers
range: all real numbers

77.
$$y = \sqrt{9x-27} + 4$$
$$y = \sqrt{9(x-3)} + 4$$
$$y = 3\sqrt{x-3} + 4$$

$y = 3\sqrt{x-3} + 4$ is the graph of $y = 3\sqrt{x}$ translated 3 units to the right and 4 units up.

79.
$$y = \sqrt[3]{8x+24}$$
$$y = \sqrt[3]{8(x+3)}$$
$$y = 2\sqrt[3]{x+3}$$

$y = 2\sqrt[3]{x+3}$ is the graph of $y = 2\sqrt[3]{x}$ translated 3 units to the left.

81.


no solution