Chapter 26
The Magnetic Field

Conceptual Problems

1 • **[SSM]** When the axis of a cathode-ray tube is horizontal in a region in which there is a magnetic field that is directed vertically upward, the electrons emitted from the cathode follow one of the dashed paths to the face of the tube in Figure 26-30. The correct path is (*a*) 1, (*b*) 2, (*c*) 3, (*d*) 4, (*e*) 5.

Determine the Concept Because the electrons are initially moving at 90° to the magnetic field, they will be deflected in the direction of the magnetic force acting on them. Use the right-hand rule based on the expression for the magnetic force acting on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$, remembering that, for a negative charge, the force is in the direction opposite that indicated by the right-hand rule, to convince yourself that the particle will follow the path whose terminal point on the screen is 2. $\boxed{(b)}$ is correct.

2 •• We define the direction of the electric field to be the same as the direction of the force on a positive test charge. Why then do we *not* define the direction of the magnetic field to be the same as the direction of the magnetic force on a moving positive test charge?

Determine the Concept The direction of the force depends on the direction of the velocity. We do not define the direction of the magnetic field to be in the direction of the force because if we did, the magnetic field would be in a different direction each time the velocity was in a different direction. If this were the case, the magnetic field would not be a useful construct.

3 • **[SSM]** A *flicker bulb* is a light bulb that has a long, thin flexible filament. It is meant to be plugged into an ac outlet that delivers current at a frequency of 60 Hz. There is a small permanent magnet inside the bulb. When the bulb is plugged in the filament oscillates back and forth. At what frequency does it oscillate? Explain.

Determine the Concept Because the alternating current running through the filament is changing direction every 1/60 s, the filament experiences a force that changes direction at the frequency of the current.

4 • In a cyclotron, the potential difference between the dees oscillates with a period given by $T = 2\pi m/(qB)$. Show that the expression to the right of the equal sign has units of seconds if q, B and m have units of coulombs, teslas and kilograms, respectively.

Determine the Concept Substituting the SI units for q, B, and m yields:

$$\frac{C \cdot T}{kg} = \frac{C\left(\dfrac{N}{A \cdot m}\right)}{kg} = \frac{C\left(\dfrac{N \cdot s}{C \cdot m}\right)}{kg} = \frac{\dfrac{kg \cdot m}{s^2} \cdot s}{m} = \frac{\dfrac{kg \cdot m}{s}}{kg \cdot m} = \boxed{\frac{1}{s}}$$

5   •   A ^{7}Li nucleus has a charge equal to $+3e$ and a mass that is equal to the mass of seven protons. A ^{7}Li nucleus and a proton are both moving perpendicular to a uniform magnetic field $\vec{B}$. The magnitude of the momentum of the proton is equal to the magnitude of the momentum of the nucleus. The path of the proton has a radius of curvature equal to R_p and the path of the ^{7}Li nucleus has a radius of curvature equal to R_{Li}. The ratio R_p/R_{Li} is closest to (*a*) 3/1, (*b*) 1/3, (*c*) 1/7, (*d*) 7/1, (*e*) 3/7, (*f*) 7/3.

Determine the Concept We can use Newton's 2nd law for circular motion to express the radius of curvature R of each particle in terms of its charge, momentum, and the magnetic field. We can then divide the proton's radius of curvature by that of the ^{7}Li nucleus to decide which of these alternatives is correct.

Apply Newton's 2nd law to the lithium nucleus to obtain:

$$qvB = m\frac{v^2}{R} \Rightarrow R = \frac{mv}{qB}$$

For the ^{7}Li nucleus this becomes:

$$R_{Li} = \frac{p_{Li}}{3eB} \tag{1}$$

For the proton we have:

$$R_p = \frac{p_p}{eB} \tag{2}$$

Divide equation (2) by equation (1) and simplify to obtain:

$$\frac{R_p}{R_{Li}} = \frac{\dfrac{p_p}{eB}}{\dfrac{p_{Li}}{3eB}} = 3\frac{p_p}{p_{Li}}$$

Because the momenta are equal:

$$\frac{R_p}{R_{Li}} = 3 \Rightarrow \boxed{(a)} \text{ is correct.}$$

6   •   An electron moving in the $+x$ direction enters a region that has a uniform magnetic field in the $+y$ direction. When the electron enters this region, it will (*a*) be deflected toward the $+y$ direction, (*b*) be deflected toward the $-y$ direction, (*c*) be deflected toward the $+z$ direction, (*d*) be deflected toward the $-z$ direction, (*e*) continue undeflected in the $+x$ direction.

Determine the Concept Application of the right-hand rule indicates that a positively charged body would experience a downward force and, in the absence of other forces, be deflected downward. Because the direction of the magnetic force on an electron is opposite that of the force on a positively charged object, an electron will be deflected upward. $\boxed{(c)}$ is correct.

7 •   **[SSM]**   In a velocity selector, the speed of the undeflected charged particle is given by the ratio of the magnitude of the electric field to the magnitude of the magnetic field. Show that E/B in fact does have the units of m/s if E and B are in units of volts per meter and teslas, respectively.

Determine the Concept Substituting the SI units for E and B yields:

$$\frac{\dfrac{N}{C}}{\dfrac{N}{A\cdot m}} = \frac{A\cdot m}{C} == \frac{\dfrac{C}{s}\cdot m}{C} = \boxed{\frac{m}{s}}$$

8 •   How are the properties of magnetic field lines similar to the properties of electric field lines? How are they different?

Similarities	Differences
1. Their density on a surface perpendicular to the lines is a measure of the strength of the field	1. Magnetic field lines neither begin nor end. Electric field lines begin on positive charges and end on negative charges.
2. The lines point in the direction of the field	2. Electric forces are parallel or anti-parallel to the field lines. Magnetic forces are perpendicular to the field lines.
3. The lines do not cross.	

9 •   True or false:

(*a*)   The magnetic moment of a bar magnet points from its north pole to its south pole.
(*b*)   Inside the material of a bar magnet, the magnetic field due to the bar magnet points from the magnet's south pole toward its north pole.
(*c*)   If a current loop simultaneously has its current doubled and its area cut in half, then the magnitude of its magnetic moment remains the same.
(*d*)   The maximum torque on a current loop placed in a magnetic field occurs when the plane of the loop is perpendicular to the direction of the magnetic field.

(*a*) False. By definition, the magnetic moment of a small bar magnet points from its south pole to its north pole.

(*b*) True. The external magnetic field of a bar magnet points from the north pole of the magnet to south pole. Because magnetic field lines are continuous, the magnet's internal field lines point from the south pole to the north pole.

(*c*) True. Because the magnetic dipole moment of a current loop is given by $\vec{\mu} = NIA\hat{n}$, simultaneously doubling the current and halving its area leaves the magnetic dipole moment unchanged.

(*d*) False. The magnitude of the torque acting on a magnetic dipole moment is given $\tau = \mu B \sin\theta$ where θ is the angle between the axis of the current loop and the direction of the magnetic field. When the plane of the loop is perpendicular to the field direction $\theta = 0$ and the torque is zero.

10 •• Show that the von Klitzing constant, h/e^2, gives the SI unit for resistance (the ohm) h and e are in units of joule seconds and coulombs, respectively.

Determine the Concept The von Klitzing resistance is related to the Hall resistance according to $R_K = nR_H$ where $R_K = \dfrac{h}{e^2}$.

Substituting the SI units of h and e yields: $\dfrac{J \cdot s}{C^2} = \dfrac{J/C}{C/s} = V/A = \boxed{\Omega}$

11 ••• The theory of relativity states that no law of physics can be described using the absolute velocity of an object, which is in fact impossible to define due to a lack of an absolute reference frame. Instead, the behavior of interacting objects can only be described by the relative velocities between the objects. New physical insights result from this idea. For example, in Figure 26-31 a magnet moving at high speed relative to some observer passes by an electron that is at rest relative to the same observer. Explain why you are sure that a force must be acting on the electron. In what direction will the force point at the instant the north pole of the magnet passes directly underneath the electron? Explain your answer.

Determine the Concept From relativity; this is equivalent to the electron moving from right to left at speed v with the magnet stationary. When the electron is directly over the magnet, the field points directly up, so there is a force directed out of the page on the electron.

Estimation and Approximation

12 •   Estimate the maximum magnetic force per meter that Earth's magnetic field could exert on a current-carrying wire in a 20-A circuit in your house.

Picture the Problem Because the magnetic force on a current-carrying wire is given by $\vec{F} = I\vec{L} \times \vec{B}$, the maximum force occurs when $\theta = 90°$. Under this condition, $F_{max} = ILB$.

The maximum force per unit length that the Earth's magnetic field could exert on a current-carrying wire in your home is given by:

$$\left.\frac{F}{L}\right]_{max} = IB$$

For a 20-A circuit and $B = 0.5 \times 10^{-4}$ T:

$$\left.\frac{F}{L}\right]_{max} = (20\text{ A})(0.5 \times 10^{-4}\text{ T})$$

$$= \boxed{1\text{ mN/m}}$$

13 ••   Your friend wants to be magician and intends to use Earth's magnetic field to suspend a current-carrying wire above the stage. He asks you to estimate the minimum current needed to suspend the wire just above Earth's surface at the equator (where Earth's magnetic field is horizontal). Assume the wire has a linear mass density of 10 g/m. Would you advise him to proceed with his plans for this act?

Picture the Problem Because the magnetic force on a current-carrying wire is given by $\vec{F} = I\vec{L} \times \vec{B}$, the maximum force occurs when $\theta = 90°$. Under this condition, $F_{max} = ILB$. In order to suspend the wire, this magnetic force would have to be equal in magnitude to the gravitational force exerted by Earth on the wire:

Letting the upward direction be the +y direction, apply $\sum F_y = 0$ to the wire to obtain:

$F_m - F_g = 0$

or,

$ILB - mg = 0$

Solving for I yields:

$$I = \frac{mg}{LB} = \left(\frac{m}{L}\right)\frac{g}{B}$$

where m/L is the linear density of the wire.

Substitute numerical values and evaluate I:

$$I = (10\,\text{g/m})\frac{9.81\,\text{m/s}^2}{0.5\times 10^{-4}\,\text{T}} \approx \boxed{2\,\text{kA}}$$

You should advise him to develop some other act. A current of 2000 A would overheat the wire (which is a gross understatement).

The Force Exerted by a Magnetic Field

14 • Find the magnetic force on a proton moving in the $+x$ direction at a speed of 0.446 Mm/s in a uniform magnetic field of 1.75 T in the $+z$ direction.

Picture the Problem The magnetic force acting on a charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector ("cross") product, and multiply by the scalar q to find $\vec{F}$.

The magnetic force acting on the proton is given by:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Express $\vec{v}$:

$$\vec{v} = (0.446\,\text{Mm/s})\hat{i}$$

Express $\vec{B}$:

$$\vec{B} = (1.75\,\text{T})\hat{k}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = (1.602\times 10^{-19}\,\text{C})[(0.446\,\text{Mm/s})\hat{i}\times(1.75\,\text{T})\hat{k}] = \boxed{-(0.125\,\text{pN})\hat{j}}$$

15 • A point particle has a charge equal to −3.64 nC and a velocity equal to 2.75×10^3 m/s $\hat{i}$. Find the force on the charge if the magnetic field is
(a) 0.38 T $\hat{j}$, (b) 0.75 T $\hat{i}$ + 0.75 T $\hat{j}$, (c) 0.65 T $\hat{i}$, and (d) 0.75 T $\hat{i}$ + 0.75 T $\hat{k}$.

Picture the Problem The magnetic force acting on the charge is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector (also known as the "cross") product, and multiply by the scalar q to find $\vec{F}$.

Express the force acting on the charge:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Substitute numerical values to obtain:

$$\vec{F} = (-3.64\,\text{nC})[(2.75\times 10^3\,\text{m/s})\hat{i}\times\vec{B}]$$

(*a*) Evaluate $\vec{F}$ for $\vec{B} = 0.38$ T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\big[(2.75\times10^3\ \text{m/s})\hat{i}\times(0.38\,\text{T})\hat{j}\big] = \boxed{-(3.8\,\mu\text{N})\hat{k}}$$

(*b*) Evaluate $\vec{F}$ for $\vec{B} = 0.75$ T $\hat{i}$ + 0.75 T $\hat{j}$:

$$\vec{F} = (-3.64\,\text{nC})\big[(2.75\times10^3\ \text{m/s})\hat{i}\times\{(0.75\,\text{T})\hat{i}+(0.75\,\text{T})\hat{j}\}\big] = \boxed{-(7.5\,\mu\text{N})\hat{k}}$$

(*c*) Evaluate $\vec{F}$ for $\vec{B} = 0.65$ T $\hat{i}$:

$$\vec{F} = (-3.64\,\text{nC})\big[(2.75\times10^3\ \text{m/s})\hat{i}\times(0.65\,\text{T})\hat{i}\big] = \boxed{0}$$

(*d*) Evaluate $\vec{F}$ for $\vec{B} = 0.75$ T $\hat{i}$ + 0.75 T $\hat{k}$:

$$\vec{F} = (-3.64\,\text{nC})\big[(2.75\times10^3\ \text{m/s})\hat{i}\times(0.75\,\text{T})\hat{i}+(0.75\,\text{T})\hat{k}\big] = \boxed{(7.5\,\mu\text{N})\hat{j}}$$

16 • A uniform magnetic field equal to 1.48 T is in the $+z$ direction. Find the force exerted by the field on a proton if the velocity of the proton is (*a*) 2.7 km/s $\hat{i}$, (*b*) 3.7 km/s $\hat{j}$, (*c*) 6.8 km/s $\hat{k}$, and (*d*) 4.0 km/s $\hat{i}$ + 3.0 km/s $\hat{j}$.

Picture the Problem The magnetic force acting on the proton is given by $\vec{F} = q\vec{v}\times\vec{B}$. We can express $\vec{v}$ and $\vec{B}$, form their vector product, and multiply by the scalar q to find $\vec{F}$.

The magnetic force acting on the proton is given by:

$$\vec{F} = q\vec{v}\times\vec{B}$$

(*a*) Evaluate $\vec{F}$ for $\vec{v} = 2.7$ *k*m/s $\hat{i}$:

$$\vec{F} = (1.602\times10^{-19}\ \text{C})\big[(2.7\,\text{km/s})\hat{i}\times(1.48\,\text{T})\hat{k}\big] = \boxed{-(6.4\times10^{-16}\ \text{N})\hat{j}}$$

(*b*) Evaluate $\vec{F}$ for $\vec{v} = 3.7$ km/s $\hat{j}$:

$$\vec{F} = (1.602\times10^{-19}\ \text{C})\big[(3.7\,\text{km/s})\hat{j}\times(1.48\,\text{T})\hat{k}\big] = \boxed{(8.8\times10^{-16}\ \text{N})\hat{i}}$$

(c) Evaluate $\vec{F}$ for $\vec{v} = 6.8$ km/s $\hat{k}$:

$$\vec{F} = \left(1.602 \times 10^{-19}\,\text{C}\right)\left[\left(6.8\,\text{km/s}\right)\hat{k} \times \left(1.48\,\text{T}\right)\hat{k}\right] = \boxed{0}$$

(d) Evaluate $\vec{F}$ for $\vec{v} = 4.0$ km/s $\hat{i} + 3.0$ km/s $\hat{j}$:

$$\vec{F} = \left(1.602 \times 10^{-19}\,\text{C}\right)\left[\left\{\left(4.0\,\text{km/s}\right)\hat{i} + \left(3.0\,\text{km/s}\right)\hat{j}\right\} \times \left(1.48\,\text{T}\right)\hat{k}\right]$$
$$= \boxed{\left(7.1 \times 10^{-16}\,\text{N}\right)\hat{i} - \left(9.5 \times 10^{-16}\,\text{N}\right)\hat{j}}$$

17 •   A straight wire segment that is 2.0 m long makes an angle of 30° with a uniform 0.37-T magnetic field. Find the magnitude of the force on the wire if the wire carries a current of 2.6 A.

Picture the Problem The magnitude of the magnetic force acting on a segment of wire carrying a current I is given by $F = I\ell B \sin\theta$ where ℓ is the length of the segment of wire, B is the magnetic field, and θ is the angle between direction of the current in the segment of wire and the direction of the magnetic field.

Express the magnitude of the magnetic force acting on the segment of wire:

$$F = I\ell B \sin\theta$$

Substitute numerical values and evaluate F:

$$F = \left(2.6\,\text{A}\right)\left(2.0\,\text{m}\right)\left(0.37\,\text{T}\right)\sin 30°$$
$$= \boxed{0.96\,\text{N}}$$

18   •   A straight segment of a current-carrying wire has a current element $I\vec{L}$, where $I = 2.7$ A and $\vec{L} = 3.0\,\text{cm}\,\hat{i} + 4.0\,\text{cm}\,\hat{j}$. The segment is in a region with a uniform magnetic field given by $1.3\,\text{T}\hat{i}$. Find the force on the segment of wire.

Picture the Problem We can use $\vec{F} = I\vec{L} \times \vec{B}$ to find the force acting on the wire segment.

Express the force acting on the wire segment:

$$\vec{F} = I\vec{L} \times \vec{B}$$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \left(2.7\,\text{A}\right)\left[\left(3.0\,\text{cm}\right)\hat{i} + \left(4.0\,\text{cm}\right)\hat{j}\right] \times \left(1.3\,\text{T}\right)\hat{i}$$
$$= \boxed{-\left(0.14\,\text{N}\right)\hat{k}}$$

19 •   What is the force on an electron that has a velocity equal to 2.0×10^6 m/s $\hat{i} - 3.0\times10^6\,\hat{j}$ when it is in a region with a magnetic field given by $0.80\,\mathrm{T}\,\hat{i} + 0.60\,\mathrm{T}\,\hat{k}$?

Picture the Problem The magnetic force acting on the electron is given by $\vec{F} = q\vec{v}\times\vec{B}$.

The magnetic force acting on the proton is given by: $\vec{F} = q\vec{v}\times\vec{B}$

Substitute numerical values and evaluate $\vec{F}$:

$$\vec{F} = \left(-1.602\times10^{-19}\,\mathrm{C}\right)\left[\left\{(2\,\mathrm{Mm/s})\hat{i} - (3\,\mathrm{Mm/s})\hat{j}\right\}\times\left(0.8\hat{i} + 0.6\,\hat{j} - 0.4\,\hat{k}\right)\mathrm{T}\right]$$
$$= \left(-0.192\,\mathrm{pN}\right)\hat{k} + \left(-0.128\,\mathrm{pN}\right)\hat{j} + \left(-0.384\,\mathrm{pN}\right)\hat{k} + \left(-0.192\,\mathrm{pN}\right)\hat{i}$$
$$= -(0.192\,\mathrm{pN})\hat{i} - (0.128\,\mathrm{pN})\hat{j} - (0.577\,\mathrm{pN})\hat{k}$$
$$= \boxed{-(0.19\,\mathrm{pN})\hat{i} - (0.13\,\mathrm{pN})\hat{j} - (0.58\,\mathrm{pN})\hat{k}}$$

20 ••   The section of wire shown in Figure 26-32 carries a current equal to 1.8 A from a to b. The segment is in a region that has a magnetic field whose value is $1.2\,\mathrm{T}\,\hat{k}$. Find the total force on the wire and show that the total force is the same as if the wire were in the form of a straight wire directly from a to b and carrying the same current.

Picture the Problem We can use $\vec{F} = I\vec{\ell}\times\vec{B}$ to find the force acting on the segments of the wire as well as the magnetic force acting on the wire if it were a straight segment from a to b.

Express the magnetic force acting on the wire: $\vec{F} = \vec{F}_{3\,\mathrm{cm}} + \vec{F}_{4\,\mathrm{cm}}$

Evaluate $\vec{F}_{3\,\mathrm{cm}}$:

$$\vec{F}_{3\,\mathrm{cm}} = (1.8\,\mathrm{A})\left[(3.0\,\mathrm{cm})\hat{i}\times(1.2\,\mathrm{T})\hat{k}\right]$$
$$= (0.0648\,\mathrm{N})\left(-\hat{j}\right)$$
$$= -(0.0648\,\mathrm{N})\hat{j}$$

Evaluate $\vec{F}_{4\,\mathrm{cm}}$:

$$\vec{F}_{4\,\mathrm{cm}} = (1.8\,\mathrm{A})\left[(4.0\,\mathrm{cm})\hat{j}\times(1.2\,\mathrm{T})\hat{k}\right]$$
$$= (0.0864\,\mathrm{N})\hat{i}$$

Substitute to obtain:
$$\vec{F} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i}$$
$$= \boxed{(86\,\text{mN})\hat{i} - (65\,\text{mN})\hat{j}}$$

If the wire were straight from *a* to *b*:
$$\vec{\ell} = (3.0\,\text{cm})\hat{i} + (4.0\,\text{cm})\hat{j}$$

The magnetic force acting on the wire is:

$$\vec{F} = (1.8\,\text{A})\left[(3.0\,\text{cm})\hat{i} + (4.0\,\text{cm})\hat{j}\right] \times (1.2\,\text{T})\hat{k} = -(0.0648\,\text{N})\hat{j} + (0.0864\,\text{N})\hat{i}$$
$$= \boxed{(86\,\text{mN})\hat{i} - (65\,\text{mN})\hat{j}}$$

in agreement with the result obtained above when we treated the two straight segments of the wire separately.

21 •• A straight, stiff, horizontal 25-cm-long wire that has a mass equal to 50 g is connected to a source of emf by light, flexible leads. A magnetic field of 1.33 T is horizontal and perpendicular to the wire. Find the current necessary to "float" the wire, that is, when it is released from rest it remains at rest.

Picture the Problem Because the magnetic field is horizontal and perpendicular to the wire, the force it exerts on the current-carrying wire will be vertical. Under equilibrium conditions, this upward magnetic force will be equal to the downward gravitational force acting on the wire.

Apply $\sum F_{\text{vertical}} = 0$ to the wire:
$$F_{\text{mag}} - w = 0$$

Express F_{mag}:
$$F_{\text{mag}} = I\ell B \text{ because } \theta = 90°.$$

Substitute for F_{mag} to obtain:
$$I\ell B - mg = 0 \Rightarrow I = \frac{mg}{\ell B}$$
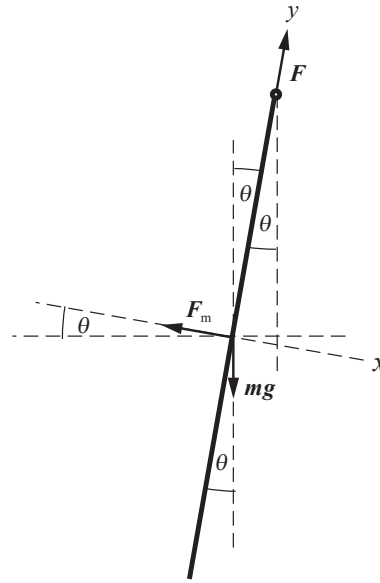
Substitute numerical values and evaluate *I*:
$$I = \frac{(50\,\text{g})(9.81\,\text{m/s}^2)}{(25\,\text{cm})(1.33\,\text{T})} = \boxed{1.5\,\text{A}}$$

22 •• In your physics laboratory class, you have constructed a simple *gaussmeter* for measuring the horizontal component of magnetic fields. The setup consists of a stiff 50-cm wire that hangs vertically from a conducting pivot so that its free end makes contact with a pool of mercury in a dish below (Figure 26-33). The mercury provides an electrical contact without constraining the movement of the wire. The wire has a mass of 5.0 g and conducts a current downward. (*a*) What is the equilibrium angular displacement of the wire from vertical if the horizontal component of the magnetic field is 0.040 T and the current is 0.20 A?

(b) What is the sensitivity of this gaussmeter? That is, what is the ratio of the output to the input (in radians per tesla).

Picture the Problem The magnetic field is out of the page. The diagram shows the gaussmeter displaced from equilibrium under the influence of the gravitational force $m\vec{g}$, the magnetic force $\vec{F}_m$, and the force exerted by the conducting pivot $\vec{F}$. We can apply the condition for translational equilibrium in the x direction to find the equilibrium angular displacement of the wire from the vertical. In Part (b) we can solve the equation derived in Part (a) for B and evaluate this expression for the given data to find the horizontal magnetic field sensitivity of this gaussmeter.

(a) Apply $\sum F_x = 0$ to the wire to obtain:

$$\sum F_x = -F_m + mg\sin\theta = 0$$

The magnitude of the magnetic force acting on the wire is given by:

$F_m = I\ell B\sin\phi$
or, because $\phi = 90°$,
$F_m = I\ell B$

Substitute for F_m to obtain:

$$-I\ell B + mg\sin\theta = 0 \qquad (1)$$

Solving for θ yields:

$$\theta = \sin^{-1}\left[\frac{I\ell B}{mg}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(0.20\,\text{A})(0.50\,\text{m})(0.040\,\text{T})}{(0.0050\,\text{kg})(9.81\,\text{m/s}^2)}\right]$$

$$= 4.679° = \boxed{4.7°} = \boxed{82\,\text{mrad}}$$

(b) The sensitivity of this gaussmeter is the ratio of the output to the input:

$$\text{sensitivity} = \frac{\theta}{B}$$

Substitute numerical values and evaluate the sensitivity of the gaussmeter:

$$\text{sensitivity} = \frac{82 \text{ mrad}}{0.040 \text{ T}} = \boxed{2.0 \text{ rad/T}}$$

23 ••   [SSM]   A 10-cm long straight wire is parallel with the x axis and carries a current of 2.0 A in the $+x$ direction. The force on this wire due to the presence of a magnetic field $\vec{B}$ is $3.0 \text{ N}\hat{j} + 2.0 \text{ N}\hat{k}$. If this wire is rotated so that it is parallel with the y axis with the current in the $+y$ direction, the force on the wire becomes $-3.0 \text{ N}\hat{i} - 2.0 \text{ N}\hat{k}$. Determine the magnetic field $\vec{B}$.

Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (1)$$

Express $\vec{F}$ in terms of $\vec{B}$:

$$\vec{F} = I\vec{\ell} \times \vec{B} = (2.0 \text{ A})\left[(0.10 \text{ m})\hat{i}\right] \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$
$$= (0.20 \text{ A} \cdot \text{m})\hat{i} \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = -(0.20 \text{ A} \cdot \text{m})B_z\hat{j} + (0.20 \text{ A} \cdot \text{m})B_y\hat{k}$$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$$-(0.20 \text{ A} \cdot \text{m})B_z = 3.0 \text{ N}$$
and
$$(0.20 \text{ A} \cdot \text{m})B_y = 2.0 \text{ N}$$

Noting that B_x is undetermined, solve for B_z and B_y:

$$B_z = -15 \text{ T and } B_y = 10 \text{ T}$$

When the wire is rotated so that the current flows in the positive y direction:

$$\vec{F} = I\vec{\ell} \times \vec{B} = (2.0 \text{ A})\left[(0.10 \text{ m})\hat{j}\right] \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$
$$= (0.20 \text{ A} \cdot \text{m})\hat{j} \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = (0.20 \text{ A} \cdot \text{m})B_z\hat{i} - (0.20 \text{ A} \cdot \text{m})B_x\hat{k}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:

$(0.20\,\text{A}\cdot\text{m})B_x = -2.0\,\text{N}$

and

$-(0.20\,\text{A}\cdot\text{m})B_z = -3.0\,\text{N}$

Solve for B_x and B_z to obtain:

$B_x = 10\,\text{T}$ and, in agreement with our results above, $B_z = -15\,\text{T}$

Substitute in equation (1) to obtain:

$\vec{B} = \boxed{(10\,\text{T})\hat{i} + (10\,\text{T})\hat{j} - (15\,\text{T})\hat{k}}$

24 ••   A 10-cm long straight wire is parallel with the z axis and carries a current of 4.0 A in the $+z$ direction. The force on this wire due to a uniform magnetic field $\vec{B}$ is $-0.20\,\text{N}\,\hat{i} + 0.20\,\text{N}\,\hat{j}$. If this wire is rotated so that it is parallel with the x axis with the current is in the $+x$ direction, the force on the wire becomes $0.20\,\hat{k}\,\text{N}$. Find $\vec{B}$.

Picture the Problem We can use the information given in the 1st and 2nd sentences to obtain an expression containing the components of the magnetic field $\vec{B}$. We can then use the information in the 1st and 3rd sentences to obtain a second equation in these components that we can solve simultaneously for the components of $\vec{B}$.

Express the magnetic field $\vec{B}$ in terms of its components:

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ (1)

Express $\vec{F}$ in terms of $\vec{B}$:

$\vec{F} = I\vec{\ell}\times\vec{B}$

$= (4.0\,\text{A})\left[(0.1\,\text{m})\hat{k}\right]\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$= (0.40\,\text{A}\cdot\text{m})\hat{k}\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$= (0.40\,\text{A}\cdot\text{m})B_y\hat{j} - (0.40\,\text{A}\cdot\text{m})B_y\hat{i}$

Equate the components of this expression for $\vec{F}$ with those given in the second sentence of the statement of the problem to obtain:

$(0.40\,\text{A}\cdot\text{m})B_y = 0.20\,\text{N}$

and

$(0.40\,\text{A}\cdot\text{m})B_x = 0.20\,\text{N}$

Noting that B_z is undetermined, solve for B_x and B_y:

$B_x = 0.50\,\text{T}$ and $B_y = 0.50\,\text{T}$

When the wire is rotated so that the current flows in the positive x direction:

$$\vec{F} = I\vec{\ell} \times \vec{B} = (4.0\,\text{A})\left[(0.10\,\text{m})\hat{i}\right] \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$
$$= (0.40\,\text{A}\cdot\text{m})\hat{i} \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = -(0.40\,\text{A}\cdot\text{m})B_z\hat{j} + (0.40\,\text{A}\cdot\text{m})B_y\hat{k}$$

Equate the components of this expression for $\vec{F}$ with those given in the third sentence of the problem statement to obtain:	$-(0.40\,\text{A}\cdot\text{m})B_z = 0$ and $(0.40\,\text{A}\cdot\text{m})B_y = 0.2\,\text{N}$
Solve for B_z and B_y to obtain:	$B_z = 0$ and, in agreement with our results above, $B_y = 0.50\,\text{T}$
Substitute in equation (1) to obtain:	$\boxed{\vec{B} = (0.50\,\text{T})\hat{i} + (0.50\,\text{T})\hat{j}}$

25 •• **[SSM]**   A current-carrying wire is bent into a closed semicircular loop of radius R that lies in the xy plane (Figure 26-34). The wire is in a uniform magnetic field that is in the $+z$ direction, as shown. Verify that the force acting on the loop is zero.

Picture the Problem With the current in the direction indicated and the magnetic field in the z direction, pointing out of the plane of the page, the force is in the radial direction and we can integrate the element of force dF acting on an element of length $d\ell$ between $\theta = 0$ and π to find the force acting on the semicircular portion of the loop and use the expression for the force on a current-carrying wire in a uniform magnetic field to find the force on the straight segment of the loop.
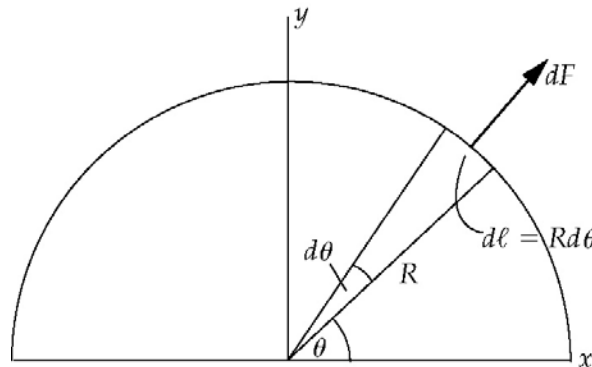
Express the net force acting on the semicircular loop of wire:	$\vec{F} = \vec{F}_{\substack{\text{semicircular}\\\text{loop}}} + \vec{F}_{\substack{\text{straight}\\\text{segment}}}$	(1)

Express the force acting on the straight segment of the loop:

$$\vec{F}_{\substack{\text{straight}\\\text{segment}}} = I\vec{\ell} \times \vec{B} = 2RI\hat{i} \times B\hat{k} = -2RIB\hat{j}$$

Express the force dF acting on the element of the wire of length $d\ell$:

$$dF = Id\ell B = IRBd\theta$$

Express the x and y components of dF:

$$dF_x = dF\cos\theta \text{ and } dF_y = dF\sin\theta$$

Because, by symmetry, the x component of the force is zero, we can integrate the y component to find the force on the wire:

$$dF_y = IRB\sin\theta\,d\theta$$

and

$$\vec{F}_{\substack{\text{semicircular}\\\text{loop}}} = F_y\hat{j} = \left(RIB\int_0^\pi \sin\theta\,d\theta\right)\hat{j}$$

$$= 2RIB\hat{j}$$

Substitute in equation (1) to obtain:

$$\vec{F} = 2RIB\hat{j} - 2RIB\hat{j} = \boxed{0}$$

26 ••• A wire bent in some arbitrary shape carries a current I. The wire is in a region with a uniform magnetic field $\vec{B}$. Show that the total force on the part of the wire from some arbitrary point on the wire (designated as a) to some other arbitrary point on the wire (designated as b) is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ is the vector from point a to point b. In other words, show that the force on an arbitrary section of the bent wire is the same as the force would be on a straight section wire carrying the same current and connecting the two endpoints of the arbitrary section.

Picture the Problem We can integrate the expression for the force $d\vec{F}$ acting on an element of the wire of length $d\vec{L}$ from a to b to show that $\vec{F} = I\vec{L} \times \vec{B}$.

Express the force $d\vec{F}$ acting on the element of the wire of length $d\vec{L}$:

$$d\vec{F} = Id\vec{L} \times \vec{B}$$

Integrate this expression to obtain:

$$\vec{F} = \int_a^b Id\vec{L} \times \vec{B}$$

Because $\vec{B}$ and I are constant:

$$\vec{F} = I\left(\int_a^b d\vec{L}\right) \times \vec{B} = \boxed{I\vec{L} \times \vec{B}}$$

where $\vec{L}$ is the vector from a to b.

Motion of a Point Charge in a Magnetic Field

27 •   **[SSM]**   A proton moves in a 65-cm-radius circular orbit that is perpendicular to a uniform magnetic field of magnitude 0.75 T. (*a*) What is the orbital period for the motion? (*b*) What is the speed of the proton? (*c*) What is the kinetic energy of the proton?

Picture the Problem We can apply Newton's 2nd law to the orbiting proton to relate its speed to its radius. We can then use $T = 2\pi r/v$ to find its period. In Part (*b*) we can use the relationship between T and v to determine v. In Part (*c*) we can use its definition to find the kinetic energy of the proton.

(*a*) Relate the period T of the motion of the proton to its orbital speed v:

$$T = \frac{2\pi r}{v} \qquad (1)$$

Apply Newton's 2nd law to the proton to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Substitute for r in equation (1) and simplify to obtain:

$$T = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi\left(1.673\times10^{-27}\text{ kg}\right)}{\left(1.602\times10^{-19}\text{ C}\right)\left(0.75\text{ T}\right)} = 87.4\,\text{ns}$$

$$= \boxed{87\,\text{ns}}$$

(*b*) From equation (1) we have:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi\left(0.65\text{ m}\right)}{87.4\,\text{ns}} = 4.67\times10^7\text{ m/s}$$

$$= \boxed{4.7\times10^7\text{ m/s}}$$

(*c*) Using its definition, express and evaluate the kinetic energy of the proton:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(1.673\times10^{-27}\text{ kg}\right)\left(4.67\times10^7\text{ m/s}\right)^2 = 1.82\times10^{-12}\text{ J}\times\frac{1\text{eV}}{1.602\times10^{-19}\text{ J}}$$

$$= \boxed{11\text{MeV}}$$

28 •   A 4.5-keV electron (an electron that has a kinetic energy equal to 4.5 keV) moves in a circular orbit that is perpendicular to a magnetic field of 0.325 T. (*a*) Find the radius of the orbit. (*b*) Find the frequency and period of the orbital motion.

Picture the Problem (*a*) We can apply Newton's 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass *m*, charge *q*, speed *v*, and the magnitude of the magnetic field *B*. Using the definition of kinetic energy will allow us to express *r* in terms of *m*, *q*, *B*, and the electron's kinetic energy *K*. (*b*) The period of the orbital motion is given by $T = 2\pi r/v$. Substituting for *r* (or *r/v*) from Part (*a*) will eliminate the orbital speed of the electron and leave us with an expression for *T* that depends only on *m*, *q*, and *B*. The frequency of the orbital motion is the reciprocal of the period of the orbital motion.

(*a*) Apply Newton's 2nd law to the orbiting electron to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Express the kinetic energy of the electron:

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substituting for *v* in the expression for *r* and simplifying yields:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{\sqrt{2(4.5\,\text{keV})(9.109\times10^{-31}\,\text{kg})\left(\dfrac{1.602\times10^{-19}\,\text{J}}{\text{eV}}\right)}}{(1.602\times10^{-19}\,\text{C})(0.325\,\text{T})} = 0.696\,\text{mm} = \boxed{0.70\,\text{mm}}$$

(*b*) Relate the period of the electron's motion to the radius of its orbit and its orbital speed:

$$T = \frac{2\pi r}{v}$$

Because $r = \dfrac{mv}{qB}$:

$$T = \frac{2\pi\dfrac{mv}{qB}}{v} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(9.109 \times 10^{-31}\,\text{kg})}{(1.602 \times 10^{-19}\,\text{C})(0.325\,\text{T})}$$

$$= 1.099 \times 10^{-10}\,\text{s} = \boxed{0.11\,\text{ns}}$$

The frequency of the motion is known as the cyclotron frequency and is the reciprocal of the period of the electron's motion:

$$f = \frac{1}{T} = \frac{1}{0.110\,\text{ns}} = \boxed{9.1\,\text{GHz}}$$

29 ••   A proton, a deuteron and an alpha particle in a region with a uniform magnetic field each follow circular paths that have the same radius. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Assume that $m_\alpha = 2m_d = 4m_p$. Compare (a) their speeds, (b) their kinetic energies, and (c) the magnitudes of their angular momenta about the centers of the orbits.

Picture the Problem We can apply Newton's 2nd law to the orbiting particles to derive an expression for their orbital speeds as a function of their charge, their mass, the magnetic field in which they are moving, and the radii of their orbits. We can then compare their speeds by expressing their ratios. In Parts (b) and (c) we can proceed similarly starting with the definitions of kinetic energy and angular momentum.

(a) Apply Newton's 2nd law to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow v = \frac{qBr}{m}$$

The speeds of the orbiting particles are given by:

$$v_p = \frac{q_p Br}{m_p}, \tag{1}$$

$$v_\alpha = \frac{q_\alpha Br}{m_\alpha}, \text{ and} \tag{2}$$

$$v_d = \frac{q_d Br}{m_d} \tag{3}$$

Divide equation (2) by equation (1) and simplify to obtain:

$$\frac{v_\alpha}{v_p} = \frac{\dfrac{q_\alpha Br}{m_\alpha}}{\dfrac{q_p Br}{m_p}} = \frac{q_\alpha m_p}{q_p m_\alpha} = \frac{2em_p}{e(4m_p)} = \tfrac{1}{2}$$

or

$$2v_\alpha = v_p$$

Divide equation (3) by equation (1) and simplify to obtain:

$$\frac{v_d}{v_p} = \frac{\dfrac{q_d Br}{m_d}}{\dfrac{q_p Br}{m_p}} = \frac{q_d m_p}{q_p m_d} = \frac{e m_p}{e(2m_p)} = \frac{1}{2}$$

or

$$2v_d = v_p$$

Combining these results yields:

$$\boxed{2v_\alpha = 2v_d = v_p}$$

(b) Using the expression for its orbital speed derived in (a), express the kinetic energy of an orbiting particle:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2 B^2 r^2}{2m}$$

The kinetic energies of the three particles are given by:

$$K_p = \frac{q_p^2 B^2 r^2}{2m_p},\qquad(4)$$

$$K_\alpha = \frac{q_\alpha^2 B^2 r^2}{2m_\alpha},\ \text{ and}\qquad(5)$$

$$K_d = \frac{q_d^2 B^2 r^2}{2m_d}\qquad(6)$$

Dividing equation (7) by equation (6) and simplifying yields:

$$\frac{K_\alpha}{K_p} = \frac{\dfrac{1}{2}\dfrac{q_\alpha^2 B^2 r^2}{m_\alpha}}{\dfrac{1}{2}\dfrac{q_p^2 B^2 r^2}{m_p}} = \frac{q_\alpha^2 m_p}{q_p^2 m_\alpha} = \frac{(2e)^2 m_p}{e^2(4m_p)}$$

$$= 1 \Rightarrow K_\alpha = K_p$$

Divide equation (8) by equation (6) and simplify to obtain:

$$\frac{K_d}{K_p} = \frac{\dfrac{q_d^2 B^2 r^2}{2m_d}}{\dfrac{q_p^2 B^2 r^2}{2m_p}} = \frac{q_d^2 m_p}{q_p^2 m_d} = \frac{e^2 m_p}{e^2(2m_p)}$$

$$= \tfrac{1}{2} \Rightarrow K_p = 2K_d$$

Combining these results yields:

$$\boxed{K_\alpha = 2K_d = K_p}$$

(c) The angular momenta of the orbiting particles are given by:

$$L_{\mathrm{p}} = m_{\mathrm{p}}v_{\mathrm{p}}r,$$
$$L_{\alpha} = m_{\alpha}v_{\alpha}r, \text{ and}$$
$$L_{\mathrm{d}} = m_{\mathrm{d}}v_{\mathrm{d}}r$$

Express the ratio of L_{α} to L_{p}:

$$\frac{L_{\alpha}}{L_{\mathrm{p}}} = \frac{m_{\alpha}v_{\alpha}r}{m_{\mathrm{p}}v_{\mathrm{p}}r} = \frac{(4m_{\mathrm{p}})(\frac{1}{2}v_{\mathrm{p}})}{m_{\mathrm{p}}v_{\mathrm{p}}} = 2$$

or

$$L_{\alpha} = 2L_{\mathrm{p}}$$

Express the ratio of L_{d} to L_{p}:

$$\frac{L_{\mathrm{d}}}{L_{\mathrm{p}}} = \frac{m_{\mathrm{d}}v_{\mathrm{d}}r}{m_{\mathrm{p}}v_{\mathrm{p}}r} = \frac{(2m_{\mathrm{p}})(\frac{1}{2}v_{\mathrm{p}})}{m_{\mathrm{p}}v_{\mathrm{p}}} = 1$$

or

$$L_{\mathrm{d}} = L_{\mathrm{p}}$$

Combining these results yields:

$$\boxed{L_{\alpha} = 2L_{\mathrm{d}} = 2L_{\mathrm{p}}}$$

30 •• A particle has a charge q, a mass m, a linear momentum of magnitude p and a kinetic energy K. The particle moves in a circular orbit of radius R perpendicular to a uniform magnetic field $\vec{B}$. Show that (a) $p = BqR$ and (b) $K = \frac{1}{2}B^2q^2R^2/m$.

Picture the Problem We can use the definition of momentum to express p in terms of v and apply Newton's 2nd law to the orbiting particle to express v in terms of q, B, R, and m. In Part (b) we can express the particle's kinetic energy in terms of its momentum and use our result from Part (a) to show that $K = \frac{1}{2}B^2q^2R^2/m$.

(a) Express the momentum of the particle:

$$p = mv \qquad\qquad (1)$$

Apply $\sum F_{\mathrm{radial}} = ma_{\mathrm{c}}$ to the orbiting particle to obtain:

$$qvB = m\frac{v^2}{R} \Rightarrow v = \frac{qBR}{m}$$

Substitute for v in equation (1) to obtain:

$$p = m\left(\frac{qBR}{m}\right) = \boxed{qBR}$$

(b) Express the kinetic energy of the orbiting particle as a function of its momentum:

$$K = \frac{p^2}{2m}$$

Substitute our result for p from Part (a) to obtain:

$$K = \frac{(qBR)^2}{2m} = \boxed{\frac{q^2 B^2 R^2}{2m}}$$

31 •• **[SSM]**   A beam of particles with velocity $\vec{v}$ enters a region that has a uniform magnetic field $\vec{B}$ in the $+x$ direction. Show that when the x component of the displacement of one of the particles is $2\pi(m/qB)v\cos\theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$, the velocity of the particle is in the same direction as it was when the particle entered the field.

Picture the Problem The particle's velocity has a component v_1 parallel to $\vec{B}$ and a component v_2 normal to $\vec{B}$. $v_1 = v\cos\theta$ and is constant, whereas $v_2 = v\sin\theta$, being normal to $\vec{B}$, will result in a magnetic force acting on the beam of particles and circular motion perpendicular to $\vec{B}$. We can use the relationship between distance, rate, and time and Newton's 2nd law to express the distance the particle moves in the direction of the field during one period of the motion.

Express the distance moved in the direction of $\vec{B}$ by the particle during one period:

$$x = v_1 T \qquad\qquad (1)$$

Express the period of the circular motion of the particles in the beam:

$$T = \frac{2\pi r}{v_2} \qquad\qquad (2)$$

Apply Newton's 2nd law to a particle in the beam to obtain:

$$qv_2 B = m\frac{v_2^2}{r} \Rightarrow v_2 = \frac{qBr}{m}$$

Substituting for v_2 in equation (2) and simplifying yields:

$$T = \frac{2\pi r}{\dfrac{qBr}{m}} = \frac{2\pi m}{qB}$$

Because $v_1 = v\cos\theta$, equation (1) becomes:

$$x = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \boxed{2\pi\left(\frac{m}{qB}\right)v\cos\theta}$$

32 ••   A proton that has a speed equal to 1.00×10^6 m/s enters a region that has a uniform magnetic field that has a magnitude of 0.800 T and points into the

page, as shown in Figure 26-35. The proton enters the region at an angle $\theta = 60°$.
Find the exit angle ϕ and the distance d.

Picture the Problem The trajectory of
the proton is shown to the right. We
know that, because the proton enters the
uniform field perpendicularly to the
field, its trajectory while in the field
will be circular. We can use symmetry
considerations to determine ϕ. The
application of Newton's 2nd law to the
proton while it is in the magnetic field
and of trigonometry will allow us to
conclude that $r = d$ and to determine the
value of d.
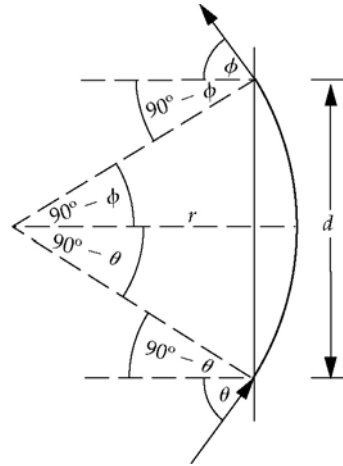
From symmetry, it is evident that the
angle θ in Figure 26-35 equals the
angle ϕ:

$$\phi = \boxed{60°}$$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton
while it is in the magnetic field to
obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 30° = \frac{1}{2} = \frac{d/2}{r}$$

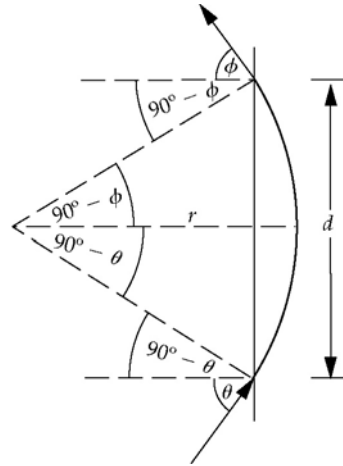Solving for d yields:

$$r = d$$

Substitute for r to obtain:

$$d = \frac{mv}{qB}$$

Substitute numerical values and
evaluate d:

$$d = r = \frac{(1.673 \times 10^{-27}\,\text{kg})(1.00 \times 10^6\,\text{m/s})}{(1.602 \times 10^{-19}\,\text{C})(0.800\,\text{T})}$$

$$= \boxed{13.1\,\text{mm}}$$

33 •• **[SSM]** Suppose that in Figure 26-35, the magnetic field has a
magnitude of 60 mT, the distance d is 40 cm, and θ is 24°. Find the speed v at
which a particle enters the region and the exit angle ϕ if the particle is a (a) proton
and (b) deuteron. Assume that $m_d = 2m_p$.

Picture the Problem The trajectory of the proton is shown to the right. We know that, because the proton enters the uniform field perpendicularly to the field, its trajectory while in the field will be circular. We can use symmetry considerations to determine ϕ. The application of Newton's 2nd law to the proton and deuteron while they are in the uniform magnetic field will allow us to determine the values of v_p and v_d.

(*a*) From symmetry, it is evident that the angle θ in Figure 26-35 equals the angle ϕ:

$\phi = \boxed{24°}$

Apply $\sum F_{\text{radial}} = ma_c$ to the proton while it is in the magnetic field to obtain:

$$q_p v_p B = m_p \frac{v_p^2}{r_p} \Rightarrow v_p = \frac{q_p r_p B}{m_p} \quad (1)$$

Use trigonometry to obtain:

$$\sin(90° - \theta) = \sin 66° = \frac{d/2}{r}$$

Solving for r yields:

$$r = \frac{d}{2\sin 66°}$$

Substituting for r in equation (1) and simplifying yields:

$$v_p = \frac{q_p B d}{2m_p \sin 66°} \quad (2)$$

Substitute numerical values and evaluate v_p:

$$v_p = \frac{\left(1.602 \times 10^{-19}\,\text{C}\right)\left(60\,\text{mT}\right)\left(0.40\,\text{m}\right)}{2\left(1.673 \times 10^{-27}\,\text{kg}\right)\sin 66°}$$

$$= \boxed{1.3 \times 10^6\,\text{m/s}}$$

(*b*) From symmetry, it is evident that the angle θ in Figure 26-35 equals the angle ϕ:

$\phi = \boxed{24°}$ independently of whether the particles are protons or deuterons.

For deuterons equation (2) becomes:

$$v_d = \frac{q_d B d}{2m_d \sin 66°}$$

Because $m_d = 2m_p$ and $q_d = q_p$:

$$v_d \approx \frac{q_p B d}{2(2m_p)\sin 66°} = \frac{q_p B d}{4m_p \sin 66°}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{(1.602 \times 10^{-19}\,\text{C})(60\,\text{mT})(0.40\,\text{m})}{4(1.673 \times 10^{-27}\,\text{kg})\sin 66°}$$

$$= \boxed{6.3 \times 10^5 \text{ m/s}}$$

34 ••   The galactic magnetic field in some region of interstellar space has a magnitude of 1.00×10^{-9} T. A particle of interstellar dust has a mass of 10.0 μg and a total charge of 0.300 nC. How many years does it take for the particle to complete revolution of the circular orbit caused by its interaction with the magnetic field?

Picture the Problem We can apply Newton's 2nd law of motion to express the orbital speed of the particle and then find the period of the dust particle from this orbital speed. Assume that the particle moves in a direction perpendicular to $\vec{B}$.

The period of the dust particle's motion is given by:

$$T = \frac{2\pi r}{v}$$

Apply $\sum F = ma_c$ to the particle:

$$qvB = m\frac{v^2}{r} \Rightarrow v = \frac{qBr}{m}$$

Substitute for v in the expression for T and simplify:

$$T = \frac{2\pi r m}{qBr} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi(10.0 \times 10^{-6}\,\text{g} \times 10^{-3}\,\text{kg/g})}{(0.300\,\text{nC})(1.00 \times 10^{-9}\,\text{T})}$$

$$= 2.094 \times 10^{11}\,\text{s} \times \frac{1\,\text{y}}{31.56\,\text{Ms}}$$

$$= \boxed{6.64 \times 10^3 \text{ y}}$$

Applications of the Magnetic Force Acting on Charged Particles

35 •   **[SSM]**   A velocity selector has a magnetic field that has a magnitude equal to 0.28 T and is perpendicular to an electric field that has a magnitude equal to 0.46 MV/m. (*a*) What must the speed of a particle be for that particle to pass through the velocity selector undeflected? What kinetic energy must (*b*) protons and (*c*) electrons have in order to pass through the velocity selector undeflected?

Picture the Problem Suppose that, for positively charged particles, their motion is from left to right through the velocity selector and the electric field is upward. Then the magnetic force must be downward and the magnetic field out of the page. We can apply the condition for translational equilibrium to relate v to E and B. In (b) and (c) we can use the definition of kinetic energy to find the energies of protons and electrons that pass through the velocity selector undeflected.

(a) Apply $\sum F_y = 0$ to the particle to obtain:

$F_{elec} - F_{mag} = 0$

or

$qE - qvB = 0 \Rightarrow v = \dfrac{E}{B}$

Substitute numerical values and evaluate v:

$v = \dfrac{0.46\,\text{MV/m}}{0.28\,\text{T}} = 1.64 \times 10^6 \text{ m/s}$

$= \boxed{1.6 \times 10^6 \text{ m/s}}$

(b) The kinetic energy of protons passing through the velocity selector undeflected is:

$K_p = \tfrac{1}{2} m_p v^2$

$= \tfrac{1}{2}\left(1.673 \times 10^{-27}\,\text{kg}\right)\left(1.64 \times 10^6 \text{ m/s}\right)^2$

$= 2.26 \times 10^{-15} \text{ J} \times \dfrac{1\,\text{eV}}{1.602 \times 10^{-19} \text{ J}}$

$= \boxed{14\,\text{keV}}$

(c) The kinetic energy of electrons passing through the velocity selector undeflected is:

$K_e = \tfrac{1}{2} m_e v^2$

$= \tfrac{1}{2}\left(9.109 \times 10^{-31}\,\text{kg}\right)\left(1.64 \times 10^6 \text{ m/s}\right)^2$

$= 1.23 \times 10^{-18} \text{ J} \times \dfrac{1\,\text{eV}}{1.602 \times 10^{-19} \text{ J}}$

$= \boxed{7.7\,\text{eV}}$

36 •• A beam of protons is moving in the $+x$ direction with a speed of 12.4 km/s through a region in which the electric field is perpendicular to the magnetic field. The beam is not deflected in this region. (a) If the magnetic field has a magnitude of 0.85 T and points in the $+y$ direction, find the magnitude and direction of the electric field. (b) Would electrons that have the same velocity as the protons be deflected by these fields? If so, in what direction would they be deflected? If not, why not?

Picture the Problem Because the beam of protons is not deflected; we can conclude that the electric force acting on them is balanced by the magnetic force. Hence, we can find the magnetic force from the given data and use its definition to express the electric field.

(*a*) Use the definition of electric field to relate it to the electric force acting on the beam of protons:

$$\vec{E}_{elec} = \frac{\vec{F}_{elec}}{q}$$

Express the magnetic force acting on the beam of protons:

$$\vec{F}_{mag} = qv\hat{i} \times B\hat{j} = qvB\hat{k}$$

Because the electric force must be equal in magnitude but opposite in direction:

$$\vec{F}_{elec} = -qvB\hat{k} = -\left(1.602 \times 10^{-19}\,\text{C}\right)\left(12.4\,\text{km/s}\right)\left(0.85\,\text{T}\right)\hat{k} == -\left(1.689 \times 10^{-15}\,\text{N}\right)\hat{k}$$

Substitute in the equation for the electric field to obtain:

$$\vec{E}_{elec} = \frac{-\left(1.689 \times 10^{-15}\,\text{N}\right)\hat{k}}{1.602 \times 10^{-19}\,\text{C}}$$

$$= \boxed{-\left(11\,\text{kV/m}\right)\hat{k}}$$

(*b*) Because both $\vec{F}_{mag}$ and $\vec{F}_{elec}$ would be reversed, electrons are not deflected either.

37   ••   The plates of a Thomson *q/m* apparatus are 6.00 cm long and are separated by 1.20 cm. The end of the plates is 30.0 cm from the tube screen. The kinetic energy of the electrons is 2.80 keV. If a potential difference of 25.0 V is applied across the deflection plates, by how much will the point where the beam strikes the screen displaced?

Picture the Problem Figure 26-18 is reproduced below. We can express the total deflection of the electron beam as the sum of the deflections while the beam is in the field between the plates and its deflection while it is in the field-free space. We can, in turn, use constant-acceleration equations to express each of these deflections. The resulting equation is in terms of v_0 and E. We can find v_0 from the kinetic energy of the beam and E from the potential difference across the plates and their separation. In Part (*b*) we can equate the electric and magnetic forces acting on an electron to express B in terms of E and v_0.
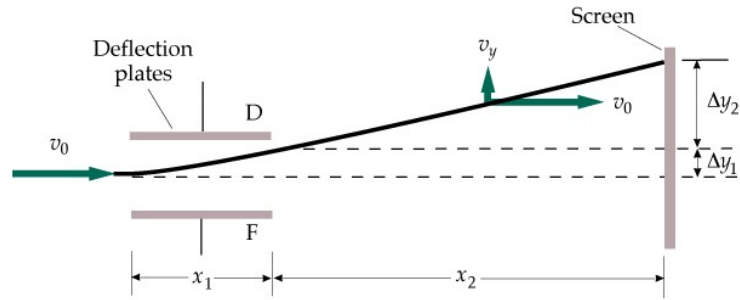
Express the total deflection Δy of the electrons:

$$\Delta y = \Delta y_1 + \Delta y_2 \qquad (1)$$

where Δy_1 is the deflection of the beam while it is in the electric field and Δy_2 is the deflection of the beam while it travels along a straight-line path outside the electric field.

Use a constant-acceleration equation to express Δy_1:

$$\Delta y_1 = \tfrac{1}{2} a_y (\Delta t)^2 \qquad (2)$$

where $\Delta t = x_1/v_0$ is the time an electron is in the electric field between the plates.

Apply Newton's 2nd law to an electron between the plates to obtain:

$$qE = ma_y \Rightarrow a_y = \frac{qE}{m}$$

Substitute for a_y in equation (2) to obtain:

$$\Delta y_1 = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 \qquad (3)$$

Express the vertical deflection Δy_2 of the electrons once they are out of the electric field:

$$\Delta y_2 = v_y \Delta t_2 \qquad (4)$$

Use a constant-acceleration equation to find the vertical speed of an electron as it leaves the electric field:

$$v_y = v_{0y} + a_y \Delta t_1$$
$$= 0 + \frac{qE}{m}\left(\frac{x_1}{v_0}\right)$$

Substitute in equation (4) to obtain:

$$\Delta y_2 = \frac{qE}{m}\left(\frac{x_1}{v_0}\right)\left(\frac{x_2}{v_0}\right) = \frac{qEx_1x_2}{mv_0^2} \qquad (5)$$

Substitute equations (3) and (5) in equation (1) to obtain:

$$\Delta y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x_1}{v_0}\right)^2 + \frac{qEx_1x_2}{mv_0^2}$$

or

$$\Delta y = \frac{qEx_1}{mv_0^2}\left(\frac{x_1}{2} + x_2\right) \qquad (6)$$

Use the definition of kinetic energy to express the square of the speed of the electrons:

$$K = \frac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{2K}{m}$$

Express the electric field between the plates in terms of their potential difference:

$$E = \frac{V}{d}$$

Substituting for E and v_0^2 in equation (6) and simplifying yields:

$$\Delta y = \frac{q\dfrac{V}{d}x_1}{m\dfrac{2K}{m}}\left(\frac{x_1}{2} + x_2\right) = \frac{qVx_1}{2dK}\left(\frac{x_1}{2} + x_2\right)$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(25.0\,\text{V}\right)\left(6.00\,\text{cm}\right)}{2\left(1.20\,\text{cm}\right)\left(2.80\,\text{keV}\right)}\left(\frac{6.00\,\text{cm}}{2} + 30.0\,\text{cm}\right) = \boxed{7.37\,\text{mm}}$$

38 •• Chlorine has two stable isotopes, ^{35}Cl and ^{37}Cl. Chlorine gas which consists of singly-ionized ions is to be separated into its isotopic components using a mass spectrometer. The magnetic field strength in the spectrometer is 1.2 T. What is the minimum value of the potential difference through which these ions must be accelerated so that the separation between them, after they complete their semicircular path, is 1.4 cm?

Picture the Problem The diagram below represents the paths of the two ions entering the magnetic field at the left. The magnetic force acting on each causes them to travel in circular paths of differing radii due to their different masses. We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for its radius and then express their final separation in terms of these radii that, in turn, depend on the energy with which the ions enter the field. We can connect their energy to the potential through which they are accelerated using the work-kinetic energy theorem and relate their separation Δs to the accelerating potential difference ΔV.
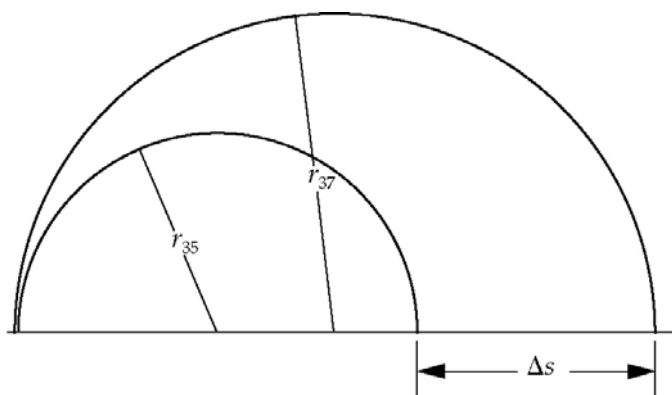
Express the separation Δs of the chlorine ions:

$$\Delta s = 2(r_{37} - r_{35}) \qquad (1)$$

Apply Newton's 2nd law to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad (2)$$

Relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute for v in equation (2) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}}$$

Use this equation to express the radii of the paths of the two chlorine isotopes to obtain:

$$r_{35} = \sqrt{\frac{2m_{35}\Delta V}{qB^2}} \text{ and } r_{37} = \sqrt{\frac{2m_{37}\Delta V}{qB^2}}$$

Substitute for r_{35} and r_{37} in equation (1) to obtain:

$$\Delta s = 2\left(\sqrt{\frac{2m_{35}\Delta V}{qB^2}} - \sqrt{\frac{2m_{35}\Delta V}{qB^2}}\right)$$

$$= 2\left(\frac{1}{B}\sqrt{\frac{2\Delta V}{q}}\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)\right)$$

Solving for ΔV yields:

$$\Delta V = \frac{qB^2\left(\dfrac{\Delta s}{2}\right)^2}{2\left(\sqrt{m_{37}} - \sqrt{m_{35}}\right)^2}$$

Substitute numerical values and evaluate ΔV:

$$\Delta V = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(1.2\,\text{T}\right)^2\left(\dfrac{1.4\,\text{cm}}{2}\right)^2}{2\left(\sqrt{37\,\text{u}} - \sqrt{35\,\text{u}}\right)^2}$$

$$= \frac{5.65\times10^{-24}\,\text{C}\cdot\text{T}^2\cdot\text{m}^2}{\left(\sqrt{37}-\sqrt{35}\right)^2\left(1.66\times10^{-27}\,\text{kg}\right)}$$

$$= \boxed{0.12\,\text{MV}}$$

39 ••   [SSM] In a mass spectrometer, a singly ionized ^{24}Mg ion has a mass equal to 3.983×10^{-26} kg and is accelerated through a 2.50-kV potential difference. It then enters a region where it is deflected by a magnetic field of 557 G. (*a*) Find the radius of curvature of the ion's orbit. (*b*) What is the difference in the orbital radii of the ^{26}Mg and ^{24}Mg ions? Assume that their mass ratio is 26:24.

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field to obtain an expression for r as a function of m, v, q, and B and use the work-kinetic energy theorem to express the kinetic energy in terms of the potential difference through which the ion has been accelerated. Eliminating v between these equations will allow us to express r in terms of m, q, B, and ΔV.

Apply Newton's 2nd law to an ion in the magnetic field of the mass spectrometer:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad (1)$$

Apply the work-kinetic energy theorem to relate the speed of an ion as it enters the magnetic field to the potential difference through which it has been accelerated:

$$q\Delta V = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$$

Substitute for v in equation (1) and simplify to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2m\Delta V}{qB^2}} \qquad (2)$$

(*a*) Substitute numerical values and evaluate equation (2) for ^{24}Mg :

$$r_{24} = \sqrt{\frac{2\left(3.983\times10^{-26}\,\text{kg}\right)\left(2.50\,\text{kV}\right)}{\left(1.602\times10^{-19}\,\text{C}\right)\left(557\times10^{-4}\,\text{T}\right)^2}}$$

$$= \boxed{63.3\,\text{cm}}$$

(b) Express the difference in the radii for ^{24}Mg and ^{26}Mg:

$$\Delta r = r_{26} - r_{24}$$

Substituting for r_{26} and r_{24} and simplifying yields:

$$\Delta r = \sqrt{\frac{2m_{26}\Delta V}{qB^2}} - \sqrt{\frac{2m_{24}\Delta V}{qB^2}} = \frac{1}{B}\sqrt{\frac{2\Delta V}{q}}\left(\sqrt{m_{26}} - \sqrt{m_{24}}\right)$$

$$= \frac{1}{B}\sqrt{\frac{2\Delta V}{q}}\left(\sqrt{\frac{26}{24}m_{24}} - \sqrt{m_{24}}\right) = \frac{1}{B}\sqrt{\frac{2\Delta V m_{24}}{q}}\left(\sqrt{\frac{26}{24}} - 1\right)$$

Substitute numerical values and evaluate Δr:

$$\Delta r = \frac{1}{557\times10^{-4}\,\text{T}}\sqrt{\frac{2(2.50\,\text{kV})(3.983\times10^{-26}\,\text{kg})}{1.602\times10^{-19}\,\text{C}}}\left(\sqrt{\frac{26}{24}} - 1\right) = \boxed{2.58\,\text{cm}}$$

40 ••   A beam of singly ionized ^{6}Li and ^{7}Li ions passes through a velocity selector and enters a region of uniform magnetic field with a velocity that is perpendicular to the direction of the field. If the diameter of the orbit of the ^{6}Li ions is 15 cm, what is the diameter of the orbit for ^{7}Li ions? Assume their mass ratio is 7:6.

Picture the Problem We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to relate the diameter of its orbit to its charge, mass, velocity, and the magnetic field. If we assume that the velocity is the same for the two ions, we can then express the ratio of the two diameters as the ratio of the masses of the ions and solve for the diameter of the orbit of ^{7}Li.

Apply Newton's 2nd law to an ion in the field of the spectrometer:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Express the diameter of the orbit:

$$d = \frac{2mv}{qB}$$

The diameters of the orbits for ^{6}Li and ^{7}Li are:

$$d_6 = \frac{2m_6 v}{qB} \text{ and } d_7 = \frac{2m_7 v}{qB}$$

Assume that the velocities of the two ions are the same and divide the 2nd of these diameters by the first to obtain:

$$\frac{d_7}{d_6} = \frac{\dfrac{2m_7 v}{qB}}{\dfrac{2m_6 v}{qB}} = \frac{m_7}{m_6}$$

Solve for and evaluate d_7:

$$d_7 = \frac{m_7}{m_6} d_6 = \frac{7}{6}(15\,\text{cm}) = \boxed{18\,\text{cm}}$$

41 ••   Using Example 26-6, determine the time required for a ^{58}Ni ion and a ^{60}Ni ion to complete the semicircular path.

Picture the Problem The time required for each of the ions to complete its semicircular paths is half its period. We can apply Newton's 2nd law to an ion in the magnetic field of the spectrometer to obtain an expression for r as a function of the charge and mass of the ion, its velocity, and the magnetic field.

Express the time for each ion to complete its semicircular path:

$$\Delta t = \tfrac{1}{2} T = \frac{\pi r}{v}$$

Apply Newton's 2nd law to an ion in the field of the spectrometer:

$$qvB = m \frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Substitute for r to obtain:

$$\Delta t = \frac{\pi m}{qB}$$

Substitute numerical values and evaluate Δt_{58} and Δt_{60}:

$$\Delta t_{58} = \frac{58\pi\left(1.6606 \times 10^{-27}\,\text{kg}\right)}{\left(1.602 \times 10^{-19}\,\text{C}\right)\left(0.120\,\text{T}\right)}$$

$$= \boxed{15.7\,\mu s}$$

and

$$\Delta t_{60} = \frac{60\pi\left(1.6606 \times 10^{-27}\,\text{kg}\right)}{\left(1.602 \times 10^{-19}\,\text{C}\right)\left(0.120\,\text{T}\right)}$$

$$= \boxed{16.3\,\mu s}$$

42 ••   Before entering a mass spectrometer, ions pass through a velocity selector consisting of parallel plates that are separated by 2.0 mm and have a potential difference of 160 V. The magnetic field strength is 0.42 T in the region between the plates. The magnetic field strength in the mass spectrometer is 1.2 T. Find (*a*) the speed of the ions entering the mass spectrometer and (*b*) the difference in the diameters of the orbits of singly ionized ^{238}U and ^{235}U. The mass of a ^{235}U ion is 3.903×10^{-25} kg.

Picture the Problem We can apply a condition for equilibrium to ions passing through the velocity selector to obtain an expression relating E, B, and v that we can solve for v. We can, in turn, express E in terms of the potential difference V between the plates of the selector and their separation d. In (*b*) we can apply

Newton's 2nd law to an ion in the bending field of the spectrometer to relate its diameter to its mass, charge, velocity, and the magnetic field.

(*a*) Apply $\sum F_y = 0$ to the ions in the crossed fields of the velocity selector to obtain:

$F_{elec} - F_{mag} = 0$

or

$qE - qvB = 0 \Rightarrow v = \dfrac{E}{B}$

Express the electric field between the plates of the velocity selector in terms of their separation and the potential difference across them:

$E = \dfrac{V}{d}$

Substituting for *E* yields:

$v = \dfrac{V}{dB}$

Substitute numerical values and evaluate *v*:

$v = \dfrac{160\,\text{V}}{(2.0\,\text{mm})(0.42\,\text{T})} = 1.905 \times 10^5\ \text{m/s}$

$= \boxed{1.9 \times 10^5\ \text{m/s}}$

(*b*) Express the difference in the diameters of the orbits of singly ionized ^{238}U and ^{235}U:

$\Delta d = d_{238} - d_{235}$ \hfill (1)

Apply $\sum F_{radial} = ma_c$ to an ion in the spectrometer's magnetic field:

$qvB = m\dfrac{v^2}{r} \Rightarrow r = \dfrac{mv}{qB}$

Express the diameter of the orbit:

$d = \dfrac{2mv}{qB}$

The diameters of the orbits for ^{238}U and ^{235}U are:

$d_{238} = \dfrac{2m_{238}v}{qB}$ and $d_{235} = \dfrac{2m_{235}v}{qB}$

Substitute in equation (1) to obtain:

$\Delta d = \dfrac{2m_{238}v}{qB} - \dfrac{2m_{235}v}{qB}$

$= \dfrac{2v}{qB}(m_{238} - m_{235})$

Substitute numerical values and evaluate Δd:

$$\Delta d = \frac{2\left(1.905\times10^5\,\text{m/s}\right)\left(238\,\text{u} - 235\,\text{u}\right)\left(\dfrac{1.6606\times10^{-27}\,\text{kg}}{\text{u}}\right)}{\left(1.602\times10^{-19}\,\text{C}\right)\left(1.2\,\text{T}\right)} = \boxed{1\,\text{cm}}$$

43 •• **[SSM]** A cyclotron for accelerating protons has a magnetic field strength of 1.4 T and a radius of 0.70 m. (*a*) What is the cyclotron's frequency? (*b*) Find the kinetic energy of the protons when they emerge. (*c*) How will your answers change if deuterons are used instead of protons?

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons/deuterons. By applying Newton's 2nd law, we can relate the radius of the particle's orbit to its speed and, hence, express the cyclotron frequency as a function of the particle's mass and charge and the cyclotron's magnetic field. In Part (*b*) we can use the definition of kinetic energy and their maximum speed to find the maximum energy of the emerging protons.

(*a*) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r} \qquad (1)$$

Apply Newton's 2nd law to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r} \;\Rightarrow\; r = \frac{mv}{qB} \qquad (2)$$

Substitute for r in equation (1) and simplify to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m} \qquad (3)$$

Substitute numerical values and evaluate f:

$$f = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(1.4\,\text{T}\right)}{2\pi\left(1.673\times10^{-27}\,\text{kg}\right)} = 21.3\,\text{MHz}$$

$$= \boxed{21\,\text{MHz}}$$

(*b*) Express the maximum kinetic energy of a proton:

$$K_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2$$

From equation (2), v_{max} is given by:

$$v_{\text{max}} = \frac{qBr_{\text{max}}}{m}$$

Substitute for v_{max} and simplify to obtain:

$$K_{max} = \tfrac{1}{2}m\left(\frac{qBr_{max}}{m}\right)^2 = \tfrac{1}{2}\left(\frac{q^2B^2}{m}\right)r_{max}^2$$

Substitute numerical values and evaluate K_{max}:

$$K_{max} = \tfrac{1}{2}\left(\frac{(1.602\times10^{-19}\,\mathrm{C})^2(1.4\,\mathrm{T})^2}{1.673\times10^{-27}\,\mathrm{kg}}\right)(0.7\,\mathrm{m})^2 = 7.37\times10^{-12}\,\mathrm{J}\times\frac{1\,\mathrm{eV}}{1.602\times10^{-19}\,\mathrm{J}}$$

$$= 46.0\,\mathrm{MeV} = \boxed{46\,\mathrm{MeV}}$$

(c) From equation (3) we see that doubling m halves f:

$$f_{deuterons} = \tfrac{1}{2}f_{protons} = \boxed{11\,\mathrm{MHz}}$$

From our expression for K_{max} we see that doubling m halves K:

$$K_{deuterons} = \tfrac{1}{2}K_{protons} = \boxed{23\,\mathrm{MeV}}$$

44 ••   A certain cyclotron that has a magnetic field whose magnitude is 1.8 T is designed to accelerate protons to a kinetic energy of 25 MeV. (a) What is the cyclotron frequency for this cyclotron? (b) What must the minimum radius of the magnet be to achieve this energy? (c) If the alternating potential difference applied to the dees has a maximum value of 50 kV, how many revolutions must the protons make before emerging with kinetic energies of 25 MeV?

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of the protons be accelerated in the cyclotron. By applying Newton's 2nd law, we can relate the radius of the proton's orbit to its speed and, hence, express the cyclotron frequency as a function of the its mass and charge and the cyclotron's magnetic field. In Part (b) we can use the definition of kinetic energy express the minimum radius required to achieve the desired emergence energy. In Part (c) we can find the number of revolutions required to achieve this emergence energy from the energy acquired during each revolution.

(a) Express the cyclotron frequency in terms of the proton's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply Newton's 2nd law to a proton in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad (1)$$

Substitute for *v* and simplify to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{qB}{2\pi m}$$

Substitute numerical values and evaluate *f*:

$$f = \frac{\left(1.602 \times 10^{-19}\,\text{C}\right)\left(1.8\,\text{T}\right)}{2\pi\left(1.673 \times 10^{-27}\,\text{kg}\right)}$$

$$= \boxed{27\,\text{MHz}}$$

(*b*) Using the definition of kinetic energy, relate emergence energy of the protons to their velocity:

$$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substitute for *v* in equation (1) and simplify to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Substitute numerical values and evaluate $r_{\min}$:

$$r = \frac{\sqrt{2(25\,\text{MeV})\left(1.673 \times 10^{-27}\,\text{kg}\right)}}{\left(1.602 \times 10^{-19}\,\text{C}\right)\left(1.8\,\text{T}\right)}$$

$$= \boxed{40\,\text{cm}}$$

(*c*) Express the required number of revolutions *N* in terms of the energy gained per revolution:

$$N = \frac{25\,\text{MeV}}{E_{\text{rev}}}$$

Because the beam is accelerated through a potential difference of 50 kV twice during each revolution:

$$E_{\text{rev}} = 2q\Delta V = 100\,\text{keV}$$

Substitute the numerical value of E_{rev} and evaluate *N*:

$$N = \frac{25\,\text{MeV}}{100\,\text{keV/rev}} = \boxed{2.5 \times 10^2\,\text{rev}}$$

45 ••   Show that for a given cyclotron the cyclotron frequency for accelerating deuterons is the same as the frequency for accelerating alpha particles is half the frequency for accelerating protons in the same magnetic field. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Assume that $m_\alpha = 2m_d = 4m_p$.

Picture the Problem We can express the cyclotron frequency in terms of the maximum orbital radius and speed of a particle being accelerated in the cyclotron. By applying Newton's 2$^{\text{nd}}$ law, we can relate the radius of the particle's orbit to its

speed and, hence, express the cyclotron frequency as a function of its charge-to-mass ratio and the cyclotron's magnetic field. We can then use data for the relative charges and masses of deuterons, alpha particles, and protons to establish the ratios of their cyclotron frequencies.

Express the cyclotron frequency in terms of a particle's orbital speed and radius:

$$f = \frac{1}{T} = \frac{1}{2\pi r/v} = \frac{v}{2\pi r}$$

Apply Newton's 2nd law to a particle in the magnetic field of the cyclotron:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Substitute for r to obtain:

$$f = \frac{qBv}{2\pi mv} = \frac{B}{2\pi}\frac{q}{m} \qquad (1)$$

Evaluate equation (1) for deuterons:

$$f_d = \frac{B}{2\pi}\frac{q_d}{m_d} = \frac{B}{2\pi}\frac{e}{m_d}$$

Evaluate equation (1) for alpha particles:

$$f_\alpha = \frac{B}{2\pi}\frac{q_\alpha}{m_\alpha} = \frac{B}{2\pi}\frac{2e}{2m_d} = \frac{B}{2\pi}\frac{e}{m_d}$$

and

$$\boxed{f_d = f_\alpha}$$

Evaluate equation (1) for protons:

$$f_p = \frac{B}{2\pi}\frac{q_p}{m_p} = \frac{B}{2\pi}\frac{e}{\frac{1}{2}m_d} = 2\left(\frac{B}{2\pi}\frac{e}{m_d}\right)$$

$$= 2f_d$$

and

$$\tfrac{1}{2}f_p = \boxed{f_d = f_\alpha}$$

46 •••   Show that the radius of the orbit of a charged particle in a cyclotron is proportional to the square root of the number of orbits completed.

Picture the Problem We can apply Newton's 2nd law to the orbiting charged particle to obtain an expression for its radius as a function of its particle's kinetic energy. Because the energy gain per revolution is constant, we can express this kinetic energy as the product of the number of orbits completed and the energy gained per revolution and, hence, show that the radius is proportional to the square root of the number of orbits completed.

| Apply Newton's 2nd law to a particle in the magnetic field of the cyclotron: | $qvB = m\dfrac{v^2}{r} \Rightarrow r = \dfrac{mv}{qB}$ | (1) |

| Express the kinetic energy of the particle in terms of its speed and solve for v: | $K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2K}{m}}$ | (2) |

| Noting that the energy gain per revolution is constant, express the kinetic energy in terms of the number of orbits N completed by the particle and energy E_{rev} gained by the particle each revolution: | $K = NE_{rev}$ | (3) |

Substitute equations (2) and (3) in equation (1) to obtain:

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2mK}$$

$$= \frac{1}{qB}\sqrt{2mNE_{rev}} = \frac{\sqrt{2mE_{rev}}}{qB}N^{1/2}$$

or $\boxed{r \propto N^{1/2}}$

Torques on Current Loops, Magnets, and Magnetic Moments

47 • [SSM]   A small circular coil consisting of 20 turns of wire lies in a region with a uniform magnetic field whose magnitude is 0.50 T. The arrangement is such that the normal to the plane of the coil makes an angle of 60° with the direction of the magnetic field. The radius of the coil is 4.0 cm, and the wire carries a current of 3.0 A. (*a*) What is the magnitude of the magnetic moment of the coil? (*b*) What is the magnitude of the torque exerted on the coil?

Picture the Problem We can use the definition of the magnetic moment of a coil to evaluate μ and the expression for the torque exerted on the coil $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the magnitude of τ.

| (*a*) Using its definition, express the magnetic moment of the coil: | $\mu = NIA = NI\pi r^2$ |

| Substitute numerical values and evaluate μ: | $\mu = (20)(3.0\,\text{A})\pi(0.040\,\text{m})^2$ $= 0.302\,\text{A}\cdot\text{m}^2 = \boxed{0.30\,\text{A}\cdot\text{m}^2}$ |

(b) Express the magnitude of the
torque exerted on the coil:

$$\tau = \mu B \sin \theta$$

Substitute numerical values and
evaluate τ:

$$\tau = (0.302\,\text{A}\cdot\text{m}^2)(0.50\,\text{T})\sin 60°$$
$$= \boxed{0.13\,\text{N}\cdot\text{m}}$$

48 •   What is the maximum torque on a 400-turn circular coil of radius
0.75 cm that carries a current of 1.6 mA and is in a region with a uniform
magnetic field of 0.25 T?

Picture the Problem The coil will experience the maximum torque when the
plane of the coil makes an angle of 90° with the direction of $\vec{B}$. The magnitude of
the maximum torque is then given by $\tau_{max} = \mu B$.

The maximum torque acting on the
coil is:

$$\tau_{max} = \mu B$$

Use its definition to express the
magnetic moment of the coil:

$$\mu = NIA = NI\pi r^2$$

Substitute to obtain:

$$\tau_{max} = NI\pi r^2 B$$

Substitute numerical values and
evaluate τ:

$$\tau_{max} = (400)(1.6\,\text{mA})\pi(0.75\,\text{cm})^2(0.25\,\text{T})$$
$$= \boxed{28\,\mu\text{N}\cdot\text{m}}$$

49 •   **[SSM]**   A current-carrying wire is in the shape of a square of edge-
length 6.0 cm. The square lies in the $z = 0$ plane. The wire carries a current of
2.5 A. What is the magnitude of the torque on the wire if it is in a region with a
uniform magnetic field of magnitude 0.30 T that points in the (a) +z direction and
(b) +x direction?

Picture the Problem We can use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque on the coil in the
two orientations of the magnetic field.

Express the torque acting on the
coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Express the magnetic moment of
the coil:

$$\vec{\mu} = \pm IA\hat{k} = \pm IL^2\hat{k}$$

(a) Evaluate $\vec{\tau}$ for $\vec{B}$ in the $+z$ direction:

$$\vec{\tau} = \pm IL^2 \hat{k} \times B\hat{k} = \pm IL^2 B(\hat{k} \times \hat{k}) = \boxed{0}$$

(b) Evaluate $\vec{\tau}$ for $\vec{B}$ in the $+x$ direction:

$$\vec{\tau} = \pm IL^2 \hat{k} \times B\hat{i} = \pm IL^2 B(\hat{k} \times \hat{i})$$
$$= \pm(2.5\,\text{A})(0.060\,\text{m})^2(0.30\,\text{T})\hat{j}$$
$$= \pm(2.7\,\text{mN}\cdot\text{m})\hat{j}$$

and

$$|\vec{\tau}| = \boxed{2.7 \times 10^{-3}\,\text{N}\cdot\text{m}}$$

50 •   A current-carrying wire is in the shape of an equilateral triangle of edge-length 8.0 cm. The triangle lies in the $z = 0$ plane. The wire carries a current of 2.5 A. What is the magnitude of the torque on the wire if it is in a region with a uniform magnetic field of magnitude 0.30 T that points in the (a) $+z$ direction and (b) $+x$ direction?

Picture the Problem We can use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque on the equilateral triangle in the two orientations of the magnetic field.

Express the torque acting on the coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Express the magnetic moment of the coil:

$$\vec{\mu} = \pm IA\hat{k}$$

Relate the area of the equilateral triangle to the length of its side:

$$A = \tfrac{1}{2}\,\text{base} \times \text{altitude}$$
$$= \frac{1}{2}(L)\left(\frac{\sqrt{3}L}{2}\right) = \frac{\sqrt{3}}{4}L^2$$

Substitute to obtain:

$$\vec{\mu} = \pm\frac{\sqrt{3}L^2 I}{4}\hat{k}$$

(a) Evaluate $\vec{\tau}$ for $\vec{B}$ in the $+z$ direction:

$$\vec{\tau} = \pm\frac{\sqrt{3}L^2 I}{4}\hat{k} \times B\hat{k}$$
$$= \pm\frac{\sqrt{3}L^2 IB}{4}(\hat{k} \times \hat{k}) = \boxed{0}$$

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(b) Evaluate $\vec{\tau}$ for $\vec{B}$ in the $+x$ direction:

$$\vec{\tau} = \pm \frac{\sqrt{3}L^2 I}{4} \hat{k} \times B\hat{i} = \pm \frac{\sqrt{3}L^2 IB}{4} \left(\hat{k} \times \hat{i} \right)$$

$$= \pm \frac{\sqrt{3}(0.080\,\text{m})^2 (2.5\,\text{A})(0.30\,\text{T})}{4} \hat{j}$$

$$= \pm \left(2.1 \times 10^{-3}\,\text{N} \cdot \text{m} \right) \hat{j}$$

and

$$\boxed{|\vec{\tau}| = 2.1 \times 10^{-3}\,\text{N} \cdot \text{m}}$$

51 •• A rigid wire is in the shape of a square of edge-length L. The square has mass m and the wire carries current I. The square lies on flat horizontal surface in a region where there is a magnetic field of magnitude B that is parallel to two edges of the square. What is the minimum value of B so that one edge of the square will lift off the surface?

Picture the Problem One edge of the square will lift off the surface when the magnitude of the magnetic torque acting on it equals the magnitude of the gravitational torque acting on it.

The condition for liftoff is that the magnitudes of the torques must be equal:

$$\tau_{\text{mag}} = \tau_{\text{grav}} \qquad (1)$$

Express the magnetic torque acting on the square:

$$\tau_{\text{mag}} = \mu B = IL^2 B$$

Express the gravitational torque acting on one edge of the square:

$$\tau_{\text{grav}} = mgL$$

Substituting in equation (1) yields:

$$IL^2 B_{\text{min}} = mgL \Rightarrow \boxed{B_{\text{min}} = \frac{mg}{IL}}$$

52 •• A rectangular current-carrying 50-turn coil, as shown in Figure 26-36, is pivoted about the z axis. (a) If the wires in the $z = 0$ plane make an angle $\theta = 37°$ with the y axis, what angle does the magnetic moment of the coil make with the unit vector $\hat{i}$? (b) Write an expression for $\hat{n}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$, where $\hat{n}$ is a unit vector in the direction of the magnetic moment. (c) What is the magnetic moment of the coil? (d) Find the torque on the coil when there is a uniform magnetic field $\vec{B} = 1.5\,\text{T}\hat{j}$ in the region occupied by the coil. (e) Find the potential energy of the coil in this field. (The potential energy is zero when $\theta = 0$.)

Picture the Problem The diagram shows the coil as it would appear from along the positive z axis. The right-hand rule for determining the direction of $\hat{n}$ has been used to establish $\hat{n}$ as shown. We can use the geometry of this figure to determine θ and to express the unit normal vector $\hat{n}$. The magnetic moment of the coil is given by $\vec{\mu} = NIA\hat{n}$ and the torque exerted on the coil by $\vec{\tau} = \vec{\mu} \times \vec{B}$. Finally, we can find the potential energy of the coil in this field from $U = -\vec{\mu} \cdot \vec{B}$.
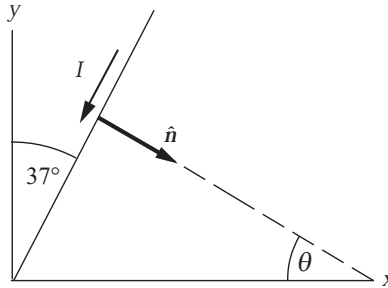
(a) Noting that θ and the angle whose measure is 37° have their right and left sides mutually perpendicular, we can conclude that:

$$\theta = \boxed{37°}$$

(b) Use the components of $\hat{n}$ to express $\hat{n}$ in terms of $\hat{i}$ and $\hat{j}$:

$$\hat{n} = n_x\hat{i} + n_y\hat{j} = \cos 37°\hat{i} - \sin 37°\hat{j}$$
$$= 0.799\hat{i} - 0.602\hat{j}$$
$$= \boxed{0.80\hat{i} - 0.60\hat{j}}$$

(c) Express the magnetic moment of the coil:

$$\vec{\mu} = NIA\hat{n}$$

Substitute numerical values and evaluate $\vec{\mu}$:

$$\vec{\mu} = (50)(1.75\,\text{A})(48.0\,\text{cm}^2)(0.799\hat{i} - 0.602\hat{j}) = (0.335\,\text{A}\cdot\text{m}^2)\,\hat{i} - (0.253\,\text{A}\cdot\text{m}^2)\hat{j}$$
$$= \boxed{(0.34\,\text{A}\cdot\text{m}^2)\,\hat{i} - (0.25\,\text{A}\cdot\text{m}^2)\hat{j}}$$

(d) Express the torque exerted on the coil:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Substitute for $\vec{\mu}$ and $\vec{B}$ to obtain:

$$\vec{\tau} = \left\{ \left(0.335\,\text{A} \cdot \text{m}^2 \right)\hat{i} - \left(0.253\,\text{A} \cdot \text{m}^2 \right)\hat{j} \right\} \times \left(1.5\,\text{T} \right)\hat{j}$$
$$= \left(0.503\,\text{N} \cdot \text{m} \right)\left(\hat{i} \times \hat{j} \right) - \left(0.379\,\text{N} \cdot \text{m} \right)\left(\hat{j} \times \hat{j} \right) = \boxed{ \left(0.50\,\text{N} \cdot \text{m} \right)\hat{k} }$$

(*e*) Express the potential energy of the coil in terms of its magnetic moment and the magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$

Substitute for $\vec{\mu}$ and $\vec{B}$ and evaluate U:

$$U = -\left\{ \left(0.335\,\text{A} \cdot \text{m}^2 \right)\hat{i} - \left(0.253\,\text{A} \cdot \text{m}^2 \right)\hat{j} \right\} \cdot \left(1.5\,\text{T} \right)\hat{j}$$
$$= -\left(0.503\,\text{N} \cdot \text{m} \right)\left(\hat{i} \cdot \hat{j} \right) + \left(0.379\,\text{N} \cdot \text{m} \right)\left(\hat{j} \cdot \hat{j} \right) = \boxed{ 0.38\,\text{J} }$$

53 •• **[SSM]** For the coil in Problem 52 the magnetic field is now $\vec{B} = 2.0\,\text{T}\hat{j}$. Find the torque exerted on the coil when $\hat{n}$ is equal to (*a*) $\hat{i}$, (*b*) $\hat{j}$, (*c*) $-\hat{j}$, and (*d*) $\dfrac{\hat{i}}{\sqrt{2}} + \dfrac{\hat{j}}{\sqrt{2}}$.

Picture the Problem We can use the right-hand rule for determining the direction of $\hat{n}$ to establish the orientation of the coil for value of $\hat{n}$ and $\vec{\tau} = \vec{\mu} \times \vec{B}$ to find the torque exerted on the coil in each orientation.
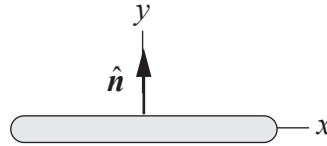
(*a*) The orientation of the coil is shown to the right:
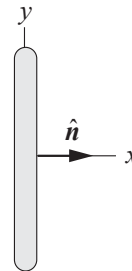
Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\,\text{T}\,\hat{j}$ and $\hat{n} = \hat{i}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NIA\hat{n} \times \vec{B}$$
$$= \left(50 \right)\left(1.75\,\text{A} \right)\left(48.0\,\text{cm}^2 \right)\hat{i} \times \left(2.0\,\text{T} \right)\hat{j}$$
$$= \left(0.840\,\text{N} \cdot \text{m} \right)\left(\hat{i} \times \hat{j} \right) = \left(0.840\,\text{N} \cdot \text{m} \right)\hat{k}$$
$$= \boxed{ \left(0.84\,\text{N} \cdot \text{m} \right)\hat{k} }$$

(*b*) The orientation of the coil is shown to the right:

Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\,\text{T}\,\hat{j}$ and $\hat{n} = \hat{j}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NIA\hat{n} \times \vec{B}$$
$$= (50)(1.75\,\text{A})(48.0\,\text{cm}^2)\hat{j} \times (2.0\,\text{T})\hat{j}$$
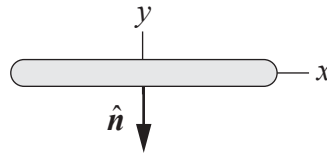$$= (0.840\,\text{N} \cdot \text{m})(\hat{j} \times \hat{j})$$
$$= \boxed{0}$$

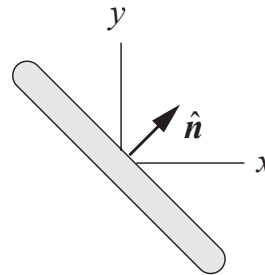(*c*) The orientation of the coil is shown to the right:

Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\,\text{T}\,\hat{j}$ and $\hat{n} = -\hat{j}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NIA\hat{n} \times \vec{B}$$
$$= -(50)(1.75\,\text{A})(48.0\,\text{cm}^2)\hat{j} \times (2.0\,\text{T})\hat{j}$$
$$= (-0.840\,\text{N} \cdot \text{m})(\hat{j} \times \hat{j})$$
$$= \boxed{0}$$

(*d*) The orientation of the coil is shown to the right:

Evaluate $\vec{\tau}$ for $\vec{B} = 2.0\,\text{T}\,\hat{j}$ and $\hat{n} = (\hat{i} + \hat{j})/\sqrt{2}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NIA\hat{n} \times \vec{B}$$
$$= \frac{(50)(1.75\,\text{A})(48.0\,\text{cm}^2)}{\sqrt{2}}(\hat{i} + \hat{j}) \times (2.0\,\text{T})\hat{j}$$
$$= (0.594\,\text{N} \cdot \text{m})(\hat{i} \times \hat{j})$$
$$\quad + (0.594\,\text{N} \cdot \text{m})(\hat{j} \times \hat{j})$$
$$= \boxed{(0.59\,\text{N} \cdot \text{m})\hat{k}}$$

54   ••   A small bar magnet has a length equal to 6.8 cm and its magnetic moment is aligned with a uniform magnetic field of magnitude 0.040 T. The bar magnet is then rotated through an angle of 60 about an axis perpendicular to its length The observed torque on the bar magnet has a magnitude of 0.10 N·m.

(*a*) Find the magnetic moment of the magnet. (*b*) Find the potential energy of the magnet.

Picture the Problem Because the small magnet can be modeled as a magnetic dipole; we can use the equation for the torque on a current loop to find its magnetic moment.

(*a*) Express the magnitude of the torque acting on the magnet:

$$\tau = \mu B \sin\theta$$

Solve for μ to obtain:

$$\mu = \frac{\tau}{B \sin\theta}$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{0.10\,\text{N} \cdot \text{m}}{(0.040\,\text{T})\sin 60°} = \boxed{2.9\,\text{A} \cdot \text{m}^2}$$

(*b*) The potential energy of the magnet is given by:

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

Substitute numerical values and evaluate U:

$$U = -(2.887\,\text{A} \cdot \text{m}^2)(0.040\,\text{T})\cos 60°$$
$$= \boxed{-58\,\text{mJ}}$$

55 •• A wire loop consists of two semicircles connected by straight segments (Figure 26-37). The inner and outer radii are 0.30 m and 0.50 m, respectively. A current of 1.5 A is in this wire and the current in the outer semicircle is in the clockwise direction. What is the magnetic moment of this current loop?

Picture the Problem We can use the definition of the magnetic moment to find the magnetic moment of the given current loop and a right-hand rule to find its direction.

Using its definition, express the magnetic moment of the current loop:

$$\mu = IA$$

Express the area bounded by the loop:

$$A = \tfrac{1}{2}\left(\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right) = \frac{\pi}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute for A to obtain:

$$\mu = \frac{\pi I}{2}\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$$

Substitute numerical values and evaluate μ:

$$\mu = \frac{\pi(1.5\,\text{A})}{2}\left[(0.50\,\text{m})^2 - (0.30\,\text{m})^2\right]$$

$$= \boxed{0.38\,\text{A}\cdot\text{m}^2}$$

Apply the right-hand rule for determining the direction of the unit normal vector (the direction of μ) to conclude that $\vec{\mu}$ points into the page.

56 •• A wire of length L is wound into a circular coil that has N turns. Show that when the wire carries a current I, the magnetic moment of the coil has a magnitude given by $IL^2/(4\pi N)$.

Picture the Problem We can use the definition of the magnetic moment of a coil to find the magnetic moment of a wire of length L that is wound into a circular coil of N loops. We can find the area of the coil from its radius R and we can find R by dividing the length of the wire by the number of turns.

Use its definition to express the magnetic moment of the coil:

$$\mu = NIA \qquad\qquad (1)$$

Express the circumference of each loop:

$$\frac{L}{N} = 2\pi R \Rightarrow R = \frac{L}{2\pi N}$$

where R is the radius of a loop.

The area of the coil is given by:

$$A = \pi R^2$$

Substituting for A and simplifying yields:

$$A = \pi\left(\frac{L}{2\pi N}\right)^2 = \frac{L^2}{4\pi N^2}$$

Substitute for A in equation (1) and simplify to obtain:

$$\mu = NI\left(\frac{L^2}{4\pi N^2}\right) = \boxed{\frac{IL^2}{4\pi N}}$$

57 •• **[SSM]** A particle that has a charge q and a mass m moves with angular velocity ω in a circular path of radius r. (*a*) Show that the average current created by this moving particle is $\omega q/(2\pi)$ and that the magnetic moment of its orbit has a magnitude of $\frac{1}{2}q\omega r^2$. (*b*) Show that the angular momentum of this particle has the magnitude of $mr^2\omega$ and that the magnetic moment and angular momentum vectors are related by $\vec{\mu} = \left(\frac{q}{2m}\right)\vec{L}$, where $\vec{L}$ is the angular momentum about the center of the circle.

Picture the Problem We can use the definition of current and the relationship between the frequency of the motion and its period to show that $I = q\omega/2\pi$. We can use the definition of angular momentum and the moment of inertia of a point particle to show that the magnetic moment has the magnitude $\mu = \frac{1}{2}q\omega r^2$. Finally, we can express the ratio of μ to L and the fact that $\vec{\mu}$ and $\vec{L}$ are both parallel to $\vec{\omega}$ to conclude that $\vec{\mu} = (q/2m)\vec{L}$.

(a) Using its definition, relate the average current to the charge passing a point on the circumference of the circle in a given period of time:

$$I = \frac{\Delta q}{\Delta t} = \frac{q}{T} = qf$$

Relate the frequency of the motion to the angular frequency of the particle:

$$f = \frac{\omega}{2\pi}$$

Substitute for f to obtain:

$$I = \boxed{\frac{q\omega}{2\pi}}$$

From the definition of the magnetic moment we have:

$$\mu = IA = \left(\frac{q\omega}{2\pi}\right)\left(\pi r^2\right) = \boxed{\frac{1}{2}q\omega r^2}$$

(b) Express the angular momentum of the particle:

$$L = I\omega$$

The moment of inertia of the particle is:

$$I = mr^2$$

Substituting for I yields:

$$L = \left(mr^2\right)\omega = \boxed{mr^2\omega}$$

Express the ratio of μ to L and simplify to obtain:

$$\frac{\mu}{L} = \frac{\frac{1}{2}q\omega r^2}{mr^2\omega} = \frac{q}{2m} \Rightarrow \mu = \frac{q}{2m}L$$

Because $\vec{\mu}$ and $\vec{L}$ are both parallel to $\vec{\omega}$:

$$\vec{\mu} = \boxed{\frac{q}{2m}\vec{L}}$$

58 ••• A hollow non-conducting cylinder has length L and inner and outer radii R_i A uniformly charged non-conducting cylindrical shell (Figure 26-38) has length L, inner and outer radii R_i and R_o, respectively, a charge density ρ and an

angular velocity ω about its axis. Derive an expression for the magnetic moment of the cylinder.

Picture the Problem We can express the magnetic moment of an element of charge dq in a cylinder of length L, radius r, and thickness dr, relate this charge to the length, radius, and thickness of the cylinder, express the current due to this rotating charge, substitute for A and dI in our expression for μ and then integrate to complete our derivation for the magnetic moment of the rotating cylinder as a function of its angular velocity.

Express the magnetic moment of an element of charge dq in a cylinder of length L, radius r, and thickness dr:

$$d\mu = A\,dI = \pi r^2\,dI \qquad (1)$$

Relate the charge dq in the cylinder to the length of the cylinder, its radius, and thickness:

$$dq = 2\pi L \rho r\,dr$$

The current due to this rotating charge is given by:

$$dI = \frac{\omega}{2\pi}\,dq = \frac{\omega}{2\pi}(2\pi L \rho r\,dr) = L\rho r \omega\,dr$$

Substitute for dI in equation (1) and simplify to obtain:

$$d\mu = \pi r^2(L\rho\omega r\,dr) = L\rho\pi\omega r^3\,dr$$

Integrate r from R_i to R_0 to obtain:

$$\mu = L\rho\pi\omega \int_{R_i}^{R_0} r^3\,dr = \tfrac{1}{4}L\rho\pi\omega\left(R_0^4 - R_i^4\right)$$

Because $\vec{\mu}$ and $\vec{\omega}$ are parallel:

$$\boxed{\vec{\mu} = \tfrac{1}{4}L\rho\pi\left(R_0^4 - R_i^4\right)\vec{\omega}}$$

59 ••• [SSM]   A uniform non-conducting thin rod of mass m and length L has a uniform charge per unit length λ and rotates with angular speed ω about an axis through one end and perpendicular to the rod. (*a*) Consider a small segment of the rod of length dx and charge $dq = \lambda dr$ at a distance r from the pivot (Figure 26-40). Show that the average current created by this moving segment is $\omega dq/(2\pi)$ and show that the magnetic moment of this segment is $\tfrac{1}{2}\lambda\omega r^2 dx$. (*b*) Use this to show that the magnitude of the magnetic moment of the rod is $\tfrac{1}{6}\lambda\omega L^3$. (*c*) Show that the magnetic moment $\vec{\mu}$ and angular momentum $\vec{L}$ are related by $\vec{\mu} = \left(\dfrac{Q}{2m}\right)\vec{L}$, where Q is the total charge on the rod.

Picture the Problem We can follow the step-by-step outline provided in the problem statement to establish the given results.

(*a*) Express the magnetic moment of the rotating element of charge:

$$d\mu = A\,dI \qquad\qquad (1)$$

The area enclosed by the rotating element of charge is:

$$A = \pi x^2$$

Express dI in terms of dq and Δt:

$$dI = \frac{dq}{\Delta t} = \frac{\lambda\,dx}{\Delta t} \quad \text{where } \Delta t \text{ is the time}$$

required for one revolution.

The time Δt required for one revolution is:

$$\Delta t = \frac{1}{f} = \frac{2\pi}{\omega}$$

Substitute for Δt and simplify to obtain:

$$dI = \frac{\lambda\omega}{2\pi}\,dx$$

Substituting for dI in equation (1) and simplifying yields:

$$d\mu = \left(\pi x^2\right)\left(\frac{\lambda\omega}{2\pi}\,dx\right) = \boxed{\tfrac{1}{2}\lambda\omega x^2\,dx}$$

(*b*) Integrate $d\mu$ from $x = 0$ to $x = L$ to obtain:

$$\mu = \tfrac{1}{2}\lambda\omega\int_0^L x^2\,dx = \boxed{\tfrac{1}{6}\lambda\omega L^3}$$

(*c*) Express the angular momentum of the rod:

$$L = I\omega$$

where L is the angular momentum of the rod and I is the moment of inertia of the rod with respect to the point about which it is rotating.

Express the moment of inertia of the rod with respect to an axis through its end:

$$I = \tfrac{1}{3}mL^2$$

where L is now the length of the rod.

Substitute to obtain:

$$L = \tfrac{1}{3}mL^2\omega$$

Divide the expression for μ by L to obtain:

$$\frac{\mu}{L} = \frac{\frac{1}{6}\lambda\omega L^3}{\frac{1}{3}mL^2\omega} = \frac{\lambda L}{2m}$$

or, because $Q = \lambda L$,

$$\mu = \frac{Q}{2m}L$$

Because $\vec{\omega}$ and $\vec{L} = I\vec{\omega}$ are parallel:

$$\boxed{\vec{\mu} = \frac{Q}{2M}\vec{L}}$$

60   •••   A non-uniform, non-conducting thin disk of mass m, radius R, and total charge Q has a charge per unit area σ that varies as $\sigma_0 r/R$ and a mass per unit area σ_m that is given by $(m/Q)\,\sigma$. The disk rotates with angular speed ω about its central axis. (*a*) Show that the magnetic moment of the disk has a magnitude $\frac{1}{5}\pi\omega\sigma_0 R^4$ which can be alternatively rewritten as $\frac{3}{10}\omega QR^2$. (*b*) Show that the magnetic moment $\vec{\mu}$ and angular momentum $\vec{L}$ are related by $\vec{\mu} = \frac{Q}{2M}\vec{L}$.

Picture the Problem We can express the magnetic moment of an element of current dI due to a ring of radius r, and thickness dr with charge dq. Integrating this expression from $r = 0$ to $r = R$ will give us the magnetic moment of the disk. We can integrate the charge on the ring between these same limits to find the total charge on the disk and divide μ by Q to establish the relationship between them. In Part (*b*) we can find the angular momentum of the disk by first finding the moment of inertia of the disk by integrating $r^2 dm$ between the same limits used above.
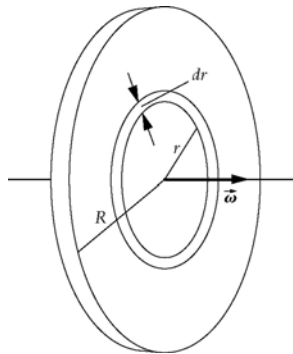
(*a*) Express the magnetic moment of an element of the disk:

$$d\mu = A\,dI$$

The area enclosed by the rotating element of charge is:

$$A = \pi x^2$$

Express the element of current dI:

$$dI = \frac{dq}{\Delta t} = \frac{\sigma dA}{\Delta t} = f\sigma dA$$

$$= \frac{\omega}{2\pi}\left(\sigma_0 \frac{r}{R}\right)(2\pi \, rdr) = \frac{\sigma_0 \omega}{R} r^2 dr$$

Substitute for A and dI and simplify to obtain:

$$d\mu = \pi r^2 \frac{\sigma_0 \omega}{R} r^2 dr = \frac{\sigma_0 \pi \omega}{R} r^4 dr$$

Integrate $d\mu$ from $r = 0$ to $r = R$ to obtain:

$$\mu = \frac{\sigma_0 \pi \omega}{R} \int_0^R r^4 dr = \boxed{\tfrac{1}{5}\sigma_0 \pi \omega R^4} \qquad (1)$$

The charge dq within a distance r of the center of the disk is given by:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{r}{R}\right)dr$$

$$= \frac{2\pi\sigma_0}{R} r^2 dr$$

Integrate dq from $r = 0$ to $r = R$ to obtain:

$$Q = \frac{2\pi\sigma_0}{R} \int_0^R r^2 dr = \tfrac{2}{3}\pi\sigma_0 R^2 \qquad (2)$$

Divide equation (1) by Q to obtain:

$$\frac{\mu}{Q} = \frac{\tfrac{1}{5}\sigma_0 \pi \omega R^4}{\tfrac{2}{3}\pi\sigma_0 R^2} = \frac{3\omega R^2}{10}$$

and

$$\mu = \boxed{\tfrac{3}{10} Q\omega R^2} \qquad (3)$$

(b) Express the moment of inertia of an element of mass dm of the disk:

$$dI = r^2 dm = r^2 \sigma_m dA$$

$$= r^2\left(\frac{m}{Q}\sigma\right)(2\pi r dr)$$

$$= \frac{2\pi m\left(\frac{r}{R}\sigma_0\right)}{Q} r^3 dr$$

$$= \frac{2\pi m\sigma_0}{QR} r^4 dr$$

Integrate dI from $r = 0$ to $r = R$ to obtain:

$$I = \frac{2\pi m\sigma_0}{QR} \int_0^R r^4 dr = \frac{2\pi m\sigma_0}{5Q} R^4$$

Divide I by equation (2) and simplify to obtain:

$$\frac{I}{Q} = \frac{\frac{2\pi n \sigma_0}{5Q} R^4}{\frac{2}{3}\pi\sigma_0 R^2} = \frac{3m}{5Q} R^2$$

and

$$I = \frac{3m}{5} R^2$$

Express the angular momentum of the disk:

$$L = I\omega = \tfrac{3}{5} m R^2 \omega$$

Divide equation (3) by L and simplify to obtain:

$$\frac{\mu}{L} = \frac{\frac{3}{10} Q\omega R^2}{\frac{3}{5} m R^2 \omega} = \frac{Q}{2m} \Rightarrow \mu = \frac{Q}{2m} L$$

Because $\vec{\mu}$ is in the same direction as $\vec{\omega}$:

$$\boxed{\vec{\mu} = \frac{Q}{2m} \vec{L}}$$

61 ••• **[SSM]** A spherical shell of radius R carries a constant surface charge density σ. The shell rotates about its diameter with angular speed ω. Find the magnitude of the magnetic moment of the rotating shell.

Picture the Problem We can use the result of Problem 57 to express μ as a function of Q, M, and L. We can then use the definitions of surface charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating shell.

Express the magnetic moment of the spherical shell in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M} L$$

Use the definition of surface charge density to express the charge on the spherical shell:

$$Q = \sigma A = 4\pi\sigma R^2$$

Express the angular momentum of the spherical shell:

$$L = I\omega = \tfrac{2}{3} M R^2 \omega$$

Substitute for L and simplify to obtain:

$$\mu = \left(\frac{4\pi\sigma R^2}{2M}\right)\left(\frac{2}{3} M R^2 \omega\right) = \boxed{\tfrac{4}{3}\pi\sigma R^4 \omega}$$

62 ••• A uniform solid uniformly charged sphere of radius R has a volume charge density ρ. The sphere rotates about an axis through its center with angular speed ω. Find the magnitude of the magnetic moment of this rotating sphere.

Picture the Problem We can use the result of Problem 57 to express μ as a function of Q, M, and L. We can then use the definitions of volume charge density and angular momentum to substitute for Q and L to obtain the magnetic moment of the rotating sphere.

Express the magnetic moment of the solid sphere in terms of its mass, charge, and angular momentum:

$$\mu = \frac{Q}{2M}L$$

Use the definition of volume charge density to express the charge of the sphere:

$$Q = \rho V = \tfrac{4}{3}\pi\rho R^3$$

Express the angular momentum of the solid sphere:

$$L = I\omega = \tfrac{2}{5}MR^2\omega$$

Substitute for Q and L and simplify to obtain:

$$\mu = \left(\frac{\tfrac{4}{3}\pi\rho R^3}{2M}\right)\left(\frac{2}{5}MR^2\omega\right) = \boxed{\tfrac{4}{15}\pi\rho R^5\omega}$$

63 ••• A uniform thin uniformly charged disk of mass m, radius R, and uniform surface charge density σ rotates with angular speed ω about an axis through its center and perpendicular to the disk (Figure 26-40). The disk in in a region with a uniform magnetic field $\vec{B}$ that makes an angle θ with the rotation axis. Calculate (*a*) the magnitude of the torque exerted on the disk by the magnetic field and (*b*) the precession frequency of the disk in the magnetic field.

Picture the Problem We can use its definition to express the torque acting on the disk, Example 26-11 to express the magnetic moment of the disk, and the definition of the precession frequency to find the precession frequency of the disk.

(*a*) The magnitude of the net torque acting on the disk is:

$\tau = \mu B\sin\theta$
where μ is the magnetic moment of the disk.

From Example 26-11:

$$\mu = \tfrac{1}{4}\pi\sigma r^4\omega$$

Substitute for μ in the expression for τ to obtain:

$$\tau = \boxed{\tfrac{1}{4}\pi\sigma r^4\omega B\sin\theta}$$

(b) The precession frequency Ω is equal to the ratio of the torque divided by the spin angular momentum:

$$\Omega = \frac{\tau}{I\omega}$$

For a solid disk, the moment of inertia is given by:

$$I = \tfrac{1}{2}mr^2$$

Substitute for τ and I to obtain:

$$\Omega = \frac{\tfrac{1}{4}\pi\sigma r^4 \omega B \sin\theta}{\tfrac{1}{2}mr^2\omega} = \boxed{\frac{\pi\sigma r^2 B}{2m}\sin\theta}$$

Remarks: Note that the precession frequency is independent of ω.

The Hall Effect

64 •   A metal strip that is 2.00-cm wide and 0.100-cm thick carries a current of 20.0 A in region with a uniform magnetic field of 2.00 T, as shown in Figure 26-41. The Hall voltage is measured to be 4.27 μV. (a) Calculate the drift speed of the free electrons in the strip. (b) Find the number density of the free electrons in the strip. (c) Is point a or point b at the higher potential? Explain your answer.

Picture the Problem We can use the Hall effect equation to find the drift speed of the electrons and the relationship between the current and the number density of charge carriers to find n. In (c) we can use a right-hand rule to decide whether a or b is at the higher potential.

(a) Express the Hall voltage as a function of the drift speed of the electrons in the strip:

$$V_{\mathrm{H}} = v_{\mathrm{d}}Bw \Rightarrow v_{\mathrm{d}} = \frac{V_{\mathrm{H}}}{Bw}$$

Substitute numerical values and evaluate v_{d}:

$$v_{\mathrm{d}} = \frac{4.27\,\mu\mathrm{V}}{(2.00\,\mathrm{T})(2.00\,\mathrm{cm})} = 0.1068\,\mathrm{mm/s}$$
$$= \boxed{0.107\,\mathrm{mm/s}}$$

(b) Express the current as a function of the number density of charge carriers:

$$I = nAqv_{\mathrm{d}} \Rightarrow n = \frac{I}{Aqv_{\mathrm{d}}}$$

Substitute numerical values and evaluate n:

$$n = \frac{20.0\,\mathrm{A}}{(2.00\,\mathrm{cm})(0.100\,\mathrm{cm})(1.602\times10^{-19}\,\mathrm{C})(0.1068\,\mathrm{mm/s})} = \boxed{5.85\times10^{28}\,\mathrm{m}^{-3}}$$

(c) Apply a right-hand rule to $I\vec{\ell}$ and $\vec{B}$ to conclude that positive charge will accumulate at a and negative charge at b and therefore $\boxed{V_a > V_b}$. The Hall effect electric field is directed from a toward b.

65 •• **[SSM]** The number density of free electrons in copper is 8.47×10^{22} electrons per cubic centimeter. If the metal strip in Figure 26-41 is copper and the current is 10.0 A, find (a) the drift speed v_d and (b) the potential difference $V_a - V_b$. Assume that the magnetic field strength is 2.00 T.

Picture the Problem We can use $I = nqv_d A$ to find the drift speed and $V_H = v_d Bw$ to find the potential difference $V_a - V_b$.

(a) Express the current in the metal strip in terms of the drift speed of the electrons:

$$I = nqv_d A \Rightarrow v_d = \frac{I}{nqA}$$

Substitute numerical values and evaluate v_d:

$$v_d = \frac{10.0\,\text{A}}{\left(8.47 \times 10^{22}\,\text{cm}^{-3}\right)\left(1.602 \times 10^{-19}\,\text{C}\right)\left(2.00\,\text{cm}\right)\left(0.100\,\text{cm}\right)} = 3.685 \times 10^{-5}\,\text{m/s}$$

$$= \boxed{3.68 \times 10^{-5}\,\text{m/s}}$$

(b) The potential difference $V_a - V_b$ is the Hall voltage and is given by:

$$V_a - V_b = V_H = v_d Bw$$

Substitute numerical values and evaluate $V_a - V_b$:

$$V_a - V_b = \left(3.685 \times 10^{-5}\,\text{m/s}\right)\left(2.00\,\text{T}\right)\left(2.00\,\text{cm}\right) = \boxed{1.47\,\mu\text{V}}$$

66 •• A copper strip has 8.47×10^{22} electrons per cubic centimeter is 2.00-cm wide, is 0.100-cm thick, and is used to measure the magnitudes of unknown magnetic fields that are perpendicular to it. Find the magnitude of B when the current is 20.0 A and the Hall voltage is (a) 2.00 μV, (b) 5.25 μV, and (c) 8.00 μV.

Picture the Problem We can use $V_H = v_d Bw$ to express B in terms of V_H and $I = nqv_d A$ to eliminate the drift velocity v_d and derive an expression for B in terms of V_H, n, and t.

Relate the Hall voltage to the drift velocity and the magnetic field:

$$V_{H} = v_{d}Bw \Rightarrow B = \frac{V_{H}}{v_{d}w}$$

Express the current in the metal strip in terms of the drift velocity of the electrons:

$$I = nqv_{d}A \Rightarrow v_{d} = \frac{I}{nqA}$$

Substitute for v_d and simplify to obtain:

$$B = \frac{V_{H}}{\frac{I}{nqA}w} = \frac{nqAV_{H}}{Iw} = \frac{nqwtV_{H}}{Iw}$$

$$= \frac{nqt}{I}V_{H}$$

Substitute numerical values and simplify to obtain:

$$B = \frac{(8.47 \times 10^{22}\ \text{cm}^{-3})(1.602 \times 10^{-19}\ \text{C})(0.100\,\text{cm})V_{H}}{20.0\,\text{A}} = (6.7845 \times 10^{5}\ \text{s/m}^{2})V_{H}$$

(*a*) Evaluate *B* for $V_H = 2.00\ \mu V$:

$$B = (6.7845 \times 10^{5}\ \text{s/m}^{2})(2.00\ \mu V)$$
$$= \boxed{1.36\,\text{T}}$$

(*b*) Evaluate *B* for $V_H = 5.25\ \mu V$:

$$B = (6.7845 \times 10^{5}\ \text{s/m}^{2})(5.25\ \mu V)$$
$$= \boxed{3.56\,\text{T}}$$

(*c*) Evaluate *B* for $V_H = 8.00\ \mu V$:

$$B = (6.7845 \times 10^{5}\ \text{s/m}^{2})(8.00\ \mu V)$$
$$= \boxed{5.43\,\text{T}}$$

67 •• Because blood contains ions, moving blood develops a Hall voltage across the diameter of an artery. A large artery that has a diameter of 0.85 cm can have blood flowing through it with a maximum speed of 0.60 m/s. If a section of this artery is in a magnetic field of 0.20 T, what is the maximum potential difference across the diameter of the artery?

Picture the Problem We can use $V_{H} = v_{d}Bw$ to find the Hall voltage developed across the diameter of the artery.

Relate the Hall voltage to the flow speed of the blood v_d, the diameter of the artery w, and the magnetic field B:

$$V_{H} = v_{d}Bw$$

Substitute numerical values and evaluate V_H:

$$V_H = (0.60\,\text{m/s})(0.20\,\text{T})(0.85\,\text{cm})$$
$$= \boxed{1.0\,\text{mV}}$$

68 •• The Hall coefficient R_H is a property of conducting material (just as resistivity is). It is defined as $R_H = E_y/(J_x B_z)$, where J_x is x component of the current density in the material, B_z is the z component of the magnetic field, and E_y is the y component resulting Hall electric field. Show that the Hall coefficient is equal to $1/(nq)$, where q is the charge of the charge carriers ($-e$ if they are electrons). (The Hall coefficients of monovalent metals, such as copper, silver, and sodium are therefore negative.)

Picture the Problem Let the width of the slab be w and its thickness t. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to show that the Hall coefficient is also given by $1/(nq)$.

The Hall coefficient is:

$$R = \frac{E_y}{J_x B_z}$$

Using its definition, express the Hall electric field in the slab:

$$E_y = \frac{V_H}{w}$$

The current density in the slab is:

$$J_x = \frac{I}{wt} = nqv_d$$

Substitute for E_y and J_x and simplify to obtain:

$$R = \frac{\dfrac{V_H}{w}}{nqv_d B_z} = \frac{V_H}{nqv_d w B_z}$$

Express the Hall voltage in terms of v_d, B, and w:

$$V_H = v_d B_z w$$

Substitute for V_H and simplify to obtain:

$$R = \frac{v_d B_z w}{nqv_d w B_z} = \boxed{\frac{1}{nq}}$$

69 •• **[SSM]** Aluminum has a density of 2.7×10^3 kg/m^3 and a molar mass of 27 g/mol. The Hall coefficient of aluminum is $R = -0.30 \times 10^{-10}$ m^3/C. (See Problem 68 for the definition of R.) What is the number of conduction electrons per aluminum atom?

Picture the Problem We can determine the number of conduction electrons per atom from the quotient of the number density of charge carriers and the number of charge carriers per unit volume. Let the width of a slab of aluminum be w and its thickness t. We can use the definition of the Hall electric field in the slab, the expression for the Hall voltage across it, and the definition of current density to find n in terms of R and q and $n_a = \rho N_A / M$, to express n_a.

Express the number of electrons per atom N:	$$N = \dfrac{n}{n_a}$$	(1)
	where n is the number density of charge carriers and n_a is the number of atoms per unit volume.	
From the definition of the Hall coefficient we have:	$$R = \dfrac{E_y}{J_x B_z}$$	
Express the Hall electric field in the slab:	$$E_y = \dfrac{V_H}{w}$$	
The current density in the slab is:	$$J_x = \dfrac{I}{wt} = nqv_d$$	
Substitute for E_y and J_x in the expression for R to obtain:	$$R = \dfrac{\dfrac{V_H}{w}}{nqv_d B_z} = \dfrac{V_H}{nqv_d w B_z}$$	
Express the Hall voltage in terms of v_d, B, and w:	$$V_H = v_d B_z w$$	
Substitute for V_H and simplify to obtain:	$$R = \dfrac{v_d B_z w}{nqv_d w B_z} = \dfrac{1}{nq} \Rightarrow n = \dfrac{1}{Rq}$$	(2)
Express the number of atoms n_a per unit volume:	$$n_a = \rho \dfrac{N_A}{M}$$	(3)
Substitute equations (2) and (3) in equation (1) to obtain:	$$N = \dfrac{M}{qR\rho N_A}$$	

Substitute numerical values and evaluate N:

$$N = \frac{27\dfrac{\text{g}}{\text{mol}}}{\left(-1.602\times10^{-19}\,\text{C}\right)\left(-0.30\times10^{-10}\,\dfrac{\text{m}^3}{\text{C}}\right)\left(2.7\times10^3\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(6.022\times10^{23}\,\dfrac{\text{atoms}}{\text{mol}}\right)}$$

$$\approx \boxed{4}$$

General Problems

70 •   A long wire parallel to the x axis carries a current of 6.50 A in the $+x$ direction. The wire occupies a region that has a uniform magnetic field $\vec{B} = 1.35\ \text{T}\,\hat{j}$. Find the magnetic force per unit length on the wire.

Picture the Problem We can use the expression for the magnetic force acting on a wire ($\vec{F} = I\vec{\ell}\times\vec{B}$) to find the force per unit length on the wire.

Express the magnetic force on the wire:

$$\vec{F} = I\vec{\ell}\times\vec{B}$$

Substitute for $I\vec{\ell}$ and $\vec{B}$ to obtain:

$$\vec{F} = (6.50\,\text{A})\ell\hat{i}\times(1.35\,\text{T})\hat{j}$$
and
$$\frac{\vec{F}}{\ell} = (6.50\,\text{A})\hat{i}\times(1.35\,\text{T})\hat{j}$$

Simplify to obtain:

$$\frac{\vec{F}}{\ell} = (8.78\,\text{N/m})(\hat{i}\times\hat{j}) = \boxed{(8.78\,\text{N/m})\hat{k}}$$

71 •   An alpha particle (charge $+2e$) travels in a circular path of radius 0.50 m in a region with a magnetic field whose magnitude is 0.10 T. Find (*a*) the period, (*b*) the speed, and (*c*) the kinetic energy (in electron volts) of the alpha particle. (The mass of an alpha particle is 6.65×10^{-27} kg.)

Picture the Problem We can express the period of the alpha particle's motion in terms of its orbital speed and use Newton's 2nd law to express its orbital speed in terms of known quantities. Knowing the particle's period and the radius of its motion we can find its speed and kinetic energy.

(a) Relate the period of the alpha particle's motion to its orbital speed:

$$T = \frac{2\pi r}{v} \qquad (1)$$

Apply Newton's 2nd law to the alpha particle to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow v = \frac{qBr}{m}$$

Substitute for v in equation (1) and simplify to obtain:

$$T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate T:

$$T = \frac{2\pi\left(6.65\times10^{-27}\,\text{kg}\right)}{2\left(1.602\times10^{-19}\,\text{C}\right)\left(0.10\,\text{T}\right)} = 1.30\,\mu s$$

$$= \boxed{1.3\,\mu s}$$

(b) Solve equation (1) for v:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi\left(0.50\,\text{m}\right)}{1.30\,\mu s} = 2.409\times10^6\,\text{m/s}$$

$$= \boxed{2.4\times10^6\,\text{m/s}}$$

(c) The kinetic energy of the alpha particle is:

$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}\left(6.65\times10^{-27}\,\text{kg}\right)\left(2.409\times10^6\,\text{m/s}\right)^2$$

$$= 1.930\times10^{-14}\,\text{J}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\,\text{J}}$$

$$= \boxed{0.12\,\text{MeV}}$$

72 •• The pole strength q_m of a bar magnet is defined by $\vec{\mu} = q_m\vec{\ell}$, where $\vec{\mu}$ is the magnetic moment of the magnet and $\vec{\ell}$ is the position of the north-pole end of the magnet relative to the south-pole end. Show that the torque exerted on a bar magnet in a uniform magnetic field $\vec{B}$ is the same as if a force $+q_m\vec{B}$ is exerted on the north-pole of the magnetic and a force $-q_m\vec{B}$ is exerted on the south-pole.

Picture the Problem The configuration of the magnet and field are shown in the figure. We'll assume that a force $+q_m\vec{B}$ is exerted on the north-pole end and a force $-q_m\vec{B}$ is exerted on the south-pole end and show that this assumption leads to the familiar expression for the torque acting on a magnetic dipole.
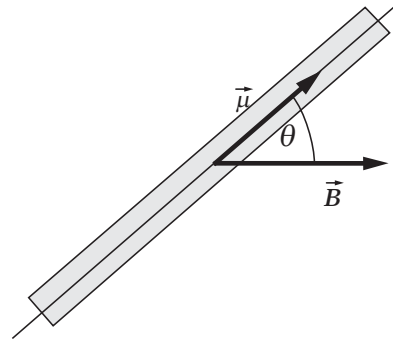
Assuming that a force $+q_m\vec{B}$ is exerted on the north-pole end and a force $-q_m\vec{B}$ is exerted on the south-pole end, express the net torque acting on the bar magnet:

$$\tau = \frac{Bq_m\ell}{2}\sin\theta - \frac{-Bq_m\ell}{2}\sin\theta$$
$$= Bq_m\ell\sin\theta$$

Substitute for q_m to obtain:

$$\tau = B\frac{|\vec{\mu}|}{\ell}\ell\sin\theta = \mu B\sin\theta$$

or

$$\boxed{\vec{\tau} = \vec{\mu}\times\vec{B}}$$

73 •• [SSM] A particle of mass m and charge q enters a region where there is a uniform magnetic field $\vec{B}$ parallel with the x axis. The initial velocity of the particle is $\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j}$, so the particle moves in a helix. (*a*) Show that the radius of the helix is $r = mv_{0y}/qB$. (*b*) Show that the particle takes a time $\Delta t = 2\pi m/qB$ to complete each turn of the helix. (*c*) What is the x component of the displacement of the particle during time given in Part (*b*)?

Picture the Problem We can use $\vec{F} = q\vec{v}\times\vec{B}$ to show that motion of the particle in the x direction is not affected by the magnetic field. The application of Newton's 2nd law to motion of the particle in yz plane will lead us to the result that $r = mv_{0y}/qB$. By expressing the period of the motion in terms of v_{0y} we can show that the time for one complete orbit around the helix is $t = 2\pi m/qB$.

(*a*) Express the magnetic force acting on the particle:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Substitute for $\vec{v}$ and $\vec{B}$ and simplify to obtain:

$$\vec{F} = q\left(v_{0x}\hat{i} + v_{0y}\hat{j}\right) \times B\hat{i}$$
$$= qv_{0x}B\left(\hat{i} \times \hat{i}\right) + qv_{0y}B\left(\hat{j} \times \hat{i}\right)$$
$$= 0 - qv_{0y}B\hat{k} = -qv_{0y}B\hat{k}$$

i.e., the motion in the direction of the magnetic field (the x direction) is not affected by the field.

Apply Newton's 2nd law to the particle in the plane perpendicular to $\hat{i}$ (i.e., the yz plane):

$$qv_{0y}B = m\frac{v_{0y}^2}{r} \tag{1}$$

Solving for r yields:

$$r = \boxed{\frac{mv_{0y}}{qB}}$$

(b) Relate the time for one orbit around the helix to the particle's orbital speed:

$$\Delta t = \frac{2\pi r}{v_{0y}}$$

Solve equation (1) for v_{0y}:

$$v_{0y} = \frac{qBr}{m}$$

Substitute for v_{0y} and simplify to obtain:

$$\Delta t = \frac{2\pi r}{\dfrac{qBr}{m}} = \boxed{\frac{2\pi m}{qB}}$$

(c) Because, as was shown in Part (a), the motion in the direction of the magnetic field (the x direction) is not affected by the field, the x component of the displacement of the particle as a function of t is:

$$x(t) = v_{ox}t$$

For $t = \Delta t$:

$$x(\Delta t) = v_{ox}\left(\frac{2\pi m}{qB}\right) = \boxed{\frac{2\pi mv_{ox}}{qB}}$$

74 ••   A metal crossbar of mass m rides on a parallel pair of long horizontal conducting rails separated by a distance L and connected to a device that supplies constant current I to the circuit, as shown in Figure 26-42. The circuit is in a region with a uniform magnetic field $\vec{B}$ whose direction is vertically downward.

There is no friction and the bar starts from rest at $t = 0$. (*a*) In which direction will the bar start to move? (*b*) Show that at time t the bar has a speed of $(BIL/m)t$.

Picture the Problem We can use a constant-acceleration equation to relate the velocity of the crossbar to its acceleration and Newton's 2nd law to express the acceleration of the crossbar in terms of the magnetic force acting on it. We can determine the direction of motion of the crossbar using a right-hand rule or, equivalently, by applying $\vec{F} = I\vec{\ell} \times \vec{B}$.

(*a*) Using a constant-acceleration equation, express the velocity of the bar as a function of its acceleration and the time it has been in motion:

$$v = v_0 + at$$
or, because $v_0 = 0$,
$$v = at$$

Use Newton's 2nd law to express the acceleration of the rail:

$$a = \frac{F}{m}$$
where F is the magnitude of the magnetic force acting in the direction of the crossbar's motion.

Substitute for a to obtain:

$$v = \frac{F}{m}t$$

Express the magnetic force acting on the current-carrying crossbar:

$$F = ILB$$

Substitute to obtain:

$$v = \boxed{\frac{ILB}{m}t}$$

(*b*) Because the magnetic force is to the right and the crossbar starts from rest, the motion of the crossbar will also be toward the right.

75 ••   **[SSM]** Assume that the rails Problem 74 are frictionless but tilted upward so that they make an angle θ with the horizontal, and with the current source attached to the low end of the rails. The magnetic field is still directed vertically downward. (*a*) What minimum value of B is needed to keep the bar from sliding down the rails? (*b*) What is the acceleration of the bar if B is twice the value found in Part (*a*)?

Picture the Problem Note that with the rails tilted, $\vec{F}$ still points horizontally to the right (I, and hence $\vec{\ell}$, is out of the page). Choose a coordinate system in which down the incline is the positive x direction. Then we can apply a condition

for translational equilibrium to find the vertical magnetic field $\vec{B}$ needed to keep the bar from sliding down the rails. In Part (b) we can apply Newton's 2nd law to find the acceleration of the crossbar when B is twice its value found in (a).
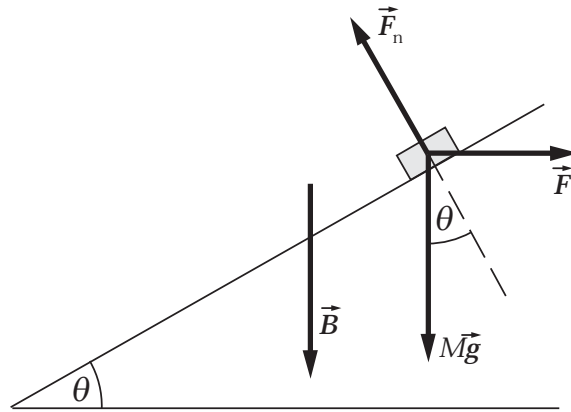
(a) Apply $\sum F_x = 0$ to the crossbar to obtain:

$mg \sin\theta - I\ell B \cos\theta = 0$

Solving for B yields:

$B = \dfrac{mg}{I\ell} \tan\theta$ and $\boxed{\vec{B} = -\dfrac{mg}{I\ell} \tan\theta \, \hat{u}_v}$

where $\hat{u}_v$ is a unit vector in the vertical direction.

(b) Apply Newton's 2nd law to the crossbar to obtain:

$I\ell B' \cos\theta - mg \sin\theta = ma$

Solving for a yields:

$a = \dfrac{I\ell B'}{m} \cos\theta - g \sin\theta$

Substitute $B' = 2B$ and simplify to obtain:

$a = \dfrac{2I\ell \dfrac{mg}{I\ell} \tan\theta}{m} \cos\theta - g \sin\theta$

$= 2g \sin\theta - g \sin\theta = \boxed{g \sin\theta}$

Note that the direction of the acceleration is up the incline.

76 ••   A long, narrow bar magnet that has magnetic moment $\vec{\mu}$ parallel to its long axis is suspended at its center as a frictionless compass needle. When placed in region with a horizontal magnetic field $\vec{B}$, the needle lines up with the field. If it is displaced by a small angle θ, show that the needle will oscillate about

its equilibrium position with frequency $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu B}{I}}$, where I is the moment of inertia of the needle about the point of suspension.

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the bar magnet executes simple harmonic motion. To show its motion is SHM we need to show that the bar magnet experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the differential equation of motion we can identify ω and express f.

Apply Newton's 2nd law to the bar magnet:

$$-\mu B \sin\theta = I\dfrac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the magnet with the magnetic field and I is the moment of inertia of the magnet.

For small displacements from equilibrium, $\theta << 1$ and:

$$\sin\theta \approx \theta$$

Hence our differential equation of motion becomes:

$$I\dfrac{d^2\theta}{dt^2} = -\mu B\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the bar magnet is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\dfrac{d^2\theta}{dt^2} = -\dfrac{\mu B}{I}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\dfrac{\mu B}{I}}$

Relate f to ω to obtain:

$$f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{\mu B}{I}}}$$

77 •• A straight conducting wire whose length is 20 m is parallel to the y axis and is moving in the $+x$ direction with a speed of 20 m/s in a region with a magnetic field given by $0.50\,\text{T}\,\hat{k}$. (*a*) Because of this magnetic force, electrons move to one end of the wire leaving the other end positively charged, until the

electric field due to this charge separation exerts a force on the conduction electrons that balances the magnetic force. Find the magnitude and direction of this electric field in the steady state situation. (b) Which end of the wire is positively charged and which end is negatively charged? (c) Suppose the moving wire is 2.0-m long. What is the potential difference between its two ends due to this electric field?

Picture the Problem (a) We can use a condition for translational equilibrium to relate $\vec{E}$ to $\vec{F}$. In Part (c) we can apply the definition of electric field in terms of potential difference to evaluate the difference in potential between the ends of the moving wire.

(a) Sum the forces acting on an electron under steady-state conditions to obtain:

$$q\vec{E} + \vec{F} = 0 \Rightarrow \vec{E} = -\frac{\vec{F}}{q}$$

The magnetic force on an electron in the conductor is given by:

$$\vec{F} = q\vec{v} \times \vec{B} = qv\hat{i} \times B\hat{k}$$
$$= qvB\left(\hat{i} \times \hat{k}\right) = -qvB\hat{j}$$

Substituting for $\vec{F}$ and simplifying yields:

$$\vec{E} = -\frac{-qvB\hat{j}}{q} = vB\hat{j}$$

Substitute numerical values and evaluate $\vec{E}$:

$$\vec{E} = (20\,\text{m/s})(0.50\,\text{T})\hat{j} = \boxed{(10\,\text{V/m})\hat{j}}$$

(b) Because the electric force acting on the conduction electrons is in the $+y$ direction, the end of the wire that is in the $+y$ direction becomes negatively charged and the end of the wire that is in the $-y$ direction becomes positively charged. The positive end has the lesser y coordinate.

(c) The potential difference between the ends of the wire is:

$$\Delta V = E\Delta y = (10.0\,\text{V/m})(2.0\,\text{m})$$
$$= \boxed{20\,\text{V}}$$

78 ••• A circular loop of wire that has a mass m and carries a constant current I is in a region with a uniform magnetic field. It is initially in equilibrium and its magnetic moment is aligned with the magnetic field. The loop is given a small angular displacement about an axis through it center and perpendicular to the magnetic field and then released. What is the period of the subsequent motion? (Assume that the only torque exerted on the loop is due to the magnetic field and that there are no other forces acting on the loop.)

Picture the Problem We're being asked to show that, for small displacements from equilibrium, the circular loop executes simple harmonic motion. To show its motion is SHM we must show that the loop experiences a linear restoring torque when displaced from equilibrium. We can accomplish this by applying Newton's 2nd law in rotational form and using a small angle approximation to obtain the differential equation for simple harmonic motion. Once we have the differential equation we can identify ω and express the period T of the motion.

Apply Newton's 2nd law to the loop:

$$-IAB\sin\theta = I_{\text{inertia}}\frac{d^2\theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the loop with the magnetic field and I_{inertia} is the moment of inertia of the loop.

For small displacements from equilibrium, $\theta \ll 1$ and:

$$\sin\theta \approx \theta$$

Hence, our differential equation of motion becomes:

$$I_{\text{inertia}}\frac{d^2\theta}{dt^2} = -IAB\theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the current loop is the differential equation of simple harmonic motion. Solve this equation for $d^2\theta/dt^2$ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{IAB}{I_{\text{inertia}}}\theta$$

.

Noting that the moment of inertia of a hoop about its diameter is $\frac{1}{2}mR^2$, substitute for I_{inertia} and simplify to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{I\pi R^2 B}{\frac{1}{2}mR^2}\theta = -\frac{2I\pi B}{m}\theta = -\omega^2\theta$$

where $\omega = \sqrt{\dfrac{2\pi IB}{m}}$

The period T of the motion is related to the angular frequency ω:

$$T = \frac{2\pi}{\omega}$$

Substituting for ω and simplifying yields:

$$T = \boxed{\sqrt{\frac{2\pi m}{IB}}}$$

79 •••   A small bar magnet has a magnetic moment $\vec{\mu}$ that makes an angle θ with the x axis. The magnet is in a region that has a *non-uniform* magnetic field given by $\vec{B} = B_x(x)\hat{i} + B_y(y)\hat{j}$. Using $F_x = -\partial U/\partial x$, $F_y = -\partial U/\partial y$ and $F_z = -\partial U/\partial z$, show that there is a net magnetic force on the magnet that is given by $\vec{F} = \mu_x \dfrac{\partial B_x}{\partial x}\hat{i} + \mu_y \dfrac{\partial B_y}{\partial y}\hat{j}$.

Picture the Problem We can express $\vec{\mu}$ in terms of its components and calculate U from $\vec{\mu}$ and $\vec{B}$ using $U = -\vec{\mu} \cdot \vec{B}$. Knowing U we can calculate the components of $\vec{F}$ using $F_x = -dU/dx$ and $F_y = -dU/dy$.

Express the net force acting on the magnet in terms of its components:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} \qquad\qquad (1)$$

Express $\vec{\mu}$ in terms of its components:

$$\vec{\mu} = \mu_x\hat{i} + \mu_y\hat{j} + \mu_z\hat{k}$$

Express the potential energy of the bar magnetic in the nonuniform magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$
$$= -\left(\mu_x\hat{i} + \mu_y\hat{j} + \mu_z\hat{k}\right) \cdot \left(B_x(x)\hat{i} + B_y(y)\hat{j}\right)$$
$$= -\mu_x B_x(x) - \mu_y B_y(y)$$

Because $\vec{\mu}$ is constant but $\vec{B}$ depends on x and y:

$$F_x = -\frac{dU}{dx} = \mu_x\left(\frac{\partial B_x}{\partial x}\right)$$

and

$$F_y = -\frac{dU}{dy} = \mu_y\left(\frac{\partial B_y}{\partial y}\right)$$

Substitute in equation (1) to obtain:

$$\boxed{\vec{F} = \mu_x\frac{\partial B_x}{\partial x}\hat{i} + \mu_y\frac{\partial B_y}{\partial y}\hat{j}}$$

80 ••   A proton, a deuteron and an alpha particle all have the same kinetic energy. They are moving in a region with a uniform magnetic field that is perpendicular to each of their velocities. Let R_p, R_d, and R_α be the radii of their circular orbits, respectively. The deuteron has a charge that is equal to the charge a proton has, and an alpha particle has a charge that is equal to twice the charge a proton has. Find the ratios R_d/R_p and R_α/R_p. Assume that $m_\alpha = 2m_d = 4m_p$.

Picture the Problem We can apply Newton's 2nd law to an orbiting particle to obtain an expression for the radius of its orbit R as a function of its mass m, charge q, speed v, and the magnitude of the magnetic field B.

Apply Newton's 2nd law to an orbiting particle to obtain:	$qvB = m\dfrac{v^2}{r} \Rightarrow r = \dfrac{mv}{qB}$

Express the kinetic energy of the particle:	$K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2K}{m}}$

Substitute for v in the expression for r and simplify to obtain:	$r = \dfrac{m}{qB}\sqrt{\dfrac{2K}{m}} = \dfrac{1}{qB}\sqrt{2Km}$ (1)

Using equation (1), express the ratio R_d/R_p:	$\dfrac{R_d}{R_p} = \dfrac{\frac{1}{q_dB}\sqrt{2Km_d}}{\frac{1}{q_pB}\sqrt{2Km_p}} = \dfrac{q_p}{q_d}\sqrt{\dfrac{m_d}{m_p}}$
	$= \dfrac{e}{e}\sqrt{\dfrac{2m_p}{m_p}} = \boxed{\sqrt{2}}$

Using equation (1), express the ratio R_α/R_p:	$\dfrac{R_\alpha}{R_p} = \dfrac{\frac{1}{q_\alpha B}\sqrt{2Km_\alpha}}{\frac{1}{q_pB}\sqrt{2Km_p}} = \dfrac{q_p}{q_\alpha}\sqrt{\dfrac{m_\alpha}{m_p}}$
	$= \dfrac{e}{2e}\sqrt{\dfrac{4m_p}{m_p}} = \boxed{1}$

81 ••• Your forensic chemistry group, working closely with the local law enforcement agencies, has acquired a mass spectrometer similar to that discussed in the text. It employs a uniform magnetic field that has a magnitude of 0.75 T. To calibrate the mass spectrometer, you decide to measure the masses of various carbon isotopes by measuring the position of impact of the various singly ionized carbon ions that have entered the spectrometer with a kinetic energy of 25 keV. A wire chamber with position sensitivity of 0.50 mm is part of the apparatus. What will be the limit on its mass resolution (in kg) for ions in this mass range, that is those whose mass is on the order of that of a carbon atom?

Picture the Problem We can apply Newton's 2nd law, with the force on a moving charged particle in a magnetic field as the net force, to an ion in the spectrometer to obtain an expression for the radius of its trajectory as a function of its

momentum. We can then use the definition of kinetic energy to eliminate the speed of the ion from the expression for the radius of its trajectory. Differentiating the expression for the range (twice the radius of curvature) of the ions with respect to their mass will yield the mass resolution for ions whose masses are roughly 19.9×10^{-27} kg. We'll assume that the carbon atoms are singly ionized.

Apply Newton's 2$^{\text{nd}}$ law to an ion in the spectrometer to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad (1)$$

where q is the charge of the ion, m is its mass, and r is the radius of curvature of its path.

From the definition of kinetic energy we have:

$$E = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$$

Substituting for v in equation (1) and simplifying yields:

$$r = \frac{m\sqrt{\dfrac{2E}{m}}}{qB} = \frac{\sqrt{2mE}}{qB} \qquad (2)$$

The range R of the ions is twice their radius of curvature:

$$R = \frac{2\sqrt{2mE}}{qB} = \frac{\sqrt{8mE}}{qB} \qquad (3)$$

Differentiate R with respect to m to obtain:

$$\frac{dR}{dm} = \frac{d}{dm}\left(\frac{\sqrt{8mE}}{qB}\right) = \frac{\sqrt{8E}}{qB}\frac{d}{dm}\left(\sqrt{m}\right)$$

$$= \frac{\sqrt{8E}}{qB}\frac{1}{2\sqrt{m}} = \frac{\sqrt{\dfrac{2E}{m}}}{qB} = \sqrt{\frac{2E}{mq^2B^2}}$$

Solving for dm yields:

$$dm = \frac{dR}{\sqrt{\dfrac{2E}{mq^2B^2}}} = dR\sqrt{\frac{mq^2B^2}{2E}}$$

Substitute numerical values and evaluate dm:

$$dm = (0.50\,\text{mm})\sqrt{\frac{\left(19.9\times10^{-27}\,\text{kg}\right)\left(1.602\times10^{-19}\,\text{C}\right)^2\left(0.80\,\text{T}\right)^2}{2\left(25\,\text{keV}\times\dfrac{1.602\times10^{-19}\,\text{C}}{\text{eV}}\right)}} = \boxed{1.0\times10^{-28}\,\text{kg}}$$