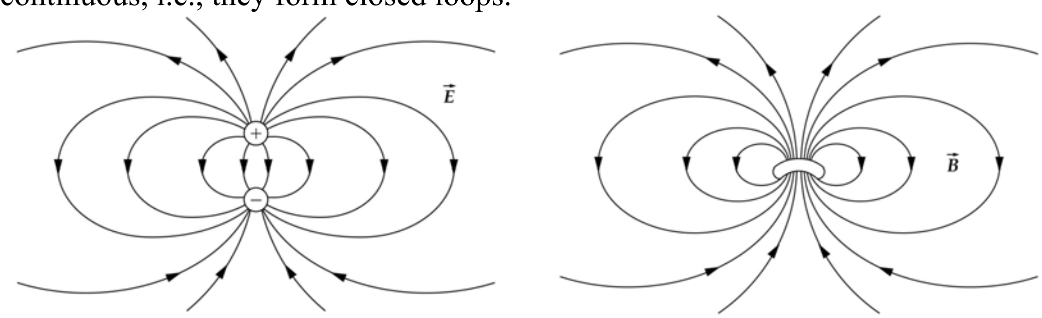
Chapter 27
Sources of the Magnetic Field

Conceptual Problems

1 • Sketch the field lines for the electric dipole and the magnetic dipole shown in Figure 27-47. How do the field lines differ in appearance close to the center of each dipole?

Picture the Problem Note that, while the two far fields (the fields far from the dipoles) are the same, the two near fields (the fields near to the dipoles) are not. At the center of the electric dipole, the electric field is antiparallel to the direction of the far field above and below the dipole, and at the center of the magnetic dipole, the magnetic field is parallel to the direction of the far field above and below the dipole. It is especially important to note that while the electric field lines begin and terminate on electric charges, the magnetic field lines are continuous, i.e., they form closed loops.



2 • Two wires lie in the plane of the page and carry equal currents in opposite directions, as shown in Figure 27- 48. At a point midway between the wires, the magnetic field is (*a*) zero, (*b*) into the page, (*c*) out of the page, (*d*) toward the top or bottom of the page, (*e*) toward one of the two wires.

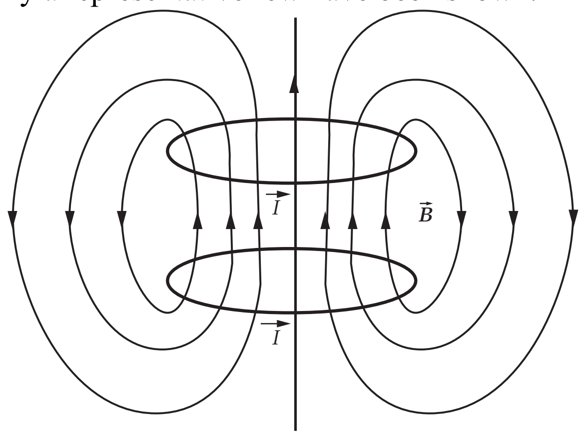
Determine the Concept Applying the right-hand rule to the wire to the left we see that the magnetic field due to its current is out of the page at the midpoint. Applying the right-hand rule to the wire to the right we see that the magnetic field due to its current is also out of the page at the midpoint. Hence, the sum of the magnetic fields is out of the page as well. 1is correct.

3 • Parallel wires 1 and 2 carry currents I_1 and I_2, respectively, where $I_2 = 2I_1$. The two currents are in the same direction. The magnitudes of the magnetic force by current 1 on wire 2 and by current 2 on wire 1 are F_{12} and F_{21}, respectively. These magnitudes are related by (*a*) $F_{21} = F_{21}$, (*b*) $F_{21} = 2F_{12}$, (*c*) $2F_{21} = F_{12}$, (*d*) $F_{21} = 4F_{12}$, (*e*) $4F_{21} = F_{12}$.

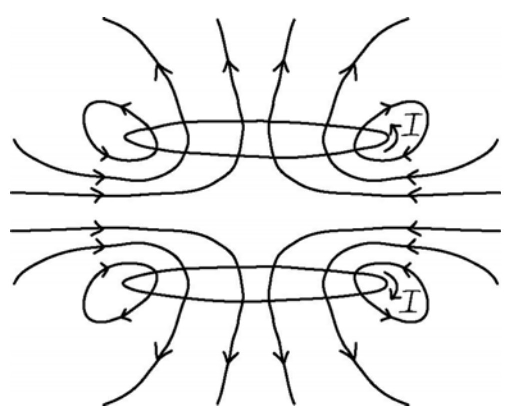
Determine the Concept While we could express the force wire 1 exerts on wire 2 and compare it to the force wire 2 exerts on wire 1 to show that they are the same, it is simpler to recognize that these are action and reaction forces. $\boxed{(a)}$ is correct.

4 • Make a field-line sketch of the magnetic field due to the currents in the pair of identical coaxial coils shown in Figure 27-49. Consider two cases: (*a*) the currents in the coils have the same magnitude and have the same direction and (*b*) the currents in the coils have the same magnitude and have the opposite directions.

Picture the Problem (*a*) The field-line sketch follows. An assumed direction for the current in the coils is shown in the diagram. Note that the field is stronger in the region between the coaxial coils and that the field lines have neither beginning nor ending points as do electric-field lines. Because there are an uncountable infinity of lines, only a representative few have been shown.



(*b*) The field-line sketch is shown below. An assumed direction for the current in the coils is shown in the diagram. Note that the field lines never begin or end and that they do not touch or cross each other. Because there are an uncountable infinity of lines, only a representative few have been shown.



5 • **[SSM]** Discuss the differences and similarities between Gauss's law for magnetism and Gauss's law for electricity.

Determine the Concept Both tell you about the respective fluxes through closed surfaces. In the electrical case, the flux is proportional to the net charge enclosed. In the magnetic case, the flux is always zero because there is no such thing as magnetic charge (a magnetic monopole). The source of the magnetic field is NOT the equivalent of electric charge; that is, it is NOT a thing called magnetic charge, but rather it is moving electric charges.

6 • Explain how you would modify Gauss's law if scientists discovered that single, isolated magnetic poles actually existed.

Determine the Concept Gauss' law for magnetism now reads: ″The flux of the magnetic field through any closed surface is equal to zero.″ Just like each electric pole has an electric pole strength (an amount of electric charge), each magnetic pole would have a magnetic pole strength (an amount of magnetic charge). Gauss' law for magnetism would read: ″The flux of the magnetic field through any closed surface is proportional to the total amount of magnetic charge inside.″

7 • **[SSM]** You are facing directly into one end of a long solenoid and the magnetic field inside of the solenoid points away from you. From your perspective, is the direction of the current in the solenoid coils clockwise or counterclockwise? Explain your answer.

Determine the Concept Application of the right-hand rule leads one to conclude that the current is clockwise.

8 • Opposite ends of a helical metal spring are connected to the terminals of a battery. Do the spacings between the coils of the spring tend increase, decrease, or remain the same when the battery is connected? Explain your answer.

Determine the Concept The coils attract each other and tend to move closer together when there is current in the spring. The current elements in the same direction will attract each other, and a current element of one segment of a coil are close to the current elements in adjacent coils that are in the same direction as it is.

9 • The current density is constant and uniform in a long straight wire that has a circular cross section. True or false:

(*a*) The magnitude of the magnetic field produced by this wire is greatest at the surface of the wire.
(*b*) The magnetic field strength in the region surrounding the wire varies inversely with the square of the distance from the wire's central axis.
(*c*) The magnetic field is zero at all points on the wire's central axis.
(*d*) The magnitude of the magnetic field inside the wire increases linearly with the distance from the wire's central axis.

(*a*) True. The magnetic field due to an infinitely long, straight wire is given by $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$, where *R* is the perpendicular distance to the field point. Because the magnetic field decreases linearly as the distance from the wire's central axis, the maximum field produced by this current is at the surface of the wire.

(*b*) False. Because the magnetic field due to an infinitely long, straight wire is given by $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$, where *R* is the perpendicular distance to the field point, the magnetic field outside the wire decreases linearly as the distance from the wire's central axis.

(*c*) True. Because $I_C = 0$ at the center of the wire, Ampere's law ($\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_C$) tells us that the magnetic field is zero at the center of the wire.

(*d*) True. Application of Ampere's law shows that, inside the wire, $B = \frac{\mu_0}{2\pi}\frac{I}{R^2}r$, where *R* is the radius of the wire and *r* is the distance from the center of the wire.

10 • If the magnetic susceptibility of a material is positive,
(*a*) paramagnetic effects or ferromagnetic effects must be greater than diamagnetic effects, (*b*) diamagnetic effects must be greater than paramagnetic effects,
(*c*) diamagnetic effects must be greater than ferromagnetic effects,
(*d*) ferromagnetic effects must be greater than paramagnetic effects,
(*e*) paramagnetic effects must be greater than ferromagnetic effects.

Determine the Concept The magnetic susceptibility χ_m is defined by the equation $\vec{M} = \chi_m \frac{\vec{B}_{app}}{\mu_0}$, where $\vec{M}$ is the magnetization vector and $\vec{B}_{app}$ is the applied magnetic field. For paramagnetic materials, χ_m is a small positive number that depends on temperature, whereas for diamagnetic materials, it is a small negative constant independent of temperature. $\boxed{(a)}$ is correct.

11 • **[SSM]** Of the four gases listed in Table 27-1, which are diamagnetic and which are paramagnetic?

Determine the Concept H_2, CO_2, and N_2 are diamagnetic ($\chi_m < 0$); O_2 is paramagnetic ($\chi_m > 0$).

12 • When a current is passed through the wire in Figure 27- 50, will the wire tend to bunch up or will it tend to form a circle? Explain your answer.

Determine the Concept It will tend to form a circle. Oppositely directed current elements will repel each other, and so opposite sides of the loop will repel.

The Magnetic Field of Moving Point Charges

13 • [SSM] At time $t = 0$, a particle has a charge of 12 μC, is located in the $z = 0$ plane at $x = 0$, $y = 2.0$ m, and has a velocity equal to $30\text{ m/s}\,\hat{\boldsymbol{i}}$. Find the magnetic field in the $z = 0$ plane at (*a*) the origin, (*b*) $x = 0$, $y = 1.0$ m, (*c*) $x = 0$, $y = 3.0$ m, and (*d*) $x = 0$, $y = 4.0$ m.

Picture the Problem We can substitute for $\vec{v}$ and q in the equation describing the magnetic field of the moving charged particle ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the given points of interest, and then find $\vec{B}$.

The magnetic field of the moving charged particle is given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Substitute numerical values and simplify to obtain:

$$\vec{B} = \left(10^{-7}\text{ N/A}^2\right)\left(12\,\mu\text{C}\right)\frac{\left(30\text{ m/s}\right)\hat{\boldsymbol{i}}\times\hat{r}}{r^2}$$

$$= \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the particle at (0, 2.0 m) and the point of interest at the origin:

$$\vec{r} = -\left(2.0\text{ m}\right)\hat{\boldsymbol{j}},\ r = 2.0\text{ m},\text{ and }\hat{r} = -\hat{\boldsymbol{j}}$$

Evaluating $\vec{B}(0,0)$ yields:

$$\vec{B}(0,0) = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\left(-\hat{\boldsymbol{j}}\right)}{\left(2.0\text{ m}\right)^2}$$

$$= \boxed{-\left(9.0\text{ pT}\right)\hat{\boldsymbol{k}}}$$

(*b*) Find r and $\hat{r}$ for the particle at (0, 2.0 m) and the point of interest at (0, 1.0 m):

$$\vec{r} = -\left(1.0\text{ m}\right)\hat{\boldsymbol{j}},\ r = 1.0\text{ m},\text{ and }\hat{r} = -\hat{\boldsymbol{j}}$$

Evaluate $\vec{B}(0,1.0\text{ m})$ to obtain:

$$\vec{B}(0,1.0\text{ m}) = \left(36.0\text{ pT}\cdot\text{m}^2\right)\frac{\hat{\boldsymbol{i}}\times\left(-\hat{\boldsymbol{j}}\right)}{\left(1.0\text{ m}\right)^2}$$

$$= \boxed{-\left(36\text{ pT}\right)\hat{\boldsymbol{k}}}$$

(*c*) Find r and $\hat{\boldsymbol{r}}$ for the particle at (0, 2.0 m) and the point of interest at (0, 3.0 m):

$$\vec{\boldsymbol{r}} = (1.0\,\text{m})\hat{\boldsymbol{j}},\ r = 1.0\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Evaluating $\vec{\boldsymbol{B}}(0,3.0\,\text{m})$ yields:

$$\vec{\boldsymbol{B}}(0,3.0\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}}{(1.0\,\text{m})^2} = \boxed{(36\,\text{pT})\hat{\boldsymbol{k}}}$$

(*d*) Find r and $\hat{\boldsymbol{r}}$ for the particle at (0, 2.0 m) and the point of interest at (0, 4.0 m):

$$\vec{\boldsymbol{r}} = (2.0\,\text{m})\hat{\boldsymbol{j}},\ r = 2.0\,\text{m},\ \text{and}\ \hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Evaluate $\vec{\boldsymbol{B}}(0,4.0\,\text{m})$ to obtain:

$$\vec{\boldsymbol{B}}(0,4.0\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}}{(2.0\,\text{m})^2} = \boxed{(9.0\,\text{pT})\hat{\boldsymbol{k}}}$$

14 • At time $t = 0$, a particle has a charge of 12 μC, is located in the $z = 0$ plane at $x = 0$, $y = 2.0$ m, and has a velocity equal to $30\,\text{m/s}\,\hat{\boldsymbol{i}}$. Find the magnetic field in the $z = 0$ plane at (*a*) $x = 1.0$ m, $y = 3.0$ m, (*b*) $x = 2.0$ m, $y = 2.0$ m, and (*c*) $x = 2.0$ m, $y = 3.0$ m.

Picture the Problem We can substitute for $\vec{\boldsymbol{v}}$ and q in the equation describing the magnetic field of the moving charged particle ($\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}$), evaluate r and $\hat{\boldsymbol{r}}$ for each of the given points of interest, and substitute to find $\vec{\boldsymbol{B}}$.

The magnetic field of the moving charged particle is given by:

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2}$$

Substitute numerical values and simplify to obtain:

$$\vec{\boldsymbol{B}} = (10^{-7}\,\text{N/A}^2)(12\,\mu\text{C})\frac{(30\,\text{m/s})\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2} = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$$

(*a*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2.0 m) and the point of interest at (1.0 m, 3.0 m):

$\vec{r} = (1.0\,\text{m})\hat{\boldsymbol{i}} + (1.0\,\text{m})\hat{\boldsymbol{j}}$, $r = \sqrt{2}\,\text{m}$, and $\hat{\boldsymbol{r}} = \frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}$

Substitute for $\hat{\boldsymbol{r}}$ and *r* in $\vec{\boldsymbol{B}} = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$ and evaluate $\vec{\boldsymbol{B}}(1.0\,\text{m},3.0\,\text{m})$:

$$\vec{\boldsymbol{B}}(1.0\,\text{m},3.0\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{j}}\right)}{\left(\sqrt{2}\,\text{m}\right)^2} = \frac{(36.0\,\text{pT}\cdot\text{m}^2)}{\sqrt{2}}\frac{\hat{\boldsymbol{k}}}{\left(\sqrt{2}\,\text{m}\right)^2}$$

$$= \boxed{(13\,\text{pT})\hat{\boldsymbol{k}}}$$

(*b*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2.0 m) and the point of interest at (2.0 m, 2.0 m):

$\vec{r} = (2.0\,\text{m})\hat{\boldsymbol{i}}$, $r = 2.0\,\text{m}$, and $\hat{\boldsymbol{r}} = \hat{\boldsymbol{i}}$

Substitute for $\hat{\boldsymbol{r}}$ and *r* in $\vec{\boldsymbol{B}} = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$ and evaluate $\vec{\boldsymbol{B}}(2.0\,\text{m},2.0\,\text{m})$:

$$\vec{\boldsymbol{B}}(2.0\,\text{m},2.0\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}}{(2.0\,\text{m})^2} = \boxed{0}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the particle at (0, 2.0 m) and the point of interest at (2.0 m, 3.0 m):

$\vec{r} = (2.0\,\text{m})\hat{\boldsymbol{i}} + (1.0\,\text{m})\hat{\boldsymbol{j}}$, $r = \sqrt{5}\,\text{m}$, and $\hat{\boldsymbol{r}} = \frac{2}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}$

Substitute for $\hat{\boldsymbol{r}}$ and *r* in $\vec{\boldsymbol{B}} = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\hat{\boldsymbol{r}}}{r^2}$ and evaluate $\vec{\boldsymbol{B}}(2.0\,\text{m},3.0\,\text{m})$:

$$\vec{\boldsymbol{B}}(2.0\,\text{m},3.0\,\text{m}) = (36.0\,\text{pT}\cdot\text{m}^2)\frac{\hat{\boldsymbol{i}}\times\left(\frac{2}{\sqrt{5}}\hat{\boldsymbol{i}} + \frac{1}{\sqrt{5}}\hat{\boldsymbol{j}}\right)}{\left(\sqrt{5}\,\text{m}\right)^2} = \boxed{(3.2\,\text{pT})\hat{\boldsymbol{k}}}$$

15 • A proton has a velocity of 1.0×10^2 m/s $\hat{\boldsymbol{i}} + 2.0\times10^2$ m/s $\hat{\boldsymbol{j}}$ and is located in the *z* = 0 plane at *x* = 3.0 m, *y* = 4.0 m at some time *t*. Find the magnetic field in the *z* = 0 plane at (*a*) *x* = 2.0 m, *y* = 2.0 m, (*b*) *x* = 6.0 m, *y* = 4.0 m, and (*c*) *x* = 3.0 m, *y* = 6.0 m.

Picture the Problem We can substitute for $\vec{v}$ and q in the equation describing the magnetic field of the moving proton ($\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the given points of interest, and substitute to find $\vec{B}$.

The magnetic field of the moving proton is given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Substituting numerical values yields:

$$\vec{B} = \left(10^{-7}\ \text{N/A}^2\right)\left(1.602\times10^{-19}\,\text{C}\right)\frac{\left[\left(1.0\times10^2\ \text{m/s}\right)\hat{i} + \left(2.0\times10^2\ \text{m/s}\right)\hat{j}\right]\times\hat{r}}{r^2}$$

$$= \left(1.60\times10^{-24}\ \text{T}\cdot\text{m}^2\right)\frac{\left(1.0\,\hat{i} + 2.0\,\hat{j}\right)\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the proton at (3.0 m, 4.0 m) and the point of interest at (2.0 m, 2.0 m):

$$\vec{r} = -(1.0\,\text{m})\hat{i} - (2.0\,\text{m})\hat{j},\ r = \sqrt{5}\,\text{m},$$
and $\hat{r} = -\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$

Substitute for $\hat{r}$ and r in $\vec{B} = \left(1.60\times10^{-24}\ \text{T}\cdot\text{m}^2\right)\frac{\left(1.0\,\hat{i} + 2.0\,\hat{j}\right)\times\hat{r}}{r^2}$ and evaluate $\vec{B}(2.0\,\text{m}, 2.0\,\text{m})$:

$$\vec{B}(2.0\,\text{m}, 2.0\,\text{m}) = \left(1.60\times10^{-24}\ \text{T}\cdot\text{m}^2\right)\frac{\left(1.0\hat{i} + 2.0\hat{j}\right)\times\left(-\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right)}{r^2}$$

$$= \frac{\left(1.60\times10^{-24}\ \text{T}\cdot\text{m}^2\right)}{\sqrt{5}}\left[\frac{-2.0\hat{k} + 2.0\hat{k}}{\left(\sqrt{5}\,\text{m}\right)^2}\right] = \boxed{0}$$

(*b*) Find r and $\hat{r}$ for the proton at (3.0 m, 4.0 m) and the point of interest at (6.0 m, 4.0 m):

$$\vec{r} = (3.0\,\text{m})\hat{i},\ r = 3.0\,\text{m},\ \text{and}\ \hat{r} = \hat{i}$$

Substitute for $\hat{r}$ and r in $\vec{B} = \left(1.60\times10^{-24}\ \text{T}\cdot\text{m}^2\right)\frac{\left(1.0\,\hat{i} + 2.0\,\hat{j}\right)\times\hat{r}}{r^2}$ and evaluate $\vec{B}(6.0\,\text{m}, 4.0\,\text{m})$:

$$\vec{B}(6.0\,\text{m},4.0\,\text{m}) = \left(1.60\times10^{-24}\,\text{T}\cdot\text{m}^2\right)\frac{\left(1.0\,\hat{\boldsymbol{i}}+2.0\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{i}}}{(3.0\,\text{m})^2}$$

$$= \left(1.60\times10^{-22}\,\text{T}\cdot\text{m}^2\right)\left(\frac{-2.0\,\hat{\boldsymbol{k}}}{9.0\,\text{m}^2}\right) = \boxed{-\left(3.6\times10^{-25}\,\text{T}\right)\hat{\boldsymbol{k}}}$$

(*c*) Find *r* and $\hat{\boldsymbol{r}}$ for the proton at (3.0 m, 4.0 m) and the point of interest at the (3.0 m, 6.0 m):

$$\vec{r} = (2.0\,\text{m})\hat{\boldsymbol{j}}\,,\; r = 2.0\,\text{m}\,,\text{ and }\hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$$

Substitute for $\hat{\boldsymbol{r}}$ and *r* in $\vec{B} = \left(1.60\times10^{-24}\,\text{T}\cdot\text{m}^2\right)\dfrac{\left(1.0\,\hat{\boldsymbol{i}}+2.0\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{r}}}{r^2}$ and evaluate $\vec{B}(3.0\,\text{m},6.0\,\text{m})$:

$$\vec{B}(3.0\,\text{m},6.0\,\text{m}) = \left(1.60\times10^{-24}\,\text{T}\cdot\text{m}^2\right)\frac{\left(1.0\,\hat{\boldsymbol{i}}+2.0\hat{\boldsymbol{j}}\right)\times\hat{\boldsymbol{j}}}{(2.0\,\text{m})^2}$$

$$= \left(1.60\times10^{-24}\,\text{T}\cdot\text{m}^2\right)\left(\frac{\hat{\boldsymbol{k}}}{4.0\,\text{m}^2}\right) = \boxed{\left(4.0\times10^{-25}\,\text{T}\right)\hat{\boldsymbol{k}}}$$

16 •• In a pre–quantum-mechanical model of the hydrogen atom, an electron orbits a proton at a radius of 5.29×10^{-11} m. According to this model, what is the magnitude of the magnetic field at the proton due to the orbital motion of the electron? Neglect any motion of the proton.

Picture the Problem The centripetal force acting on the orbiting electron is the Coulomb force between the electron and the proton. We can apply Newton's 2nd law to the electron to find its orbital speed and then use the expression for the magnetic field of a moving charge to find *B*.

Express the magnetic field due to the motion of the electron:

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2}$$

Apply $\sum F_{\text{radial}} = ma_{\text{c}}$ to the electron:

$$\frac{ke^2}{r^2} = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{ke^2}{mr}}$$

Substitute for *v* in the expression for *B* and simplify to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{e}{r^2}\sqrt{\frac{ke^2}{mr}} = \frac{\mu_0 e^2}{4\pi r^2}\sqrt{\frac{k}{mr}}$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(1.602\times 10^{-19}\ \text{C})^2}{4\pi (5.29\times 10^{-11}\ \text{m})^2}\sqrt{\frac{8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(9.109\times 10^{-31}\ \text{kg})(5.29\times 10^{-11}\ \text{m})}}$$
$$= \boxed{12.5\,\text{T}}$$

17 •• Two equal point charges q are, at some instant, located at (0, 0, 0) and at (0, b, 0). They are both moving with speed v in the +x direction (assume $v << c$). Find the ratio of the magnitude of the magnetic force to the magnitude of the electric force on each charge.

Picture the Problem We can find the ratio of the magnitudes of the magnetic and electrostatic forces by using the expression for the magnetic field of a moving charge and Coulomb's law. Note that $\vec{v}$ and $\vec{r}$, where $\vec{r}$ is the vector from one charge to the other, are at right angles. The field $\vec{B}$ due to the charge at the origin at the location (0, b, 0) is perpendicular to $\vec{v}$ and $\vec{r}$.

Express the magnitude of the magnetic force on the moving charge at (0, b, 0):

$$F_B = qvB = \frac{\mu_0}{4\pi}\frac{q^2v^2}{b^2}$$

and, applying the right hand rule, we find that the direction of the force is toward the charge at the origin; i.e., the magnetic force between the two moving charges is attractive.

Express the magnitude of the repulsive electrostatic interaction between the two charges:

$$F_E = \frac{1}{4\pi \in_0}\frac{q^2}{b^2}$$

Express the ratio of F_B to F_E and simplify to obtain:

$$\frac{F_B}{F_E} = \frac{\dfrac{\mu_0}{4\pi}\dfrac{q^2v^2}{b^2}}{\dfrac{1}{4\pi \in_0}\dfrac{q^2}{b^2}} = \boxed{\in_0 \mu_0 v^2}$$

The Magnetic Field Using the Biot–Savart Law

18 • A small current element at the origin has a length of 2.0 mm and carries a current of 2.0 A in the +z direction. Find the magnetic field due to the current element: (a) on the x axis at x = 3.0 m, (b) on the x axis at x = –6.0 m, (c) on the z axis at z = 3.0 m, and (d) on the y axis at y = 3.0 m.

Picture the Problem We can substitute for I and $d\vec{\ell} \approx \Delta\vec{\ell}$ in the Biot-Savart relationship ($d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for each of the points of interest, and substitute to find $d\vec{B}$.

Express the Biot-Savart law for the given current element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2} = \left(10^{-7}\ \text{N/A}^2\right)\frac{(2.0\,\text{A})(2.0\,\text{mm})\hat{k}\times\hat{r}}{r^2} = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

(*a*) Find r and $\hat{r}$ for the point whose coordinates are (3.0 m, 0, 0):

$$\vec{r} = (3.0\,\text{m})\hat{i}\,,\ r = 3.0\,\text{m}\,,\ \text{and}\ \hat{r} = \hat{i}$$

Evaluate $d\vec{B}$ at (3.0 m, 0, 0):

$$d\vec{B}(3.0\,\text{m},0,0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{i}}{(3.0\,\text{m})^2} = \boxed{(44\,\text{pT})\hat{j}}$$

(*b*) Find r and $\hat{r}$ for the point whose coordinates are (–6.0 m, 0, 0):

$$\vec{r} = -(6.0\,\text{m})\hat{i}\,,\ r = 6.0\,\text{m}\,,\ \text{and}\ \hat{r} = -\hat{i}$$

Evaluate $d\vec{B}$ at (–6.0 m, 0, 0):

$$d\vec{B}(-6.0\,\text{m},0,0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\cdot\frac{\hat{k}\times\left(-\hat{i}\right)}{(6.0\,\text{m})^2} = \boxed{-(11\,\text{pT})\hat{j}}$$

(*c*) Find r and $\hat{r}$ for the point whose coordinates are (0, 0, 3.0 m):

$$\vec{r} = (3.0\,\text{m})\hat{k}\,,\ r = 3.0\,\text{m}\,,\ \text{and}\ \hat{r} = \hat{k}$$

Evaluate $d\vec{B}$ at (0, 0, 3.0 m):

$$d\vec{B}(0,0,3.0\,\text{m}) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{k}}{(3.0\,\text{m})^2} = \boxed{0}$$

(*d*) Find r and $\hat{r}$ for the point whose coordinates are (0, 3.0 m, 0):

$$\vec{r} = (3.0\,\text{m})\hat{j}\,,\ r = 3.0\,\text{m}\,,\ \text{and}\ \hat{r} = \hat{j}$$

Evaluate $d\vec{B}$ at (0, 3.0 m, 0):

$$d\vec{B}(0,3.0\,\text{m},0) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{j}}{(3.0\,\text{m})^2} = \boxed{-(44\,\text{pT})\hat{i}}$$

19 • **[SSM]** A small current element at the origin has a length of 2.0 mm and carries a current of 2.0 A in the +z direction. Find the magnitude and direction of the magnetic field due to the current element at the point (0, 3.0 m, 4.0 m).

Picture the Problem We can substitute for I and $d\vec{\ell} \approx \Delta\vec{\ell}$ in the Biot-Savart law ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for (0, 3.0 m, 4.0 m), and substitute to find $d\vec{B}$.

The Biot-Savart law for the given current element is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$$

Substituting numerical values yields:

$$d\vec{B} = \left(1.0\times10^{-7}\,\text{N/A}^2\right)\frac{(2.0\,\text{A})(2.0\,\text{mm})\hat{k}\times\hat{r}}{r^2} = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\hat{r}}{r^2}$$

Find r and $\hat{r}$ for the point whose coordinates are (0, 3.0 m, 4.0 m):

$$\vec{r} = (3.0\,\text{m})\hat{j} + (4.0\,\text{m})\hat{k},$$

$$r = 5.0\,\text{m}, \text{ and } \hat{r} = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$$

Evaluate $d\vec{B}$ at (0, 3.0 m, 4.0 m):

$$d\vec{B}(0,3.0\,\text{m},4.0\,\text{m}) = \left(0.400\,\text{nT}\cdot\text{m}^2\right)\frac{\hat{k}\times\left(\frac{3}{5}\hat{j}+\frac{4}{5}\hat{k}\right)}{(5.0\,\text{m})^2} = \boxed{-(9.6\,\text{pT})\hat{i}}$$

20 • A small current element at the origin has a length of 2.0 mm and carries a current of 2.0 A in the +z direction. Find the magnitude of the magnetic field due to this element and indicate its direction on a diagram at (*a*) x = 2.0 m, y = 4.0 m, z = 0 and (*b*) x = 2.0 m, y = 0, z = 4.0 m.

Picture the Problem We can substitute for I and $d\vec{\ell} \approx \Delta\vec{\ell}$ in the Biot-Savart law ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$), evaluate r and $\hat{r}$ for the given points, and substitute to find $d\vec{B}$.

Apply the Biot-Savart law to the given current element to obtain:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$
$$= \left(10^{-7}\ \text{N/A}^2\right)\frac{(2.0\ \text{A})(2.0\ \text{mm})\hat{k} \times \hat{r}}{r^2}$$
$$= \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k} \times \hat{r}}{r^2}$$

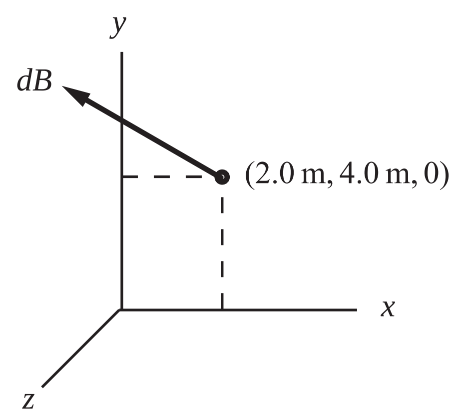
(*a*) Find r and $\hat{r}$ for the point whose coordinates are (2.0 m, 4.0 m, 0):

$$\vec{r} = (2.0\ \text{m})\hat{i} + (4.0\ \text{m})\hat{j},$$
$$r = 2.0\sqrt{5}\ \text{m},$$
and
$$\hat{r} = \frac{2.0}{2\sqrt{5}}\hat{i} + \frac{4.0}{2\sqrt{5}}\hat{j} = \frac{1.0}{\sqrt{5}}\hat{i} + \frac{2.0}{\sqrt{5}}\hat{j}$$

Evaluate $d\vec{B}$ at (2.0 m, 4.0 m, 0):

$$d\vec{B}(2.0\ \text{m}, 4.0\ \text{m}, 0) = \left(0.400\ \text{nT}\cdot\text{m}^2\right)\frac{\hat{k} \times \left(\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right)}{\left(2\sqrt{5}\ \text{m}\right)^2} = \boxed{-(18\ \text{pT})\hat{i} + (8.9\ \text{pT})\hat{j}}$$

The diagram is shown to the right:



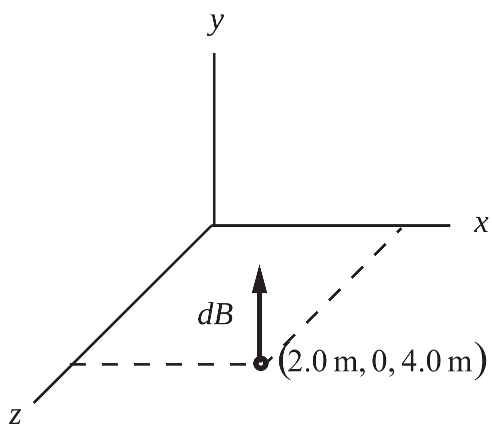
(*b*) Find r and $\hat{r}$ for the point whose coordinates are (2.0 m, 0, 4.0 m):

$$\vec{r} = (2.0\ \text{m})\hat{i} + (4.0\ \text{m})\hat{k},$$
$$r = 2.0\sqrt{5}\ \text{m},$$
and
$$\hat{r} = \frac{2.0}{2.0\sqrt{5}}\hat{i} + \frac{4.0}{2.0\sqrt{5}}\hat{k} = \frac{1.0}{\sqrt{5}}\hat{i} + \frac{2.0}{\sqrt{5}}\hat{k}$$

Evaluate $d\vec{B}$ at (2.0 m, 0, 4.0 m):

$$d\vec{B}(2.0\,\text{m},0,4.0\,\text{m}) = (0.400\,\text{nT}\cdot\text{m}^2)\frac{\hat{k}\times\left(\frac{1.0}{\sqrt{5}}\hat{i}+\frac{2.0}{\sqrt{5}}\hat{k}\right)}{\left(2.0\sqrt{5}\,\text{m}\right)^2} = \boxed{(8.9\,\text{pT})\hat{j}}$$

The diagram is shown to the right:



The Magnetic Field Due to Current Loops and Coils

21 • A single conducting loop has a radius equal to 3.0 cm and carries a current equal to 2.6 A. What is the magnitude of the magnetic field on the line through the center of the loop and perpendicular to the plane of the loop (*a*) the center of the loop, (*b*) 1.0 cm from the center, (*c*) 2.0 cm from the center, and (*d*) 35 cm from the center?

Picture the Problem We can use $B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$ to find B on the axis of the current loop.

B on the axis of a current loop is given by:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

Substitute numerical values to obtain:

$$B_x = \left(10^{-7}\ \text{N/A}^2\right)\frac{2\pi(0.030\,\text{m})^2(2.6\,\text{A})}{\left(x^2+(0.030\,\text{m})^2\right)^{3/2}}$$

$$= \frac{1.470\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left(x^2+(0.030\,\text{m})^2\right)^{3/2}}$$

(*a*) Evaluate B at the center of the loop:

$$B(0) = \frac{1.470\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left(0+(0.030\,\text{m})^2\right)^{3/2}} = \boxed{54\,\mu\text{T}}$$

(*b*) Evaluate B at x = 1.0 cm:

$$B(0.010\,\text{m}) = \frac{1.470\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.010\,\text{m})^2 + (0.030\,\text{m})^2\right)^{3/2}} = \boxed{46\,\mu\text{T}}$$

(*c*) Evaluate B at x = 2.0 cm:

$$B(0.020\,\text{m}) = \frac{1.470\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.020\,\text{m})^2 + (0.030\,\text{m})^2\right)^{3/2}} = \boxed{31\,\mu\text{T}}$$

(*d*) Evaluate B at x = 35 cm:

$$B(0.35\,\text{m}) = \frac{1.470\times10^{-9}\ \text{T}\cdot\text{m}^3}{\left((0.35\,\text{m})^2 + (0.030\,\text{m})^2\right)^{3/2}} = \boxed{34\,\text{nT}}$$

22 ••• A pair of identical coils, each having a radius of 30 cm, are separated by a distance equal to their radii, that is 30 cm. Called *Helmholtz coils*, they are coaxial and carry equal currents in directions such that their axial fields are in the same direction. A feature of Helmholtz coils is that the resultant magnetic field in the region between the coils is very uniform. Assume the current in each is 15 A and there are 250 turns for each coil. Using a **spreadsheet** program calculate and graph the magnetic field as a function of z, the distance from the center of the coils along the common axis, for – 30 cm < z < + 30 cm. Over what range of z does the field vary by less than 20%?

Picture the Problem Let the origin be midway between the coils so that one of them is centered at $z = -R/2$ and the other is centered at $z = R/2$, where R is the radius of the coils. Let the numeral 1 denote the coil centered at $z = -R/2$ and the numeral 2 the coil centered at $z = R/2$. We can express the magnetic field in the region between the coils as the sum of the magnetic fields B_1 and B_2 due to the two coils.

The magnetic field on the z is the sum of the magnetic fields due to the currents in coils 1 and 2:

$$B_z = B_1(z) + B_2(z) \qquad (1)$$

Express the magnetic field on the z axis due to the coil centered at $z = -R/2$:

$$B_1(z) = \frac{\mu_0 N R^2 I}{2\left[z + \left(\tfrac{1}{2}R\right)^2 + R^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the z axis due to the coil centered at $z = R/2$:

$$B_2(z) = \frac{\mu_0 N R^2 I}{2\left[\left(z - \tfrac{1}{2}R\right)^2 + R^2\right]^{3/2}}$$

Substitute for $B_1(z)$ and $B_2(z)$ in equation (1) to express the total magnetic field along the z axis:

$$B_z = \frac{\mu_0 N R^2 I}{2\left[\left(z+\frac{1}{2}R\right)^2+R^2\right]^{3/2}} + \frac{\mu_0 N R^2 I}{2\left[\left(z-\frac{1}{2}R\right)^2+R^2\right]^{3/2}}$$
$$= \frac{\mu_0 N R^2 I}{2}\left(\left[\left(z+\frac{1}{2}R\right)^2+R^2\right]^{-3/2} + \left[\left(z-\frac{1}{2}R\right)^2+R^2\right]^{-3/2}\right)$$

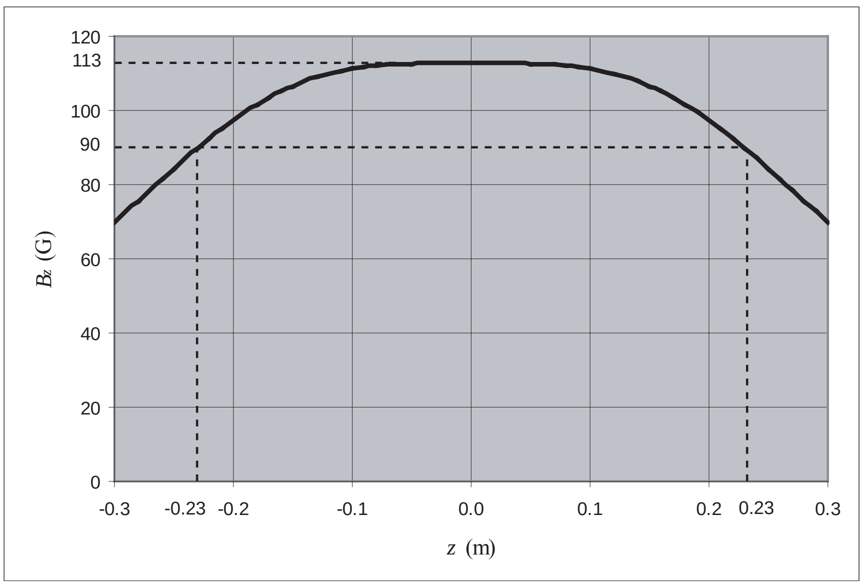
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.13×10^{-7}	μ_0
B2	0.30	R
B3	250	N
B3	15	I
B5	0.5*$B$1*$B$3*($B$2^2)*$B$4	$\text{Coeff} = \frac{\mu_0 N R^2 I}{2}$
A8	−0.30	$-R$
B8	$B$5*(($B$2/2+A8)^2+$B$2^2)^(−3/2)	$\frac{\mu_0 N R^2 I}{2}\left[\left(z-\frac{1}{2}R\right)^2+R^2\right]^{-3/2}$
C8	$B$5* (($B$2/2−A8)^2+$B$2^2)^(−3/2)	$\frac{\mu_0 N R^2 I}{2}\left[\left(z+\frac{1}{2}R\right)^2+R^2\right]^{-3/2}$
D8	10^4(B8+C8)	$B_z = 10^4\left(B_1(z)+B_2(z)\right)$

	A	B	C	D
1	μ_0=	1.26E-06	N/A^2	
2	R=	0.30	m	
3	N=	250	turns	
4	I=	15	A	
5	Coeff=	2.13E−04		
6				
7	z	B_1	B_2	$B(z)$
8	−0.30	5.63E−03	1.34E−03	70
9	−0.29	5.86E−03	1.41E−03	73
10	−0.28	6.08E−03	1.48E−03	76
11	−0.27	6.30E−03	1.55E−03	78
12	−0.26	6.52E−03	1.62E−03	81
13	−0.25	6.72E−03	1.70E−03	84
14	−0.24	6.92E−03	1.78E−03	87
15	−0.23	7.10E−03	1.87E−03	90

61	0.23	1.87E–03	7.10E–03	90
62	0.24	1.78E–03	6.92E–03	87
63	0.25	1.70E–03	6.72E–03	84
64	0.26	1.62E–03	6.52E–03	81
65	0.27	1.55E–03	6.30E–03	78
66	0.28	1.48E–03	6.08E–03	76
67	0.29	1.41E–03	5.86E–03	73
68	0.30	1.34E–03	5.63E–03	70

The following graph of B_z as a function of z was plotted using the data in the above table.



The maximum value of B_z is 113 G. Eighty percent of this maximum value is 90 G. We see that the field is within 20 percent of 113 G in the interval $\boxed{-0.23\,\text{m} < z < 0.23\,\text{m}.}$

23 ••• A pair of Helmholtz coils that have radii R have their axes along the z axis (see Problem 22). One coil is in the $z = -\frac{1}{2}R$ plane and the second coil is in the $z = \frac{1}{2}R$ plane. Show that on the z axis at $z = 0$ $dB_z/dz = 0$, $d^2B_z/dz^2 = 0$, and $d^3B_z/dz^3 = 0$. (*Note:* These results show that the magnitude and direction of the magnetic field in the region to either side of the midpoint is approximately equal to the magnitude and direction of the magnetic field at the midpoint.)

Picture the Problem Let the numeral 1 denote the coil centered at $z = -\frac{1}{2}R$ and the numeral 2 the coil centered at $z = \frac{1}{2}R$. We can express the magnetic field strength in the region between the coils as the sum of the magnetic field strengths due to the two coils and then evaluate the derivatives of this function to show that $dB_z/dz = 0$, $d^2B_z/dz^2 = 0$, and $d^3B_z/dz^3 = 0$ at $z = 0$.

Express the magnetic field strength on the z axis due to the coil centered at $z = -\frac{1}{2}R$:

$$B_1(z) = \frac{\mu_0 NR^2 I}{2z_1^3}$$

where $z_1 = \sqrt{\left(z + \frac{1}{2}R\right)^2 + R^2}$ and N is the number of turns and.

Express the magnetic field strength on the z axis due to the coil centered at $z = \frac{1}{2}R$:

$$B_2(x) = \frac{\mu_0 NR^2 I}{2z_2^3}$$

where $z_2 = \sqrt{\left(z - \frac{1}{2}R\right)^2 + R^2}$

Add these equations to express the total magnetic field along the x axis:

$$B_z(z) = B_1(z) + B_2(z) = \frac{\mu_0 NR^2 I}{2z_1^3} + \frac{\mu_0 NR^2 I}{2z_2^3} = \frac{\mu_0 NR^2 I}{2}\left(z_1^{-3} + z_2^{-3}\right)$$

Differentiate B_z with respect to z to obtain:

$$\frac{dB_z}{dz} = \frac{\mu_0 NR^2 I}{2}\frac{d}{dz}\left(z_1^{-3} + z_2^{-3}\right) = \frac{\mu_0 NR^2 I}{2}\left(-3z_1^{-4}\frac{dz_1}{dz} - 3z_2^{-4}\frac{dz_2}{dz}\right)$$

Because $z_1 = \sqrt{\left(z + \frac{1}{2}R\right)^2 + R^2}$:

$$\frac{dz_1}{dz} = \frac{1}{2}\left[\left(z + \frac{1}{2}R\right)^2 + R^2\right]^{-1/2}\left[2\left(z + \frac{1}{2}R\right)\right] = \frac{z + \frac{1}{2}R}{z_1}$$

Because $z_2 = \sqrt{\left(z - \frac{1}{2}R\right)^2 + R^2}$:

$$\frac{dz_2}{dz} = \frac{1}{2}\left[\left(z - \frac{1}{2}R\right)^2 + R^2\right]^{-1/2}\left[2\left(z - \frac{1}{2}R\right)\right] = \frac{z - \frac{1}{2}R}{z_2}$$

Evaluating $z_1(0)$ and $z_2(0)$ yields:

$$z_1(0) = \sqrt{\left(\tfrac{1}{2}R\right)^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

and

$$z_2(0) = \sqrt{\left(\tfrac{1}{2}R\right)^2 + R^2} = \left(\tfrac{5}{4}R^2\right)^{1/2}$$

Substituting for $\frac{dz_1}{dz}$ and $\frac{dz_2}{dz}$ in the expression for $\frac{dB_z}{dz}$ and simplifying yields:

$$\begin{aligned}\frac{dB_z}{dz} &= \frac{\mu_0 NR^2 I}{2}\left[\left(-3z_1^{-4}\left(\frac{z+\frac{1}{2}R}{z_1}\right) + -3z_2^{-4}\left(\frac{z-\frac{1}{2}R}{z_2}\right)\right)\right] \\ &= \frac{\mu_0 NR^2 I}{2}\left[\frac{-3\left(z+\frac{1}{2}R\right)}{z_1^5} + \frac{-3\left(z-\frac{1}{2}R\right)}{z_2^5}\right]\end{aligned}$$

Substitute for $z_1(0)$ and $z_2(0)$ and evaluate $\frac{dB_z}{dz}$ at $z = 0$:

$$\left.\frac{dB_z}{dz}\right|_{z=0} = \frac{\mu_0 NR^2 I}{2}\left(\frac{-3\left(\frac{1}{2}R\right)}{\left(\frac{5}{4}R^2\right)^{5/2}} + \frac{-3\left(-\frac{1}{2}R\right)}{\left(\frac{5}{4}R^2\right)^{5/2}}\right) = \boxed{0}$$

Differentiate $\frac{dB_z}{dz}$ with respect to z to obtain:

$$\begin{aligned}\frac{d^2B_z}{dz^2} &= \frac{\mu_0 NR^2 I}{2}\frac{d}{dz}\left[\frac{-3\left(z+\frac{1}{2}R\right)}{z_1^5} + \frac{-3\left(z-\frac{1}{2}R\right)}{z_2^5}\right] \\ &= \frac{\mu_0 NR^2 I}{2}(-3)\left[\frac{1}{z_1^5} - \frac{5\left(z+\frac{1}{2}R\right)^2}{z_1^7} + \frac{1}{z_2^5} - \frac{5\left(z-\frac{1}{2}R\right)^2}{z_2^7}\right]\end{aligned}$$

Evaluate $\frac{d^2B_z}{dz^2}$ at $z = 0$ to obtain:

$$\left.\frac{d^2B_z}{dz^2}\right|_{z=0} = \frac{\mu_0 NR^2 I}{2}(-3)\left[\frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{5\left(\frac{1}{2}R\right)^2}{\left(\frac{5}{4}R\right)^{7/2}} + \frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{5\left(-\frac{1}{2}R\right)^2}{\left(\frac{5}{4}R\right)^{7/2}}\right]$$

$$= \frac{\mu_0 NR^2 I}{2}(-3)\left[\frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R\right)^{7/2}} + \frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{\frac{5}{4}R^2}{\left(\frac{5}{4}R\right)^{7/2}}\right]$$

$$= \frac{\mu_0 NR^2 I}{2}(-3)\left[\frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{1}{\left(\frac{5}{4}R\right)^{5/2}} + \frac{1}{\left(\frac{5}{4}R\right)^{5/2}} - \frac{1}{\left(\frac{5}{4}R\right)^{5/2}}\right]$$

$$= \boxed{0}$$

Differentiate $\frac{d^2B_z}{dz^2}$ with respect to z to obtain:

$$\frac{d^3B_z}{dz^3} = \frac{\mu_0 NR^2 I}{2}(-3)\frac{d}{dz}\left[\frac{1}{z_1^5} - \frac{5\left(z+\frac{1}{2}R\right)^2}{z_1^7} + \frac{1}{z_2^5} - \frac{5\left(z-\frac{1}{2}R\right)^2}{z_2^7}\right]$$

$$= \frac{\mu_0 NR^2 I}{2}(-3)\left(\frac{35\left(z+\frac{1}{2}R\right)^3}{z_1^9} - \frac{15\left(z+\frac{1}{2}R\right)}{z_1^7} - \frac{15\left(z-\frac{1}{2}R\right)}{z_2^7} + \frac{35\left(z-\frac{1}{2}R\right)^3}{z_2^9}\right)$$

Evaluate $\frac{d^3B_z}{dz^3}$ at $z = 0$ to obtain:

$$\left.\frac{d^3B_z}{dz^3}\right|_{z=0} = \frac{\mu_0 NR^2 I}{2}(-3)\left(\frac{35\left(\frac{1}{2}R\right)^3}{z_1^9} - \frac{15\left(\frac{1}{2}R\right)}{z_1^7} - \frac{15\left(-\frac{1}{2}R\right)}{z_2^7} + \frac{35\left(-\frac{1}{2}R\right)^3}{z_2^9}\right)$$

$$= \frac{\mu_0 NR^2 I}{2}(-3)\left(\frac{35\left(\frac{1}{2}R\right)^3}{\left(\frac{5}{4}R^2\right)^{9/2}} - \frac{15\left(\frac{1}{2}R\right)}{\left(\frac{5}{4}R^2\right)^{7/2}} + \frac{15\left(\frac{1}{2}R\right)}{\left(\frac{5}{4}R^2\right)^{7/2}} - \frac{35\left(\frac{1}{2}R\right)^3}{\left(\frac{5}{4}R^2\right)^{9/2}}\right)$$

$$= \boxed{0}$$

24 ••• *Anti-Helmholtz* coils are used in many physics applications, such as laser cooling and trapping, where a field with a uniform gradient is desired. These coils have the same construction as a Helmholtz coil, except that the currents have opposite directions, so that the axial fields are in opposite directions, and the coil separation is $\sqrt{3}R$ rather than R. Graph the magnetic field as a function of z, the axial distance from the center of the coils, for an anti-Helmholtz coil using the same parameters as in Problem 22. Over what interval of the z axis is dB_z/dz within one percent of its value at the midpoint between the coils?

Picture the Problem Let the origin be midway between the coils so that one of them is centered at $z=-\frac{\sqrt{3}}{2}R$ and the other is centered at $z=\frac{\sqrt{3}}{2}R$. Let the numeral 1 denote the coil centered at $z=-\frac{\sqrt{3}}{2}R$ and the numeral 2 the coil centered at $z=\frac{\sqrt{3}}{2}R$. We can express the magnetic field in the region between the coils as the difference of the magnetic fields B_1 and B_2 due to the two coils and use the two-point formula to approximate the slope of the graph of B_z as a function of z.

The magnetic field on the z is the difference between the magnetic fields due to the currents in coils 1 and 2:

$$B_z = B_1(z) - B_2(z) \qquad (1)$$

Express the magnetic field on the z axis due to the coil centered at $z=-R\sqrt{3}/2$:

$$B_1(z) = \frac{\mu_0 NR^2 I}{2\left[\left(z+\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{3/2}}$$

where N is the number of turns.

Express the magnetic field on the z axis due to the coil centered at $z=R\sqrt{3}/2$:

$$B_2(z) = \frac{\mu_0 NR^2 I}{2\left[\left(z-\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{3/2}}$$

Substitute for $B_1(z)$ and $B_2(z)$ in equation (1) to express the total magnetic field along the z axis:

$$\begin{aligned} B_z &= \frac{\mu_0 NR^2 I}{2\left[\left(z+\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{3/2}} - \frac{\mu_0 NR^2 I}{2\left[\left(z-\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{3/2}} \\ &= \frac{\mu_0 NR^2 I}{2}\left(\left[\left(z+\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{-3/2} - \left[\left(z-\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{-3/2}\right) \end{aligned} \qquad (1)$$

The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

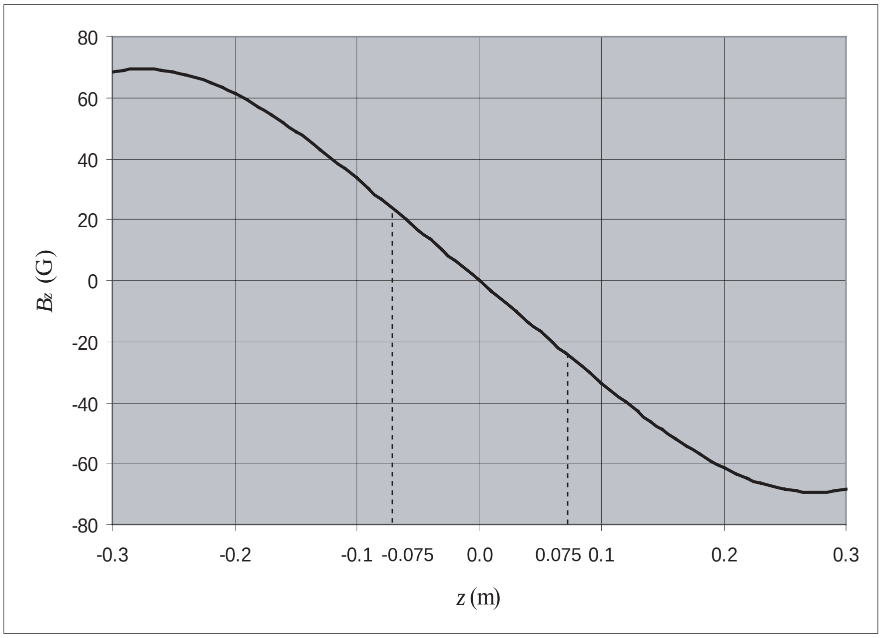
Cell	Formula/Content	Algebraic Form
B1	1.26×10^{-6}	μ_0
B2	0.30	R
B3	250	N
B3	15	I

B5	0.5*B$1*B$3*(B$2^2)*B$4	$\text{Coeff} = \dfrac{\mu_0 NR^2 I}{2}$
A8	−0.30	$-R$
B8	B$5*((B$2*SQRT(3)/2+A8)^2 +B$2^2)^(−3/2)	$\dfrac{\mu_0 NR^2 I}{2}\left[\left(z+\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{-3/2}$
C8	B$5* ((B$2*SQRT(3)/2−A8)^2 +B$2^2)^(−3/2)	$\dfrac{\mu_0 NR^2 I}{2}\left[\left(z-\frac{\sqrt{3}}{2}R\right)^2+R^2\right]^{-3/2}$
D8	10^4*(B8−C8)	$B_x = B_1(z) - B_2(z)$
E9	(D10 − D8)/(A10 − A8)	$\Delta B_z/\Delta z$
F9	ABS(100*(E9 − E$38)/E$38)	% diff

	A	B	C	D	E	F
1	μ_0=	1.26E−06	N/A^2			
2	R=	0.30	m			
3	N=	250	turns			
4	I=	15	A			
5	Coeff=	2.13E−04				
6						
7	z	B_1	B_2	$B(z)$	slope	% diff
8	−0.30	7.67E−03	8.30E-04	68.4		
9	−0.29	7.76E−03	8.65E-04	68.9	41	112.1
10	−0.28	7.82E−03	9.03E-04	69.2	14	104.1
11	−0.27	7.86E−03	9.42E-04	69.2	−14	95.9
12	−0.26	7.87E−03	9.84E-04	68.9	−42	87.5
30	−0.08	4.97E−03	2.28E−03	26.9	−332	1.3
31	−0.07	4.75E−03	2.40E−03	23.5	−334	0.8
32	−0.06	4.54E−03	2.52E−03	20.2	−335	0.4
33	−0.05	4.33E−03	2.65E−03	16.8	−336	0.2
34	−0.04	4.13E−03	2.79E−03	13.5	−336	0.1
35	−0.03	3.94E−03	2.93E−03	10.1	−337	0.0
36	−0.02	3.75E−03	3.08E−03	6.7	−337	0.0
37	−0.01	3.57E−03	3.24E−03	3.4	−337	0.0
38	0.00	3.40E−03	3.40E−03	0.0	−337	0.0
39	0.01	3.24E−03	3.57E−03	−3.4	−337	0.0
40	0.02	3.08E−03	3.75E−03	−6.7	−337	0.0
41	0.03	2.93E−03	3.94E−03	−10.1	−337	0.0
42	0.04	2.79E−03	4.13E−03	−13.5	−336	0.1
43	0.05	2.65E−03	4.33E−03	−16.8	−336	0.2
44	0.06	2.52E−03	4.54E−03	−20.2	−335	0.4
45	0.07	2.40E−03	4.75E−03	−23.5	−334	0.8
46	0.08	2.28E−03	4.97E−03	−26.9	−332	1.3

64	0.26	9.84E–04	7.87E–03	–68.9	–42	87.5
65	0.27	9.42E–04	7.86E–03	–69.2	–14	95.9
66	0.28	9.03E–04	7.82E–03	–69.2	14	104.1
67	0.29	8.65E–04	7.76E–03	–68.9	41	112.1
68	0.30	8.30E–04	7.67E–03	–68.4		

The following graph of B_z as a function of z was plotted using the data in the above table.



Inspection of the table reveals that the slope of the graph of B_z, evaluated at $z = 0$, is -337 G. 1% of this value corresponds approximately to $-0.075\text{ m} < z < 0.075\text{ m}$ or $\boxed{-0.25R < z < 0.25R}$.

The Magnetic Field Due to Straight-Line Currents

25 •• [SSM] If the currents are both in the $-x$ direction, find the magnetic field at the following points on the y axis: (a) $y = -3.0$ cm, (b) $y = 0$, (c) $y = +3.0$ cm, and (d) $y = +9.0$ cm.

Picture the Problem Let + denote the wire (and current) at $y = +6.0$ cm and $-$ the wire (and current) at $y = -6.0$ cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current-carrying wires and superimpose the magnetic fields due to the currents in these wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at $y = -3.0$ cm:

$$\vec{B}(-3.0\,\text{cm}) = \vec{B}_+(-3.0\,\text{cm}) + \vec{B}_-(-3.0\,\text{cm}) \qquad (1)$$

Find the magnitudes of the magnetic fields at $y = -3.0$ cm due to each wire:

$$B_+(-3.0\,\text{cm}) = (10^{-7}\,\text{T}\cdot\text{m/A})\frac{2(20\,\text{A})}{0.090\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3.0\,\text{cm}) = (10^{-7}\,\text{T}\cdot\text{m/A})\frac{2(20\,\text{A})}{0.030\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3.0\,\text{cm}) = (44.4\,\mu\text{T})\hat{k}$$

and

$$\vec{B}_-(-3.0\,\text{cm}) = -(133\,\mu\text{T})\hat{k}$$

Substituting in equation (1) yields:

$$\vec{B}(-3.0\,\text{cm}) = (44.4\,\mu\text{T})\hat{k} - (133\,\mu\text{T})\hat{k} = \boxed{-(89\,\mu\text{T})\hat{k}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0) = \vec{B}_+(0) + \vec{B}_-(0)$$

Because $\vec{B}_+(0) = -\vec{B}_-(0)$:

$$\vec{B}(0) = \boxed{0}$$

(*c*) Proceed as in (*a*) to obtain:

$$\vec{B}_+(3.0\,\text{cm}) = (133\,\mu\text{T})\hat{k}\,,$$

$$\vec{B}_-(3.0\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k}\,,$$

and

$$\mathbf{B}(3.0\,\text{cm}) = (133\,\mu\text{T})\hat{\mathbf{k}} - (44.4\,\mu\text{T})\hat{\mathbf{k}} = \boxed{(89\,\mu\text{T})\hat{\mathbf{k}}}$$

(*d*) Proceed as in (*a*) with $y = 9.0$ cm to obtain:

$$\vec{B}_+(9.0\,\text{cm}) = -(133\,\mu\text{T})\hat{k}\,,$$

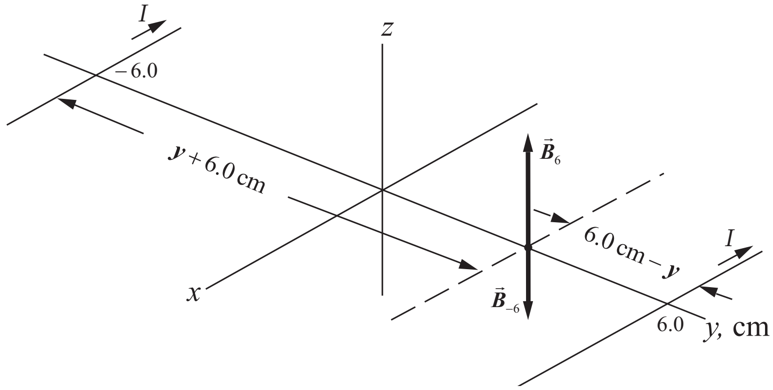
$$\vec{B}_-(9.0\,\text{cm}) = -(26.7\,\mu\text{T})\hat{k}\,,$$

and

$$\vec{B}(9.0\,\text{cm}) = -(133\,\mu\text{T})\hat{k} - (26.7\,\mu\text{T})\hat{k} = \boxed{-(160\,\mu\text{T})\hat{k}}$$

26 •• Using a **spreadsheet** program or graphing calculator, graph B_z versus y when both currents are in the $-x$ direction.

Picture the Problem The diagram shows the two wires with the currents flowing in the negative *x* direction. We can use the expression for *B* due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at $y = 6.0$ cm is:

$$\boldsymbol{B}_6 = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{I}}{0.060\,\text{m} - \boldsymbol{y}}$$

The field due to the current in the wire located at $y = -6.0$ cm is:

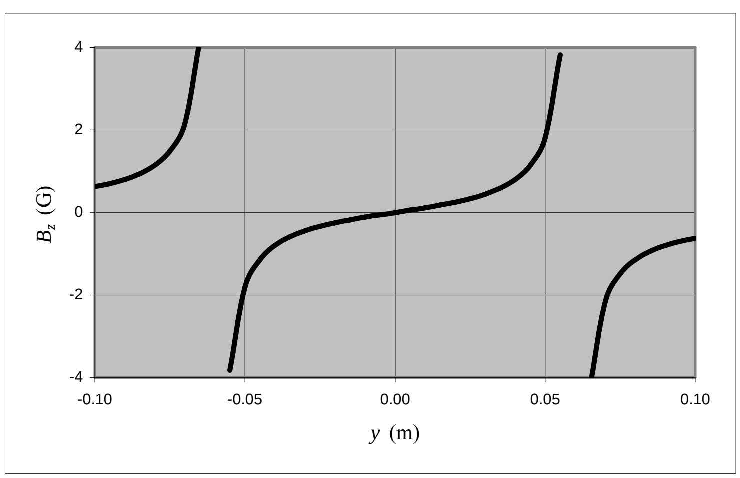
$$\boldsymbol{B}_{-6} = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{I}}{0.060\,\text{m} + \boldsymbol{y}}$$

The resultant field B_z is the difference between B_6 and B_{-6}:

$$\boldsymbol{B}_z = \boldsymbol{B}_6 - \boldsymbol{B}_{-6} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{I}}{0.060\,\text{m} - \boldsymbol{y}} - \frac{\mu_0}{4\pi}\frac{\boldsymbol{I}}{0.060\,\text{m} - \boldsymbol{y}}$$

$$= \frac{\mu_0 \boldsymbol{I}}{4\pi}\left(\frac{1}{0.060\,\text{m} - \boldsymbol{y}} - \frac{1}{0.060\,\text{m} + \boldsymbol{y}}\right)$$

The following graph of B_z as a function of *y* was plotted using a spreadsheet program:



27 •• The current in the wire at $y = -6.0$ cm is in the $-x$ direction and the current in the wire at $y = +6.0$ cm is in the $+x$ direction. Find the magnetic field at the following points on the y axis: (*a*) $y = -3.0$ cm, (*b*) $y = 0$, (*c*) $y = +3.0$ cm, and (*d*) $y = +9.0$ cm.

Picture the Problem Let + denote the wire (and current) at $y = +6$ cm and − the wire (and current) at $y = -6.0$ cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the y axis. We can apply the right-hand rule to find the direction of each of the fields and, hence, of $\vec{B}$.

(*a*) Express the resultant magnetic field at $y = -3.0$ cm:

$$\vec{B}(-3.0\,\text{cm}) = \vec{B}_+(-3.0\,\text{cm}) + \vec{B}_-(-3.0\,\text{cm}) \quad (1)$$

Find the magnitudes of the magnetic fields at $y = -3.0$ cm due to each wire:

$$B_+(-3.0\,\text{cm}) = \left(10^{-7}\,\text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.090\,\text{m}} = 44.4\,\mu\text{T}$$

and

$$B_-(-3.0\,\text{cm}) = \left(10^{-7}\,\text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.030\,\text{m}} = 133\,\mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(-3.0\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k}$$

and

$$\vec{B}_-(-3.0\,\text{cm}) = -(133\,\mu\text{T})\hat{k}$$

Substituting in equation (1) yields:

$$\vec{B}(-3.0\,\text{cm}) = -(44.4\,\mu\text{T})\hat{k} - (133\,\mu\text{T})\hat{k} = \boxed{-(0.18\,\text{mT})\hat{k}}$$

(*b*) Express the resultant magnetic field at $y = 0$:

$$\vec{B}(0) = \vec{B}_+(0) + \vec{B}_-(0)$$

Find the magnitudes of the magnetic fields at y = 0 cm due to each wire:

$$B_+(0) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.060\,\text{m}} = 66.7\ \mu\text{T}$$

and

$$B_-(0) = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(20\,\text{A})}{0.060\,\text{m}} = 66.7\ \mu\text{T}$$

Apply the right-hand rule to find the directions of $\vec{B}_+$ and $\vec{B}_-$:

$$\vec{B}_+(0) = -(66.7\ \mu\text{T})\hat{k}$$

and

$$\vec{B}_-(0) = -(66.7\ \mu\text{T})\hat{k}$$

Substitute to obtain:

$$\vec{B}(0) = -(66.7\ \mu\text{T})\hat{k} - (66.7\ \mu\text{T})\hat{k} = \boxed{-(0.13\,\text{mT})\hat{k}}$$

(*c*) Proceed as in (*a*) with y = +3.0 cm to obtain:

$$\vec{B}_+(3.0\,\text{cm}) = -(133\ \mu\text{T})\hat{k}\,,$$

$$\vec{B}_-(3.0\,\text{cm}) = -(44.4\ \mu\text{T})\hat{k}\,,$$

and

$$\vec{B}(3.0\,\text{cm}) = -(133\ \mu\text{T})\hat{k} - (44.4\ \mu\text{T})\hat{k} = \boxed{-(0.18\,\text{mT})\hat{k}}$$

(*d*) Proceed as in (*a*) with y = +9.0 m to obtain:

$$\vec{B}_+(9.0\,\text{cm}) = (133\ \mu\text{T})\hat{k}\,,$$

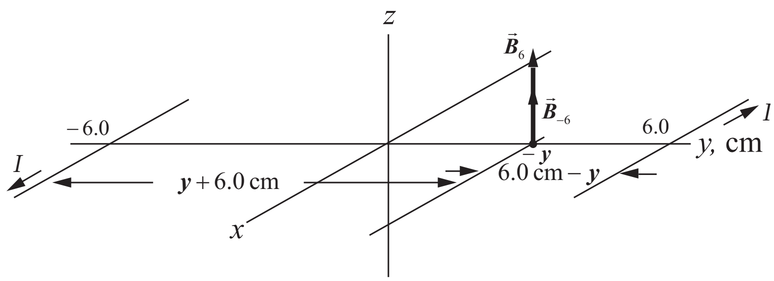
$$\vec{B}_-(9.0\,\text{cm}) = -(26.7\ \mu\text{T})\hat{k}\,,$$

and

$$\vec{B}(9.0\,\text{cm}) = (133\ \mu\text{T})\hat{k} - (26.7\ \mu\text{T})\hat{k} = \boxed{(0.11\,\text{mT})\hat{k}}$$

28 •• The current in the wire at y = –6.0 cm is in the +x direction and the current in the wire at y = +6.0 cm is in the –x direction. Using a **spreadsheet** program or graphing calculator, graph B_z versus y.

Picture the Problem The diagram shows the two wires with the currents flowing in the negative x direction. We can use the expression for B due to a long, straight wire to express the difference of the fields due to the two currents. We'll denote each field by the subscript identifying the position of each wire.



The field due to the current in the wire located at $y = 6.0$ cm is:

$$\boldsymbol{B}_6 = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{I}}{0.060\,\text{m} - \boldsymbol{y}}$$

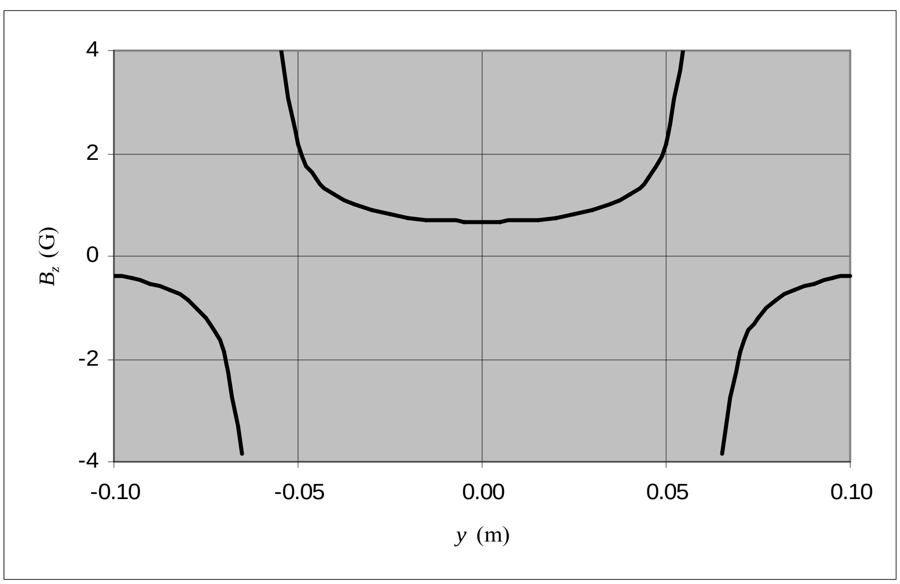
The field due to the current in the wire located at $y = -6.0$ cm is:

$$\boldsymbol{B}_{-6} = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{I}}{0.060\,\text{m} + \boldsymbol{y}}$$

The resultant field B_z is the sum of B_6 and B_{-6}:

$$B_z = B_6 + B_{-6} = \frac{\mu_0}{4\pi}\frac{I}{0.060\,\text{m} - y} + \frac{\mu_0}{4\pi}\frac{I}{0.060\,\text{m} - y}$$
$$= \frac{\mu_0 I}{4\pi}\left(\frac{1}{0.060\,\text{m} - y} + \frac{1}{0.060\,\text{m} + y}\right)$$

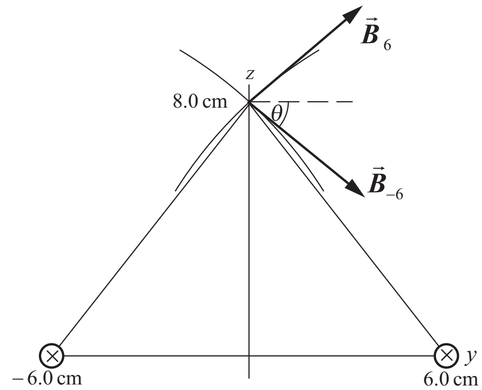
The following graph of B_z as a function of y was plotted using a spreadsheet program:



29 • Find the magnetic field on the z axis at $z = +8.0$ cm if (a) the currents are both in the $-x$ direction, and (b) the current in the wire at $y = -6.0$ cm is in the $-x$ direction and the current in the wire at $y = +6.0$ cm is in the $+x$ direction.

Picture the Problem Let + denote the wire (and current) at $y = +6.0$ cm and − the wire (and current) at $y = -6.0$ cm. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnetic field due to each of the current carrying wires and superimpose the magnetic fields due to the currents in the wires to find B at the given points on the z axis.

(*a*) Apply the right-hand rule to show that, for the currents parallel and in the negative x direction, the directions of the fields are as shown to the right:



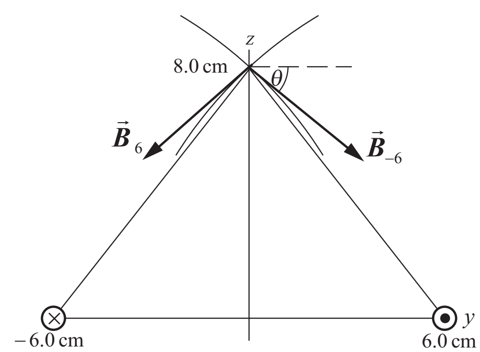
Express the magnitudes of the magnetic fields at $z = +8.0$ cm due to the current-carrying wires at $y = -6.0$ cm and $y = +6.0$ cm:

$$\boldsymbol{B}_{z-} = \boldsymbol{B}_{z+} = \left(10^{-7}\ \mathrm{T\cdot m/A}\right) \times \frac{2(20\,\mathrm{A})}{\sqrt{(0.060\,\mathrm{m})^2 + (0.080\,\mathrm{m})^2}} = 40.0\,\boldsymbol{\mu}\mathrm{T}$$

Noting that the z components add to zero, express the resultant magnetic field at $z = +8.0$ cm:

$$\vec{\boldsymbol{B}}(8.0\,\mathrm{cm}) = 2(40.0\,\mu\mathrm{T})\cos\theta\,\hat{\boldsymbol{j}} = 2(40.0\,\mu\mathrm{T})(0.60)\hat{\boldsymbol{j}} = \boxed{(48\,\mu\mathrm{T})\hat{\boldsymbol{j}}}$$

(*b*) Apply the right-hand rule to show that, for the currents antiparallel with the current in the wire at $y = -6.0$ cm in the negative x direction, the directions of the fields are as shown to the right:



Noting that the y components add to zero, express the resultant magnetic field at $z = +8.0$ cm:

$$\vec{\boldsymbol{B}}(8.0\,\mathrm{cm}) = -2(40.0\,\mu\mathrm{T})\sin\theta\,\hat{\boldsymbol{k}} = -2(40.0\,\mu\mathrm{T})(0.80)\hat{\boldsymbol{k}} = \boxed{-(64\,\mu\mathrm{T})\hat{\boldsymbol{k}}}$$

30 • Find the magnitude of the force per unit length exerted by one wire on the other.

Picture the Problem Let + denote the wire (and current) at $y = +6.0$ cm and − the wire (and current) at $y = -6.0$ cm. The forces per unit length the wires exert on each other are action and reaction forces and hence are equal in magnitude. We can use $F = I\ell B$ to express the force on either wire and $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to express the magnetic field at the location of either wire due to the current in the other.

Express the force exerted on either wire:

$$F = I\ell B$$

Express the magnetic field at either location due to the current in the wire at the other location:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Substitute for B to obtain:

$$F = I\ell\left(\frac{\mu_0}{4\pi}\frac{2I}{R}\right) = \frac{2\ell\mu_0}{4\pi}\frac{I^2}{R}$$

Divide both sides of the equation by ℓ to obtain:

$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$$

Substitute numerical values and evaluate F/ℓ:

$$\frac{F}{\ell} = \frac{2\left(10^{-7}\ \mathrm{T\cdot m/A}\right)(20\ \mathrm{A})^2}{0.12\ \mathrm{m}} = \boxed{0.67\ \mathrm{mN/m}}$$

31 • Two long, straight parallel wires 8.6 cm apart carry equal currents. The wires repel each other with a force per unit length of 3.6 nN/m. (*a*) Are the currents parallel or anti-parallel? Explain your answer. (*b*) Determine the current in each wire.

Picture the Problem We can use $\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R}$ to relate the force per unit length each current-carrying wire exerts on the other to their common current.

(*a*) Because the currents repel, they are antiparallel.

(*b*)The force per unit length experienced by each wire is given by:

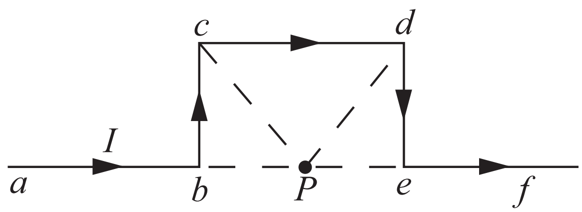
$$\frac{F}{\ell} = \frac{2\mu_0}{4\pi}\frac{I^2}{R} \Rightarrow I = \sqrt{\frac{4\pi R}{2\mu_0}\frac{F}{\ell}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{(8.6\,\text{cm})}{2(10^{-7}\,\text{T}\cdot\text{m/A})}(3.6\,\text{nN/m})} = \boxed{39\,\text{mA}}$$

32 •• The current in the wire shown in Figure 27-52 is 8.0 A. Find the magnetic field at point P.

Picture the Problem Note that the current segments *a-b* and *e-f* do not contribute to the magnetic field at point P. The current in the segments *b-c*, *c-d*, and *d-e* result in a magnetic field at P that points into the plane of the paper. Note that the angles *bPc* and *ePd* are 45° and use the expression for B due to a straight wire segment to find the contributions to the field at P of segments *bc*, *cd*, and *de*.



Express the resultant magnetic field at P:

$$B = B_{bc} + B_{cd} + B_{de}$$

Express the magnetic field due to a straight line segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_1 + \sin\theta_2) \qquad (1)$$

Use equation (1) to express B_{bc} and B_{de}:

$$B_{bc} = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin 45° + \sin 0°) = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Use equation (1) to express B_{cd}:

$$B_{cd} = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin 45° + \sin 45°) = 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute to obtain:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + 2\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° + \frac{\mu_0}{4\pi}\frac{I}{R}\sin 45° = 4\frac{\mu_0}{4\pi}\frac{I}{R}\sin 45°$$

Substitute numerical values and evaluate B:

$$B = 4\left(10^{-7}\ \mathrm{T\cdot m/A}\right)\frac{8.0\,\mathrm{A}}{0.010\,\mathrm{m}}\sin 45° = \boxed{0.23\,\mathrm{mT\ into\ the\ page}}$$

33 •• **[SSM]** As a student technician, you are preparing a lecture demonstration on ″magnetic suspension. ″ You have a 16-cm long straight rigid wire that will be suspended by flexible conductive lightweight leads above a long straight wire. Currents that are equal but are in opposite directions will be established in the two wires so the 16-cm wire ″floats″ a distance *h* above the long wire with no tension in its suspension leads. If the mass of the 16-cm wire is 14 g and if *h* (the distance between the central axes of the two wires) is 1.5 mm, what should their common current be?

Picture the Problem The forces acting on the wire are the upward magnetic force F_B and the downward gravitational force *mg*, where *m* is the mass of the wire. We can use a condition for translational equilibrium and the expression for the force per unit length between parallel current-carrying wires to relate the required current to the mass of the wire, its length, and the separation of the two wires.

Apply $\sum F_y = 0$ to the floating wire to obtain:

$$F_B - mg = 0$$

Express the repulsive force acting on the upper wire:

$$F_B = 2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R}$$

Substitute to obtain:

$$2\frac{\mu_0}{4\pi}\frac{I^2\ell}{R} - mg = 0 \Rightarrow I = \sqrt{\frac{4\pi mgR}{2\mu_0\ell}}$$

Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{\left(14\times10^{-3}\ \mathrm{kg}\right)\left(9.81\,\mathrm{m/s^2}\right)\left(1.5\times10^{-3}\ \mathrm{m}\right)}{2\left(10^{-7}\ \mathrm{T\cdot m/A}\right)\left(0.16\,\mathrm{m}\right)}} = \boxed{80\,\mathrm{A}}$$

34 •• Three long, parallel straight wires pass through the vertices of an equilateral triangle that has sides equal to 10 cm, as shown in Figure 27-53. The dot indicates that the direction of the current is out of the page and a cross indicates that the direction of the current is into the page. If each current is 15 A, find (*a*) the magnetic field at the location of the upper wire due to the currents in the two lower wires and (*b*) the force per unit length on the upper wire.

Picture the Problem (*a*) We can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B=\frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents. (*b*) Note that the forces on the upper wire are away from and directed along the lines to the lower wire and that their horizontal components cancel. We can use $\frac{F}{\ell}=2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the upward direction (the *y* direction) acting on the top wire.

(*a*) Noting, from the geometry of the wires, the magnetic field vectors both are at an angle of 30° with the horizontal and that their *y* components cancel, express the resultant magnetic field:

$$\vec{\boldsymbol{B}}=2\frac{\mu_0}{4\pi}\frac{2I}{R}\cos 30°\hat{\boldsymbol{i}}$$

Substitute numerical values and evaluate *B*:

$$B=2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(15\ \text{A})}{0.10\ \text{m}}\cos 30°$$
$$=\boxed{52\ \mu\text{T toward the right}}$$

(*b*) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell}=2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the horizontal components add up to zero, express the net upward force per unit length on the upper wire:

$$\sum\frac{F_y}{\ell}=2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30°+2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30°$$
$$=4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 30°$$

Substitute numerical values and evaluate $\sum\frac{F_y}{\ell}$:

$$\sum\frac{F_y}{\ell}=4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(15\ \text{A})^2}{0.10\ \text{m}}\cos 30°$$
$$=\boxed{7.8\times10^{-4}\ \text{N/m up the page}}$$

35 •• Rework Problem 34 with the current in the lower right corner of Figure 27- 53 reversed.

Picture the Problem (*a*) We can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ to find the magnitude of the resultant field due to these currents. (*b*) Note that the forces on the upper wire are away from and directed along the lines to the lower wire and that their horizontal components cancel. We can use $\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$ to find the resultant force in the upward direction (the *y* direction) acting on the top wire.

(*a*) Noting, from the geometry of the wires, that the magnetic field vectors both are at an angle of 30° with the horizontal and that their *x* components cancel, express the resultant magnetic field:

$$\vec{\boldsymbol{B}} = -2\frac{\mu_0}{4\pi}\frac{2I}{R}\sin 30°\,\hat{\boldsymbol{j}}$$

Substitute numerical values and evaluate *B*:

$$B = 2\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{2(15\,\text{A})}{0.10\,\text{m}}\sin 30° = \boxed{30\,\mu\text{T down the page}}$$

(*b*) Express the force per unit length each of the lower wires exerts on the upper wire:

$$\frac{F}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}$$

Noting that the vertical components add up to zero, express the net force per unit length acting to the right on the upper wire:

$$\sum\frac{F_x}{\ell} = 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60° + 2\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60° = 4\frac{\mu_0}{4\pi}\frac{I^2}{R}\cos 60°$$

Substitute numerical values and evaluate $\sum\frac{F_x}{\ell}$:

$$\sum\frac{F_x}{\ell} = 4\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(15\,\text{A})^2}{0.10\,\text{m}}\cos 60° = \boxed{4.5\times10^{-4}\ \text{N/m toward the right}}$$

36 •• An infinitely long wire lies along the x axis, and carries current I in the $+x$ direction. A second infinitely long wire lies along the y axis, and carries current I in the $+y$ direction. At what points in the $z = 0$ plane is the resultant magnetic field zero?

Picture the Problem Let the numeral 1 denote the current flowing in the positive x direction and the magnetic field resulting from it and the numeral 2 denote the current flowing in the positive y direction and the magnetic field resulting from it. We can express the magnetic field anywhere in the xy plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the set of points that satisfy this condition.

Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \qquad (1)$$

Express the magnetic field due to the current flowing in the positive x direction:

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{y}\hat{k}$$

Express the magnetic field due to the current flowing in the positive y direction:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_2}{x}\hat{k}$$

Substitute for $\vec{B}_1$ and $\vec{B}_2$ in equation (1) and simplify to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{y}\hat{k} - \frac{\mu_0}{4\pi}\frac{2I}{x}\hat{k} = \left(\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x}\right)\hat{k}$$

because $I = I_1 = I_2$.

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\frac{2I}{y} - \frac{\mu_0}{4\pi}\frac{2I}{x} = 0 \Rightarrow x = y.$$

$\vec{B} = 0$ everywhere on the plane that contains both the z axis and the line $y = x$ in the $z = 0$ plane.

37 •• **[SSM]** An infinitely long wire lies along the z axis and carries a current of 20 A in the $+z$ direction. A second infinitely long wire that is parallel to the z and intersects the x axis at x = 10.0 cm. (a) Find the current in the second wire if the magnetic field is zero at (2.0 cm, 0, 0) is zero. (b) What is the magnetic field at (5.0 cm, 0, 0)?

Picture the Problem Let the numeral 1 denote the current flowing along the positive *z* axis and the magnetic field resulting from it and the numeral 2 denote the current flowing in the wire located at $x = 10$ cm and the magnetic field resulting from it. We can express the magnetic field anywhere in the *xy* plane using $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule and then impose the condition that $\vec{B} = 0$ to determine the current that satisfies this condition.

(*a*) Express the resultant magnetic field due to the two current-carrying wires:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \qquad (1)$$

Express the magnetic field at $x = 2.0$ cm due to the current flowing in the positive *z* direction:

$$\vec{B}_1(2.0\,\text{cm}) = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{2.0\,\text{cm}}\right)\hat{j}$$

Express the magnetic field at $x = 2.0$ cm due to the current flowing in the wire at $x = 10.0$ cm:

$$\vec{B}_2(2.0\,\text{cm}) = -\frac{\mu_0}{4\pi}\left(\frac{2I_2}{8.0\,\text{cm}}\right)\hat{j}$$

Substitute for $\vec{B}_1$ and $\vec{B}_2$ in equation (1) and simplify to obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{2.0\,\text{cm}}\right)\hat{j} - \frac{\mu_0}{4\pi}\left(\frac{2I_2}{8.0\,\text{cm}}\right)\hat{j} = \left(\frac{\mu_0}{4\pi}\frac{2I_1}{2.0\,\text{cm}} - \frac{\mu_0}{4\pi}\frac{2I_2}{8.0\,\text{cm}}\right)\hat{j}$$

For $\vec{B} = 0$:

$$\frac{\mu_0}{4\pi}\left(\frac{2I_1}{2.0\,\text{cm}}\right) - \frac{\mu_0}{4\pi}\left(\frac{2I_2}{8.0\,\text{cm}}\right) = 0$$

or

$$\frac{\boldsymbol{I}_1}{2.0} - \frac{\boldsymbol{I}_2}{8.0} = 0 \Rightarrow I_2 = 4I_1$$

Substitute numerical values and evaluate I_2:

$$I_2 = 4(20\,\text{A}) = \boxed{80\,\text{A}}$$

(*b*) Express the magnetic field at $x = 5.0$ cm:

$$\vec{B}(5.0\text{ cm}) = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{5.0\,\text{cm}}\right)\hat{j} - \frac{\mu_0}{4\pi}\left(\frac{2I_2}{5.0\,\text{cm}}\right)\hat{j} = \frac{2\mu_0}{4\pi(5.0\,\text{cm})}(I_1 - I_2)\hat{j}$$

Substitute numerical values and evaluate $\vec{B}(5.0\,\text{cm})$:

$$\vec{B}(5.0\,\text{cm}) = \frac{2\left(10^{-7}\,\text{T}\cdot\text{m/A}\right)}{5.0\,\text{cm}}(20\,\text{A} - 80\,\text{A})\hat{j} = \boxed{-(0.24\,\text{mT})\hat{j}}$$

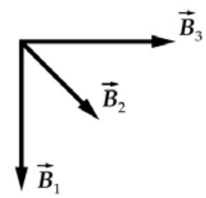
38 •• Three long parallel wires are at the corners of a square, as shown in Figure 27- 54. The wires each carry a current I. Find the magnetic field at the unoccupied corner of the square when (*a*) all the currents are into the page, (*b*) I_1 and I_3 are into the page and I_2 is out, and (*c*) I_1 and I_2 are into the page and I_3 is out. Your answers should be in terms of I and L.

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive x axis to the right and the positive y axis upward. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at the unoccupied corner due to each of the currents, and superimpose these fields to find the resultant field.

(*a*) Express the resultant magnetic field at the unoccupied corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the unoccupied corner are as shown to the right:



Express the magnetic field at the unoccupied corner due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

Express the magnetic field at the unoccupied corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(\hat{i} - \hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i} - \hat{j}\right)$$

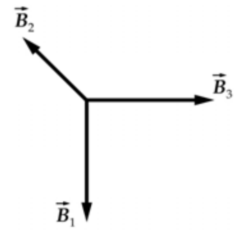
Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\begin{aligned}\vec{B} &= -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i}-\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j}+\frac{1}{2}\left(\hat{i}-\hat{j}\right)+\hat{i}\right)\\ &= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1+\frac{1}{2}\right)\hat{i}+\left(-1-\frac{1}{2}\right)\hat{j}\right] = \boxed{\frac{3\mu_0 I}{4\pi L}\left[\hat{i}-\hat{j}\right]}\end{aligned}$$

(*b*) When I_2 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



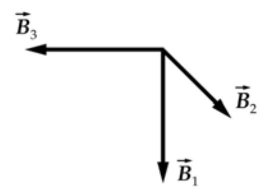
Express the magnetic field at the unoccupied corner due to the current I_2:

$$\begin{aligned}\vec{B}_2 &= \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(-\hat{i}+\hat{j}\right)\\ &= \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i}+\hat{j}\right)\end{aligned}$$

Substitute in equation (1) and simplify to obtain:

$$\begin{aligned}\vec{B} &= -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(-\hat{i}+\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j}+\frac{1}{2}\left(-\hat{i}+\hat{j}\right)+\hat{i}\right)\\ &= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(1-\frac{1}{2}\right)\hat{i}+\left(-1+\frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\frac{1}{2}\hat{i}-\frac{1}{2}\hat{j}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[\hat{i}-\hat{j}\right]}\end{aligned}$$

(*c*) When I_1 and I_2 are in and I_3 is out of the paper the magnetic fields at the unoccupied corner are as shown to the right:



From (*a*) or (*b*) we have:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j}$$

From (*a*) we have:

$$\begin{aligned}\vec{B}_2 &= \frac{\mu_0}{4\pi}\frac{2I}{L\sqrt{2}}\cos 45°\left(\hat{i}-\hat{j}\right)\\ &= \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i}-\hat{j}\right)\end{aligned}$$

Express the magnetic field at the unoccupied corner due to the current I_3:

$$\vec{B}_3 = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{L}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2L}\left(\hat{i}-\hat{j}\right) - \frac{\mu_0}{4\pi}\frac{2I}{L}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{L}\left(-\hat{j}+\frac{1}{2}\left(\hat{i}-\hat{j}\right)-\hat{i}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{L}\left[\left(-1+\frac{1}{2}\right)\hat{i}+\left(-1-\frac{1}{2}\right)\hat{j}\right] = \boxed{\frac{\mu_0 I}{4\pi L}\left[-\hat{i}-3\hat{j}\right]}$$

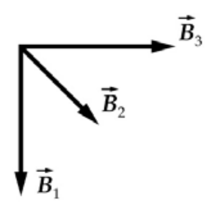
39 •• [SSM] Four long, straight parallel wires each carry current *I*. In a plane perpendicular to the wires, the wires are at the corners of a square of side length *a*. Find the magnitude of the force per unit length on one of the wires if (*a*) all the currents are in the same direction and (*b*) the currents in the wires at adjacent corners are oppositely directed.

Picture the Problem Choose a coordinate system with its origin at the lower left-hand corner of the square, the positive *x* axis to the right and the positive *y* axis upward. Let the numeral 1 denote the wire and current in the upper left-hand corner of the square, the numeral 2 the wire and current in the lower left-hand corner (at the origin) of the square, and the numeral 3 the wire and current in the lower right-hand corner of the square. We can use $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$ and the right-hand rule to find the magnitude and direction of the magnetic field at, say, the upper right-hand corner due to each of the currents, superimpose these fields to find the resultant field, and then use $F = I\ell B$ to find the force per unit length on the wire.

(*a*) Express the resultant magnetic field at the upper right-hand corner:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \qquad (1)$$

When all the currents are into the paper their magnetic fields at the upper right-hand corner are as shown to the right:



Express the magnetic field due to the current I_1:

$$\vec{B}_1 = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j}$$

Express the magnetic field due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(\hat{i}-\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{i}-\hat{j}\right)$$

Express the magnetic field due to the current I_3:

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(\hat{i}-\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j}+\frac{1}{2}\left(\hat{i}-\hat{j}\right)+\hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1+\frac{1}{2}\right)\hat{i}+\left(-1-\frac{1}{2}\right)\hat{j}\right] = \frac{3\mu_0 I}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

Using the expression for the magnetic force on a current-carrying wire, express the force per unit length on the wire at the upper right-hand corner:

$$\frac{F}{\ell} = BI \qquad (2)$$

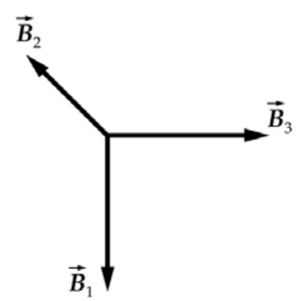
Substitute to obtain:

$$\frac{\vec{F}}{\ell} = \frac{3\mu_0 I^2}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

and

$$\frac{F}{\ell} = \sqrt{\left(\frac{3\mu_0 I^2}{4\pi a}\right)^2 + \left(\frac{3\mu_0 I^2}{4\pi a}\right)^2} = \boxed{\frac{3\sqrt{2}\mu_0 I^2}{4\pi a}}$$

(*b*) When the current in the upper right-hand corner of the square is out of the page, and the currents in the wires at adjacent corners are oppositely directed, the magnetic fields at the upper right-hand are as shown to the right:



Express the magnetic field at the upper right-hand corner due to the current I_2:

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{a\sqrt{2}}\cos 45°\left(-\hat{i}+\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i}+\hat{j}\right)$$

Using $\vec{B}_1$ and $\vec{B}_3$ from (a), substitute in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{2I}{a}\hat{j} + \frac{\mu_0}{4\pi}\frac{2I}{2a}\left(-\hat{i}+\hat{j}\right) + \frac{\mu_0}{4\pi}\frac{2I}{a}\hat{i} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left(-\hat{j}+\frac{1}{2}\left(-\hat{i}+\hat{j}\right)+\hat{i}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\left(1-\frac{1}{2}\right)\hat{i}+\left(-1+\frac{1}{2}\right)\hat{j}\right] = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\frac{1}{2}\hat{i}-\frac{1}{2}\hat{j}\right] = \frac{\mu_0 I}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

Substitute in equation (2) to obtain:

$$\frac{\vec{F}}{\ell} = \frac{\mu_0 I^2}{4\pi a}\left[\hat{i}-\hat{j}\right]$$

and

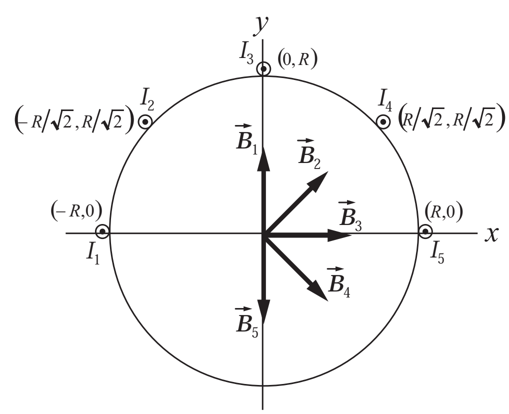
$$\frac{F}{\ell} = \sqrt{\left(\frac{\mu_0 I^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 I^2}{4\pi a}\right)^2} = \boxed{\frac{\sqrt{2}\mu_0 I^2}{4\pi a}}$$

40 •• Five long straight current-carrying wires are parallel to the z axis, and each carries a current I in the $+z$ direction. The wires each are a distance R from the z axis. Two of the wires intersect the x axis, one at $x = R$ and the other at $x = -R$. Another wire intersects the y axis at $y = R$. One of the remaining wires intersects the $z = 0$ plane at the point $\left(R/\sqrt{2}, R/\sqrt{2}\right)$ and the last remaining wire intersects the $z = 0$ plane at the point $\left(-R/\sqrt{2}, R/\sqrt{2}\right)$. Find the magnetic field on the z axis.

Picture the Problem The configuration is shown in the adjacent figure. Here the z axis points out of the plane of the paper, the x axis points to the right, the y axis points up. We can use

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

and the right-hand rule to find the magnetic field due to the current in each wire and add these magnetic fields vectorially to find the resultant field.



Express the resultant magnetic field on the z axis:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 \quad (1)$$

$\vec{B}_1$ is given by: $\vec{B}_1 = B\hat{j}$

$\vec{B}_2$ is given by: $\vec{B}_2 = (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j}$

$\vec{B}_3$ is given by: $\vec{B}_3 = B\hat{i}$

$\vec{B}_4$ is given by: $\vec{B}_4 = (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j}$

$\vec{B}_5$ is given by: $\vec{B}_5 = -B\hat{j}$

Substitute for $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, $\vec{B}_4$, and $\vec{B}_5$ in equation (1) and simplify to obtain:

$$\begin{aligned}\vec{B} &= B\hat{j} + (B\cos 45°)\hat{i} + (B\sin 45°)\hat{j} + B\hat{i} + (B\cos 45°)\hat{i} - (B\sin 45°)\hat{j} - B\hat{j}\\ &= (B\cos 45°)\hat{i} + B\hat{i} + (B\cos 45°)\hat{i} = (B + 2B\cos 45°)\hat{i} = \left(1+\sqrt{2}\right)B\hat{i}\end{aligned}$$

Express B due to each current at $z = 0$: $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$

Substitute for B to obtain: $\vec{B} = \boxed{\left(1+\sqrt{2}\right)\frac{\mu_0 I}{2\pi R}\hat{i}}$

Magnetic Field Due to a Current-carrying Solenoid

41 •• **[SSM]** A solenoid that has length 30 cm, radius 1.2 cm, and 300 turns carries a current of 2.6 A. Find the magnitude of the magnetic field on the axis of the solenoid (*a*) at the center of the solenoid, (*b*) at one end of the solenoid.

Picture the Problem We can use $B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}} + \frac{a}{\sqrt{a^2+R^2}}\right)$ to find B at any point on the axis of the solenoid. Note that the number of turns per unit length for this solenoid is 300 turns/0.30 m = 1000 turns/m.

Express the magnetic field at any point on the axis of the solenoid: $B_x = \frac{1}{2}\mu_0 nI\left(\frac{b}{\sqrt{b^2+R^2}} + \frac{a}{\sqrt{a^2+R^2}}\right)$

Substitute numerical values to obtain:

$$B_x = \tfrac{1}{2}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(1000)(2.6\,\mathrm{A})\left(\frac{b}{\sqrt{b^2+(0.012\,\mathrm{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\,\mathrm{m})^2}}\right)$$
$$= (1.634\,\mathrm{mT})\left(\frac{b}{\sqrt{b^2+(0.012\,\mathrm{m})^2}}+\frac{a}{\sqrt{a^2+(0.012\,\mathrm{m})^2}}\right)$$

(*a*) Evaluate B_x for $a = b = 0.15$ m:

$$B_x(0.15\,\mathrm{m}) = (1.634\,\mathrm{mT})\left(\frac{0.15\,\mathrm{m}}{\sqrt{(0.15\,\mathrm{m})^2+(0.012\,\mathrm{m})^2}}+\frac{0.15\,\mathrm{m}}{\sqrt{(0.15\,\mathrm{m})^2+(0.012\,\mathrm{m})^2}}\right)$$
$$= \boxed{3.3\,\mathrm{mT}}$$

(*b*) Evaluate B_x (= B_{end}) for $a = 0$ and $b = 0.30$ m:

$$B_x(0.30\,\mathrm{m}) = (1.634\,\mathrm{mT})\left(\frac{0.30\,\mathrm{m}}{\sqrt{(0.30\,\mathrm{m})^2+(0.012\,\mathrm{m})^2}}\right) = \boxed{1.6\,\mathrm{mT}}$$

Note that $B_{\mathrm{end}} \approx \tfrac{1}{2}B_{\mathrm{center}}$.

42 • A solenoid is 2.7-m long, has a radius of 0.85 cm, and has 600 turns. It carries a current I of 2.5 A. What is the magnitude of the magnetic field B inside the solenoid and far from either end?

Picture the Problem We can use $B = \mu_0 nI$ to find the magnetic field inside the solenoid and far from either end.

The magnetic field inside the solenoid and far from either end is given by:

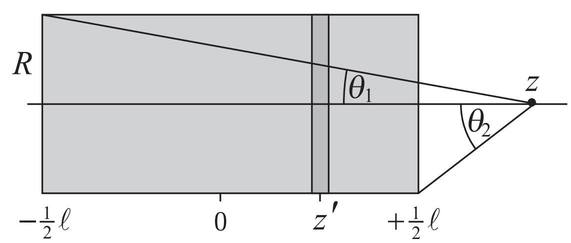
$$B = \mu_0 nI$$

Substitute numerical values and evaluate B:

$$B = \left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)\left(\frac{600}{2.7\,\mathrm{m}}\right)(2.5\,\mathrm{A})$$
$$= \boxed{0.70\,\mathrm{mT}}$$

43 •• A solenoid has n turns per unit length, has a radius R, and carries a current I. Its axis coincides with the z axis with one end at $z=-\frac{1}{2}\ell$ and the other end at $z=+\frac{1}{2}\ell$. Show that the magnitude of the magnetic field at a point on the z axis in the interval $z > \frac{1}{2}\ell$ is given by $B=\frac{1}{2}\mu_0 nI\left(\cos\theta_1-\cos\theta_2\right)$, where the angles are related to the geometry by: $\cos\theta_1=\left(z+\frac{1}{2}\ell\right)\Big/\sqrt{\left(z+\frac{1}{2}\ell\right)^2+R^2}$ and $\cos\theta_2=\left(z-\frac{1}{2}\ell\right)\Big/\sqrt{\left(z-\frac{1}{2}\ell\right)^2+R^2}$.

Picture the Problem The solenoid, extending from $z=-\ell/2$ to $z=\ell/2$, with the origin at its center, is shown in the following diagram. To find the field at the point whose coordinate is z outside the solenoid we can determine the field at z due to an infinitesimal segment of the solenoid of width dz' at z', and then integrate from $z=-\ell/2$ to $z=\ell/2$. We'll treat the segment as a coil of thickness ndz' carrying a current I.



Express the field dB at the axial point whose coordinate is z:

$$dB=\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left[\left(z-z'\right)^2+R^2\right]^{3/2}}dz'$$

Integrate dB from $z=-\ell/2$ to $z=\ell/2$ to obtain:

$$B=\frac{\mu_0 nIR^2}{2}\int_{-\ell/2}^{\ell/2}\frac{dz'}{\left[\left(z-z'\right)^2+R^2\right]^{3/2}}=\frac{\mu_0 nI}{2}\left(\frac{z+\ell/2}{\sqrt{\left(z+\ell/2\right)^2+R^2}}-\frac{z-\ell/2}{\sqrt{\left(z-\ell/2\right)^2+R^2}}\right)$$

Refer to the diagram to express $\cos\theta_1$ and $\cos\theta_2$:

$$\cos\theta_1=\frac{z+\frac{1}{2}\ell}{\left[R^2+\left(z+\frac{1}{2}\ell\right)^2\right]^{1/2}}$$

and

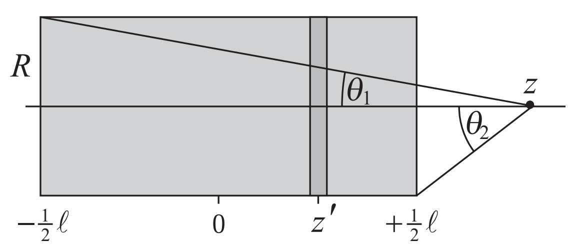
$$\cos\theta_2=\frac{z-\frac{1}{2}\ell}{\left[R^2+\left(z-\frac{1}{2}\ell\right)^2\right]^{1/2}}$$

Substitute for the terms in parentheses in the expression for B to obtain:

$$B=\boxed{\frac{1}{2}\mu_0 nI\left(\cos\theta_1-\cos\theta_2\right)}$$

44 ••• In Problem 43, an expression for the magnitude of the magnetic field along the axis of a solenoid is given. For $z >> \ell$ and $\ell >> R$, the angles θ_1 and θ_2 are very small, so the small-angle approximations $\cos\theta \approx 1-\frac{1}{2}\theta^2$ and $\sin\theta \approx \tan\theta \approx \theta$ are highly accurate. (*a*) Draw a diagram and use it to show that, for these conditions, the angles can be approximated as $\theta_1 \approx R/\left(z+\frac{1}{2}\ell\right)$ and $\theta_2 \approx R/\left(z-\frac{1}{2}\ell\right)$. (*b*) Using these approximations, show that the magnetic field at a points on the *z* axis where $z >> \ell$ can be written as $B=\frac{\mu_0}{4\pi}\left(\frac{q_n}{r_2^2}-\frac{q_n}{r_1^2}\right)$ where $r_2 = z-\frac{1}{2}\ell$ is the distance to the near end of the solenoid, $r_1 = z+\frac{1}{2}\ell$ is the distance to the far end, and the quantity q_m is defined by $q_m = nI\pi R^2 = \mu/\ell$, where $\mu = NI\pi R^2$ is the magnitude of the magnetic moment of the solenoid.

Picture the Problem (*a*) We can use the results of Problem 43, together with the small angle approximation for the cosine and tangent functions, to show that θ_1 and θ_2 are as given in the problem statement and that (*b*) $B=\frac{\mu_0}{4\pi}\left(\frac{q_m}{r_2^2}-\frac{q_m}{r_1^2}\right)$. The angles θ_1 and θ_2 are shown in the diagram. Note that $\tan\theta_1 = R/\left(z+\ell/2\right)$ and $\tan\theta_2 = R/\left(z-\ell/2\right)$.



(*a*) Apply the small angle approximation $\tan\theta \approx \theta$ to obtain:

$$\theta_1 \approx \boxed{\frac{R}{z+\frac{1}{2}\ell}} \text{ and } \theta_2 \approx \boxed{\frac{R}{z-\frac{1}{2}\ell}}$$

(*b*) Express the magnetic field outside the solenoid:

$$B=\tfrac{1}{2}\mu_0 nI\left(\cos\theta_1-\cos\theta_2\right)$$

Apply the small angle approximation for the cosine function to obtain:

$$\cos\theta_1 = 1-\tfrac{1}{2}\left(\frac{R}{z+\frac{1}{2}\ell}\right)^2$$

and

$$\cos\theta_2 = 1-\tfrac{1}{2}\left(\frac{R}{z-\frac{1}{2}\ell}\right)^2$$

Substitute and simplify to obtain:

$$B = \tfrac{1}{2}\mu_0 nI\left[1-\tfrac{1}{2}\left(\frac{R}{z+\frac{1}{2}\ell}\right)^2 - 1 + \tfrac{1}{2}\left(\frac{R}{z-\frac{1}{2}\ell}\right)^2\right] = \tfrac{1}{4}\mu_0 nIR^2\left[\frac{1}{\left(z-\frac{1}{2}\ell\right)^2} - \frac{1}{\left(z+\frac{1}{2}\ell\right)^2}\right]$$

Let $r_1 = z + \frac{1}{2}\ell$ be the distance to the near end of the solenoid, $r_2 = z - \frac{1}{2}\ell$ the distance to the far end, and $q_m = nI\pi R^2 = \mu/\ell$, where $\mu = nI\pi R^2$ is the magnetic moment of the solenoid to obtain:

$$B = \boxed{\frac{\mu_0}{4\pi}\left(\frac{q_\mathrm{m}}{r_2^2} - \frac{q_\mathrm{m}}{r_1^2}\right)}$$

Using Ampère's Law

45 • **[SSM]** A long, straight, thin-walled cylindrical shell of radius R carries a current I parallel to the central axis of the shell. Find the magnetic field (including direction) both inside and outside the shell.

Picture the Problem We can apply Ampère's law to a circle centered on the axis of the cylinder and evaluate this expression for $r < R$ and $r > R$ to find B inside and outside the cylinder. We can use the right-hand rule to determine the direction of the magnetic fields.

Apply Ampère's law to a circle centered on the axis of the cylinder:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

Note that, by symmetry, the field is the same everywhere on this circle.

Evaluate this expression for $r < R$:

$$\oint_C \vec{B}_{\text{inside}} \cdot d\vec{\ell} = \mu_0(0) = 0$$

Solve for B_{inside} to obtain:

$$B_{\text{inside}} = \boxed{0}$$

Evaluate this expression for $r > R$:

$$\oint_C \vec{B}_{\text{outside}} \cdot d\vec{\ell} = B(2\pi R) = \mu_0 I$$

Solve for B_{outside} to obtain:

$$B_{\text{outside}} = \boxed{\frac{\mu_0 I}{2\pi R}}$$

The direction of the magnetic field is in the direction of the curled fingers of your right hand when you grab the cylinder with your right thumb in the direction of the current.

46 • In Figure 27-55, one current is 8.0 A into the page, the other current is 8.0 A out of the page, and each curve is a circular path. (*a*) Find $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}}$ for each path, assuming that each integral is to be evaluated in the counterclockwise direction. (*b*) Which path, if any, can be used to find the combined magnetic field of these currents?

Picture the Problem We can use Ampère's law, $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = \mu_0 I_C$, to find the line integral $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell}$ for each of the three paths.

(*a*) Noting that the angle between $\vec{\boldsymbol{B}}$ and $d\vec{\ell}$ is 180°, evaluate $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell}$ for C_1:

$$\oint_{C_1} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boxed{-\mu_0(8.0\,\mathrm{A})}$$

The positive tangential direction on C_1 is counterclockwise. Therefore, in accord with convention (a right-hand rule), the positive normal direction for the flat surface bounded by C_1 is out of the page. $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell}$ is negative because the current through the surface is in the negative direction (into the page).

Noting that the net current bounded by C_2 is zero, evaluate $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell}$:

$$\oint_{C_2} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0(8.0\,\mathrm{A} - 8.0\,\mathrm{A}) = \boxed{0}$$

Noting that the angle between $\vec{B}$ and $d\vec{\ell}$ is 0°, Evaluate $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell}$ for C_3:

$$\oint_{C_3} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boxed{+\mu_0(8.0\,\mathrm{A})}$$

(*b*) None of the paths can be used to find $\vec{\boldsymbol{B}}$ because the current configuration does not have cylindrical symmetry, which means that $\vec{\boldsymbol{B}}$ cannot be factored out of the integral.

47 •• **[SSM]** Show that a uniform magnetic field that has no fringing field, such as that shown in Figure 27- 56 is impossible because it violates Ampère's law. Do this calculation by applying Ampère's law to the rectangular curve shown by the dashed lines.

Determine the Concept The contour integral consists of four portions, two horizontal portions for which $\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = 0$, and two vertical portions. The portion within the magnetic field gives a nonvanishing contribution, whereas the portion outside the field gives no contribution to the contour integral. Hence, the contour integral has a finite value. However, it encloses no current; thus, it appears that Ampère's law is violated. What this demonstrates is that there must be a fringing field so that the contour integral does vanish.

48 •• A coaxial cable consists of a solid conducting cylinder that has a radius equal to 1.00 mm and a conducting cylindrical shell that has an inner radius equal to 2.00 mm and an outer radius equal to 3.00 mm. The solid cylinder carries a current of 15.0 A parallel to the central axis. The cylindrical shell carries and current of 15.0 A in the opposite direction. Assume that the current densities are uniformly distributed in both conductors. (*a*) Using a **spreadsheet** program or graphing calculator, graph the magnitude of the magnetic field as a function of the radial distance *r* from the central axis for $0 < R < 3.00$ mm. (*b*) What is the magnitude of the field for $R > 3.00$ mm?

Picture the Problem Let $r_1 = 1.00$ mm, $r_2 = 2.00$ mm, and $r_3 = 3.00$ mm and apply Ampère's law in each of the three regions to obtain expressions for *B* in each part of the coaxial cable and outside the coaxial cable.

(*a*) Apply Ampère's law to a circular path of radius $r < r_1$ to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the inner wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi r_1^2} \Rightarrow I_C = \frac{r^2}{r_1^2} I$$

Substitute for I_C to obtain:

$$B_{r<r_1}(2\pi r) = \mu_0 \frac{r^2}{r_1^2} I$$

Solving for $B_{r<r_1}$ yields:

$$B_{r<r_1} = \frac{2\mu_0 I}{4\pi} \frac{r}{r_1^2} \qquad (1)$$

Apply Ampère's law to a circular path of radius $r_1 < r < r_2$ to obtain:

$$B_{r_1<r<r_2}(2\pi r) = \mu_0 I$$

Solving for $B_{r_1<r<r_2}$ yields:

$$B_{r_1<r<r_2} = \frac{2\mu_0 I}{4\pi} \frac{1}{r} \qquad (2)$$

Apply Ampère's law to a circular path of radius $r_2 < r < r_3$ to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0 I_C = \mu_0(I - I')$$

where I' is the current in the outer conductor at a distance less than r from the center of the inner conductor.

Because the current is uniformly distributed over the cross section of the outer conductor:

$$\frac{I'}{\pi r^2 - \pi r_2^2} = \frac{I}{\pi r_3^2 - \pi r_2^2}$$

Solving for I' yields:

$$I' = \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I$$

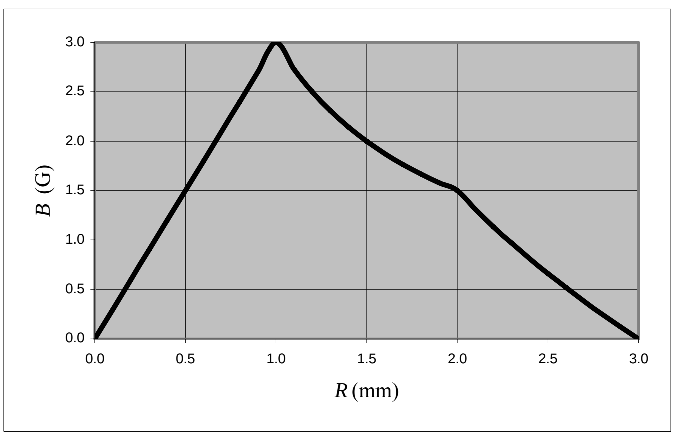
Substitute for I' to obtain:

$$B_{r_2<r<r_3}(2\pi r) = \mu_0\left(I - \frac{r^2 - r_2^2}{r_3^2 - r_2^2} I\right)$$

Solving for $B_{r_1<r<r_2}$ yields:

$$B_{r_2<r<r_3} = \frac{2\mu_0 I}{4\pi r}\left(1 - \frac{r^2 - r_2^2}{r_3^2 - r_2^2}\right) \quad (3)$$

A spreadsheet program was used to plot the following graph of equations (1), (2), and (3).



(*b*) Apply Ampère's law to a circular path of radius $r > r_3$ to obtain:

$$B_{r>r_3}(2\pi r) = \mu_0 I_C = \mu_0(I - I) = 0$$

and $B_{r>r_3} = \boxed{0}$

49 •• **[SSM]** A long cylindrical shell has an inner radius a and an outer radius b and carries a current I parallel to the central axis. Assume that within the material of the shell the current density is uniformly distributed. Find an expression for the magnitude of the magnetic field for (*a*) $0 < R < a$, (*b*) $a < R < b$, and (*c*) $R > b$.

Picture the Problem We can use Ampère's law to calculate B because of the high degree of symmetry. The current through C depends on whether R is less than the inner radius a, greater than the inner radius a but less than the outer radius b, or greater than the outer radius b.

(*a*) Apply Ampère's law to a circular path of radius $R < a$ to obtain:

$$\oint_C \vec{\boldsymbol{B}}_{r<a} \cdot d\vec{\ell} = \mu_0 I_C = \mu_0(0) = 0$$

and $B_{r<a} = \boxed{0}$

(*b*) Use the uniformity of the current over the cross-section of the conductor to express the current I' enclosed by a circular path whose radius satisfies the condition $a < R < b$:

$$\frac{I'}{\pi(R^2 - a^2)} = \frac{I}{\pi(b^2 - a^2)}$$

Solving for $I_C = I'$ yields:

$$I_C = I' = I\frac{R^2 - a^2}{b^2 - a^2}$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{\boldsymbol{B}}_{a<R<b} \cdot d\vec{\ell} = B_{a<r<b}(2\pi R)$$

$$= \mu_0 I' = \mu_0 I\frac{R^2 - a^2}{b^2 - a^2}$$

Solving for $B_{a<r<b}$ yields:

$$B_{a<r<b} = \boxed{\frac{\mu_0 I}{2\pi r}\frac{R^2 - a^2}{b^2 - a^2}}$$

(*c*) Express I_C for $R > b$:

$$I_C = I$$

Substituting in Ampère's law yields:

$$\oint_C \vec{\boldsymbol{B}}_{R>b} \cdot d\vec{\ell} = B_{r>b}(2\pi R) = \mu_0 I$$

Solve for $B_{R>b}$ to obtain:

$$B_{R>b} = \boxed{\frac{\mu_0 I}{2\pi R}}$$

50 •• Figure 27-57 shows a solenoid that has n turns per unit length and carries a current I. Apply Ampère's law to the rectangular curve shown in the figure to derive an expression for the magnitude of the magnetic field. Assume that inside the solenoid the magnetic field is uniform and parallel with the central axis, and that outside the solenoid there is no magnetic field.

Picture the Problem The number of turns enclosed within the rectangular area is *na*. Denote the corners of the rectangle, starting in the lower left-hand corner and proceeding counterclockwise, as 1, 2, 3, and 4. We can apply Ampère's law to each side of this rectangle in order to evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$.

Express the integral around the closed path *C* as the sum of the integrals along the sides of the rectangle:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = \int_{1\to2} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{2\to3} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{3\to4} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{4\to1} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}}$$

Evaluate $\int_{1\to2} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}}$:

$$\int_{1\to2} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{aB}$$

For the paths $2 \to 3$ and $4 \to 1$, $\vec{\boldsymbol{B}}$ is either zero (outside the solenoid) or is perpendicular to $d\vec{\ell}$ and so:

$$\int_{2\to3} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = \int_{4\to1} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = 0$$

For the path $3 \to 4$, $\vec{\boldsymbol{B}}$ =0 and:

$$\int_{3\to4} \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{\ell}} = 0$$

Substitute in Ampère's law to obtain:

$$\oint_C \vec{\boldsymbol{B}}\cdot d\vec{\ell} = aB + 0 + 0 + 0 = aB = \mu_0 I_C = \mu_0 naI$$

Solving for *B* yields:

$$B = \boxed{\mu_0 nI}$$

51 •• [SSM] A tightly wound 1000-turn toroid has an inner radius 1.00 cm and an outer radius 2.00 cm, and carries a current of 1.50 A. The toroid is centered at the origin with the centers of the individual turns in the $z = 0$ plane. In the $z = 0$ plane: (*a*) What is the magnetic field strength at a distance of 1.10 cm from the origin? (*b*) What is the magnetic field strength at a distance of 1.50 cm from the origin?

Picture the Problem The magnetic field inside a tightly wound toroid is given by $B = \mu_0 NI/(2\pi r)$, where $a < r < b$ and *a* and *b* are the inner and outer radii of the toroid.

Express the magnetic field of a toroid:

$$B = \frac{\mu_0 NI}{2\pi r}$$

(a) Substitute numerical values and evaluate B(1.10 cm):

$$\boldsymbol{B}(1.10\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(1000)(1.50\,\text{A})}{2\pi(1.10\,\text{cm})} = \boxed{27.3\,\text{mT}}$$

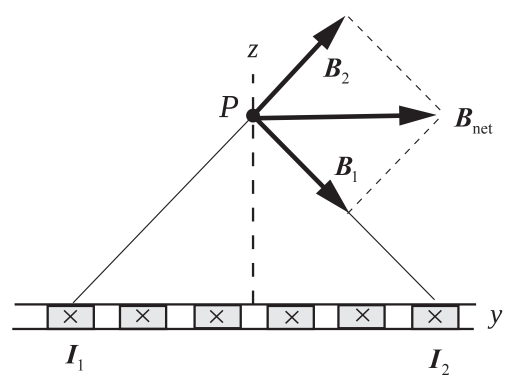
(b) Substitute numerical values and evaluate B(1.50 cm):

$$\boldsymbol{B}(1.50\,\text{cm}) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(1000)(1.50\,\text{A})}{2\pi(1.50\,\text{cm})} = \boxed{20.0\,\text{mT}}$$

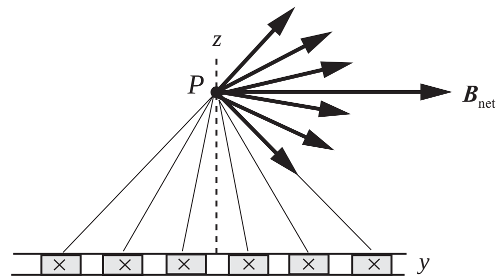
52 ••• A thin conducting sheet in the $z = 0$ plane carries current in the $-x$ direction (Figure 27-88a). The sheet extends indefinitely in all directions and the current is uniformly distributed throughout the sheet. To find the direction of the magnetic field at point P consider the field due only to the currents I_1 and I_2 in the two narrow strips shown. The strips are identical, so $I_1 = I_2$. (a) What is the direction of the magnetic field at point P due to just I_1 and I_2? Explain your answer using a sketch. (b) What is the direction of the magnetic field at point P due to the *entire* sheet? Explain your answer. (c) What is the direction of the field at a point to the right of point P (where $y \neq 0$)? Explain your answer. (d) What is the direction of the field at a point below the sheet (where $z < 0$)? Explain your answer using a sketch. (e) Apply Ampere's law to the rectangular curve (Figure 27-88b) to show that the magnetic field strength at point P is given by $B = \frac{1}{2}\mu_0\lambda$, where $\lambda = dI/dy$ is the current per unit length along the y axis.

Picture the Problem In Parts (a), (b), and (c) we can use a right-hand rule to determine the direction of the magnetic field at points above and below the infinite sheet of current. In Part (d) we can evaluate $\oint_C \vec{B}\cdot d\vec{\ell}$ around the specified path and equate it to $\mu_0 I_C$ and solve for B.

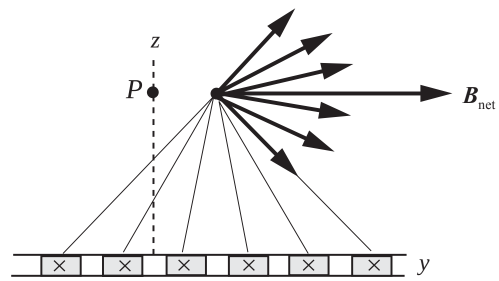
(a) Because its vertical components cancel at P, the magnetic field points to the right (i.e., in the $+y$ direction).



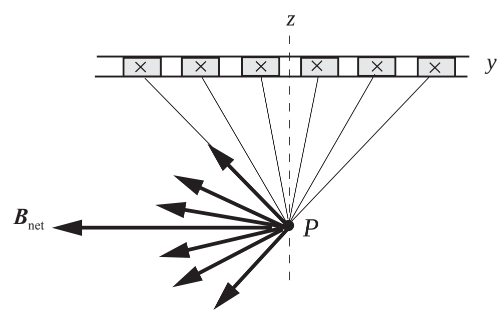
(*b*) The vertical components of the field cancel in pairs. The magnetic field is in the $+y$ direction.



(*c*) The magnetic field is in the $+y$ direction. This result follows from the same arguments that were used in (*a*) and (*b*).



(*d*) Below the sheet the magnetic field points to the left; i.e., in the $-y$ direction. The vertical components cancel in pairs.



(*e*) Express $\oint_C \vec{B}\cdot d\vec{\ell}$, in the counterclockwise direction, for the given path:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} + 2\int_{\perp} \vec{B}\cdot d\vec{\ell} \quad (1)$$

For the paths perpendicular to the sheet, $\vec{B}$ and $d\vec{\ell}$ are perpendicular to each other and:

$$\int_{\perp} \vec{B}\cdot d\vec{\ell} = 0$$

For the paths parallel to the sheet, $\vec{B}$ and $d\vec{\ell}$ are in the same direction and:

$$\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = Bw$$

Substituting in equation (1) and simplifying yields:

$$\oint_C \vec{B}\cdot d\vec{\ell} = 2\int_{\text{parallel}} \vec{B}\cdot d\vec{\ell} = 2Bw$$
$$= \mu_0 I_C = \mu_0(\lambda w)$$

Solving for B yields:

$$B = \boxed{\tfrac{1}{2}\mu_0\lambda}$$

Magnetization and Magnetic Susceptibility

53 • **[SSM]** A tightly wound solenoid is 20.0-cm long, has 400 turns, and carries a current of 4.00 A so that its axial field is in the +z direction. Find B and B_{app} at the center when (a) there is no core in the solenoid, and (b) there is a soft iron core that has a magnetization of 1.2×10^6 A/m.

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}} + \mu_0 M$ when there is an iron core with a magnetization $M = 1.2 \times 10^6$ A/m.

(a) Express the magnetic field, in the absence of a core, in the solenoid :

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app}:

$$B = B_{\text{app}} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{400}{0.200\,\text{m}}\right)(4.00\,\text{A}) = \boxed{10.1\,\text{mT}}$$

(b) With an iron core with a magnetization $M = 1.2 \times 10^6$ A/m present:

$$B_{\text{app}} = \boxed{10.1\,\text{mT}}$$

and

$$B = B_{\text{app}} + \mu_0 M = 10.1\,\text{mT} + \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(1.2 \times 10^6\ \text{A/m}\right) = \boxed{1.5\,\text{T}}$$

54 • A long tungsten-core solenoid carries a current. (a) If the core is removed while the current is held constant, does the magnetic field strength in the region inside the solenoid decrease or increase? (b) By what percentage does the magnetic field strength in the region inside the solenoid decrease or increase?

Picture the Problem We can use $B = B_{\text{app}}(1 + \chi_{\text{m}})$ to relate B and B_{app} to the magnetic susceptibility of tungsten. Dividing both sides of this equation by B_{app} and examining the value of $\chi_{\text{m, tungsten}}$ will allow us to decide whether the field inside the solenoid decreases or increases when the core is removed.

Express the magnetic field inside the solenoid with the tungsten core present B in terms of B_{app} and χ_{m}:

$$B = B_{\text{app}}(1 + \chi_{\text{m}})$$

where B_{app} is the magnetic field in the absence of the tungsten core.

Express the ratio of B to B_{app}:

$$\frac{B}{B_{\text{app}}} = 1 + \chi_{\text{m}} \quad (1)$$

(*a*) Because $\chi_{\text{m, tungsten}} > 0$:

$B > B_{\text{app}}$ and B will decrease when the tungsten core is removed.

(*b*) From equation (1), the fractional change is:

$$\chi_{\text{m}} = 6.8\times10^{-5} = \boxed{6.8\times10^{-3}\%}$$

55 • As a liquid fills the interior volume of a solenoid that carries a constant current, the magnetic field inside the solenoid *decreases* by 0.0040 percent. Determine the magnetic susceptibility of the liquid.

Picture the Problem We can use $B = B_{\text{app}}(1+\chi_{\text{m}})$ to relate B and B_{app} to the magnetic susceptibility of liquid sample.

Express the magnetic field inside the solenoid with the liquid sample present B in terms of B_{app} and $\chi_{\text{m, sample}}$:

$$B = B_{\text{app}}(1+\chi_{\text{m, sample}})$$

where B_{app} is the magnetic field in the absence of the liquid sample.

The fractional change in the magnetic field in the core is:

$$\frac{\Delta B}{B_{\text{app}}} = \chi_{\text{m, sample}}$$

Substitute numerical values and evaluate $\chi_{\text{m, sample}}$:

$$\chi_{\text{m, sample}} = \frac{\Delta B}{B_{\text{app}}} = -0.0040\% = \boxed{-4.0\times10^{-5}}$$

56 • A long thin solenoid carrying a current of 10 A has 50 turns per centimeter of length. What is the magnetic field strength in the region occupied by the interior of the solenoid when the interior is (*a*) a vacuum, (*b*) filled with aluminum, and (*c*) filled with silver?

Picture the Problem We can use $B = B_{\text{app}} = \mu_0 nI$ to find B and B_{app} at the center when there is no core in the solenoid and $B = B_{\text{app}}(1+\chi_{\text{m}})$ when there is an aluminum or silver core.

(*a*) Express the magnetic field, in the absence of a core, in the solenoid:

$$B = B_{\text{app}} = \mu_0 nI$$

Substitute numerical values and evaluate B and B_{app} :

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{\text{cm}}\right)(10\,\text{A}) = \boxed{63\,\text{mT}}$$

(*b*) With an aluminum core: $B = B_{app}(1 + \chi_m)$

Use Table 27-1 to find the value of χ_m for aluminum:

$\chi_{m,\,Al} = 2.3 \times 10^{-5}$
and
$1 + \chi_{m,\,Al} = 1 + 2.3 \times 10^{-5} \approx 1$

Substitute numerical values and evaluate B and B_{app} :

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{\text{cm}}\right)(10\,\text{A}) = \boxed{63\,\text{mT}}$$

(*c*) With a silver core: $B = B_{app}(1 + \chi_m)$

Use Table 27-1 to find the value of χ_m for silver:

$\chi_{m,\,Ag} = -2.6 \times 10^{-5}$
and
$1 + \chi_{m,\,Ag} = 1 - 2.6 \times 10^{-5} \approx 1$

Substitute numerical values and evaluate B and B_{app} :

$$B = B_{app} = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(\frac{50}{\text{cm}}\right)(10\,\text{A}) = \boxed{63\,\text{mT}}$$

57 •• [SSM] A cylinder of iron, initially unmagnetized, is cooled to 4.00 K. What is the magnetization of the cylinder at that temperature due to the influence of Earth's magnetic field of 0.300 G? Assume a magnetic moment of 2.00 Bohr magnetons per atom.

Picture the Problem We can use Curie's law to relate the magnetization M of the cylinder to its saturation magnetization M_S. The saturation magnetization is the product of the number of atoms n in the cylinder and the magnetic moment of each molecule. We can find n using the proportion $\frac{n}{N_A} = \frac{\rho_{Fe}}{M_{Fe}}$ where M_{Fe} is the molar mass of iron.

The magnetization of the cylinder is given by Curie's law:

$$M = \frac{1}{3}\frac{\mu B_{\text{app}}}{kT}M_S$$

Assuming a magnetic moment of 2.00 Bohr magnetons per atom:

$$M = \frac{1}{3}\frac{(2.00\mu_B)B_{\text{Earth}}}{kT}M_S \qquad (1)$$

The saturation magnetization is given by:

$$M_S = n\mu = n(2.00\mu_B)$$

where n is the number of atoms and μ is the magnetic moment of each molecule.

The number of atoms of iron per unit volume n can be found from the molar mass M_{Fe} of iron, the density ρ_{Fe} of iron, and Avogadro's number N_A:

$$n = \frac{\rho_{\text{Fe}}}{M_{\text{Fe}}}N_A \Rightarrow M_S = \frac{2.00\rho_{\text{Fe}}N_A\mu_B}{M_{\text{Fe}}}$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{2.00\left(7.96\times10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(6.022\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2\right)}{55.85\frac{\text{g}}{\text{mol}}\times\frac{1\,\text{kg}}{10^3\ \text{g}}}$$

$$=1.591\times10^6\ \frac{\text{A}}{\text{m}}$$

Substitute numerical values in equation (1) and evaluate M:

$$M = \frac{2.00\left(5.788\times10^{-5}\ \frac{\text{eV}}{\text{T}}\right)\left(0.300\ \text{G}\times\frac{1\,\text{T}}{10^4\ \text{G}}\right)}{3\left(8.617\times10^{-5}\ \frac{\text{eV}}{\text{K}}\right)(4.00\ \text{K})}\left(1.591\times10^6\ \frac{\text{A}}{\text{m}}\right) = \boxed{5.34\frac{\text{A}}{\text{m}}}$$

58 •• A cylinder of silver at a temperature of 77 K has a magnetization equal to 0.075% of its saturation magnetization. Assume a magnetic moment of one Bohr magneton per atom. The density of silver is 1.05×10^4 kg/m^3. (*a*) What value of applied magnetic field parallel to the central axis of the cylinder is required to reach this magnetization? (*b*) What is the magnetic field strength at the center of the cylinder?

Picture the Problem We can use Curie's law to relate the magnetization M of the cylinder to its saturation magnetization M_S. The saturation magnetization is the product of the number of atoms n in the cylinder and the magnetic moment of each atom. We can find n using the proportion $\dfrac{n}{N_A}=\dfrac{\rho_{Ag}}{M_{Ag}}$ where M_{Ag} is the molar mass of silver. In Part (*b*), we can use Equation 27-22 to find the magnetic field at the center of the cylinder, on the axis defined by the magnetic field.

(*a*) The magnetization of the cylinder is given by Curie's law:

$$M=\frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

Assuming a magnetic moment of 1Bohr magnetons per atom:

$$M=\frac{(\mu_B)B_{app}}{3kT}M_S\Rightarrow B_{app}=\frac{3kT}{\mu_B}\left(\frac{M}{M_S}\right)$$

Substitute numerical values and evaluate B_{app}:

$$B_{app}=\frac{3\left(8.617\times10^{-5}\ \frac{\text{eV}}{\text{K}}\right)(77\ \text{K})}{5.788\times10^{-5}\ \frac{\text{eV}}{\text{T}}}(0.00075)=\boxed{0.26\ \text{T}}$$

(*b*) The magnetic field at the center of the cylinder on the axis defined by the magnetic field is:

$$B=B_{app}+\mu_0 M_S \qquad (1)$$

The saturation magnetization is given by:

$$M_S=n\mu=n(\mu_B)$$

where n is the number of atoms and μ is the magnetic moment of each molecule.

The number of atoms of silver per unit volume n can be found from the molar mass M_{Ag} of silver, the density ρ_{Ag} of silver, and Avogadro's number N_A:

$$n=\frac{\rho_{Ag}}{M_{Ag}}N_A\Rightarrow M_S=\frac{\rho_{Ag}N_A\mu_B}{M_{Ag}}$$

Substitute numerical values and evaluate M_S:

$$M_S=\frac{\left(10.5\times10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(6.022\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2\right)}{107.870\frac{\text{g}}{\text{mol}}\times\frac{1\,\text{kg}}{10^3\ \text{g}}}=5.434\times10^5\ \frac{\text{A}}{\text{m}}$$

Substitute numerical values in equation (1) and evaluate B:

$$B = 0.26\ \text{T} + \left(4\pi \times 10^{-7}\ \text{T}\cdot\text{A}\right)\left(5.434 \times 10^{5}\ \frac{\text{A}}{\text{m}}\right) = \boxed{0.94\ \text{T}}$$

59 •• During a solid-state physics lab, you are handed a cylindrically shaped sample of unknown magnetic material. You and your lab partners place the sample in a long solenoid that has n turns per unit length and a current I. The values for magnetic field B within the material versus nI, where B_{app} is the field due to the current I and K_{m} is the relative permeability of the sample, are given below. Use these values to plot B versus B_{app} and K_{m} versus nI.

nI, A/m	0	50	100	150	200	500	1000	10 000
B, T	0	0.04	0.67	1.00	1.2	1.4	1.6	1.7

Picture the Problem We can use the data in the table and $B_{\text{app}} = \mu_0 nI$ to plot B versus B_{app}. We can find K_{m} using $B = K_{\text{m}} B_{\text{app}}$.

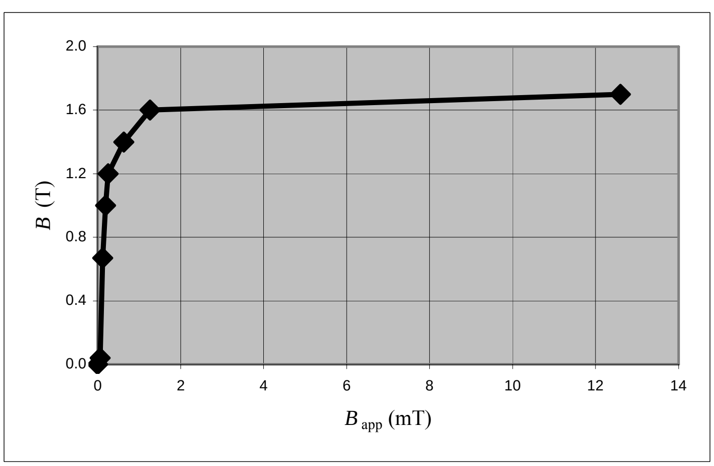
We can find the applied field B_{app} for a long solenoid using:

$$B_{\text{app}} = \mu_0 nI$$

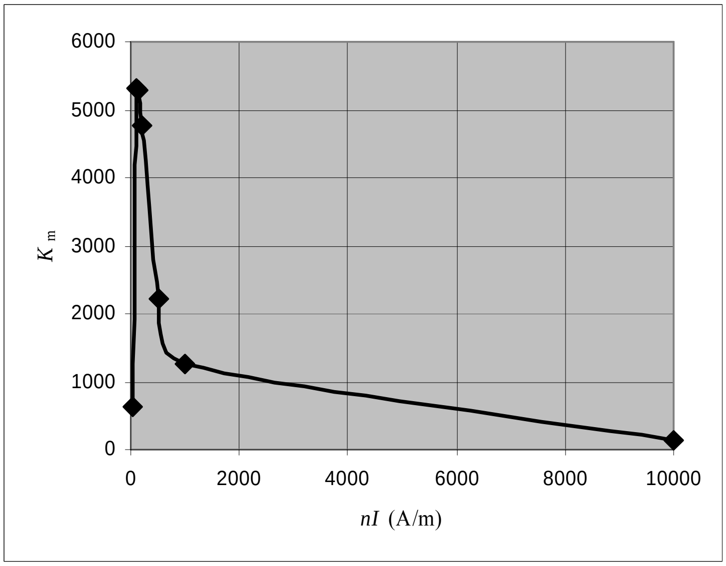
K_{m} can be found from B_{app} and B using:

$$K_{\text{m}} = \frac{B}{B_{\text{app}}}$$

The following graph was plotted using a spreadsheet program. The abscissa values for the graph were obtained by multiplying nI by μ_0. B initially rises rapidly, and then becomes nearly flat. This is characteristic of a ferromagnetic material.



The following graph of K_m versus nI was also plotted using a spreadsheet program. Note that K_m becomes quite large for small values of nI but then diminishes. A more revealing graph would be to plot $B/(nI)$, which would be quite large for small values of nI and then drop to nearly zero at nI = 10,000 A/m, corresponding to saturation of the magnetization.



Atomic Magnetic Moments

60 •• Nickel has a density of 8.70 g/cm^3 and a molar mass of 58.7 g/mol. Nickel's saturation magnetization is 0.610 T. Calculate the magnetic moment of a nickel atom in Bohr magnetons.

Picture the Problem We can find the magnetic moment of a nickel atom μ from its relationship to the saturation magnetization M_S using $M_S = n\mu$ where n is the number of molecules per unit volume. n, in turn, can be found from Avogadro's number, the density of nickel, and its molar mass using $n = \frac{N_A \rho}{M}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$\boldsymbol{M}_S = \boldsymbol{n\mu} \Rightarrow \mu = \frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A \rho}{M}$$

Substitute for n in the equation for μ and simplify to obtain:

$$\mu = \frac{M_S}{\frac{N_A\rho}{M}} = \frac{\mu_0 M_S}{\frac{\mu_0 N_A \rho}{M}} = \frac{\mu_0 M_S M}{\mu_0 N_A \rho}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{(0.610\,\mathrm{T})(58.7\times10^{-3}\ \mathrm{kg/mol})}{(4\pi\times10^{-7}\ \mathrm{N/A^2})(6.022\times10^{23}\ \mathrm{atoms/mol})(8.70\,\mathrm{g/cm^3})} = 5.439\times10^{-24}\ \mathrm{A\cdot m^2}$$

Divide μ by $\mu_B = 9.27\times10^{-24}\ \mathrm{A\cdot m^2}$ to obtain:

$$\frac{\mu}{\mu_B} = \frac{5.439\times10^{-24}\ \mathrm{A\cdot m^2}}{9.27\times10^{-24}\ \mathrm{A\cdot m^2}} = 0.587$$

or $\mu = \boxed{0.587\mu_B}$

61 •• Repeat Problem 60 for cobalt, which has a density of 8.90 g/cm^3, a molar mass of 58.9 g/mol, and a saturation magnetization of 1.79 T.

Picture the Problem We can find the magnetic moment of a cobalt atom μ from its relationship to the saturation magnetization M_S using $M_S = n\mu$, where n is the number of molecules per unit volume. n, in turn, can be found from Avogadro's number, the density of cobalt, and its molar mass using $n = \frac{N_A\rho}{M}$.

Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu \Rightarrow \mu = \frac{M_S}{n}$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molar mass M, and the density ρ:

$$n = \frac{N_A\rho}{M}$$

Substitute for n in the equation for μ and simplify to obtain:

$$\mu = \frac{M_S}{\frac{N_A\rho}{M}} = \frac{\mu_0 M_S}{\frac{\mu_0 N_A \rho}{M}} = \frac{\mu_0 M_S M}{\mu_0 N_A \rho}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{(1.79\,\mathrm{T})(58.9\times10^{-3}\ \mathrm{kg/mol})}{(4\pi\times10^{-7}\ \mathrm{N/A^2})(6.022\times10^{23}\ \mathrm{atoms/mol})(8.90\,\mathrm{g/cm^3})} = 1.57\times10^{-23}\ \mathrm{A\cdot m^2}$$

Divide μ by $\mu_B = 9.27\times10^{-24}\ \text{A}\cdot\text{m}^2$ to obtain:

$$\frac{\mu}{\mu_B} = \frac{1.57\times10^{-23}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = 1.69$$

or $\mu = \boxed{1.69\mu_B}$

Paramagnetism

62 • Show that Curie's law predicts that the magnetic susceptibility of a paramagnetic substance is given by $\chi_m = \mu\mu_0 M_s/3kT$.

Picture the Problem We can show that $\chi_m = \mu\mu_0 M_s/3kT$ by equating Curie's law and the equation that defines χ_m ($M = \chi_m \dfrac{B_{app}}{\mu_0}$) and solving for χ_m.

Express Curie's law:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

where M_S is the saturation value.

Express the magnetization of the substance in terms of its magnetic susceptibility χ_m:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Equate these expressions to obtain:

$$\chi_m \frac{B_{app}}{\mu_0} = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

and

$$\frac{\chi_m}{\mu_0} = \frac{1}{3}\frac{\mu}{kT}M_S \Rightarrow \chi_m = \boxed{\frac{\mu_0\mu M_S}{3kT}}$$

63 •• In a simple model of paramagnetism, we can consider that some fraction f of the molecules have their magnetic moments aligned with the external magnetic field and that the rest of the molecules are randomly oriented and therefore do not contribute to the magnetic field. (*a*) Use this model and Curie's law to show that at temperature T and external magnetic field B, the fraction of aligned molecules f is given by $\mu B/(3kT)$. (*b*) Calculate this fraction for a sample temperature of 300 K, an external field of 1.00 T. Assume that μ has a value of 1.00 Bohr magneton.

Picture the Problem We can use the assumption that $M = fM_S$ and Curie's law to solve these equations simultaneously for the fraction f of the molecules have their magnetic moments aligned with the external magnetic field.

(*a*) Assume that some fraction *f* of the molecules have their magnetic moments aligned with the external magnetic field and that the rest of the molecules are randomly oriented and so do not contribute to the magnetic field:

$$M = fM_S$$

From Curie's law we have:

$$M = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S$$

Equating these expressions yields:

$$fM_S = \frac{1}{3}\frac{\mu B_{app}}{kT}M_S \Rightarrow f = \boxed{\frac{\mu B}{3kT}}$$

because B given in the problem statement is the external magnetic field B_{app} .

(*b*) Substitute numerical values and evaluate *f*:

$$f = \frac{(9.27\times10^{-24}\ \mathrm{A\cdot m^2})(1.00\ \mathrm{T})}{3(1.381\times10^{-23}\ \mathrm{J/K})(300\ \mathrm{K})} = \boxed{7.46\times10^{-4}}$$

64 •• Assume that the magnetic moment of an aluminum atom is 1.00 Bohr magneton. The density of aluminum is 2.70 g/cm^3 and its molar mass is 27.0 g/mol. (*a*) Calculate the value of the saturation magnetization and the saturation magnetic field for aluminum. (*b*) Use the result of Problem 62 to calculate the magnetic susceptibility at 300 K. (*c*) Explain why the result for Part (*b*) is larger than the value listed in Table 27-1.

Picture the Problem In (*a*) we can express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule and use $n = N_A\rho/\mathrm{M}$ to express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ. We can use $\chi_m = \mu_0\mu M_S/3kT$ from Problem 86 to calculate χ_m.

(*a*) Express the saturation magnetic field in terms of the number of molecules per unit volume and the magnetic moment of each molecule:

$$M_S = n\mu_B$$

Express the number of molecules per unit volume in terms of Avogadro's number N_A, the molecular mass M, and the density ρ:

$$n = \frac{N_A \rho}{M}$$

Substitute for n to obtain:

$$M_S = \frac{N_A \rho}{M}\mu_B$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{(6.022\times10^{23}\ \text{atoms/mol})(2.70\times10^{3}\ \text{kg/m}^3)(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2)}{27.0\,\text{g/mol}}$$
$$= 5.582\times10^{5}\ \text{A/m} = \boxed{5.58\times10^{5}\ \text{A/m}}$$

and

$$B_S = \mu_0 M_S = (4\pi\times10^{-7}\ \text{N/A}^2)(5.582\times10^{5}\ \text{A/m}) = \boxed{0.702\,\text{T}}$$

(*b*) From Problem 62 we have:

$$\chi_m = \frac{\mu_0 \mu M_S}{3kT}$$

Substitute numerical values and evaluate χ_m:

$$\chi_m = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(9.27\times10^{-24}\ \text{A}\cdot\text{m}^2)(5.582\times10^{5}\ \text{A/m})}{3(1.381\times10^{-23}\ \text{J/K})(300\,\text{K})} = \boxed{5.23\times10^{-4}}$$

(*c*) In calculating χ_m in Part (*b*) we neglected any diamagnetic effects.

65 •• [SSM] A toroid has N turns, carries a current I, has a mean radius R, and has a cross-sectional radius r, where $r << R$ (Figure 27-59). When the toroid is filled with material, it is called a *Rowland ring*. Find B_{app} and B in such a ring, assuming a magnetization that is everywhere parallel to $\vec{B}_{app}$.

Picture the Problem We can use $B_{app} = \dfrac{\mu_0 NI}{2\pi a}$ to express B_{app} and $B = B_{app} + \mu_0 M$ to express B in terms of B_{app} and M.

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \boxed{\frac{\mu_0 NI}{2\pi a}} \text{ for } R - r < a < R + r$$

The resultant field B in the ring is the sum of B_{app} and $\mu_0 M$:

$$B = B_{app} + \mu_0 M = \boxed{\frac{\mu_0 NI}{2\pi a} + \mu_0 M}$$

66 •• A toroid is filled with liquid oxygen that has a magnetic susceptibility of 4.00×10^{-3}. The toroid has 2000 turns and carries a current of 15.0 A. Its mean radius is 20.0 cm, and the radius of its cross section is 8.00 mm. (*a*) What is the magnetization? (*b*) What is the magnetic field? (*c*) What is the percentage change in magnetic field produced by the liquid oxygen?

Picture the Problem We can find the magnetization using $M = \chi_m B_{app}/\mu_0$ and the magnetic field using $B = B_{app}(1+\chi_m)$.

(*a*) The magnetization M in terms of χ_m and B_{app} is given by:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Express B_{app} inside a tightly wound toroid:

$$B_{app} = \frac{\mu_0 NI}{2\pi r_{mean}}$$

Substitute for B_{app} to obtain:

$$M = \chi_m \frac{\dfrac{\mu_0 NI}{2\pi r_{mean}}}{\mu_0} = \chi_m \frac{NI}{2\pi r_{mean}}$$

Substitute numerical values and evaluate M:

$$M = \frac{(4.00\times10^{-3})(2000)(15.0\,\text{A})}{2\pi(0.200\,\text{m})} = \boxed{95.5\,\text{A/m}}$$

(*b*) Express B in terms of B_{app} and χ_m:

$$B = B_{app}(1+\chi_m)$$

Substitute for B_{app} to obtain:

$$B = \frac{\mu_0 NI}{2\pi r_{mean}}(1+\chi_m)$$

Substitute numerical values and evaluate B:

$$B = \frac{(4\pi\times10^{-7}\,\text{N/A}^2)(2000)(15.0\,\text{A})}{2\pi(0.200\,\text{m})}(1+4.00\times10^{-3}) = \boxed{30.1\,\text{mT}}$$

(*c*) Express the fractional increase in B produced by the liquid oxygen:

$$\frac{\Delta B}{B}=\frac{B-B_{\text{app}}}{B}=\frac{B_{\text{app}}(1+\chi_{\text{m}})-B_{\text{app}}}{B}=\frac{\chi_{\text{m}}B_{\text{app}}}{B}=\frac{\chi_{\text{m}}}{1+\chi_{\text{m}}}=\frac{1}{\dfrac{1}{\chi_{\text{m}}}+1}$$

Substitute numerical values and evaluate $\Delta B/B$:

$$\frac{\Delta B}{B}=\frac{1}{\dfrac{1}{4.00\times10^{-3}}+1}=3.98\times10^{-3}=\boxed{0.398\%}$$

67 •• The centers of the turns of a toroid form a circle with a radius of 14.0 cm. The cross-sectional area of each turn is 3.00 cm^2. It is wound with 5278 turns of fine wire, and the wire carries a current of 4.00 A. The core is filled with a paramagnetic material of magnetic susceptibility 2.90×10^{-4}. (*a*) What is the magnitude of the magnetic field within the substance? (*b*) What is the magnitude of the magnetization? (*c*) What would the magnitude of the magnetic field be if there were no paramagnetic core present?

Picture the Problem We can use $B=B_{\text{app}}(1+\chi_{\text{m}})$ and $B_{\text{app}}=\dfrac{\mu_0NI}{2\pi r_{\text{mean}}}$ to find B within the substance and $M=\chi_{\text{m}}\dfrac{B_{\text{app}}}{\mu_0}$ to find the magnitude of the magnetization.

(*a*) Express the magnetic field B within the substance in terms of B_{app} and χ_{m}:

$$B=B_{\text{app}}(1+\chi_{\text{m}})$$

Express B_{app} inside the toroid:

$$B_{\text{app}}=\frac{\mu_0NI}{2\pi r_{\text{mean}}}$$

Substitute to obtain:

$$B=\frac{\mu_0NI(1+\chi_{\text{m}})}{2\pi r_{\text{mean}}}$$

Substitute numerical values and evaluate B:

$$B=\frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(5278)(4.00\ \text{A})\left(1+2.90\times10^{-4}\right)}{2\pi(14.0\ \text{cm})}=\boxed{30.2\ \text{mT}}$$

(*b*) Express the magnetization M in terms of χ_m and B_app:

$$M=\chi_\text{m}\frac{B_\text{app}}{\mu_0}$$

Substitute for B_app and simplify to obtain:

$$M=\frac{\chi_\text{m}NI}{2\pi r_\text{mean}}$$

Substitute numerical values and evaluate M:

$$M=\frac{\left(2.90\times10^{-4}\right)(5278)(4.00\ \text{A})}{2\pi(14.0\ \text{cm})}=\boxed{6.96\ \text{A/m}}$$

(*c*) If there were no paramagnetic core present:

$$B=B_\text{app}=\boxed{30.2\ \text{mT}}$$

Ferromagnetism

68 • For annealed iron, the relative permeability K_m has its maximum value of approximately 5500 at $B_\text{app}=1.57\times10^{-4}$ T. Find the magnitude of the magnetization and magnetic field in annealed iron when K_m is maximum.

Picture the Problem We can use $B=K_\text{m}B_\text{app}$ to find B and $M=(K_\text{m}-1)B_\text{app}/\mu_0$ to find M.

Express B in terms of M and K_m:

$$B=K_\text{m}B_\text{app}$$

Substitute numerical values and evaluate B:

$$B=(5500)\left(1.57\times10^{-4}\ \text{T}\right)=\boxed{0.86\ \text{T}}$$

Relate M to K_m and B_app:

$$M=(K_\text{m}-1)\frac{B_\text{app}}{\mu_0}\approx\frac{K_\text{m}B_\text{app}}{\mu_0}$$

Substitute numerical values and evaluate M:

$$M=\frac{(5500)\left(1.57\times10^{-4}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{N/A}^2}=\boxed{6.9\times10^5\ \text{A/m}}$$

69 •• [SSM] The saturation magnetization for annealed iron occurs when B_{app} = 0.201 T. Find the permeability and the relative permeability of annealed iron at saturation. (See Table 27-2)

Picture the Problem We can relate the permeability μ of annealed iron to χ_m using $\mu = (1+\chi_m)\mu_0$, find χ_m using $M = \chi_m \dfrac{B_{app}}{\mu_0}$, and use its definition ($K_m = 1+\chi_m$) to evaluate K_m.

Express the permeability μ of annealed iron in terms of its magnetic susceptibility χ_m:

$$\mu = (1+\chi_m)\mu_0 \qquad (1)$$

The magnetization M in terms of χ_m and B_{app} is given by:

$$M = \chi_m \frac{B_{app}}{\mu_0}$$

Solve for and evaluate χ_m (see Table 27-2 for the product of μ_0 and M):

$$\chi_m = \frac{\mu_0 M}{B_{app}} = \frac{2.16\,\text{T}}{0.201\,\text{T}} = 10.75$$

Use its definition to express and evaluate the relative permeability K_m:

$$K_m = 1+\chi_m = 1+10.75 = 11.746 = \boxed{11.7}$$

Substitute numerical values in equation (1) and evaluate μ:

$$\mu = (1+10.746)\left(4\pi\times10^{-7}\ \text{N/A}^2\right) = \boxed{1.48\times10^{-5}\,\text{N/A}^2}$$

70 •• The *coercive force* (which is a misnomer because it is really a magnetic field value) is defined as the applied magnetic field needed to bring the magnetic field back to zero along the hysteresis curve (which is point *c* in Figure 27- 38). For a certain permanent bar magnet, the coercive force is known to be 5.53×10^{-2} T. The bar magnet is to be demagnetized by placing it inside a 15.0-cm-long solenoid that has 600 turns. What minimum current is needed in the solenoid to demagnetize the magnet?

Picture the Problem We can use the relationship between the magnetic field on the axis of a solenoid and the current in the solenoid to find the minimum current is needed in the solenoid to demagnetize the magnet.

Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 nI \Rightarrow I = \frac{B_x}{\mu_0 n}$$

Let $B_{\text{app}} = B_x$ to obtain:

$$I = \frac{B_{\text{app}}}{\mu_0 n}$$

Substitute numerical values and evaluate I:

$$I = \frac{5.53\times10^{-2}\ \text{T}}{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(\dfrac{600}{0.150\,\text{m}}\right)} = \boxed{11.0\,\text{A}}$$

71 •• A long thin solenoid has 50 turns/cm and carries a current of 2.00 A. The solenoid is filled with iron and the magnetic field is measured to be 1.72 T. (*a*) Neglecting end effects, what is the magnitude of the applied magnetic field? (*b*) What is the magnetization? (*c*) What is the relative permeability?

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{\text{app}} + \mu_0 M$ to find M and $B = K_{\text{m}} B_{\text{app}}$ to evaluate K_{m}.

(*a*) Relate the magnetic field on the axis of a solenoid to the current in the solenoid:

$$B_x = \mu_0 nI$$

Substitute numerical values and evaluate B_{app}:

$$B_{\text{app}} = \left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(50\,\text{cm}^{-1}\right)\left(2.00\,\text{A}\right) = \boxed{12.6\,\text{mT}}$$

(*b*) Relate M to B and B_{app}:

$$B = B_{\text{app}} + \mu_0 M \Rightarrow M = \frac{B - B_{\text{app}}}{\mu_0}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.72\,\text{T} - 12.6\,\text{mT}}{4\pi\times10^{-7}\ \text{N/A}^2} = \boxed{1.36\times10^6\ \text{A/m}}$$

(*c*) Express B in terms of K_{m} and B_{app}:

$$B = K_{\text{m}} B_{\text{app}} \Rightarrow K_{\text{m}} = \frac{B}{B_{\text{app}}}$$

Substitute numerical values and evaluate K_m:

$$K_m = \frac{1.72\,\text{T}}{12.6\,\text{mT}} = \boxed{137}$$

72 •• When the current in Problem 71 is 0.200 A, the magnetic field is measured to be 1.58 T. (*a*) Neglecting end effects, what is the applied magnetic field? (*b*) What is the magnetization? (*c*) What is the relative permeability?

Picture the Problem We can use the equation describing the magnetic field on the axis of a solenoid, as a function of the current in the solenoid, to find B_{app}. We can then use $B = B_{\text{app}} + \mu_0 M$ to find *M* and $B = K_m B_{\text{app}}$ to evaluate K_m.

(*a*) Relate the magnetic field on the axis of the solenoid to the current in the solenoid:

$$B_x = \mu_0 n I$$

Substitute numerical values and evaluate B_{app}:

$$B_{\text{app}} = \left(4\pi\times10^{-7}\,\frac{\text{N}}{\text{A}^2}\right)\left(50\,\text{cm}^{-1}\right)\left(0.200\,\text{A}\right) = 1.257\,\text{mT} = \boxed{1.26\,\text{mT}}$$

(*b*) Relate *M* to *B* and B_{app}:

$$B = B_{\text{app}} + \mu_0 M \Rightarrow M = \frac{B - B_{\text{app}}}{\mu_0}$$

Substitute numerical values and evaluate *M*:

$$M = \frac{1.58\,\text{T} - 1.257\,\text{mT}}{4\pi\times10^{-7}\,\text{N/A}^2} = \boxed{1.26\times10^6\,\text{A/m}}$$

(*c*) Express *B* in terms of K_m and B_{app}:

$$B = K_m B_{\text{app}} \Rightarrow K_m = \frac{B}{B_{\text{app}}}$$

Substitute numerical values and evaluate K_m:

$$K_m = \frac{1.58\,\text{T}}{1.257\,\text{mT}} = \boxed{1.26\times10^3}$$

73 •• [SSM] A toroid has *N* turns, carries a current *I*, has a mean radius *R*, and has a cross-sectional radius *r*, where *r* << *R* (Figure 27-53). The core of the toroid of is filled with iron. When the current is 10.0 A, the magnetic field in the region where the iron is has a magnitude of 1.80 T. (*a*) What is the magnetization? (*b*) Find the values for the relative permeability, the permeability, and magnetic susceptibility for this iron sample.

Picture the Problem We can use $B = B_{\text{app}} + \mu_0 M$ and the expression for the magnetic field inside a tightly wound toroid to find the magnetization M. We can find K_m from its definition, $\mu = K_m\mu_0$ to find μ, and $K_m = 1 + \chi_m$ to find χ_m for the iron sample.

(*a*) Relate the magnetization to B and B_{app}:

$$B = B_{\text{app}} + \mu_0 M \Rightarrow M = \frac{B - B_{\text{app}}}{\mu_0}$$

Express the magnetic field inside a tightly wound toroid:

$$B_{\text{app}} = \frac{\mu_0 NI}{2\pi r}$$

Substitute for B_{app} and simplify to obtain:

$$M = \frac{B - \frac{\mu_0 NI}{2\pi r}}{\mu_0} = \frac{B}{\mu_0} - \frac{NI}{2\pi r}$$

Substitute numerical values and evaluate M:

$$M = \frac{1.80\,\text{T}}{4\pi \times 10^{-7}\ \text{N/A}^2} - \frac{2000(10.0\,\text{A})}{2\pi(0.200\,\text{m})}$$
$$= \boxed{1.42 \times 10^6\ \text{A/m}}$$

(*b*) Use its definition to express K_m:

$$K_m = \frac{B}{B_{\text{app}}} = \frac{B}{\frac{\mu_0 NI}{2\pi r}} = \frac{2\pi rB}{\mu_0 NI}$$

Substitute numerical values and evaluate K_m:

$$K_m = \frac{2\pi(0.200\,\text{m})(1.80\,\text{T})}{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(2000)(10.0\,\text{A})}$$
$$= \boxed{90.0}$$

Now that we know K_m we can find μ using:

$$\mu = K_m\mu_0 = 90\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)$$
$$= \boxed{1.13 \times 10^{-4}\ \text{T}\cdot\text{m/A}}$$

Relate χ_m to K_m:

$$K_m = 1 + \chi_m \Rightarrow \chi_m = K_m - 1$$

Substitute the numerical value of K_m and evaluate χ_m:

$$\chi_m = 90 - 1 = \boxed{89}$$

74 •• The centers of the turns of a toroid form a circle with a radius of 14.0 cm. The cross-sectional area of each turn is 3.00 cm^2. It is wound with 5278 turns of fine wire, and the wire carries a current of 0.200 A. The core is filled with soft iron, which has a relative permeability of 500. What is the magnetic field strength in the core?

Picture the Problem We can substitute the expression for applied magnetic field ($B_{\text{app}} = \dfrac{\mu_0 NI}{2\pi r}$) in the defining equation for K_{m} ($B = K_{\text{m}} B_{\text{app}}$) to obtain an expression for the magnetic field B in the toroid.

Relate the magnetic field in the toroid to the relative permeability of its core:

$$B = K_{\text{m}} B_{\text{app}}$$

Express the applied magnetic field in the toroid in terms of the current in its winding:

$$B_{\text{app}} = \frac{\mu_0 NI}{2\pi r}$$

Substitute for B_{app} to obtain:

$$B = \frac{K_{\text{m}} \mu_0 NI}{2\pi r}$$

Substitute numerical values and evaluate B:

$$B = \frac{500\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(5278)(0.200\ \text{A})}{2\pi(14.0\ \text{cm})} = \boxed{0.754\ \text{T}}$$

75 ••• A long straight wire that has a radius of 1.00 mm is coated with an insulating ferromagnetic material that has a thickness of 3.00 mm and a relative magnetic permeability of 400. The coated wire is in air and the wire itself is nonmagnetic. The wire carries a current of 40.0 A. (*a*) Find the magnetic field in the region occupied by the inside of the wire as a function of the perpendicular distance, *r*, from the central axis of the wire. (*b*) Find the magnetic field in the region occupied by the inside of the ferromagnetic material as a function of the perpendicular distance, *r*, from the central axis of the wire. (*c*) Find the magnetic field in the region surrounding the wire and coating as a function of the perpendicular distance, *r*, from the central axis of the wire. (*d*)What must the magnitudes and directions of the Amperian currents be on the surfaces of the ferromagnetic material to account for the magnetic fields observed?

Picture the Problem We can use Ampère's law to obtain expressions for the magnetic field inside the wire, inside the ferromagnetic material, and in the region surrounding the wire and coating.

(*a*) Apply Ampère's law to a circle of radius r < 1.00 mm and concentric with the center of the wire:

$$\oint_C \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C \qquad (1)$$

Assuming that the current is distributed uniformly over the cross-sectional area of the wire (uniform current density), express I_C in terms of the total current I:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi R^2} \Rightarrow I_C = \frac{r^2}{R^2} I$$

Substitute for I_C in equation (1) to obtain:

$$B(2\pi r) = \frac{\mu_0 I r^2}{R^2} \Rightarrow B = \frac{\mu_0 I}{2\pi R^2} r$$

Substitute numerical values and evaluate B:

$$B = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(40.0\ \text{A})}{2\pi(1.00\ \text{mm})^2} r = \boxed{(8.00\ \text{T/m})r}$$

(*b*) Relate the magnetic field inside the ferromagnetic material to the magnetic field due to the current in the wire:

$$B = K_\text{m} B_\text{app} \qquad (1)$$

Apply Ampère's law to a circle of radius 1.00 mm < r < 4.00 mm and concentric with the center of the wire:

$$\int_C \vec{B}\cdot d\vec{\ell} = B_\text{app}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solving for B_app yields:

$$B_\text{app} = \frac{\mu_0 I}{2\pi r}$$

Substitute for B_app in equation (1) to obtain:

$$B = \frac{K_\text{m}\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B:

$$B = \frac{400\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(40.0\ \text{A})}{2\pi r} = \boxed{\left(3.20\times10^{-3}\ \text{T}\cdot\text{m}\right)\frac{1}{r}}$$

(*c*) Apply Ampère's law to a circle of radius $r > 4.00$ mm and concentric with the center of the wire:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solving for *B* yields:

$$B = \frac{\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{(4\pi \times 10^{-7}\ \mathrm{N/A^2})(40.0\ \mathrm{A})}{2\pi r}$$

$$= \boxed{(8.00 \times 10^{-6}\ \mathrm{T \cdot m})\frac{1}{r}}$$

(*d*) Note that the field in the ferromagnetic region is that which would be produced in a nonmagnetic region by a current of $400I = 1600$ A. The ampèrian current on the inside of the surface of the ferromagnetic material must therefore be (1600 − 40) A = 1560 A in the direction of *I*. On the outside surface there must then be an ampèrian current of 1560 A in the opposite direction.

General Problems

76 • Find the magnetic field at point *P* in Figure 27-60.

Picture the Problem Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P*. Hence, we can use the expression for the magnetic field at the center of a current loop to find B_P.

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where *R* is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Substitute numerical values and evaluate *B*:

$$B = \frac{(4\pi \times 10^{-7}\ \mathrm{N/A^2})(15\ \mathrm{A})}{4(0.20\ \mathrm{m})}$$

$$= \boxed{24\ \mu\mathrm{T\ out\ of\ the\ page}}$$

77 • **[SSM]** Using Figure 27-61, find the magnetic field (in terms of the parameters given in the figure) at point P, the common center of the two arcs.

Picture the Problem Let out of the page be the positive x direction. Because point P is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at P. Hence, the resultant magnetic field at P will be the sum of the magnetic fields due to the current in the two semicircles, and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at P:

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 \qquad (1)$$

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Express $\vec{B}_1$ and $\vec{B}_2$:

$$\vec{B}_1 = \frac{\mu_0 I}{4R_1}\hat{\boldsymbol{i}} \text{ and } \vec{B}_2 = -\frac{\mu_0 I}{4R_2}\hat{\boldsymbol{i}}$$

Substitute in equation (1) to obtain:

$$\vec{B}_P = \frac{\mu_0 I}{4R_1}\hat{\boldsymbol{i}} - \frac{\mu_0 I}{4R_2}\hat{\boldsymbol{i}} = \boxed{\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{\boldsymbol{i}} \text{ out of the page}}$$

78 •• A wire of length ℓ, is wound into a circular coil of N turns, and carries a current I. Show that the magnetic field strength in the region occupied by the center of the coil is given by $\mu_0 \pi N^2 I/\ell$.

Picture the Problem We can express the magnetic field strength B as a function of N, I, and R using $B = \dfrac{\mu_0 NI}{2R}$ and eliminate R by relating ℓ to R.

Express the magnetic field at the center of a coil of N turns and radius R:

$$B = \frac{\mu_0 NI}{2R}$$

Relate ℓ to the number of turns N:

$$\ell = 2\pi RN \Rightarrow R = \frac{\ell}{2\pi N}$$

Substitute for R in the expression for B and simplify to obtain:

$$B = \frac{\mu_0 NI}{2\left(\frac{\ell}{2\pi N}\right)} = \boxed{\frac{\mu_0 \pi N^2 I}{\ell}}$$

79 •• A very long wire carrying a current I is bent into the shape shown in Figure 27-62. Find the magnetic field at point P.

Picture the Problem The magnetic field at P (which is out of the page) is the sum of the magnetic fields due to the three parts of the wire. Let the numerals 1, 2, and 3 denote the left-hand, center (short), and right-hand wires. We can then use the expression for B due to a straight wire segment to find each of these fields and their sum.

Express the resultant magnetic field at point P:

$$B_P = B_1 + B_2 + B_3$$

Because $B_1 = B_3$:

$$B_P = 2B_1 + B_2 \qquad (1)$$

Express the magnetic field due to a straight wire segment:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_1 + \sin\theta_2)$$

For wires 1 and 3 (the long wires), $\theta_1 = 90°$ and $\theta_2 = 45°$:

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 90° + \sin 45°) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)$$

For wire 2, $\theta_1 = \theta_2 = 45°$:

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 45° + \sin 45°) = \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right)$$

Substitute for B_1 and B_2 in equation (1) and simplify to obtain:

$$B_P = 2\left[\frac{\mu_0}{4\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}}\right)\right] + \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{2}{\sqrt{2}}\right) = \frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \boxed{\frac{\mu_0}{2\pi}\frac{I}{a}\left(1 + \sqrt{2}\right)\text{out of the page}}$$

80 •• A power cable carrying 50 A is 2.0 m below Earth's surface, but the cable's direction and precise position are unknown. Explain how you could locate the cable using a compass. Assume that you are at the equator, where Earth's magnetic field is horizontal and 0.700 G due north.

Picture the Problem Depending on the direction of the wire, the magnetic field due to its current (provided this field is a large enough fraction of Earth's magnetic field) will either add to or subtract from Earth's field and moving the compass over the ground in the vicinity of the wire will indicate the direction of the current.

Apply Ampère's law to a circle of radius r and concentric with the center of the wire:

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\ell} = B_{\text{wire}}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solve for B to obtain:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$

Substitute numerical values and evaluate B_{wire}:

$$B_{\text{wire}} = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50\,\text{A})}{2\pi(2.0\,\text{m})} = 0.0500\,\text{G}$$

Express the ratio of B_{wire} to B_{Earth}:

$$\frac{B_{\text{wire}}}{B_{\text{Earth}}} = \frac{0.05\,\text{G}}{0.7\,\text{G}} \approx 7\%$$

Thus, the field of the current-carrying wire should be detectable with a good compass.

If the cable runs in a direction other than east-west, its magnetic field is in a direction different than that of Earth's, and by moving the compass about one should observe a change in the direction of the compass needle.

If the cable runs east-west, its magnetic field is in the north-south direction and thus either adds to or subtracts from Earth's field, depending on the current direction and location of the compass. If the magnetic field is toward the north, the two fields add and the resultant field is stronger. If perturbed, the compass needle will oscillate about its equilibrium position. The stronger the field, the higher the frequency of oscillation. By moving from place to place and systematically perturbing the needle one should be able to detect a change frequency, and thus a change in magnetic field strength.

81 •• [SSM] A long straight wire carries a current of 20.0 A, as shown in Figure 27-63. A rectangular coil that has two sides parallel to the straight wire has sides that are 5.00-cm long and 10.0-cm long. The side nearest to the wire is 2.00 cm from the wire. The coil carries a current of 5.00 A. (*a*) Find the force on each segment of the rectangular coil due to the current in the long straight wire. (*b*) What is the net force on the coil?

Picture the Problem Let I_1 and I_2 represent the currents of 20 A and 5.0 A, $\vec{F}_{\text{top}}$, $\vec{F}_{\text{left side}}$, $\vec{F}_{\text{bottom}}$, and $\vec{F}_{\text{right side}}$ the forces that act on the horizontal wire, and $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ the magnetic fields at these wire segments due to I_1. We'll need to take into account the fact that $\vec{B}_1$ and $\vec{B}_3$ are not constant over the segments 1 and 3 of the rectangular coil. Let the +*x* direction be to the right and the +*y* direction be upward. Then the +*z* direction is toward you (i.e., out of the page). Note that only the components of $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ into or out of the page contribute to the forces acting on the rectangular coil. The +*x* and +*y* directions are up the page and to the right.

(*a*) Express the force $d\vec{F}_1$ acting on a current element $I_2 d\vec{\ell}$ in the top segment of wire:

$$d\vec{F}_{\text{top}} = I_2 d\vec{\ell} \times \vec{B}_1$$

Because $I_2 d\vec{\ell} = I_2 d\ell\left(-\hat{i}\right)$ in this segment of the coil and the magnetic field due to I_1 is given by $\vec{B}_1 = \dfrac{\mu_0 I_1}{2\pi\,\ell}\left(-\hat{k}\right)$:

$$\vec{F}_{\text{top}} = I_2 d\ell\left(-\hat{i}\right) \times \frac{\mu_0 I_1}{2\pi\,\ell}\left(-\hat{k}\right) = -\frac{\mu_0 I_1 I_2}{2\pi}\frac{d\ell}{\ell}\hat{j}$$

Integrate $d\vec{F}_{\text{top}}$ to obtain:

$$\vec{F}_{\text{top}} = -\frac{\mu_0 I_1 I_2}{2\pi}\int_{2.0\,\text{cm}}^{7.0\,\text{cm}}\frac{d\ell}{\ell}\hat{j} = -\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{7.0\,\text{cm}}{2.0\,\text{cm}}\right)\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{\text{top}}$:

$$\vec{F}_{\text{top}} = -\frac{\left(4\pi\times10^{-7}\,\frac{\text{N}}{\text{A}^2}\right)(20\,\text{A})(5.0\,\text{A})}{2\pi}\ln\left(\frac{7.0\,\text{cm}}{2.0\,\text{cm}}\right)\hat{j} = \boxed{-\left(2.5\times10^{-5}\,\text{N}\right)\hat{j}}$$

Express the force $d\vec{F}_{\text{bottom}}$ acting on a current element $I_2 d\vec{\ell}$ in the horizontal segment of wire at the bottom of the coil:

$$d\vec{F}_{\text{bottom}} = I_2 d\vec{\ell} \times \vec{B}_3$$

Because $I_2 d\vec{\ell} = I_2 d\ell\left(\hat{i}\right)$ in this segment of the coil and the magnetic field due to I_1 is given by $\vec{B}_1 = \frac{\mu_0 I_1}{2\pi \ell}\left(-\hat{k}\right)$:

$$d\vec{F}_{\text{bottom}} = I_2 d\ell\hat{i} \times \frac{\mu_0 I_1}{2\pi\ell}\left(-\hat{k}\right) = \frac{\mu_0 I_1 I_2}{2\pi}\frac{d\ell}{\ell}\hat{j}$$

Integrate $d\vec{F}_{\text{bottom}}$ to obtain:

$$d\vec{F}_{\text{bottom}} = \frac{\mu_0 I_1 I_2}{2\pi}\int_{2.0\,\text{cm}}^{7.0\,\text{cm}}\frac{d\ell}{\ell}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{7.0\,\text{cm}}{2.0\,\text{cm}}\right)\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{\text{bottom}}$:

$$\vec{F}_{\text{bottom}} = \frac{\left(4\pi\times10^{-7}\,\frac{\text{N}}{\text{A}^2}\right)(20\,\text{A})(5.0\,\text{A})}{2\pi}\ln\left(\frac{7.0\,\text{cm}}{2.0\,\text{cm}}\right)\hat{j} = \boxed{\left(2.5\times10^{-5}\,\text{N}\right)\hat{j}}$$

Express the forces $\vec{F}_{\text{left side}}$ and $\vec{F}_{\text{right side}}$ in terms of I_2 and $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{F}_{\text{left side}} = I_2\vec{\ell}_2 \times \vec{B}_2$$

and

$$\vec{F}_{\text{right side}} = I_2\vec{\ell}_4 \times \vec{B}_4$$

Express $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k} \text{ and } \vec{B}_4 = -\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}$$

Substitute for $\vec{B}_2$ and $\vec{B}_4$ to obtain:

$$\vec{F}_{\text{left side}} = -I_2\ell_2\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_1}\hat{k}\right) = \frac{\mu_0\ell_2 I_1 I_2}{2\pi R_2}\hat{i}$$

and

$$\vec{F}_{\text{right side}} = I_2\ell_4\hat{j}\times\left(-\frac{\mu_0}{4\pi}\frac{2I_1}{R_4}\hat{k}\right) = -\frac{\mu_0\ell_4 I_1 I_2}{2\pi R_4}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}_{\text{left side}}$ and $\vec{F}_{\text{right side}}$:

$$\vec{F}_{\text{left side}} = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(0.100\,\text{m})(20.0\,\text{A})(5.00\,\text{A})}{2\pi(0.0200\,\text{m})}\hat{i} = \boxed{\left(1.0\times10^{-4}\ \text{N}\right)\hat{i}}$$

and

$$\vec{F}_{\text{right side}} = -\frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(0.100\,\text{m})(20.0\,\text{A})(5.00\,\text{A})}{2\pi(0.0700\,\text{m})}\hat{i} = \boxed{\left(-0.29\times10^{-4}\ \text{N}\right)\hat{i}}$$

(*b*) Express the net force acting on the coil:

$$\vec{F}_{\text{net}} = \vec{F}_{\text{top}} + \vec{F}_{\text{left side}} + \vec{F}_{\text{bottom}} + \vec{F}_{\text{right side}}$$

Substitute for $\vec{F}_{\text{top}}$, $\vec{F}_{\text{left side}}$, $\vec{F}_{\text{bottom}}$, and $\vec{F}_{\text{right side}}$ and simplify to obtain:

$$\begin{aligned}\vec{F}_{\text{net}} &= \left(-2.5\times10^{-5}\ \text{N}\right)\hat{j} + \left(1.0\times10^{-4}\ \text{N}\right)\hat{i} + \left(2.5\times10^{-5}\ \text{N}\right)\hat{j} + \left(-0.29\times10^{-4}\ \text{N}\right)\hat{i}\\ &= \boxed{\left(0.71\times10^{-4}\ \text{N}\right)\hat{i}}\end{aligned}$$

82 •• The closed loop shown in Figure 27-64 carries a current of 8.0 A in the counterclockwise direction. The radius of the outer arc is 0.60 m and that of the inner arc is 0.40 m. Find the magnetic field at point *P*.

Picture the Problem Let out of the page be the positive *x* direction and the numerals 40 and 60 refer to the circular arcs whose radii are 40 cm and 60 cm. Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P*. Hence the resultant magnetic field at *P* will be the sum of the magnetic fields due to the current in the two circular arcs and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at *P*:

$$\vec{B}_P = \vec{B}_{40} + \vec{B}_{60} \qquad (1)$$

Express the magnetic field at the center of a current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of one-sixth of a current loop:

$$B = \frac{1}{6}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{12R}$$

Express $\vec{B}_{40}$ and $\vec{B}_{60}$:

$$\vec{B}_{40} = -\frac{\mu_0 I}{12R_{40}}\hat{i} \text{ and } \vec{B}_{60} = \frac{\mu_0 I}{12R_{60}}\hat{i}$$

Substitute for $\vec{B}_{40}$ and $\vec{B}_{60}$ in equation (1) and simplify to obtain:

$$\vec{B}_P = -\frac{\mu_0 I}{12R_{40}}\hat{i} + \frac{\mu_0 I}{12R_{60}}\hat{i} = \frac{\mu_0 I}{12}\left(\frac{1}{R_{60}} - \frac{1}{R_{40}}\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{B}_P$:

$$\vec{B}_P = \frac{\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(8.0\ \mathrm{A})}{12}\left(\frac{1}{0.60\ \mathrm{m}} - \frac{1}{0.40\ \mathrm{m}}\right)\hat{i} = (-0.70\ \mu\mathrm{T})\hat{i}$$

or

$$B_P = \boxed{0.70\ \mu\mathrm{T}\ \text{into the page}}$$

83 •• A closed circuit consists of two semicircles of radii 40 cm and 20 cm that are connected by straight segments, as shown in Figure 27-65. A current of 3.0 A exists in this circuit and has a clockwise direction. Find the magnetic field at point *P*.

Picture the Problem Let the +*x* direction be into the page and the numerals 20 and 40 refer to the circular arcs whose radii are 20 cm and 40 cm. Because point *P* is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at *P* and the resultant field at *P* is the sum of the fields due to the two semicircular current loops.

Express the resultant magnetic field at *P*:

$$\vec{B}_P = \vec{B}_{20} + \vec{B}_{40} \qquad (1)$$

Express the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R}$$

where R is the radius of the loop.

Express the magnetic field at the center of half a circular current loop:

$$B = \frac{1}{2}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$$

Express $\vec{B}_{20}$ and $\vec{B}_{40}$:

$$\vec{B}_{20} = \frac{\mu_0 I}{4R_{20}}\hat{i} \text{ and } \vec{B}_{40} = \frac{\mu_0 I}{4R_{40}}\hat{i}$$

Substitute for $\vec{B}_{20}$ and $\vec{B}_{40}$ in equation (1) and simplify to obtain:

$$\vec{B}_P = \frac{\mu_0 I}{4R_{20}}\hat{i} + \frac{\mu_0 I}{4R_{40}}\hat{i} = \frac{\mu_0 I}{4}\left(\frac{1}{R_{20}} + \frac{1}{R_{40}}\right)\hat{i}$$

Substitute numerical values and evaluate B_P:

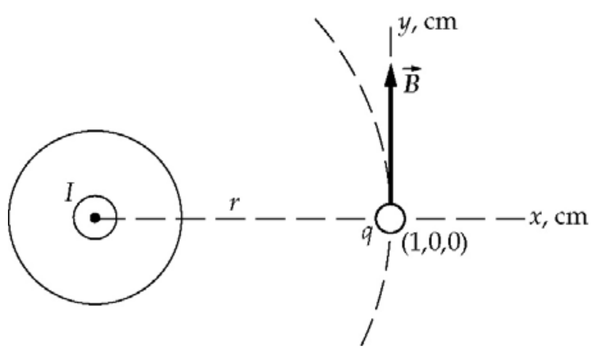
$$\vec{B}_P = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(3.0\,\text{A})}{4}\left(\frac{1}{0.20\,\text{m}} + \frac{1}{0.40\,\text{m}}\right)\hat{i} = \boxed{(7.1\,\mu\text{T})\hat{i}}$$

or

$$B_P = \boxed{7.1\,\mu\text{T into the page}}$$

84 •• A very long straight wire carries a current of 20.0 A. An electron outside the wire is 1.00 cm from the central axis of the wire is moving with a speed of 5.00×10^6 m/s. Find the force on the electron when it moves (*a*) directly away from the wire, (*b*) parallel to the wire in the direction of the current, and (*c*) perpendicular to the central axis of wire and tangent to a circle that is coaxial with the wire.

Picture the Problem Chose the coordinate system shown to the right. Then the current is in the +z direction. Assume that the electron is at (1.00 cm, 0, 0). We can use $\vec{F} = q\vec{v}\times\vec{B}$ to relate the magnetic force on the electron to $\vec{v}$ and $\vec{B}$ and $\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$ to express the magnetic field at the location of the electron. We'll need to express $\vec{v}$ for each of the three situations described in the problem in order to evaluate $\vec{F} = q\vec{v}\times\vec{B}$.



Express the magnetic force acting on the electron:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Express the magnetic field due to the current in the wire as a function of distance from the wire:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j}$$

Substitute for $\vec{B}$ and simplifying yields:

$$\vec{F} = q\vec{v} \times \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{j} = \frac{2q\mu_0 I}{4\pi r}\left(\vec{v} \times \hat{j}\right) \quad (1)$$

(*a*) Express the velocity of the electron when it moves directly away from the wire:

$$\vec{v} = v\hat{i}$$

Substitute for $\vec{v}$ in equation (1) and simplify to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{i} \times \hat{j}\right) = \frac{2q\mu_0 Iv}{4\pi r}\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \frac{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(-1.602 \times 10^{-19}\ \text{C}\right)\left(5.00 \times 10^{6}\ \text{m/s}\right)\left(20.0\ \text{A}\right)\hat{k}}{4\pi\left(0.0100\ \text{m}\right)}$$
$$= \left(-3.20 \times 10^{-16}\ \text{N}\right)\hat{k}$$

and

$$F = \boxed{3.20 \times 10^{-16}\ \text{N into the page}}$$

(*b*) Express $\vec{v}$ when the electron is traveling parallel to the wire in the direction of the current:

$$\vec{v} = v\hat{k}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{k} \times \hat{j}\right) = -\frac{2q\mu_0 Iv}{4\pi r}\hat{i}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = -\frac{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(-1.602 \times 10^{-19}\ \text{C}\right)\left(5.00 \times 10^{6}\ \text{m/s}\right)\left(20.0\ \text{A}\right)\hat{i}}{4\pi\left(0.0100\ \text{m}\right)}$$
$$= \left(3.20 \times 10^{-16}\ \text{N}\right)\hat{i}$$

and

$$F = \boxed{3.20 \times 10^{-16}\ \text{N toward the right}}$$

(*c*) Express $\vec{v}$ when the electron is traveling perpendicular to the wire and tangent to a circle around the wire:

$$\vec{v} = v\hat{j}$$

Substitute for $\vec{v}$ in equation (1) and simplify to obtain:

$$\vec{F} = \frac{2q\mu_0 I}{4\pi r}\left(v\hat{j}\times\hat{j}\right) = \boxed{0}$$

85 •• A current of 5.00 A is uniformly distributed over the cross section of a long straight wire of radius $R_o = 2.55$ mm. Using a **spreadsheet** program, graph the magnetic field strength as a function of *R*, the distance from the central axis of the wire, for $0 \le R \le R_0$.

Picture the Problem We can apply Ampère's law to derive expressions for the magnetic field strength as a function of the distance from the center of the wire.

Apply Ampère's law to a closed circular path of radius $r \le R_0$ to obtain:

$$B_{r\le R_0}(2\pi r) = \mu_0 I_C$$

Because the current is uniformly distributed over the cross section of the wire:

$$\frac{I_C}{\pi r^2} = \frac{I}{\pi R_0^2} \Rightarrow I_C = \frac{r^2}{R_0^2} I$$

Substitute for I_C to obtain:

$$B_{r\le R_0}(2\pi r) = \frac{\mu_0 r^2 I}{R_0^2}$$

Solving for $B_{r\le R_0}$ yields:

$$B_{r\le R_0} = \frac{\mu_0 r I}{2\pi R_0^2} = \frac{\mu_0}{4\pi}\frac{2I}{R_0^2} r \qquad (1)$$

Apply Ampère's law to a closed circular path of radius $r > R_0$ to obtain:

$$B_{r>R_0}(2\pi r) = \mu_0 I_C = \mu_0 I$$

Solving for $B_{r>R_0}$ yields:

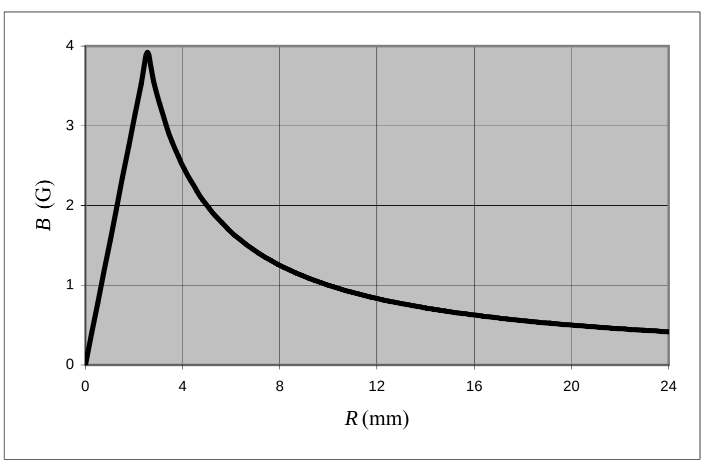
$$B_{r>r_0} = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad (2)$$

The spreadsheet program to calculate *B* as a function of *r* in the interval $0 \le r \le 10R_0$ is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B1	1.00E−07	$\frac{\mu_0}{4\pi}$
B2	5.00	I
B3	2.55E−03	R_0
C6	10^4*$B$1*2*$B$2*A6/$B$3^2	$\frac{\mu_0}{4\pi}\frac{2I}{R_0^2}r$
C17	10^4*$B$1*2*$B$2*A6/A17	$\frac{\mu_0}{4\pi}\frac{2I}{r}$

	A	B	C
1	$\mu/4\pi$=	1.00E−07	N/A^2
2	I=	5	A
3	R_0=	2.55E−03	m
4			
5	R (m)	R (mm)	B (T)
6	0.00E+00	0.00E+00	0.00E+00
7	2.55E−04	2.55E−01	3.92E−01
8	5.10E−04	5.10E−01	7.84E−01
9	7.65E−04	7.65E−01	1.18E+00
10	1.02E−03	1.02E+00	1.57E+00
102	2.45E−02	2.45E+01	4.08E−01
103	2.47E−02	2.47E+01	4.04E−01
104	2.50E−02	2.50E+01	4.00E−01
105	2.52E−02	2.52E+01	3.96E−01
106	2.55E−02	2.55E+01	3.92E−01

A graph of B as a function of r follows.



86 •• A 50-turn coil of radius 10.0 cm carries a current of 4.00 A and a concentric 20-turn coil of radius 0.500 cm carries a current of 1.00 A. The planes of the two coils are perpendicular. Find the magnitude of the torque exerted by the large coil on the small coil. (Neglect any variation in magnetic field due to the current in the large coil over the region occupied by the small coil.)

Picture the Problem We can use $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ to find the torque exerted on the small coil (magnetic moment = $\vec{\boldsymbol{\mu}}$) by the magnetic field $\vec{\boldsymbol{B}}$ due to the current in the large coil.

Relate the torque exerted by the large coil on the small coil to the magnetic moment $\vec{\boldsymbol{\mu}}$ of the small coil and the magnetic field $\vec{\boldsymbol{B}}$ due to the current in the large coil:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$$

or, because the planes of the two coils are perpendicular, $\tau = \mu B$

Express the magnetic moment of the small coil:

$$\mu = NIA$$

where I is the current in the coil, N is the number of turns in the coil, and A is the cross-sectional area of the coil.

Express the magnetic field at the center of the large coil:

$B = \dfrac{N'\mu_0 I'}{2R}$ where I' is the current in the large coil, N' is the number of turns in the coil, and R is its radius.

Substitute for B and μ in the expression for τ to obtain:

$$\tau = \frac{NN'II'A\mu_0}{2R}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{(50)(20)(4.00\,\text{A})(1.00\,\text{A})\pi(0.500\,\text{cm})^2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)}{2(10.0\,\text{cm})} = \boxed{1.97\,\mu\text{N}\cdot\text{m}}$$

87 •• The magnetic needle of a compass is a uniform rod with a length of 3.00 cm, a radius of 0.850 mm, and a density of 7.96×10^3 kg/m^3. The needle is free to rotate in a horizontal plane, where the horizontal component of Earth's magnetic field is 0.600 G. When disturbed slightly, the compass executes simple harmonic motion about its midpoint with a frequency of 1.40 Hz. (*a*) What is the magnetic dipole moment of the needle? (*b*) What is the magnetization of the needle? (*c*) What is the amperian current on the surface of the needle?

Picture the Problem (*a*) We can solve the equation for the frequency f of the compass needle for the magnetic dipole moment of the needle. In Parts (*b*) and (*c*) we can use their definitions to find the magnetization M and the amperian current I_{amperian}.

(*a*) The frequency of the compass needle is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}} \Rightarrow \mu = \frac{4\pi^2 f^2 I}{B}$$

where I is the moment of inertia of the needle.

The moment of inertia of the needle is:

$$I = \tfrac{1}{12} mL^2 = \tfrac{1}{12}\rho V L^2 = \tfrac{1}{12}\rho \pi r^2 L^3$$

Substitute for I to obtain:

$$\mu = \frac{\pi^3 f^2 \rho r^2 L^3}{3B}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi^3 \left(1.40\,\text{s}^{-1}\right)^2 \left(7.96\times 10^3\,\text{kg/m}^3\right)\left(0.850\times 10^{-3}\,\text{m}\right)^2 \left(0.0300\,\text{m}\right)^3}{3\left(0.600\times 10^{-4}\,\text{T}\right)} = \boxed{5.24\times 10^{-2}\,\text{A}\cdot\text{m}^2}$$

(*b*) Use its definition to express the magnetization M:

$$M = \frac{\mu}{V}$$

Substitute to obtain:

$$M = \frac{\mu}{V} = \frac{\pi^3 f^2 \rho r^2 L^3}{3BV} = \frac{\pi^2 f^2 \rho L^2}{3B}$$

Substitute numerical values and evaluate M:

$$M = \frac{\pi^2 \left(1.40\,\text{s}^{-1}\right)^2 \left(7.96\times 10^3\,\text{kg/m}^3\right)\left(0.0300\,\text{m}\right)^2}{3\left(0.600\times 10^{-4}\,\text{T}\right)} = \boxed{7.70\times 10^5\,\text{A/m}}$$

(*c*) The amperian current on the surface of the needle is:

$$I_{\text{amperian}} = ML = \left(7.70\times 10^5\,\text{A/m}\right)\left(0.0300\,\text{m}\right) = \boxed{23.1\,\text{kA}}$$

88 •• A relatively inexpensive ammeter, called a *tangent galvanometer*, can be made using Earth's magnetic field. A plane circular coil that has N turns and a radius R is oriented so the magnetic field B_c it produces in the center of the coil is either east or west. A compass is placed at the center of the coil. When there is no current in the coil, assume the compass needle points due north. When there is a current in the coil (I), the compass needle points in the direction of the resultant magnetic field at an angle θ to the north. Show that the current I is related to θ and to the horizontal component of Earth's magnetic field B_e by $I = \dfrac{2RB_e}{\mu_0 N}\tan\theta$.

Picture the Problem Note that B_e and B_c are perpendicular to each other and that the resultant magnetic field is at an angle θ with north. We can use trigonometry to relate B_c and B_e and express B_c in terms of the geometry of the coil and the current flowing in it.

Express B_c in terms of B_e:

$$B_c = B_e \tan\theta$$

where θ is the angle of the resultant field from north.

Express the field B_c due to the current in the coil:

$$B_c = \frac{N\mu_0 I}{2R}$$

where N is the number of turns.

Substitute for B_c to obtain:

$$\frac{N\mu_0 I}{2R} = B_e \tan\theta \Rightarrow I = \boxed{\frac{2RB_e}{\mu_0 N}\tan\theta}$$

89 •• Earth's magnetic field is about 0.600 G at the magnetic poles, and is pointed vertically downward at the magnetic pole in the northern hemisphere. If the magnetic field were due to an electric current circulating in a loop at the radius of the inner iron core of Earth (approximately 1300 km), (*a*) what would be the magnitude of the current required? (*b*) What direction would this current have—the same as Earth's spin, or opposite? Explain your answer.

Picture the Problem The current required can be found by solving the equation for the magnetic field on the axis of a current loop for the current in the loop. We can use the right-hand rule to determine the direction of this current.

(*a*) The magnetic field on the axis of a current loop is given by:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}}$$

Solving for I yields:

$$I = \frac{4\pi\left(x^2 + R^2\right)^{3/2} B_x}{2\pi\mu_0 R^2}$$

Substitute numerical values and evaluate I:

$$I = \frac{4\pi\left((6370\text{ km})^2 + (1300\text{ km})^2\right)^{3/2}\left(0.600\text{ G}\times\dfrac{1\text{ T}}{10^4\text{ G}}\right)}{2\pi\left(4\pi\times10^{-7}\ \dfrac{\text{N}}{\text{A}^2}\right)(1300\text{ km})^2} = \boxed{15.5\text{ GA}}$$

(b) Because Earth's magnetic field points down at the north pole, application of the right-hand rule indicates that the current is counterclockwise when viewed from above the north pole.

90 •• A long, narrow bar magnet has its magnetic moment $\vec{\boldsymbol{\mu}}$ parallel to its long axis and is suspended at its center—in essence becoming a frictionless compass needle. When the magnet is placed in a magnetic field $\vec{\boldsymbol{B}}$, it lines up with the field. If it is displaced by a small angle and released, show that the magnet will oscillate about its equilibrium position with frequency given by $\dfrac{1}{2\pi}\sqrt{\dfrac{\mu B}{I}}$, where I is the moment of inertia about the point of suspension.

Picture the Problem We can apply Newton's 2nd law for rotational motion to obtain the differential equation of motion of the bar magnet. While this equation is not linear, we can use a small-angle approximation to render it linear and obtain an expression for the square of the angular frequency that we can solve for the frequency f of the motion.

Apply Newton's 2nd law to the bar magnet to obtain the differential equation of motion for the magnet:

$$-\mu B\sin\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about an axis through its point of suspension.

For small displacements from equilibrium ($\theta << 1$):

$$-\mu B\theta \approx I\frac{d^2\theta}{dt^2}$$

Rewrite the differential equation as:

$$I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0$$

Because the coefficient of the linear term is the square of the angular frequency, we have:

$$\omega^2 = \frac{\mu B}{I} \Rightarrow \omega = \sqrt{\frac{\mu B}{I}}$$

Because $\omega = 2\pi f$:

$$2\pi f = \sqrt{\frac{\mu B}{I}} \Rightarrow f = \boxed{\frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}}$$

91 •• An infinitely long straight wire is bent, as shown in Figure 27-66. The circular portion has a radius of 10.0 cm and its center a distance *r* from the straight part. Find *r* so that the magnetic field at the region occupied by the center of the circular portion is zero.

Picture the Problem Let the positive *x* direction be out of the page. We can use the expressions for the magnetic fields due to an infinite straight line and a circular loop to express the net magnetic field at the center of the circular loop. We can set this net field to zero and solve for *r*.

Express the net magnetic field at the center of circular loop:

$$\vec{B} = \vec{B}_{\text{loop}} + \vec{B}_{\text{line}} \qquad (1)$$

Letting *R* represent the radius of the loop, express $\vec{B}_{\text{loop}}$:

$$\vec{B}_{\text{loop}} = -\frac{\mu_0 I}{2R}\hat{i}$$

Express the magnetic field due to the current in the infinite straight line:

$$\vec{B}_{\text{line}} = \frac{\mu_0 I}{2\pi r}\hat{i}$$

Substitute for $\vec{B}_{\text{loop}}$ and $\vec{B}_{\text{line}}$ in equation (1) and simplify to obtain:

$$\vec{B} = -\frac{\mu_0 I}{2R}\hat{i} + \frac{\mu_0 I}{2\pi r}\hat{i} = \left(-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r}\right)\hat{i}$$

If $\vec{B} = 0$, then:

$$-\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi r} = 0 \Rightarrow -\frac{1}{R} + \frac{1}{\pi r} = 0$$

Solving for *r* yields:

$$r = \frac{10.0\,\text{cm}}{\pi} = \boxed{3.18\,\text{cm}}$$

92 •• (*a*) Find the magnetic field strength point *P* on the perpendicular bisector of a wire segment carrying current *I*, as shown in Figure 27-67. (*b*) Use your result from Part (*a*) to find the magnetic field strength at the center of a regular polygon of *N* sides. (*c*) Show that when *N* is very large, your result approaches that for the magnetic field strength at the center of a circle.

Picture the Problem (*a*) We can use the expression for *B* due to a straight wire segment, to find the magnetic field strength at P. Note that the current in the wires whose lines contain point P do not contribute to the magnetic field strength at point P. In Part (b) we can use our result from (a), together with the value for θ when the polygon has N sides, to obtain an expression for B at the center of a polygon of N sides. (c) Letting N grow without bound will yield the equation for the magnetic field strength at the center of a circle.

(a) Express the magnetic field strength at P due to the straight wire segment:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

Because $\theta_1 = \theta_2 = \theta$:

$$B_P = \frac{\mu_0}{4\pi}\frac{I}{R}\left(2\sin\theta\right) = \left(\frac{\mu_0}{2\pi}\frac{I}{R}\right)\sin\theta$$

Refer to the figure to obtain:

$$\sin\theta = \frac{a}{\sqrt{a^2+R^2}}$$

Substituting for $\sin\theta$ in the expression for B_P yields:

$$B_P = \boxed{\frac{\mu_0 aI}{2\pi R\sqrt{a^2+R^2}}}$$

(b) θ for an N-sided polygon is given by:

$$\theta = \frac{\pi}{N}$$

Because each side of the polygon contributes to B an amount equal to that obtained in (a):

$$B = \boxed{\left(\frac{N\mu_0 I}{2\pi R}\right)\sin\left(\frac{\pi}{N}\right)},\ N = 3, 4, \ldots K$$

(c) For large N, π/N is small, so $\sin\left(\frac{\pi}{N}\right) \approx \frac{\pi}{N}$. Hence:

$$B_\infty = \text{Limit}_{N\to\infty} N\frac{\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{N}\right)$$

$$= \frac{\mu_0 I}{2\pi R}\text{Limit}_{N\to\infty} N\frac{\pi}{N} = \boxed{\frac{\mu_0 I}{2R}}$$

the expression for the magnetic field strength at the center of a current-carrying circular loop.

93 •• The current in a long cylindrical conductor of radius 10 cm varies with distance from the axis of the cylinder according to the relation $I(r) = (50\ \text{A/m})r$. Find the magnetic field at the following perpendicular distances from the wire's central axis (a) 5.0 cm, (b) 10 cm, and (c) 20 cm.

Picture the Problem We can use Ampère's law to derive expressions for $B(r)$ for

$r \le R$ and $r > R$ that we can evaluate for the given distances from the center of the cylindrical conductor.

Apply Ampère's law to a closed circular path a distance $r \le R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(r)$$

Solve for $B(r)$ to obtain:

$$B(r) = \frac{\mu_0 I(r)}{2\pi r}$$

Substitute for $I(r)$:

$$B(r) = \frac{\mu_0 (50\,\text{A/m})r}{2\pi\, r} = \frac{\mu_0 (50\,\text{A/m})}{2\pi}$$

(a) and (b) Noting that B is independent of r, substitute numerical values and evaluate $B(5.0\text{ cm})$ and $B(10\text{ cm})$:

$$B(5.0\,\text{cm}) = B(10\,\text{cm}) = \frac{(4\pi\times10^{-7}\,\text{N/A}^2)(50\,\text{A/m})}{2\pi} = \boxed{10\,\mu\text{T}}$$

(c) Apply Ampère's law to a closed circular path a distance $r > R$ from the center of the cylindrical conductor to obtain:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B(r)(2\pi r) = \mu_0 I_C = \mu_0 I(R)$$

Solving for $B(r)$ yields:

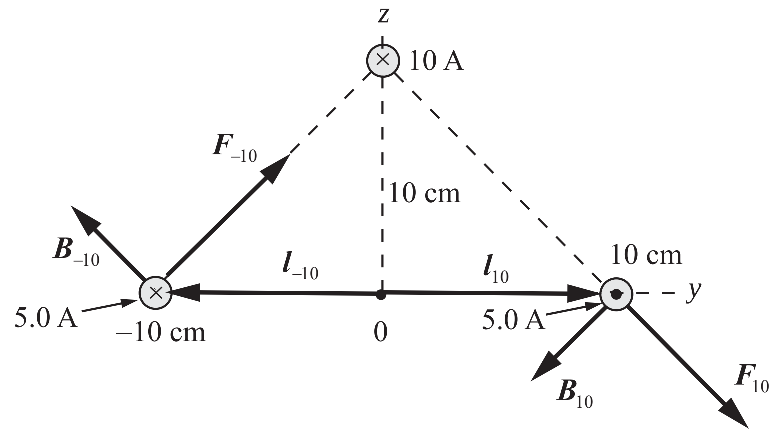
$$B(r) = \frac{\mu_0 I(R)}{2\pi r}$$

Substitute numerical values and evaluate $B(20\text{ cm})$:

$$B(20\,\text{cm}) = \frac{(4\pi\times10^{-7}\,\text{N/A}^2)(50\,\text{A/m})(0.10\,\text{m})}{2\pi(0.20\,\text{m})} = \boxed{5.0\,\mu\text{T}}$$

94 •• Figure 27- 68 shows a square loop that has 20-cm long sides and is in the $z = 0$ plane with its center at the origin. The loop carries a current of 5.0 A. An infinitely long wire that is parallel to the x axis and carries a current of 10 A intersects the z axis at $z = 10$ cm. The directions of the currents are shown in the figure. (a) Find the net torque on the loop. (b) Find the net force on the loop.

Picture the Problem The field $\vec{B}$ due to the 10-A current is in the yz plane. The net force on the wires of the square in the y direction cancel and do not contribute to a net torque or force. We can use $\vec{\tau} = \vec{l} \times \vec{F}$, $\vec{F} = I\vec{\ell} \times \vec{B}$, and the expression for the magnetic field due to a long straight wire to express the torque acting on each of the wires and hence, the net torque acting on the loop.



(*a*) The net torque about the *x* axis is the sum of the torques due to the forces $\vec{F}_{10}$ and $\vec{F}_{-10}$:

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{10} + \vec{\tau}_{-10}$$

Substituting for $\vec{\tau}_{10}$ and $\vec{\tau}_{-10}$ yields:

$$\vec{\tau}_{\text{net}} = \vec{l}_{10} \times \vec{F}_{10} + \vec{l}_{-10} \times \vec{F}_{-10}$$

where the subscripts refer to the positions of the current-carrying wires.

The forces acting on the wires are given by:

$$\vec{F}_{10} = \left(I\vec{\ell}\right)_{10} \times \vec{B}_{10}$$

and

$$\vec{F}_{-10} = \left(I\vec{\ell}\right)_{-10} \times \vec{B}_{-10}$$

Substitute for $\vec{F}_{10}$ and $\vec{F}_{-10}$ to obtain:

$$\vec{\tau}_{\text{net}} = \vec{l}_{10} \times \left[\left(I\vec{\ell}\right)_{10} \times \vec{B}_{10}\right] + \vec{l}_{-10} \times \left[\left(I\vec{\ell}\right)_{-10} \times \vec{B}_{-10}\right] \qquad (1)$$

The lever arms for the forces acting on the wires at $y = 10$ cm and $y = -10$ cm are:

$$\vec{l}_{10} = (0.10\text{ m})\hat{j} \text{ and } \vec{l}_{-10} = -(0.10\text{ m})\hat{j}$$

The magnetic field at the wire at $y = 10$ cm is given by:

$$\vec{B}_{10} = \frac{\mu_0}{4\pi}\frac{2I}{R}\frac{1}{\sqrt{2}}\left(-\hat{j} - \hat{k}\right)$$

where

$$R = \sqrt{(0.10\text{m})^2 + (0.10\text{m})^2} = 0.141\text{m}.$$

Substitute numerical values and evaluate $\vec{B}_{10}$:

$$\vec{B}_{10}=\frac{4\pi\times10^{-7}\ \text{N/A}^2}{4\pi\sqrt{2}}\frac{2(10\ \text{A})}{0.141\,\text{m}}\left(-\hat{\boldsymbol{j}}-\hat{\boldsymbol{k}}\right)=(10.0\ \mu\text{T})\left(-\hat{\boldsymbol{j}}-\hat{\boldsymbol{k}}\right)$$

Proceed similarly to obtain:

$$\vec{B}_{-10}=(10.0\ \mu\text{T})\left(-\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)$$

Substitute in equation (1) and simplify to obtain:

$$\begin{aligned}\vec{\boldsymbol{\tau}}_{\text{net}}&=(0.10\ \text{m})\hat{j}\times\left[(5.0\ \text{A})(0.20\ \text{m})\hat{i}\times(10.0\ \mu\text{T})\left(-\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)\right]\\&\quad-(0.10\ \text{m})\hat{j}\times\left[(5.0\ \text{A})(0.20\ \text{m})\left(-\hat{i}\right)\times(10.0\ \mu\text{T})\left(-\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)\right]\\&=\boxed{-(2.0\ \mu\text{N}\cdot\text{m})\hat{i}}\end{aligned}$$

(*b*) The net force acting on the loop is the sum of the forces acting on its four sides:

$$\vec{F}_{net}=\vec{F}_{10}+\vec{F}_{-10}\qquad(2)$$

Evaluate $\vec{F}_{10}$ to obtain:

$$\begin{aligned}\vec{F}_{10}&=\left(\boldsymbol{I}\vec{\ell}\right)_{10}\times\vec{B}_{10}=(5.0\ \text{A})(0.20\,\text{m})\hat{\boldsymbol{i}}\times(10.0\ \mu\text{T})\left(-\hat{\boldsymbol{j}}-\hat{\boldsymbol{k}}\right)\\&=(10\ \mu\text{N})\left[\hat{\boldsymbol{i}}\times\left(-\hat{\boldsymbol{j}}-\hat{\boldsymbol{k}}\right)\right]\\&=(10\ \mu\text{N})\left(-\hat{\boldsymbol{k}}+\hat{\boldsymbol{j}}\right)\end{aligned}$$

Evaluating $\vec{F}_{-10}$ yields:

$$\begin{aligned}\vec{F}_{-10}&=\left(\boldsymbol{I}\vec{\ell}\right)_{-10}\times\vec{B}_{-10}=(5.0\ \text{A})(-0.20\,\text{m})\hat{\boldsymbol{i}}\times(10.0\ \mu\text{T})\left(-\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)\\&=(-10\ \mu\text{N})\left[\hat{\boldsymbol{i}}\times\left(-\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)\right]\\&=(10\ \mu\text{N})\left(\hat{\boldsymbol{k}}+\hat{\boldsymbol{j}}\right)\end{aligned}$$

Substitute for $\vec{F}_{10}$ and $\vec{F}_{-10}$ in equation (2) and simplify to obtain:

$$\vec{F}_{\text{net}}=(10\ \mu\text{N})\left(-\hat{\boldsymbol{k}}+\hat{\boldsymbol{j}}\right)+(10\ \mu\text{N})\left(\hat{\boldsymbol{k}}+\hat{\boldsymbol{j}}\right)=\boxed{(20\ \mu\text{N})\hat{\boldsymbol{j}}}$$

95 •• **[SSM]** A current balance is constructed in the following way: A straight 10.0-cm-long section of wire is placed on top of the pan of an electronic balance (Figure 27-69). This section of wire is connected in series with a power supply and a long straight horizontal section of wire that is parallel to it and positioned directly above it. The distance between the central axes of the two wires is 2.00 cm. The power supply provides a current in the wires. When the power supply is switched on, the reading on the balance increases by 5.00 mg. What is the current in the wire?

Picture the Problem The force acting on the lower wire is given by $F_{\text{lower wire}} = I\ell B$, where I is the current in the lower wire, ℓ is the length of the wire on the balance, and B is the magnetic field strength at the location of the lower wire due to the current in the upper wire. We can apply Ampere's law to find B at the location of the wire on the pan of the balance.

The force experienced by the lower wire is given by:

$$F_{\text{lower wire}} = I\ell B$$

Apply Ampere's law to a closed circular path of radius r centered on the upper wire to obtain:

$$B(2\pi r) = \mu_0 I_C = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Substituting for B in the expression for the force on the lower wire and simplifying yields:

$$F_{\text{lower wire}} = I\ell\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 \ell I^2}{2\pi r}$$

Solve for I to obtain:

$$I = \sqrt{\frac{2\pi r F_{\text{lower wire}}}{\mu_0 \ell}}$$

Note that the force on the lower wire is the increase in the reading of the balance. Substitute numerical values and evaluate I:

$$I = \sqrt{\frac{2\pi(2.00\text{ cm})(5.00\times10^{-6}\text{ kg})}{(4\pi\times10^{-7}\text{ N/A}^2)(10.0\text{ cm})}} = 2.236\text{ A} = \boxed{2.24\text{ A}}$$

96 •• Consider the current balance of Problem 95. If the sensitivity of the balance is 0.100 mg, what is the minimum current detectable using this current balance?

Picture the Problem We can use a proportion to relate minimum current detectible using this balance to its sensitivity and to the current and change in balance reading from Problem 95.

The minimum current I_{min} detectible is to the sensitivity of the balance as the current in Problem 95 is to the change in the balance reading in Problem 95:

$$\frac{I_{\text{min}}}{0.100\text{ mg}} = \frac{2.236\text{ A}}{5.00\text{ mg}}$$

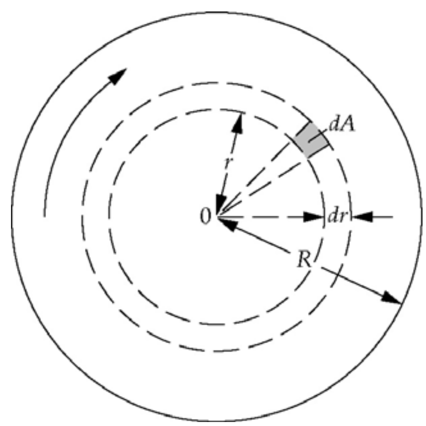
Solving for I_{min} yields:

$$I_{min} = (0.100\,\text{mg})\left(\frac{2.236\,\text{A}}{5.00\,\text{mg}}\right) = \boxed{44.7\,\text{mA}}$$

The "standard" current balance can be made very sensitive by increasing the length (i.e., moment arm) of the wire balance, which one cannot do with this kind; however, this is compensated somewhat by the high sensitivity of the electronic balance.

97 ••• [SSM] A non-conducting disk that has radius R, carries a uniform surface charge density σ, and rotates with angular speed ω. (*a*) Consider an annular strip that has a radius r, a width dr, and a charge dq. Show that the current (dI) produced by this rotating strip is given by $\omega\sigma r dr$. (b) Use your result from Part (*a*) to show that the magnetic field strength at the center of the disk is given by the expression $\frac{1}{2}\mu_0\sigma\omega R$. (*c*) Use your result from Part (*a*) to find an expression for the magnetic field strength at a point on the central axis of the disk a distance z from its center.

Picture the Problem The diagram shows the rotating disk and the circular strip of radius r and width dr with charge dq. We can use the definition of surface charge density to express dq in terms of r and dr and the definition of current to show that $dI = \omega\sigma r\ dr$. We can then use this current and expression for the magnetic field on the axis of a current loop to obtain the results called for in Parts (*b*) and (*c*).



(*a*) Express the total charge dq that passes a given point on the circular strip once each period:

$$dq = \sigma dA = 2\pi\sigma r dr$$

Letting q be the total charge that passes along a radial section of the disk in a period of time T, express the current in the element of width dr:

$$dI = \frac{dq}{dt} = \frac{2\pi\sigma r dr}{\frac{2\pi}{\omega}} = \boxed{\omega\sigma r dr}$$

(*c*) Express the magnetic field dB_x at a distance *z* along the axis of the disk due to the current loop of radius *r* and width *dr*:

$$dB_x = \frac{\mu_0}{4\pi}\frac{2\pi r^2\,dI}{\left(z^2+r^2\right)^{3/2}} = \frac{\mu_0\omega\sigma r^3}{2\left(z^2+r^2\right)^{3/2}}dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

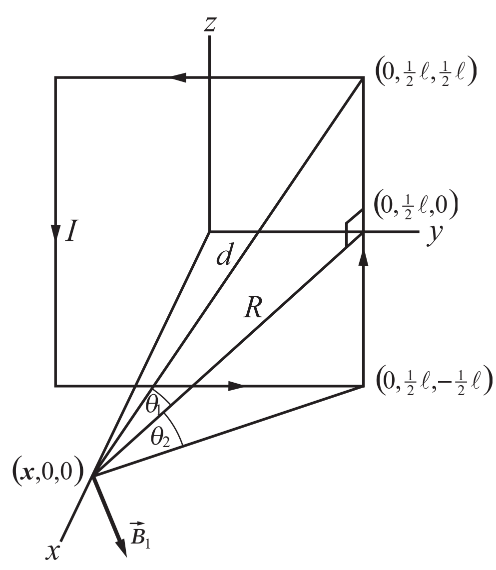
$$B_x = \frac{\mu_0\omega\sigma}{2}\int_0^R \frac{r^3}{\left(z^2+r^2\right)^{3/2}}dr = \boxed{\frac{\mu_0\omega\sigma}{2}\left(\frac{R^2+2z^2}{\sqrt{R^2+z^2}}-2x\right)}$$

(*b*) Evaluate B_x for $x = 0$:

$$B_x(0) = \frac{\mu_0\omega\sigma}{2}\left(\frac{R^2}{\sqrt{R^2}}\right) = \boxed{\tfrac{1}{2}\mu_0\sigma\omega R}$$

98 ••• A square loop that has sides of length ℓ lies in the $z = 0$ plane with its center at the origin. The loop carries a current *I*. (*a*) Derive an expression for the magnetic field strength at any point on the *z* axis. (*b*) Show that for *z* much larger than ℓ, your result from Part (*a*) becomes $B \approx \mu\mu_0/\left(2\pi z^3\right)$, where μ is the magnitude of the magnetic moment of the loop.

Picture the Problem From the symmetry of the system it is evident that the magnetic fields due to each segment have the same magnitude. We can express the magnetic field at (*x*,0,0) due to one side (segment) of the square, find its component in the *x* direction, and then multiply by four to find the resultant field.



(*a*) *B* due to a straight wire segment is given by:

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_1 + \sin\theta_2\right)$$

where *R* is the perpendicular distance from the wire segment to the field point.

Use $\theta_1 = \theta_2$ and $R = \sqrt{x^2 + \ell^2/4}$ to express *B* due to one side at (x,0,0):

$$B_1(x,0,0) = \frac{\mu_0}{4\pi}\frac{I}{\sqrt{x^2 + \frac{\ell^2}{4}}}\left(2\sin\theta_1\right)$$

$$= \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2 + \frac{\ell^2}{4}}}\left(\sin\theta_1\right)$$

Referring to the diagram, express $\sin\theta_1$:

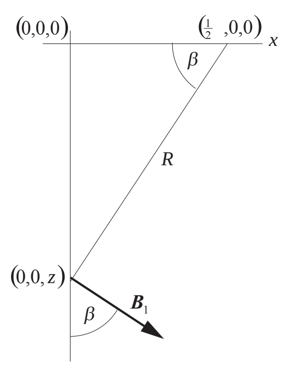
$$\sin\theta_1 = \frac{\frac{\ell}{2}}{d} = \frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

Substituting for $\sin\theta_1$ and simplifying yields:

$$B_1(x,0,0) = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2 + \frac{\ell^2}{4}}}\frac{\frac{\ell}{2}}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

$$= \frac{\mu_0 I}{4\pi\sqrt{x^2 + \frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2 + \frac{\ell^2}{2}}}$$

By symmetry, the sum of the *y* and *z* components of the fields due to the four wire segments must vanish, whereas the *x* components will add. The diagram to the right is a view of the *xy* plane showing the relationship between $\vec{B}_1$ and the angle β it makes with the *x* axis.



Express B_{1x}:

$$B_{1x} = B_1\cos\beta$$

Substituting for $\cos\beta$ and simplifying yields:

$$B_{1x} = \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{\ell^2}{4}}}\frac{\ell}{\sqrt{x^2+\frac{\ell^2}{2}}}\frac{\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}$$

$$= \frac{\mu_0 I\ell^2}{8\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}$$

The resultant magnetic field is the sum of the fields due to the four wire segments (sides of the square):

$$\vec{B} = 4B_{1x}\hat{i}$$

$$= \boxed{\frac{\mu_0 I\ell^2}{2\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}\hat{i}}$$

(*b*) Factor x^2 from the two factors in the denominator to obtain:

$$\vec{B} = \frac{\mu_0 I\ell^2}{2\pi x^2\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{x^2\left(1+\frac{\ell^2}{2x^2}\right)}}\hat{i}$$

$$= \frac{\mu_0 I\ell^2}{2\pi x^3\left(1+\frac{\ell^2}{4x^2}\right)\sqrt{1+\frac{\ell^2}{2x^2}}}\hat{i}$$

For $x >> \ell$:

$$\vec{B} \approx \frac{\mu_0 I\ell^2}{2\pi x^3}\hat{i} = \boxed{\frac{\mu_0 \mu}{2\pi x^3}\hat{i}}$$

where $\mu = I\ell^2$.