## Chapter 25 <br> Electric Current and Direct-Current Circuits

## Conceptual Problems

1 - In our study of electrostatics, we concluded that no electric field exists within the material of a conductor in electrostatic equilibrium. Why can we discuss electric fields within the material of conductors in this chapter?

Determine the Concept In earlier chapters the conductors are constrained to be in electrostatic equilibrium. In this chapter this constraint is no longer in place.

2 - Figure 25-12 shows a mechanical analog of a simple electric circuit. Devise another mechanical analog in which the current is represented by a flow of water instead of marbles. In the water circuit, what would be analogous to the battery? What would be analogous to the wire? What would be analogous to the resistor?

Determine the Concept The analog is a wind-up water pump that pumps water through a tube with a necked down section. One end of the tube is connected to the output port of the pump, and the other end of the tube is connected to the input port of the pump. The pump, including the spring, is analogous to the battery. The tube is analogous to the wires. The necked down section is analogous to the resistor.

3 - Wires A and B are both made of copper. The wires are connected in series, so we know they carry the same current. However, the diameter of wire A is twice the diameter of wire B . Which wire has the higher number density (number per unit volume) of charge carriers? (a) A, (b) B, (c) They have the same number density of charge carriers.

Determine the Concept Equation 25-3 ( $I=q n A v_{\mathrm{d}}$ ) relates the current $I$ in a wire to the charge $q$ of the charge carriers, the number density $n$ of charge carriers, the cross-sectional area $A$ of the wire, and the drift speed $v_{\mathrm{d}}$ of the charge carriers. Both the number density of the charge carriers and the resistivity are properties of the substance, and do not depend on the current. Because the number density of carriers may, like resistivity, vary slowly with temperature and the drift speeds are not equal in the two wires, the correct answer is $(c)$.

4 - The diameters of copper wires A and B are equal. The current carried by wire $A$ is twice the current carried by wire $B$. In which wire do the charge carriers have the higher drift speed? (a) A (b) B (c) They have the same drift speed.

Determine the Concept Equation 25-3 ( $I=q n A v_{\mathrm{d}}$ ) relates the current $I$ in a wire to the charge $q$ of the charge carriers, the number density $n$ of charge carriers, the cross-sectional area $A$ of the wire, and the drift velocity $v_{\mathrm{d}}$ of the charge carriers. Both the number density of the charge carriers and the resistivity are properties of the substance, and do not depend on the current. Because the cross-sectional areas, the charge of the charge carriers, and the number densities of the charge carriers are the same for the two wires, the drift speed of the charge carriers will be higher in the wire carrying the larger current. Because wire A carries the larger current, $(a)$ is correct.

5 - Wire A and wire B are identical copper wires. The current carried by wire A is twice the current carried by wire B . Which wire has the higher current density? (a) A, (b) B, (c) They have the same current density. (d) None of the above

Determine the Concept The current density is the ratio of the current to the cross-sectional area of the conductor. Because the wires are identical, their crosssectional areas are the same and their current densities are directly proportional to their currents. Because wire A carries the larger current, $(a)$ is correct.

6 - Consider a metal wire that has each end connected to a different terminal of the same battery. Your friend argues that no matter how long the wire is, the drift speed of the charge carriers in the wire is the same. Evaluate your friend's claim.

Determine the Concept The longer the wire the higher its resistance is. Thus, the longer the wire the smaller the current in the wire. The smaller the current the smaller the drift speed of the charge carriers. The claim that the drift speed is independent of length is bogus.

7 - In a resistor, the direction of the current must always be in the "downhill" direction, that is, in the direction of decreasing electric potential. Is it also the case that in a battery, the direction of the current must always be "downhill"? Explain your answer.

Determine the Concept No, it is not necessarily true for a battery. Under normal operating conditions the current in the battery is in the direction away from the negative battery terminal and toward the positive battery terminal. That is, it opposite to the direction of the electric field.

8 - Discuss the distinction between an emf and a potential difference.
Determine the Concept An emf is a source of energy that gives rise to a potential difference between two points and may result in current flow if there is a conducting path whereas a potential difference is the consequence of two points in space being at different potentials.

9 - Wire A and wire B are made of the same material and have the same length. The diameter of wire A is twice the diameter of wire $B$. If the resistance of wire B is $R$, then what is the resistance of wire A ? (Neglect any effects that temperature may have on resistance.) (a) $R$, (b) $2 R$, (c) $R / 2$, (d) $4 R$, (e) $R / 4$,

Picture the Problem The resistances of the wires are given by $R=\rho L / A$, where $L$ is the length of the wire and $A$ is its cross-sectional area. Because the resistance of a wire varies inversely with its cross-sectional area and its cross-sectional area varies with the square of its diameter, the wire that has the larger diameter will have a resistance that is one fourth the resistance of the wire that has the smaller diameter. Because wire A has a diameter that is twice that of wire B , its resistance is one-fourth the resistance of wire B. (e) is correct.

10 - Two cylindrical copper wires have the same mass. Wire A is twice as long as wire B . (Neglect any effects that temperature may have on resistance.) Their resistances are related by (a) $R_{\mathrm{A}}=8 R_{\mathrm{B}}$, (b) $R_{\mathrm{A}}=4 R_{\mathrm{B}},(c) R_{\mathrm{A}}=2 R_{\mathrm{B}}$, (d) $R_{\mathrm{A}}=R_{\mathrm{B}}$.

Determine the Concept The resistance of a wire is given by $R=\rho L / A$, where $L$ is the length of the wire and $A$ is its cross-sectional area. Because the wires have the same masses, their volumes must be the same. Because their volumes are the same, the cross-sectional area of wire A must be half the cross-sectional area of wire B. Because the cross sectional area of anything that is circular varies with the square of the diameter of the circle, wire A must have a resistance, based solely on its diameter, that is four times the resistance of wire B. Because it is also twice as long as wire $B$, its resistance will be eight times the resistance of wire $B$. (b) is correct.

11 - If the current in a resistor is $I$, the power delivered to the resistor is $P$. If the current in the resistor is increased to $3 I$, what is the power then delivered to the resistor? (Assume the resistance of the resistor does not change.) (a) $P$, (b) $3 P$, (c) $P / 3$, (d) $9 P$, (e) $P / 9$

Picture the Problem The power dissipated in the resistor is given by $P=I^{2} R$. Because of this quadratic dependence of the power dissipated on the current, tripling the current in the resistor increases its resistance by a factor of $9 .(d)$ is correct.

12 - If the potential drop across the resistor is $V$, the power delivered to the resistor is $P$. If the potential drop is increased to $2 V$, what is the power delivered to the resistor then equal to? (a) $P$, (b) $2 P$, (c) $4 P$, (d) $P / 2$, (e) $P / 4$

Picture the Problem Assuming the current (which depends on the resistance) to be constant, the power dissipated in a resistor is directly proportional to the square of the potential drop across it. Hence, doubling the potential drop across a resistor increases the power delivered to the resistor by a factor of $4 .(c)$ is correct.

13 - [SSM] A heater consists of a variable resistor (a resistor whose resistance can be varied) connected across an ideal voltage supply. (An ideal voltage supply is one that has a constant emf and a negligible internal resistance.) To increase the heat output, should you decrease the resistance or increase the resistance? Explain your answer.

Determine the Concept You should decrease the resistance. The heat output is given by $P=V^{2} / R$. Because the voltage across the resistor is constant, decreasing the resistance will increase $P$.

14 - One resistor has a resistance $R_{1}$ and another resistor has a resistance $R_{2}$. The resistors are connected in parallel. If $R_{1} \gg R_{2}$, the equivalent resistance of the combination is approximately $(a) R_{1},(b) R_{2},(c) 0,(d)$ infinity.

Determine the Concept The equivalent resistance of the two resistors connected in parallel is given by $\frac{1}{R_{\text {eq }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$. If $R_{1} \gg R_{2}$, then the reciprocal of $R_{1}$ is very small and $\frac{1}{R_{\mathrm{eq}}} \approx \frac{1}{R_{2}}$. Hence $R_{\mathrm{eq}} \approx R_{2}$ and $(b)$ is correct.

15 - One resistor has a resistance $R_{1}$ and another resistor has a resistance $R_{2}$. The resistors are connected in series. If $R_{1} \gg R_{2}$, the equivalent resistance of the combination is approximately (a) $R_{1}$, (b) $R_{2},(c) 0,(d)$ infinity.

Determine the Concept The equivalent resistance of the two resistors connected in series is given by $R_{\mathrm{eq}}=R_{1}+R_{2}=R_{1}\left(1+\frac{R_{2}}{R_{1}}\right)$. If $R_{1} \gg R_{2}$, then $R_{2} / R_{1}$ is very small and $R_{\mathrm{eq}} \approx R_{1}$. Hence $(a)$ is correct.

16 - A parallel combination consisting of resistors $A$ and $B$ is connected across the terminals of a battery. The resistor A has twice the resistance of resistor B. If the current carried by resistor A is $I$, then what is the current carried by resistor B? (a) I, (b) 2I, (c) I/2, (d) 4I, (e) I/4

Picture the Problem Ohm's law states that the current in a resistor is proportional to the potential drop across the resistor. Because the potential difference across resistors A and connected in parallel is the same for each resistor, the resistor with half the resistance of the other resistor will carry twice the current carried by the resistor with the larger resistance. Because resistor A has twice the resistance of resistor B, resistor B will carry twice the current of resistor A. (b) is correct.

17 - A series combination consisting of resistors $A$ and $B$ is connected across the terminals of a battery. The resistor A has twice the resistance of resistor B. If the current carried by resistor A is $I$, then what is the current carried by resistor B? (a) I, (b) $2 I$, (c) $I / 2$, (d) $4 I$, (e) $I / 4$

Determine the Concept In a series circuit, because there are no alternative pathways, all resistors carry the same current. $(a)$ is correct.

18 - Kirchhoff's junction rule is considered to be a consequence of (a) conservation of charge, (b) conservation of energy, (c) Newton's laws, (d) Coulomb's law, (e) quantization of charge.

Determine the Concept While Kirchhoff's junction rule is a statement about current, recall that current is the rate at which charge passes some point in space. Hence, the junction rule is actually a statement that charge is conserved. (a) is correct.

19 - True or False:
(a) An ideal voltmeter has a zero internal resistance.
(b) An ideal ammeter has a zero internal resistance.
(c) An ideal voltage source has a zero internal resistance
(a) False. An ideal voltmeter would have infinite resistance. A voltmeter consists of a galvanometer movement connected in series with a large resistance. The large resistor accomplishes two purposes; 1) it protects the galvanometer movement by limiting the current drawn by it, and 2) minimizes the loading of the circuit by the voltmeter by placing a large resistance in parallel with the circuit element across which the potential difference is being measured.
(b) True. An ideal ammeter would have zero resistance. An ammeter consists of a
very small resistance in parallel with a galvanometer movement. The small resistance accomplishes two purposes: 1) It protects the galvanometer movement by shunting most of the current in the circuit around the galvanometer movement, and 2) It minimizes the loading of the circuit by the ammeter by minimizing the resistance of the ammeter.
(c) True. An ideal voltage source would have zero internal resistance. The terminal potential difference of a voltage source is given by $V=E-I r$, where $\varepsilon$ is the emf of the source, $I$ is the current drawn from the source, and $r$ is the internal resistance of the source.

20 - Before you and your classmates run an experiment, your professor lectures about safety. She reminds you that to measure the voltage across a resistor you connect a voltmeter in parallel with the resistor, and to measure the current in a resistor you connect an ammeter in series with the resistor. She also states that connecting a voltmeter in series with a resistor will not measure the voltage across the resistor, but also cannot do any damage to the circuit or the instrument. In addition, connecting an ammeter in parallel with a resistor will not measure the current in the resistor, but could cause significant damage to the circuit and the instrument. Explain why connecting a voltmeter in series with a resistor causes no damage while connecting an ammeter in parallel with a resistor can cause significant damage.

Determine the Concept Because of the voltmeter's high resistance, if you connect a voltmeter in series with a circuit element, the current, both in the voltmeter and in the rest of the circuit, will be very small. This means that there is little chance of heating the voltmeter and causing damage. However, because of the ammeter's low resistance, if you connect an ammeter in parallel with a circuit element, the current, both in the ammeter and in the entire circuit, excluding any elements in parallel with the ammeter, will be very large. This means that there is a good chance of overheating and causing damage, maybe even a fire. For this reason, ammeters are often equipped with fuses or circuit breakers.

21 - The capacitor in Figure 25-49 is initially uncharged. Just after the switch S is closed, (a) the voltage across $C$ equals $\mathcal{E},(b)$ the voltage across $R$ equals $\mathcal{E},(c)$ the current in the circuit is zero, $(d)$ both $(a)$ and $(c)$ are correct.

Determine the Concept If we apply Kirchhoff's loop rule with the switch closed, we obtain $\mathcal{E}-I R-V_{\mathrm{C}}=0$. Immediately after the switch is closed, $I=I_{\max }$ and $V_{C}=0$. Hence $\mathcal{E}=I_{\max } R .(b)$ is correct.

22 •• A capacitor is discharging through a resistor. If it takes a time $T$ for the charge on a capacitor to drop to half its initial value, how long (in terms of $T$ ) does it take for the stored energy to drop to half its initial value?

Picture the Problem We can express the variation of charge on the discharging capacitor as a function of time to find the time $T$ it takes for the charge on the capacitor to drop to half its initial value. We can also express the energy remaining in the electric field of the discharging capacitor as a function of time and find the time $t^{\prime}$ for the energy to drop to half its initial value in terms of $T$.

Express the time-dependence of the charge stored on a capacitor:

For $Q(t)=\frac{1}{2} Q_{0}$ :

Take the natural logarithm of both sides of the equation and solve for $T$ to obtain:

Express the dependence of the

$$
Q(t)=Q_{0} e^{-t / \tau}
$$

where $\tau=R C$.

$$
\frac{1}{2} Q_{0}=Q_{0} e^{-T / \tau} \text { or } \frac{1}{2}=e^{-T / \tau}
$$

$$
T=\tau \ln (2)
$$

energy stored in a capacitor on the potential difference $V_{\mathrm{C}}$ across its terminals:

Express the potential difference across a discharging capacitor as a function of time:

Substitute for $V_{\mathrm{C}}$ in equation (1) and simplify to obtain:

$$
\begin{aligned}
U(t) & =\frac{1}{2} C\left(V_{0} e^{-t / R C}\right)^{2}=\frac{1}{2} C V_{0}^{2} e^{-2 t / R C} \\
& =U_{0} e^{-2 t / R C}
\end{aligned}
$$

For $\boldsymbol{U}(\boldsymbol{t})=\frac{1}{2} \boldsymbol{U}_{0}$ :

Take the natural logarithm of both sides of the equation and solve for $t^{\prime}$

$$
\begin{equation*}
U(t)=\frac{1}{2} C V_{\mathrm{C}}^{2} \tag{1}
\end{equation*}
$$

$$
V_{\mathrm{C}}=V_{0} e^{-t / R C}
$$

to obtain:

$$
\frac{1}{2} U_{0}=U_{0} e^{-2 t^{\prime} / R C} \Rightarrow \frac{1}{2}=e^{-2 t^{\prime} / R C}
$$

$$
t^{\prime}=\frac{1}{2} \tau \ln (2)=\frac{1}{2} T
$$

23 •• [SSM] In Figure 25-50, the values of the resistances are related as follows: $R_{2}=R_{3}=2 R_{1}$. If power $P$ is delivered to $R_{1}$, what is the power delivered to $R_{2}$ and $R_{3}$ ?

Determine the Concept The power delivered to a resistor varies with the square of the current in the resistor and is directly proportional to the resistance of the resistor ( $\left.\boldsymbol{P}=\boldsymbol{I}^{2} \boldsymbol{R}\right)$. The current in either resistor 2 or 3 is half the current in resistor 1 , and the resistance of either 2 or 3 is half that of resistor 1 . Hence the
power delivered to either resistor 2 or 3 is one eighth the power delivered to resistor 1 .

24 •• The capacitor in Figure 25-49 is initially uncharged. The switch $S$ is closed and remains closed for a very long time. During this time, (a) the energy supplied by the battery is $\frac{1}{2} \boldsymbol{C} \boldsymbol{\mathcal { E }}^{2},(b)$ the energy dissipated in the resistor is $\frac{1}{2} \boldsymbol{C} \boldsymbol{\mathcal { E }}^{2}$, (c) energy in the resistor is dissipated at a constant rate, (d) the total charge passing through the resistor is $\frac{1}{2} \boldsymbol{C} \mathcal{E}$.

Determine the Concept The energy stored in the fully charged capacitor is $\boldsymbol{U}=\frac{1}{2} \boldsymbol{C} \boldsymbol{\varepsilon}^{2}$. During the charging process, a total charge $Q_{\mathrm{f}}=\boldsymbol{\varepsilon} C$ flows through the battery. The battery therefore does work $W=Q_{\mathrm{f}} \boldsymbol{\varepsilon}=C \boldsymbol{\varepsilon}^{2}$. The energy dissipated in the resistor is the difference between $W$ and $U . \boxed{(b)}$ is correct.

## Estimation and Approximation

25 •• It is not a good idea to stick the ends of a paper clip into the two rectangular slots of a household electrical wall outlet in the United States.
Explain why by estimating the current that a paper clip would carry until either the fuse blows or the breaker trips.

Picture the Problem The current drawn by the paper clip is the ratio of the potential difference $(120 \mathrm{~V})$ between the two holes and the resistance of the paper clip. We can use $R=\rho L / A$ to find the resistance of the paper clip. Paper clips are made of steel, which has a resistivity in the range 10 to $100 \times 10^{-8} \Omega \cdot \mathrm{~m}$,

From the definition of resistance: $\quad I=\frac{\mathcal{E}}{R}$

The resistance of the paper clip is given by:

$$
R=\rho \frac{L}{A}
$$

Substituting for $R$ in the expression for $I$ and simplifying yields

$$
I=\frac{\varepsilon}{\rho \frac{L}{A}}=\frac{\varepsilon A}{\rho L}=\frac{\varepsilon \pi r^{2}}{\rho L}=\frac{\varepsilon \pi d^{2}}{4 \rho L}
$$

Assuming that the length of a paper clip is 10 cm and that its diameter is

$$
I=\frac{(120 \mathrm{~V}) \pi(1.0 \mathrm{~mm})^{2}}{4\left(50 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(10 \mathrm{~cm})}=1.9 \mathrm{kA}
$$ 1.0 mm , substitute numerical values and evaluate $I$ :

26 •• (a) Estimate the resistance of an automobile jumper cable. (b) Look up the current required to start a typical car. At that current, what is the potential drop that occurs across the jumper cable? (c) How much power is dissipated in the jumper cable when it carries this current?

Picture the Problem (a) We can use the definition of resistivity to find the resistance of the jumper cable. In Part (b), the application of Ohm's law will yield the potential difference across the jumper cable when it is starting a car, and, in Part (c), we can use the expression for the power dissipated in a conductor to find the power dissipation in the jumper cable.
$\begin{aligned} & \text { (a) Noting that a jumper cable has } \\ & \text { two leads, express the resistance of }\end{aligned} \quad R=\rho \frac{L}{A}$
the cable in terms of the wire's
resistivity and the cable's length, and cross-sectional area:

Assuming the length of the jumper cables to be 3.0 m and its crosssectional area to be $10 \mathrm{~mm}^{2}$, substitute numerical values (see

$$
\begin{aligned}
R & =\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{3.0 \mathrm{~m}}{10 \mathrm{~mm}^{2}} \\
& =5.1 \mathrm{~m} \Omega
\end{aligned}
$$

Table 25-1 for the resistivity of copper) and evaluate $R$ :
(b) Apply Ohm's law to the cable with a starting current of 90 A to obtain:
(c) Use the expression for the power dissipated in a conductor to obtain:

27 •• Your manager wants you to design a new super-insulated hot-water heater for the residential market. A coil of Nichrome wire is to be used as the heating element. Estimate the length of wire required. HINT: You will need to determine the size of a typical hot water heater and a reasonable time period for creating hot water.

Picture the Problem We can combine the expression for the rate at which energy is delivered to the water to warm it $\left(P=\varepsilon^{2} / R\right)$ and the expression for the resistance of a conductor ( $R=\rho L / A$ ) to obtain an expression for the required length $L$ of Nichrome wire.

From Equation 25-10, the length of Nichrome wire required is given by:

Use an expression for the power dissipated in a resistor to relate the required resistance to rate at which energy is delivered to generate the warm water:

Substituting for $R$ and simplifying yields:

The power required to heat the water is the rate at which the wire will deliver energy to the water:

Substitute for $P$ to obtain:

The mass of water to be heated is the product of its density and the volume of the water tank:

Finally, substituting for $m$ and simplifying yields:

$$
L=\frac{\mathcal{E}^{2} A}{\rho_{\mathrm{Ni}} m c \frac{\Delta T}{\Delta t}}
$$

$L=\frac{R A}{\rho_{\mathrm{Ni}}}$ where $A$ is the cross-sectional area of the wire.

$$
P=\frac{\varepsilon^{2}}{R} \Rightarrow R=\frac{\varepsilon^{2}}{P}
$$

$$
L=\frac{\varepsilon^{2} A}{\rho_{\mathrm{Ni}} P}
$$

$$
P=\frac{\Delta E}{\Delta t}=\frac{\Delta(m c \Delta T)}{\Delta t}=m c \frac{\Delta T}{\Delta t}
$$

$$
m=\rho_{\mathrm{H}_{2} \mathrm{O}} V=\rho_{\mathrm{H}_{2} \mathrm{O}} A^{\prime} d
$$

where $A^{\prime}$ is the cross-sectional area of the water tank.

$$
\begin{aligned}
L & =\frac{\varepsilon^{2} A}{\rho_{\mathrm{Ni}} \rho_{\mathrm{H}_{2} \mathrm{O}} A^{\prime} d c \frac{\Delta T}{\Delta t}} \\
& =\frac{\varepsilon^{2} \pi r_{\text {wire }}^{2} \Delta t}{\rho_{\mathrm{Ni}} \rho_{\mathrm{H}_{2} \mathrm{O}} \pi r_{\text {tank }}^{2} d c \Delta T} \\
& =\frac{\varepsilon^{2} r_{\text {wire }}^{2} \Delta t}{\rho_{\mathrm{Ni}} \rho_{\mathrm{H}_{2} \mathrm{O}} r_{\text {tank }}^{2} d c \Delta T}
\end{aligned}
$$

The diameter of a typical 40 -gal water heater is about 50 cm and its height is approximately 1.3 m . Assume that the diameter of the Nichrome wire is 2.0 mm and that the water, initially at $20^{\circ} \mathrm{C}$, is to be heated to $80^{\circ} \mathrm{C}$ in 1.0 h . Substitute numerical values (see Table 25-1 for the resistivity of Nichrome and Table 18-1 for the heat capacity of water) and evaluate $L$ :

$$
\boldsymbol{L}=\frac{(120 \mathrm{~V})^{2}\left(2.0 \times 10^{-3} \mathrm{~m}\right)^{2}\left(1.0 \mathrm{~h} \times \frac{3600 \mathrm{~s}}{\mathrm{~h}}\right)}{\left(100 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(1000 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)(0.25 \mathrm{~m})^{2}(0.50 \mathrm{~m})\left(4.18 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{C}^{\circ}}\right)\left(60 \mathrm{C}^{\circ}\right)}=26 \mathrm{~m}
$$

28 •• A compact fluorescent light bulb costs about $\$ 6.00$ each and has a typical lifetime of 10000 h . These bulbs use 20 W of power, but produce illumination equivalent to that of $75-\mathrm{W}$ incandescent bulbs. An incandescent bulb costs about $\$ 1.50$ and has a typical lifetime of 1000 h . Your family wonders whether it should buy fluorescent light bulbs. Estimate the amount of money your household would save each year by using compact fluorescent light bulbs instead of the incandescent bulbs.

Picture the Problem We can find the annual savings by taking into account the costs of the two types of bulbs, the rate at which they consume energy and the cost of that energy, and their expected lifetimes. We'll assume that the household lights are on for an average of 8 hours per day.

Express the yearly savings: $\quad \Delta \$=\operatorname{Cost}_{\text {incandescent }}-$ Cost $_{\text {fluorescent }}$

Express the annual costs with the incandescent and fluorescent bulbs:

$$
\operatorname{Cost}_{\text {incandescent }}=\text { Cost }_{\text {bulbs }}+\operatorname{Cost}_{\text {energy }}
$$

and

$$
\operatorname{Cost}_{\text {fluorescent }}=\operatorname{Cost}_{\text {bulbs }}+\operatorname{Cost}_{\text {energy }}
$$

The annual cost of the incandescent bulbs is the product of the number of bulbs in use, the annual consumption of bulbs, and the cost per bulb:

$$
\text { Cost }_{\text {bulbs }}=(6)\left(\frac{365.24 \mathrm{~d} \times \frac{8 \mathrm{~h}}{\mathrm{~d}}}{1000 \mathrm{~h}}\right)(\$ 1.50)=\$ 26.30
$$

The cost of operating the incandescent bulbs for one year is the product of the energy consumed and the cost per unit of energy:

$$
\text { Cost }_{\text {energy }}=6(75 \mathrm{~W})\left(365.24 \frac{\mathrm{~d}}{\mathrm{y}}\right)\left(\frac{8 \mathrm{~h}}{\mathrm{~d}}\right)\left(\frac{\$ 0.115}{\mathrm{~kW} \cdot \mathrm{~h}}\right)=\$ 151.21
$$

The annual cost of the fluorescent bulbs is the product of the number of bulbs in use, the annual consumption of bulbs, and the cost per bulb:

$$
\text { Cost }_{\text {bulbs }}=(6)\left(\frac{365.24 \mathrm{~d} \times \frac{8 \mathrm{~h}}{\mathrm{~d}}}{10000 \mathrm{~h}}\right)(\$ 6)=\$ 10.52
$$

The cost of operating the fluorescent bulbs for one year is the product of the energy consumed and the cost per unit of energy:

$$
\operatorname{Cost}_{\text {energy }}=6(20 \mathrm{~W})\left(365.24 \mathrm{~d} \times \frac{8 \mathrm{~h}}{\mathrm{~d}}\right)\left(\frac{\$ 0.115}{\mathrm{~kW} \cdot \mathrm{~h}}\right)=\$ 40.32
$$

Substitute in equation (1) and evaluate the cost savings $\Delta \$$ :

$$
\begin{aligned}
\Delta \$ & =\text { Cost }_{\text {incandescent }}-\text { Cost }_{\text {fluorescent }}=(\$ 26.30+\$ 151.20)-(\$ 10.52+\$ 40.32) \\
& =\$ 127
\end{aligned}
$$

29 •• The wires in a house must be large enough in diameter so that they do not get hot enough to start a fire. While working for a building contractor during the summer, you are involved in remodeling a house. The local building code states that the joule heating of the wire used in houses should not exceed 2.0 $\mathrm{W} / \mathrm{m}$. Estimate the maximum gauge of the copper wire that you can use during the rewiring of the house with 20-A circuits.

Picture the Problem We can use an expression for the power dissipated in a resistor to relate the Joule heating in the wire to its resistance and the definition of resistivity to relate the resistance to the length and cross-sectional area of the wire. We can find the diameter of the wire from its cross-sectional area and then use Table 25-2 to find the maximum gauge you can use during the rewiring of the house.

Express the power the wires must $\quad P=I^{2} R$ dissipate in terms of the current they carry and their resistance:

Divide both sides of the equation by
$L$ to express the power dissipation

$$
\frac{P}{L}=\frac{I^{2} R}{L}
$$ per unit length:

Using the definition of resistivity, relate the resistance of the wire to its resistivity, length and cross-sectional area:

Substitute for $R$ to obtain:

$$
R=\rho \frac{L}{A}=\rho \frac{L}{\frac{\pi}{4} d^{2}}=\frac{4 \rho L}{\pi d^{2}}
$$

$$
\begin{aligned}
& \frac{P}{L}=\frac{4 \rho I^{2}}{\pi d^{2}} \Rightarrow d=2 I \sqrt{\frac{\rho}{\pi(P / L)}} \\
& d=2(20 \mathrm{~A}) \sqrt{\frac{1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}}{\pi(2.0 \mathrm{~W} / \mathrm{m})}}=2.1 \mathrm{~mm}
\end{aligned}
$$

Substitute numerical values (see Table 25-1 for the resistivity of copper wire) and evaluate $d$ :

Consulting Table 25-2, we note that the maximum gauge you can use is 12 .

30 •• A laser diode used in making a laser pointer is a highly nonlinear circuit element. Its behavior is as follows: for any voltage drop across it that is less than about 2.30 V , it behaves as if it has an infinite internal resistance, but for voltages higher than 2.30 V it has a very low internal resistance-effectively zero. (a) A laser pointer is made by putting two 1.55 V watch batteries in series across the laser diode. If the batteries each have an internal resistance between $1.00 \Omega$ and $1.50 \Omega$, estimate the current in the laser beam. (b) About half of the power delivered to the laser diode goes into radiant energy. Using this fact, estimate the power of the laser beam, and compare this value to typical quoted values of about 3.00 mW . (c) If the batteries each have a capacity of $20.0-\mathrm{mA} \cdot \mathrm{h}$ (i.e., they can deliver a constant current of 20.0 mA for approximately one hour before discharging), estimate how long one can continuously operate the laser pointer before replacing the batteries.

Picture the Problem Let $r$ be the internal resistance of each battery and apply Kirchhoff's loop rule to the circuit consisting of the two batteries and the laser diode to relate the current in laser diode to $r$ and the potential differences across the batteries and the diode. We can find the power of the laser diode from the product of the potential difference across the internal resistance of the batteries and the current $I$ delivered by them and the time-to-discharge from the combined capacities of the two batteries and $I$.
(a) Apply Kirchhoff's loop rule to the circuit consisting of the two batteries and the laser diode to obtain:

Solving for $I$ yields:

$$
I=\frac{\varepsilon-V_{\text {laser }} \text { diode }}{\text { dit }}
$$

Assuming that $r=125 \Omega$, substitute numerical values and evaluate $I$ :

$$
\begin{aligned}
I & =\frac{2(1.55)-2.30 \mathrm{~V}}{2(125 \Omega)}=3.20 \mathrm{~mA} \\
& =3.2 \mathrm{~mA}
\end{aligned}
$$

(b) The power delivered by the

$$
P=I V=(3.20 \mathrm{~mA})(2.30 \mathrm{~V})=7.36 \mathrm{~mW}
$$ batteries is given by:

The power of the laser is half this value:

$$
\begin{aligned}
P_{\text {laser }} & =\frac{1}{2} P=\frac{1}{2}(7.36 \mathrm{~mW})=3.68 \mathrm{~mW} \\
& =3.7 \mathrm{~mW}
\end{aligned}
$$

Express the ratio of $P_{\text {laser }}$ to $P_{\text {quoted }}$ :

$$
\begin{aligned}
& \frac{P_{\text {laser }}}{P_{\text {quoted }}}=\frac{3.68 \mathrm{~mW}}{3.00 \mathrm{~mW}}=1.23 \\
& \text { or } \\
& P_{\text {laser }}=1.23 P_{\text {quoted }}
\end{aligned}
$$

(c) Express the time-to-discharge:

$$
\Delta t_{\text {discharge }}=\frac{\text { Capacity }}{I}
$$

Because each battery has a capacity of $20.0 \mathrm{~mA} \cdot \mathrm{~h}$, the series combination

$$
\Delta t_{\text {discharge }}=\frac{40.0 \mathrm{~mA} \cdot \mathrm{~h}}{3.20 \mathrm{~mA}} \approx 12.5 \mathrm{~h}
$$ has a capacity of $40.0 \mathrm{~mA} \cdot \mathrm{~h}$ and:

## Current, Current Density, Drift Speed and the Motion of Charges

31 - [SSM] A 10-gauge copper wire carries a current equal to 20 A. Assuming copper has one free electron per atom, calculate the drift speed of the free electrons in the wire.

Picture the Problem We can relate the drift velocity of the electrons to the current density using $I=n e v_{\mathrm{d}} A$. We can find the number density of charge carriers $n$ using $n=\rho N_{\mathrm{A}} / M$, where $\rho$ is the mass density, $N_{\mathrm{A}}$ Avogadro's number, and $M$ the molar mass. We can find the cross-sectional area of 10-gauge wire in Table 25-2.

Use the relation between current and drift velocity to relate $I$ and $n$ :

$$
I=n e v_{\mathrm{d}} A \Rightarrow v_{\mathrm{d}}=\frac{I}{n e A}
$$

The number density of charge carriers $n$ is related to the mass

$$
n=\frac{\rho N_{\mathrm{A}}}{M}
$$

density $\rho$, Avogadro's number $N_{\mathrm{A}}$, and the molar mass $M$ :

For copper, $\rho=8.93 \mathrm{~g} / \mathrm{cm}^{3}$ and $M=63.55 \mathrm{~g} / \mathrm{mol}$. Substitute and evaluate $n$ :

$$
\begin{aligned}
\boldsymbol{n} & =\frac{\left(8.93 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(6.022 \times 10^{23} \text { atoms } / \mathrm{mol}\right)}{63.55 \mathrm{~g} / \mathrm{mol}} \\
& =8.459 \times 10^{28} \text { atoms } / \mathrm{m}^{3}
\end{aligned}
$$

Using Table 25-2, find the cross- $\quad A=5.261 \mathrm{~mm}^{2}$ sectional area of 10 -gauge wire:

Substitute numerical values and evaluate $v_{\mathrm{d}}$ :

$$
\boldsymbol{v}_{\mathrm{d}}=\frac{20 \mathrm{~A}}{\left(8.459 \times 10^{28} \mathrm{~m}^{-3}\right)\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(5.261 \mathrm{~mm}^{2}\right)}=0.28 \mathrm{~mm} / \mathrm{s}
$$

$32 \quad$ A thin nonconducting ring that has a radius $a$ and a linear charge density $\lambda$ rotates with angular speed $\omega$ about an axis through its center and perpendicular to the plane of the ring. Find the current of the ring.

Picture the Problem We can use the definition of current, the definition of charge density, and the relationship between period and frequency to derive an expression for the current as a function of $a, \lambda$, and $\omega$.

Use the definition of current to relate the charge $\Delta Q$ associated with a

$$
I=\frac{\Delta Q}{\Delta t}
$$ segment of the ring to the time $\Delta t$ it takes the segment to pass a given point:

Because each segment carries a charge $\Delta Q$ and the time for one

$$
\begin{equation*}
I=\frac{\Delta Q}{T}=\Delta Q f \tag{1}
\end{equation*}
$$ revolution is $T$ :

Use the definition of the charge density $\lambda$ to relate the charge $\Delta Q$ to

$$
\lambda=\frac{\Delta Q}{2 \pi a} \Rightarrow \Delta Q=2 \pi a \lambda
$$

the radius $a$ of the ring:
Substitute for $\Delta Q$ in equation (1) to

$$
I=2 \pi a \lambda f
$$

obtain:

Because $\omega=2 \pi f: \quad I=a \lambda \omega$
33 •• [SSM] A length of 10-gauge copper wire and a length of 14-gauge copper wire are welded together end to end. The wires carry a current of 15 A . (a) If there is one free electron for each copper atom in each wire, find the drift speed of the electrons in each wire. (b) What is the ratio of the magnitude of the current density in the length of10-gauge wire to the magnitude of the current density in the length of 14 -gauge wire?

Picture the Problem (a) The current will be the same in the two wires and we can relate the drift velocity of the electrons in each wire to their current densities and the cross-sectional areas of the wires. We can find the number density of charge carriers $n$ using $n=\rho N_{\mathrm{A}} / M$, where $\rho$ is the mass density, $N_{\mathrm{A}}$ Avogadro's number, and $M$ the molar mass. We can find the cross-sectional area of 10- and 14 -gauge wires in Table 25-2. In Part (b) we can use the definition of current density to find the ratio of the magnitudes of the current densities in the 10 -gauge and 14-gauge wires.
(a) Relate the current density to the drift velocity of the electrons in the 10 -gauge wire:

$$
\frac{I_{10 \text { gauge }}}{A_{10 \text { gauge }}}=n e v_{\mathrm{d}, 10} \Rightarrow v_{\mathrm{d}, 10}=\frac{I_{10 \text { gauge }}}{n e A_{10 \text { gauge }}}
$$

The number density of charge carriers $n$ is related to the mass density $\rho$,

$$
n=\frac{\rho N_{\mathrm{A}}}{M}
$$

Avogadro's number $N_{\mathrm{A}}$, and the molar mass $M$ :

For copper, $\rho=8.93 \mathrm{~g} / \mathrm{cm}^{3}$ and $M=63.55 \mathrm{~g} / \mathrm{mol}$. Substitute
numerical values and evaluate $n$ :

Use Table 25-2 to find the cross-

$$
\begin{aligned}
n & =\frac{\left(8.93 \frac{\mathrm{~g}}{\mathrm{~cm}^{3}}\right)\left(6.022 \times 10^{23} \frac{\mathrm{atoms}}{\mathrm{~mol}}\right)}{63.55 \frac{\mathrm{~g}}{\mathrm{~mol}}} \\
& =8.462 \times 10^{28} \frac{\text { atoms }}{\mathrm{m}^{3}}
\end{aligned}
$$

sectional area of 10 -gauge wire:

Substitute numerical values and evaluate $v_{\text {d. } 10}$ :

$$
v_{\mathrm{d}, 10}=\frac{15 \mathrm{~A}}{\left(8.462 \times 10^{28} \mathrm{~m}^{-3}\right)\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(5.261 \mathrm{~mm}^{2}\right)}=0.210 \mathrm{~mm} / \mathrm{s}=0.21 \mathrm{~mm} / \mathrm{s}
$$

Express the continuity of the current in the two wires:

Solve for $v_{\mathrm{d}, 14}$ to obtain:

$$
I_{10 \text { gauge }}=I_{14 \text { gauge }}
$$

or
$n e v_{\mathrm{d}, 10} A_{10 \text { gauge }}=n e v_{\mathrm{d}, 14} A_{14 \text { gauge }}$

$$
v_{\mathrm{d}, 14}=v_{\mathrm{d}, 10} \frac{A_{10 \text { gauge }}}{A_{14 \text { gauge }}}
$$

Use Table 25-2 to find the crosssectional area of 14 -gauge wire: Substitute numerical values and evaluate $v_{\mathrm{d}, 14}$ :

$$
\begin{aligned}
A_{14} & =2.081 \mathrm{~mm}^{2} \\
v_{\mathrm{d}, 14} & =(0.210 \mathrm{~mm} / \mathrm{s}) \frac{5.261 \mathrm{~mm}^{2}}{2.081 \mathrm{~mm}^{2}} \\
& =0.53 \mathrm{~mm} / \mathrm{s}
\end{aligned}
$$

(b) The ratio of current density, 10gauge wire to 14 -gauge wire, is given by:

$$
\frac{J_{10}}{J_{14}}=\frac{\frac{I_{10}}{A_{10}}}{\frac{I_{14}}{A_{14}}}=\frac{I_{10} A_{14}}{I_{14} A_{10}}
$$

Because $I_{10}=I_{14}$ :

$$
\frac{J_{10}}{J_{14}}=\frac{A_{14}}{A_{10}}
$$

Substitute numerical values and evaluate $\frac{J_{10}}{J_{14}}$ :

$$
\frac{J_{10}}{J_{14}}=\frac{2.081 \mathrm{~mm}^{2}}{5.261 \mathrm{~mm}^{2}}=0.396
$$

34 •• An accelerator produces a beam of protons with a circular cross section that is 2.0 mm in diameter and has a current of 1.0 mA . The current density is uniformly distributed through the beam. The kinetic energy of each proton is 20 MeV . The beam strikes a metal target and is absorbed by the target. (a) What is the number density of the protons in the beam? (b) How many protons strike the target each minute? (c) What is the magnitude of the current density in this beam?

Picture the Problem We can use $I=n e A v$ to relate the number $n$ of protons per unit volume in the beam to current $I$. We can find the speed of the particles in the beam from their kinetic energy. In Part ( $b$ ) we can express the number of protons $N$ striking the target per unit time as the product of the number of protons per unit volume $n$ in the beam and the volume of the cylinder containing those protons that will strike the target in an elapsed time $\Delta t$ and solve for $N$. Finally, we can use the definition of current to express the charge arriving at the target as a function of time.
(a) Use the relation between current and drift velocity (Equation 25-3) to

$$
I=n e A v \Rightarrow n=\frac{I}{e A v}
$$

relate $I$ and $n$ :

The kinetic energy of the protons is given by:

$$
K=\frac{1}{2} m_{\mathrm{p}} v^{2} \Rightarrow v=\sqrt{\frac{2 K}{m_{\mathrm{p}}}}
$$

Relate the cross-sectional area $A$

$$
A=\frac{1}{4} \pi D^{2}
$$

of the beam to its diameter $D$ :
Substitute for $v$ and $A$ and simplify to obtain:

$$
n=\frac{I}{\frac{1}{4} \pi e D^{2} \sqrt{\frac{2 K}{m_{\mathrm{p}}}}}=\frac{4 I}{\pi e D^{2}} \sqrt{\frac{m_{\mathrm{p}}}{2 K}}
$$

Substitute numerical values and evaluate $n$ :

$$
\begin{aligned}
n & =\frac{4(1.0 \mathrm{~mA})}{\pi\left(1.602 \times 10^{-19} \mathrm{C}\right)(2 \mathrm{~mm})^{2}} \sqrt{\frac{1.673 \times 10^{-27} \mathrm{~kg}}{2(20 \mathrm{MeV})\left(1.602 \times 10^{-19} \mathrm{~J} / \mathrm{eV}\right)}}=3.21 \times 10^{13} \mathrm{~m}^{-3} \\
& =3.2 \times 10^{13} \mathrm{~m}^{-3}
\end{aligned}
$$

(b) Express the number of protons $N$ striking the target per unit time as the

$$
\frac{N}{\Delta t}=n(v A) \Rightarrow N=n v A \Delta t
$$

product of the number $n$ of protons
per unit volume in the beam and the
volume of the cylinder containing those protons that will strike the target in an elapsed time $\Delta t$ and solve for $N$ :

Substitute for $v$ and $A$ to obtain:

$$
N=\frac{1}{4} \pi D^{2} n \Delta t \sqrt{\frac{2 K}{m_{\mathrm{p}}}}
$$

Substitute numerical values and evaluate $N$ :

$$
\begin{aligned}
N & =\frac{1}{4} \pi(2 \mathrm{~mm})^{2}\left(3.21 \times 10^{13} \mathrm{~m}^{-3}\right)(1 \mathrm{~min}) \sqrt{\frac{2(20 \mathrm{MeV})\left(1.602 \times 10^{-19} \mathrm{~J} / \mathrm{eV}\right)}{1.673 \times 10^{-27} \mathrm{~kg}}} \\
& =3.7 \times 10^{17}
\end{aligned}
$$

(c) The magnitude of the current density in this beam is given by:

$$
\begin{aligned}
J & =\frac{I}{A}=\frac{1.0 \mathrm{~mA}}{\pi\left(1.0 \times 10^{-3} \mathrm{~m}\right)^{2}} \\
& =0.32 \mathrm{kA} / \mathrm{m}^{2}
\end{aligned}
$$

35 •• [SSM] In one of the colliding beams of a planned proton supercollider, the protons are moving at nearly the speed of light and the beam current is $5.00-\mathrm{mA}$. The current density is uniformly distributed throughout the beam. (a) How many protons are there per meter of length of the beam? (b) If the cross-sectional area of the beam is $1.00 \times 10^{-6} \mathrm{~m}^{2}$, what is the number density of protons? (c) What is the magnitude of the current density in this beam?

Picture the Problem We can relate the number of protons per meter $N$ to the number $n$ of free charge-carrying particles per unit volume in a beam of crosssectional area $A$ and then use the relation between current and drift velocity to relate $n$ to $I$.
(a) Express the number of protons $\quad N=n A$
per meter $N$ in terms of the number
$n$ of free charge-carrying particles
per unit volume in a beam of cross-
sectional area $A$ :

Use the relation between current and drift velocity to relate $I$ and $n$ :

$$
I=e n A v \Rightarrow n=\frac{I}{e A v}
$$

Substitute for $n$ and simplify to obtain:

$$
N=\frac{I A}{e A v}=\frac{I}{e v}
$$

Substitute numerical values and evaluate $N$ :
(b) From equation (1) we have:
(c) The magnitude of the current density in this beam is given by:

$$
\begin{aligned}
N & =\frac{5.00 \mathrm{~mA}}{\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)} \\
& =1.041 \times 10^{8} \mathrm{~m}^{-1} \\
& =1.04 \times 10^{8} \mathrm{~m}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
n & =\frac{N}{A}=\frac{1.041 \times 10^{8} \mathrm{~m}^{-1}}{1.00 \times 10^{-6} \mathrm{~m}^{2}} \\
& =1.04 \times 10^{14} \mathrm{~m}^{-3}
\end{aligned}
$$

$$
J=\frac{I}{A}=\frac{5.00 \mathrm{~mA}}{1.00 \times 10^{-6} \mathrm{~m}^{2}}=5.00 \mathrm{kA} / \mathrm{m}^{2}
$$

36 •• The solar wind consists of protons from the Sun moving toward Earth ( the wind actually consists of about $95 \%$ protons). The number density of protons at a distance from the Sun equal to the orbital radius of Earth is about 7.0 protons per cubic centimeter. Your research team monitors a satellite that is in orbit around the Sun at a distance from the Sun equal to equal to Earth's orbital radius. You are in charge of the satellite's mass spectrometer, an instrument used to measure the composition and intensity of the solar wind. The aperture of your spectrometer is a circle of radius 25 cm . The rate of collection of protons by the spectrometer is such that they constitute a measured current of 85 nA . What is the speed of the protons in the solar wind? (Assume the protons enter the aperture at normal incidence.)

Picture the Problem We can use Equation 25-3 to express the drift speed of the protons in the solar wind in terms of the proton current, the number density of the protons, and the cross-sectional area of the aperture of the spectrometer.

From Equation 25-3, the drift speed of the protons in the solar winds is

$$
v_{\mathrm{d}}=\frac{I}{q n A}
$$

given by:
Substitute numerical values and evaluate $v_{\mathrm{d}}$ :

$$
\boldsymbol{v}_{\mathrm{d}}=\frac{85 \mathrm{nA}}{\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(\frac{7.0}{\mathrm{~cm}^{3}}\right) \pi(25 \mathrm{~cm})^{2}}=3.9 \times 10^{5} \mathrm{~m} / \mathrm{s}
$$

37 •• A gold wire has a $0.10-\mathrm{mm}$-diameter cross section. Opposite ends of this wire are connected to the terminals of a $1.5-\mathrm{V}$ battery. If the length of the wire is 7.5 cm , how much time, on average, is required for electrons leaving the
negative terminal of the battery to reach the positive terminal? Assume the resistivity of gold is $2.44 \times 10^{-8} \Omega \cdot \mathrm{~m}$.

Picture the Problem We can use the definition of average speed, Equation 25-3 ( $I=q n A v_{\mathrm{d}}$ ), Equation 25-10 $(R=\rho L / A)$ and the relationship between the number density of electrons, Avogadro's number, and the molar mass of gold to derive an expression for the average travel time of the electrons.

The average transit time for the electrons is the ratio of the distance they travel to their drift speed:

From Equation 25-3:

$$
v_{\mathrm{d}}=\frac{I}{q n A}
$$

Substituting for $v_{\mathrm{d}}$ and $I$ and simplifying yields:

Use Equation 25-10 to express the resistance of the gold wire:

Substitute for $R$ and simplify to obtain:

The number density of free electrons is given by:

$$
R=\rho_{\mathrm{Au}} \frac{L}{A}
$$

$$
\Delta t=\frac{q n A L \rho_{\mathrm{Au}} \frac{L}{A}}{V}=\frac{q n L^{2} \rho_{\mathrm{Au}}}{V}
$$

$$
n=\left(\frac{\text { mass }}{\text { molar mass }}\right)\left(\frac{1}{\text { volume }}\right) N_{\mathrm{A}}
$$

$$
=\left(\frac{\rho_{\text {density }, \mathrm{Au}}}{M_{\mathrm{mol}, \mathrm{Au}}}\right) N_{\mathrm{A}}=\left(\frac{N_{\mathrm{A}}}{M_{\mathrm{mol}, \mathrm{Au}}}\right) \rho_{\mathrm{density}, \mathrm{Au}}
$$

Finally, substituting for $n$ yields:

$$
\Delta t=\frac{q\left(\frac{N_{\mathrm{A}}}{M_{\mathrm{mol}, \mathrm{Au}}}\right) \rho_{\mathrm{density,Au}} L^{2} \rho_{\mathrm{Au}}}{V}
$$

Substitute numerical values and evaluate $\Delta t$ :

$$
\begin{gathered}
\Delta \boldsymbol{t}=\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(\frac{6.022 \times 10^{23} \mathrm{~mol}^{-1}}{0.19697 \frac{\mathrm{~kg}}{\mathrm{~mol}}}\right)\left(19.3 \times 10^{3} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)(7.5 \mathrm{~cm})^{2} \\
\times \frac{\left(2.44 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)}{1.5 \mathrm{~V}}=0.86 \mathrm{~s}
\end{gathered}
$$

## Resistance, Resistivity and Ohm's Law

38 - A 10-m-long wire has a resistance equal to $0.20 \Omega$ and carries a current equal to 5.0 A . (a) What is the potential difference across the entire length of the wire? (b) What is the electric-field strength in the wire?

Picture the Problem We can use Ohm's law to find the potential difference between the ends of the wire and $V=E L$ to find the magnitude of the electric field in the wire.
(a) Apply Ohm's law to obtain:

$$
\begin{aligned}
& V=R I=(0.20 \Omega)(5.0 \mathrm{~A})=1.0 \mathrm{~V} \\
& E=\frac{V}{L}=\frac{1.0 \mathrm{~V}}{10 \mathrm{~m}}=0.10 \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

(b) Relate the electric field to the potential difference across the wire and the length of the wire:

39 - [SSM] A potential difference of 100 V across the terminals of a resistor produces a current of 3.00 A in the resistor. (a) What is the resistance of the resistor? (b) What is the current in the resistor when the potential difference is only 25.0 V ? (Assume the resistance of the resistor remains constant.)

Picture the Problem We can apply Ohm's law to both parts of this problem, solving first for $R$ and then for $I$.
(a) Apply Ohm's law to obtain:

$$
R=\frac{V}{I}=\frac{100 \mathrm{~V}}{3.00 \mathrm{~A}}=33.3 \Omega
$$

(b) Apply Ohm's law a second time to obtain:

$$
I=\frac{V}{R}=\frac{25.0 \mathrm{~V}}{33.3 \Omega}=0.750 \mathrm{~A}
$$

40 - A block of carbon is 3.0 cm long and has a square cross-section whose sides are $0.50-\mathrm{cm}$ long. A potential difference of 8.4 V is maintained across its length. (a) What is the resistance of the block? (b) What is the current in this resistor?

Picture the Problem We can use $R=\rho L / A$ to find the resistance of the block and Ohm's law to find the current in it for the given potential difference across its length.
(a) Relate the resistance of the block to its resistivity $\rho$, cross-sectional

$$
R=\rho \frac{L}{A}
$$

area $A$, and length $L$ :

Substitute numerical values (see Table 25-1 for the resistivity of carbon) and evaluate $R$ :

$$
\begin{aligned}
R & =\left(3500 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{3.0 \mathrm{~cm}}{(0.50 \mathrm{~cm})^{2}} \\
& =42.0 \mathrm{~m} \Omega=42 \mathrm{~m} \Omega
\end{aligned}
$$

(b) Apply Ohm's law to obtain:

$$
I=\frac{V}{R}=\frac{8.4 \mathrm{~V}}{42.0 \mathrm{~m} \Omega}=0.20 \mathrm{kA}
$$

41 - [SSM] An extension cord consists of a pair of 30-m-long 16-gauge copper wires. What is the potential difference that must be applied across one of the wires if it is to carry a current of 5.0 A ?

Picture the Problem We can use Ohm's law in conjunction with $R=\rho L / A$ to find the potential difference across one wire of the extension cord.

Using Ohm's law, express the

$$
V=I R
$$

potential difference across one wire of the extension cord:

Relate the resistance of the wire to its resistivity $\rho$, cross-sectional area

$$
R=\rho \frac{L}{A}
$$

$A$, and length $L$ :

Substitute for $R$ to obtain:

$$
V=\rho \frac{L I}{A}
$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the crosssectional area of 16 -gauge wire) and

$$
\begin{aligned}
\boldsymbol{V} & =\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{(30 \mathrm{~m})(5.0 \mathrm{~A})}{1.309 \mathrm{~mm}^{2}} \\
& =1.9 \mathrm{~V}
\end{aligned}
$$ evaluate $V$ :

42 - (a) How long is a 14-gauge copper wire that has a resistance of $12.0 \Omega$ ? (b) How much current will it carry if a $120-\mathrm{V}$ potential difference is applied across its length?

Picture the Problem We can use $R=\rho L / A$ to find the length of a 14-gauge copper wire that has a resistance of $12.0 \Omega$ and Ohm's law to find the current in the wire.
(a) Relate the resistance of the wire to its resistivity $\rho$, cross-sectional

$$
R=\rho \frac{L}{A} \Rightarrow L=\frac{R A}{\rho}
$$

area $A$, and length $L$ :

Substitute numerical values (see Table 25-1 for the resistivity of

$$
\boldsymbol{L}=\frac{(12.0 \Omega)\left(2.081 \mathrm{~mm}^{2}\right)}{1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}}=1.5 \mathrm{~km}
$$ copper and Table 25-2 for the crosssectional area of 14 -gauge wire) and evaluate $L$ :

(b) Apply Ohm's law to find the current in the wire:

$$
I=\frac{V}{R}=\frac{120 \mathrm{~V}}{12.0 \Omega}=10.0 \mathrm{~A}
$$

43 - A cylinder of glass is 1.00 cm long and has a resistivity of $1.01 \times 10^{12}$ $\Omega \cdot \mathrm{m}$. What length of copper wire that has the same cross-sectional area will have the same resistance as the glass cylinder?

Picture the Problem We can use $R=\rho L / A$ to express the resistances of the glass cylinder and the copper wire. Expressing their ratio will eliminate the common cross-sectional areas and leave us with an expression we can solve for the length of the copper wire.

Relate the resistance of the glass cylinder to its resistivity, cross-

$$
R_{\text {glass }}=\rho_{\text {glass }} \frac{L_{\text {glass }}}{A_{\text {glass }}}
$$

sectional area, and length:

Relate the resistance of the copper wire to its resistivity, cross-sectional area, and length:

Divide the second of these equations
by the first to obtain:

$$
R_{\mathrm{Cu}}=\rho_{\mathrm{Cu}} \frac{L_{\mathrm{Cu}}}{A_{\mathrm{Cu}}}
$$ -

$$
\frac{R_{\mathrm{Cu}}}{R_{\text {glass }}}=\frac{\rho_{\mathrm{Cu}} \frac{L_{\mathrm{Cu}}}{A_{\mathrm{Cu}}}}{\rho_{\text {glass }} \frac{L_{\text {glass }}}{A_{\text {glass }}}}=\frac{\rho_{\mathrm{Cu}}}{\rho_{\text {glass }}} \cdot \frac{L_{\mathrm{Cu}}}{L_{\text {glass }}}
$$

Because $A_{\text {glass }}=A_{\mathrm{Cu}}$ and $R_{\mathrm{Cu}}=R_{\text {glass }}$ :

$$
1=\frac{\rho_{\mathrm{Cu}}}{\rho_{\text {glass }}} \cdot \frac{L_{\mathrm{Cu}}}{L_{\text {glass }}} \Rightarrow L_{\mathrm{Cu}}=\frac{\rho_{\text {glass }}}{\rho_{\mathrm{Cu}}} L_{\text {glass }}
$$

Substitute numerical values (see
Table 25-1 for the resistivities of glass and copper) and evaluate $L_{\mathrm{Cu}}$ :

$$
\begin{aligned}
L_{\mathrm{Cu}} & =\frac{1.01 \times 10^{12} \Omega \cdot \mathrm{~m}}{1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}}(1.00 \mathrm{~cm}) \\
& =5.94 \times 10^{17} \mathrm{~m} \times \frac{1 c \cdot \mathrm{y}}{9.461 \times 10^{15} \mathrm{~m}} \\
& =63 \mathrm{c} \cdot \mathrm{y}
\end{aligned}
$$

44 •• While remodeling your garage, you need to temporarily splice, end to end, an $80-\mathrm{m}$-long copper wire that is 1.00 mm in diameter with a $49-\mathrm{m}$-long aluminum wire that has the same diameter. The maximum current in the wires is 2.00 A. (a) Find the potential drop across each wire of this system when the current is 2.00 A. (b) Find the electric field in each wire when the current is 2.00 A .

Picture the Problem We can use Ohm's law to relate the potential differences across the two wires to their resistances and $R=\rho L / A$ to relate their resistances to their lengths, resistivities, and cross-sectional areas. Once we've found the potential differences across each wire, we can use $E=V / L$ to find the electric field in each wire.
(a) Apply Ohm's law to express the $\quad V_{\mathrm{Cu}}=I R_{\mathrm{Cu}}$ and $V_{\mathrm{Fe}}=I R_{\mathrm{Fe}}$ potential drop across each wire:

Relate the resistances of the wires to their resistivity, cross-sectional area,

$$
R_{\mathrm{Cu}}=\rho_{\mathrm{Cu}} \frac{L_{\mathrm{Cu}}}{A_{\mathrm{Cu}}} \text { and } R_{\mathrm{Fe}}=\rho_{\mathrm{Cu}} \frac{L_{\mathrm{Fe}}}{A_{\mathrm{Fe}}}
$$ and length:

Substitute for $R_{\mathrm{Cu}}$ and $R_{\mathrm{Fe}}$ to obtain:

$$
\begin{aligned}
V_{\mathrm{Cu}} & =\frac{\rho_{\mathrm{Cu}} I_{\mathrm{Cu}}}{A_{\mathrm{Cu}}} I \text { and } V_{\mathrm{Fe}}=\frac{\rho_{\mathrm{Fe}} I_{\mathrm{Fe}}}{A_{\mathrm{Fe}}} I \\
V_{\mathrm{Cu}} & =\frac{\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(80 \mathrm{~m})}{\frac{1}{4} \pi(1.00 \mathrm{~mm})^{2}}(2.00 \mathrm{~A}) \\
& =3.463 \mathrm{~V}=3.5 \mathrm{~V}
\end{aligned}
$$

Substitute numerical values (see Table 25-1 for the resistivities of copper and iron) and evaluate the potential differences:
and

$$
\begin{aligned}
V_{\mathrm{Fe}} & =\frac{\left(10 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(49 \mathrm{~m})}{\frac{1}{4} \pi(1.00 \mathrm{~mm})^{2}}(2.00 \mathrm{~A}) \\
& =12.48 \mathrm{~V}=12 \mathrm{~V}
\end{aligned}
$$

(b) Express the electric field in each conductor in terms of its length and the potential difference across it:

Substitute numerical values and evaluate the electric fields:

$$
E_{\mathrm{Cu}}=\frac{V_{\mathrm{Cu}}}{I_{\mathrm{Cu}}} \text { and } E_{\mathrm{Fe}}=\frac{V_{\mathrm{Fe}}}{I_{\mathrm{Fe}}}
$$

$$
E_{\mathrm{Cu}}=\frac{3.463 \mathrm{~V}}{80 \mathrm{~m}}=43 \mathrm{mV} / \mathrm{m}
$$

and

$$
E_{\mathrm{Fe}}=\frac{12.48 \mathrm{~V}}{49 \mathrm{~m}}=0.25 \mathrm{~V} / \mathrm{m}
$$

$45 \quad$ •• [SSM] A 1.00-m-long wire has a resistance equal to $0.300 \Omega$. A second wire made of identical material has a length of 2.00 m and a mass equal to the mass of the first wire. What is the resistance of the second wire?

Picture the Problem We can use $R=\rho L / A$ to relate the resistance of the wires to their lengths, resistivities, and cross-sectional areas. To find the resistance of the second wire, we can use the fact that the volumes of the two wires are the same to relate the cross-sectional area of the first wire to the cross-sectional area of the second wire.

Relate the resistance of the first wire to its resistivity, cross-sectional area,

$$
R=\rho \frac{L}{A}
$$ and length:

Relate the resistance of the second wire to its resistivity, cross-sectional area, and length:

Divide the second of these equations by the first to obtain:

$$
R^{\prime}=\rho \frac{L^{\prime}}{A^{\prime}}
$$

$$
\begin{equation*}
\frac{R^{\prime}}{R}=\frac{\rho \frac{L^{\prime}}{A^{\prime}}}{\rho \frac{L}{A}}=\frac{L^{\prime}}{L} \frac{A}{A^{\prime}} \Rightarrow R^{\prime}=2 \frac{A}{A^{\prime}} R \tag{1}
\end{equation*}
$$

Express the relationship between the volume $V$ of the first wire and the

$$
V=V^{\prime} \text { or } L A=L^{\prime} A^{\prime} \Rightarrow \frac{A}{A^{\prime}}=\frac{L^{\prime}}{L}=2
$$

volume $V^{\prime}$ of the second wire:
Substituting for $\frac{A}{A^{\prime}}$ in equation (1)

$$
R^{\prime}=2(2) R=4 R
$$

yields:
Because $R=3.00 \Omega$ :

$$
R^{\prime}=4(0.300 \Omega)=1.20 \Omega
$$

$46 \quad \bullet \quad$ A 10-gauge copper wire can safely carry currents up to 30.0 A . (a) What is the resistance of a $100-\mathrm{m}$ length of the wire? (b) What is the electric field in the wire when the current is 30.0 A ? (c) How long does it take for an electron to travel 100 m in the wire when the current is 30.0 A ?

Picture the Problem We can use $R=\rho L / A$ to find the resistance of the wire from its length, resistivity, and cross-sectional area. The electric field can be found using $E=V / L$ and Ohm's law to eliminate $V$. The time for an electron to travel the length of the wire can be found from $L=v_{\mathrm{d}} \Delta t$, with $v_{\mathrm{d}}$ expressed in term of $I$ using $I=n e A v_{\mathrm{d}}$.
(a) Relate the resistance of the unstretched wire to its resistivity,

$$
R=\rho \frac{L}{A}
$$ cross-sectional area, and length:

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the cross-sectional area of 10 -gauge

$$
\begin{aligned}
R & =\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{100 \mathrm{~m}}{5.261 \mathrm{~mm}^{2}} \\
& =0.323 \Omega=0.32 \Omega
\end{aligned}
$$

wire) and evaluate $R$ :
(b) Relate the electric field in the wire to the potential difference

$$
E=\frac{V}{L}
$$

between its ends:

$$
E=\frac{I R}{L}
$$

Substitute numerical values and evaluate $E$ :
(c) Express the time $\Delta t$ for an electron to travel a distance $L$ in the wire in terms of its drift speed
$v_{\mathrm{d}}$ :

Relate the current in the wire to the drift speed of the charge carriers:

$$
I=n e A v_{\mathrm{d}} \Rightarrow v_{\mathrm{d}}=\frac{I}{n e A}
$$

Substitute for $v_{\mathrm{d}}$ to obtain:

$$
\Delta t=\frac{n e A L}{I}
$$

Substitute numerical values (in Example 25-1 it is shown that $n=8.47 \times 10^{28} \mathrm{~m}^{-3}$ ) and evaluate $\Delta t$ :

$$
\begin{aligned}
\Delta t & =\frac{\left(8.47 \times 10^{28} \mathrm{~m}^{-3}\right)\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(5.261 \mathrm{~mm}^{2}\right)(100 \mathrm{~m})}{30.0 \mathrm{~A}} \\
& =2.38 \times 10^{5} \mathrm{~s}=2.75 \mathrm{~d}
\end{aligned}
$$

47 •• A cube of copper has edges that are $2.00-\mathrm{cm}$ long. If copper in the cube is drawn to form a length of 14 -gauge wire, what will the resistance of the length of wire be? Assume the density of the copper does not change.

Picture the Problem We can use $R=\rho L / A$ to find express the resistance of the wire in terms of its length, resistivity, and cross-sectional area. The fact that the volume of the copper does not change as the cube is drawn out to form the wire will allow us to eliminate either the length or the cross-sectional area of the wire and solve for its resistance.

Express the resistance of the wire in terms of its resistivity, cross-

$$
R=\rho \frac{L}{A}
$$

sectional area, and length:

Relate the volume $V$ of the wire to its length and cross-sectional area:

$$
V=A L \Rightarrow L=\frac{V}{A}
$$

Substitute for $L$ to obtain:

$$
R=\rho \frac{V}{A^{2}}
$$

Substitute numerical values (see Table 25-1 for the resistivity of copper and Table 25-2 for the crosssectional area of 14 -gauge wire) and

$$
\begin{aligned}
R & =\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{(2.00 \mathrm{~cm})^{3}}{\left(2.081 \mathrm{~mm}^{2}\right)^{2}} \\
& =31 \mathrm{~m} \Omega
\end{aligned}
$$ evaluate $R$ :

$48 \quad$ ••• Find an expression for the resistance between the ends of the half ring shown in Figure 25-51. The resistivity of the material constituting the half-ring is $\rho$. Hint: Model the half ring as a parallel combination of a large number of thin half-rings. Assume the current is uniformly distributed on a cross section of the half ring.

Picture the Problem We can use, as our element of resistance, the semicircular strip of height $t$, radius $r$, and thickness $d r$ shown below. Then $d R=(\pi r \rho / t) d r$.
Because the strips are in parallel, integrating over them will give us the reciprocal of the resistance of half ring.


The resistance $d R$ between the ends of the infinitesimal strip is given by:

$$
d R=\rho \frac{\pi r}{t d r}
$$

Because the infinitesimal strips are in parallel, their equivalent

$$
\frac{1}{R}=\frac{t}{\pi \rho} \int_{a}^{b} \frac{d r}{r}
$$ resistance is:

Integrating $d R_{\text {eq }}$ from $r=a$ to $r=b$ yields:

$$
\frac{1}{R}=\frac{t}{\pi \rho} \ln \left(\frac{b}{a}\right) \Rightarrow R=\frac{\rho \pi}{t \ln \left(\frac{b}{a}\right)}
$$

49 [SSM] Consider a wire of length $L$ in the shape of a truncated cone. The radius of the wire varies with distance $x$ from the narrow end according to $r=a+[(b-a) / L] x$, where $0<x<L$. Derive an expression for the resistance of this wire in terms of its length $L$, radius $a$, radius $b$ and resistivity $\rho$. Hint: Model the wire as a series combination of a large number of thin disks. Assume the current is uniformly distributed on a cross section of the cone.

Picture the Problem The element of resistance we use is a segment of length $d x$ and cross-sectional area $\pi[a+(b-a) x / L]^{2}$. Because these resistance elements are in series, integrating over them will yield the resistance of the wire.


Express the resistance of the chosen element of resistance:

$$
d R=\rho \frac{d x}{A}=\frac{\rho}{\pi[a+(b-a)(x / L)]^{2}} d x
$$

Integrate $d R$ from $x=0$ to $x=L$ and simplify to obtain:

$$
\begin{aligned}
R & =\frac{\rho}{\pi} \int_{0}^{L} \frac{d x}{[a+(b-a)(x / L)]^{2}} \\
& =\frac{\rho L}{\pi(b-a)}\left(\frac{1}{a}-\frac{1}{a+(b-a)}\right) \\
& =\frac{\rho L}{\pi a b}
\end{aligned}
$$

50 ••• The space between two metallic concentric spherical shells is filled with a material that has a resistivity of $3.50 \times 10^{-5} \Omega \cdot \mathrm{~m}$. If the inner metal shell has an outer radius of 1.50 cm and the outer metal shell has an inner radius of 5.00 cm , what is the resistance between the conductors? Hint: Model the material as a series combination of a large number of thin spherical shells.

Picture the Problem The diagram shows a cross-sectional view of the concentric spheres of radii $a$ and $b$ as well as a spherical-shell element of radius $r$. We can express the resistance $d R$ of the spherical-shell element and then integrate over the volume filled with the material whose resistivity $\rho$ is given to find the resistance between the conductors. Note that the elements of resistance are in series.


Express the element of resistance $d R$ :

$$
\begin{aligned}
& d R=\rho \frac{d r}{A}=\rho \frac{d r}{4 \pi r^{2}} \\
& R=\frac{\rho}{4 \pi} \int_{a}^{b} \frac{d r}{r^{2}}=\frac{\rho}{4 \pi}\left(\frac{1}{a}-\frac{1}{b}\right)
\end{aligned}
$$

Substitute numerical values and evaluate $R$ :

$$
R=\frac{3.50 \times 10^{-5} \Omega \cdot \mathrm{~m}}{4 \pi}\left(\frac{1}{1.50 \mathrm{~cm}}-\frac{1}{5.00 \mathrm{~cm}}\right)=130 \mu \Omega
$$

51 ... The space between two metallic coaxial cylinders that have the same length $L$ is completely filled with a nonmetalic material having a resistivity $\rho$. The inner metal shell has an outer radius $a$ and the outer metal shell has an inner radius $b$. (a) What is the resistance between the two cylinders? Hint: Model the material as a series combination of a large number of thin cylindrical shells. (b) Find the current between the two metallic cylinders if $\rho=30.0 \Omega \cdot \mathrm{~m}, a=1.50 \mathrm{~cm}$, $b=2.50 \mathrm{~cm}, L=50.0 \mathrm{~cm}$, and a potential difference of 10.0 V is maintained between the two cylinders.

Picture the Problem The diagram shows a cross-sectional view of the coaxial cylinders of radii $a$ and $b$ as well as a cylindrical-shell element of radius $r$. We can express the resistance $d R$ of the cylindrical-shell element and then integrate over the volume filled with the material whose resistivity $\rho$ is given to find the resistance between the two cylinders. Note that the elements of resistance are in series.

(a) Express the element of resistance $d R$ :

$$
d R=\rho \frac{d r}{A}=\rho \frac{d r}{2 \pi r L}
$$

Integrate $d R$ from $r=a$ to $r=b$ to obtain:

$$
R=\frac{\rho}{2 \pi L} \int_{a}^{b} \frac{d r}{r}=\frac{\rho}{2 \pi L} \ln \left(\frac{b}{a}\right)
$$

(b) Apply Ohm's law to obtain:

$$
I=\frac{V}{R}=\frac{V}{\frac{\rho}{2 \pi L} \ln \left(\frac{b}{a}\right)}=\frac{2 \pi L V}{\rho \ln \left(\frac{b}{a}\right)}
$$

Substitute numerical values and evaluate $I$ :

$$
I=\frac{2 \pi(50.0 \mathrm{~cm})(10.0 \mathrm{~V})}{(30.0 \Omega \cdot \mathrm{~m}) \ln \left(\frac{2.50 \mathrm{~cm}}{1.50 \mathrm{~cm}}\right)}=2.05 \mathrm{~A}
$$

## Temperature Dependence of Resistance

52 - A tungsten rod is 50 cm long and has a square cross-section that has $1.0-\mathrm{mm}$-long edges. (a) What is its resistance at $20^{\circ} \mathrm{C}$ ? (b) What is its resistance at $40^{\circ} \mathrm{C}$ ?

Picture the Problem We can use $R=\rho L / A$ to find the resistance of the rod at $20^{\circ} \mathrm{C}$. Ignoring the effects of thermal expansion, we can we apply the equation defining the temperature coefficient of resistivity, $\alpha$, to relate the resistance at $40^{\circ} \mathrm{C}$ to the resistance at $20^{\circ} \mathrm{C}$.
(a) Express the resistance of the rod at $20^{\circ} \mathrm{C}$ as a function of its resistivity,

$$
R_{20}=\rho_{20} \frac{L}{A}
$$

length, and cross-sectional area:

Substitute numerical values and evaluate $R_{20}$ :

$$
\begin{aligned}
R_{20} & =\left(5.5 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{0.50 \mathrm{~m}}{(1.0 \mathrm{~mm})^{2}} \\
& =27.5 \mathrm{~m} \Omega=28 \mathrm{~m} \Omega
\end{aligned}
$$

(b) Express the resistance of the rod at $40^{\circ} \mathrm{C}$ as a function of its resistance at $20^{\circ} \mathrm{C}$ and the temperature coefficient of resistivity $\alpha$ :

$$
\begin{aligned}
R_{40} & =\rho_{40} \frac{L}{A} \\
& =\rho_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right)\right] \frac{L}{A} \\
& =\rho_{20} \frac{L}{A}+\rho_{20} \frac{L}{A} \alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right) \\
& =R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right)\right]
\end{aligned}
$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of tungsten) and evaluate $R_{40}$ :

$$
R_{40}=(27.5 \mathrm{~m} \Omega)\left[1+\left(4.5 \times 10^{-3} \mathrm{~K}^{-1}\right)\left(20 \mathrm{C}^{\circ}\right)\right]=30 \mathrm{~m} \Omega
$$

53 - [SSM] At what temperature will the resistance of a copper wire be 10 percent greater than its resistance at $20^{\circ} \mathrm{C}$ ?

Picture the Problem The resistance of the copper wire increases with temperature according to $R_{t_{\mathrm{C}}}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right)\right]$. We can replace $R_{t_{\mathrm{C}}}$ by $1.1 R_{20}$ and solve for $t_{\mathrm{C}}$ to find the temperature at which the resistance of the wire will be $110 \%$ of its value at $20^{\circ} \mathrm{C}$.

Express the resistance of the wire at

$$
1.10 R_{20}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right)\right]
$$ $1.10 R_{20}$ :

Simplifying this expression yields:

$$
0.10=\alpha\left(t_{\mathrm{C}}-20 \mathrm{C}^{\circ}\right)
$$

Solve to $t_{\mathrm{C}}$ to obtain:

$$
t_{\mathrm{C}}=\frac{0.10}{\alpha}+20 \mathrm{C}^{\circ}
$$

Substitute numerical values (see Table 25-1 for the temperature

$$
\boldsymbol{t}_{\mathrm{C}}=\frac{0.10}{3.9 \times 10^{-3} \mathrm{~K}^{-1}}+20 \mathrm{C}^{\circ}=46^{\circ} \mathrm{C}
$$ coefficient of resistivity of copper) and evaluate $t_{\mathrm{C}}$ :

54 •• You have a toaster that uses a Nichrome wire as a heating element. You need to determine the temperature of the Nichrome wire under operating conditions. First, you measure the resistance of the heating element at $20^{\circ} \mathrm{C}$ and find it to be $80.0 \Omega$. Then you measure the current immediately after you plug the toaster into a wall outlet - before the temperature of the Nichrome wire increases significantly. You find this startup current to be 8.70 A . When the heating element reaches its operating temperature, you measure the current to be 7.00 A . Use your data to determine the maximum operating temperature of the heating element.

Picture the Problem Let the primed quantities denote the current and resistance at the final temperature of the heating element. We can express $R^{\prime}$ in terms of $R_{20}$ and the final temperature of the wire $t_{\mathrm{C}}$ using $R^{\prime}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20^{\circ} \mathrm{C}\right)\right]$ and relate $I^{\prime}, R^{\prime}, I_{20}$, and $R_{20}$ using Ohm's law.

Express the resistance of the heating

$$
\begin{equation*}
R^{\prime}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20^{\circ} \mathrm{C}\right)\right] \tag{1}
\end{equation*}
$$

element at its final temperature as a function of its resistance at $20^{\circ} \mathrm{C}$ and the temperature coefficient of resistivity for Nichrome:

Apply Ohm's law to the heating

$$
V=I_{20} R_{20}
$$

element when it is first turned on:

Apply Ohm's law to the heating

$$
V=I^{\prime} R^{\prime}
$$

element when it has reached its final temperature:

Because the voltage is constant, we have:

$$
I^{\prime} R^{\prime}=I_{20} R_{20} \text { or } R^{\prime}=\frac{I_{20}}{I^{\prime}} R_{20}
$$

Substitute in equation (1) to obtain:

$$
\begin{aligned}
& \frac{I_{20}}{I^{\prime}} R_{20}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20^{\circ} \mathrm{C}\right)\right] \\
& \text { or } \\
& \frac{I_{20}}{I^{\prime}}=1+\alpha\left(t_{\mathrm{C}}-20^{\circ} \mathrm{C}\right)
\end{aligned}
$$

Solve for $t_{\mathrm{C}}$ to obtain:

$$
t_{\mathrm{C}}=\frac{\frac{I_{20}}{I^{\prime}}-1}{\alpha}+20^{\circ} \mathrm{C}
$$

Substitute numerical values (see
$t_{\mathrm{C}}=\frac{\frac{8.70 \mathrm{~A}}{7.00 \mathrm{~A}}-1}{0.4 \times 10^{-3} \mathrm{~K}^{-1}}+20^{\circ} \mathrm{C} \approx 6 \times 10^{2 \circ} \mathrm{C}$ coefficient of resistivity of Nichrome) and evaluate $t_{\mathrm{C}}$ :
where the answer has only one significant figure because the temperature coefficient is only given to one significant figure.

## Remarks: At $600^{\circ} \mathrm{C}$ the wire glows red.

55 •• Your electric space heater has a Nichrome heating element that has a resistance of $8.00 \Omega$ at $20.0^{\circ} \mathrm{C}$. When 120 V are applied, the electric current heats the Nichrome wire to $1000^{\circ} \mathrm{C}$. (a) What is the initial current in the heating element at $20.0^{\circ} \mathrm{C}$ ? (b) What is the resistance of the heating element at $1000^{\circ} \mathrm{C}$ ? (c) What is the operating power of this heater?

Picture the Problem We can apply Ohm's law to find the initial current drawn by the cold heating element. The resistance of the wire at $1000^{\circ} \mathrm{C}$ can be found using $R_{1000}=R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20.0^{\circ} \mathrm{C}\right)\right]$ and the power consumption of the heater at this temperature from $P=V^{2} / R_{1000}$.
(a) Apply Ohm's law to find the initial current $I_{20}$ drawn by the

$$
I=\frac{V}{R_{20}}=\frac{120 \mathrm{~V}}{8.00 \Omega}=15.0 \mathrm{~A}
$$

heating element:
(b) Express the resistance of the heating element at $1000^{\circ} \mathrm{C}$ as a function of its resistance at $20.0^{\circ} \mathrm{C}$ and the temperature coefficient of resistivity for Nichrome:

Substitute numerical values (see
Table 25-1 for the temperature coefficient of resistivity of Nichrome) and evaluate $R_{1000}$ :

$$
\begin{aligned}
R_{1000}= & (8.00 \Omega)\left[1+\left(0.4 \times 10^{-3} \mathrm{~K}^{-1}\right)\right. \\
& \left.\times\left(1000^{\circ} \mathrm{C}-20.0^{\circ} \mathrm{C}\right)\right] \\
= & 11.1 \Omega
\end{aligned}
$$

(c) The operating power of this heater at $1000^{\circ} \mathrm{C}$ is:

$$
P=\frac{V^{2}}{R_{1000}}=\frac{(120 \mathrm{~V})^{2}}{11.1 \Omega}=1.30 \mathrm{~kW}
$$

56 •• A 10.0- $\Omega$ Nichrome resistor is wired into an electronic circuit using copper leads (wires) that have diameters equal to 0.600 mm . The copper leads have a total length of 50.0 cm . (a) What additional resistance is due to the copper leads? (b) What percentage error in the total resistance is produced by neglecting the resistance of the copper leads? (c) What change in temperature would produce a change in resistance of the Nichrome wire equal to the resistance of the copper leads? Assume that the Nichrome section is the only one whose temperature is changed.

Picture the Problem We can find the resistance of the copper leads using $R_{\mathrm{Cu}}=\rho_{\mathrm{Cu}} L / A$ and express the percentage error in neglecting the resistance of the leads as the ratio of $R_{\text {Cu }}$ to $R_{\text {Nichrome. }}$. In Part (c) we can express the change in resistance in the Nichrome wire corresponding to a change $\Delta t_{\mathrm{C}}$ in its temperature and then find $\Delta t_{\mathrm{C}}$ by substitution of the resistance of the copper wires in this equation.
(a) Relate the resistance of the copper leads to their resistivity, length, and cross-sectional area:

Substitute numerical values (see Table 25-1 for the resistivity of copper) and evaluate $R_{\mathrm{Cu}}$ :
(b) Express the percentage error as the ratio of $R_{\mathrm{Cu}}$ to $R_{\text {Nichrome }}$ :
(c) Express the change in the resistance of the Nichrome wire as its temperature changes from $t_{\mathrm{C}}$ to $t_{\mathrm{C}}$ :

Solve for $\Delta t_{\mathrm{C}}$ to obtain:

Set $\Delta R$ equal to the resistance of the copper wires (see Table 25-1 for the temperature coefficient of resistivity of Nichrome wire) and evaluate $\Delta t_{\mathrm{C}}$ :

$$
\begin{aligned}
\% \text { error } & =\frac{R_{\mathrm{Cu}}}{R_{\text {Nichrome }}}=\frac{30.1 \mathrm{~m} \Omega}{10.0 \Omega} \\
& =0.30 \%
\end{aligned}
$$

$$
\begin{aligned}
\Delta R= & R^{\prime}-R \\
= & R_{20}\left[1+\alpha\left(t_{\mathrm{C}}{ }^{\prime}-20^{\circ} \mathrm{C}\right)\right] \\
& -R_{20}\left[1+\alpha\left(t_{\mathrm{C}}-20^{\circ} \mathrm{C}\right)\right] \\
= & R_{20} \alpha \Delta t_{\mathrm{C}}
\end{aligned}
$$

$$
\Delta t_{\mathrm{C}}=\frac{\Delta R}{R_{20} \alpha}
$$

$$
R_{\mathrm{Cu}}=\rho_{\mathrm{Cu}} \frac{L}{A}
$$

$$
\begin{aligned}
R_{\mathrm{Cu}} & =\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{50.0 \mathrm{~cm}}{\frac{1}{4} \pi(0.600 \mathrm{~mm})^{2}} \\
& =30.1 \mathrm{~m} \Omega=30 \mathrm{~m} \Omega
\end{aligned}
$$

$$
\Delta \boldsymbol{t}_{\mathrm{C}}=\frac{30.1 \mathrm{~m} \Omega}{(10.0 \Omega)\left(0.4 \times 10^{-3} \mathrm{~K}^{-1}\right)}=8^{\circ} \mathrm{C}
$$

57 •• [SSM] A wire that has a cross-sectional area $A$, a length $L_{1}$, a resistivity $\rho_{1}$, and a temperature coefficient $\alpha_{1}$ is connected end to end to a second wire that has the same cross-sectional area, a length $L_{2}$, a resistivity $\rho_{2}$, and a temperature coefficient $\alpha_{2}$, so that the wires carry the same current.
(a) Show that if $\rho_{1} L_{1} \alpha_{1}+\rho_{2} L_{2} \alpha_{2}=0$, then the total resistance is independent of temperature for small temperature changes. (b) If one wire is made of carbon and the other wire is made of copper, find the ratio of their lengths for which the total resistance is approximately independent of temperature.

Picture the Problem Expressing the total resistance of the two current-carrying (and hence warming) wires connected in series in terms of their resistivities, temperature coefficients of resistivity, lengths and temperature change will lead us to an expression in which, if $\rho_{1} L_{1} \alpha_{1}+\rho_{2} L_{2} \alpha_{2}=0$, the total resistance is temperature independent. In Part (b) we can apply the condition that

$$
\rho_{1} L_{1} \alpha_{1}+\rho_{2} L_{2} \alpha_{2}=0
$$

to find the ratio of the lengths of the carbon and copper wires.
(a) Express the total resistance of these two wires connected in series:

$$
\begin{aligned}
R & =R_{1}+R_{2}=\rho_{1} \frac{L_{1}}{A}\left(1+\alpha_{1} \Delta T\right)+\rho_{2} \frac{L_{2}}{A}\left(1+\alpha_{2} \Delta T\right) \\
& =\frac{1}{A}\left[\rho_{1} L_{1}\left(1+\alpha_{1} \Delta T\right)+\rho_{2} L_{2}\left(1+\alpha_{2} \Delta T\right)\right]
\end{aligned}
$$

Expand and simplify this expression to obtain:

$$
R=\frac{1}{A}\left[\rho_{1} L_{1}+\rho_{2} L_{2}+\left(\rho_{1} L_{1} \alpha_{1}+\rho_{1} L_{1} \alpha_{2}\right) \Delta T\right]
$$

If $\rho_{1} L_{1} \alpha_{1}+\rho_{2} L_{2} \alpha_{2}=0$, then: $\quad R=\frac{1}{A}\left[\rho_{1} L_{1}+\rho_{2} L_{2}\right]$ independently of
the temperature.
(b) Apply the condition for

$$
\rho_{\mathrm{C}} L_{\mathrm{C}} \alpha_{\mathrm{C}}+\rho_{\mathrm{Cu}} L_{\mathrm{Cu}} \alpha_{\mathrm{Cu}}=0
$$ temperature independence obtained in (a) to the carbon and copper wires:

Solve for the ratio of $L_{\mathrm{Cu}}$ to $L_{\mathrm{C}}$ :

$$
\frac{L_{\mathrm{Cu}}}{L_{\mathrm{C}}}=-\frac{\rho_{\mathrm{C}} \alpha_{\mathrm{C}}}{\rho_{\mathrm{Cu}} \alpha_{\mathrm{Cu}}}
$$

Substitute numerical values (see Table 25-1 for the temperature coefficient of resistivity of carbon and copper) and evaluate the ratio of $L_{\mathrm{Cu}}$ to $L_{\mathrm{C}}$ :

$$
\frac{L_{\mathrm{Cu}}}{L_{\mathrm{C}}}=-\frac{\left(3500 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(-0.5 \times 10^{-3} \mathrm{~K}^{-1}\right)}{\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(3.9 \times 10^{-3} \mathrm{~K}^{-1}\right)} \approx 3 \times 10^{2}
$$

58 ••• The resistivity of tungsten increases approximately linearly with temperature from $56.0 \mathrm{n} \Omega \cdot \mathrm{m}$ at 293 K to $1.10 \mu \Omega \cdot \mathrm{~m}$ at 3500 K . A light bulb is powered by a a $100-\mathrm{V}$ dc power supply. Under these operating conditions the temperature of the tungsten filament is 2500 K , the length of the filament is equal to 5.00 cm and the power delivered to the filament is $40-\mathrm{W}$. Estimate (a) the resistance of the filament and $(b)$ the diameter of the filament.

Picture the Problem We can use the relationship between the rate at which energy is transformed into heat and light in the filament and the resistance of and potential difference across the filament to estimate the resistance of the filament. The linear dependence of the resistivity on temperature will allow us to find the resistivity of the filament at 2500 K . We can then use the relationship between the resistance of the filament, its resistivity, and cross-sectional area to find its diameter.
(a) Express the wattage of the lightbulb as a function of its resistance $R$ and the voltage $V$ supplied by the source:

Substitute numerical values and evaluate $R$ :
(b) Relate the resistance $R$ of the filament to its resistivity $\rho$, radius $r$, and length $\ell$ :

The diameter $d$ of the filament is:

$$
P=\frac{V^{2}}{R} \Rightarrow R=\frac{V^{2}}{P}
$$

$$
R=\frac{(100 \mathrm{~V})^{2}}{40 \mathrm{~W}}=0.25 \mathrm{k} \Omega
$$

$$
R=\frac{\rho \ell}{\pi r^{2}} \Rightarrow r=\sqrt{\frac{\rho \ell}{\pi R}}
$$

$$
\begin{equation*}
d=2 \sqrt{\frac{\rho \ell}{\pi R}} \tag{1}
\end{equation*}
$$

Because the resistivity varies linearly with temperature, we can use a proportion to find its value at 2500 K :

$$
\begin{aligned}
\frac{\rho_{2500 \mathrm{~K}}-\rho_{293 \mathrm{~K}}}{\rho_{3500 \mathrm{~K}}-\rho_{293 \mathrm{~K}}} & =\frac{2500 \mathrm{~K}-293 \mathrm{~K}}{3500 \mathrm{~K}-293 \mathrm{~K}} \\
& =\frac{2207}{3207}
\end{aligned}
$$

Solve for $\rho_{2500 \mathrm{~K}}$ to obtain:

$$
\rho_{2500 \mathrm{~K}}=\frac{2207}{3207}\left(\rho_{3500 \mathrm{~K}}-\rho_{293 \mathrm{~K}}\right)+\rho_{293 \mathrm{~K}}
$$

Substitute numerical values and evaluate $\rho_{2500 \mathrm{~K}}$ :

$$
\rho_{2500 \mathrm{~K}}=\frac{2207}{3207}(1.10 \mu \Omega \cdot \mathrm{~m}-56.0 \mathrm{n} \Omega \cdot \mathrm{~m})+56.0 \mathrm{n} \Omega \cdot \mathrm{~m}=774.5 \mu \Omega \cdot \mathrm{~m}
$$

Substitute numerical values in equation (1) and evaluate $d$ :

$$
\begin{aligned}
d & =2 \sqrt{\frac{(774.5 \mathrm{n} \Omega \cdot \mathrm{~m})(5.00 \mathrm{~cm})}{\pi(250 \Omega)}} \\
& =14.0 \mu \mathrm{~m}
\end{aligned}
$$

59 ••• A 5.00-V light bulb used in an electronics class has a carbon filament that has a length of 3.00 cm and a diameter of $40.0 \mu \mathrm{~m}$. At temperatures between 500 K and 700 K , the resistivity of the carbon used in making small light bulb filaments is about $3.00 \times 10^{-5} \Omega \cdot \mathrm{~m}$. (a) Assuming that the bulb is a perfect blackbody radiator, calculate the temperature of the filament under operating conditions. (b) One concern about carbon filament bulbs, unlike tungsten filament bulbs, is that the resistivity of carbon decreases with increasing temperature. Explain why this decrease in resistivity is a concern.

Picture the Problem We can use the relationship between the rate at which an object radiates and its temperature to find the temperature of the bulb.
(a) At a temperature $T$, the power emitted by a perfect blackbody is:

Solve for $T$ and substitute for $P$ to obtain:

$$
T=\sqrt[4]{\frac{P}{\sigma A}}=\sqrt[4]{\frac{P}{\sigma \pi d L}}=\sqrt[4]{\frac{V^{2}}{\sigma \pi d L R}}
$$

Relate the resistance $R$ of the filament to its resistivity $\rho$ :

$$
R=\frac{\rho L}{A}=\frac{4 \rho L}{\pi d^{2}}
$$

Substitute for $R$ in the expression for $T$ to obtain:

$$
T=\sqrt[4]{\frac{V^{2}}{\sigma \pi d L \frac{4 \rho L}{\pi d^{2}}}}=\sqrt[4]{\frac{V^{2} d}{4 \sigma L^{2} \rho}}
$$

Substitute numerical values and evaluate $T$ :

$$
\boldsymbol{T}=\sqrt[4]{\frac{(5.00 \mathrm{~V})^{2}\left(40.0 \times 10^{-6} \mathrm{~m}\right)}{4\left(5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{~K}^{4}\right)(0.0300 \mathrm{~m})^{2}\left(3.00 \times 10^{-5} \Omega \cdot \mathrm{~m}\right)}}=636 \mathrm{~K}
$$

(b) As the filament heats up, its resistance decreases. This results in more power being dissipated, further heat, higher temperature, etc. If not controlled, this thermal runaway can burn out the filament.

## Energy in Electric Circuits

60 - A $1.00-\mathrm{kW}$ heater is designed to operate at 240 V . (a) What is the heater's resistance and what is the current in the wires that supply power to the heater? (b) What is the power delivered to the heater if it operates at 120 V ? Assume that its resistance remains the same.

Picture the Problem We can use $P=V^{2} / R$ to find the resistance of the heater and Ohm's law to find the current it draws.
(a) Express the power output of the heater in terms of its resistance and

$$
P=\frac{V^{2}}{R} \Rightarrow R=\frac{V^{2}}{P}
$$ its operating voltage:

Substitute numerical values and evaluate $R$ :

$$
R=\frac{(240 \mathrm{~V})^{2}}{1.00 \mathrm{~kW}}=57.60 \Omega=57.6 \Omega
$$

Apply Ohm's law to find the current drawn by the heater:

$$
I=\frac{V}{R}=\frac{240 \mathrm{~V}}{57.60 \Omega}=4.17 \mathrm{~A}
$$

(b) The power delivered to the heater operating at 120 V is:

$$
P=\frac{(120 \mathrm{~V})^{2}}{57.60 \Omega}=250 \mathrm{~W}
$$

61 - A battery has an emf of 12 V . How much work does it do in 5.0 s if it delivers a current of 3.0 A ?

Picture the Problem We can use the definition of power and the relationship between the battery's power output and its emf to find the work done by it under the given conditions.

Use the definition of power to relate the work done by the battery to the

$$
P=\frac{\Delta W}{\Delta t} \Rightarrow \Delta W=P \Delta t
$$ time current is drawn from it:

Express the power output of the $\quad P=\varepsilon I$ battery as a function of the battery's emf:

Substituting for $P$ yields:

Substitute numerical values and evaluate $\Delta W$ :

$$
\Delta W=\varepsilon I \Delta t
$$

$$
\Delta W=(12 \mathrm{~V})(3.0 \mathrm{~A})(5.0 \mathrm{~s})=0.18 \mathrm{~kJ}
$$

62 - An automotive battery has an emf of 12.0 V . When supplying power to the starter motor, the current in the battery is 20.0 A and the terminal voltage of the battery is 11.4 V . What is the internal resistance of the battery?

Picture the Problem We can relate the terminal voltage of the battery to its emf, internal resistance, and the current delivered by it and then solve this relationship for the internal resistance.

Express the terminal potential difference of the battery in terms of its emf and internal resistance:

Substitute numerical values and evaluate $r$ :

$$
V_{a}-V_{b}=\varepsilon-I r \Rightarrow r=\frac{\varepsilon-\left(V_{a}-V\right)_{b}}{I}
$$

$$
r=\frac{12.0 \mathrm{~V}-11.4 \mathrm{~V}}{20.0 \mathrm{~A}}=0.03 \Omega
$$

63 - [SSM] (a) How much power is delivered by the battery in Problem 62 due to the chemical reactions within the battery when the current in the battery is 20 A ? (b) How much of this power is delivered to the starter when the current in the battery is 20 A ? (c) By how much does the chemical energy of the battery decrease if the current in the starter is 20 A for 7.0 s ? (d) How much energy is dissipated in the battery during these 7.0 seconds?

Picture the Problem We can find the power delivered by the battery from the product of its emf and the current it delivers. The power delivered to the battery can be found from the product of the potential difference across the terminals of the starter (or across the battery when current is being drawn from it) and the current being delivered to it. In Part (c) we can use the definition of power to relate the decrease in the chemical energy of the battery to the power it is delivering and the time during which current is drawn from it. In Part (d) we can use conservation of energy to relate the energy delivered by the battery to the heat developed in the battery and the energy delivered to the starter
(a) Express the power delivered by the battery as a function of its emf and the current it delivers:

$$
\begin{aligned}
P & =\varepsilon I=(12.0 \mathrm{~V})(20 \mathrm{~A})=240 \mathrm{~W} \\
& =0.24 \mathrm{~kW}
\end{aligned}
$$

(b) Relate the power delivered to the starter to the potential difference across its terminals:
(c) Use the definition of power to express the decrease in the chemical energy of the battery as it delivers current to the starter:
(d) Use conservation of energy to relate the energy delivered by the battery to the heat developed in the battery and the energy delivered to the starter:

Express the energy delivered by the battery and the energy delivered to the starter in terms of the rate at which this energy is delivered:

Substitute numerical values and evaluate $Q$ :

$$
\begin{aligned}
P_{\text {starter }} & =V_{\text {starter }} I=(11.4 \mathrm{~V})(20 \mathrm{~A}) \\
& =228 \mathrm{~W}=0.23 \mathrm{~kW}
\end{aligned}
$$

$$
\begin{aligned}
\Delta E & =P \Delta t \\
& =(240 \mathrm{~W})(7.0 \mathrm{~s})=1680 \mathrm{~J} \\
& =1.7 \mathrm{~kJ}
\end{aligned}
$$

$$
\begin{aligned}
E_{\text {delivered }}^{\text {by battery }} & =\underset{\substack{\text { transformed } \\
\text { into heat }}}{ }+E_{\text {delivered }}^{\text {to starter }} \\
& =Q+E_{\text {delivered }}^{\text {to starter }}
\end{aligned}
$$

$$
P \Delta t=Q+P_{\mathrm{s}} \Delta t \Rightarrow Q=\left(P-P_{\mathrm{s}}\right) \Delta t
$$

$$
Q=(240 \mathrm{~W}-228 \mathrm{~W})(7.0 \mathrm{~s})=84 \mathrm{~J}
$$

64 - A battery that has an emf of 6.0 V and an internal resistance of $0.30 \Omega$ is connected to a variable resistor with resistance $R$. Find the current and power delivered by the battery when $R$ is $(a) 0,(b) 5.0 \Omega,(c) 10 \Omega$, and (d) infinite.

Picture the Problem We can use conservation of energy to relate the emf of the battery to the potential differences across the variable resistor and the energy converted to heat within the battery. Solving this equation for $I$ will allow us to find the current for the four values of $R$ and we can use $P=I^{2} R$ to find the power delivered by the battery for the four values of $R$.

Apply conservation of energy (Kirchhoff's loop rule) to the

$$
\mathcal{E}=I R+I r \Rightarrow I=\frac{\mathcal{E}}{R+r}
$$ circuit:

Express the power delivered by the

$$
P=I^{2} R
$$ battery as a function of the current drawn from it:

(a) For $R=0$ :

$$
I=\frac{\varepsilon}{R+r}=\frac{6.0 \mathrm{~V}}{0.30 \Omega}=20 \mathrm{~A}
$$

and

$$
P=(20 \mathrm{~A})^{2}(0)=0 \mathrm{~W}
$$

(b) For $R=5.0 \Omega$ :

$$
\begin{aligned}
I & =\frac{\varepsilon}{R+r}=\frac{6.0 \mathrm{~V}}{5.0 \Omega+0.30 \Omega}=1.13 \mathrm{~A} \\
& =1.1 \mathrm{~A}
\end{aligned}
$$

and

$$
P=(1.13 \mathrm{~A})^{2}(5.0 \Omega)=6.4 \mathrm{~W}
$$

(c) For $R=10 \Omega$ :
(d) For $R=\infty$ :

$$
\begin{aligned}
I & =\frac{\varepsilon}{R+r}=\frac{6.0 \mathrm{~V}}{10 \Omega+0.30 \Omega}=0.583 \mathrm{~A} \\
& =0.58 \mathrm{~A}
\end{aligned}
$$

and

$$
P=(0.583 \mathrm{~A})^{2}(10 \Omega)=3.4 \mathrm{~W}
$$

$$
I=\frac{\varepsilon}{R+r}=\lim _{R \rightarrow \infty} \frac{6.0 \mathrm{~V}}{R+0.30 \Omega}=0 \mathrm{~A}
$$

and

$$
P=0 \mathrm{~W}
$$

65 •• A $12.0-\mathrm{V}$ automobile battery that has a negligible internal resistance can deliver a total charge of $160 \mathrm{~A} \cdot \mathrm{~h}$. (a) What is the amount of energy stored in the battery? (b) After studying all night for a calculus test, you try to drive to class to take the test. However, you find that the car's battery is "dead" because you had left the headlights on! Assuming the battery was able to produce current at a constant rate until it died, how long were your lights on? Assume the pair of headlights together operates at a power of 150 W .

Picture the Problem We can express the total stored energy $\Delta U$ in the battery in terms of its emf and the product $I \Delta t$ of the current it can deliver for a period of time $\Delta t$. We can apply the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights.
(a) Express $\Delta U$ in terms of $\mathcal{E}$ and

$$
\Delta U=\varepsilon I \Delta t
$$ the product $I \Delta t$ :

Substitute numerical values and evaluate $\Delta U$ :
(b) Use the definition of power to relate the lifetime of the battery to the rate at which it is providing energy to the pair of headlights:

Substitute numerical values and evaluate $\Delta t$ :

$$
\begin{aligned}
\Delta U & =(12.0 \mathrm{~V})(160 \mathrm{~A} \cdot \mathrm{~h})=1.92 \mathrm{~kW} \cdot \mathrm{~h} \\
& =1.92 \mathrm{~kW} \cdot \mathrm{~h} \times \frac{3.6 \mathrm{MJ}}{\mathrm{~kW} \cdot \mathrm{~h}} \\
& =6.9 \mathrm{MJ}
\end{aligned}
$$

$$
\Delta t=\frac{\Delta U}{P}
$$

The measured current in a circuit in your uncle's house is 12.5 A . In this circuit, the only appliance that is on is a space heater that is being used to heat the bathroom. A pair of 12-gauge copper wires carries the current from the supply panel in your basement to the wall outlet in the bathroom, a distance of 30.0 m . You measure the voltage at the supply panel to be exactly 120 V . What is the voltage at the wall outlet in the bathroom that the space heater is connected to?

Picture the Problem We can use conservation of energy (Kirchhoff's loop rule) to relate the emf at the fuse box and the voltage drop in the wires to the voltage at the wall outlet in the bathroom.

Apply Kirchhoff's loop rule to the circuit to obtain:

$$
\begin{aligned}
& \mathcal{E}-V_{\text {wires }}-V_{\text {outlet }}=0 \\
& \text { or } \\
& \mathcal{E}-I R_{\text {wires }}-V_{\text {outlet }}=0
\end{aligned}
$$

Solve for $V_{\text {outlet }}$ to obtain:

Relate the resistance of the copper wires to the resistivity of copper, the length of the wires, and the crosssectional area of 12-gauge wire:

Substituting for $R_{\text {wires }}$ yields:

$$
V_{\text {outlet }}=\varepsilon-\frac{I \rho_{\mathrm{Cu}} L}{A}
$$

Substitute numerical values (see Table $25-1$ for the resistivity of copper and Table 25-2 for the cross-sectional area of 12-gauge wire) and evaluate $V_{\text {outlet: }}$

$$
V_{\text {outlet }}=120 \mathrm{~V}-\frac{(12.5 \mathrm{~A})\left(1.7 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(60.0 \mathrm{~m})}{3.309 \mathrm{~mm}^{2}}=116 \mathrm{~V}
$$

67 •• [SSM] A lightweight electric car is powered by a series combination of ten $12.0-\mathrm{V}$ batteries, each having negligible internal resistance. Each battery can deliver a charge of $160 \mathrm{~A} \cdot \mathrm{~h}$ before needing to be recharged. At a speed of $80.0 \mathrm{~km} / \mathrm{h}$, the average force due to air drag and rolling friction is 1.20 kN .
(a) What must be the minimum power delivered by the electric motor if the car is to travel at a speed of $80.0 \mathrm{~km} / \mathrm{h}$ ? (b) What is the total charge, in coulombs, that can be delivered by the series combination of ten batteries before recharging is required? (c) What is the total electrical energy delivered by the ten batteries before recharging? (d) How far can the car travel (at $80.0 \mathrm{~km} / \mathrm{h}$ ) before the batteries must be recharged? (e) What is the cost per kilometer if the cost of recharging the batteries is 9.00 cents per kilowatt-hour?

Picture the Problem We can use $P=f v$ to find the power the electric motor must develop to move the car at $80 \mathrm{~km} / \mathrm{h}$ against a frictional force of 1200 N . We can find the total charge that can be delivered by the 10 batteries using $\Delta Q=N I \Delta t$. The total electrical energy delivered by the 10 batteries before recharging can be found using the definition of emf. We can find the distance the car can travel from the definition of work and the cost per kilometer of driving the car this distance by dividing the cost of the required energy by the distance the car has traveled.
(a) Express the power the electric motor must develop in terms of the speed of the car and the friction force:
(b) Because the batteries are in series, the total charge that can be delivered before charging is the same as the charge from a single battery:
(c) Use the definition of emf to express the total electrical energy available in the batteries:

$$
\begin{aligned}
P & =f v=(1.20 \mathrm{kN})(80.0 \mathrm{~km} / \mathrm{h}) \\
& =26.7 \mathrm{~kW}
\end{aligned}
$$

$$
\begin{aligned}
\Delta Q & =I \Delta t=(160 \mathrm{~A} \cdot \mathrm{~h})\left(\frac{3600 \mathrm{~s}}{\mathrm{~h}}\right) \\
& =576 \mathrm{kC}
\end{aligned}
$$

$$
\begin{aligned}
W & =Q \varepsilon=10(576 \mathrm{kC})(12.0 \mathrm{~V}) \\
& =69.12 \mathrm{MJ}=69.1 \mathrm{MJ}
\end{aligned}
$$

(d) Relate the amount of work the batteries can do to the work required

$$
W=f d \Rightarrow d=\frac{W}{f}
$$ to overcome friction:

Substitute numerical values and evaluate $d$ :

$$
d=\frac{69.12 \mathrm{MJ}}{1.20 \mathrm{kN}}=57.6 \mathrm{~km}
$$

(e) The cost per kilometer is the ratio of the cost of the energy to the distance traveled before recharging:

Cost $/ \mathrm{km}=\frac{\left(\frac{\$ 0.0900}{\mathrm{~kW} \cdot \mathrm{~h}}\right) \varepsilon I t}{d}$
Substitute numerical values and calculate the cost per kilometer:

$$
\mathrm{Cost} / \mathrm{km}=\frac{\left(\frac{\$ 0.0900}{\mathrm{~kW} \cdot \mathrm{~h}}\right)(120 \mathrm{~V})(160 \mathrm{~A} \cdot \mathrm{~h})}{5.76 \mathrm{~km}}=\$ 0.300 / \mathrm{km}
$$

68 ... A 100-W heater is designed to operate with an applied voltage of 120 V. (a) What is the heater's resistance, and what current does the heater carry? (b) Show that if the potential difference $V$ across the heater changes by a small amount $\Delta V$, the power $P$ changes by a small amount $\Delta P$, where $\Delta P / P \approx 2 \Delta V / V$. Hint: Approximate the changes by modeling them as differentials, and assume the resistance is constant. (c) Using the Part-(b) result, find the approximate power dissipated in the heater, if the potential difference is decreased to 115 V . Compare your result to the exact answer.

Picture the Problem We can use the definition of power to find the current drawn by the heater and Ohm's law to find its resistance. In Part (b) we'll use the hint to show that $\Delta P / P \approx 2 \Delta V / V$ and in Part (c) use this result to find the approximate power dissipated in the heater if the potential difference is decreased to 115 V .
(a) Use the definition of power to relate the current $I$ drawn by the heater to its power rating $P$ and the potential difference across it $V$ :

Substitute numerical values and evaluate $I$ :

$$
P=I V \Rightarrow I=\frac{P}{V}
$$

$$
I=\frac{100 \mathrm{~W}}{120 \mathrm{~V}}=0.833 \mathrm{~A}
$$

Apply Ohm's law to relate the resistance of the heater to the voltage across it and the current it draws:
(b) Approximating $d P / d V$ as a differential yields:

$$
R=\frac{V}{I}=\frac{120 \mathrm{~V}}{0.833 \mathrm{~A}}=144 \Omega
$$

Express the dependence of $P$ on $V$ :

$$
P=\frac{V^{2}}{R}
$$

Assuming $R$ to be constant, evaluate $d P / d V$ :

$$
\frac{d P}{d V}=\frac{d}{d V}\left[\frac{V^{2}}{R}\right]=\frac{2 V}{R}
$$

Substitute to obtain:

$$
\Delta P \approx \frac{2 V}{R} \Delta V=2 \frac{V^{2}}{R} \frac{\Delta V}{V}=2 P \frac{\Delta V}{V}
$$

Divide both sides of the equation by $P$ to obtain:

$$
\frac{\Delta P}{P}=2 \frac{\Delta V}{V}
$$

(c) Express the approximate power

$$
\begin{align*}
P & \approx P_{0}+\Delta P \\
& =P_{0}+2 P_{0} \frac{\Delta V}{V}  \tag{1}\\
& =P_{0}\left(1+2 \frac{\Delta V}{V}\right)
\end{align*}
$$

its power consumption and the

$$
\frac{d P}{d V} \approx \frac{\Delta P}{\Delta V} \Rightarrow \Delta P \approx \frac{d P}{d V} \Delta V
$$

## Combinations of Resistors

69 - [SSM] If the potential drop from point $a$ to point $b$ (Figure 25-52) is 12.0 V, find the current in each resistor.

Picture the Problem We can apply Ohm's law to find the current through each resistor.

Apply Ohm's law to each of the resistors to find the current

$$
\begin{aligned}
& I_{4}=\frac{V}{4.00 \Omega}=\frac{12.0 \mathrm{~V}}{4.00 \Omega}=3.00 \mathrm{~A} \\
& I_{3}=\frac{V}{3.00 \Omega}=\frac{12.0 \mathrm{~V}}{3.00 \Omega}=4.00 \mathrm{~A}
\end{aligned}
$$ flowing through each:

and

$$
I_{6}=\frac{V}{6.00 \Omega}=\frac{12.0 \mathrm{~V}}{6.00 \Omega}=2.00 \mathrm{~A}
$$

## Remarks: You would find it instructive to use Kirchhoff's junction rule (conservation of charge) to confirm our values for the currents through the three resistors.

70 - If the potential drop between point $a$ and point $b$ (Figure 25-53) is 12.0 V , find the current in each resistor.

Picture the Problem We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We can then add that resistance and the $3.00-\Omega$ resistance to find the equivalent resistance between points $a$ and $b$. Denoting the currents through each resistor with subscripts corresponding to the resistance through which the current flows, we can apply Ohm's law to find those currents.

Express the equivalent resistance of the two resistors in parallel and solve for $R_{\text {eq }, 1}$ :

$$
\frac{1}{R_{\mathrm{eq}, 1}}=\frac{1}{R_{6}}+\frac{1}{R_{2}}=\frac{1}{6.00 \Omega}+\frac{1}{2.00 \Omega}
$$

and

$$
R_{\mathrm{eq}, 1}=1.50 \Omega
$$

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{3}+R_{\mathrm{eq}, 1}=3.00 \Omega+1.50 \Omega \\
& =4.50 \Omega
\end{aligned}
$$

Apply Ohm's law to the network to find $I_{3}$ :

$$
\begin{aligned}
I_{3} & =\frac{V_{a b}}{R_{\mathrm{eq}}}=\frac{12.0 \mathrm{~V}}{4.50 \Omega}=2.667 \mathrm{~A} \\
& =2.67 \mathrm{~A}
\end{aligned}
$$

Find the potential difference across the parallel resistors:

$$
\begin{aligned}
V_{6 \& 2} & =V_{a b}-V_{3} \\
& =12.0 \mathrm{~V}-(2.667 \mathrm{~A})(3.00 \Omega) \\
& =3.999 \mathrm{~V}
\end{aligned}
$$

Use the common potential difference across the resistors in parallel to find the current through each of them:

$$
I_{6}=\frac{V_{6}}{R_{6}}=\frac{3.999 \mathrm{~V}}{6.00 \Omega}=0.667 \mathrm{~A}
$$

and

$$
I_{2}=\frac{V_{6 \& 2}}{R_{2}}=\frac{3.999 \mathrm{~V}}{2.00 \Omega}=2.00 \mathrm{~A}
$$

Remarks: We could have found the currents through the $6.00-\Omega$ and $2.00-\Omega$ resistors by using the fact that the current at the junction between the $3.00-\Omega$ resistor and the parallel branch divides inversely with the resistance in each branch of the parallel network.

71 - (a) Show that the equivalent resistance between point $a$ and point $b$ in Figure 25-54 is $R$. (b) How would adding a fifth resistor that has resistance $R$ between point $c$ and point $d$ effect the equivalent resistance between point $a$ and point $b$ ?

Picture the Problem Note that the resistors between $a$ and $c$ and between $c$ and $b$ are in series as are the resistors between $a$ and $d$ and between $d$ and $b$. Hence, we have two branches in parallel, each branch consisting of two resistors $R$ in series. In Part (b) it will be important to note that the potential difference between point $c$ and point $d$ is zero.
(a) Express the equivalent resistance between points $a$ and $b$

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{R_{a c b}}+\frac{1}{R_{a d b}}=\frac{1}{2 R}+\frac{1}{2 R}
$$

in terms of the equivalent resistances between $a c b$ and $a d b$ :

Solve for $R_{\text {eq }}$ to obtain:

$$
R_{\mathrm{eq}}=R
$$

(b) It would not affect it. Because the potential difference between points c and d is zero, no current would flow through the resistor connected between these two points, and the addition of that resistor would not change the network.

72 •• The battery in Figure 25-55 has negligible internal resistance. Find (a) the current in each resistor and $(b)$ the power delivered by the battery.

Picture the Problem Note that the $2.00-\Omega$ resistors are in parallel with each other and with the $4.00-\Omega$ resistor. We can Apply Kirchhoff's loop rule to relate the current $I_{3}$ drawn from the battery to the emf of the battery and equivalent resistance $R_{\text {eq }}$ of the resistor network. We can find the current through the resistors connected in parallel by applying Kirchhoff's loop rule a second time. In Part (b) we can find the power delivered by the battery from the product of its emf and the current it delivers to the circuit.

## (a) Apply Kirchhoff's loop rule to

 obtain:Find the equivalent resistance of the three resistors in parallel:

$$
\begin{equation*}
\varepsilon-I_{3} R_{\mathrm{eq}}=0 \Rightarrow I_{3}=\frac{\mathcal{E}}{R_{\mathrm{eq}}} \tag{1}
\end{equation*}
$$

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}, 1}} & =\frac{1}{R_{2}}+\frac{1}{R_{2}}+\frac{1}{R_{4}} \\
& =\frac{1}{2.00 \Omega}+\frac{1}{2.00 \Omega}+\frac{1}{4.00 \Omega}
\end{aligned}
$$

and

$$
R_{\mathrm{eq}, 1}=0.800 \Omega
$$

Find the equivalent resistance of $R_{\text {eq }, 1}$ and $R_{3}$ in series:

Substitute numerical values in equation (1) and evaluate $I_{3}$ :

Express the current through each of the parallel resistors in terms of their common potential difference $V$ :

Apply Kirchhoff's loop rule to obtain:

Substituting for $V$ in the equations for $I_{2}$ and $I_{4}$ yields:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{3}+R_{\mathrm{eq}, 1}=3.00 \Omega+0.800 \Omega \\
& =3.80 \Omega
\end{aligned}
$$

$$
\boldsymbol{I}_{3}=\frac{6.00 \mathrm{~V}}{3.80 \Omega}=1.579 \mathrm{~V}=1.58 \mathrm{~A}
$$

$$
I_{2}=\frac{V}{R_{2}} \text { and } I_{4}=\frac{V}{R_{4}}
$$

$$
\varepsilon-I_{3} R_{3}-V=0 \Rightarrow V=\varepsilon-I_{3} R_{3}
$$

$I_{2}=\frac{\varepsilon-I_{3} R_{3}}{R_{2}}$ and $I_{4}=\frac{\varepsilon-I_{3} R_{3}}{R_{4}}$

Substitute numerical values and evaluate $I_{2}$ and $I_{4}$ :

$$
\begin{aligned}
I_{2} & =\frac{6.00 \mathrm{~V}-(1.579 \mathrm{~A})(3.00 \Omega)}{2.00 \Omega} \\
& =0.632 \mathrm{~A}
\end{aligned}
$$

and

$$
\begin{aligned}
I_{4} & =\frac{6.00 \mathrm{~V}-(1.579 \mathrm{~A})(3.00 \Omega)}{4.00 \Omega} \\
& =0.316 \mathrm{~A}
\end{aligned}
$$

(b) $P$ is the product of $\mathcal{E}$ and $I_{3}$ :

$$
P=\varepsilon I_{3}=(6.00 \mathrm{~V})(1.579 \mathrm{~A})=9.47 \mathrm{~W}
$$

## Remarks: Note that the currents $I_{3}, I_{2}$, and $I_{4}$ satisfy Kirchhoff's junction rule.

73 •• [SSM] A 5.00-V power supply has an internal resistance of $50.0 \Omega$. What is the smallest resistor that can be put in series with the power supply so that the voltage drop across the resistor is larger than 4.50 V ?

Picture the Problem Let $r$ represent the resistance of the internal resistance of the power supply, $\varepsilon$ the emf of the power supply, $R$ the resistance of the external resistor to be placed in series with the power supply, and $I$ the current drawn from the power supply. We can use Ohm's law to express the potential difference across $R$ and apply Kirchhoff's loop rule to express the current through $R$
 in terms of $\mathcal{E}, r$, and $R$.

Express the potential difference $\quad V_{R}=I R$
across the resistor whose resistance is $R$ :

Apply Kirchhoff's loop rule to the circuit to obtain:

$$
\varepsilon-I r-I R=0 \Rightarrow I=\frac{\varepsilon}{r+R}
$$

Substitute in equation (1) to obtain:

Substitute numerical values and evaluate $R$ :

$$
V_{R}=\left(\frac{\varepsilon}{r+R}\right) R \Rightarrow R=\frac{V_{R} r}{\varepsilon-V_{R}}
$$

$$
R=\frac{(4.50 \mathrm{~V})(50.0 \Omega)}{5.00 \mathrm{~V}-4.50 \mathrm{~V}}=0.45 \mathrm{k} \Omega
$$

74 •• You have been handed an unknown battery. Using your multimeter, you determine that when a $5.00-\Omega$ resistor is connected across the battery's terminals, the current in the battery is 0.500 A . When this resistor is replaced by an $11.0-\Omega$ resistor, the current drops to 0.250 A . From this data, find (a) the emf and $(b)$ internal resistance of your battery.

Picture the Problem We can apply Kirchhoff's loop rule to the two circuits described in the problem statement and solve the resulting equations simultaneously for $r$ and $\mathcal{E}$.
(a) and (b) Apply Kirchhoff's loop rule to the two circuits to obtain:

$$
\mathcal{E}-I_{1} r-I_{1} R_{5}=0
$$

and

$$
\varepsilon-I_{2} r-I_{2} R_{11}=0
$$

Substitute numerical values and simplify to obtain:

$$
\begin{equation*}
\mathcal{E}-(0.500 \mathrm{~A}) r=2.50 \mathrm{~V} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{E}-(0.250 \mathrm{~A}) r=2.75 \mathrm{~V} \tag{2}
\end{equation*}
$$

Solve equations (1) and (2)

$$
\mathcal{E}=3.00 \mathrm{~V} \text { and } r=1.00 \Omega
$$

Find the equivalent resistance of the $12.0-\Omega$ resistor in series with the

$$
\begin{aligned}
R_{\mathrm{eq}, 3} & =R_{6}+R_{12} \\
& =6.00 \Omega+12.0 \Omega=18.0 \Omega
\end{aligned}
$$ $6.00-\Omega$ resistor:

Finally, find the equivalent resistance of $R_{\text {eq, } 3}$ in parallel with $R_{\text {eq }, 2}$ :

$$
\begin{aligned}
R_{\mathrm{eq}} & =\frac{R_{\mathrm{eq}, 2} R_{\mathrm{eq}, 3}}{R_{\mathrm{eq}, 2}+R_{\mathrm{eq}, 3}}=\frac{(9.00 \Omega)(18.0 \Omega)}{9.00 \Omega+18.0 \Omega} \\
& =6.00 \Omega
\end{aligned}
$$

(b) Apply Ohm's law to the upper branch to find the current

$$
I_{\substack{\text { upper } \\ \text { branch }}}=I_{12}=I_{6}:
$$

$$
\begin{aligned}
I_{\text {upper }}^{\text {branch }} & =I_{12}=I_{6}=\frac{V_{a b}}{R_{\text {eq, } 3}}=\frac{12.0 \mathrm{~V}}{18.0 \Omega} \\
& =667 \mathrm{~mA}
\end{aligned}
$$

Apply Ohm's law to the lower branch to find the current

$$
I_{\substack{\text { lower } \\ \text { branch }}}=I_{6.00-\Omega \text { resistor }}^{\text {in series }}
$$

Express the current through the $6.00-\Omega$ resistors in parallel:

$$
\begin{aligned}
I_{\substack{\text { lower } \\
\text { branch }}} & =I_{\substack{6.00-\Omega \text { resistor } \\
\text { in series }}}=\frac{V_{a b}}{R_{\text {eq }, 2}}=\frac{12.0 \mathrm{~V}}{9.00 \Omega} \\
& =1.33 \mathrm{~A}
\end{aligned}
$$

$$
\begin{aligned}
I_{\substack{6.00 \Omega \text { resistors } \\
\text { in parallel }}} & =\frac{1}{2} I_{l}=\frac{1}{2}(1.33 \mathrm{~A}) \\
& =667 \mathrm{~mA}
\end{aligned}
$$

In summary, the current in both the $6.00-\Omega$ and the $12.0-\Omega$ resistor in the upper branch is 667 mA . The current in each $6.00-\Omega$ resistor in the parallel combination in the lower branch is 667 mA . The current in the $6.00-\Omega$ resistor on the right in the lower branch is 1.33 A .
$76 \quad \bullet \quad(a)$ Find the equivalent resistance between point $a$ and point $b$ in Figure 25-57. (b) If the potential drop between point $a$ and point $b$ is 12.0 V , find the current in each resistor.

Picture the Problem Assign currents in each of the resistors as shown in the diagram. We can simplify the network by first replacing the resistors that are in parallel by their equivalent resistance. We'll then have a parallel network with two resistors in series in each branch and can use the expressions for resistors in series to simplify the network to two resistors in parallel. The equivalent resistance between points $a$ and $b$ will be the single resistor equivalent to these two resistors. In Part (b) we'll use the fact that the potential difference across the upper branch is the same as the potential difference across the lower branch, in conjunction with Ohm's law, to find the currents through each resistor.

(a) Find the equivalent resistance of the resistors in parallel in the upper branch:

$$
\begin{aligned}
R_{\mathrm{eq}, 1} & =\frac{\left(R_{2}+R_{4}\right) R_{4}}{\left(R_{2}+R_{4}\right)+R_{4}} \\
& =\frac{(6.00 \Omega)(4.00 \Omega)}{2.00 \Omega+4.00 \Omega+4.00 \Omega} \\
& =2.40 \Omega
\end{aligned}
$$

Find the equivalent resistance of the $6.00-\Omega$ resistor is in series with $R_{\mathrm{eq}, 1}$ :

$$
\begin{aligned}
R_{\mathrm{eq}, 2} & =R_{6}+R_{\mathrm{eq}, 1} \\
& =6.00 \Omega+2.40 \Omega=8.40 \Omega
\end{aligned}
$$

Express and evaluate the equivalent resistance of the resistors in parallel in the lower branch and solve for $R_{\text {eq, } 2}$ :

Find the equivalent resistance of the $4.00-\Omega$ resistor is in series with $R_{\text {eq }, 2}$ :

$$
\begin{aligned}
R_{\mathrm{eq}, 3} & =R_{4}+R_{\mathrm{eq}, 2} \\
& =4.00 \Omega+4.00 \Omega=8.00 \Omega
\end{aligned}
$$

Finally, find the equivalent resistance of $R_{\mathrm{eq}, 2}$ in parallel with $R_{\text {eq, },}$ :

$$
\begin{aligned}
R_{\mathrm{eq}} & =\frac{R_{\mathrm{eq}, 2} R_{\mathrm{eq}, 3}}{R_{\mathrm{eq}, 2}+R_{\mathrm{eq}, 3}}=\frac{(8.40 \Omega)(8.00 \Omega)}{8.40 \Omega+8.00 \Omega} \\
& =4.10 \Omega
\end{aligned}
$$

(b) Apply Ohm's law to the upper branch to find the current $I_{1}$ through the $6.00-\Omega$ resistor:

$$
I_{1}=\frac{V_{a b}}{R_{\mathrm{eq}, 2}}=\frac{12.0 \mathrm{~V}}{8.40 \Omega}=1.43 \mathrm{~A}
$$

Find the potential difference across the $4.00-\Omega$ and $6.00-\Omega$ parallel combination in the upper branch:

$$
\begin{aligned}
V_{4 \text { and } 6} & =12.0 \mathrm{~V}-V_{6}=12.0 \mathrm{~V}-I_{1} R_{6} \\
& =12.0 \mathrm{~V}-(1.43 \mathrm{~A})(6.00 \Omega) \\
& =3.43 \mathrm{~V}
\end{aligned}
$$

Apply Ohm's law to find $I_{4}$ :

$$
I_{4}=\frac{V_{6}}{R_{6}}=\frac{3.43 \mathrm{~V}}{6.00 \Omega}=0.57 \mathrm{~A}
$$

Apply Ohm's law to find $I_{3}$ :

$$
I_{3}=\frac{V_{4}}{R_{4}}=\frac{3.43 \mathrm{~V}}{4.00 \Omega}=0.86 \mathrm{~A}
$$

Apply Ohm's law to the lower branch to find $I_{2}$ :

$$
I_{2}=\frac{V_{a b}}{R_{\mathrm{eq}, 2}}=\frac{12.0 \mathrm{~V}}{8.00 \Omega}=1.50 \mathrm{~A}
$$

Find the potential difference across the $8.00-\Omega$ and $8.00-\Omega$ parallel combination in the lower branch:

$$
\begin{aligned}
V_{8 \text { and } 8} & =12.0 \mathrm{~V}-I_{2} R_{4} \\
& =12.0 \mathrm{~V}-(1.50 \mathrm{~A})(4.00 \Omega) \\
& =6.0 \mathrm{~V}
\end{aligned}
$$

Apply Ohm's law to find $I_{5}=I_{6}$ :

$$
I_{5}=I_{6}=\frac{V_{8 \text { and } 8}}{8.00 \Omega}=\frac{6.0 \mathrm{~V}}{8.00 \Omega}=0.75 \mathrm{~A}
$$

In summary, the current through the $6.00-\Omega$ resistor is 1.43 A , the current through the lower $4.00-\Omega$ resistor is 1.50 A , the current through the $4.00-\Omega$ resistor that is in parallel with the $2.00-\Omega$ and $4.00-\Omega$ resistors is 0.86 A , the current through the $2.00-\Omega$ and $4.00-\Omega$ resistors that are in series is 0.57 A , and the currents through the two $8.00-\Omega$ resistors are 0.75 A .

77 •• [SSM] A length of wire has a resistance of $120 \Omega$. The wire is cut into pieces that have the same length, and then the wires are connected in parallel. The resistance of the parallel arrangement is $1.88 \Omega$. Find the number of pieces into which the wire was cut.

Picture the Problem We can use the equation for $N$ identical resistors connected in parallel to relate $N$ to the resistance $R$ of each piece of wire and the equivalent resistance $R_{\text {eq }}$.

Express the resistance of the $N$
pieces connected in parallel:
$\frac{1}{R_{\mathrm{eq}}}=\frac{N}{R}$
where $R$ is the resistance of one of the $N$ pieces.

Relate the resistance of one of the $N$ pieces to the resistance of the wire:

$$
R=\frac{R_{\text {wire }}}{N}
$$

Substitute for $R$ to obtain:

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{N^{2}}{R_{\mathrm{wire}}} \Rightarrow N=\sqrt{\frac{R_{\mathrm{wire}}}{R_{\mathrm{eq}}}}
$$

Substitute numerical values and evaluate $N$ :

$$
N=\sqrt{\frac{120 \Omega}{1.88 \Omega}}=8 \text { pieces }
$$

78 •• A parallel combination of an $8.00-\Omega$ resistor and a resistor of unknown resistance is connected in series with a $16.0-\Omega$ resistor and an ideal battery. This circuit is then disassembled and the three resistors are then connected in series with each other and the same battery. In both arrangements, the current through the $8.00-\Omega$ resistor is the same. What is the resistance of the unknown resistor?

Picture the Problem Assigning currents as shown in the diagram of the first arrangement of resistors will allow us to apply Kirchhoff's loop rule and obtain an expression for $I_{2}$.


The same current $I_{2}$ flows in all three resistors when they are connected in series (see the following diagram). Applying Kirchhoff's loop rule to this circuit will yield a second expression for $I_{2}$ that we can solve simultaneously with the expression obtained for $I_{2}$ from the circuit shown above. We can solve the resulting equation for $R$.


Apply Kirchhoff's loop rule to the first arrangement of the resistors:

Find the equivalent resistance of the first arrangement of the resistors:

Substitute for $R_{\text {eq }}$ to obtain:

Apply Kirchhoff's loop rule to the loop containing $R$ and the $8.00-\Omega$ resistor:

Solving for $I_{2}$ yields:

Substitute for $I_{1}$ and simplify to obtain:

## second arrangement of the resistors:

Apply Kirchhoff's loop rule to the

Solve for $I_{2}$ to obtain:

Find the equivalent resistance of the second arrangement of the resistors:
$\mathcal{E}-I_{1} R_{\mathrm{eq}}=0 \Rightarrow I_{1}=\frac{\mathcal{E}}{R_{\mathrm{eq}}}$
where $I_{1}$ is the current supplied by the battery.

$$
\begin{aligned}
R_{\mathrm{eq}} & =\frac{(8.00 \Omega) R}{8.00 \Omega+R}+16.0 \Omega \\
& =\frac{(24.0 \Omega) R+128 \Omega^{2}}{R+8.00 \Omega}
\end{aligned}
$$

$$
\begin{aligned}
I_{1} & =\frac{\varepsilon}{\frac{(24.0 \Omega) R+128 \Omega^{2}}{R+8.00 \Omega}} \\
& =\frac{\varepsilon(R+8.00 \Omega)}{(24.0 \Omega) R+128 \Omega^{2}}
\end{aligned}
$$

$-(8.00 \Omega) I_{2}+R I_{3}=0$
or, because $I_{1}=I_{2}+I_{3}$,
$-(8.00 \Omega) I_{2}+R\left(I_{1}-I_{2}\right)=0$
$I_{2}=\frac{R}{R+8.00 \Omega} I_{1}$

$$
\begin{aligned}
I_{2} & =\left(\frac{R}{R+8.00 \Omega}\right)\left(\frac{\varepsilon(R+8.00 \Omega)}{(24.0 \Omega) R+128 \Omega^{2}}\right) \\
& =\frac{\varepsilon R}{(24.0 \Omega) R+128 \Omega^{2}}
\end{aligned}
$$

$\mathcal{E}-I_{2} R_{\text {eq }}^{\prime}=0$
where $I_{2}$ is the current supplied by the battery and $R_{\text {eq }}^{\prime}$ is the equivalent resistance of the series arrangement of the resistors.

$$
I_{2}=\frac{\varepsilon}{R_{\mathrm{eq}}^{\prime}}
$$

$$
\begin{aligned}
R_{\mathrm{eq}}^{\prime} & =8.00 \Omega+R+16.0 \Omega \\
& =R+24.0 \Omega
\end{aligned}
$$

Substitute for $R_{\text {eq }}^{\prime}$ to obtain:

$$
I_{2}=\frac{\varepsilon}{R+24.0 \Omega}
$$

Equating the two expressions for $I_{2}$ yields:

$$
\frac{\varepsilon R}{(24.0 \Omega) R+128 \Omega^{2}}=\frac{\varepsilon}{R+24.0 \Omega}
$$

Solving for $R$ yields:

$$
R=11.3 \Omega
$$

79 •• For the network shown in Figure 25-58, let $R_{\mathrm{ab}}$ denote the equivalent resistance between terminals $a$ and $b$. Find (a) $R_{3}$, so that $R_{\mathrm{ab}}=R_{1}$, (b) $R_{2}$, so that $R_{\mathrm{ab}}=R_{3}$; and (c) $R_{1}$, so that $R_{\mathrm{ab}}=R_{1}$.

Picture the Problem We can find the equivalent resistance $R_{a b}$ between points $a$ and $b$ and then set this resistance equal, in turn, to $R_{1}, R_{3}$, and $R_{1}$ and solve for $R_{3}$, $R_{2}$, and $R_{1}$, respectively.
(a) Express the equivalent resistance between points $a$ and

$$
R_{a b}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3}
$$

$b$ :

Equate this expression to $R_{1}$ :

$$
R_{1}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3}
$$

Solving for $R_{3}$ yields:

$$
R_{3}=\frac{R_{1}^{2}}{R_{1}+R_{2}}
$$

(b) Set $R_{3}$ equal to $R_{a b}$ :

$$
R_{3}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3}
$$

Solve for $R_{2}$ to obtain:

$$
R_{2}=0
$$

(c) Set $R_{1}$ equal to $R_{a b}$ and rewrite the result as a quadratic equation in $R_{1}$ :

$$
R_{1}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3}
$$

or

$$
R_{1}^{2}-R_{3} R_{1}-R_{2} R_{3}=0
$$

Solving the quadratic equation for $R_{1}$ yields:

$$
R_{1}=\sqrt{\frac{R_{3}+\sqrt{R_{3}^{2}+4 R_{2} R_{3}}}{2}}
$$

where we've used the positive sign because resistance is a non-negative quantity.
$80 \quad$ - Check your results for Problem 79 using the following specific values:
(a) $R_{1}=4.00 \Omega, R_{2}=6.00 \Omega$; (b) $R_{1}=4.00 \Omega, R_{3}=3.00 \Omega$; and
(c) $R_{2}=6.00 \Omega, R_{3}=3.00 \Omega$.

Picture the Problem We can substitute the given resistances in the equations derived in Problem 79.
(a) For $R_{1}=4.00 \Omega$ and $R_{2}=6.00 \Omega$ :

$$
\begin{aligned}
R_{3} & =\frac{R_{1}^{2}}{R_{1}+R_{2}}=\frac{(4.00 \Omega)^{2}}{4.00 \Omega+6.00 \Omega} \\
& =1.60 \Omega
\end{aligned}
$$

and

$$
\begin{aligned}
R_{a b} & =\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3} \\
& =\frac{(4.00 \Omega)(6.00 \Omega)}{4.00 \Omega+6.00 \Omega}+1.60 \Omega \\
& =4.00 \Omega
\end{aligned}
$$

(b) For $R_{1}=4.00 \Omega$ and $R_{3}=3.00 \Omega: \quad R_{2}=0$
and

$$
\begin{aligned}
R_{a b} & =\frac{R_{1}(0)}{R_{1}+0}+R_{3}=0+3.00 \Omega \\
& =3.00 \Omega
\end{aligned}
$$

(c) For $R_{2}=6.00 \Omega$ and $R_{3}=3.00 \Omega$ :

$$
R_{1}=\frac{3.00 \Omega+\sqrt{(3.00 \Omega)^{2}+4(6.00 \Omega)(3.00 \Omega)}}{2}=\frac{3.00 \Omega+9.00 \Omega}{2}=6.00 \Omega
$$

and

$$
R_{a b}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}+R_{3}=\frac{(6.00 \Omega)(6.00 \Omega)}{6.00 \Omega+6.00 \Omega}+3.00 \Omega=6.00 \Omega
$$

## Kirchhoff's Rules

81 - [SSM] In Figure 25-59, the battery's emf is 6.00 V and $R$ is $0.500 \Omega$. The rate of Joule heating in $R$ is 8.00 W . (a) What is the current in the circuit? (b) What is the potential difference across $R$ ? (c) What is the resistance $r$ ?

Picture the Problem We can relate the current provided by the source to the rate of Joule heating using $P=I^{2} R$ and use Ohm's law and Kirchhoff's rules to find the potential difference across $R$ and the value of $r$.
(a) Relate the current $I$ in the circuit to rate at which energy is being

$$
P=I^{2} R \Rightarrow I=\sqrt{\frac{P}{R}}
$$

dissipated in the form of Joule heat:
Substitute numerical values and evaluate $I$ :

$$
I=\sqrt{\frac{8.00 \mathrm{~W}}{0.500 \Omega}}=4.00 \mathrm{~A}
$$

(b) Apply Ohm's law to find $V_{R}$ :

$$
V_{R}=I R=(4.00 \mathrm{~A})(0.500 \Omega)=2.00 \mathrm{~V}
$$

(c) Apply Kirchhoff's loop rule to obtain:

$$
\varepsilon-I r-I R=0 \Rightarrow r=\frac{\varepsilon-I R}{I}=\frac{\varepsilon}{I}-R
$$

Substitute numerical values and evaluate $r$ :

$$
r=\frac{6.00 \mathrm{~V}}{4.00 \mathrm{~A}}-0.500 \Omega=1.00 \Omega
$$

82 - The batteries in the circuit in Figure 25-60 have negligible internal resistance. (a) Find the current using Kirchhoff's loop rule. (b) Find the power delivered to or supplied by each battery. (c) Find the rate of Joule heating in each resistor.

Picture the Problem Assume that the current flows clockwise in the circuit and let $\varepsilon_{1}$ represent the $12.0-\mathrm{V}$ source and $\mathcal{E}_{2}$ the $6.00-\mathrm{V}$ source. We can apply Kirchhoff's loop rule (conservation of energy) to this series circuit to relate the current to the emfs of the sources and the resistance of the circuit. In Part (b) we can find the power delivered or absorbed by each source using $P=\boldsymbol{E I}$ and in Part (c) the rate of Joule heating using $P=I^{2} R$.
(a) Apply Kirchhoff’s loop rule to

$$
\mathcal{E}_{1}-I R_{2}-\mathcal{E}_{2}-I R_{4}=0
$$ the circuit to obtain:

Solving for $I$ yields:

$$
I=\frac{\mathcal{E}_{1}-\mathcal{E}_{2}}{R_{2}+R_{4}}
$$

Substitute numerical values and evaluate $I$ :

$$
I=\frac{12.0 \mathrm{~V}-6.00 \mathrm{~V}}{2.00 \Omega+4.00 \Omega}=1.0 \mathrm{~A}
$$

(b) Express the power delivered /absorbed by each source in terms of its emf and the current drawn from or forced through it:
(c) Express the rate of Joule heating in terms of the current through and the resistance of each resistor:

$$
P_{12}=\varepsilon_{12} I=(12.0 \mathrm{~V})(1.0 \mathrm{~A})=12 \mathrm{~W}
$$

and

$$
P_{6}=\mathcal{E}_{6} I=(-6.00 \mathrm{~V})(1.0 \mathrm{~A})=-6.0 \mathrm{~W}
$$

where the minus sign means that this source is absorbing power.

$$
P_{2}=I^{2} R_{2}=(1.0 \mathrm{~A})^{2}(2.00 \Omega)=2.0 \mathrm{~W}
$$

and

$$
P_{4}=I^{2} R_{4}=(1.0 \mathrm{~A})^{2}(4.00 \Omega)=4.0 \mathrm{~W}
$$

83 •• An old car battery that has an emf of $\mathcal{E}=11.4 \mathrm{~V}$ and an internal resistance of $50.0 \mathrm{~m} \Omega$ is connected to a $2.00-\Omega$ resistor. In an attempt to recharge the battery, you connect a second battery that has an emf of $\mathcal{E}=12.6 \mathrm{~V}$ and an internal resistance of $10.0 \mathrm{~m} \Omega$ in parallel with the first battery and the resistor with a pair of jumper cables. (a) Draw a diagram of your circuit. (b) Find the current in each branch of this circuit. (c) Find the power supplied by the second battery and discuss where this power is delivered. Assume that the emfs and internal resistances of both batteries remain constant.

Picture the Problem The circuit is shown in the diagram for Part (a). (b) Let $\mathcal{E}_{1}$ and $r_{1}$ denote the emf of the old battery and its internal resistance, $\varepsilon_{2}$ and $r_{2}$ the emf of the second battery and its internal resistance, and $R$ the load resistance. Let $I_{1}, I_{2}$, and $I_{\mathrm{R}}$ be the currents. We can apply Kirchhoff's rules to determine the unknown currents. (c) The power supplied by the second battery is given by $P_{2}=\left(\varepsilon_{2}-I_{2} r_{2}\right) I_{2}$.
(a) The circuit diagram is shown to the right:

(b) Apply Kirchhoff's junction rule $\quad I_{1}+I_{2}=I_{R}$ to junction 1 to obtain:

Apply Kirchhoff's loop rule to loop 1 to obtain:
$\varepsilon_{1}-r_{1} I_{1}-R I_{R}=0$
or
$11.4 \mathrm{~V}-(0.0500 \Omega) I_{1}-(2.00 \Omega) I_{R}=0$

Apply Kirchhoff’s loop rule to loop 2
$\mathcal{E}_{2}-r_{2} I_{2}-R I_{R}=0$
to obtain:

Solve the equations in $I_{1}, I_{2}$, and $I_{3}$ simultaneously to obtain:
(c) The power supplied by the second battery is given by:

Substitute numerical values and evaluate $P_{2}$ :

The power delivered to the first battery is given by:
$I_{1}=-18.971 \mathrm{~A}=-19.0 \mathrm{~A}$,
$I_{2}=25.145 \mathrm{~A}=25.1 \mathrm{~A}$, and
$I_{R}=6.174 \mathrm{~A}=6.17 \mathrm{~A}$
where the minus sign for $I_{1}$ means that the current flows in the direction opposite to the direction we arbitrarily chose, i.e., the battery is being charged.

$$
P_{2}=\left(\varepsilon_{2}-I_{2} r_{2}\right) I_{2}
$$

$$
\begin{aligned}
P_{2} & =\left\lfloor 12.6 \mathrm{~V}-(25.145 \mathrm{~A})^{2}(50.0 \mathrm{~m} \Omega)\right\rfloor \\
& \times(25.145 \mathrm{~A}) \\
& =311 \mathrm{~W}
\end{aligned}
$$

$$
P_{1}=\varepsilon_{1} I_{1}+I_{1}^{2} r_{1}
$$

Substitute numerical values and evaluate $P_{1}$ :

$$
\begin{aligned}
P_{1}= & (11.4 \mathrm{~V})(18.971 \mathrm{~A}) \\
& +(18.971 \mathrm{~A})^{2}(0.0500 \Omega) \\
= & 216 \mathrm{~W}
\end{aligned}+18.0 \mathrm{~W}=234 \mathrm{~W}
$$

Find the power dissipated in the load resistance $R$ :

$$
\begin{aligned}
P_{R} & =I_{R}^{2} R=(6.174 \mathrm{~A})^{2}(2.00 \Omega) \\
& =76.2 \mathrm{~W}
\end{aligned}
$$

In summary, battery 2 supplies 311 W .234 W is delivered to Battery 1 , and of that $234 \mathrm{~W}, 216 \mathrm{~W}$ goes into recharging battery 1 and 18.0 W is dissipated by the internal resistance. In addition, 76.2 W is delivered to the $2.00-\Omega$ resistor.

> Remarks: Note that the sum of the power dissipated in the internal and load resistances and that absorbed by the second battery is the same (within round off errors) as that delivered by the first battery ... just as we would expect from conservation of energy.

84 •• In the circuit in Figure 25-61, the reading of the ammeter is the same when both switches are open and when both switches are closed. What is the unknown resistance $R$ ?

Picture the Problem Note that when both switches are closed the $50.0-\Omega$ resistor is shorted. With both switches open, we can apply Kirchhoff's loop rule to find the current $I$ in the $100-\Omega$ resistor. With the switches closed, the $100-\Omega$ resistor and $R$ are in parallel. Hence, the potential difference across them is the same and we can express the current $I_{100}$ in terms of the current $I_{\text {tot }}$ flowing into the parallel branch whose resistance is $R$, and the resistance of the $100-\Omega$ resistor. $I_{\mathrm{tot}}$, in turn, depends on the equivalent resistance of the closed-switch circuit, so we can express $I_{100}=I$ in terms of $R$ and solve for $R$.

Apply Kirchhoff's loop rule to a loop around the outside of the circuit with both switches open:

Solving for $I$ yields:

Relate the potential difference across the $100-\Omega$ resistor to the potential difference across $R$ when both switches are closed:

$$
\varepsilon-(300 \Omega) I-(100 \Omega) I-(50.0 \Omega) I=0
$$

$$
I=\frac{\varepsilon}{450 \Omega}=\frac{1.50 \mathrm{~V}}{450 \Omega}=3.33 \mathrm{~mA}
$$

$$
\begin{equation*}
(100 \Omega) I_{100}=R I_{R} \tag{1}
\end{equation*}
$$

Apply Kirchhoff's junction rule at the junction to the left of the $100-\Omega$ resistor and $R$ :

Substituting for $I_{\mathrm{R}}$ in equation (1) yields:

Express the current $I_{\text {tot }}$ drawn from the source with both switches closed:

Express the equivalent resistance when both switches are closed:

Substitute for $R_{\text {eq }}$ to obtain:

Substituting for $I_{\text {tot }}$ in equation (2) yields:

$$
\begin{aligned}
& I_{\mathrm{tot}}=I_{100}+I_{R} \\
& \text { or } \\
& I_{R}=I_{\text {tot }}-I_{100}
\end{aligned}
$$

where $I_{\mathrm{tot}}$ is the current drawn from the source when both switches are closed.

$$
(100 \Omega) I_{100}=R\left(I_{\text {tot }}-I_{100}\right)
$$

or

$$
\begin{equation*}
I_{100}=\frac{R I_{\mathrm{tot}}}{R+100 \Omega} \tag{2}
\end{equation*}
$$

$$
I_{\mathrm{tot}}=\frac{\mathcal{E}}{R_{\mathrm{eq}}}
$$

$$
R_{\mathrm{eq}}=\frac{(100 \Omega) R}{R+100 \Omega}+300 \Omega
$$

$$
I_{\text {tot }}=\frac{1.5 \mathrm{~V}}{\frac{(100 \Omega) R}{R+100 \Omega}+300 \Omega}
$$

$$
\begin{aligned}
I_{100} & =\frac{R}{R+100 \Omega}\left(\frac{1.50 \mathrm{~V}}{\frac{(100 \Omega) R}{R+100 \Omega}+300 \Omega}\right) \\
& =\frac{(1.50 \mathrm{~V}) R}{(400 \Omega) R+30,000 \Omega^{2}}=3.33 \mathrm{~mA}
\end{aligned}
$$

Solving for $R$ yields:

$$
R=600 \Omega
$$

Remarks: Note that we can also obtain the result in the third step by applying Kirchhoff's loop rule to the parallel branch of the circuit.

85 •• [SSM] In the circuit shown in Figure 25-62, the batteries have negligible internal resistance. Find (a) the current in each branch of the circuit, $(b)$ the potential difference between point $a$ and point $b$, and $(c)$ the power supplied by each battery.

Picture the Problem Let $I_{1}$ be the current delivered by the left battery, $I_{2}$ the current delivered by the right battery, and $I_{3}$ the current through the $6.00-\Omega$ resistor, directed down. We can apply Kirchhoff's rules to obtain three equations
that we can solve simultaneously for $I_{1}, I_{2}$, and $I_{3}$. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points $a$ and $b$ and the power delivered by both the sources.
(a) Apply Kirchhoff's junction rule $\quad I_{4 \Omega}+I_{3 \Omega}=I_{6 \Omega}$ at junction $a$ :

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

$$
\begin{gathered}
12.0 \mathrm{~V}-(4.00 \Omega) I_{4 \Omega}+(3.00 \Omega) I_{3 \Omega} \\
-12.0 \mathrm{~V}=0
\end{gathered}
$$

$$
\begin{align*}
& \text { or }  \tag{2}\\
& -(4.00 \Omega) I_{4 \Omega}+(3.00 \Omega) I_{3 \Omega}=0
\end{align*}
$$

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$
\begin{align*}
& 12.0 \mathrm{~V}-(4.00 \Omega) I_{4 \Omega} \\
& \quad-(6.00 \Omega) I_{6 \Omega}=0 \tag{3}
\end{align*}
$$

Solving equations (1), (2), and (3) simultaneously yields:

$$
I_{4 \Omega}=0.667 \mathrm{~A}, I_{3 \Omega}=0.889 \mathrm{~A},
$$

and

$$
I_{6 \Omega}=1.56 \mathrm{~A}
$$

(b) Apply Ohm's law to find the potential difference between points $a$ and $b$ :

$$
\begin{aligned}
V_{a b} & =(6.00 \Omega) I_{6 \Omega}=(6.00 \Omega)(1.56 \mathrm{~A}) \\
& =9.36 \mathrm{~V}
\end{aligned}
$$

(c) Express the power delivered by the $12.0-\mathrm{V}$ battery in the left-hand branch of the circuit:

Express the power delivered by the $12.0-\mathrm{V}$ battery in the right-hand branch of the circuit:

$$
\begin{aligned}
P_{\text {left }} & =\mathcal{E} I_{4 \Omega} \\
& =(12.0 \mathrm{~V})(0.667 \mathrm{~A})=8.00 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P_{\text {right }} & =\mathcal{E} I_{3 \Omega} \\
& =(12.0 \mathrm{~V})(0.889 \mathrm{~A})=10.7 \mathrm{~W}
\end{aligned}
$$

86 •• In the circuit shown in Figure 25-63, the batteries have negligible internal resistance. Find $(a)$ the current in each branch of the circuit, $(b)$ the potential difference between point $a$ and point $b$, and (c) the power supplied by each battery.

Picture the Problem Let $I_{2 \Omega}$ be the current delivered by the $7.00-\mathrm{V}$ battery, $I_{3 \Omega}$ the current delivered by the $5-\mathrm{V}$ battery, and $I_{1 \Omega}$, directed up, the current through the $1.00-\Omega$ resistor. We can apply Kirchhoff's rules to obtain three equations that we can solve simultaneously for $I_{1}, I_{2}$, and $I_{3}$. Knowing the currents in each branch, we can use Ohm's law to find the potential difference between points $a$ and $b$ and the power delivered by both the sources.
(a) Apply Kirchhoff's junction rule $\quad I_{2 \Omega}=I_{3 \Omega}+I_{1 \Omega}$ at junction $a$ :

Apply Kirchhoff's loop rule to a loop around the outside of the circuit to obtain:

Apply Kirchhoff's loop rule to a loop around the left-hand branch of the circuit to obtain:

$$
\begin{align*}
& 7.00 \mathrm{~V}-(2.00 \Omega) I_{2 \Omega} \\
& -(1.00 \Omega) I_{1 \Omega}=0 \tag{2}
\end{align*}
$$

$$
\begin{gathered}
7.00 \mathrm{~V}-(2.00 \Omega) I_{2 \Omega}-(3.00 \Omega) I_{3 \Omega} \\
+5.00 \mathrm{~V}=0
\end{gathered}
$$

or

$$
\begin{equation*}
(2.00 \Omega) I_{2 \Omega}+(3.00 \Omega) I_{3 \Omega}=12.0 \mathrm{~V} \tag{3}
\end{equation*}
$$

Solve equations (1), (2), and (3) simultaneously to obtain:

$$
I_{2 \Omega}=3.00 \mathrm{~A}, I_{3 \Omega}=2.00 \mathrm{~A},
$$

and

$$
I_{1 \Omega}=1.00 \mathrm{~A}
$$

(b) Apply Ohm's law to find the potential difference between points $a$ and $b$ :

$$
\begin{aligned}
V_{a b} & =-5.00 \mathrm{~V}+(3.00 \Omega) I_{3 \Omega} \\
& =-5.00 \mathrm{~V}+(3.00 \Omega)(2.00 \mathrm{~A}) \\
& =1.00 \mathrm{~V}
\end{aligned}
$$

(c) Express the power delivered by the $7.00-\mathrm{V}$ battery:

$$
\begin{aligned}
P_{7 \mathrm{~V}} & =\varepsilon I_{2 \Omega}=(7.00 \mathrm{~V})(3.00 \mathrm{~A}) \\
& =21.0 \mathrm{~W}
\end{aligned}
$$

(c) Express the power delivered by the $5.00-\mathrm{V}$ battery:

$$
\begin{aligned}
P_{5 \mathrm{~V}} & =\varepsilon I_{3 \Omega}=(5.00 \mathrm{~V})(2.00 \mathrm{~A}) \\
& =10.0 \mathrm{~W}
\end{aligned}
$$

87 ••• Two identical batteries, each having an emf $\boldsymbol{\varepsilon}$ and an internal resistance $r$, can be connected across a resistance $R$ with the batteries connected either in series or in parallel. In each situation, determine explicitly whether the power supplied to $R$ is greater when $R$ is less than $r$ or when $R$ is greater than $r$.

Picture the Problem The series and parallel connections are shown below. The power supplied to a resistor whose resistance is $R$ when the current through it is $I$ is given by $P=I^{2} R$. We can use Kirchhoff's rules to find the currents $I_{\mathrm{p}}$ and $I_{\mathrm{s}}$ and, hence, $P_{\mathrm{s}}$ and $P_{\mathrm{p}}$, and then use the inequalities $R>r$ and $R<r$ to show that if $R>r$, then $P_{\mathrm{p}}<P_{\mathrm{s}}$ and if $R<r$, then $P_{\mathrm{p}}>P_{\mathrm{s}}$. The steps in the proofs that follow were motivated by having started with the desired outcome and then working backward; for example, assuming that $P_{\mathrm{p}}<P_{\mathrm{s}}$ and showing that $R>r$.


The power $P_{\mathrm{s}}$ supplied to $R$ in the series circuit is given by:

Apply Kirchhoff's loop rule to obtain:

Solving for $I_{\mathrm{s}}$ yields:

Substitute for $I_{\mathrm{s}}$ and simplify to obtain:

The power $P_{\mathrm{p}}$ supplied to $R$ in the series circuit is given by:

Apply Kirchhoff's junction rule to point $a$ to obtain:

Apply Kirchhoff's loop rule to loop 1 to obtain:

$$
I_{\mathrm{s}}=\frac{2 \varepsilon}{2 r+R}
$$



$$
P_{\mathrm{s}}=I_{\mathrm{s}}^{2} R
$$

$$
-r I_{\mathrm{s}}+\mathcal{E}-r I_{\mathrm{s}}+\mathcal{E}-R I_{\mathrm{s}}=0
$$

$$
\begin{equation*}
P_{\mathrm{s}}=\left(\frac{2 \varepsilon}{2 r+R}\right)^{2} R=\frac{4 \varepsilon^{2} R}{(2 r+R)^{2}} \tag{1}
\end{equation*}
$$

$$
P_{\mathrm{p}}=I_{\mathrm{p}}^{2} R
$$

$$
I_{\mathrm{p}}=I_{1}+I_{2}
$$

$$
-r I_{1}+\varepsilon-\varepsilon+r I_{2}=0
$$

or
$I_{1}=I_{2}=\frac{1}{2} I_{\mathrm{p}}$

Apply Kirchhoff's loop rule to the outer loop to obtain:

Substituting for $I_{\mathrm{p}}$ in the expression for $P_{\mathrm{p}}$ yields:

To prove that if $R>r$, then $P_{\mathrm{p}}<P_{\mathrm{s}}, \quad 3 R^{2}>3 r^{2}$ suppose that $R>r$. Then:

Adding $r^{2}+4 r R+R^{2}$ to both sides of the inequality yields:

Factor 4 from the left-hand side and recognize that the right-hand side is the square of a binomial to obtain:

Taking the reciprocal of both sides of the inequality reverses the sense of the inequality:

Multiplying both sides of the inequality by $\varepsilon^{2} R$ yields:

$$
\mathcal{E}-R I_{\mathrm{p}}-r I_{1}=0
$$

or

$$
\varepsilon-R I_{\mathrm{p}}-\frac{1}{2} r I_{\mathrm{p}}=0 \Rightarrow I_{\mathrm{p}}=\frac{\varepsilon}{\frac{1}{2} r+R}
$$

$$
\begin{equation*}
P_{\mathrm{p}}=\left(\frac{\mathcal{E}}{\frac{1}{2} r+R}\right)^{2} R=\frac{\mathcal{E}^{2} R}{\left(\frac{1}{2} r+R\right)^{2}} \tag{2}
\end{equation*}
$$

$$
3 R^{2}>3 r^{2}
$$

$$
r^{2}+4 r R+4 R^{2}>4 r^{2}+4 r R+R^{2}
$$

$$
4\left(\frac{1}{2} r+R\right)^{2}>(2 r+R)^{2}
$$

or

$$
\left(\frac{1}{2} r+R\right)^{2}>\frac{(2 r+R)^{2}}{4}
$$

$$
\frac{1}{\left(\frac{1}{2} r+R\right)^{2}}<\frac{4}{(2 r+R)^{2}}
$$

$$
\frac{\mathcal{E}^{2} R}{\left(\frac{1}{2} r+R\right)^{2}}<\frac{4 \mathcal{E}^{2} R}{(2 r+R)^{2}}
$$

For the series combination, the power delivered to the load is greater if $R>r$ and is greatest when $R=2 r$. If $R=r$, both arrangements provide the same power to the load.

To prove that if $R<r$, then $P_{\mathrm{p}}>P_{\mathrm{s}}, \quad 3 R^{2}<3 r^{2}$ suppose that $R<r$. Then:

Adding $r^{2}+4 r R+R^{2}$ to both sides $\quad r^{2}+4 r R+4 R^{2}<4 r^{2}+4 r R+R^{2}$ of the inequality yields:

Factor 4 from the left-hand side and recognize that the right-hand side is the square of a binomial to obtain:

$$
4\left(\frac{1}{2} r+R\right)^{2}<(2 r+R)^{2}
$$

or

$$
\left(\frac{1}{2} r+R\right)^{2}<\frac{(2 r+R)^{2}}{4}
$$

Taking the reciprocal of both sides of the inequality reverses the sense of the inequality:

Multiplying both sides of the inequality by $\varepsilon^{2} R$ yields:

$$
\frac{1}{\left(\frac{1}{2} r+R\right)^{2}}>\frac{4}{(2 r+R)^{2}}
$$

$$
\frac{\varepsilon^{2} R}{\left(\frac{1}{2} r+R\right)^{2}}>\frac{4 \varepsilon^{2} R}{(2 r+R)^{2}}
$$

For the parallel combination, the power delivered to the load is greater if $R<r$ and is a maximum when $R=\frac{1}{2} r$.
$88 \quad$ •• The circuit fragment shown in Figure 25-64 is called a voltage divider. (a) If $R_{\text {load }}$ is not attached, show that $V_{\text {out }}=V R_{2} /\left(R_{1}+R_{2}\right)$. (b) If $R_{1}=R_{2}=10 \mathrm{k} \Omega$, what is the smallest value of $R_{\text {load }}$ that can be used so that $V_{\text {out }}$ drops by less than 10 percent from its unloaded value? ( $V_{\text {out }}$ is measured with respect to ground.)

Picture the Problem Let the current drawn from the source be $I$. We can use Ohm's law in conjunction with Kirchhoff's loop rule to express the output voltage as a function of $V, R_{1}$, and $R_{2}$. In (b) we can use the result of $(a)$ to express the condition on the output voltages in terms of the effective resistance of the loaded output and the resistances $R_{1}$ and $R_{2}$.
(a) Use Ohm's law to express $V_{\text {out }}$ in

$$
V_{\text {out }}=I R_{2}
$$ terms of $R_{2}$ and $I$ :

Apply Kirchhoff's loop rule to the circuit to obtain:

$$
V-I R_{1}-I R_{2}=0 \Rightarrow I=\frac{V}{R_{1}+R_{2}}
$$

Substitute for $I$ in the expression for $V_{\text {out }}$ to obtain:

$$
V_{\text {out }}=\left(\frac{V}{R_{1}+R_{2}}\right) R_{2}=V\left(\frac{R_{2}}{R_{1}+R_{2}}\right)
$$

(b) Relate the effective resistance of the loaded circuit $R_{\text {eff }}$ to $R_{2}$ and

$$
\frac{1}{R_{\mathrm{eff}}}=\frac{1}{R_{2}}+\frac{1}{R_{\text {load }}}
$$

$R_{\text {load }}$ :
Solving for $R_{\text {load }}$ yields:

$$
\begin{equation*}
R_{\text {load }}=\frac{R_{2} R_{\text {eff }}}{R_{2}-R_{\text {eff }}} \tag{1}
\end{equation*}
$$

Letting $V_{\text {out }}^{\prime}$ represent the output voltage under load, express the

$$
\begin{equation*}
\frac{V_{\text {out }}-V_{\text {out }}^{\prime}}{V_{\text {out }}}=1-\frac{V_{\text {out }}^{\prime}}{V_{\text {out }}}<0.1 \tag{2}
\end{equation*}
$$

condition that $V_{\text {out }}$ drops by less than 10 percent of its unloaded value:

Using the result from (a), express
$V_{\text {out }}^{\prime}$ in terms of the effective output load $R_{\text {eff }}$ :

$$
V_{\mathrm{out}}^{\prime}=V\left(\frac{R_{\mathrm{eff}}}{R_{1}+R_{\mathrm{eff}}}\right)
$$

Substitute for $V_{\text {out }}$ and $V_{\text {out }}^{\prime}$ in

$$
1-\frac{\frac{R_{\mathrm{eff}}}{R_{1}+R_{\text {eff }}}}{\frac{R_{2}}{R_{1}+R_{2}}}<0.1
$$

or

$$
1-\frac{R_{\text {eff }}\left(R_{1}+R_{2}\right)}{R_{2}\left(R_{1}+R_{\text {eff }}\right)}<0.1
$$

Solving for $R_{\text {eff }}$ yields:

$$
R_{\text {eff }}>\frac{0.9 R_{1} R_{2}}{R_{1}+0.1 R_{2}}
$$

Substitute numerical values and evaluate $R_{\text {eff }}$ :

$$
R_{\mathrm{eff}}>\frac{(0.90)(10 \mathrm{k} \Omega)(10 \mathrm{k} \Omega)}{10 \mathrm{k} \Omega+(0.10)(10 \mathrm{k} \Omega)}=8.18 \mathrm{k} \Omega
$$

Finally, substitute numerical values in equation (1) and evaluate $R_{\text {load }}$ :

$$
R_{\text {load }}<\frac{(10 \mathrm{k} \Omega)(8.18 \mathrm{k} \Omega)}{10 \mathrm{k} \Omega-8.18 \mathrm{k} \Omega}=45 \mathrm{k} \Omega
$$

89 ••• [SSM] For the circuit shown in Figure 25-65, find the potential difference between point $a$ and point $b$.

Picture the Problem Let $I_{1}$ be the current in the left branch resistor, directed up; let $I_{3}$ be the current, directed down, in the middle branch; and let $I_{2}$ be the current in the right branch, directed up. We can apply Kirchhoff's rules to find $I_{3}$ and then the potential difference between points $a$ and $b$.

Relate the potential at $a$ to the potential at $b$ :

$$
\begin{aligned}
& V_{a}-R_{4} I_{3}-4 \mathrm{~V}=V_{b} \\
& \text { or } \\
& V_{a}-V_{b}=R_{4} I_{3}+4 \mathrm{~V}
\end{aligned}
$$

Apply Kirchhoff's junction rule at $a$

$$
\begin{equation*}
I_{1}+I_{2}=I_{3} \tag{1}
\end{equation*}
$$ to obtain:

Apply the loop rule to a loop around the outside of the circuit to obtain:

$$
\begin{aligned}
& 2.00 \mathrm{~V}-(1.00 \Omega) I_{1}+(1.00 \Omega) I_{2} \\
& \quad-2.00 \mathrm{~V}+(1.00 \Omega) I_{2}-(1.00 \Omega) I_{1}=0
\end{aligned}
$$

$$
\begin{align*}
& \text { or } \\
& I_{1}-I_{2}=0 \tag{2}
\end{align*}
$$

Apply the loop rule to the left side of the circuit to obtain:

Solve equations (1), (2), and (3) simultaneously to obtain:

Substitute to obtain:

$$
\begin{aligned}
2.00 \mathrm{~V}- & (1.00 \Omega) I_{1}-(4.00 \Omega) I_{3} \\
& -4.00 \mathrm{~V}-(1.00 \Omega) I_{1}=0
\end{aligned}
$$

or

$$
\begin{equation*}
-(1.00 \Omega) \boldsymbol{I}_{1}-(2.00 \Omega) \boldsymbol{I}_{3}=1.00 \mathrm{~V} \tag{3}
\end{equation*}
$$

$I_{1}=-0.200 \mathrm{~A}, I_{2}=-0.200 \mathrm{~A}$, and $I_{3}=-0.400 \mathrm{~A}$
where the minus signs indicate that the currents flow in opposite directions to the directions chosen.

$$
\begin{aligned}
V_{a}-V_{b} & =(4.00 \Omega)(-0.400 \mathrm{~A})+4.00 \mathrm{~V} \\
& =2.40 \mathrm{~V}
\end{aligned}
$$

## Remarks: Note that point $a$ is at the higher potential.

$90 \quad$ ••• For the circuit shown in Figure 25-66, find (a) the current in each resistor, $(b)$ the power supplied by each source of emf, and (c) the power delivered to each resistor.

Picture the Problem Let $I_{1,2 \Omega}$ be the current in the $1.00-\Omega$ and $2.00-\Omega$ resistors either side of the $4.00-\mathrm{V}$ source, directed to the right; let $I_{2 \Omega}$ be the current, directed up, in the middle branch; and let $I_{6 \Omega}$ be the current in the $6.00-\Omega$ resistor, directed down. We can apply Kirchhoff's rules to find these currents, the power supplied by each source, and the power dissipated in each resistor.
(a) Apply Kirchhoff's junction rule $\quad I_{1,2 \Omega}+I_{2 \Omega}=I_{6 \Omega}$ at the top junction to obtain:

Apply Kirchhoff's loop rule to the outside loop of the circuit to obtain:

$$
\begin{align*}
& 8.00 \mathrm{~V}-(1.00 \Omega) I_{1,2 \Omega}+4.00 \mathrm{~V} \\
& \quad-(2.00 \Omega) I_{1,2 \Omega}-(6.00 \Omega) I_{6 \Omega}=0 \\
& \text { or } \\
& (3.00 \Omega) I_{1,2 \Omega}+(6.00 \Omega) I_{6 \Omega}=12.0 \mathrm{~V} \tag{2}
\end{align*}
$$

Apply the loop rule to the inside loop at the left-hand side of the circuit to obtain:

Solve equations (1), (2), and (3) simultaneously to obtain:
(b) The power delivered by the $8.00-\mathrm{V}$ source is:

The power delivered by the $4.00-\mathrm{V}$ source is:
(c) Express the power dissipated in the $1.00-\Omega$ resistor:

Express the power dissipated in the $2.00-\Omega$ resistor in the left branch:

Express the power dissipated in the $2.00-\Omega$ resistor in the middle branch:

Express the power dissipated in the $6.00-\Omega$ resistor:

$$
\begin{align*}
& 8.00 \mathrm{~V}-(1.00 \Omega) I_{1,2 \Omega}+4.00 \mathrm{~V} \\
& \quad-(2.00 \Omega) I_{1,2 \Omega}+(2.00 \Omega) I_{2 \Omega} \\
& \quad-4.00 \mathrm{~V}=0 \\
& \text { or } \\
& \begin{array}{c}
8.00 \mathrm{~V}- \\
\\
\\
\quad(3.00 \Omega) I_{1,2 \Omega} \\
\\
+(2.00 \Omega) I_{2 \Omega}=0
\end{array}
\end{align*}
$$

$I_{1,2 \Omega}=2.00 \mathrm{~A}, I_{2 \Omega}=-1.00 \mathrm{~A}$,
and $I_{6 \Omega}=1.00 \mathrm{~A}$
where the minus sign indicates that the current flows downward rather than upward as we had assumed.

$$
\begin{aligned}
P_{8 \mathrm{~V}} & =\mathcal{E}_{8 \mathrm{~V}} I_{1,2 \Omega}=(8.00 \mathrm{~V})(2.00 \mathrm{~A}) \\
& =16.0 \mathrm{~W} \\
P_{4 \mathrm{~V}} & =\varepsilon_{4 \mathrm{~V}} I_{2 \Omega}=(4.00 \mathrm{~V})(-1.00 \mathrm{~A}) \\
& =-4.00 \mathrm{~W}
\end{aligned}
$$

where the minus sign indicates that this source is having current forced through it and is absorbing power.

$$
\begin{aligned}
P_{1 \Omega} & =I_{1,2 \Omega}^{2} R_{1 \Omega}=(2.00 \mathrm{~A})^{2}(1.00 \Omega) \\
& =4.00 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P_{2 \Omega, \mathrm{left}} & =I_{1,2 \Omega}^{2} R_{2 \Omega}=(2.00 \mathrm{~A})^{2}(2.00 \Omega) \\
& =8.00 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P_{2 \Omega, \text { middle }} & =I_{2 \Omega}^{2} R_{2 \Omega}=(1.00 \mathrm{~A})^{2}(2.00 \Omega) \\
& =2.00 \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P_{6 \Omega} & =I_{6 \Omega}^{2} R_{6 \Omega}=(1.00 \mathrm{~A})^{2}(6.00 \Omega) \\
& =6.00 \mathrm{~W}
\end{aligned}
$$

## Ammeters and Voltmeters

91 •• [SSM] The voltmeter shown in Figure 25-67 can be modeled as an ideal voltmeter (a voltmeter that has an infinite internal resistance) in parallel with a $10.0 \mathrm{M} \Omega$ resistor. Calculate the reading on the voltmeter when (a) $R=1.00 \mathrm{k} \Omega$, (b) $R=10.0 \mathrm{k} \Omega$, (c) $R=1.00 \mathrm{M} \Omega$, (d) $R=10.0 \mathrm{M} \Omega$, and (e) $R=100 \mathrm{M} \Omega$.
$(f)$ What is the largest value of $R$ possible if the measured voltage is to be within 10 percent of the true voltage (that is, the voltage drop across $R$ without the voltmeter in place)?

Picture the Problem Let $I$ be the current drawn from source and $R_{\text {eq }}$ the resistance equivalent to $R$ and $10 \mathrm{M} \Omega$ connected in parallel and apply Kirchhoff's loop rule to express the measured voltage $V$ across $R$ as a function of $R$.

The voltage measured by the voltmeter is given by:

$$
\begin{equation*}
V=I R_{\mathrm{eq}} \tag{1}
\end{equation*}
$$

Apply Kirchhoff's loop rule to the circuit to obtain:

$$
10.0 \mathrm{~V}-I R_{\mathrm{eq}}-I(2 R)=0
$$

Solving for $I$ yields:

$$
I=\frac{10.0 \mathrm{~V}}{R_{\mathrm{eq}}+2 R}
$$

Express $R_{\text {eq }}$ in terms of $R$ and $10.0-\mathrm{M} \Omega$ resistance in parallel with it:

Solving for $R_{\text {eq }}$ yields:

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{10.0 \mathrm{M} \Omega}+\frac{1}{R}
$$

$$
R_{\mathrm{eq}}=\frac{(10.0 \mathrm{M} \Omega) R}{R+10.0 \mathrm{M} \Omega}
$$

Substitute for $I$ in equation (1) and simplify to obtain:

$$
V=\left(\frac{10.0 \mathrm{~V}}{R_{\mathrm{eq}}+2 R}\right) R_{\mathrm{eq}}=\frac{10.0 \mathrm{~V}}{1+\frac{2 R}{R_{\mathrm{eq}}}}
$$

Substitute for $R_{\mathrm{eq}}$ and simplify to obtain:

$$
\begin{equation*}
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{R+15.0 \mathrm{M} \Omega} \tag{2}
\end{equation*}
$$

(a) Evaluate equation (2) for $R=1.00 \mathrm{k} \Omega$ :

$$
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{1.00 \mathrm{k} \Omega+15.0 \mathrm{M} \Omega}=3.3 \mathrm{~V}
$$

(b) Evaluate equation (2) for $R=10.0 \mathrm{k} \Omega$ :

$$
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{10.0 \mathrm{k} \Omega+15.0 \mathrm{M} \Omega}=3.3 \mathrm{~V}
$$

(c) Evaluate equation (2) for $R=1.00 \mathrm{M} \Omega$ :

$$
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{1.00 \mathrm{M} \Omega+15.0 \mathrm{M} \Omega}=3.1 \mathrm{~V}
$$

(d) Evaluate equation (2) for $R=10.0 \mathrm{M} \Omega$ :

$$
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{10.0 \mathrm{M} \Omega+15.0 \mathrm{M} \Omega}=2.0 \mathrm{~V}
$$

(e) Evaluate equation (2) for $R=100 \mathrm{M} \Omega$ :

$$
V=\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{100 \mathrm{M} \Omega+15.0 \mathrm{M} \Omega}=0.43 \mathrm{~V}
$$

(f) Express the condition that the measured voltage to be within 10 percent of the true voltage $V_{\text {true }}$ :

$$
\frac{V_{\text {true }}-V}{V_{\text {true }}}=1-\frac{V}{V_{\text {true }}}<0.1
$$

Substitute for $V$ and $V_{\text {true }}$ to obtain:

$$
1-\frac{\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{R+15.0 \mathrm{M} \Omega}}{I R}<0.1
$$

Because $I=10.0 \mathrm{~V} / 3 R$ :

$$
1-\frac{\frac{(10.0 \mathrm{~V})(5.0 \mathrm{M} \Omega)}{R+15.0 \mathrm{M} \Omega}}{\frac{10.0}{3} \mathrm{~V}}<0.1
$$

Solving for $R$ yields:

$$
R<\frac{1.5 \mathrm{M} \Omega}{0.90}=1.67 \mathrm{M} \Omega
$$

92 •• You are given a D'Arsonval galvanometer that will deflect full scale if a current of $50.0 \mu \mathrm{~A}$ runs through the galvanometer. At this current, there is a voltage drop of 0.250 V across the meter. What is the internal resistance of the galvanometer?

Picture the Problem The diagram shows a voltmeter connected in parallel with a galvanometer movement whose internal resistance is $R$. We can apply Kirchhoff's loop rule to express $R$ in terms of $I$ and $V$.


Apply Kirchhoff's loop rule to the loop that includes the galvanometer movement and the voltmeter:

Substitute numerical values and evaluate $R$ :

$$
V-I R=0 \Rightarrow R=\frac{V}{I}
$$

$$
R=\frac{0.250 \mathrm{~V}}{50.0 \mu \mathrm{~A}}=5.00 \mathrm{k} \Omega
$$

93 •• You are given a D'Arsonval galvanometer that will deflect full scale if a current of $50.0 \mu \mathrm{~A}$ runs through the galvanometer. At this current, there is a voltage drop of 0.250 V across the meter. You wish to use this galvanometer to construct an ammeter that can measure currents up to 100 mA . Show that this can be done by placing a resistor in parallel with the meter, and find the value of its resistance.

Picture the Problem When there is a voltage drop of 0.250 V across this galvanometer, the meter reads full scale. The diagram shows the galvanometer movement with a resistor of resistance $r$ in parallel. The purpose of this resistor is to limit the current through the movement to $I_{\mathrm{g}}=50.0 \mu \mathrm{~A}$. We can apply Kirchhoff's loop rule to the circuit fragment containing the galvanometer movement and the shunt resistor to derive an expression for $r$.

Apply Kirchhoff's loop rule to the circuit fragment to obtain:

Apply Kirchhoff's junction rule at point $a$ to obtain:

Substitute for $I_{\mathrm{r}}$ in the loop equation:


$$
-R I_{\mathrm{g}}+r I_{\mathrm{r}}=0
$$

$$
I_{\mathrm{r}}=I-I_{\mathrm{g}}
$$

$$
-R I_{\mathrm{g}}+r\left(I-I_{\mathrm{g}}\right)=0 \Rightarrow r=\frac{R I_{\mathrm{g}}}{I-I_{\mathrm{g}}}
$$

Noting that $R I_{\mathrm{g}}=0.250 \mathrm{~V}$, substitute numerical values and evaluate $r$ :

$$
r=\frac{0.250 \mathrm{~V}}{100 \mathrm{~mA}-50.0 \mu \mathrm{~A}}=2.5 \Omega
$$

94 •• You are given a D'Arsonval galvanometer that will deflect full scale if a current of $50.0 \mu \mathrm{~A}$ runs through the galvanometer. At this current, there is a voltage drop of 0.250 V across the meter. You wish to use this galvanometer to construct a voltmeter that can measure potential differences up to 10.0 V . Show that this can be done by placing a large resistance in series with the meter movement, and find the resistance needed.

Picture the Problem The circuit diagram shows a fragment of a circuit in which a resistor of resistance $r$ is connected in series with the meter movement of Problem 92. The purpose of this resistor is to limit the current through the galvanometer movement to $50 \mu \mathrm{~A}$ and to produce a deflection of the galvanometer movement that is a measure of the potential difference $V$. We can apply Kirchhoff's loop rule to express $r$ in terms of $V_{\mathrm{g}}, I_{\mathrm{g}}$, and $R$.


Apply Kirchhoff's loop rule to the

$$
V-r I_{\mathrm{g}}-R I_{\mathrm{g}}=0
$$

circuit fragment to obtain:
Solving for $r$ yields:

$$
\begin{equation*}
r=\frac{V-R I_{\mathrm{g}}}{I_{\mathrm{g}}}=\frac{V}{I_{\mathrm{g}}}-R \tag{1}
\end{equation*}
$$

Use Ohm's law to relate the current $I_{\mathrm{g}}$ through the galvanometer movement to the potential difference

$$
I_{\mathrm{g}}=\frac{V_{\mathrm{g}}}{R} \Rightarrow R=\frac{V_{\mathrm{g}}}{I_{\mathrm{g}}}
$$ $V_{\mathrm{g}}$ across it:

Use the values for $V_{\mathrm{g}}$ and $I_{\mathrm{g}}$ given in Problem 114 to evaluate $R$ :

$$
R=\frac{0.250 \mathrm{~V}}{50.0 \mu \mathrm{~A}}=5000 \Omega
$$

Substitute numerical values in equation (1) and evaluate $r$ :

$$
r=\frac{10.0 \mathrm{~V}}{50.0 \mu \mathrm{~A}}-5000 \Omega=195 \mathrm{k} \Omega
$$

## Remarks: The total series resistance is the sum of $r$ and $R$ or $200 \mathrm{k} \Omega$.

## RC Circuits

95 - For the circuit shown in Figure 25-69, $C=6.00-\mu \mathrm{F}, \mathcal{E}=100 \mathrm{~V}$ and $R=500 \Omega$. After having been at contact $a$ for a long time, the switch throw is rotated to contact $b$. (a) What is the charge on the upper plate capacitor just as the switch throw is moved to contact $a$ ? (b) What is the initial current just after the switch throw is rotated to contact $a$ ? (c) What is the time constant of this circuit? (d) How much charge is on the upper plate of the capacitor 6.00 ms after the switch throw is rotated to contact $b$ ?

Picture the Problem We can use the definition of capacitance to find the initial charge on the capacitor and Ohm's law to find the initial current in the circuit. We can find the time constant of the circuit using its definition and the charge on the capacitor after 6 ms using $Q(t)=Q_{0} e^{-t / \tau}$.
(a) Use the definition of capacitance to find the initial charge on the capacitor:

$$
\begin{aligned}
Q_{0} & =C V_{0}=(6.00 \mu \mathrm{~F})(100 \mathrm{~V}) \\
& =600 \mu \mathrm{C}
\end{aligned}
$$

(b) Apply Ohm's law to the resistor to obtain:
$I_{0}=\frac{V_{0}}{R}=\frac{100 \mathrm{~V}}{500 \Omega}=0.200 \mathrm{~A}$
(c) Use its definition to find the time constant of the circuit:
(d) Express the charge on the

$$
Q(t)=Q_{0} e^{-t / \tau}
$$

capacitor as a function of time:

Substitute numerical values and evaluate $Q(6 \mathrm{~ms})$ :

$$
\begin{aligned}
Q(6.00 \mathrm{~ms}) & =(600 \mu \mathrm{C}) e^{-6.00 \mathrm{~ms} / 3.00 \mathrm{~ms}} \\
& =81.2 \mu \mathrm{C}
\end{aligned}
$$

96 - At $t=0$ the switch throw in Figure 25-68 is rotated to contact $b$ after having been at contact $a$ for a long time. (a) Find the energy stored in the capacitor before the switch throw is rotated away from contact $a$. (b) Find the energy stored in the capacitor as a function of time. (c) Sketch a plot of the energy stored in the capacitor versus time $t$.

Picture the Problem We can use $U_{0}=\frac{1}{2} C V_{0}^{2}$ to find the initial energy stored in the capacitor and $U(t)=\frac{1}{2} C\left(V_{\mathrm{C}}(t)\right)^{2}$ with $V_{\mathrm{C}}(t)=V_{0} e^{-t / \tau}$ to find the energy stored in the capacitor as a function of time.
(a) The initial energy stored in the capacitor is given by:
(b) Express the energy stored in the discharging capacitor as a function of time:

Substitute for $\boldsymbol{V}_{\mathrm{C}}(\boldsymbol{t})$ and simplify to obtain:

$$
U_{0}=\frac{1}{2} C \boldsymbol{\varepsilon}^{2}
$$

$$
U(t)=\frac{1}{2} C\left(V_{\mathrm{C}}(t)\right)^{2}
$$

where

$$
V_{\mathrm{C}}(t)=\varepsilon e^{-t / \tau}
$$

$$
\begin{aligned}
U(t) & =\frac{1}{2} C\left(\varepsilon e^{-t / \tau}\right)^{2}=\frac{1}{2} C \varepsilon^{2} e^{-2 t / \tau} \\
& =U_{0} e^{-2 t / \tau}
\end{aligned}
$$

(c) A graph of $U$ versus $t$ is shown below. $U$ is in units of $U_{0}$ and $t$ is in units of $\tau$.


97 •• [SSM] In the circuit in Figure 25-69, the emf equals 50.0 V and the capacitance equals $2.00 \mu \mathrm{~F}$. Switch $S$ is opened after having been closed for a long time, and 4.00 s later the voltage drop across the resistor is 20.0 V . Find the resistance of the resistor.

Picture the Problem We can find the resistance of the circuit from its time constant and use Ohm's law and the expression for the current in a charging $R C$ circuit to express $\tau$ as a function of time, $V_{0}$, and $V(t)$.

Express the resistance of the resistor in terms of the time constant of the

$$
\begin{equation*}
R=\frac{\tau}{C} \tag{1}
\end{equation*}
$$

circuit:

Using Ohm's law, express the
voltage drop across the resistor as a function of time:

Express the current in the circuit as a function of the elapsed time after the switch is closed:

Substitute for $I(t)$ to obtain:

Take the natural logarithm of both sides of the equation and solve for $\tau$ to obtain:

$$
V(t)=I_{0} e^{-t / \tau} R=\left(I_{0} R\right) e^{-t / \tau}=V_{0} e^{-t / \tau}
$$

$$
\tau=-\frac{t}{\ln \left[\frac{V(t)}{V_{0}}\right]}
$$

Substitute for $\tau$ in equation (1) to obtain:

$$
R=-\frac{t}{C \ln \left[\frac{V(t)}{V_{0}}\right]}
$$

Substitute numerical values and evaluate $R$ using the data given for $t=4.00 \mathrm{~s}$ :

$$
\begin{aligned}
R & =-\frac{4.00 \mathrm{~s}}{(2.00 \mu \mathrm{~F}) \ln \left(\frac{20.0 \mathrm{~V}}{50.0 \mathrm{~V}}\right)} \\
& =2.18 \mathrm{M} \Omega
\end{aligned}
$$

98 •• For the circuit shown in Figure $25-68, C=0.120 \mu \mathrm{~F}$ and $\mathcal{E}=100 \mathrm{~V}$. The switch throw is rotated to contact $b$ after having been at contact $a$ for a long time, and 4.00 s later the potential difference across the capacitor is equal to $\frac{1}{2} \mathcal{E}$. What is the value of $R$ ?

Picture the Problem We can find the resistance of the circuit from its time constant and use the expression for the charge on a discharging capacitor as a function of time to express $\tau$ as a function of time, $V_{0}$, and $V(t)$.

Express the effective resistance
across the capacitor in terms of the

$$
\begin{equation*}
R=\frac{\tau}{C} \tag{1}
\end{equation*}
$$ time constant of the circuit:

Express the voltage across the capacitor as a function of the elapsed time after the switch is closed:

Take the natural logarithm of both sides of the equation and solve for $\tau$ to obtain:

$$
\tau=-\frac{t}{\ln \left(\frac{V(t)}{V_{0}}\right)}
$$

Substitute for $\tau$ in equation (1) to obtain:

$$
R=-\frac{t}{C \ln \left(\frac{V(t)}{V_{0}}\right)}
$$

Substitute numerical values and evaluate $R$ :

$$
R=-\frac{4.00 \mathrm{~s}}{(0.120 \mu \mathrm{~F}) \ln \left(\frac{\frac{1}{2} V_{0}}{V_{0}}\right)}=48.1 \mathrm{M} \Omega
$$

99 •• In the circuit in Figure 25-70, the emf equals 6.00 V and has negligible internal resistance. The capacitance equals $1.50 \mu \mathrm{~F}$ and the resistance equals $2.00-\mathrm{M} \Omega$. Switch S has been closed for a long time. Switch S is opened. After a time interval equal to one time constant of the circuit has elapsed, find: (a) the charge on the capacitor plate on the right, $(b)$ the rate at which the charge is increasing, $(c)$ the current, $(d)$ the power supplied by the battery, $(e)$ the power delivered to the resistor, and $(f)$ the rate at which the energy stored in the capacitor is increasing.

Picture the Problem We can use $Q(t)=Q_{\mathrm{f}}\left(1-e^{-t / \tau}\right)=C \boldsymbol{\mathcal { E }}\left(1-e^{-t / \tau}\right)$ to find the charge on the capacitor at $t=\tau$ and differentiate this expression with respect to time to find the rate at which the charge is increasing (the current). The power supplied by the battery is given by $P_{\tau}=I_{\tau} \varepsilon$ and the power dissipated in the resistor by $P_{R, \tau}=I_{\tau}^{2} R$. In Part ( $f$ ) we can differentiate $U(t)=Q^{2}(t) / 2 C$ with respect to time and evaluate the derivative at $t=\tau$ to find the rate at which the energy stored in the capacitor is increasing.
(a) Express the charge $Q$ on the capacitor as a function of time:

Evaluate $Q(\tau)$ to obtain:
(b) and (c) Differentiate equation (1) with respect to $t$ to obtain:

Apply Kirchhoff's loop rule to the circuit just after the circuit is completed to obtain:

Because $V_{\mathrm{C} 0}=0$ we have:

Substituting for $I_{0}$ yields:

Substitute numerical values and
evaluate $\frac{d Q(t)}{d t}=I(t)$ :

$$
I_{0}=\frac{\varepsilon}{R}
$$

$$
\begin{equation*}
Q(t)=Q_{\mathrm{f}}\left(1-e^{-t / \tau}\right)=C \boldsymbol{\varepsilon}\left(1-e^{-t / \tau}\right) \tag{1}
\end{equation*}
$$

where $\tau=R C$.

$$
\varepsilon-R I_{0}-V_{\mathrm{C} 0}=0
$$

$$
\frac{d Q(t)}{d t}=I(t)=\frac{\varepsilon}{R} e^{-t / \tau}
$$

$$
\begin{aligned}
Q(\tau) & =(1.50 \mu \mathrm{~F})(6.00 \mathrm{~V})\left(1-e^{-1}\right) \\
& =5.689 \mu \mathrm{C}=5.69 \mu \mathrm{C}
\end{aligned}
$$

$$
\frac{d Q(t)}{d t}=I(t)=I_{0} e^{-t / \tau}
$$

$$
\begin{aligned}
\frac{d Q(t)}{d t} & =I(t)=\frac{6.00 \mathrm{~V}}{2.00 \mathrm{M} \Omega} e^{-1} \\
& =1.104 \mu \mathrm{C} / \mathrm{s}=1.10 \mu \mathrm{C} / \mathrm{s}
\end{aligned}
$$

(d) Express the power supplied by the battery as the product of its emf and the current drawn from it at
$t=\tau$.
(e) The power dissipated in the resistor is given by:

$$
\begin{aligned}
P(\tau) & =I(\tau) \varepsilon=(1.104 \mu \mathrm{~A})(6.00 \mathrm{~V}) \\
& =6.62 \mu \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P_{R}(\tau) & =I^{2}(\tau) R=(1.104 \mu \mathrm{~A})^{2}(2.00 \mathrm{M} \Omega) \\
& =2.44 \mu \mathrm{~W}
\end{aligned}
$$

(f) Express the energy stored in the capacitor as a function of time:

Differentiate this expression with respect to time to obtain:

$$
U(t)=\frac{Q^{2}(t)}{2 C}
$$

$$
\begin{aligned}
\frac{d U(t)}{d t} & =\frac{1}{2 C} \frac{d}{d t}\left[Q^{2}(t)\right] \\
& =\frac{1}{2 C}(2 Q(t)) \frac{d Q(t)}{d t} \\
& =\frac{Q(t)}{C} I(t)
\end{aligned}
$$

Evaluate $\frac{d U(t)}{d t}$ when $t=\tau$ to
obtain:

$$
\begin{aligned}
\frac{d U(\tau)}{d t} & =\frac{5.689 \mu \mathrm{C}}{1.50 \mu \mathrm{~F}}(1.104 \mu \mathrm{~A}) \\
& =4.19 \mu \mathrm{~W}
\end{aligned}
$$

Remarks: Note that our answer for Part ( $f$ ) is the difference between the power delivered by the battery at $t=\tau$ and the rate at which energy is dissipated in the resistor at the same time.

100 •• A constant charge of 1.00 mC is on the positively charged plate of the $5.00-\mu \mathrm{F}$ capacitor in the circuit shown in Figure 25-70. Find (a) the battery current, and (b) the resistances $R_{1}, R_{2}$, and $R_{3}$.

Picture the Problem We can apply Kirchhoff's junction rule to find the current in each branch of this circuit and then use the loop rule to obtain equations solvable for $R_{1}, R_{2}$, and $R_{3}$.
(a) Apply Kirchhoff's junction rule $\quad I_{\text {bat }}=I_{10 \Omega}+5.00 \mathrm{~A}$
at the junction of the $5.00-\mu \mathrm{F}$
capacitor and the $10.0-\Omega$ and $50.0-\Omega$
resistors under steady-state conditions:

Because the potential differences across the $5.00-\mu \mathrm{F}$ capacitor and the $10.0-\Omega$ resistor are the same:

Express the potential difference across the capacitor to its steadystate charge:

Substitute for $V_{\mathrm{C}}$ to obtain:

Substitute in equation (1) to obtain:

Substitute numerical values and evaluate $I_{\text {bat }}$ :
(b) Use Kirchhoff's junction rule to find the currents $I_{5 \Omega}, I_{\mathrm{R} 3}$, and $I_{\mathrm{R} 1}$ :

Apply the loop rule to the loop that includes the battery, $R_{1}$, and the $50.0-\Omega$ and $5.00-\Omega$ resistors:

Solve for $R_{1}$ to obtain:

Apply the loop rule to the loop that includes the battery, $R_{1}$, the $10.0-\Omega$ resistor and $R_{3}$ :

Solving for $R_{3}$ yields:

Apply the loop rule to the loop that includes the $10.0-\Omega$ and $50.0-\Omega$ resistors and $R_{2}$ :

Solving for $R_{2}$ yields:

$$
I_{10 \Omega}=\frac{V_{10 \Omega}}{10.0 \Omega}=\frac{V_{\mathrm{C}}}{10.0 \Omega}
$$

$$
V_{\mathrm{C}}=\frac{Q_{\mathrm{f}}}{C}
$$

$$
I_{10 \Omega}=\frac{Q_{\mathrm{f}}}{(10.0 \Omega) C}
$$

$$
I_{\text {bat }}=\frac{Q_{\mathrm{f}}}{(10.0 \Omega) C}+5.00 \mathrm{~A}
$$

$$
\begin{aligned}
I_{\text {bat }} & =\frac{1000 \mu \mathrm{C}}{(10.0 \Omega)(5.00 \mu \mathrm{~F})}+5.00 \mathrm{~A} \\
& =25.0 \mathrm{~A}
\end{aligned}
$$

$$
I_{5 \Omega}=10.0 \mathrm{~A}, I_{R 3}=15.0 \mathrm{~A}, \text { and }
$$

$$
I_{R 1}=I_{\mathrm{bat}}=25.0 \mathrm{~A}
$$

$$
\begin{gathered}
310 \mathrm{~V}-(25.0 \mathrm{~A}) R_{1}-(5.00 \mathrm{~A})(50.0 \Omega) \\
-(10.0 \mathrm{~A})(5.00 \Omega)=0
\end{gathered}
$$

$$
\begin{aligned}
& R_{1}= 0.400 \Omega \\
& 310 \mathrm{~V}-(25.0 \mathrm{~A})(0.400 \Omega) \\
&-(20.0 \mathrm{~A})(10.0 \Omega)-(15.0 \mathrm{~A}) R_{3}=0
\end{aligned}
$$

$$
R_{3}=6.67 \Omega
$$

$$
-(20.0 \mathrm{~A})(10.0 \Omega)-(5.00 \mathrm{~A}) R_{2}
$$

$$
+(5.00 \mathrm{~A})(50.0 \Omega)=0
$$

$$
R_{2}=10.0 \Omega
$$

101 •• Show that Equation 25-39 can be rearranged and written as $\frac{d Q}{\varepsilon C-Q}=\frac{d t}{R C}$. Integrate this equation to derive the solution given by Equation 25-40.

Picture the Problem We can separate the variables in Equation 25-39 to obtain the equation given in the problem statement. Integrating this differential equation will yield Equation 25-40.

Solve Equation 25-39 for $d Q / d t$ to obtain:

$$
\frac{d Q}{d t}=\frac{\varepsilon C-Q}{R C}
$$

Separate the variables to obtain:

Integrate $d Q^{\prime}$ from 0 to $Q$ and $d t^{\prime}$ from 0 to $t$ :

$$
\frac{d Q}{\varepsilon C-Q}=\frac{d t}{R C}
$$

$$
\int_{0}^{Q} \frac{d Q^{\prime}}{\varepsilon C-Q^{\prime}}=\frac{1}{R C} \int_{0}^{t} d t^{\prime}
$$

and

$$
\ln \left(\frac{\varepsilon C}{\varepsilon C-Q}\right)=\frac{t}{R C}
$$

Transform from logarithmic to exponential form to obtain:

Solve for $Q$ to obtain Equation

$$
\frac{\varepsilon C}{\varepsilon C-Q}=e^{\frac{t}{R C}}
$$ 25-40:

$$
Q=\varepsilon C\left(1-e^{-t / R C}\right)=Q_{\mathrm{f}}\left(1-e^{-t / R C}\right)
$$

102 •• Switch S, shown in Figure 25-71, is closed after having been open for a long time. (a) What is the initial value of the battery current just after switch S is closed? (b) What is the battery current a long time after switch S is closed?
(c) What are the charges on the plates of the capacitors a long time after switch S is closed? (d) Switch S is reopened. What are the charges on the plates of the capacitors a long time after switch $S$ is reopened?

Picture the Problem When the switch is closed, the initial potential differences across the capacitors are zero (they have no charge) and the resistors in the bridge portion of the circuit are in parallel. When a long time has passed, the current through the capacitors will be zero and the resistors will be in series. In both cases, the application of Kirchhoff's loop rule to the entire circuit will yield the current in the circuit. To find the final charges on the capacitors we can use the definition of capacitance and apply Kirchhoff's loop rule to the loops containing two resistors and a capacitor to find the potential differences across the capacitors.
(a) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

Solving for $I_{0}$ yields:

Find the equivalent resistance of $15.0 \Omega, 12.0 \Omega$, and $15.0 \Omega$ in parallel:

Substitute for $R_{\text {eq }}$ and evaluate $I_{0}$ :
(b) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed:

Solving for $I_{\infty}$ yields:

Find the equivalent resistance of $15.0 \Omega, 12.0 \Omega$, and $15.0 \Omega$ in series:

Substitute for $R_{\text {eq }}$ and evaluate $I_{\infty}$ :
(c) Using the definition of capacitance, express the charge on the capacitors in terms of their final potential differences:

Apply Kirchhoff's loop rule to the loop containing the $15.0-\Omega$ and $12.0-\Omega$ resistors and the $10.0 \mu \mathrm{~F}$ capacitor to obtain:

Solving for $V_{10 \mu \mathrm{~F}}$ yields:

$$
50.0 \mathrm{~V}-I_{0}(10.0 \Omega)-I_{0} R_{\mathrm{eq}}=0
$$

$$
I_{0}=\frac{50.0 \mathrm{~V}}{10.0 \Omega+R_{\mathrm{eq}}}
$$

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{15.0 \Omega}+\frac{1}{12.0 \Omega}+\frac{1}{15.0 \Omega}
$$

and

$$
R_{\mathrm{eq}}=4.615 \Omega
$$

$$
I_{0}=\frac{50.0 \mathrm{~V}}{10.0 \Omega+4.615 \Omega}=3.42 \mathrm{~A}
$$

$$
50.0 \mathrm{~V}-I_{\infty}(10.0 \Omega)-I_{\infty} R_{\mathrm{eq}}=0
$$

$$
I_{\infty}=\frac{50.0 \mathrm{~V}}{10.0 \Omega+R_{\mathrm{eq}}}
$$

$$
R_{\mathrm{eq}}=15.0 \Omega+12.0 \Omega+15.0 \Omega=42.0 \Omega
$$

$$
I_{\infty}=\frac{50.0 \mathrm{~V}}{10.0 \Omega+42.0 \Omega}=0.962 \mathrm{~A}
$$

$$
\begin{equation*}
Q_{10 \mu \mathrm{~F}}=C_{10 \mu \mathrm{~F}} V_{10 \mu \mathrm{~F}} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{5 \mu \mathrm{~F}}=C_{5 \mu \mathrm{~F}} V_{5 \mu \mathrm{~F}} \tag{2}
\end{equation*}
$$

$$
V_{10 \mu \mathrm{~F}}-(15.0 \Omega) I_{\infty}-(12.0 \Omega) I_{\infty}=0
$$

$$
V_{10 \mu \mathrm{~F}}=(27.0 \Omega) I_{\infty}
$$

Substitute numerical values in equation (1) and evaluate $Q_{10 \mu \mathrm{~F}}$ :

Apply Kirchhoff's loop rule to the loop containing the $15.0-\Omega$ and $12.0-\Omega$ resistors and the $5.00 \mu \mathrm{~F}$ capacitor to obtain:

Solve for $V_{5 \mu \mathrm{~F}}$ :

Substitute numerical values in equation (2) and evaluate $Q_{5 \mu \mathrm{~F}}$ :

$$
\begin{aligned}
Q_{10 \mu \mathrm{~F}} & =(10.0 \mu \mathrm{~F})(27.0 \Omega)(0.962 \mathrm{~A}) \\
& =260 \mu \mathrm{C}
\end{aligned}
$$

$$
V_{5 \mu \mathrm{~F}}-(15.0 \Omega) I_{\infty}-(12.0 \Omega) I_{\infty}=0
$$

$$
V_{5 \mu \mathrm{~F}}=(27.0 \Omega) I_{\infty}
$$

$$
Q_{5 \mu \mathrm{~F}}=(5.00 \mu \mathrm{~F})(27.0 \Omega)(0.962 \mathrm{~A})
$$

$$
=130 \mu \mathrm{C}
$$

(d) The charges on the plates of the capacitors a long time after switch S is reopened will be 0 .

103 ••• In the circuit shown in Figure 25-72, switch $S$ has been open for a long time. At time $t=0$ the switch is then closed. (a) What is the battery current just after switch S is closed? (b) What is the battery current a long time after switch S is closed? (c) What is the current in the $600-\Omega$ resistor as a function of time?

Picture the Problem Let $R_{1}=200 \Omega, R_{2}=600 \Omega, I_{1}$ and $I_{2}$ their currents, and $I_{3}$ the current into the capacitor. We can apply Kirchhoff's loop rule to find the initial battery current $I_{0}$ and the battery current $I_{\infty}$ a long time after the switch is closed. In Part (c) we can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in the $600-\Omega$ resistor as a function of time. We can solve this differential equation by assuming a solution of a given form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution.
(a) Apply Kirchhoff's loop rule to $\quad \mathcal{E}-(200 \Omega) I_{0}-V_{\mathrm{C} 0}=0$ the circuit at the instant the switch is closed:

Because the capacitor is initially

$$
V_{\mathrm{C} 0}=0
$$ uncharged:

Solve for and evaluate $I_{0}$ :
(b) Apply Kirchhoff's loop rule to the circuit after a long time has passed:

Solve for $I_{\infty}$ to obtain:
(c) Apply the junction rule at the junction between the $200-\Omega$
resistor and the capacitor to obtain:

Apply the loop rule to the loop containing the source, the $200-\Omega$ resistor and the capacitor to obtain:

Apply the loop rule to the loop containing the $600-\Omega$ resistor and the capacitor to obtain:

Differentiate equation (2) with respect to time to obtain:

Differentiate equation (3) with respect to time to obtain:

$$
\begin{equation*}
\frac{Q}{C}-R_{2} I_{2}=0 \tag{3}
\end{equation*}
$$

$$
\begin{aligned}
\frac{d}{d t}\left[\varepsilon-R_{1} I_{1}-\frac{Q}{C}\right] & =0-R_{1} \frac{d I_{1}}{d t}-\frac{1}{C} \frac{d Q}{d t} \\
& =-R_{1} \frac{d I_{1}}{d t}-\frac{1}{C} I_{3}=0
\end{aligned}
$$

or
$R_{1} \frac{d I_{1}}{d t}=-\frac{1}{C} I_{3}$
$\frac{d}{d t}\left[\frac{Q}{C}-R_{2} I_{2}\right]=\frac{1}{C} \frac{d Q}{d t}-R_{2} \frac{d I_{2}}{d t}=0$
or

$$
\begin{equation*}
R_{2} \frac{d I_{2}}{d t}=\frac{1}{C} I_{3} \tag{5}
\end{equation*}
$$

Using equation (1), substitute for $I_{3}$ in equation (5) to obtain:

$$
I_{0}=\frac{\varepsilon}{200 \Omega}=\frac{50.0 \mathrm{~V}}{200 \Omega}=0.250 \mathrm{~A}
$$

$$
50.0 \mathrm{~V}-(200 \Omega) I_{\infty}-(600 \Omega) I_{\infty}=0
$$

$$
I_{\infty}=\frac{50.0 \mathrm{~V}}{800 \Omega}=62.5 \mathrm{~mA}
$$

$$
\begin{equation*}
I_{1}=I_{2}+I_{3} \tag{1}
\end{equation*}
$$

Solve equation (2) for $I_{1}$ :

$$
I_{1}=\frac{\varepsilon-Q / C}{R_{1}}=\frac{\varepsilon-R_{2} I_{2}}{R_{1}}
$$

Substitute for $I_{1}$ in equation (6) and simplify to obtain the differential equation for $I_{2}$ :

$$
\begin{aligned}
\frac{d I_{2}}{d t} & =\frac{1}{R_{2} C}\left(\frac{\varepsilon-R_{2} I_{2}}{R_{1}}-I_{2}\right) \\
& =\frac{\varepsilon}{R_{1} R_{2} C}-\left(\frac{R_{1}+R_{2}}{R_{1} R_{2} C}\right) I_{2}
\end{aligned}
$$

To solve this linear differential

$$
\begin{equation*}
I_{2}(t)=a+b e^{-t / \tau} \tag{7}
\end{equation*}
$$ equation with constant coefficients we can assume a solution of the form:

Differentiate $I_{2}(t)$ with respect to time to obtain:

$$
\frac{d I_{2}}{d t}=\frac{d}{d t}\left[a+b e^{-t / \tau}\right]=-\frac{b}{\tau} e^{-t / \tau}
$$

Substitute for $I_{2}$ and $d I_{2} / d t$ to obtain:

$$
-\frac{b}{\tau} e^{-t / \tau}=\frac{\varepsilon}{R_{1} R_{2} C}-\left(\frac{R_{1}+R_{2}}{R_{1} R_{2} C}\right)\left(a+b e^{-t / \tau}\right)
$$

Equate coefficients of $e^{-t / \tau}$ to obtain: $\quad \tau=\frac{R_{1} R_{2} C}{R_{1}+R_{2}}$

Requiring the equation to hold for all values of $a$ yields:

$$
a=\frac{\varepsilon}{R_{1}+R_{2}}
$$

If $I_{2}$ is to be zero when $t=0$ :

$$
\begin{aligned}
& 0=a+b \\
& \text { or } \\
& b=-a=-\frac{\varepsilon}{R_{1}+R_{2}}
\end{aligned}
$$

Substitute in equation (7) to obtain:

$$
\begin{aligned}
I_{2}(t) & =\frac{\varepsilon}{R_{1}+R_{2}}-\frac{\varepsilon}{R_{1}+R_{2}} e^{-t / \tau} \\
& =\frac{\varepsilon}{R_{1}+R_{2}}\left(1-e^{-t / \tau}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\tau & =\frac{R_{1} R_{2} C}{R_{1}+R_{2}}=\frac{(200 \Omega)(600 \Omega)(5.00 \mu \mathrm{~F})}{200 \Omega+600 \Omega} \\
& =0.750 \mathrm{~ms}
\end{aligned}
$$

Substitute numerical values and evaluate $I_{2}(t)$ :

$$
\begin{aligned}
I_{2}(t) & =\frac{50.0 \mathrm{~V}}{200 \Omega+600 \Omega}\left(1-e^{-t / 0.750 \mathrm{~ms}}\right) \\
& =(62.5 \mathrm{~mA})\left(1-e^{-t / 0.750 \mathrm{~ms}}\right)
\end{aligned}
$$

104 ••• In the circuit shown in Figure 25-72, switch $S$ has been open for a long time. At time $t=0$ the switch is then closed. (a) What is the battery current just after switch S is closed? (b) What is the battery current a long time after switch S is closed? (c) The switch has been closed for a long time. At $t=0$ the switch is then opened. Find the current through the $600-\mathrm{k} \Omega$ resistor as a function of time.

Picture the Problem Let $R_{1}$ represent the $1.20-\mathrm{M} \Omega$ resistor and $R_{2}$ the $600-\mathrm{k} \Omega$ resistor. Immediately after switch S is closed, the capacitor has zero charge and so the potential difference across it (and the $600 \mathrm{k} \Omega$-resistor) is zero. A long time after the switch is closed, the capacitor will be fully charged and the potential difference across it will be given by both $Q / C$ and $I_{\infty} R_{2}$. When the switch is opened after having been closed for a long time, both the source and the $1.20-\mathrm{M} \Omega$ resistor will be out of the circuit and the fully charged capacitor will discharge through $R_{2}$. We can use Kirchhoff's loop to find the currents drawn from the source immediately after the switch is closed and a long time after the switch is closed, as well as the current in the $R C$ circuit when the switch is again opened and the capacitor discharges through $R_{2}$.
(a) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed to obtain:

Substitute numerical values and evaluate $I_{0}$ :
(b) Apply Kirchhoff's loop rule to the circuit a long time after the switch is closed to obtain:

Substitute numerical values and evaluate $I_{\infty}$ :

$$
\varepsilon-I_{0} R_{1}-V_{\mathrm{C} 0}=0
$$

$$
\text { or, because } V_{\mathrm{C} 0}=0 \text {, }
$$

$$
\varepsilon-I_{0} R_{1}=0 \Rightarrow I_{0}=\frac{\varepsilon}{R_{1}}
$$

$$
I_{0}=\frac{50.0 \mathrm{~V}}{1.20 \mathrm{M} \Omega}=41.7 \mu \mathrm{~A}
$$

$$
\mathcal{E}-I_{\infty} R_{1}-I_{\infty} R_{2}=0 \Rightarrow I_{\infty}=\frac{\varepsilon}{R_{1}+R_{2}}
$$

$$
I_{\infty}=\frac{50.0 \mathrm{~V}}{1.20 \mathrm{M} \Omega+600 \mathrm{k} \Omega}=27.8 \mu \mathrm{~A}
$$

(c) Apply Kirchhoff's loop rule to the $R C$ circuit sometime after the switch is opened and solve for $I(t)$ to obtain:

Substituting for $V_{\mathrm{C}}(t)$ yields:

Substitute numerical values to obtain:

$$
V_{\mathrm{C}}(t)-R_{2} I(t)=0 \Rightarrow I(t)=\frac{V_{\mathrm{C}}(t)}{R_{2}}
$$

$$
I(t)=\frac{V_{\mathrm{C} \infty}}{R_{2}} e^{-t / \tau}=I_{\infty} e^{-t / \tau}
$$

where $\tau=R_{2} C$.

$$
\begin{aligned}
I(t) & =(27.8 \mu \mathrm{~A}) e^{-t /(600 \mathrm{k} \Omega)(2.50 \mu \mathrm{~F})} \\
& =(27.8 \mu \mathrm{~A}) e^{-t / 1.50 \mathrm{~s}}
\end{aligned}
$$

105 ••• [SSM] In the circuit shown in Figure 25-74, the capacitor has a capacitance of $2.50 \mu \mathrm{~F}$ and the resistor has a resistance of $0.500 \mathrm{M} \Omega$. Before the switch is closed, the potential drop across the capacitor is 12.0 V , as shown. Switch S is closed at $t=0$. (a) What is the current immediately after switch S is closed? (b) At what time $t$ is the voltage across the capacitor 24.0 V ?

Picture the Problem We can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current $I_{0}$. We can find the time at which the voltage across the capacitor is 24.0 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t)=I_{0} e^{-t / \tau}$ for the current through the resistor as a function of time and solving for $t$.
(a) Apply Kirchhoff's loop rule to the circuit immediately after the switch is closed:

Solving for $I_{0}$ yields:

$$
\mathcal{E}-12.0 \mathrm{~V}-I_{0} R=0
$$

$$
I_{0}=\frac{\varepsilon-12.0 \mathrm{~V}}{R}
$$

Substitute numerical values and evaluate $I_{0}$ :
(b) Apply Kirchhoff's loop rule to the circuit when $V_{\mathrm{C}}=24.0 \mathrm{~V}$ and solve for $V_{R}$ :

$$
I_{0}=\frac{36.0 \mathrm{~V}-12.0 \mathrm{~V}}{0.500 \mathrm{M} \Omega}=48.0 \mu \mathrm{~A}
$$

$$
36.0 \mathrm{~V}-24.0 \mathrm{~V}-I(t) R=0
$$

and

$$
I(t) R=12.0 \mathrm{~V}
$$

Express the current through the

$$
I(t)=I_{0} e^{-t / \tau} \text { where } \tau=R C \text {. }
$$

resistor as a function of $I_{0}$ and $\tau$ :

Substitute to obtain:

$$
R I_{0} e^{-t / \tau}=12.0 \mathrm{~V} \Rightarrow e^{-t / \tau}=\frac{12.0 \mathrm{~V}}{R I_{0}}
$$

Take the natural logarithm of both sides of the equation to obtain:

$$
-\frac{t}{\tau}=\ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)
$$

Solving for $t$ yields:

$$
t=-\tau \ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)=-R C \ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)
$$

Substitute numerical values and evaluate $t$ :

$$
t=-(0.500 \mathrm{M} \Omega)(2.50 \mu \mathrm{~F}) \ln \left[\frac{12.0 \mathrm{~V}}{(0.500 \mathrm{M} \Omega)(48.0 \mu \mathrm{~A})}\right]=0.866 \mathrm{~s}
$$

106 ••• Repeat Problem 105 if the initial polarity of the capacitor opposite to that shown in Figure 25-74.

Picture the Problem We can apply Kirchhoff's loop rule to the circuit immediately after the switch is closed in order to find the initial current $I_{0}$. We can find the time at which the voltage across the capacitor is 24.0 V by again applying Kirchhoff's loop rule to find the voltage across the resistor when this condition is satisfied and then using the expression $I(t)=I_{0} e^{-t / \tau}$ for the current through the resistor as a function of time and solving for $t$.
(a) Apply Kirchhoff's loop rule to the circuit immediately after the

$$
\varepsilon+12.0 \mathrm{~V}-I_{0} R=0 \Rightarrow I_{0}=\frac{\varepsilon+12.0 \mathrm{~V}}{R}
$$ switch is closed:

Substitute numerical values and evaluate $I_{0}$ :
(b) Apply Kirchhoff's loop rule to the circuit when $V_{\mathrm{C}}=24.0 \mathrm{~V}$ and solve for $V_{R}$ :

$$
I_{0}=\frac{36.0 \mathrm{~V}+12.0 \mathrm{~V}}{0.500 \mathrm{M} \Omega}=96.0 \mu \mathrm{~A}
$$

$$
36.0 \mathrm{~V}-24.0 \mathrm{~V}-I(t) R=0
$$

and

$$
I(t) R=12.0 \mathrm{~V}
$$

Express the current through the

$$
I(t)=I_{0} e^{-t / \tau} \text { where } \tau=R C \text {. }
$$

resistor as a function of $I_{0}$ and $\tau$ :

Substitute for $I(t)$ to obtain:

$$
\begin{aligned}
& R I_{0} e^{-t / \tau}=12.0 \mathrm{~V} \Rightarrow e^{-t / \tau}=\frac{12.0 \mathrm{~V}}{R I_{0}} \\
& -\frac{t}{\tau}=\ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)
\end{aligned}
$$

Taking the natural logarithm of both sides of the equation yields:

Solving for $t$ yields:

$$
t=-\tau \ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)=-R C \ln \left(\frac{12.0 \mathrm{~V}}{R I_{0}}\right)
$$

Substitute numerical values and evaluate $t$ :

$$
t=-(0.500 \mathrm{M} \Omega)(2.50 \mu \mathrm{~F}) \ln \left[\frac{12.0 \mathrm{~V}}{(0.500 \mathrm{M} \Omega)(96.0 \mu \mathrm{~A})}\right]=1.73 \mathrm{~s}
$$

## General Problems

107 •• [SSM] In Figure 25-75, $R_{1}=4.00 \Omega, R_{2}=6.00 \Omega, R_{3}=12.0 \Omega$, and the battery emf is 12.0 V . Denote the currents through these resistors as $I_{1}, I_{2}$ and $I_{3}$, respectively, (a) Decide which of the following inequalities holds for this circuit. Explain your answer conceptually. (1) $I_{1}>I_{2}>I_{3}$, (2) $I_{2}=I_{3}$, (3) $I_{3}>I_{2}$, (4) None of the above (b) To verify that your answer to Part $(a)$ is correct, calculate all three currents.

Determine the Concept We can use Kirchhoff's rules in Part (b) to confirm our choices in Part (a).
(a) 1. The potential drops across $R_{2}$ and $R_{3}$ are equal, so $I_{2}>I_{3}$. The current in $R_{1}$ equals the sum of the currents $I_{2}$ and $I_{3}$, so $I_{1}$ is greater than either $I_{2}$ or $I_{3}$.
(b) Apply Kirchhoff's junction rule $\quad I_{1}-I_{2}-I_{3}=0$ to obtain:

Applying Kirchhoff's loop rule in the clockwise direction to the loop defined by the two resistors in

$$
R_{3} I_{3}-R_{2} I_{2}=0
$$ parallel yields:

or

$$
\begin{equation*}
0 I_{1}-R_{2} I_{2}+R_{3} I_{3}=0 \tag{2}
\end{equation*}
$$

Apply Kirchhoff's loop rule in the clockwise direction to the loop around the perimeter of the circuit to obtain:

Substituting numerical values in equations (1), (2), and (3) yields:

Solve this system of three equations in three unknowns for the currents in the branches of the circuit to obtain:

$$
\mathcal{E}-R_{1} I_{1}-R_{2} I_{2}=0
$$

or

$$
\begin{equation*}
R_{1} I_{1}+R_{2} I_{2}-0 I_{3}=\varepsilon \tag{3}
\end{equation*}
$$

$$
\begin{aligned}
& I_{1}-I_{2}-I_{3}=0 \\
& 0 I_{1}-(6.00 \Omega) I_{2}+(12.0 \Omega) I_{3}=0 \\
& (4.00 \Omega) I_{1}+(6.00 \Omega) I_{2}-0 I_{3}=12.0 \mathrm{~V}
\end{aligned}
$$

$$
I_{1}=1.50 \mathrm{~A}, I_{2}=1.00 \mathrm{~A}
$$

$$
\text { and } I_{3}=0.50 \mathrm{~A} \text {, confirming our }
$$ choice in Part (a).

108 •• A $120-\mathrm{V}, 25.0-\mathrm{W}$ light bulb is connected in series with a $120-\mathrm{V}$, $100-\mathrm{W}$ light bulb and a potential difference of 120 V is placed across the combination. Assume the bulbs have constant resistance. (a) Which bulb should be brighter under these conditions? Explain your answer conceptually. Hint: What does the phrase "25.0-W light bulb" mean? That is, under what conditions is 25-W of power delivered to the bulb? (b) Determine the power delivered to each bulb under these conditions. Do your results support your answer to Part (a)?

Picture the Problem In Part (b) we need to find the current drawn by the series combination of the two light bulbs in order to calculate the actual power output of each bulb. To find this current, we first need to determine the resistances of the bulbs. These resistances are given by $R=\mathcal{E}^{2} / P$, one of the three forms of Equation 25-14.
(a) The $25-\mathrm{W}$ bulb will be brighter. The more power delivered to a bulb the brighter the bulb. The resistance of the $25-\mathrm{W}$ bulb is 4 times greater than that of the $100-\mathrm{W}$ bulb, and in the series combination, the same current $I$ flows through the bulbs. Hence, $I^{2} R_{25}>I^{2} R_{100}$.
(b) The actual power output of each

$$
\begin{equation*}
P_{25}=I^{2} R_{25} \tag{1}
\end{equation*}
$$ bulb is given by:

and

$$
\begin{equation*}
P_{100}=I^{2} R_{100} \tag{2}
\end{equation*}
$$

We need to find the current drawn by the series combination of the two light bulbs. To find this current, we first need to determine the resistances of the bulbs. There resistances are given by:

Use Ohm's law to express the current drawn by the series combination of the light bulbs:

Substitute numerical values and evaluate $I$ :

Substitute numerical values in equations (1) and (2) and evaluate $P_{25}$ and $P_{100}$ :

$$
R_{25}=\frac{\varepsilon^{2}}{P_{25}}=\frac{(120 \mathrm{~V})^{2}}{25 \mathrm{~W}}=576 \Omega
$$

and

$$
R_{100}=\frac{\varepsilon^{2}}{P_{100}}=\frac{(120 \mathrm{~V})^{2}}{100 \mathrm{~W}}=144 \Omega
$$

$I=\frac{\varepsilon}{R_{\text {eq }}}=\frac{\varepsilon}{R_{25}+R_{100}}$
$I=\frac{120 \mathrm{~V}}{576 \Omega+144 \Omega}=0.1667 \mathrm{~A}$
$P_{25}=(0.1667 \mathrm{~A})^{2}(576 \Omega)=16 \mathrm{~W}$
and

$$
P_{100}=(0.1667 \mathrm{~A})^{2}(144 \Omega)=4 \mathrm{~W}
$$

confirming our choice in Part (a).

109 •• The circuit shown in Figure 25-76 is a Wheatstone bridge, and the variable resistor is being used as a slide-wire potentiometer. The resistance $R_{0}$ is known. This "bridge" is used to determine an unknown resistance $R_{x}$. The resistances $R_{1}$ and $R_{2}$ comprise a wire 1.00 m long. Point $a$ is a sliding contact that is moved along the wire to vary these resistances. Resistance $R_{1}$ is proportional to the distance from the left end of the wire (labeled 0.00 cm ) to point $a$, and $R_{2}$ is proportional to the distance from point $a$ to the right end of the wire (labeled 100 cm ). The sum of $R_{1}$ and $R_{2}$ remains constant. When points $a$ and $b$ are at the same potential, there is no current in the galvanometer and the bridge is said to be balanced. (Because the galvanometer is used to detect the absence of a current, it is called a null detector.) If the fixed resistance $R_{0}=200 \Omega$, find the unknown resistance $R_{x}$ if $(a)$ the bridge balances at the $18.0-\mathrm{cm}$ mark, $(b)$ the bridge balances at the $60.0-\mathrm{cm}$ mark, and (c) the bridge balances at the $95.0-\mathrm{cm}$ mark.

Picture the Problem Let the current flowing through the galvanometer be $I_{\mathrm{G}}$. By applying Kirchhoff's rules to the loops including 1) $R_{1}$, the galvanometer, and $R_{x}$, and 2) $R_{2}$, the galvanometer, and $R_{0}$, we can obtain two equations relating the unknown resistance to $R_{1}, R_{2}$ and $R_{0}$. Using $R=\rho L / A$ will allow us to express $R_{x}$ in terms of the length of wire $L_{1}$ that corresponds to $R_{1}$ and the length of wire $L_{2}$ that corresponds to $R_{2}$.

Apply Kirchhoff's loop rule to the $\quad-R_{1} I_{1}+R_{x} I_{2}=0$
loop that includes $R_{1}$, the galvanometer, and $R_{x}$ to obtain:

Apply Kirchhoff's loop rule to the $\quad-R_{2}\left(I_{1}-I_{\mathrm{G}}\right)+R_{0}\left(I_{2}+I_{\mathrm{G}}\right)=0$
loop that includes $R_{2}$, the galvanometer, and $R_{0}$ to obtain:

When the bridge is balanced, $I_{\mathrm{G}}=0$ and equations (1) and (2) become:

Divide equation (3) by equation (4) and solve for $R_{x}$ to obtain:

Express $R_{1}$ and $R_{2}$ in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

Substitute in equation (5) to obtain:
(a) When the bridge balances at the $18.0-\mathrm{cm}$ mark, $L_{1}=18.0 \mathrm{~cm}$,

$$
\begin{equation*}
R_{1} I_{1}=R_{x} I_{2} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{2} I_{1}=R_{0} I_{2} \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
R_{x}=R_{0} \frac{R_{1}}{R_{2}} \tag{5}
\end{equation*}
$$

$$
R_{1}=\rho \frac{L_{1}}{A} \text { and } R_{2}=\rho \frac{L_{2}}{A}
$$

$$
R_{x}=R_{0} \frac{L_{1}}{L_{2}}
$$

$L_{2}=82.0 \mathrm{~cm}$ and:
(b) When the bridge balances at the $60.0-\mathrm{cm}$ mark, $L_{1}=60.0 \mathrm{~cm}$, $L_{2}=40.0 \mathrm{~cm}$ and:
(c) When the bridge balances at the $95.0-\mathrm{cm}$ mark, $L_{1}=95.0 \mathrm{~cm}$,

$$
R_{x}=(200 \Omega) \frac{18.0 \mathrm{~cm}}{82.0 \mathrm{~cm}}=43.9 \Omega
$$

$L_{2}=5.0 \mathrm{~cm}$ and:

$$
\begin{aligned}
& R_{x}=(200 \Omega) \frac{60.0 \mathrm{~cm}}{40.0 \mathrm{~cm}}=300 \Omega \\
& R_{x}=(200 \Omega) \frac{95.0 \mathrm{~cm}}{5.0 \mathrm{~cm}}=3.8 \mathrm{k} \Omega
\end{aligned}
$$

110 •• For the Wheatstone bridge in Problem 109, suppose the bridge balances at the $98.0-\mathrm{cm}$ mark. (a) What is the unknown resistance? (b) What is the percentage error in the measured value of $R_{x}$ if there is an error of 2.00 mm in the location of the balance point? (c) To what value should $R_{0}$ be changed to so that the balance point for this unknown resistor will be nearer the $50.0-\mathrm{cm}$ mark? (d) If the balance point is at the $50.0-\mathrm{cm}$ mark, what is the percentage error in the measured value of $R_{x}$ if there is an error of 2.00 mm in the location of the balance point?

Picture the Problem Let the current flowing through the galvanometer by $I_{\mathrm{G}}$. By applying Kirchhoff's rules to the loops including 1) $R_{1}$, the galvanometer, and $R_{x}$, and 2) $R_{2}$, the galvanometer, and $R_{0}$, we can obtain two equations relating the unknown resistance to $R_{1}, R_{2}$ and $R_{0}$. Using $R=\rho L / A$ will allow us to express $R_{x}$ in terms of the length of wire $L_{1}$ that corresponds to $R_{1}$ and the length of wire $L_{2}$ that corresponds to $R_{2}$. To find the effect of an error of 2.00 mm in the location of the balance point we can use the relationship $\Delta R_{x}=\left(d R_{x} / d L\right) \Delta L$ to determine $\Delta R_{x}$ and then divide by $R_{x}=R_{0} L /(1-L)$ to find the fractional change (error) in $R_{x}$ resulting from a given error in the determination of the balance point.

Apply Kirchhoff's loop rule to the $\quad-R_{1} I_{1}+R_{x} I_{2}=0$ loop that includes $R_{1}$, the galvanometer, and $R_{x}$ to obtain:

Apply Kirchhoff's loop rule to the $\quad-R_{2}\left(I_{1}-I_{\mathrm{G}}\right)+R_{0}\left(I_{2}+I_{\mathrm{G}}\right)=0$ loop that includes $R_{2}$, the galvanometer, and $R_{0}$ to obtain:

When the bridge is balanced, $I_{\mathrm{G}}=0$ and equations (1) and (2) become:

$$
\begin{equation*}
R_{1} I_{1}=R_{x} I_{2} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{2} I_{1}=R_{0} I_{2} \tag{4}
\end{equation*}
$$

Divide equation (3) by equation (4) and solve for $R_{x}$ to obtain:

$$
\begin{equation*}
R_{x}=R_{0} \frac{R_{1}}{R_{2}} \tag{5}
\end{equation*}
$$

Express $R_{1}$ and $R_{2}$ in terms of their lengths, cross-sectional areas, and the resistivity of their wire:

Substitute in equation (5) to obtain:
(a) When the bridge balances at the $98.0-\mathrm{cm}$ mark, $L_{1}=98.0 \mathrm{~cm}$, $L_{2}=2.0 \mathrm{~cm}$ and:
(b) Express $R_{x}$ in terms of the distance to the balance point:

$$
R_{1}=\rho \frac{L_{1}}{A} \text { and } R_{2}=\rho \frac{L_{2}}{A}
$$

$$
\begin{equation*}
R_{x}=R_{0} \frac{L_{1}}{L_{2}} \tag{6}
\end{equation*}
$$

$$
R_{x}=(200 \Omega) \frac{98.0 \mathrm{~cm}}{2.0 \mathrm{~cm}}=9.8 \mathrm{k} \Omega
$$

Express the error $\Delta R_{x}$ in $R_{x}$ resulting from an error $\Delta L$ in $L$ :

$$
\begin{aligned}
\Delta R_{x} & =\frac{d R_{x}}{d L} \Delta L=R_{0} \frac{d}{d L}\left[\frac{L}{1-L}\right] \Delta L \\
& =R_{0} \frac{1}{(1-L)^{2}} \Delta L
\end{aligned}
$$

Divide $\Delta R_{x}$ by $R_{x}$ to obtain:

$$
\frac{\Delta R_{x}}{R_{x}}=\frac{R_{0} \frac{1}{(1-L)^{2}} \Delta L}{R_{0} \frac{L}{1-L}}=\frac{1}{1-L} \frac{\Delta L}{L}
$$

Evaluate $\Delta R_{x} / R_{x}$ for $L=98.0 \mathrm{~cm}$ and $\Delta L=2.00 \mathrm{~mm}$ :

$$
\begin{aligned}
\frac{\Delta R_{x}}{R_{x}} & =\left(\frac{1.00 \mathrm{~m}}{1.00 \mathrm{~m}-0.98 \mathrm{~m}}\right)\left(\frac{2.00 \mathrm{~mm}}{1.00 \mathrm{~m}}\right) \\
& =10 \%
\end{aligned}
$$

(c) Solve equation (6) for the ratio of $L_{1}$ to $L_{2}$ :

$$
\frac{L_{1}}{L_{2}}=\frac{R_{x}}{R_{0}}
$$

For $L_{1}=50.0 \mathrm{~cm}, L_{2}=50.0 \mathrm{~cm}, R_{0}=R_{\mathrm{x}}=9.8 \mathrm{k} \Omega$. Hence, a resistor of approximately $10 \mathrm{k} \Omega$ will cause the bridge to balance near the $50.0-\mathrm{cm}$ mark.
(d) From Part (b) we have:

$$
\frac{\Delta R_{x}}{R_{x}}=\frac{1}{1-L} \frac{\Delta L}{L}
$$

Evaluate $\Delta R_{x} / R_{x}$ for $L=50.0 \mathrm{~cm}$ and $\Delta L=2.00 \mathrm{~mm}$ :

$$
\begin{aligned}
\frac{\Delta R_{x}}{R_{x}} & =\left(\frac{1.00 \mathrm{~m}}{1.00 \mathrm{~m}-0.50 \mathrm{~m}}\right)\left(\frac{2.00 \mathrm{~mm}}{1.00 \mathrm{~m}}\right) \\
& =0.40 \%
\end{aligned}
$$

111 •• [SSM] You are running an experiment that uses an accelerator that produces a $3.50-\mu \mathrm{A}$ proton beam. Each proton in the beam has $60.0-\mathrm{MeV}$ of kinetic energy. The protons impinge upon, and come to rest inside, a $50.0-\mathrm{g}$ copper target within a vacuum chamber. You are concerned that the target will get too hot and melt the solder on some connecting wires that are crucial to the experiment. (a) Determine the number of protons that strike the target per second. (b) Find the amount of energy delivered to the target each second. (c) Determine how much time elapses before the target temperature increases to $300^{\circ} \mathrm{C}$ ? (Neglect any heat released by the target.)

Picture the Problem Knowing the beam current and charge per proton, we can use $I=n e$ to determine the number of protons striking the target per second. The energy deposited per second is the power delivered to the target and is given by
$P=I V$. We can find the elapsed time before the target temperature rises $300^{\circ} \mathrm{C}$ using $\Delta Q=P \Delta t=m c_{\mathrm{Cu}} \Delta T$.
(a) Relate the current to the number of protons per second $n$ arriving at

$$
I=n e \Rightarrow n=\frac{I}{e}
$$

the target:
Substitute numerical values and evaluate $n$ :

$$
n=\frac{3.50 \mu \mathrm{~A}}{1.602 \times 10^{-19} \mathrm{C}}=2.18 \times 10^{13} \mathrm{~s}^{-1}
$$

(b) Express the power of the beam in terms of the beam current and energy:

$$
\begin{aligned}
P & =I V=(3.50 \mu \mathrm{~A})\left(60.0 \frac{\mathrm{MeV}}{\text { proton }}\right) \\
& =210 \mathrm{~W}
\end{aligned}
$$

(c) Relate the energy delivered to the

$$
\Delta Q=P \Delta t=C_{\mathrm{Cu}} \Delta T=m c_{\mathrm{Cu}} \Delta T
$$ target to its heat capacity and temperature change:

Solving for $\Delta t$ yields:

$$
\Delta t=\frac{m c_{\mathrm{cu}} \Delta T}{P}
$$

Substitute numerical values (see Table 19-1 for the specific heat of copper) and evaluate $\Delta t$ :

$$
\begin{aligned}
\Delta t & =\frac{(50.0 \mathrm{~g})(0.386 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})\left(300^{\circ} \mathrm{C}\right)}{210 \mathrm{~J} / \mathrm{s}} \\
& =27.6 \mathrm{~s}
\end{aligned}
$$

112 •• The belt of a Van de Graaff generator carries a surface charge density of $5.00 \mathrm{mC} / \mathrm{m}^{2}$. The belt is 0.500 m wide and moves at $20.0 \mathrm{~m} / \mathrm{s}$. (a) What current does the belt carry? (b) If the potential of the dome of the generator is 100 kV above ground, what is the minimum power of the motor needed to drive the belt?

Picture the Problem We can use the definition of current to express the current delivered by the belt in terms of the surface charge density, width, and speed of the belt. The minimum power needed to drive the belt can be found from $P=I V$.
(a) Use its definition to express the current carried by the belt:

$$
\begin{aligned}
I & =\frac{d Q}{d t}=\sigma w \frac{d x}{d t}=\sigma w v \\
I & =\left(5.00 \mathrm{mC} / \mathrm{m}^{2}\right)(0.500 \mathrm{~m})(20.0 \mathrm{~m} / \mathrm{s}) \\
& =50.0 \mathrm{~mA}
\end{aligned}
$$

(b) Express the minimum power of $\quad P=I V$ the motor in terms of the current delivered and the potential of the charge:

Substitute numerical values and evaluate $P$ :

$$
P=(50.0 \mathrm{~mA})(100 \mathrm{kV})=5.00 \mathrm{~kW}
$$

113 •- Large conventional electromagnets use water cooling to prevent excessive heating of the magnet coils. A large laboratory electromagnet carries a current equal to 100 A when a voltage of 240 V is applied to the terminals of the energizing coils. To cool the coils, water at an initial temperature of $15^{\circ} \mathrm{C}$ is circulated around the coils. How many liters of water must circulate by the coils each second if the temperature of the coils is not to exceed $50^{\circ} \mathrm{C}$ ?

Picture the Problem We can differentiate the expression relating the amount of heat required to produce a given temperature change with respect to time to express the mass flow-rate required to maintain the temperature of the coils at $50^{\circ} \mathrm{C}$. We can then use the definition of density to find the necessary volume flow rate.

Express the heat that must be

$$
Q=m c_{\text {water }} \Delta T
$$

dissipated in terms of the specific
heat and mass of the water and
the desired temperature change
of the water:

Differentiate this expression with respect to time to obtain an

$$
P=\frac{d Q}{d t}=\frac{d m}{d t} c_{\text {water }} \Delta T
$$

expression for the power dissipation:
Solving for $\frac{d m}{d t}$ yields:

$$
\frac{d m}{d t}=\frac{P}{c_{\text {water }} \Delta T}
$$

Substitute for the power dissipated to obtain:

$$
\frac{d m}{d t}=\frac{I V}{c_{\text {water }} \Delta T}
$$

Substitute numerical values and evaluate $\frac{d m}{d t}$ :

$$
\begin{aligned}
\frac{d m}{d t} & =\frac{(100 \mathrm{~A})(240 \mathrm{~V})}{\left(4.18 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}}\right)\left(50^{\circ} \mathrm{C}-15^{\circ} \mathrm{C}\right)} \\
& =0.164 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

Using the definition of density, express the volume flow rate in

$$
\frac{d V}{d t}=\frac{1}{\rho} \frac{d m}{d t}
$$ terms of the mass flow rate to obtain:

Substitute numerical values and evaluate $\frac{d V}{d t}$ :

$$
\begin{aligned}
\frac{d V}{d t} & =\left(\frac{0.164 \frac{\mathrm{~kg}}{\mathrm{~s}}}{1.00 \times 10^{3} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}}\right)\left(\frac{1 \mathrm{~L}}{10^{-3} \mathrm{~m}^{3}}\right) \\
& =0.16 \mathrm{~L} / \mathrm{s}
\end{aligned}
$$

114 ••• (a) Give support to the assertion that a leaky capacitor (one for which the resistance of the dielectric is finite) can be modeled as a capacitor that has an infinite resistance in parallel with a resistor. (b) Show that the time constant for discharging this capacitor is given by $\tau=\kappa \epsilon_{0} \rho$. (For simplicity, assume the capacitor is a parallel plate variety filled completely with a leaky dielectric.) (c) Mica has a dielectric constant equal to about 5.0 and a resistivity equal to about $9.0 \times 10^{13} \Omega \cdot \mathrm{~m}$. Calculate the time it takes for the charge of a mica-filled capacitor to decrease to 10 percent of its initial value.

Picture the Problem We'll assume that the capacitor is fully charged initially and apply Kirchhoff's loop rule to the circuit fragment to obtain the differential equation describing the discharge of the leaky capacitor. We'll show that the solution to this equation is the familiar expression for an exponential decay with time constant
 $\tau=\epsilon_{0} \rho \kappa$.
(a) If we think of the leaky capacitor as a resistor/capacitor combination, the voltage drop across the resistor must be the same as the voltage drop across the capacitor. Hence they must be in parallel.
(b) Assuming that the capacitor is initially fully charged, apply Kirchhoff's loop rule to the circuit fragment to obtain:

$$
\frac{Q}{C}-R I=0
$$

or, because $I=-\frac{d Q}{d t}$,

$$
\frac{Q}{C}+R \frac{d Q}{d t}=0
$$

Separate variables in this differential equation to obtain:

$$
\begin{equation*}
\frac{d Q}{Q}=-\frac{1}{R C} d t \tag{1}
\end{equation*}
$$

The capacitance of a dielectric-filled parallel-plate capacitor with plate

$$
C=\frac{\kappa \epsilon_{0} A}{d}
$$

area $A$ and plate separation $d$ is given
by:

The resistance of a conductor with the same dimensions is given by:

$$
R=\frac{\rho d}{A}
$$

The product of $R$ and $C$ is:

$$
R C=\epsilon_{0} \rho \kappa
$$

Substituting for $R C$ in the differential equation yields:

$$
\frac{d Q}{Q}=-\frac{1}{\epsilon_{0} \rho \kappa} d t
$$

Integrate this equation from $Q^{\prime}=Q_{0}$ to $Q$ to obtain:
(c) Because $Q / Q_{0}=0.10$ :

Solve for $t$ by taking the natural logarithm of both sides of the equation:

Substitute numerical values and evaluate $t$ :

$$
t=-\left(8.854 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}\right)\left(9.0 \times 10^{13} \Omega \cdot \mathrm{~m}\right)(5) \ln (0.10)=9.17 \times 10^{3} \mathrm{~s} \approx 2.5 \mathrm{~h}
$$

115 ... Figure $25-77$ shows the basis of the sweep circuit used in an oscilloscope. Switch S is an electronic switch that closes whenever the potential across the switch $S$ increases to a value $V_{c}$ and opens when the potential across switch S decreases to 0.200 V . The emf $\mathcal{E}$ ], which is much greater than $V_{c}$, charges the capacitor $C$ through a resistor $R_{1}$. The resistor $R_{2}$ represents the small but finite resistance of the electronic switch. In a typical circuit, $\mathcal{E}=800 \mathrm{~V}$, $V_{c}=4.20 \mathrm{~V}, R_{2}=1.00 \mathrm{~m} \Omega, R_{1}=0.500 \mathrm{M} \Omega$, and $C=20.0 \mathrm{nF}$. (a) What is the time constant for charging of the capacitor $C$ ? (b) Show that as the potential across switch S increases from 0.200 V to 4.20 V , the potential across the capacitor increases almost linearly with time. Hint: Use the approximation $e^{x} \approx 1+x$, for $|x| \ll 1$. (This approximation of $e^{x}$ can be derived using the differential approximation.) (c) What should the value of $R_{1}$ be changed to so that the capacitor charges from 0.200 V to 4.20 V in 0.100 s ? (d) How much time elapses during the discharge of the capacitor when switch S closes? (e) At what average
rate is energy dissipated in the resistor $R_{1}$ during charging and in the switch resistance $R_{2}$ during discharge?

Picture the Problem We can use its definition to find the time constant of the charging circuit in Part $(a)$. In Part $(b)$ we can use the expression for the potential difference as a function of time across a charging capacitor and utilize the hint given in the problem statement to show that the voltage across the capacitor increases almost linearly over the time required to bring the potential across the switch to its critical value. In Part $(c)$ we can use the result derived in Part (b) to find the value of $R_{1}$ such that $C$ charges from 0.200 V to 4.20 V in 0.100 s . In Part (d) we can use the expression for the potential difference as a function of time across a discharging capacitor to find the discharge time. Finally, in Part (e) we can integrate $I^{2} R_{1}$ over the discharge time to find the rate at which energy is dissipated in $R_{1}$ during the discharge of the capacitor and use the difference in the energy stored in the capacitor initially and when the switch opens to find the rate of energy dissipation in resistance of the capacitor.
(a) When the capacitor is charging, the switch is open and the resistance in the charging circuit is $R_{1}$. Hence:
(b) Express the voltage across the charging capacitor as a function of time:

Use the power series for $e^{x}$ to expand $e^{t / \tau}$ :

$$
\begin{aligned}
\tau & =R_{1} C=(0.500 \mathrm{M} \Omega)(20.0 \mathrm{nF}) \\
& =10.0 \mathrm{~ms}
\end{aligned}
$$

$$
V(t)=\varepsilon\left(1-e^{-t / \tau}\right)
$$

$$
\begin{aligned}
& e^{t / \tau}=1+\frac{t}{\tau}+\frac{1}{2!}\left(\frac{t}{\tau}\right)^{2}+\ldots \\
& \approx 1+\frac{1}{\tau} t \\
& \text { provided } t / \tau \ll 1
\end{aligned}
$$

$$
V(t)=\boldsymbol{\varepsilon}\left\{1-\left(1+\frac{1}{\tau} t\right)\right\}=\frac{\mathcal{E}}{\tau} t
$$

(c) Using the result derived in (b), relate the time $\Delta t$ required to change the voltage across the capacitor by an amount $\Delta V$ to $\Delta V(t)$ :

$$
\Delta V(t)=\frac{\varepsilon}{\tau} \Delta t
$$

or

$$
\tau=R_{1} C=\frac{\varepsilon}{\Delta V(t)} \Delta t \Rightarrow R_{1}=\frac{\varepsilon}{C \Delta V(t)} \Delta t
$$

Substitute numerical values and evaluate $R_{1}$ :
(d) Express the potential difference across the capacitor as a function of time:

Solve for $t$ to obtain:

Substitute numerical values and evaluate $t$ :
(e) Express the rate at which energy is dissipated in $R_{1}$ as a function of its resistance and the current through it:

Because the current varies with time, we need to integrate over time to find $\Delta E_{1}$ :

Substitute numerical values and evaluate $P_{1}$ :

Express the rate at which energy is dissipated in the switch resistance:

$$
\begin{aligned}
R_{1} & =\frac{(800 \mathrm{~V})(0.100 \mathrm{~s})}{(20.0 \mathrm{nF})(4.20 \mathrm{~V}-0.200 \mathrm{~V})} \\
& =1.00 \mathrm{G} \Omega
\end{aligned}
$$

$$
V_{\mathrm{C}}(t)=V_{\mathrm{C} 0} e^{-t / \tau^{t}}
$$

$$
\begin{aligned}
\Delta E_{1} & =\int_{t_{1}}^{t_{2}} I^{2} R_{1} d t=\int_{t_{1}}^{t_{2}}\left(\frac{V(t)}{R_{1}}\right)^{2} R_{1} d t \\
& =\int_{t_{1}}^{t_{2}}\left(\frac{\varepsilon t}{\tau R_{1}}\right)^{2} R_{1} d t \\
& =\left(\frac{\varepsilon}{\tau}\right)^{2} \frac{1}{R_{1}} \int_{0.005 \mathrm{~s}}^{0.105 \mathrm{~s}} t^{2} d t \\
& =\left(\frac{800 \mathrm{~V}}{20.0 \mathrm{~s}}\right)^{2} \frac{1}{1.00 \mathrm{G} \Omega}\left[\frac{t^{3}}{3}\right]_{0.00500 \mathrm{~s}}^{0.105 \mathrm{~s}} \\
& =6.17 \times 10^{-10} \mathrm{~J}
\end{aligned}
$$

where

$$
\tau^{\prime}=R_{2} C .
$$

$$
t=-\tau^{\prime} \ln \left(\frac{V_{\mathrm{C}}(t)}{V_{\mathrm{C} 0}}\right)=-R_{2} C \ln \left(\frac{V_{\mathrm{C}}(t)}{V_{\mathrm{C} 0}}\right)
$$

$$
\begin{aligned}
t & =-(1.00 \mathrm{~m} \Omega)(20.0 \mathrm{nF}) \ln \left(\frac{0.200 \mathrm{~V}}{4.20 \mathrm{~V}}\right) \\
& =60.89 \mathrm{ps}=60.9 \mathrm{ps}
\end{aligned}
$$

$$
P_{1}=\frac{\Delta E_{1}}{\Delta t}=I^{2} R_{1}
$$

$$
P_{1}=\frac{6.17 \times 10^{-10} \mathrm{~J}}{0.100 \mathrm{~s}}=6.17 \mathrm{nW}
$$

$$
\begin{aligned}
P_{2} & =\frac{\Delta U_{\mathrm{C}}}{\Delta t}=\frac{U_{\mathrm{Ci}}-U_{\mathrm{Cf}}}{\Delta t} \\
& =\frac{\frac{1}{2} C V_{\mathrm{i}}^{2}-\frac{1}{2} C V_{\mathrm{f}}^{2}}{\Delta t}=\frac{\frac{1}{2} C\left(V_{\mathrm{i}}^{2}-V_{\mathrm{f}}^{2}\right)}{\Delta t}
\end{aligned}
$$

Substitute numerical values and evaluate $P_{2}$ :

$$
P_{2}=\frac{\left.\frac{1}{2}(20.0 \mathrm{nF})(4.20 \mathrm{~V})^{2}-(0.200 \mathrm{~V})^{2}\right\rfloor}{60.89 \mathrm{ps}}=2.89 \mathrm{~kW}
$$

116 ••• In the circuit shown in Figure $25-78, R_{1}=2.00 \mathrm{M} \Omega, R_{2}=5.00 \mathrm{M} \Omega$, and $C=1.00 \mu \mathrm{~F}$. The capacitor is initially without charge on either plate. At $t=0$, switch S is closed, and at $t=2.00 \mathrm{~s}$ switch S is opened. (a) Sketch a graph of the voltage across $C$ and the current in $R_{2}$ between $t=0$ and $t=10.0$ s. (b) Find the voltage across the capacitor at $t=2.00 \mathrm{~s}$ and at $t=8.00 \mathrm{~s}$.

Picture the Problem We can apply both the loop and junction rules to obtain equations that we can use to obtain a linear differential equation with constant coefficients describing the current in $R_{2}$ as a function of time. We can solve this differential equation by assuming a solution of an appropriate form, differentiating this assumed solution and substituting it and its derivative in the differential equation. Equating coefficients, requiring the solution to hold for all values of the assumed constants, and invoking an initial condition will allow us to find the constants in the assumed solution. Once we know how the current varies with time in $R_{2}$, we can express the potential difference across it (as well as across $C$ because they are in parallel). To find the voltage across the capacitor at $t=8.00 \mathrm{~s}$, we can express the dependence of the voltage on time for a discharging capacitor ( $C$ is discharging after $t=2.00 \mathrm{~s}$ ) and evaluate this function, with a time constant differing from that found in $(a)$, at $t=6.00 \mathrm{~s}$. The diagram shows the circuit shortly after the switch is closed. The directions of the currents in the resistors and the capacitor have been chosen as shown.

(a) Apply the junction rule at $\quad I_{1}=I_{2}+I_{3}$ junction $a$ to obtain:

Apply the loop rule to loop 1 to obtain:

$$
\begin{equation*}
\varepsilon-\frac{Q}{C}-R_{1} I_{1}=0 \tag{2}
\end{equation*}
$$

Apply the loop rule to loop 2 to obtain:

Differentiate equation (2) with respect to time to obtain:

Differentiate equation (3) with respect to time to obtain:

Using equation (1), substitute for $I_{3}$ in equation (5) to obtain:

Solve equation (2) for $I_{1}$ :

Substitute for $I_{1}$ in equation (6) and simplify to obtain the differential equation for $I_{2}$ :

$$
\begin{equation*}
\frac{d I_{2}}{d t}=\frac{1}{R_{2} C}\left(I_{1}-I_{2}\right) \tag{6}
\end{equation*}
$$

$$
\begin{align*}
& \text { or } \\
& R_{1} \frac{d I_{1}}{d t}=-\frac{1}{C} I_{3} \tag{4}
\end{align*}
$$

$$
\frac{d}{d t}\left[\frac{Q}{C}-R_{2} I_{2}\right]=\frac{1}{C} \frac{d Q}{d t}-R_{2} \frac{d I_{2}}{d t}=0
$$

or

$$
\begin{equation*}
R_{2} \frac{d I_{2}}{d t}=\frac{1}{C} I_{3} \tag{5}
\end{equation*}
$$

$$
I_{1}=\frac{\varepsilon-Q / C}{R_{1}}=\frac{\varepsilon-R_{2} I_{2}}{R_{1}}
$$

$$
\begin{aligned}
\frac{d I_{2}}{d t} & =\frac{1}{R_{2} C}\left(\frac{\varepsilon-R_{2} I_{2}}{R_{1}}-I_{2}\right) \\
& =\frac{\varepsilon}{R_{1} R_{2} C}-\left(\frac{R_{1}+R_{2}}{R_{1} R_{2} C}\right) I_{2}
\end{aligned}
$$

To solve this linear differential equation with constant coefficients we can assume a solution of the form:

Differentiate $I_{2}(t)$ with respect to time to obtain:

Substitute for $I_{2}$ and $d I_{2} / d t$ to obtain:

$$
\begin{equation*}
I_{2}(t)=a+b e^{-t / \tau} \tag{7}
\end{equation*}
$$

Subtitute for $I_{2}$ and ${ }_{2}$ dt to obtain.

$$
-\frac{b}{\tau} e^{-t / \tau}=\frac{\varepsilon}{R_{1} R_{2} C}-\left(\frac{R_{1}+R_{2}}{R_{1} R_{2} C}\right)\left(a+b e^{-t / \tau}\right)
$$

Equate coefficients of $e^{-t / \tau}$ to obtain: $\quad \tau=\frac{R_{1} R_{2} C}{R_{1}+R_{2}}$

Requiring the equation to hold for all values of $a$ yields:

$$
a=\frac{\varepsilon}{R_{1}+R_{2}}
$$

If $I_{2}$ is to be zero when $t=0$ :

$$
0=a+b \Rightarrow b=-a=-\frac{\varepsilon}{R_{1}+R_{2}}
$$

Substitute in equation (7) to obtain:

$$
\begin{aligned}
& \begin{aligned}
I_{2}(t) & =\frac{\varepsilon}{R_{1}+R_{2}}-\frac{\varepsilon}{R_{1}+R_{2}} e^{-t / \tau} \\
& =\frac{\varepsilon}{R_{1}+R_{2}}\left(1-e^{-t / \tau}\right)
\end{aligned} \\
& \text { where } \tau=\frac{R_{1} R_{2} C}{R_{1}+R_{2}}
\end{aligned}
$$

Substitute numerical values and evaluate $\tau$ :

$$
\begin{aligned}
\tau & =\frac{(2.00 \mathrm{M} \Omega)(5.00 \mathrm{M} \Omega)(1.00 \mu \mathrm{~F})}{2.00 \mathrm{M} \Omega+5.00 \mathrm{M} \Omega} \\
& =1.43 \mathrm{~s}
\end{aligned}
$$

Substitute numerical values and evaluate $I_{2}(t)$ :-

$$
\begin{aligned}
I_{2}(t) & =\frac{10.0 \mathrm{~V}}{2.00 \mathrm{M} \Omega+5.00 \mathrm{M} \Omega}\left(1-e^{-t / 1.429 \mathrm{~s}}\right) \\
& =(1.429 \mu \mathrm{~A})\left(1-e^{-t / 1.43 \mathrm{~s}}\right)
\end{aligned}
$$

Because $C$ and $R_{2}$ are in parallel, they have a common potential difference given by:

$$
V_{\mathrm{C}}(t)=V_{2}(t)=I_{2}(t) R_{2}=(1.429 \mu \mathrm{~A})(5.00 \mathrm{M} \Omega)\left(1-e^{-\mathrm{t} / 1.429 \mathrm{~s}}\right)=(7.143 \mathrm{~V})\left(1-e^{-\mathrm{t} / 1.429 \mathrm{~s}}\right)
$$

Evaluate $V_{\mathrm{C}}$ at $t=2.00 \mathrm{~s}$ :

$$
\begin{aligned}
V_{\mathrm{C}}(2 \mathrm{~s}) & =(7.143 \mathrm{~V})\left(1-e^{-2.00 \mathrm{~s} / 1.429 \mathrm{~s}}\right) \\
& =5.381 \mathrm{~V}
\end{aligned}
$$

The voltage across the capacitor as a function of time is shown in the following graph. The current through the $5.00-\mathrm{M} \Omega$ resistor $R_{2}$ follows the same time course, its value being $V_{\mathrm{C}} /\left(5.00 \times 10^{6}\right) \mathrm{A}$.

(b) The value of $V_{\mathrm{C}}$ at $t=2.00 \mathrm{~s}$ has

$$
V_{\mathrm{C}}(2.00 \mathrm{~s})=5.381 \mathrm{~V}=5.38 \mathrm{~V}
$$

already been determined to be:
When S is opened at $t=2.00 \mathrm{~s}, C$

$$
\tau^{\prime}=R_{2} C=(5.00 \mathrm{M} \Omega)(1.00 \mu \mathrm{~F})=5.00 \mathrm{~s}
$$

discharges through $R_{2}$ with a time constant given by:

Express the potential difference across $C$ as a function of time:

$$
\begin{aligned}
V_{\mathrm{C}}(t) & =V_{\mathrm{C} 0} e^{-(t-2.00 \mathrm{~s}) / \tau^{\prime}} \\
& =(5.381 \mathrm{~V}) e^{-(t-2.00 \mathrm{~s}) / 5.00 \mathrm{~s}}
\end{aligned}
$$

Evaluate $V_{\mathrm{C}}$ at $t=8.00 \mathrm{~s}$ to obtain:

$$
\begin{aligned}
V_{\mathrm{C}}(8.00 \mathrm{~s}) & =(5.381 \mathrm{~V}) e^{-6.00 \mathrm{~s} / 5.00 \mathrm{~s}} \\
& =1.62 \mathrm{~V}
\end{aligned}
$$

in good agreement with the graph.

117 •.. Two batteries that have emfs $\mathcal{E}_{1}$ and $\mathcal{E}_{2}$ and internal resistances $r_{1}$ and $r_{2}$ are connected in parallel. Prove that if a resistor of resistance $R$ is connected in parallel with combination, the optimal load resistance (the value of $R$ at which maximum power is delivered) is given by $R=r_{1} r_{2} /\left(r_{1}+r_{2}\right)$.

Picture the Problem Let $I_{1}$ be the current supplied by the battery whose emf is $\mathcal{E}_{1}, I_{2}$ the current supplied by the battery whose emf is $\mathcal{E}_{2}$, and $I_{3}$ the current through the resistor $R$. We can apply Kirchhoff's rules to obtain three equations in the unknowns $I_{1}, I_{2}$, and $I_{3}$ that we can solve simultaneously to find $I_{3}$. We can then express the power delivered by the sources to $R$. Setting the derivative of this expression equal to zero will allow us to solve for the value of $R$ that maximizes the power delivered by the sources.


Apply Kirchhoff's junction rule at $a \quad I_{1}+I_{2}=I_{3}$ to obtain:

Apply the loop rule around the $\quad \mathcal{E}_{1}-I_{3} R-r_{1} I_{1}=0$ outside of the circuit to obtain:

Apply the loop rule to loop 1 to
$\varepsilon_{2}-I_{3} R-r_{2} I_{2}=0$ obtain:

Eliminate $I_{1}$ from equations (1) and

$$
\begin{equation*}
\mathcal{E}_{1}-I_{3} R-r_{1}\left(I_{3}-I_{2}\right)=0 \tag{4}
\end{equation*}
$$

(2) to obtain:

Solving equation (3) for $I_{2}$ yields: $\quad I_{2}=\frac{\mathcal{E}_{2}-I_{3} R}{r_{2}}$

Substitute for $I_{2}$ in equation (4) to obtain:

$$
\mathcal{E}_{1}-I_{3} R-r_{1}\left(I_{3}-\frac{\varepsilon_{2}-I_{3} R}{r_{2}}\right)=0
$$

Solving for $I_{3}$ yields:

$$
I_{3}=\frac{\mathcal{E}_{1} r_{2}+\mathcal{E}_{2} r_{1}}{r_{1} r_{2}+R\left(r_{1}+r_{2}\right)}
$$

Express the power delivered to $R$ :

$$
\begin{aligned}
& P=I_{3}^{2} R=\left(\frac{\mathcal{E}_{1} r_{2}+\mathcal{E}_{2} r_{1}}{r_{1} r_{2}+R\left(r_{1}+r_{2}\right)}\right)^{2} R \\
& \\
& \quad=\left(\frac{\mathcal{E}_{1} r_{2}+\mathcal{E}_{2} r_{1}}{r_{1}+r_{2}}\right)^{2}\left[\frac{R}{(R+A)^{2}}\right] \\
& \text { where } A=\frac{r_{1} r_{2}}{r_{1}+r_{2}}
\end{aligned}
$$

Noting that the quantity in parentheses is independent of $R$ and that therefore we can ignore it, differentiate $P$ with respect to $R$ and set the derivative equal to zero:

Solving for $R$ yields:

To establish that this value for $R$ corresponds to a maximum, we need to evaluate the second derivative of $P$ with respect to $R$ at $R=A$ and show that this quantity is negative; that is, concave downward:

Because $\left.\frac{d^{2} P}{d R^{2}}\right|_{R=A}<0$ we can conclude that:

$$
\begin{aligned}
\frac{d P}{d R} & =\frac{d}{d R}\left[\frac{R}{(R+A)^{2}}\right] \\
& =\frac{(R+A)^{2}-R \frac{d}{d R}(R+A)^{2}}{(R+A)^{4}} \\
& =\frac{(R+A)^{2}-2 R(R+A)}{(R+A)^{4}} \\
& =0 \text { for extrema }
\end{aligned}
$$

$$
R=A=\frac{r_{1} r_{2}}{r_{1}+r_{2}}
$$

$$
\begin{aligned}
\frac{d^{2} P}{d R^{2}} & =\frac{d}{d R}\left[\frac{(R+A)^{2}-2 R(R+A)}{(R+A)^{4}}\right] \\
& =\frac{2 R-4 A}{(R+A)^{4}}
\end{aligned}
$$

and

$$
\left.\frac{d^{2} P}{d R^{2}}\right|_{R=A}=\frac{-2 A}{(R+A)^{4}}<0
$$

$$
R=\frac{r_{1} r_{2}}{r_{1}+r_{2}} \text { maximizes the power }
$$

delivered by the sources.

118 ••• Capacitors $C_{1}$ and $C_{2}$ are connected to a resistor of resistance $R$ and an ideal battery that has terminal voltage $V_{0}$ as shown in Figure 25-79. Initially the throw of switch $S$ is at contact $a$ and both capacitors are without charge. The throw is then rotated to contact $b$ and left there for a long time. Finally, at time $t$ $=0$, the throw is returned to contact $a$. (a) Quantitatively compare the total energy stored in the two capacitors at $t=0$ and a long time later. (b) Find the current through $R$ as a function of time. (c) Find the energy dissipated in the resistor as a function of time. (d) Find the total energy dissipated in the resistor and compare it with the loss of stored energy found in Part (a).

Picture the Problem (a) Let $Q_{1}$ and $Q_{2}$ represent the final charges on the capacitors $C_{1}$ and $C_{2}$. Knowing that charge is conserved as it is redistributed to the two capacitors and that the final-state potential differences across the two capacitors will be the same, we can obtain two equations in the unknowns $Q_{1}$ and $Q_{2}$ that we can solve simultaneously. We can compare the initial and final energies stored in this system by examining their ratio. (b) Let $q_{1}$ and $q_{2}$ be the
time-dependent charges on the two capacitors after the switches are closed. We can use Kirchhoff's loop rule and the conservation of charge to obtain a firstorder linear differential equation describing the current $I_{2}$ through $R$ after the switches are closed. We can solve this differential equation by assuming a solution of the form $q_{2}(t)=a+b e^{-t / \tau}$ and requiring that the solution satisfy the initial condition $q_{2}(0)=0$ and the differential equation be satisfied for all values of $t$. (c) Once we know $I_{2}$, we can find the energy dissipated in the resistor as a function of time and $(d)$ the total energy dissipated in the resistor.
(a) The initial and final energies stored in the capacitors are:

Relate the total charge stored

$$
\begin{equation*}
Q=C_{1} V_{0}=Q_{1}+Q_{2} \tag{3}
\end{equation*}
$$ initially to the final charges $Q_{1}$ and $Q_{2}$ on $C_{1}$ and $C_{2}$ :

Because, in their final state, the potential differences across the two capacitors will be the same:

Solve equation (4) for $Q_{1}$ and substitute in equation (3) to obtain:

Substitute for $Q_{2}$ in either equation
(3) or equation (4) and solve for $Q_{1}$ to obtain:

Substitute for $Q_{1}$ and $Q_{2}$ in equation (2) and simplify to obtain:

$$
\begin{equation*}
\frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
U_{\mathrm{f}}=\frac{1}{2} \frac{Q_{1}^{2}}{C_{1}}+\frac{1}{2} \frac{Q_{2}^{2}}{C_{2}} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
U_{\mathrm{i}}=\frac{1}{2} C_{1} V_{0}^{2} \tag{1}
\end{equation*}
$$

suostitute ill equation (ग) to ovtanl.

$$
\frac{C_{1}}{C_{2}} Q_{2}+Q_{2}=C_{1} V_{0} \Rightarrow Q_{2}=\frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}
$$

$$
Q_{1}=\frac{C_{1}^{2}}{C_{1}+C_{2}} V_{0}
$$

$$
\begin{aligned}
U_{\mathrm{f}} & =\frac{1}{2} \frac{\left(\frac{C_{1}^{2}}{C_{1}+C_{2}} V_{0}\right)^{2}}{C_{1}}+\frac{1}{2} \frac{\left(\frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}\right)^{2}}{C_{2}} \\
& =\frac{1}{2} \frac{C_{1}^{2}}{C_{1}+C_{2}} V_{0}^{2}
\end{aligned}
$$

Expressing the difference between $U_{\mathrm{i}}$ and $U_{\mathrm{f}}$ yields:

$$
\begin{aligned}
U_{\mathrm{i}}-U_{\mathrm{f}} & =\frac{1}{2} C_{1} V_{0}^{2}-\frac{1}{2} \frac{C_{1}^{2}}{C_{1}+C_{2}} V_{0}^{2} \\
& =\frac{1}{2} \frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}^{2}
\end{aligned}
$$

Note that, if we express the ratio of $U_{\mathrm{i}}$ and $U_{\mathrm{f}}$ we obtain:

$$
\begin{aligned}
\frac{U_{\mathrm{i}}}{U_{\mathrm{f}}} & =\frac{C_{1} V_{0}^{2}}{\frac{\left(\frac{C_{1}^{2}}{C_{1}+C_{2}} V_{0}\right)^{2}}{C_{1}}+\frac{\left(\frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}\right)^{2}}{C_{2}}} \\
& =1+\frac{C_{2}}{C_{1}}
\end{aligned}
$$

That is, $U_{\mathrm{i}}$ is greater than $U_{\mathrm{f}}$ by a factor of $1+C_{2} / C_{1}$.
(b) Apply Kirchhoff’s loop rule to the circuit when the switch is in $\frac{q_{1}}{C_{1}}-I R-\frac{q_{2}}{C_{2}}=0$ position $a$ to obtain:
or, because $I=\frac{d q_{2}}{d t}$,
$\frac{q_{1}}{C_{1}}-R \frac{d q_{2}}{d t}-\frac{q_{2}}{C_{2}}=0$

Apply conservation of charge during
$q_{1}=Q-q_{2}=C_{1} V_{0}-q_{2}$ the redistribution of charge to obtain:

Substituting for $q_{1}$ yields:

$$
V_{0}-\frac{q_{2}}{C_{1}}-R \frac{d q_{2}}{d t}-\frac{q_{2}}{C_{2}}=0
$$

Rearrange to obtain a first- order differential equation:

$$
R \frac{d q_{2}}{d t}+\left(\frac{C_{1}+C_{2}}{C_{1} C_{2}}\right) q_{2}=V_{0}
$$

Assume a solution of the form:

$$
\begin{equation*}
q_{2}(t)=a+b e^{-t / \tau} \tag{4}
\end{equation*}
$$

Differentiate the assumed
solution with respect to time to $\frac{d q_{2}(t)}{d t}=\frac{d}{d t}\left[a+b e^{-t / \tau}\right]=-\frac{b}{\tau} e^{-t / \tau}$ obtain:

Substitute for $d q_{2} / d t$ and $q_{2}$ in the differential equation to obtain:

$$
\begin{gathered}
R\left(-\frac{b}{\tau} e^{-t / \tau}\right)+\left(\frac{C_{1}+C_{2}}{C_{1} C_{2}}\right)\left(a+b e^{-t / \tau}\right) \\
=V_{0}
\end{gathered}
$$

$$
\begin{aligned}
& {\left[-\frac{R}{\tau} e^{-t / \tau}\right] b+\left(\frac{C_{1}+C_{2}}{C_{1} C_{2}}\right) a} \\
& \quad+\left[\left(\frac{C_{1}+C_{2}}{C_{1} C_{2}}\right) e^{-t / \tau}\right] b=V_{0}
\end{aligned}
$$

If this equation is to be satisfied for all values of $t$ :

$$
a=\frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}=C_{\mathrm{eq}} V_{0}
$$

and

$$
\left[-\frac{R}{\tau} e^{-t / \tau}\right] b+\left[\left(\frac{C_{1}+C_{2}}{C_{1} C_{2}}\right) e^{-t / \tau}\right] b=0
$$

Simplifying further yields:

$$
-\frac{R}{\tau}+\frac{C_{1}+C_{2}}{C_{1} C_{2}}=0
$$

Solve for $\tau$ to obtain:

$$
\tau=R \frac{C_{1} C_{2}}{C_{1}+C_{2}}=R C_{\mathrm{eq}}
$$

Applying the initial condition $q_{2}(0)=0$ to equation (4) yields:

Substitute for $a$ and $b$ in equation (4) to obtain:

$$
\begin{aligned}
q_{2}(t) & =C_{\mathrm{eq}} V_{0}-C_{\mathrm{eq}} V_{0} e^{-t / \tau} \\
& =C_{\mathrm{eq}} V_{0}\left(1-e^{-t / \tau}\right)
\end{aligned}
$$

Differentiate $q_{2}(t)$ with respect to time to find the current through $R$ as a function of time:

$$
\begin{aligned}
& 0=a+b \\
& \text { or } \\
& b=-a=-C_{\mathrm{eq}} V_{0}
\end{aligned}
$$

$$
\begin{aligned}
I(t) & =\frac{d q_{2}(t)}{d t}=C_{\mathrm{eq}} V_{0} \frac{d}{d t}\left(1-e^{-t / \tau}\right) \\
& =C_{\mathrm{eq}} V_{0}\left(-e^{-t / \tau}\right)\left(-\frac{1}{\tau}\right)=\frac{C_{\mathrm{eq}} V_{0}}{\tau} e^{-t / \tau} \\
& =\frac{V_{0}}{R} e^{-t / \tau}
\end{aligned}
$$

where $\tau=R \frac{C_{1} C_{2}}{C_{1}+C_{2}}$.
(c) Express the energy dissipated in the resistor as a function of

$$
P(t)=I^{2} R=\left(\frac{V_{0}}{R} e^{-t / \tau}\right)^{2} R=\frac{V_{0}^{2}}{R} e^{-2 t / \tau}
$$ time:

$$
\text { where } \tau=R \frac{C_{1} C_{2}}{C_{1}+C_{2}}
$$

(d) The total energy dissipated in the resistor is the integral of $P(t)$ between $t=0$ and $t=\infty$ :

Because the capacitors are in series:

Substituting for $C_{\text {eq }}$ yields:

$$
E=\frac{V_{0}^{2}}{R} \int_{0}^{\infty} e^{-2 t / R C_{\mathrm{cq}}} d t=\frac{1}{2} V_{0}^{2} C_{\mathrm{eq}}
$$

$$
C_{\mathrm{eq}}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}
$$

$$
E=\frac{1}{2} \frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}^{2}
$$

This energy, dissipated as Joule heating in the resistor, is exactly the difference between the initial and final energies found in Part (a).

119 ••• (a) Calculate the equivalent resistance (in terms of $R$, the resistance of each individual resistor) between points $a$ and $b$ for the infinite ladder of resistors shown in Figure 25-80 assuming the resistors are identical. That is assuming $R=R_{1}=R_{2}$. (b) Repeat Part (a) but do not assume that $R_{1}=R_{2}$ and express your answer in terms of $R_{1}$ and $R_{2}$.(c), Check your results by showing that your result from Part (b) agrees with your result from Part (a) if you substitute $R$ for both $R_{1}$ and $R_{2}$.

Picture the Problem Let $R$ be the resistance of each resistor in the ladder and let $R_{\text {eq }}$ be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in $R_{\text {eq }}$ that we can solve for the equivalent resistance between points $a$ and $b$.
(a) The equivalent resistance of the series combination of $R$ and ( $R \| R_{\text {eq }}$ ) is $R_{\text {eq }}$, so:


$$
R_{\mathrm{eq}}=R+R \| R_{\mathrm{eq}}=R+\frac{R R_{\mathrm{eq}}}{R+R_{\mathrm{eq}}}
$$

Simplify to obtain:

$$
R_{\mathrm{eq}}^{2}-R R_{\mathrm{eq}}-R^{2}=0
$$

Solve for $R_{\mathrm{eq}}$ to obtain:

$$
R_{\mathrm{eq}}=\left(\frac{1+\sqrt{5}}{2}\right) R \approx 1.618 R
$$

(b) The equivalent resistance of the series combination of $R_{1}$ and ( $R_{2} \| R_{\text {eq }}$ ) is $R_{\text {eq }}$, so:

$$
R_{\mathrm{eq}}=R_{1}+R_{2} \| R_{\mathrm{eq}}=R_{1}+\frac{R_{2} R_{\mathrm{eq}}}{R_{2}+R_{\mathrm{eq}}}
$$

Simplify to obtain:

$$
R_{\mathrm{eq}}^{2}-R_{1} R_{\mathrm{eq}}-R_{1} R_{2}=0
$$

Solve for the positive value of $R_{\text {eq }}$ to obtain:

$$
R_{\mathrm{eq}}=\frac{R_{1}+\sqrt{R_{1}^{2}+4 R_{1} R_{2}}}{2}
$$

(c) If $R_{1}=R_{2}=R$, then:

$$
R_{\mathrm{eq}}=\frac{R+\sqrt{R^{2}+4 R R}}{2}=\left(\frac{1+\sqrt{5}}{2}\right) R
$$

in agreement with $\operatorname{Part}(a)$.
120 ••• A graph of current as a function of voltage for an Esaki diode is shown in Figure 25-81. (a) Make a graph of the differential resistance of the diode as a function of voltage. The differential resistance $R_{\mathrm{d}}$ of a circuit element is defined as $R_{\mathrm{d}}=d V / d I$, where $V$ is the voltage drop across the element and $I$ is the current in the element. (b) At what value of the voltage drop does the differential resistance become negative? (c) What is the maximum differential resistance for this diode in the range shown and at what voltage does it occur? (d) Are there any places in the voltage range shown where the diode exhibits a differential resistance equal to zero? If so, under value(s) of the voltage does this (do these) occur?

Picture the Problem We can approximate the slope of the graph in Figure 25-81 and take its reciprocal to obtain values for $R_{\mathrm{d}}$ that we can plot as a function of $V$.
(a) Use the graph in Figure 25-81 to complete the table to the right.

| $V(\mathrm{~V})$ | $R_{\mathrm{d}}(\Omega)$ |
| :---: | :---: |
| 0 | 6.67 |
| 0.1 | 17.9 |
| 0.3 | -75.2 |
| 0.4 | 42.9 |
| 0.5 | 8 |

The following graph was plotted using a spreadsheet program.

(b) The differential resistance becomes negative at approximately 0.14 V .
(c) The maximum differential resistance for this diode is approximately $45 \Omega$ and occurs at about 0.42 V .
(d) The diode exhibits no resistance where the curve crosses the $V$ axis; that is at $V=0.14 \mathrm{~V}$ and 0.36 V .

