

Chapter 21

The Electric Field I: Discrete Charge Distributions

Conceptual Problems

1 • Objects are composed of atoms which are composed of charged particles (protons and electrons); however, we rarely observe the effects of the electrostatic force. Explain why we do not observe these effects.

Determine the Concept The net charge on large objects is always very close to zero. Hence the most obvious force is the gravitational force.

2 • A carbon atom can become a carbon *ion* if it has one or more of its electrons removed during a process called *ionization*. What is the net charge on a carbon atom that has had two of its electrons removed? (a) $+e$, (b) $-e$, (c) $+2e$, (d) $-2e$

Determine the Concept If two electrons are removed from a carbon atom, it will have a net positive charge of $+2e$. (c) is correct.

3 •• You do a simple demonstration for your high school physics teacher in which you claim to disprove Coulomb's law. You first run a rubber comb through your dry hair, then use it to attract tiny neutral pieces of paper on the desk. You then say "Coulomb's law states that for there to be electrostatic forces of attraction between two objects, both objects need to be charged. However, the paper was not charged. So according to Coulomb's law, there should be no electrostatic forces of attraction between them, yet there clearly was." You rest your case. (a) What is wrong with your assumptions? (b) Does attraction between the paper and the comb require that the net charge on the comb be negative? Explain.

Determine the Concept

(a) Coulomb's law is only valid for point particles. The paper bits cannot be modeled as point particles because the paper bits become polarized.

(b) No, the attraction does not depend on the sign of the charge on the comb. The induced charge on the paper that is closest to the comb is always opposite in sign to of the charge on the comb, and thus the net force on the paper is always attractive.

4 •• You have a positively charged insulating rod and two metal spheres on insulating stands. Give step-by-step directions of how the rod, without actually touching either sphere, can be used to give one of the spheres (a) a negative charge, and (b) a positive charge.

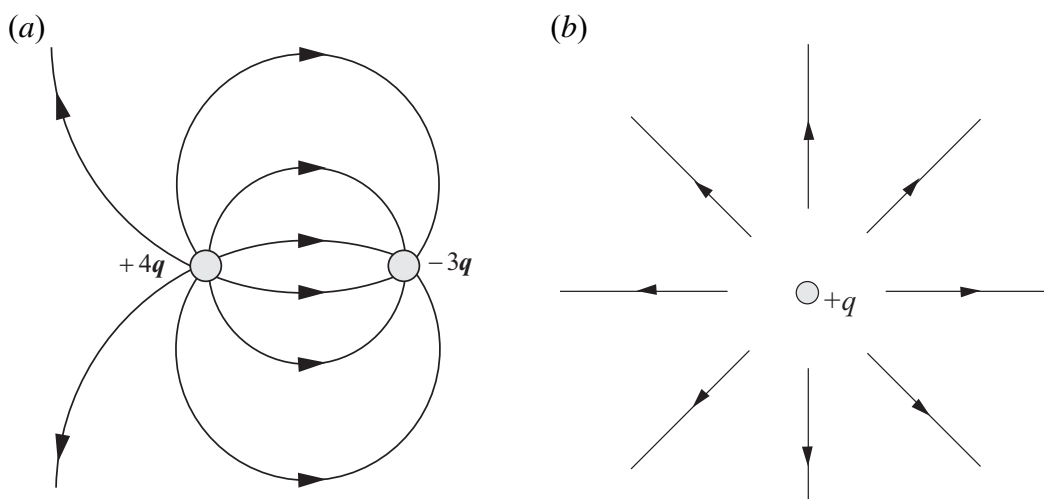
Determine the Concept

(a) Connect the metal sphere to ground; bring the insulating rod near the metal sphere and disconnect the sphere from ground; then remove the insulating rod. The sphere will be negatively charged.

(b) Bring the insulating rod in contact with the metal sphere; some of the positive charge on the rod will be transferred to the metal sphere.

5 •• (a) Two point particles that have charges of $+4q$ and $-3q$ are separated by distance d . Use field lines to draw a visualization of the electric field in the neighborhood of this system. (b) Draw the field lines at distances much greater than d from the charges.

Determine the Concept (a) We can use the rules for drawing electric field lines to draw the electric field lines for this system. Two field lines have been assigned to each charge q . (b) At distances much greater than the separation distance between the two charges, the system of two charged bodies will "look like" a single charge of $+q$ and the field pattern will be that due to a point charge of $+q$. Eight field lines have been assigned to the single charge.



6 •• A metal sphere is positively charged. Is it possible for the sphere to electrically attract another positively charged ball? Explain your answer.

Determine the Concept Yes. Because a metal sphere is a conductor, the proximity of a positively charged ball (not necessarily a conductor), will induce a redistribution of charges on the metal sphere with the surface nearer the positively charged ball becoming negatively charged. Because the negative charges on the metal sphere are closer to the positively charged ball than are the positive charges on the metal sphere, the net force will be attractive.

7 •• A simple demonstration of electrostatic attraction can be done simply by dangling a small ball of crumpled aluminum foil on a string and bringing a charged rod near the ball. The ball initially will be attracted to the rod, but once they touch, the ball will be strongly repelled from it. Explain these observations.

Determine the Concept Assume that the rod has a negative charge. When the charged rod is brought near the aluminum foil, it induces a redistribution of charges with the side nearer the rod becoming positively charged, and so it swings toward the rod. When it touches the rod, some of the negative charge is transferred to the foil, which, as a result, acquires a net negative charge and is now repelled by the rod.

8 •• Two positive point charges that are equal in magnitude are fixed in place, one at $x = 0.00$ m and the other at $x = 1.00$ m, on the x axis. A third positive point charge is placed at an equilibrium position. (a) Where is this equilibrium position? (b) Is the equilibrium position stable if the third particle is constrained to move parallel with the x axis? (c) What about if it is constrained to move parallel with the y axis? Explain.

Determine the Concept

(a) A third positive charge can be placed midway between the fixed positive charges. This is the only location.

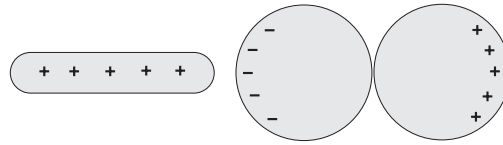
(b) Yes. The position identified in (a) is one of stable equilibrium. It is stable in the x -direction because regardless of whether you displace the third positive charge to the right or to the left, the net force acting on it is back toward the midpoint between the two fixed charges.

(c) If the third positive charge is displaced in the y direction, the net force acting on it will be away from its equilibrium position. Hence the position midway between the fixed positive charges is one of unstable equilibrium in the y direction.

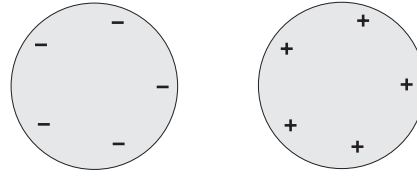
9 •• Two neutral conducting spheres are in contact and are supported on a large wooden table by insulated stands. A positively charged rod is brought up close to the surface of one of the spheres on the side opposite its point of contact with the other sphere. (a) Describe the induced charges on the two conducting spheres, and sketch the charge distributions on them. (b) The two spheres are separated and then the charged rod is removed. The spheres are then separated far apart. Sketch the charge distributions on the separated spheres.

Determine the Concept Because the spheres are conductors, there are free electrons on them that will reposition themselves when the positively charged rod is brought nearby.

(a) On the sphere near the positively charged rod, the induced charge is negative and near the rod. On the other sphere, the net charge is positive and on the side far from the rod. This is shown in the diagram.

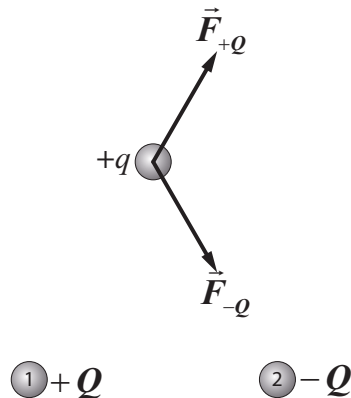


(b) When the spheres are separated and far apart and the rod has been removed, the induced charges are distributed uniformly over each sphere. The charge distributions are shown in the diagram.



10 •• Three point charges, $+q$, $+Q$, and $-Q$, are placed at the corners of an equilateral triangle as shown in Figure 21-33. No other charged objects are nearby. (a) What is the direction of the net force on charge $+q$ due to the other two charges? (b) What is the total electric force on the system of three charges? Explain.

Determine the Concept The forces acting on point charge $+q$ are shown in the diagram. The force acting on point charge $+q$ due to point charge $-Q$ is along the line joining them and directed toward $-Q$. The force acting on point charge $+q$ due to point charge $+Q$ is along the line joining them and directed away from point charge $+Q$.



(a) Because point charges $+Q$ and $-Q$ are equal in magnitude, the forces due to these charges are equal and their sum (the net force on charge $+q$) will be to the right. Note that the vertical components of these forces add up to zero.

(b) Because no other charged objects are nearby, the forces acting on this system of three point charges are internal forces and the net force acting on the system is zero.

11 •• A positively charged particle is free to move in a region with an electric field \vec{E} . Which statements must be true?

(a) The particle is accelerating in the direction perpendicular to \vec{E} .

- (b) The particle is accelerating in the direction of \vec{E} .
- (c) The particle is moving in the direction of \vec{E} .
- (d) The particle could be momentarily at rest.
- (e) The force on the particle is opposite the direction of \vec{E} .
- (f) The particle is moving opposite the direction of \vec{E} .

Determine the Concept

- (a) False. The only force acting on the particle is in the direction of \vec{E} .
- (b) True. The electrical force experienced by the particle is, by definition, in the direction of \vec{E} .
- (c) False. We don't know whether the particle is moving or momentarily at rest. All we know is that the net force acting on it is in the direction of \vec{E} .
- (d) Possibly. Whether the particle is ever at rest depends on how it was initially placed in the electric field. That is, it depends on whether its initial velocity was zero, in the direction of \vec{E} , or opposite \vec{E} .
- (e) False. By definition, the electric force acting on a positively charged particle in an electric field is in the direction of the field.
- (f) True. All we know for sure is that the electric force and, hence, the acceleration of the particle, is in the direction of \vec{E} . The particle could be moving in the direction of the field, in the direction opposite the field, or it could be momentarily at rest. We do know that, if it is at any time at rest, it will not stay at rest.

12 •• Four charges are fixed in place at the corners of a square as shown in Figure 21-34. No other charges are nearby. Which of the following statements is true?

- (a) \vec{E} is zero at the midpoints of all four sides of the square.
- (b) \vec{E} is zero at the center of the square.
- (c) \vec{E} is zero midway between the top two charges and midway between the bottom two charges.

Determine the Concept \vec{E} is zero wherever the net force acting on a test charge is zero.

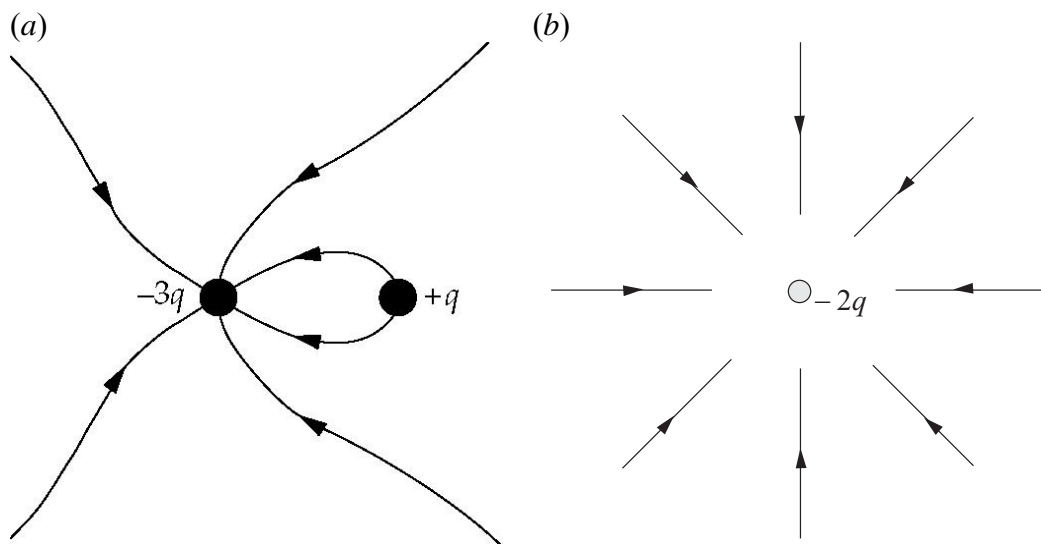
- (a) False. A test charge placed at these locations will experience a net force.

(b) True. At the center of the square the two positive charges alone produce a net electric field of zero, and the two negative charges alone also produce a net electric field of zero. Thus, the net force acting on a test charge at the midpoint of the square is zero.

(c) False. A test charge placed at any of these locations will experience a net force.

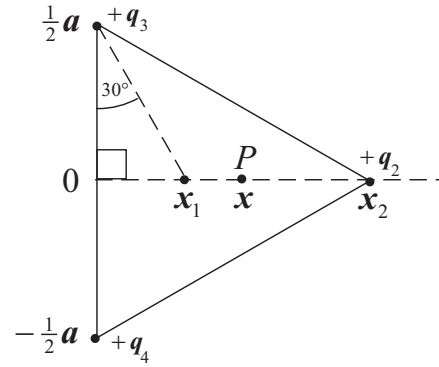
13 •• [SSM] Two point particles that have charges of $+q$ and $-3q$ are separated by distance d . (a) Use field lines to sketch the electric field in the neighborhood of this system. (b) Draw the field lines at distances much greater than d from the charges.

Determine the Concept (a) We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch we've assigned 2 field lines to each charge q . (b) At distances much greater than the separation distance between the two charges, the system of two charged bodies will "look like" a single charge of $-2q$ and the field pattern will be that due to a point charge of $-2q$. Four field lines have been assigned to each charge $-q$.



14 •• Three equal positive point charges (each charge $+q$) are fixed at the vertices of an equilateral triangle with sides of length a . The origin is at the midpoint of one side the triangle, the center of the triangle on the x axis at $x = x_1$ and the vertex opposite the origin is on the x axis at $x = x_2$. (a) Express x_1 and x_2 in terms of a . (b) Write an expression for the electric field on the x axis a distance x from the origin on the interval $0 \leq x < x_2$. (c) Show that the expression you obtained in (b) gives the expected results for $x = 0$ and for $x = x_1$.

Picture the Problem (a) We can use the geometry of an equilateral triangle to express x_1 and x_2 in terms of the side a of the triangle. (b) The electric field on the x axis a distance x from the origin on the interval $0 \leq x < x_2$ is the superposition of the electric fields due to the point charges q_2 , q_3 , and q_4 located at the vertices of the triangle.



(a) Apply the Pythagorean theorem to the triangle whose vertices are at $(0,0)$, $(0, \frac{1}{2}a)$, and $(x_2, 0)$ to obtain:

$$x_2^2 + \left(\frac{1}{2}a\right)^2 = a^2 \Rightarrow x_2 = \boxed{\frac{\sqrt{3}}{2}a}$$

Referring to the triangle whose vertices are at $(0,0)$, $(0, \frac{1}{2}a)$, and $(x_1, 0)$, note that:

$$\tan 30^\circ = \frac{x_1}{\frac{1}{2}a} \Rightarrow x_1 = \frac{1}{2}a \tan 30^\circ$$

Substitute for $\tan 30^\circ$ and evaluate x_1 :

$$x_1 = \boxed{\frac{\sqrt{3}}{6}a}$$

(b) The electric field at P is the superposition of the electric fields due to the charges q_2 , q_3 , and q_4 :

$$\vec{E}_P = \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \quad (1)$$

Express the electric field at P due to q_2 :

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2} \hat{r}_{2,P} = \frac{kq_2}{r_{2,P}^3} \vec{r}_{2,P}$$

where

$$\vec{r}_{2,P} = (x - x_2)\hat{i} = \left(x - \frac{\sqrt{3}}{2}a\right)\hat{i}$$

and

$$r_{2,P} = x_2 - x$$

Substituting for q_2 , $\vec{r}_{2,P}$, and $r_{2,P}$ yields:

$$\begin{aligned} \vec{E}_2 &= \frac{kq}{\left(\frac{\sqrt{3}}{2}a - x\right)^3} \left(x - \frac{\sqrt{3}}{2}a\right)\hat{i} \\ &= -\frac{kq}{\left(\frac{\sqrt{3}}{2}a - x\right)^2} \hat{i} \end{aligned}$$

Express the electric field at P due to q_3 :

$$\vec{E}_3 = \frac{kq_3}{r_{3,P}^2} \hat{r}_{3,P} = \frac{kq_3}{r_{3,P}^3} \vec{r}_{3,P}$$

where

$$\vec{r}_{3,P} = (x-0)\hat{i} + (0-\frac{1}{2}a)\hat{j} = x\hat{i} - \frac{1}{2}a\hat{j}$$

and

$$r_{3,P} = \sqrt{(x-0)^2 + (0-\frac{1}{2}a)^2} = \sqrt{x^2 + \frac{1}{4}a^2}$$

Substituting for q_3 , $\vec{r}_{3,P}$, and $r_{3,P}$ yields:

$$\vec{E}_3 = \frac{kq}{(x^2 + \frac{1}{4}a^2)^{3/2}} (x\hat{i} - \frac{1}{2}a\hat{j})$$

Proceed similarly for \vec{E}_4 to obtain:

$$\vec{E}_4 = \frac{kq}{(x^2 + \frac{1}{4}a^2)^{3/2}} (x\hat{i} + \frac{1}{2}a\hat{j})$$

Substituting for \vec{E}_2 , \vec{E}_3 , and \vec{E}_4 in equation (1) and simplifying yields:

$$\begin{aligned} \vec{E}_P &= -\frac{kq}{\left(\frac{\sqrt{3}}{2}a - x\right)^2} \hat{i} + \frac{kq}{(x^2 + \frac{1}{4}a^2)^{3/2}} (x\hat{i} - \frac{1}{2}a\hat{j}) + \frac{kq}{(x^2 + \frac{1}{4}a^2)^{3/2}} (x\hat{i} + \frac{1}{2}a\hat{j}) \\ &= \boxed{kq \left[-\frac{1}{\left(\frac{\sqrt{3}}{2}a - x\right)^2} + \frac{2x}{(x^2 + \frac{1}{4}a^2)^{3/2}} \right] \hat{i}} \end{aligned}$$

(c) Evaluating $\vec{E}_P(0)$ yields:

$$\vec{E}_P(0) = kq \left[-\frac{1}{\left(\frac{\sqrt{3}}{2}a\right)^2} + \frac{2(0)}{\left(\frac{1}{4}a^2\right)^{3/2}} \right] \hat{i} = \boxed{-\frac{kq}{\left(\frac{\sqrt{3}}{2}a\right)^2} \hat{i}}$$

Note that, because the electric fields due to q_3 and q_4 cancel each other at the origin and the resultant field is that due to q_2 , this is the expected result.

Evaluate $\vec{E}_P\left(\frac{\sqrt{3}}{6}a\right)$ to obtain:

$$\vec{E}_P\left(\frac{\sqrt{3}}{6}a\right) = kq \left[-\frac{1}{\left(\frac{\sqrt{3}}{2}a - \frac{\sqrt{3}}{6}a\right)^2} + \frac{2\left(\frac{\sqrt{3}}{6}a\right)}{\left(\left(\frac{\sqrt{3}}{6}a\right)^2 + \frac{1}{4}a^2\right)^{3/2}} \right] \hat{i} = kq \left[-\frac{3}{a^2} + \frac{3}{a^2} \right] \hat{i} = \boxed{0}$$

Note that, because the point at the center of the equilateral triangle is equidistant from the three vertices, the electric fields due to the charges at the vertices cancel and this is the expected result.

15 •• A molecule has a dipole moment given by \vec{p} . The molecule is momentarily at rest with \vec{p} making an angle θ with a uniform electric field \vec{E} . Describe the subsequent motion of the dipole moment.

Determine the Concept The dipole moment rotates back and forth in oscillatory motion. The dipole moment gains angular speed as it rotates toward the direction of the electric field, and losing angular speed as it rotates away from the direction of the electric field.

16 •• True or false:

- (a) The electric field of a point charge always points away from the charge.
- (b) The electric force on a charged particle in an electric field is always in the same direction as the field.
- (c) Electric field lines never intersect.
- (d) All molecules have dipole moments in the presence of an external electric field.

(a) False. The direction of the field is toward a negative charge.

(b) False. The direction of the electric force acting on a point charge depends on the sign of the point charge.

(c) False. Electric field lines intersect any point in space occupied by a point charge.

(d) True. An electric field partially polarizes the molecules; resulting in the separation of their charges and the creation of electric dipole moments.

17 ••• [SSM] Two molecules have dipole moments of equal magnitude. The dipole moments are oriented in various configurations as shown in Figure 21-34. Determine the electric-field direction at each of the numbered locations. Explain your answers.

Determine the Concept Figure 21-23 shows the electric field due to a single dipole, where the dipole moment is directed toward the right. The electric field due two a pair of dipoles can be obtained by superposing the two electric fields.

	1	2	3
(a)	down	up	up
(b)	up	right	left
(c)	down	up	up
(d)	down	up	up

Estimation and Approximation

18 •• Estimate the force required to bind the two protons in the He nucleus together. HINT: Model the protons as point charges. *You will need to have an estimate of the distance between them.*

Picture the Problem Because the nucleus is in equilibrium, the binding force must be equal to the electrostatic force of repulsion between the protons.

Apply $\sum \vec{F} = 0$ to a proton to obtain: $F_{\text{binding}} - F_{\text{electrostatic}} = 0$

Solve for F_{binding} : $F_{\text{binding}} = F_{\text{electrostatic}}$

Using Coulomb's law, substitute for $F_{\text{electrostatic}}$: $F_{\text{binding}} = \frac{kq^2}{r^2}$

Assuming the diameter of the He nucleus to be approximately 10^{-15} m, substitute numerical values and evaluate $F_{\text{electrostatic}}$:

$$F_{\text{binding}} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(10^{-15} \text{ m})^2} \approx \boxed{0.2 \text{ kN}}$$

19 •• A popular classroom demonstration consists of rubbing a plastic rod with fur to give the rod charge, and then placing the rod near an empty soda can that is on its side (Figure 21-36). Explain why the can will roll toward the rod.

Determine the Concept Because the can is grounded, the presence of the negatively-charged plastic rod induces a positive charge on it. The positive charges induced on the can are attracted, via the Coulomb interaction, to the negative charges on the plastic rod. Unlike charges attract, so the can will roll toward the rod.

20 ••• Sparks in air occur when ions in the air are accelerated to a such a high speed by an electric field that when they impact on neutral gas molecules the neutral molecules become ions. If the electric field strength is large enough, the ionized collision products are themselves accelerated and produce more ions on impact, and so forth. This avalanche of ions is what we call a spark. (a) Assume that an ion moves, on average, exactly one mean free path through the air before hitting a molecule. If the ion needs to acquire approximately 1.0 eV of kinetic energy in order to ionize a molecule, estimate the minimum strength of the electric field required at standard room pressure and temperature. Assume that the cross-sectional area of an air molecule is about 0.10 nm^2 . (b) How does the strength of the electric field in (a) depend on temperature? (c) How does the

strength of the electric field in (a) depend on pressure?

Picture the Problem We can use the definition of electric field to express E in terms of the work done on the ionizing electrons and the distance they travel λ between collisions. We can use the ideal-gas law to relate the number density of molecules in the gas ρ and the scattering cross-section σ to the mean free path and, hence, to the electric field.

(a) Apply conservation of energy to relate the work done on the electrons by the electric field to the change in their kinetic energy:

$$W = \Delta K = F\Delta s$$

From the definition of electric field we have:

$$F = qE$$

Substitute for F and Δs to obtain:

$W = qE\lambda$, where the mean free path λ is the distance traveled by the electrons between ionizing collisions with nitrogen atoms.

The mean free path λ of a particle is related to the scattering cross-section σ and the number density for air molecules n :

$$\lambda = \frac{1}{\sigma n}$$

Substitute for λ to obtain:

$$W = \frac{qE}{\sigma n} \Rightarrow E = \frac{\sigma n W}{q}$$

Use the ideal-gas law to obtain:

$$n = \frac{N}{V} = \frac{P}{kT}$$

Substituting for n yields:

$$E = \frac{\sigma P W}{q k T} \quad (1)$$

Substitute numerical values and evaluate E :

$$E = \frac{(0.10 \text{ nm}^2) \left(101.325 \times 10^3 \frac{\text{N}}{\text{m}^2} \right) (1.0 \text{ eV}) \left(1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)}{(1.602 \times 10^{-19} \text{ C}) \left(1.381 \times 10^{-23} \frac{\text{J}}{\text{K}} \right) (300 \text{ K})} \approx \boxed{2.4 \times 10^6 \text{ N/C}}$$

(b) From equation (1) we see that:

$$\boxed{E \propto T^{-1}}$$

(c) Also, from equation (1):

$$E \propto P$$

Charge

21 • A plastic rod is rubbed against a wool shirt, thereby acquiring a charge of $-0.80 \mu\text{C}$. How many electrons are transferred from the wool shirt to the plastic rod?

Picture the Problem The charge acquired by the plastic rod is an integral number of electronic charges, that is, $q = n_e(-e)$.

Relate the charge acquired by the plastic rod to the number of electrons transferred from the wool shirt:

$$q = n_e(-e) \Rightarrow n_e = \frac{q}{-e}$$

Substitute numerical values and evaluate n_e :

$$\begin{aligned} n_e &= \frac{-0.80 \mu\text{C}}{-1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}}} \\ &= \boxed{5.0 \times 10^{12} \text{ electrons}} \end{aligned}$$

22 • A charge equal to the charge of Avogadro's number of protons ($N_A = 6.02 \times 10^{23}$) is called a *faraday*. Calculate the number of coulombs in a faraday.

Picture the Problem One faraday $= N_A e$. We can use this definition to find the number of coulombs in a faraday.

Use the definition of a faraday to calculate the number of coulombs in a faraday:

$$1 \text{ faraday} = N_A e = (6.02 \times 10^{23} \text{ electrons})(1.602 \times 10^{-19} \text{ C/electron}) = \boxed{9.63 \times 10^4 \text{ C}}$$

23 • [SSM] What is the total charge of all of the protons in 1.00 kg of carbon?

Picture the Problem We can find the number of coulombs of positive charge there are in 1.00 kg of carbon from $Q = 6n_C e$, where n_C is the number of atoms in 1.00 kg of carbon and the factor of 6 is present to account for the presence of 6 protons in each atom. We can find the number of atoms in 1.00 kg of carbon by setting up a proportion relating Avogadro's number, the mass of carbon, and the molecular mass of carbon to n_C . See Appendix C for the molar mass of carbon.

Express the positive charge in terms of the electronic charge, the number of protons per atom, and the number of atoms in 1.00 kg of carbon:

$$Q = 6n_c e$$

Using a proportion, relate the number of atoms in 1.00 kg of carbon n_C , to Avogadro's number and the molecular mass M of carbon:

$$\frac{n_C}{N_A} = \frac{m_C}{M} \Rightarrow n_C = \frac{N_A m_C}{M}$$

Substitute for n_C to obtain:

$$Q = \frac{6N_A m_C e}{M}$$

Substitute numerical values and evaluate Q :

$$Q = \frac{6 \left(6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}} \right) (1.00 \text{ kg}) (1.602 \times 10^{-19} \text{ C})}{0.01201 \frac{\text{kg}}{\text{mol}}} = \boxed{4.82 \times 10^7 \text{ C}}$$

24 •• Suppose a cube of aluminum which is 1.00 cm on a side accumulates a net charge of +2.50 pC. (a) What percentage of the electrons originally in the cube was removed? (b) By what percentage has the mass of the cube decreased because of this removal?

Picture the Problem (a) The percentage of the electrons originally in the cube that was removed can be found from the ratio of the number of electrons removed to the number of electrons originally in the cube. (b) The percentage decrease in the mass of the cube can be found from the ratio of the mass of the electrons removed to the mass of the cube.

(a) Express the ratio of the electrons removed to the number of electrons originally in the cube:

$$\frac{N_{\text{rem}}}{N_{\text{ini}}} = \frac{\frac{Q_{\text{accumulated}}}{e}}{N_{\text{electrons per atom}} N_{\text{atoms}}} \quad (1)$$

The number of atoms in the cube is the ratio of the mass of the cube to the mass of an aluminum atom:

$$N_{\text{atoms}} = \frac{m_{\text{cube}}}{m_{\text{Al atom}}} = \frac{\rho_{\text{Al}} V_{\text{cube}}}{m_{\text{Al atom}}}$$

The mass of an aluminum atom is its molar mass divided by Avogadro's number:

$$m_{\text{Al atom}} = \frac{M_{\text{Al}}}{N_{\text{A}}}$$

Substituting and simplifying yields:

$$N_{\text{atoms}} = \frac{\rho_{\text{Al}} V_{\text{cube}}}{\frac{M_{\text{Al}}}{N_{\text{A}}}} = \frac{\rho_{\text{Al}} V_{\text{cube}} N_{\text{A}}}{M_{\text{Al}}}$$

Substitute for N_{atoms} in equation (1) and simplify to obtain:

$$\begin{aligned} \frac{N_{\text{rem}}}{N_{\text{ini}}} &= \frac{\frac{Q_{\text{accumulated}}}{e}}{N_{\text{electrons per atom}} \left(\frac{\rho_{\text{Al}} V_{\text{cube}} N_{\text{A}}}{M_{\text{Al}}} \right)} \\ &= \frac{Q_{\text{accumulated}} M_{\text{Al}}}{N_{\text{electrons per atom}} \rho_{\text{Al}} V_{\text{cube}} e N_{\text{A}}} \end{aligned}$$

Substitute numerical values and evaluate $\frac{N_{\text{rem}}}{N_{\text{ini}}}$:

$$\begin{aligned} \frac{N_{\text{rem}}}{N_{\text{ini}}} &= \frac{(2.50 \text{ pC}) \left(26.98 \frac{\text{g}}{\text{mol}} \right)}{\left(13 \frac{\text{electrons}}{\text{atom}} \right) \left(2.70 \frac{\text{g}}{\text{cm}^3} \right) (1.00 \text{ cm})^3 (1.602 \times 10^{-19} \text{ C}) \left(6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}} \right)} \\ &\approx \boxed{1.99 \times 10^{-15} \%} \end{aligned}$$

(b) Express the ratio of the mass of the electrons removed to the mass of the cube:

$$\frac{m_{\text{rem}}}{m_{\text{cube}}} = \frac{N_{\text{rem}} m_{\text{electron}}}{\rho_{\text{Al}} V_{\text{cube}}}$$

From (a), the number of electrons removed is given by:

$$N_{\text{rem}} = \frac{Q_{\text{accumulated}}}{e}$$

Substituting and simplifying yields:

$$\begin{aligned} \frac{m_{\text{rem}}}{m_{\text{cube}}} &= \frac{\frac{Q_{\text{accumulated}}}{e} m_{\text{electron}}}{\rho_{\text{Al}} V_{\text{cube}}} \\ &= \frac{Q_{\text{accumulated}} m_{\text{electron}}}{e \rho_{\text{Al}} V_{\text{cube}}} \end{aligned}$$

Substitute numerical values and evaluate $\frac{m_{\text{rem}}}{m_{\text{cube}}}$:

$$\frac{m_{\text{rem}}}{m_{\text{cube}}} = \frac{(2.50 \text{ pC})(9.109 \times 10^{-31} \text{ kg})}{(1.602 \times 10^{-19} \text{ C}) \left(2.70 \frac{\text{g}}{\text{cm}^3} \right) (1.00 \text{ cm}^3)} \approx \boxed{5.26 \times 10^{-19} \%}$$

25 •• During a process described by the *photoelectric effect*, ultra-violet light can be used to charge a piece of metal. (a) If such light is incident on a slab of conducting material and electrons are ejected with enough energy that they escape the surface of the metal, how long before the metal has a net charge of +1.50 nC if 1.00×10^6 electrons are ejected per second? (b) If 1.3 eV is needed to eject an electron from the surface, what is the power rating of the light beam? (Assume this process is 100% efficient.)

Picture the Problem (a) The required time is the ratio of the charge that accumulates to the rate at which it is delivered to the conductor. (b) We can use the definition of power to find the power rating of the light beam.

(a) The required time is the ratio of the charge that accumulates to the rate at which it is delivered:

$$\Delta t = \frac{\Delta q}{I} = \frac{\Delta q}{dq/dt}$$

Substitute numerical values and evaluate Δt :

$$\begin{aligned} \Delta t &= \frac{1.50 \text{ nC}}{\left(1.00 \times 10^6 \frac{\text{electrons}}{\text{s}} \right) \left(1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}} \right)} = 9.363 \times 10^3 \text{ s} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= \boxed{2.60 \text{ h}} \end{aligned}$$

(b) The power rating of the light beam is the rate at which it delivers energy:

$$P = \frac{\Delta E}{\Delta t}$$

The energy delivered by the beam is the product of the energy per electron, the electron current (that is, the number of electrons removed per unit time), and the elapsed time:

$$\Delta E = E_{\text{per electron}} I_{\text{electron}} \Delta t$$

Substituting for ΔE in the expression for P and simplifying yields:

$$P = \frac{E_{\text{per electron}} I_{\text{electron}} \Delta t}{\Delta t} = E_{\text{per electron}} I_{\text{electron}}$$

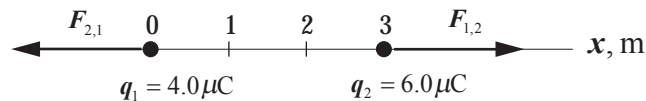
Substitute numerical values and evaluate P :

$$P = \left(1.3 \frac{\text{eV}}{\text{electron}} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}} \right) \left(1.00 \times 10^6 \frac{\text{electrons}}{\text{s}} \right) = \boxed{2.1 \times 10^{-13} \text{ W}}$$

Coulomb's Law

26 • A point charge $q_1 = 4.0 \mu\text{C}$ is at the origin and a point charge $q_2 = 6.0 \mu\text{C}$ is on the x -axis at $x = 3.0 \text{ m}$. (a) Find the electric force on charge q_2 . (b) Find the electric force on q_1 . (c) How would your answers for Parts (a) and (b) differ if q_2 were $-6.0 \mu\text{C}$?

Picture the Problem We can find the electric forces the two charges exert on each by applying Coulomb's law and Newton's 3rd law. Note that $\hat{r}_{1,2} = \hat{i}$ because the vector pointing from q_1 to q_2 is in the positive x direction. The diagram shows the situation for Parts (a) and (b).



(a) Use Coulomb's law to express the force that q_1 exerts on q_2 :

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2} \hat{r}_{1,2}$$

Substitute numerical values and evaluate $\vec{F}_{1,2}$:

$$\vec{F}_{1,2} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.0 \mu\text{C})(6.0 \mu\text{C})}{(3.0 \text{ m})^2} \hat{i} = \boxed{(24 \text{ mN})\hat{i}}$$

(b) Because these are action-and-reaction forces, we can apply Newton's 3rd law to obtain:

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{-(24 \text{ mN})\hat{i}}$$

(c) If q_2 is $-6.0 \mu\text{C}$, the force between q_1 and q_2 is attractive and both force vectors are reversed:

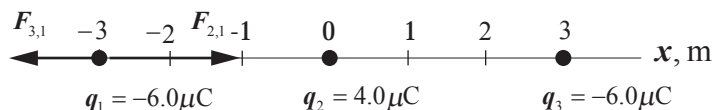
$$\vec{F}_{1,2} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.0 \mu\text{C})(-6.0 \mu\text{C})}{(3.0 \text{ m})^2} \hat{i} = \boxed{-(24 \text{ mN})\hat{i}}$$

and

$$\vec{F}_{2,1} = -\vec{F}_{1,2} = \boxed{(24 \text{ mN})\hat{i}}$$

27 • [SSM] Three point charges are on the x -axis: $q_1 = -6.0 \mu\text{C}$ is at $x = -3.0 \text{ m}$, $q_2 = 4.0 \mu\text{C}$ is at the origin, and $q_3 = -6.0 \mu\text{C}$ is at $x = 3.0 \text{ m}$. Find the electric force on q_1 .

Picture the Problem q_2 exerts an attractive electric force $\vec{F}_{2,1}$ on point charge q_1 and q_3 exerts a repulsive electric force $\vec{F}_{3,1}$ on point charge q_1 . We can find the net electric force on q_1 by adding these forces (that is, by using the superposition principle).



Express the net force acting on q_1 :

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1 :

$$\vec{F}_{2,1} = \frac{k|q_1||q_2|}{r_{2,1}^2} \hat{i}$$

Express the force that q_3 exerts on q_1 :

$$\vec{F}_{3,1} = \frac{k|q_1||q_3|}{r_{3,1}^2} (-\hat{i})$$

Substitute and simplify to obtain:

$$\begin{aligned} \vec{F}_1 &= \frac{k|q_1||q_2|}{r_{2,1}^2} \hat{i} - \frac{k|q_1||q_3|}{r_{3,1}^2} \hat{i} \\ &= k|q_1| \left(\frac{|q_2|}{r_{2,1}^2} - \frac{|q_3|}{r_{3,1}^2} \right) \hat{i} \end{aligned}$$

Substitute numerical values and evaluate \vec{F}_1 :

$$\vec{F}_1 = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.0 \mu\text{C}) \left(\frac{4.0 \mu\text{C}}{(3.0 \text{ m})^2} - \frac{6.0 \mu\text{C}}{(6.0 \text{ m})^2} \right) \hat{i} = \boxed{(1.5 \times 10^{-2} \text{ N}) \hat{i}}$$

28 •• A $2.0 \mu\text{C}$ point charge and a $4.0 \mu\text{C}$ point charge are a distance L apart. Where should a third point charge be placed so that the electric force on that third charge is zero?

Picture the Problem The third point charge should be placed at the location at which the forces on the third point charge due to each of the other two point charges cancel. There can be no such place except on the line between the two point charges. Denote the $2.0 \mu\text{C}$ and $4.0 \mu\text{C}$ point charges by the numerals 2 and 4, respectively, and the third point charge by the numeral 3. Assume that the

$2.0\ \mu\text{C}$ point charge is to the left of the $4.0\ \mu\text{C}$ point charge, let the $+x$ direction be to the right. Then the $4.0\ \mu\text{C}$ point charge is located at $x = L$.

Apply the condition for translational equilibrium to the third point charge:

$$\begin{aligned}\vec{F}_{4,3} + \vec{F}_{2,3} &= 0 \\ \text{or} \\ F_{4,3} &= F_{2,3}\end{aligned}\quad (1)$$

Letting the distance from the third point charge to the $4.0\ \mu\text{C}$ point charge be x , express the force that the $4.0\ \mu\text{C}$ point charge exerts on the third point charge:

$$F_{4,3} = \frac{kq_3q_4}{(L-x)^2}$$

The force that the $2.0\ \mu\text{C}$ point charge exerts on the third charge is given by:

$$F_{2,3} = \frac{kq_3q_2}{x^2}$$

Substitute in equation (1) to obtain:

$$\begin{aligned}\frac{kq_3q_4}{(L-x)^2} &= \frac{kq_3q_2}{x^2} \\ \text{or, simplifying,} \\ \frac{q_4}{(L-x)^2} &= \frac{q_2}{x^2}\end{aligned}$$

Rewriting this equation explicitly as a quadratic equations yields:

$$x^2 + 2Lx - L^2 = 0$$

Use the quadratic formula to obtain:

$$x = \frac{-2L \pm \sqrt{4L^2 + 4L^2}}{2} = -L \pm \sqrt{2}L$$

The root corresponding to the negative sign between the terms is extraneous because it corresponds to a position to the left of the $2.0\ \mu\text{C}$ point charge and is, therefore, not a physically meaningful root. Hence the third point charge should be placed between the point charges and a distance equal to $0.41L$ away from the $2.0\text{-}\mu\text{C}$ charge.

29 •• A $-2.0\ \mu\text{C}$ point charge and a $4.0\ \mu\text{C}$ point charge are a distance L apart. Where should a third point charge be placed so that the electric force on that third charge is zero?

Picture the Problem The third point charge should be placed at the location at which the forces on the third point charge due to each of the other two point charges cancel. There can be no such place between the two point charges. Beyond the $4.0\ \mu\text{C}$ point charge, and on the line containing the two point charges,

the force due to the $4.0 \mu\text{C}$ point charge overwhelms the force due to the $-2.0 \mu\text{C}$ point charge. Beyond the $-2.0 \mu\text{C}$ point charge, and on the line containing the two point charges, however, we can find a place where these forces cancel because they are equal in magnitude and oppositely directed. Denote the $-2.0 \mu\text{C}$ and $4.0 \mu\text{C}$ point charges by the numerals 2 and 4, respectively, and the third point charge by the numeral 3. Let the $+x$ direction be to the right with the origin at the position of the $-2.0 \mu\text{C}$ point charge and the $4.0 \mu\text{C}$ point charge be located at $x = L$.

Apply the condition for translational equilibrium to the third point charge:

$$\begin{aligned}\vec{F}_{4,3} + \vec{F}_{2,3} &= 0 \\ \text{or} \\ F_{4,3} &= F_{2,3}\end{aligned}\quad (1)$$

Letting the distance from the third point charge to the $4.0 \mu\text{C}$ point charge be x , express the force that the $4.0 \mu\text{C}$ point charge exerts on the third point charge:

$$F_{4,3} = \frac{kq_3q_4}{(L+x)^2}$$

The force that the $-2.0 \mu\text{C}$ point charge exerts on the third point charge is given by:

$$F_{2,3} = \frac{kq_3q_2}{x^2}$$

Substitute in equation (1) to obtain:

$$\frac{kq_3q_4}{(L+x)^2} = \frac{kq_3q_2}{x^2} \Rightarrow \frac{q_4}{(L+x)^2} = \frac{q_2}{x^2}$$

Rewriting this equation explicitly as a quadratic equations yields:

$$x^2 - 2Lx - L^2 = 0$$

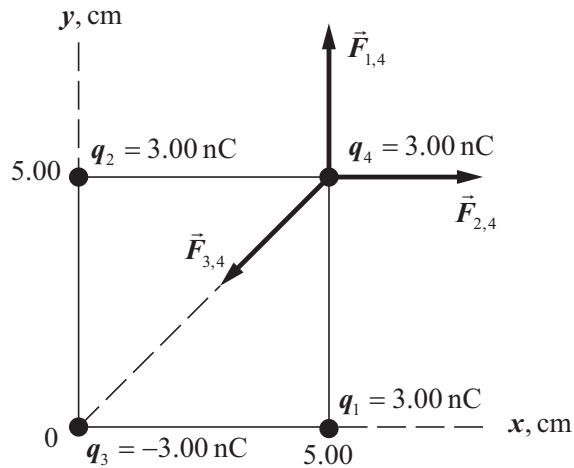
Use the quadratic formula to obtain:

$$x = \frac{2L \pm \sqrt{4L^2 + 4L^2}}{2} = L \pm \sqrt{2}L$$

The root corresponding to the positive sign between the terms is extraneous because it corresponds to a position to the right of the $2.0 \mu\text{C}$ point charge and is, therefore, not a physically meaningful root. Hence the third point charge should be placed a distance equal to $0.41L$ from the $-2.0\text{-}\mu\text{C}$ charge on the side away from the $4.0\text{-}\mu\text{C}$ charge.

30 •• Three point charges, each of magnitude 3.00 nC , are at separate corners of a square of edge length 5.00 cm . The two point charges at opposite corners are positive, and the third point charge is negative. Find the electric force exerted by these point charges on a fourth point charge $q_4 = +3.00 \text{ nC}$ at the remaining corner.

Picture the Problem The configuration of the point charges and the forces on the fourth point charge are shown in the figure ... as is a coordinate system. From the figure it is evident that the net force on the point charge q_4 is along the diagonal of the square and directed away from point charge q_3 . We can apply Coulomb's law to express $\vec{F}_{1,4}$, $\vec{F}_{2,4}$ and $\vec{F}_{3,4}$ and then add them (that is, use the principle of superposition of forces) to find the net electric force on point charge q_4 .



Express the net force acting on point charge q_4 :

$$\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4} \quad (1)$$

Express the force that point charge q_1 exerts on point charge q_4 :

$$\vec{F}_{1,4} = \frac{kq_1q_4}{r_{1,4}^2} \hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{1,4}$:

$$\vec{F}_{1,4} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (3.00 \text{ nC}) \left(\frac{3.00 \text{ nC}}{(0.0500 \text{ m})^2} \right) \hat{j} = (3.236 \times 10^{-5} \text{ N}) \hat{j}$$

Express the force that point charge q_2 exerts on point charge q_4 :

$$\vec{F}_{2,4} = \frac{kq_2q_4}{r_{2,4}^2} \hat{i}$$

Substitute numerical values and evaluate $\vec{F}_{2,4}$:

$$\vec{F}_{2,4} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (3.00 \text{ nC}) \left(\frac{3.00 \text{ nC}}{(0.0500 \text{ m})^2} \right) \hat{i} = (3.236 \times 10^{-5} \text{ N}) \hat{i}$$

Express the force that point charge q_3 exerts on point charge q_4 : $\vec{F}_{3,4} = \frac{kq_3q_4}{r_{3,4}^2} \hat{r}_{3,4}$, where $\hat{r}_{3,4}$ is a unit vector pointing from q_3 to q_4 .

Express $\vec{r}_{3,4}$ in terms of $\vec{r}_{3,1}$ and $\vec{r}_{1,4}$: $\vec{r}_{3,4} = \vec{r}_{3,1} + \vec{r}_{1,4}$
 $= (0.0500\text{ m})\hat{i} + (0.0500\text{ m})\hat{j}$

Convert $\vec{r}_{3,4}$ to $\hat{r}_{3,4}$:

$$\hat{r}_{3,4} = \frac{\vec{r}_{3,4}}{|\vec{r}_{3,4}|} = \frac{(0.0500\text{ m})\hat{i} + (0.0500\text{ m})\hat{j}}{\sqrt{(0.0500\text{ m})^2 + (0.0500\text{ m})^2}} = 0.707\hat{i} + 0.707\hat{j}$$

Substitute numerical values and evaluate $\vec{F}_{3,4}$:

$$\vec{F}_{3,4} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (-3.00\text{ nC}) \left(\frac{3.00\text{ nC}}{(0.0500\sqrt{2}\text{ m})^2} \right) (0.707\hat{i} + 0.707\hat{j})$$

$$= -(1.14 \times 10^{-5}\text{ N})\hat{i} - (1.14 \times 10^{-5}\text{ N})\hat{j}$$

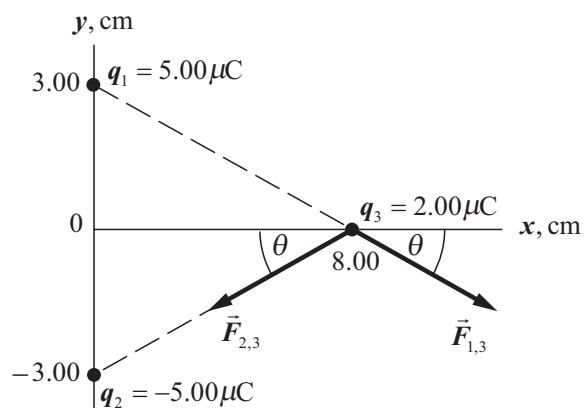
Substitute numerical values in equation (1) and simplify to find \vec{F}_4 :

$$\vec{F}_4 = (3.24 \times 10^{-5}\text{ N})\hat{j} + (3.24 \times 10^{-5}\text{ N})\hat{i} - (1.14 \times 10^{-5}\text{ N})\hat{i} - (1.14 \times 10^{-5}\text{ N})\hat{j}$$

$$= \boxed{(2.10 \times 10^{-5}\text{ N})\hat{i} + (2.10 \times 10^{-5}\text{ N})\hat{j}}$$

31 •• A point charge of $5.00\text{ }\mu\text{C}$ is on the y axis at $y = 3.00\text{ cm}$, and a second point charge of $-5.00\text{ }\mu\text{C}$ is on the y axis at $y = -3.00\text{ cm}$. Find the electric force on a point charge of $2.00\text{ }\mu\text{C}$ on the x axis at $x = 8.00\text{ cm}$.

Picture the Problem The configuration of the point charges and the forces on point charge q_3 are shown in the figure ... as is a coordinate system. From the geometry of the charge distribution it is evident that the net force on the $2.00\text{ }\mu\text{C}$ point charge is in the negative y direction. We can apply Coulomb's law to express $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and then add them (that is, use the principle of superposition of forces) to find the net force on point charge q_3 .



The net force acting on point charge q_3 is given by: $\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3}$

The force that point charge q_1 exerts on point charge q_3 is: $\vec{F}_{1,3} = F \cos \theta \hat{i} - F \sin \theta \hat{j}$

F is given by:

$$F = \frac{kq_1q_3}{r^2} = \frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(5.00 \mu\text{C})(2.00 \mu\text{C})}{(0.0300\text{m})^2 + (0.0800\text{m})^2} = 12.32 \text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{3.00 \text{ cm}}{8.00 \text{ cm}}\right) = 20.56^\circ$$

The force that point charge q_2 exerts on point charge q_3 is: $\vec{F}_{2,3} = -F \cos \theta \hat{i} - F \sin \theta \hat{j}$

Substitute for $\vec{F}_{1,3}$ and $\vec{F}_{2,3}$ and simplify to obtain:

$$\begin{aligned} \vec{F}_3 &= F \cos \theta \hat{i} - F \sin \theta \hat{j} - F \cos \theta \hat{i} \\ &\quad - F \sin \theta \hat{j} \\ &= -2F \sin \theta \hat{j} \end{aligned}$$

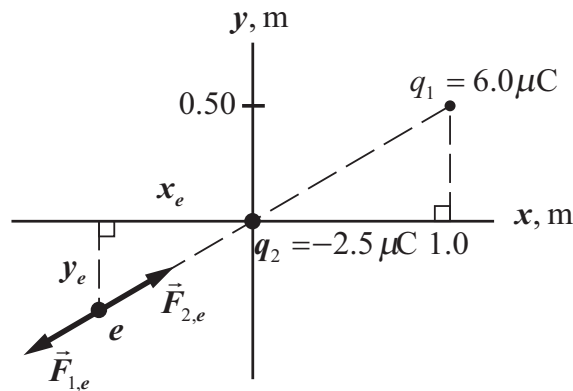
Substitute numerical values and evaluate \vec{F}_3 :

$$\begin{aligned} \vec{F}_3 &= -2(12.32 \text{ N}) \sin 20.56^\circ \hat{j} \\ &= \boxed{-(8.65 \text{ N}) \hat{j}} \end{aligned}$$

32 •• A point particle that has a charge of $-2.5 \mu\text{C}$ is located at the origin. A second point particle that has a charge of $6.0 \mu\text{C}$ is at $x = 1.0 \text{ m}$, $y = 0.50 \text{ m}$. A

third point particle, and electron, is at a point with coordinates (x, y) . Find the values of x and y such that the electron is in equilibrium.

Picture the Problem The positions of the point particles are shown in the diagram. It is apparent that the electron must be located along the line joining the two point particles. Moreover, because it is negatively charged, it must be closer to the particle with a charge of $-2.5 \mu\text{C}$ than to the particle with a charge of $6.0 \mu\text{C}$, as is indicated in the figure. We can find the x and y coordinates of the electron's position by equating the two electrostatic forces acting on it and solving for its distance from the origin. We can use similar triangles to express this radial distance in terms of the x and y coordinates of the electron.



Express the condition that must be satisfied if the electron is to be in equilibrium:

$$F_{1,e} = F_{2,e}$$

Letting r represent the distance from the origin to the electron, express the magnitude of the force that the particle whose charge is q_1 exerts on the electron:

$$F_{1,e} = \frac{kq_1e}{(r + \sqrt{1.25 \text{ m}})^2}$$

Express the magnitude of the force that the particle whose charge is q_2 exerts on the electron:

$$F_{2,e} = \frac{k|q_2|e}{r^2}$$

Substitute and simplify to obtain:

$$\frac{q_1}{(r + \sqrt{1.25 \text{ m}})^2} = \frac{|q_2|}{r^2}$$

Substitute for q_1 and q_2 and simplify to obtain:

$$\frac{6}{(r + \sqrt{1.25 \text{ m}})^2} = \frac{2.5}{r^2}$$

Solving for r yields:

$$r = 2.036 \text{ m}$$

and

$$r = -0.4386 \text{ m}$$

Because $r < 0$ is unphysical, we'll consider only the positive root.

Use the similar triangles in the diagram to establish the proportion involving the y coordinate of the electron:

$$\frac{y_e}{0.50 \text{ m}} = \frac{2.036 \text{ m}}{1.12 \text{ m}} \Rightarrow y_e = 0.909 \text{ m}$$

Use the similar triangles in the diagram to establish the proportion involving the x coordinate of the electron:

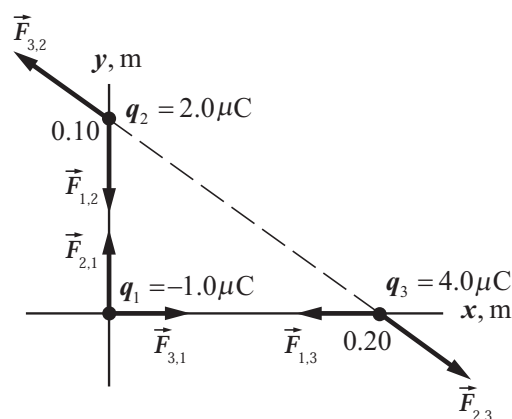
$$\frac{x_e}{1.0 \text{ m}} = \frac{2.036 \text{ m}}{1.12 \text{ m}} \Rightarrow x_e = 1.82 \text{ m}$$

The coordinates of the electron's position are:

$$(x_e, y_e) = \boxed{(-1.8 \text{ m}, -0.91 \text{ m})}$$

33 •• A point particle that has a charge of $-1.0 \mu\text{C}$ is located at the origin; a second point particle that has a charge of $2.0 \mu\text{C}$ is located at $x = 0, y = 0.10 \text{ m}$; and a third point particle that has a charge of $4.0 \mu\text{C}$ is located at $x = 0.20 \text{ m}, y = 0$. Find the total electric force on each of the three point charges.

Picture the Problem Let q_1 represent the charge of the point particle at the origin, q_2 the charge of the point particle at $(0, 0.10 \text{ m})$, and q_3 the charge of the point particle at $(0.20 \text{ m}, 0)$. The diagram shows the forces acting on each of the point particles. Note the action-and-reaction pairs. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the point particles.



Express the net force acting on the point particle whose charge is q_1 :

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that the point particle whose charge is q_2 exerts on the point particle whose charge is q_1 :

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2} \hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2} \frac{\vec{r}_{2,1}}{r_{2,1}} = \frac{kq_2q_1}{r_{2,1}^3} \vec{r}_{2,1}$$

Substitute numerical values and evaluate $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (2.0 \mu\text{C}) \frac{(-1.0 \mu\text{C})}{(0.10 \text{m})^3} (-0.10 \text{m}) \hat{j} = (1.80 \text{N}) \hat{j}$$

Express the force that the point particle whose charge is q_3 exerts on the point particle whose charge is q_1 :

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3} \vec{r}_{3,1}$$

Substitute numerical values and evaluate $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (4.0 \mu\text{C}) \frac{(-1.0 \mu\text{C})}{(0.20 \text{m})^3} (-0.20 \text{m}) \hat{i} = (0.899 \text{N}) \hat{i}$$

Substitute to find \vec{F}_1 :

$$\vec{F}_1 = \boxed{(0.90 \text{N}) \hat{i} + (1.8 \text{N}) \hat{j}}$$

Express the net force acting on the point particle whose charge is q_2 :

$$\begin{aligned} \vec{F}_2 &= \vec{F}_{3,2} + \vec{F}_{1,2} \\ &= \vec{F}_{3,2} - \vec{F}_{2,1} \\ &= \vec{F}_{3,2} - (1.80 \text{N}) \hat{j} \end{aligned}$$

because $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$ are action-and-reaction forces.

Express the force that the point particle whose charge is q_3 exerts on the point particle whose charge is q_2 :

$$\begin{aligned} \vec{F}_{3,2} &= \frac{kq_3q_2}{r_{3,2}^3} \vec{r}_{3,2} \\ &= \frac{kq_3q_2}{r_{3,2}^3} [(-0.20 \text{m}) \hat{i} + (0.10 \text{m}) \hat{j}] \end{aligned}$$

Substitute numerical values and evaluate $\vec{F}_{3,2}$:

$$\begin{aligned}\vec{F}_{3,2} &= \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (4.0 \mu\text{C}) \frac{(2.0 \mu\text{C})}{(0.224 \text{ m})^3} [(-0.20 \text{ m})\hat{i} + (0.10 \text{ m})\hat{j}] \\ &= (-1.28 \text{ N})\hat{i} + (0.640 \text{ N})\hat{j}\end{aligned}$$

Find the net force acting on the point particle whose charge is q_2 :

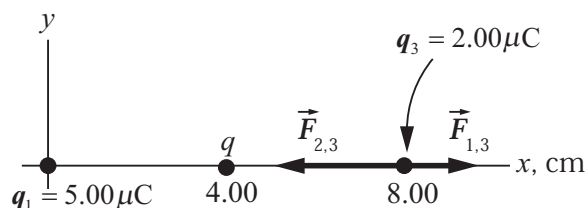
$$\begin{aligned}\vec{F}_2 &= \vec{F}_{3,2} - (1.80 \text{ N})\hat{j} = (-1.28 \text{ N})\hat{i} + (0.640 \text{ N})\hat{j} - (1.80 \text{ N})\hat{j} \\ &= \boxed{(-1.3 \text{ N})\hat{i} - (1.2 \text{ N})\hat{j}}\end{aligned}$$

Noting that $\vec{F}_{1,3}$ and $\vec{F}_{3,1}$ are an action-and-reaction pair, as are $\vec{F}_{2,3}$ and $\vec{F}_{3,2}$, express the net force acting on the point particle whose charge is q_3 :

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{1,3} + \vec{F}_{2,3} = -\vec{F}_{3,1} - \vec{F}_{3,2} = -(0.899 \text{ N})\hat{i} - [(-1.28 \text{ N})\hat{i} + (0.640 \text{ N})\hat{j}] \\ &= \boxed{(0.4 \text{ N})\hat{i} - (0.64 \text{ N})\hat{j}}\end{aligned}$$

34 •• A point particle that has a charge of $5.00 \mu\text{C}$ is located at $x = 0$, $y = 0$ and a point particle that has a charge q is located at $x = 4.00 \text{ cm}$, $y = 0$. The electric force on a point particle that has a charge of $2.00 \mu\text{C}$ at $x = 8.00 \text{ cm}$, $y = 0$ is $(-19.7 \text{ N})\hat{i}$. When this $2.00\text{-}\mu\text{C}$ charge is repositioned at $x = 17.8 \text{ cm}$, $y = 0$, the electric force on it is zero. Determine the charge q .

Picture the Problem Let q_1 represent the charge of the point particle at the origin and q_3 the charge of the point particle initially at $(8.00 \text{ cm}, 0)$ and later positioned at $(17.8 \text{ cm}, 0)$. The diagram shows the forces acting on the point particle whose charge is q_3 at $(8.00 \text{ cm}, 0)$. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the point particles.



Express the net force on the point particle whose charge is q_2 when it is at (8.00 cm, 0):

$$\begin{aligned}\vec{F}_2(8.00\text{ cm}, 0) &= \vec{F}_{1,3} + \vec{F}_{2,3} \\ &= \frac{kq_1q_3}{r_{1,3}^3}\vec{r}_{1,3} + \frac{kq_2q_3}{r_{2,3}^3}\vec{r}_{2,3} \\ &= kq_3\left(\frac{q_1}{r_{1,3}^3}\vec{r}_{1,3} + \frac{q_2}{r_{2,3}^3}\vec{r}_{2,3}\right)\end{aligned}$$

Substitute numerical values to obtain:

$$\begin{aligned}(-19.7\text{ N})\hat{i} &= \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(2.00\text{ }\mu\text{C}) \\ &\quad \times \left[\frac{5.00\text{ }\mu\text{C}}{(0.0800\text{ m})^3}(0.0800\text{ m})\hat{i} + \frac{q}{(0.0400\text{ m})^3}(0.0400\text{ m})\hat{i}\right]\end{aligned}$$

Solving for q yields:

$$q = \boxed{-3.00\text{ }\mu\text{C}}$$

Remarks: An alternative solution is to equate the electrostatic forces acting on q_2 when it is at (17.8 cm, 0).

35 •• [SSM] Five identical point charges, each having charge Q , are equally spaced on a semicircle of radius R as shown in Figure 21-37. Find the force (in terms of k , Q , and R) on a charge q located equidistant from the five other charges.

Picture the Problem By considering the symmetry of the array of charged point particles, we can see that the y component of the force on q is zero. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on q .

Express the net force acting on the point charge q :

$$\vec{F}_q = \vec{F}_{Q\text{ on } x\text{ axis}, q} + 2\vec{F}_{Q\text{ at } 45^\circ, q}$$

Express the force on point charge q due to the point charge Q on the x axis:

$$\vec{F}_{Q\text{ on } x\text{ axis}, q} = \frac{kqQ}{R^2}\hat{i}$$

Express the net force on point charge q due to the point charges at 45° :

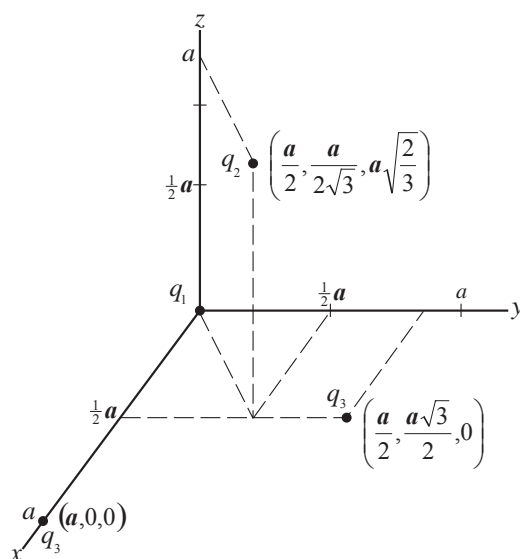
$$\begin{aligned}2\vec{F}_{Q\text{ at } 45^\circ, q} &= 2\frac{kqQ}{R^2}\cos 45^\circ\hat{i} \\ &= \frac{2}{\sqrt{2}}\frac{kqQ}{R^2}\hat{i}\end{aligned}$$

Substitute for $\vec{F}_{Q \text{ on } x \text{ axis}, q}$ and $2\vec{F}_{Q \text{ at } 45^\circ, q}$ to obtain:

$$\begin{aligned}\vec{F}_q &= \frac{kqQ}{R^2} \hat{i} + \frac{2}{\sqrt{2}} \frac{kqQ}{R^2} \hat{i} \\ &= \boxed{\frac{kqQ}{R^2} (1 + \sqrt{2}) \hat{i}}\end{aligned}$$

36 •• The structure of the NH_3 molecule is approximately that of an equilateral tetrahedron, with three H^+ ions forming the base and an N^{3-} ion at the apex of the tetrahedron. The length of each side is 1.64×10^{-10} m. Calculate the electric force that acts on each ion.

Picture the Problem Let the H^+ ions be in the x - y plane with H_1 at $(0, 0, 0)$, H_2 at $(a, 0, 0)$, and H_3 at $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}, 0\right)$. The N^{3-} ion, with charge q_4 in our notation, is then at $\left(\frac{a}{2}, \frac{a}{2\sqrt{3}}, a\sqrt{\frac{2}{3}}\right)$ where $a = 1.64 \times 10^{-10}$ m. To simplify our calculations we'll set $ke^2/a^2 = C = 8.56 \times 10^{-9}$ N. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each ion.



Express the net force acting on point charge q_1 :

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Find $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \frac{kq_1q_2}{r_{2,1}^2} \hat{r}_{2,1} = C(-\hat{i}) = -C\hat{i}$$

Find $\vec{F}_{3,1}$:

$$\begin{aligned}\vec{F}_{3,1} &= \frac{kq_3q_1}{r_{3,1}^2} \hat{r}_{3,1} \\ &= C \frac{\left(0 - \frac{a}{2}\right) \hat{i} + \left(0 - \frac{a\sqrt{3}}{2}\right) \hat{j}}{a} \\ &= -C \left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right)\end{aligned}$$

Noting that the magnitude of point charge q_4 is three times that of the other point charges and that it is negative, express $\vec{F}_{4,1}$:

$$\begin{aligned}\vec{F}_{4,1} &= 3C\hat{r}_{4,1} = -3C \frac{\left(0 - \frac{a}{2}\right) \hat{i} + \left(0 - \frac{a}{2\sqrt{3}}\right) \hat{j} + \left(0 - \frac{a\sqrt{2}}{\sqrt{3}}\right) \hat{k}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2\sqrt{3}}\right)^2 + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right)^2}} \\ &= 3C \frac{\left(\frac{a}{2}\right) \hat{i} + \left(\frac{a}{2\sqrt{3}}\right) \hat{j} + \left(\frac{a\sqrt{2}}{\sqrt{3}}\right) \hat{k}}{a} = 3C \left[\left(\frac{1}{2}\right) \hat{i} + \left(\frac{1}{2\sqrt{3}}\right) \hat{j} + \left(\sqrt{\frac{2}{3}}\right) \hat{k} \right]\end{aligned}$$

Substitute in the expression for \vec{F}_1 to obtain:

$$\begin{aligned}\vec{F}_1 &= -C\hat{i} - C \left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right) \\ &\quad + 3C \left[\left(\frac{1}{2}\right) \hat{i} + \left(\frac{1}{2\sqrt{3}}\right) \hat{j} + \left(\sqrt{\frac{2}{3}}\right) \hat{k} \right] \\ &= \boxed{C\sqrt{6}\hat{k}}\end{aligned}$$

From symmetry considerations:

$$\vec{F}_2 = \vec{F}_3 = \vec{F}_1 = \boxed{C\sqrt{6}\hat{k}}$$

Express the condition that the molecule is in equilibrium:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

Solve for and evaluate \vec{F}_4 :

$$\begin{aligned}\vec{F}_4 &= -(\vec{F}_1 + \vec{F}_2 + \vec{F}_3) = -3\vec{F}_1 \\ &= \boxed{-3C\sqrt{6}\hat{k}}\end{aligned}$$

The Electric Field

37 • [SSM] A point charge of $4.0 \mu\text{C}$ is at the origin. What is the magnitude and direction of the electric field on the x axis at (a) $x = 6.0 \text{ m}$, and (b) $x = -10 \text{ m}$? (c) Sketch the function E_x versus x for both positive and negative values of x . (Remember that E_x is negative when \vec{E} points in the $-x$ direction.)

Picture the Problem Let q represent the point charge at the origin and use Coulomb's law for \vec{E} due to a point charge to find the electric field at $x = 6.0 \text{ m}$ and -10 m .

(a) Express the electric field at a point P located a distance x from a point charge q :

$$\vec{E}(x) = \frac{kq}{x^2} \hat{r}_{p,0}$$

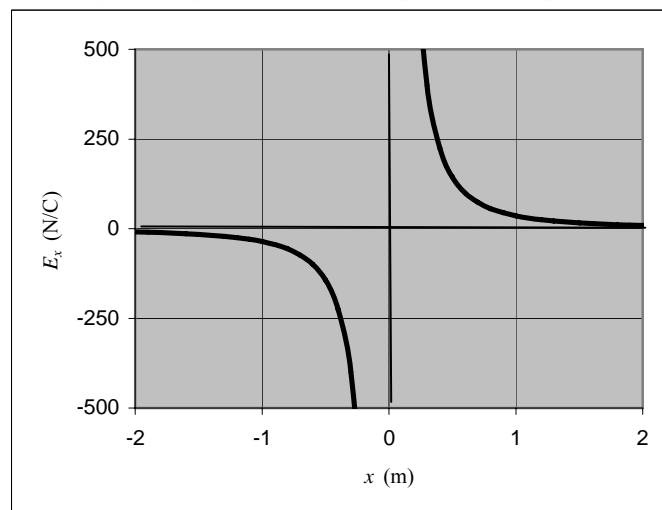
Evaluate this expression for $x = 6.0 \text{ m}$:

$$\vec{E}(6.0 \text{ m}) = \frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.0 \mu\text{C})}{(6.0 \text{ m})^2} \hat{i} = \boxed{(0.10 \text{ kN/C}) \hat{i}}$$

(b) Evaluate \vec{E} at $x = -10 \text{ m}$:

$$\vec{E}(-10 \text{ m}) = \frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(4.0 \mu\text{C})}{(10 \text{ m})^2} (-\hat{i}) = \boxed{(-0.36 \text{ kN/C}) \hat{i}}$$

(c) The following graph was plotted using a spreadsheet program:



38 • Two point charges, each $+4.0 \mu\text{C}$, are on the x axis; one point charge is at the origin and the other is at $x = 8.0 \text{ m}$. Find the electric field on the x axis at (a) $x = -2.0 \text{ m}$, (b) $x = 2.0 \text{ m}$, (c) $x = 6.0 \text{ m}$, and (d) $x = 10 \text{ m}$. (e) At what point on the x axis is the electric field zero? (f) Sketch E_x versus x for $-3.0 \text{ m} < x < 11 \text{ m}$.

Picture the Problem Let q represent the point charges of $+4.0 \mu\text{C}$ and use Coulomb's law for \vec{E} due to a point charge and the principle of superposition for fields to find the electric field at the locations specified.

Noting that $q_1 = q_2$, use Coulomb's law and the principle of superposition to express the electric field due to the given charges at point P a distance x from the origin:

$$\begin{aligned}\vec{E}(x) &= \vec{E}_{q_1}(x) + \vec{E}_{q_2}(x) = \frac{kq_1}{x^2} \hat{r}_{q_1,P} + \frac{kq_2}{(8.0 \text{ m} - x)^2} \hat{r}_{q_2,P} \\ &= kq_1 \left(\frac{1}{x^2} \hat{r}_{q_1,P} + \frac{1}{(8.0 \text{ m} - x)^2} \hat{r}_{q_2,P} \right) \\ &= (36 \text{ kN} \cdot \text{m}^2/\text{C}) \left(\frac{1}{x^2} \hat{r}_{q_1,P} + \frac{1}{(8.0 \text{ m} - x)^2} \hat{r}_{q_2,P} \right)\end{aligned}$$

(a) Apply this equation to the point at $x = -2.0 \text{ m}$:

$$\vec{E}(-2.0 \text{ m}) = (36 \text{ kN} \cdot \text{m}^2/\text{C}) \left[\frac{1}{(2.0 \text{ m})^2} (-\hat{i}) + \frac{1}{(10 \text{ m})^2} (-\hat{i}) \right] = \boxed{(-9.4 \text{ kN/C})\hat{i}}$$

(b) Evaluate \vec{E} at $x = 2.0 \text{ m}$:

$$\vec{E}(2.0 \text{ m}) = (36 \text{ kN} \cdot \text{m}^2/\text{C}) \left[\frac{1}{(2.0 \text{ m})^2} (\hat{i}) + \frac{1}{(6.0 \text{ m})^2} (-\hat{i}) \right] = \boxed{(8.0 \text{ kN/C})\hat{i}}$$

(c) Evaluate \vec{E} at $x = 6.0 \text{ m}$:

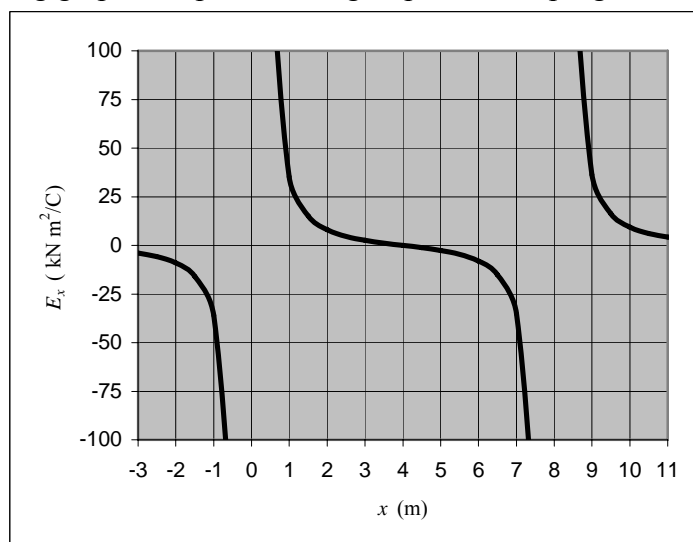
$$\vec{E}(6.0 \text{ m}) = (36 \text{ kN} \cdot \text{m}^2/\text{C}) \left[\frac{1}{(6.0 \text{ m})^2} (\hat{i}) + \frac{1}{(2.0 \text{ m})^2} (-\hat{i}) \right] = \boxed{(-8.0 \text{ kN/C})\hat{i}}$$

(d) Evaluate \vec{E} at $x = 10 \text{ m}$:

$$\vec{E}(10 \text{ m}) = (36 \text{ kN} \cdot \text{m}^2/\text{C}) \left[\frac{1}{(10 \text{ m})^2} (\hat{i}) + \frac{1}{(2.0 \text{ m})^2} (\hat{i}) \right] = \boxed{(9.4 \text{ kN/C})\hat{i}}$$

(e) From symmetry considerations: $\vec{E}(4.0\text{ m}) = \boxed{0}$

(f) The following graph was plotted using a spreadsheet program:



39 • When a 2.0-nC point charge is placed at the origin, it experiences an electric force of 8.0×10^{-4} N in the +y direction. (a) What is the electric field at the origin? (b) What would be the electric force on a -4.0-nC point charge placed at the origin? (c) If this force is due to the electric field of a point charge on the y axis at $y = 3.0$ cm, what is the value of that charge?

Picture the Problem We can find the electric field at the origin from its definition and the electric force on a point charge placed there using $\vec{F} = q\vec{E}$. We can apply Coulomb's law to find the value of the point charge placed at $y = 3$ cm.

(a) Apply the definition of electric field to obtain:

$$\begin{aligned}\vec{E}(0,0) &= \frac{\vec{F}(0,0)}{q_0} = \frac{(8.0 \times 10^{-4} \text{ N})\hat{j}}{2.0 \text{ nC}} \\ &= \boxed{(4.0 \times 10^5 \text{ N/C})\hat{j}}\end{aligned}$$

(b) The force on a point charge in an electric field is given by:

$$\begin{aligned}\vec{F}(0,0) &= q\vec{E}(0,0) \\ &= (-4.0 \text{ nC})(400 \text{ kN/C})\hat{j} \\ &= \boxed{(-1.6 \text{ mN})\hat{j}}\end{aligned}$$

(c) Apply Coulomb's law to obtain:

$$\frac{kq(-4.0 \text{ nC})}{(0.030 \text{ m})^2}(-\hat{j}) = (-1.60 \text{ mN})\hat{j}$$

Solving for q yields:

$$q = -\frac{(1.60 \text{ mN})(0.030 \text{ m})^2}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.0 \text{ nC})}$$

$$= \boxed{-40 \text{ nC}}$$

40 • The electric field near the surface of Earth points downward and has a magnitude of 150 N/C . (a) Compare the magnitude of the upward electric force on an electron with the magnitude of the gravitational force on the electron. (b) What charge should be placed on a ping pong ball of mass 2.70 g so that the electric force balances the weight of the ball near Earth's surface?

Picture the Problem We can compare the electric and gravitational forces acting on an electron by expressing their ratio. Because the ping pong ball is in equilibrium under the influence of the electric and gravitational forces acting on it, we can use the condition for translational equilibrium to find the charge that would have to be placed on it in order to balance Earth's gravitational force on it.

(a) Express the magnitude of the electric force acting on the electron:

$$F_e = eE$$

Express the magnitude of the gravitational force acting on the electron:

$$F_g = m_e g$$

Express the ratio of these forces to obtain:

$$\frac{F_e}{F_g} = \frac{eE}{mg}$$

Substitute numerical values and evaluate F_e/F_g :

$$\frac{F_e}{F_g} = \frac{(1.602 \times 10^{-19} \text{ C})(150 \text{ N/C})}{(9.109 \times 10^{-31} \text{ kg})(9.81 \text{ m/s}^2)}$$

$$= 2.69 \times 10^{12}$$

or

$$F_e = \boxed{(2.69 \times 10^{12})F_g}$$

Thus, the electric force is greater by a factor of 2.69×10^{12} .

(b) Letting the upward direction be positive, apply the condition for static equilibrium to the ping pong ball to obtain:

$$F_e - F_g = 0$$

or

$$-qE - mg = 0 \Rightarrow q = -\frac{mg}{E}$$

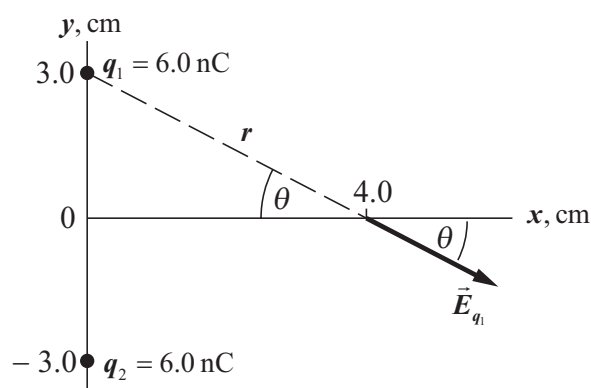
Substitute numerical values and evaluate q :

$$q = -\frac{(3.00 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)}{150 \text{ N/C}}$$

$$= \boxed{-0.196 \text{ mC}}$$

41 •• [SSM] Two point charges q_1 and q_2 both have a charge equal to $+6.0 \text{ nC}$ and are on the y axis at $y_1 = +3.0 \text{ cm}$ and $y_2 = -3.0 \text{ cm}$ respectively. (a) What is the magnitude and direction of the electric field on the x axis at $x = 4.0 \text{ cm}$? (b) What is the force exerted on a third charge $q_0 = 2.0 \text{ nC}$ when it is placed on the x axis at $x = 4.0 \text{ cm}$?

Picture the Problem The diagram shows the locations of the point charges q_1 and q_2 and the point on the x axis at which we are to find \vec{E} . From symmetry considerations we can conclude that the y component of \vec{E} at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges and the principle of superposition for fields to find the field at any point on the x axis and $\vec{F} = q\vec{E}$ to find the force on a point charge q_0 placed on the x axis at $x = 4.0 \text{ cm}$.



(a) Letting $q = q_1 = q_2$, express the x -component of the electric field due to one point charge as a function of the distance r from either point charge to the point of interest:

$$\vec{E}_x = \frac{kq}{r^2} \cos \theta \hat{i}$$

Express \vec{E}_x for both charges:

$$\vec{E}_x = 2 \frac{kq}{r^2} \cos \theta \hat{i}$$

Substitute for $\cos\theta$ and r , substitute numerical values, and evaluate to obtain:

$$\begin{aligned}\vec{E}(4.0\text{ cm})_x &= 2 \frac{kq}{r^2} \frac{0.040\text{ m}}{r} \hat{i} = \frac{2kq(0.040\text{ m})}{r^3} \hat{i} \\ &= \frac{2(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.0\text{ nC})(0.040\text{ m})}{[(0.030\text{ m})^2 + (0.040\text{ m})^2]^{3/2}} \hat{i} \\ &= (34.5\text{ kN/C}) \hat{i} = (35\text{ kN/C}) \hat{i}\end{aligned}$$

The magnitude and direction of the electric field at $x = 4.0\text{ cm}$ is:

$$35\text{ kN/C @ } 0^\circ$$

(b) Apply $\vec{F} = q\vec{E}$ to find the force on a point charge q_0 placed on the x axis at $x = 4.0\text{ cm}$:

$$\begin{aligned}\vec{F} &= (2.0\text{ nC})(34.5\text{ kN/C}) \hat{i} \\ &= (69\text{ }\mu\text{N}) \hat{i}\end{aligned}$$

42 •• A point charge of $+5.0\text{ }\mu\text{C}$ is located on the x axis at $x = -3.0\text{ cm}$, and a second point charge of $-8.0\text{ }\mu\text{C}$ is located on the x axis at $x = +4.0\text{ cm}$. Where should a third charge of $+6.0\text{ }\mu\text{C}$ be placed so that the electric field at the origin is zero?

Picture the Problem If the electric field at $x = 0$ is zero, both its x and y components must be zero. The only way this condition can be satisfied with point charges of $+5.0\text{ }\mu\text{C}$ and $-8.0\text{ }\mu\text{C}$ on the x axis is if the point charge $+6.0\text{ }\mu\text{C}$ is also on the x axis. Let the subscripts 5, -8 , and 6 identify the point charges and their fields. We can use Coulomb's law for \vec{E} due to a point charge and the principle of superposition for fields to determine where the $+6.0\text{ }\mu\text{C}$ point charge should be located so that the electric field at $x = 0$ is zero.

Express the electric field at $x = 0$ in terms of the fields due to the point charges of $+5.0\text{ }\mu\text{C}$, $-8.0\text{ }\mu\text{C}$, and $+6.0\text{ }\mu\text{C}$:

$$\begin{aligned}\vec{E}(0) &= \vec{E}_{5\mu\text{C}} + \vec{E}_{-8\mu\text{C}} + \vec{E}_{6\mu\text{C}} \\ &= 0\end{aligned}$$

Substitute for each of the fields to obtain:

$$\frac{kq_5}{r_5^2} \hat{r}_5 + \frac{kq_6}{r_6^2} \hat{r}_6 + \frac{kq_{-8}}{r_{-8}^2} \hat{r}_{-8} = 0$$

or

$$\frac{kq_5}{r_5^2} \hat{i} + \frac{kq_6}{r_6^2} (-\hat{i}) + \frac{kq_{-8}}{r_{-8}^2} (-\hat{i}) = 0$$

Divide out the unit vector \hat{i} to obtain:

$$\frac{q_5}{r_5^2} - \frac{q_6}{r_6^2} - \frac{q_{-8}}{r_{-8}^2} = 0$$

Substitute numerical values to obtain:

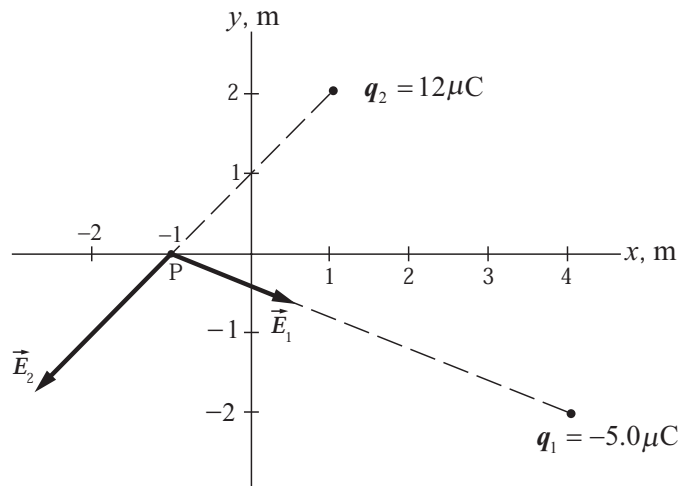
$$\frac{5}{(3.0\text{ cm})^2} - \frac{6}{r_6^2} - \frac{-8}{(4.0\text{ cm})^2} = 0$$

Solving for r_6 yields:

$$r_6 = \boxed{2.4\text{ cm}}$$

43 •• A $-5.0\text{-}\mu\text{C}$ point charge is located at $x = 4.0\text{ m}$, $y = -2.0\text{ m}$ and a $12\text{-}\mu\text{C}$ point charge is located at $x = 1.0\text{ m}$, $y = 2.0\text{ m}$. (a) Find the magnitude and direction of the electric field at $x = -1.0\text{ m}$, $y = 0$. (b) Calculate the magnitude and direction of the electric force on an electron that is placed at $x = -1.0\text{ m}$, $y = 0$.

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two point charges. We can use Coulomb's law for \vec{E} due to point charges and the superposition principle for electric fields to find \vec{E}_P . We can apply $\vec{F} = q\vec{E}$ to find the force on an electron at $(-1.0\text{ m}, 0)$.



(a) Express the electric field at $(-1.0\text{ m}, 0)$ due to the point charges q_1 and q_2 :

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

Substitute numerical values and evaluate \vec{E}_1 :

$$\begin{aligned}\vec{E}_1 &= \frac{kq_1}{r_{1,P}^2} \hat{r}_{1,P} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-5.0 \mu\text{C})}{(5.0 \text{ m})^2 + (2.0 \text{ m})^2} \left(\frac{(-5.0 \text{ m})\hat{i} + (2.0 \text{ m})\hat{j}}{\sqrt{(5.0 \text{ m})^2 + (2.0 \text{ m})^2}} \right) \\ &= (-1.55 \times 10^3 \text{ N/C})(-0.928\hat{i} + 0.371\hat{j}) \\ &= (1.44 \text{ kN/C})\hat{i} + (-0.575 \text{ kN/C})\hat{j}\end{aligned}$$

Substitute numerical values and evaluate \vec{E}_2 :

$$\begin{aligned}\vec{E}_2 &= \frac{kq_2}{r_{2,P}^2} \hat{r}_{2,P} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(12 \mu\text{C})}{(2.0 \text{ m})^2 + (2.0 \text{ m})^2} \left(\frac{(-2.0 \text{ m})\hat{i} + (-2.0 \text{ m})\hat{j}}{\sqrt{(2.0 \text{ m})^2 + (2.0 \text{ m})^2}} \right) \\ &= (13.5 \times 10^3 \text{ N/C})(-0.707\hat{i} - 0.707\hat{j}) \\ &= (-9.54 \text{ kN/C})\hat{i} + (-9.54 \text{ kN/C})\hat{j}\end{aligned}$$

Substitute for \vec{E}_1 and \vec{E}_2 and simplify to find \vec{E}_P :

$$\begin{aligned}\vec{E}_P &= (1.44 \text{ kN/C})\hat{i} + (-0.575 \text{ kN/C})\hat{j} + (-9.54 \text{ kN/C})\hat{i} + (-9.54 \text{ kN/C})\hat{j} \\ &= (-8.10 \text{ kN/C})\hat{i} + (-10.1 \text{ kN/C})\hat{j}\end{aligned}$$

The magnitude of \vec{E}_P is:

$$\begin{aligned}E_P &= \sqrt{(-8.10 \text{ kN/C})^2 + (-10.1 \text{ kN/C})^2} \\ &= \boxed{13 \text{ kN/C}}\end{aligned}$$

The direction of \vec{E}_P is:

$$\theta_E = \tan^{-1}\left(\frac{-10.1 \text{ kN/C}}{-8.10 \text{ kN/C}}\right) = \boxed{230^\circ}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-10.1 \text{ kN/C}}{-8.10 \text{ kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E .

(b) Express and evaluate the force on an electron at point P:

$$\begin{aligned}\vec{F} &= q\vec{E}_P = (-1.602 \times 10^{-19} \text{ C})[(-8.10 \text{ kN/C})\hat{i} + (-10.1 \text{ kN/C})\hat{j}] \\ &= (1.30 \times 10^{-15} \text{ N})\hat{i} + (1.62 \times 10^{-15} \text{ N})\hat{j}\end{aligned}$$

Find the magnitude of \vec{F} :

$$F = \sqrt{(1.30 \times 10^{-15} \text{ N})^2 + (1.62 \times 10^{-15} \text{ N})^2}$$

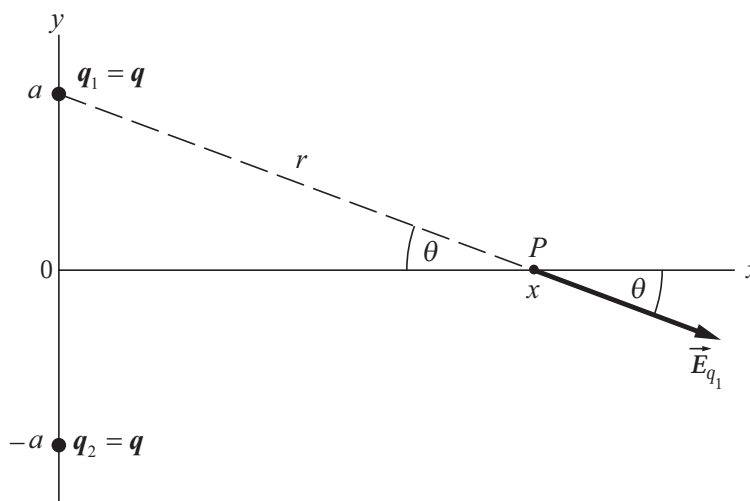
$$= \boxed{2.1 \times 10^{-15} \text{ N}}$$

Find the direction of \vec{F} :

$$\theta_F = \tan^{-1} \left(\frac{1.62 \times 10^{-15} \text{ N}}{1.3 \times 10^{-15} \text{ N}} \right) = \boxed{51^\circ}$$

44 •• Two equal positive charges q are on the y axis; one point charge is at $y = +a$ and the other is at $y = -a$. (a) Show that on the x axis the x component of the electric field is given by $E_x = 2kqx/(x^2 + a^2)^{3/2}$. (b) Show that near the origin, where x is much smaller than a , $E_x \approx 2kqx/a^3$. (c) Show that for values of x much larger than a , $E_x \approx 2kq/x^2$. Explain why a person might expect this result even without deriving it by taking the appropriate limit.

Picture the Problem The diagram shows the locations of point charges q_1 and q_2 and the point on the x axis at which we are to find \vec{E} . From symmetry considerations we can conclude that the y component of \vec{E} at any point on the x axis is zero. We can use Coulomb's law for the electric field due to point charges and the principle of superposition of fields to find the field at any point on the x axis. We can establish the results called for in Parts (b) and (c) by factoring the radicand and using the approximation $1 + \alpha \approx 1$ whenever $\alpha \ll 1$.



(a) Express the x -component of the electric field due to the point charges at $y = a$ and $y = -a$ as a function of the distance r from either charge to point P:

$$\vec{E}_x = 2 \frac{kq}{r^2} \cos \theta \hat{i}$$

Substitute for $\cos\theta$ and r to obtain:

$$\begin{aligned}\vec{E}_x &= 2 \frac{kq}{r^2} \frac{x}{r} \hat{i} = \frac{2kqx}{r^3} \hat{i} = \frac{2kqx}{(x^2 + a^2)^{3/2}} \hat{i} \\ &= \frac{2kqx}{(x^2 + a^2)^{3/2}} \hat{i}\end{aligned}$$

The magnitude of \vec{E}_x is:

$$E_x = \boxed{\frac{2kqx}{(x^2 + a^2)^{3/2}}}$$

(b) For $|x| \ll a$, $x^2 + a^2 \approx a^2$, so:

$$E_x \approx \frac{2kqx}{(a^2)^{3/2}} = \boxed{\frac{2kqx}{a^3}}$$

(c) For $x \gg a$, the charges separated by a would appear to be a single charge of magnitude $2q$. Its field would be given by $E_x = \frac{2kq}{x^2}$.

Factor the radicand to obtain:

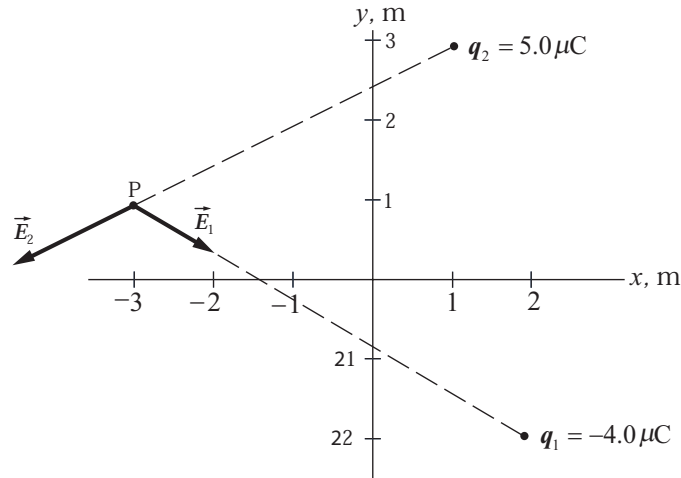
$$E_x = 2kqx \left[x^2 \left(1 + \frac{a^2}{x^2} \right) \right]^{-3/2}$$

For $a \ll x$, $1 + \frac{a^2}{x^2} \approx 1$ and:

$$E_x = 2kqx [x^2]^{-3/2} = \boxed{\frac{2kq}{x^2}}$$

45 •• A $5.0\text{-}\mu\text{C}$ point charge is located at $x = 1.0\text{ m}$, $y = 3.0\text{ m}$ and a $-4.0\text{-}\mu\text{C}$ point charge is located at $x = 2.0\text{ m}$, $y = -2.0\text{ m}$. (a) Find the magnitude and direction of the electric field at $x = -3.0\text{ m}$, $y = 1.0\text{ m}$. (b) Find the magnitude and direction of the force on a proton placed at $x = -3.0\text{ m}$, $y = 1.0\text{ m}$.

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two point charges. We can use Coulomb's law for \vec{E} due to point charges and the superposition principle for electric fields to find \vec{E}_p . We can apply $\vec{F} = q\vec{E}$ to find the force on a proton at $(-3.0\text{ m}, 1.0\text{ m})$.



(a) Express the electric field at $(-3.0 \text{ m}, 1.0 \text{ m})$ due to the point charges q_1 and q_2 :

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

Evaluate \vec{E}_1 :

$$\begin{aligned}\vec{E}_1 &= \frac{kq_1}{r_{1,P}^2} \hat{r}_{1,P} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-4.0 \mu\text{C})}{(5.0 \text{ m})^2 + (3.0 \text{ m})^2} \left(\frac{(-5.0 \text{ m})\hat{i} + (3.0 \text{ m})\hat{j}}{\sqrt{(5.0 \text{ m})^2 + (3.0 \text{ m})^2}} \right) \\ &= (-1.06 \text{ kN/C})(-0.857\hat{i} + 0.514\hat{j}) = (0.907 \text{ kN/C})\hat{i} + (-0.544 \text{ kN/C})\hat{j}\end{aligned}$$

Evaluate \vec{E}_2 :

$$\begin{aligned}\vec{E}_2 &= \frac{kq_2}{r_{2,P}^2} \hat{r}_{2,P} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \mu\text{C})}{(4.0 \text{ m})^2 + (2.0 \text{ m})^2} \left(\frac{(-4.0 \text{ m})\hat{i} + (-2.0 \text{ m})\hat{j}}{\sqrt{(4.0 \text{ m})^2 + (2.0 \text{ m})^2}} \right) \\ &= (2.25 \text{ kN/C})(-0.894\hat{i} - 0.447\hat{j}) = (-2.01 \text{ kN/C})\hat{i} + (-1.01 \text{ kN/C})\hat{j}\end{aligned}$$

Substitute and simplify to find \vec{E}_p :

$$\begin{aligned}\vec{E}_p &= (0.908 \text{ kN/C})\hat{i} + (-0.544 \text{ kN/C})\hat{j} + (-2.01 \text{ kN/C})\hat{i} + (-1.01 \text{ kN/C})\hat{j} \\ &= (-1.10 \text{ kN/C})\hat{i} + (-1.55 \text{ kN/C})\hat{j}\end{aligned}$$

The magnitude of \vec{E}_p is:

$$\begin{aligned}E_p &= \sqrt{(1.10 \text{ kN/C})^2 + (1.55 \text{ kN/C})^2} \\ &= \boxed{1.9 \text{ kN/C}}\end{aligned}$$

The direction of \vec{E}_p is:

$$\theta_E = \tan^{-1}\left(\frac{-1.55 \text{ kN/C}}{-1.10 \text{ kN/C}}\right) = \boxed{230^\circ}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-1.55 \text{ kN/C}}{-1.10 \text{ kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E .

(b) Express and evaluate the force on a proton at point P:

$$\begin{aligned}\vec{F} &= q\vec{E}_p = (1.602 \times 10^{-19} \text{ C})[(-1.10 \text{ kN/C})\hat{i} + (-1.55 \text{ kN/C})\hat{j}] \\ &= (-1.76 \times 10^{-16} \text{ N})\hat{i} + (-2.48 \times 10^{-16} \text{ N})\hat{j}\end{aligned}$$

The magnitude of \vec{F} is:

$$F = \sqrt{(-1.76 \times 10^{-16} \text{ N})^2 + (-2.48 \times 10^{-16} \text{ N})^2} = \boxed{3.0 \times 10^{-16} \text{ N}}$$

The direction of \vec{F} is:

$$\theta_F = \tan^{-1}\left(\frac{-2.48 \times 10^{-16} \text{ N}}{-1.76 \times 10^{-16} \text{ N}}\right) = \boxed{230^\circ}$$

where, as noted above, the angle returned by your calculator for $\tan^{-1}\left(\frac{-2.48 \times 10^{-16} \text{ N}}{-1.76 \times 10^{-16} \text{ N}}\right)$ is the reference angle and must be increased by 180° to yield θ_E .

46 •• Two positive point charges, each having a charge Q , are on the y axis, one at $y = +a$ and the other at $y = -a$. (a) Show that the electric field strength on the x axis is greatest at the points $x = a/\sqrt{2}$ and $x = -a/\sqrt{2}$ by computing $\partial E_x/\partial x$ and setting the derivative equal to zero. (b) Sketch the function E_x versus x using your results for Part (a) of this problem and the fact that E_x is approximately $2kqx/a^3$ when x is much smaller than a and E_x is approximately $2kq/x^2$ when x is much larger than a .

Picture the Problem In Problem 44 it is shown that the electric field on the x axis, due to equal positive charges located at $(0, a)$ and $(0, -a)$, is given by $E_x = 2kqx(x^2 + a^2)^{-3/2}$. We can identify the locations at which E_x has its greatest values by setting $\partial E_x/\partial x$ equal to zero.

(a) Evaluate $\frac{\partial E_x}{\partial x}$ to obtain:

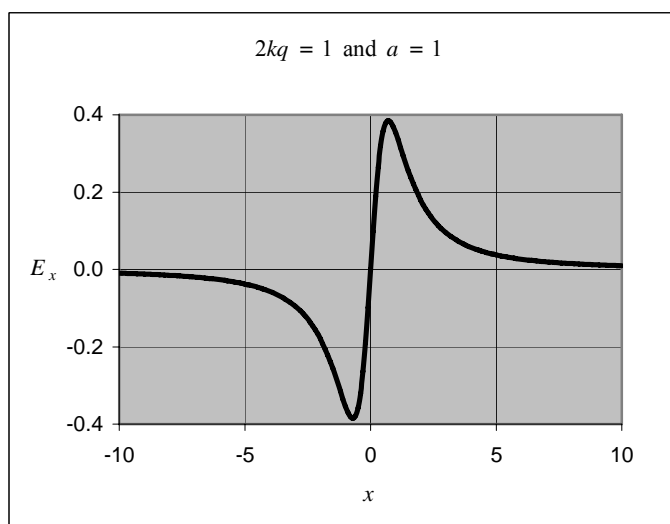
$$\begin{aligned}\frac{\partial E_x}{\partial x} &= \frac{d}{dx} \left[2kqx(x^2 + a^2)^{-3/2} \right] = 2kq \frac{d}{dx} \left[x(x^2 + a^2)^{-3/2} \right] \\ &= 2kq \left[x \frac{d}{dx} (x^2 + a^2)^{-3/2} + (x^2 + a^2)^{-3/2} \right] \\ &= 2kq \left[x \left(-\frac{3}{2} \right) (x^2 + a^2)^{-5/2} (2x) + (x^2 + a^2)^{-3/2} \right] \\ &= 2kq \left[-3x^2 (x^2 + a^2)^{-5/2} + (x^2 + a^2)^{-3/2} \right]\end{aligned}$$

Set this derivative equal to zero for extreme values: $-3x^2(x^2 + a^2)^{-5/2} + (x^2 + a^2)^{-3/2} = 0$

Solving for x yields:

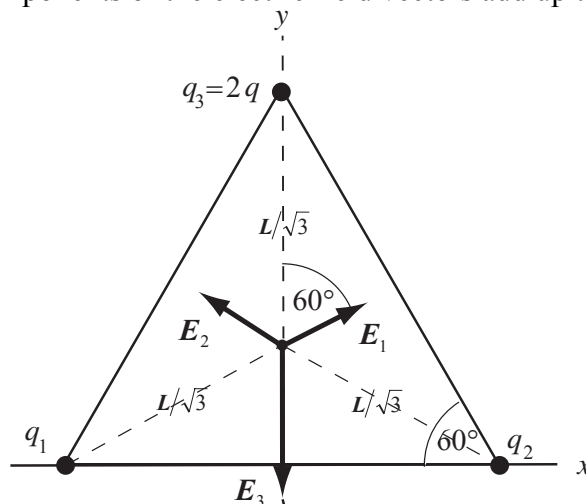
$$x = \pm \frac{a}{\sqrt{2}}$$

(b) The following graph was plotted using a spreadsheet program:



47 •• [SSM] Two point particles, each having a charge q , sit on the base of an equilateral triangle that has sides of length L as shown in Figure 21-38. A third point particle that has a charge equal to $2q$ sits at the apex of the triangle. Where must a fourth point particle that has a charge equal to q be placed in order that the electric field at the center of the triangle be zero? (The center is in the plane of the triangle and equidistant from the three vertices.)

Picture the Problem The electric field of 4th charged point particle must cancel the sum of the electric fields due to the other three charged point particles. By symmetry, the position of the 4th charged point particle must lie on the vertical centerline of the triangle. Using trigonometry, one can show that the center of an equilateral triangle is a distance $L/\sqrt{3}$ from each vertex, where L is the length of the side of the triangle. Note that the x components of the fields due to the base charged particles cancel each other, so we only need concern ourselves with the y components of the fields due to the charged point particles at the vertices of the triangle. Choose a coordinate system in which the origin is at the midpoint of the base of the triangle, the $+x$ direction is to the right, and the $+y$ direction is upward. Note that the x components of the electric field vectors add up to zero.



Express the condition that must be satisfied if the electric field at the center of the triangle is to be zero:

$$\sum_{i=1 \text{ to } 4} \vec{E}_i = 0$$

Substituting for \vec{E}_1 , \vec{E}_2 , \vec{E}_3 , and \vec{E}_4 yields:

$$\frac{k(q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \cos 60^\circ \hat{j} + \frac{k(q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \cos 60^\circ \hat{j} - \frac{k(2q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \hat{j} + \frac{kq}{y^2} \hat{j} = 0$$

Solving for y yields:

$$y = \pm \frac{L}{\sqrt{3}}$$

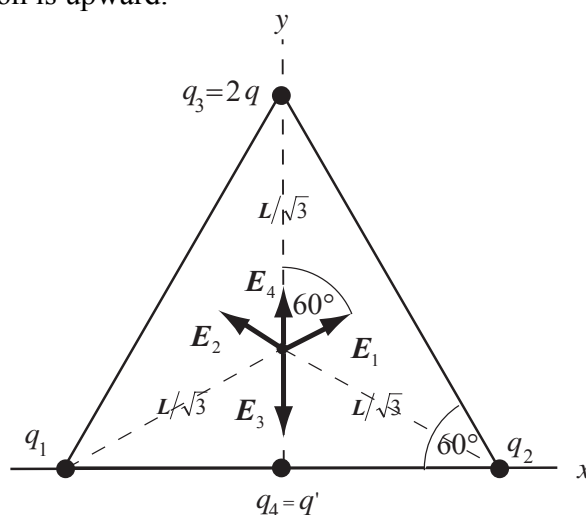
Because the positive solution corresponds to the 4th charge being at the center of the triangle, it follows that:

$$y = -\frac{L}{\sqrt{3}}$$

The charge must be placed a distance equal to the length of the side of the triangle divided by the square root of three below the midpoint of the base of the triangle, where L is the length of a side of the triangle.

48 •• Two point particles, each having a charge equal to q , sit on the base of an equilateral triangle that has sides of length L as shown in Figure 21-38. A third point particle that has a charge equal to $2q$ sits at the apex of the triangle. A fourth point particle that has charge q' is placed at the midpoint of the baseline making the electric field at the center of the triangle equal to zero. What is the value of q' ? (The center is in the plane of the triangle and equidistant from all three vertices.)

Picture the Problem The electric field of 4th charge must cancel the sum of the electric fields due to the other three charges. By symmetry, the position of the 4th charge must lie on the vertical centerline of the triangle. Using trigonometry, one can show that the center of an equilateral triangle is a distance $L/\sqrt{3}$ from each vertex, where L is the length of the side of the triangle. The distance from the center point of the triangle to the midpoint of the base is half this distance. Note that the x components of the fields due to the base charges cancel each other, so we only need concern ourselves with the y components of the fields due to the charges at the vertices of the triangle. Choose a coordinate system in which the origin is at the midpoint of the base of the triangle, the $+x$ direction is to the right, and the $+y$ direction is upward.



Express the condition that must be satisfied if the electric field at the center of the triangle is to be zero:

$$\sum_{i=1 \text{ to } 4} \vec{E}_i = 0$$

Substituting for \vec{E}_1 , \vec{E}_2 , \vec{E}_3 , and \vec{E}_4 yields:

$$\frac{k(q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \cos 60^\circ \hat{j} + \frac{k(q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \cos 60^\circ \hat{j} - \frac{k(2q)}{\left(\frac{L}{\sqrt{3}}\right)^2} \hat{j} + \frac{kq'}{\left(\frac{L}{2\sqrt{3}}\right)^2} \hat{j} = 0$$

Solve for q' to obtain:

$$q' = \boxed{\frac{1}{4}q}$$

49 •• Two equal positive point charges $+q$ are on the y axis; one is at $y = +a$ and the other is at $y = -a$. The electric field at the origin is zero. A test charge q_0 placed at the origin will therefore be in equilibrium. (a) Discuss the stability of the equilibrium for a positive test charge by considering small displacements from equilibrium along the x axis and small displacements along the y axis. (b) Repeat Part (a) for a negative test charge. (c) Find the magnitude and sign of a charge q_0 that when placed at the origin results in a net force of zero on each of the three charges.

Picture the Problem We can determine the stability of the equilibrium in Part (a) and Part (b) by considering the forces the equal point charges q at $y = +a$ and $y = -a$ exert on the test charge when it is given a small displacement along either the x or y axis. (c) The application of Coulomb's law in Part (c) will lead to the magnitude and sign of the charge that must be placed at the origin in order that a net force of zero is experienced by each of the three point charges.

(a) Because E_x is in the x direction, a positive test charge that is displaced from $(0, 0)$ in either the $+x$ direction or the $-x$ direction will experience a force pointing away from the origin and accelerate in the direction of the force. Consequently, the equilibrium at $(0,0)$ is unstable for a small displacement along the x axis.

If the positive test charge is displaced in the direction of increasing y (the positive y direction), the charge at $y = +a$ will exert a greater force than the charge at $y = -a$, and the net force is then in the $-y$ direction; i.e., it is a restoring force. Similarly, if the positive test charge is displaced in the direction of decreasing y (the negative y direction), the charge at $y = -a$ will exert a greater force than the charge at $y = +a$, and the net force is then in the $+y$ direction; i.e., it is a restoring force. Consequently, the equilibrium at $(0,0)$ is stable for a small displacement along the y axis.

(b) Following the same arguments as in Part (a), one finds that, for a negative test charge, the equilibrium is stable at (0,0) for displacements along the x axis and unstable for displacements along the y axis.

(c) Express the net force acting on the charge at $y = +a$:

$$\sum F_{q \text{ at } y=+a} = \frac{kqq_0}{a^2} + \frac{kq^2}{(2a)^2} = 0$$

Solve for q_0 to obtain:

$$q_0 = \boxed{-\frac{1}{4}q}$$

(d) If point charge q_0 is given a small displacement along the y axis, the force exerted on it by the nearest positive point charge will increase and the force exerted on it by the positive point charge that is farther away will decrease. Hence there will be a net force in the direction of its displacement that will accelerate the point charge q_0 away from the origin. The equilibrium of the system is unstable.

Remarks: In Part (c), we could just as well have expressed the net force acting on the charge at $y = -a$. Due to the symmetric distribution of the charges at $y = -a$ and $y = +a$, summing the forces acting on q_0 at the origin does not lead to a relationship between q_0 and q .

50 •• Two positive point charges $+q$ are on the y axis at $y = +a$ and $y = -a$. A bead of mass m and charge $-q$ slides without friction along a taut thread that runs along the x axis. Let x be the position of the bead. (a) Show that for $x \ll a$, the bead experiences a linear restoring force (a force that is proportional to x and directed toward the equilibrium position at $x = 0$) and therefore undergoes simple harmonic motion. (b) Find the period of the motion.

Picture the Problem In Problem 44 it is shown that the electric field on the x axis, due to equal positive point charges located at $(0, a)$ and $(0, -a)$, is given by $E_x = 2kqx(x^2 + a^2)^{-3/2}$. We can use $T = 2\pi\sqrt{m/k'}$ to express the period of the motion of the bead in terms of the restoring constant k' .

(a) Express the force acting on the bead when its displacement from the origin is x :

$$F_x = -qE_x = -\frac{2kq^2x}{(x^2 + a^2)^{3/2}}$$

Factor a^2 from the denominator to obtain:

$$F_x = -\frac{2kq^2x}{a^2\left(\frac{x^2}{a^2} + 1\right)^{3/2}}$$

For $x \ll a$:

$$F_x = -\frac{2kq^2}{a^3}x$$

That is, the bead experiences a linear restoring force.

(b) Express the period of a simple harmonic oscillator:

$$T = 2\pi\sqrt{\frac{m}{k'}}$$

Obtain k' from our result in Part (a):

$$k' = \frac{2kq^2}{a^3}$$

Substitute for k' and simplify to obtain:

$$T = 2\pi\sqrt{\frac{m}{\frac{2kq^2}{a^3}}} = 2\pi\sqrt{\frac{ma^3}{2kq^2}}$$

Point Charges in Electric Fields

51 • [SSM] The acceleration of a particle in an electric field depends on q/m (the charge-to-mass ratio of the particle). (a) Compute q/m for an electron. (b) What is the magnitude and direction of the acceleration of an electron in a uniform electric field that has a magnitude of 100 N/C? (c) Compute the time it takes for an electron placed at rest in a uniform electric field that has a magnitude of 100 N/C to reach a speed of $0.01c$. (When the speed of an electron approaches the speed of light c , relativistic kinematics must be used to calculate its motion, but at speeds of $0.01c$ or less, non-relativistic kinematics is sufficiently accurate for most purposes.) (d) How far does the electron travel in that time?

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the electron in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of $0.01c$ and the distance it travels while acquiring this speed.

(a) Use data found at the back of your text to compute e/m for an electron:

$$\begin{aligned}\frac{e}{m_e} &= \frac{1.602 \times 10^{-19} \text{ C}}{9.109 \times 10^{-31} \text{ kg}} \\ &= 1.76 \times 10^{11} \text{ C/kg}\end{aligned}$$

(b) Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}$$

Substitute numerical values and evaluate a :

$$\begin{aligned} a &= \frac{(1.602 \times 10^{-19} \text{ C})(100 \text{ N/C})}{9.109 \times 10^{-31} \text{ kg}} \\ &= 1.759 \times 10^{13} \text{ m/s}^2 \\ &= \boxed{1.76 \times 10^{13} \text{ m/s}^2} \end{aligned}$$

The direction of the acceleration of an electron is opposite the electric field.

(c) Using the definition of acceleration, relate the time required for an electron to reach $0.01c$ to its acceleration:

$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt :

$$\begin{aligned} \Delta t &= \frac{0.01(2.998 \times 10^8 \text{ m/s})}{1.759 \times 10^{13} \text{ m/s}^2} = 0.1704 \mu\text{s} \\ &= \boxed{0.2 \mu\text{s}} \end{aligned}$$

(d) Find the distance the electron travels from its average speed and the elapsed time:

$$\Delta x = v_{\text{av}} \Delta t = \frac{1}{2} [0 + 0.01(2.998 \times 10^8 \text{ m/s})] (0.1704 \mu\text{s}) = \boxed{3 \text{ mm}}$$

52 • The acceleration of a particle in an electric field depends on the charge-to-mass ratio of the particle. (a) Compute q/m for a proton, and find its acceleration in a uniform electric field that has a magnitude of 100 N/C . (b) Find the time it takes for a proton initially at rest in such a field to reach a speed of $0.01c$ (where c is the speed of light). (When the speed of an electron approaches the speed of light c , relativistic kinematics must be used to calculate its motion, but at speeds of $0.01c$ or less, non-relativistic kinematics is sufficiently accurate for most purposes.)

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the proton in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of $0.01c$ and the distance it travels while acquiring this speed.

(a) Use data found at the back of your text to compute e/m for an electron:

$$\begin{aligned} \frac{e}{m_p} &= \frac{1.602 \times 10^{-19} \text{ C}}{1.673 \times 10^{-27} \text{ kg}} \\ &= \boxed{9.58 \times 10^7 \text{ C/kg}} \end{aligned}$$

Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a = \frac{F_{\text{net}}}{m_p} = \frac{eE}{m_p}$$

Substitute numerical values and evaluate a :

$$\begin{aligned} a &= \frac{(1.602 \times 10^{-19} \text{ C})(100 \text{ N/C})}{1.673 \times 10^{-27} \text{ kg}} \\ &= 9.576 \times 10^9 \text{ m/s}^2 \\ &= \boxed{9.58 \times 10^9 \text{ m/s}^2} \end{aligned}$$

The direction of the acceleration of a proton is in the direction of the electric field.

(b) Using the definition of acceleration, relate the time required for an electron to reach $0.01c$ to its acceleration:

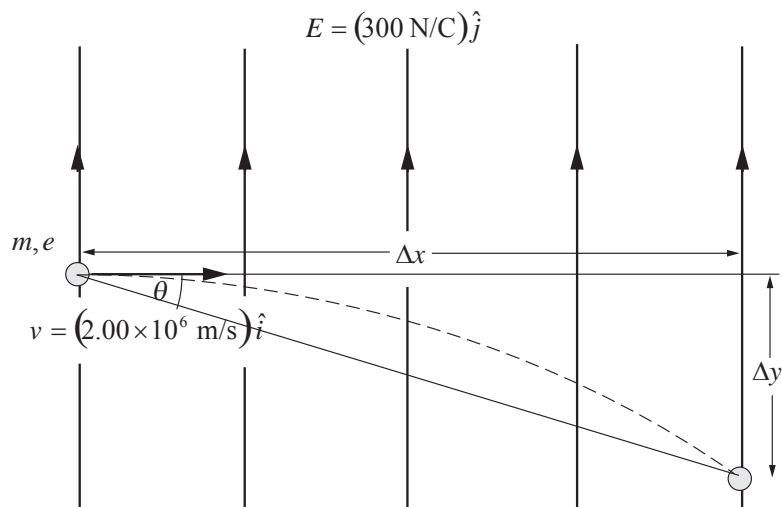
$$\Delta t = \frac{v}{a} = \frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt :

$$\Delta t = \frac{0.01(2.998 \times 10^8 \text{ m/s})}{9.576 \times 10^9 \text{ m/s}^2} = \boxed{0.3 \text{ ms}}$$

53 • An electron has an initial velocity of $2.00 \times 10^6 \text{ m/s}$ in the $+x$ direction. It enters a region that has a uniform electric field $\vec{E} = (300 \text{ N/C})\hat{j}$. (a) Find the acceleration of the electron. (b) How long does it take for the electron to travel 10.0 cm in the x direction in the region that has the field? (c) Through what angle, and in what direction, is the electron deflected while traveling the 10.0 cm in the x direction?

Picture the Problem The electric force acting on the electron is opposite the direction of the electric field. We can apply Newton's 2nd law to find the electron's acceleration and use constant acceleration equations to find how long it takes the electron to travel a given distance and its deflection during this interval of time. Finally, we can use the pictorial representation to obtain an expression for the angle through which the electron is deflected while traveling 10.0 cm in the x direction.



(a) Use Newton's 2nd law to relate the acceleration of the electron first to the net force acting on it and then the electric field in which it finds itself:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m_e} = \frac{-e\vec{E}}{m_e}$$

Substitute numerical values and evaluate \vec{a} :

$$\begin{aligned}\vec{a} &= -\frac{1.602 \times 10^{-19} \text{ C}}{9.109 \times 10^{-31} \text{ kg}} (300 \text{ N/C}) \hat{j} \\ &= (-5.276 \times 10^{13} \text{ m/s}^2) \hat{j} \\ &= \boxed{(-5.28 \times 10^{13} \text{ m/s}^2) \hat{j}}\end{aligned}$$

(b) Relate the time to travel a given distance in the x direction to the electron's speed in the x direction:

$$\Delta t = \frac{\Delta x}{v_x} = \frac{0.100 \text{ m}}{2.00 \times 10^6 \text{ m/s}} = \boxed{50.0 \text{ ns}}$$

(c) The angle through which the electron is deflected as it travels a horizontal distance Δx is given by:

$$\theta = \tan^{-1} \left[\frac{\Delta y}{\Delta x} \right]$$

where Δy is its vertical deflection.

Using a constant-acceleration equation, relate the vertical deflection of the electron to its acceleration and the elapsed time:

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2$$

That is, the electron is deflected 6.60 cm downward.

Substituting for Δy and simplifying yields:

$$\theta = \tan^{-1} \left[\frac{\frac{1}{2} a_y (\Delta t)^2}{\Delta x} \right] = \tan^{-1} \left[\frac{a_y (\Delta t)^2}{2 \Delta x} \right]$$

Substitute numerical values and evaluate θ .

$$\theta = \tan^{-1} \left[\frac{(5.28 \times 10^{13} \text{ m/s}^2)(50.0 \text{ ns})^2}{2(10.0 \text{ cm})} \right]$$

$$= \boxed{33.4^\circ}$$

Because the acceleration of the electron is downward and it was moving horizontally initially, it is deflected downward.

54 •• An electron is released from rest in a weak electric field $\vec{E} = (-1.50 \times 10^{-10} \text{ N/C})\hat{j}$. After the electron has traveled a vertical distance of $1.0 \text{ } \mu\text{m}$, what is its speed? (Do not neglect the gravitational force on the electron.)

Picture the Problem Because the electric field is in the $-y$ direction, the force it exerts on the electron is in the $+y$ direction. Applying Newton's 2nd law to the electron will yield an expression for the acceleration of the electron in the y direction. We can then use a constant-acceleration equation to relate its speed to its acceleration and the distance it has traveled.

Apply $\sum F_y = ma_y$ to the electron to obtain:

$$F_E - F_g = ma_y$$

or, because $F_E = eE$ and $F_g = mg$,

$$eE - mg = ma_y$$

Solving for a_y yields:

$$a_y = \frac{eE}{m} - g$$

Use a constant-acceleration equation to relate the speed of the electron to its acceleration and the distance it travels:

$$v_y^2 = v_0^2 + 2a_y \Delta y$$

or, because the electron starts at rest,

$$v_y^2 = 2a_y \Delta y \Rightarrow v_y = \sqrt{2a_y \Delta y}$$

Substitute for a_y in the expression for v_y :

$$v_y = \sqrt{2 \left(\frac{eE}{m} - g \right) \Delta y}$$

Substitute numerical values and evaluate v_y :

$$v_y = \sqrt{2 \left[\frac{(1.602 \times 10^{-19} \text{ C})(1.50 \times 10^{-10} \text{ N/C})}{9.109 \times 10^{-31} \text{ kg}} - 9.81 \text{ m/s}^2 \right] (1.0 \times 10^{-6} \text{ m})}$$

$$= \boxed{5.8 \text{ mm/s}}$$

55 •• A 2.00-g charged particle is released from rest in a region that has a uniform electric field $\vec{E} = (300 \text{ N/C})\hat{i}$. After traveling a distance of 0.500 m in this region, the particle has a kinetic energy of 0.120 J. Determine the charge of the particle.

Picture the Problem We can apply the work-kinetic energy theorem to relate the change in the object's kinetic energy to the net force acting on it. We can express the net force acting on the charged body in terms of its charge and the electric field.

Using the work-kinetic energy theorem, express the kinetic energy of the object in terms of the net force acting on it and its displacement:

$$W = \Delta K = F_{\text{net}} \Delta x$$

Relate the net force acting on the charged particle to the electric field:

$$F_{\text{net}} = qE$$

Substitute for F_{net} to obtain:

$$\Delta K = K_f - K_i = qE\Delta x$$

or, because $K_i = 0$,

$$K_f = qE\Delta x \Rightarrow q = \frac{K_f}{E\Delta x}$$

Substitute numerical values and evaluate q :

$$q = \frac{0.120 \text{ J}}{(300 \text{ N/C})(0.500 \text{ m})} = \boxed{800 \mu\text{C}}$$

56 •• A charged particle leaves the origin with a speed of $3.00 \times 10^6 \text{ m/s}$ at an angle of 35° above the x axis. A uniform electric field given by $\vec{E} = -E_0\hat{j}$ exists throughout the region. Find E_0 such that the particle will cross the x axis at $x = 1.50 \text{ cm}$ if the particle is (a) an electron, and (b) a proton.

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of the particle in terms of the parameter t and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for y as a function of x , q , and m that we can solve for E_y .

Express the x and y coordinates of the particle as functions of time:

$$x = (v \cos \theta)t$$

and

$$y = (v \sin \theta)t - \frac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the particle to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m} = \frac{qE_y}{m}$$

Substitute in the y-coordinate equation to obtain:

$$y = (v \sin \theta)t - \frac{qE_y}{2m}t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y = (\tan \theta)x - \frac{qE_y}{2mv^2 \cos^2 \theta}x^2$$

Set $y = 0$ and solve for E_y to obtain:

$$E_y = \frac{mv^2 \sin 2\theta}{qx}$$

Substitute the non-particle-specific data to obtain:

$$\begin{aligned} E_y &= \frac{m(3.00 \times 10^6 \text{ m/s})^2 \sin 70^\circ}{q(0.0150 \text{ m})} \\ &= (5.638 \times 10^{14} \text{ m/s}^2) \frac{m}{q} \end{aligned}$$

(a) Substitute for the mass and charge of an electron and evaluate E_y :

$$\begin{aligned} E_y &= (5.638 \times 10^{14} \text{ m/s}^2) \frac{9.109 \times 10^{-31} \text{ kg}}{1.602 \times 10^{-19} \text{ C}} \\ &= \boxed{3.2 \text{ kN/C}} \end{aligned}$$

(b) Substitute for the mass and charge of a proton and evaluate E_y :

$$\begin{aligned} E_y &= (5.64 \times 10^{14} \text{ m/s}^2) \frac{1.673 \times 10^{-27} \text{ kg}}{1.602 \times 10^{-19} \text{ C}} \\ &= \boxed{5.9 \text{ MN/C}} \end{aligned}$$

57 • [SSM] An electron starts at the position shown in Figure 21-39 with an initial speed $v_0 = 5.00 \times 10^6 \text{ m/s}$ at 45° to the x axis. The electric field is in the $+y$ direction and has a magnitude of $3.50 \times 10^3 \text{ N/C}$. The black lines in the figure are charged metal plates. On which plate and at what location will the electron strike?

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of the electron in terms of the parameter t and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for y as a function of x , q , and m . We can decide whether the electron will strike the upper plate by finding the maximum value of its y coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting $y(x) = 0$. Ignore any effects of gravitational forces.

Express the x and y coordinates of the electron as functions of time:

$$x = (v_0 \cos \theta)t$$

and

$$y = (v_0 \sin \theta)t - \frac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the electron to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}$$

Substitute in the y -coordinate equation to obtain:

$$y = (v_0 \sin \theta)t - \frac{eE_y}{2m_e}t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y(x) = (\tan \theta)x - \frac{eE_y}{2m_e v_0^2 \cos^2 \theta} x^2 \quad (1)$$

To find y_{max} , set $dy/dx = 0$ for extrema:

$$\begin{aligned} \frac{dy}{dx} &= \tan \theta - \frac{eE_y}{m_e v_0^2 \cos^2 \theta} x' \\ &= 0 \text{ for extrema} \end{aligned}$$

Solve for x' to obtain:

$$x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad (\text{See remark below.})$$

Substitute x' in $y(x)$ and simplify to obtain y_{max} :

$$y_{\text{max}} = \frac{m_e v_0^2 \sin^2 \theta}{2eE_y}$$

Substitute numerical values and evaluate y_{max} :

$$y_{\text{max}} = \frac{(9.109 \times 10^{-31} \text{ kg})(5.00 \times 10^6 \text{ m/s})^2 \sin^2 45^\circ}{2(1.602 \times 10^{-19} \text{ C})(3.50 \times 10^3 \text{ N/C})} = 1.02 \text{ cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for x to obtain:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Substitute numerical values and evaluate x :

$$x = \frac{(9.109 \times 10^{-31} \text{ kg})(5.00 \times 10^6 \text{ m/s})^2 \sin 90^\circ}{(1.602 \times 10^{-19} \text{ C})(3.50 \times 10^3 \text{ N/C})} = \boxed{4.1 \text{ cm}}$$

Remarks: x' is an extremum, that is, either a maximum or a minimum. To show that it is a maximum we need to show that d^2y/dx^2 , evaluated at x' , is negative. A simple alternative is to use your graphing calculator to show that the graph of $y(x)$ is a maximum at x' . Yet another alternative is to recognize that, because equation (1) is quadratic and the coefficient of x^2 is negative, its graph is a parabola that opens downward.

58 •• An electron that has a kinetic energy equal to $2.00 \times 10^{-16} \text{ J}$ is moving to the right along the axis of a cathode-ray tube as shown in Figure 21-41. An electric field $\vec{E} = (2.00 \times 10^4 \text{ N/C})\hat{j}$ exists in the region between the deflection plates; and no electric field ($\vec{E} = 0$) exists outside this region. (a) How far is the electron from the axis of the tube when it exits the region between the plates? (b) At what angle is the electron moving, with respect to the axis, after exiting the region between the plates? (c) At what distance from the axis will the electron strike the fluorescent screen?

Picture the Problem The trajectory of the electron while it is in the electric field is parabolic (its acceleration is downward and constant) and its trajectory, once it is out of the electric field is, if we ignore the small gravitational force acting on it, linear. We can use constant-acceleration equations and Newton's 2nd law to express the electron's x and y coordinates parametrically and then eliminate the parameter t to express $y(x)$. We can find the angle with the horizontal at which the electron leaves the electric field from the x and y components of its velocity and its total vertical deflection by summing its deflections over the first 4 cm and the final 12 cm of its flight.

(a) Using a constant-acceleration equation, express the x and y coordinates of the electron as functions of time:

$$\begin{aligned} x(t) &= v_0 t \\ \text{and} \\ y(t) &= v_{0,y} t + \frac{1}{2} a_y t^2 \end{aligned}$$

Because $v_{0,y} = 0$:

$$\begin{aligned} x(t) &= v_0 t \\ \text{and} \\ y(t) &= \frac{1}{2} a_y t^2 \end{aligned} \tag{1}$$

Using Newton's 2nd law, relate the acceleration of the electron to the electric field:

$$a_y = \frac{F_{\text{net}}}{m_e} = \frac{-eE_y}{m_e}$$

Substitute to obtain:

$$y(t) = -\frac{eE_y}{2m_e} t^2 \quad (2)$$

Eliminate the parameter t between equations (1) and (2) to obtain:

$$y(x) = -\frac{eE_y}{2m_e v_0^2} x^2 = -\frac{eE_y}{4K} x^2$$

Substitute numerical values and evaluate $y(4 \text{ cm})$:

$$y(0.04 \text{ m}) = -\frac{(1.602 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ N/C})(0.0400 \text{ m})^2}{4(2.00 \times 10^{-16} \text{ J})} = \boxed{-6.40 \text{ mm}}$$

(b) Express the horizontal and vertical components of the electron's speed as it leaves the electric field:

$$v_x = v_0 \cos \theta$$

and

$$v_y = v_0 \sin \theta$$

Divide the second of these equations by the first to obtain:

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{v_y}{v_0} \right)$$

Using a constant-acceleration equation, express v_y as a function of the electron's acceleration and its time in the electric field:

$$v_y = v_{0,y} + a_y t$$

or, because $v_{0,y} = 0$

$$v_y = a_y t = \frac{F_{\text{net},y}}{m_e} t = -\frac{eE_y}{m_e} \frac{x}{v_0}$$

Substitute to obtain:

$$\theta = \tan^{-1} \left(-\frac{eE_y x}{m_e v_0^2} \right) = \tan^{-1} \left(-\frac{eE_y x}{2K} \right)$$

Substitute numerical values and evaluate θ :

$$\theta = \tan^{-1} \left[-\frac{(1.602 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ N/C})(0.0400 \text{ m})}{2(2.00 \times 10^{-16} \text{ J})} \right] = \boxed{-17.7^\circ}$$

(c) Express the total vertical displacement of the electron:

$$y_{\text{total}} = y_{4 \text{ cm}} + y_{12 \text{ cm}}$$

Relate the horizontal and vertical distances traveled to the screen to the horizontal and vertical components of its velocity:

$$x = v_x \Delta t$$

and

$$y = v_y \Delta t$$

Eliminate Δt from these equations to obtain:

$$y = \frac{v_y}{v_x} x = (\tan \theta) x$$

Substitute numerical values and evaluate y :

$$y = [\tan(-17.7^\circ)](0.120 \text{ m}) = -3.83 \text{ cm}$$

Substitute for $y_{4 \text{ cm}}$ and $y_{12 \text{ cm}}$ and evaluate y_{total} :

$$y_{\text{total}} = -0.640 \text{ cm} - 3.83 \text{ cm}$$

$$= \boxed{-4.47 \text{ cm}}$$

That is, the electron will strike the fluorescent screen 4.47 cm below the horizontal axis.

Dipoles

59 • Two point charges, $q_1 = 2.0 \text{ pC}$ and $q_2 = -2.0 \text{ pC}$, are separated by $4.0 \text{ }\mu\text{m}$. (a) What is the magnitude of the dipole moment of this pair of charges? (b) Sketch the pair and show the direction of the dipole moment.

Picture the Problem We can use its definition to find the dipole moment of this pair of charges.

(a) Apply the definition of electric dipole moment to obtain:

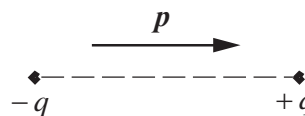
$$\vec{p} = q\vec{L}$$

and

$$p = (2.0 \text{ pC})(4.0 \text{ }\mu\text{m})$$

$$= \boxed{8.0 \times 10^{-18} \text{ C} \cdot \text{m}}$$

(b) If we assume that the dipole is oriented as shown to the right, then \vec{p} is to the right; pointing from the negative charge toward the positive charge.



60 • A dipole of moment $0.50 \text{ e} \cdot \text{nm}$ is placed in a uniform electric field that has a magnitude of $4.0 \times 10^4 \text{ N/C}$. What is the magnitude of the torque on the dipole when (a) the dipole is aligned with the electric field, (b) the dipole is transverse to (perpendicular to) the electric field, and (c) the dipole makes an

angle of 30° with the direction of the electric field? (d) Defining the potential energy to be zero when the dipole is transverse to the electric field, find the potential energy of the dipole for the orientations specified in Parts (a) and (c).

Picture the Problem The torque on an electric dipole in an electric field is given by $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy of the dipole by $U = -\vec{p} \cdot \vec{E}$.

Using its definition, express the torque on a dipole moment in a uniform electric field:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

and

$\tau = pE \sin \theta$ where θ is the angle between the electric dipole moment and the electric field.

(a) Evaluate τ for $\theta = 0^\circ$:

$$\tau(0^\circ) = pE \sin 0^\circ = \boxed{0}$$

(b) Evaluate τ for $\theta = 90^\circ$:

$$\tau(90^\circ) = (0.50 \text{ e} \cdot \text{nm})(4.0 \times 10^4 \text{ N/C}) \sin 90^\circ = \boxed{3.2 \times 10^{-24} \text{ N} \cdot \text{m}}$$

(c) Evaluate τ for $\theta = 30^\circ$:

$$\tau(30^\circ) = (0.50 \text{ e} \cdot \text{nm})(4.0 \times 10^4 \text{ N/C}) \sin 30^\circ = \boxed{1.6 \times 10^{-24} \text{ N} \cdot \text{m}}$$

(d) Using its definition, express the potential energy of a dipole in an electric field:

$$U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$$

Evaluate U for $\theta = 0^\circ$:

$$U(0^\circ) = -(0.50 \text{ e} \cdot \text{nm})(4.0 \times 10^4 \text{ N/C}) \cos 0^\circ = \boxed{-3.2 \times 10^{-24} \text{ J}}$$

Evaluate U for $\theta = 30^\circ$:

$$U(30^\circ) = -(0.50 \text{ e} \cdot \text{nm})(4.0 \times 10^4 \text{ N/C}) \cos 30^\circ = \boxed{-2.8 \times 10^{-24} \text{ J}}$$

General Problems

61 • [SSM] Show that it is only possible to place one isolated proton in an ordinary empty coffee cup by considering the following situation. Assume the

first proton is fixed at the bottom of the cup. Determine the distance directly above this proton where a second proton would be in equilibrium. Compare this distance to the depth of an ordinary coffee cup to complete the argument.

Picture the Problem Equilibrium of the second proton requires that the sum of the electric and gravitational forces acting on it be zero. Let the upward direction be the $+y$ direction and apply the condition for equilibrium to the second proton.

Apply $\sum F_y = 0$ to the second proton:

$$\vec{F}_e + \vec{F}_g = 0$$

or

$$\frac{kq_p^2}{h^2} - m_p g = 0 \Rightarrow h = \sqrt{\frac{kq_p^2}{m_p g}}$$

Substitute numerical values and evaluate h :

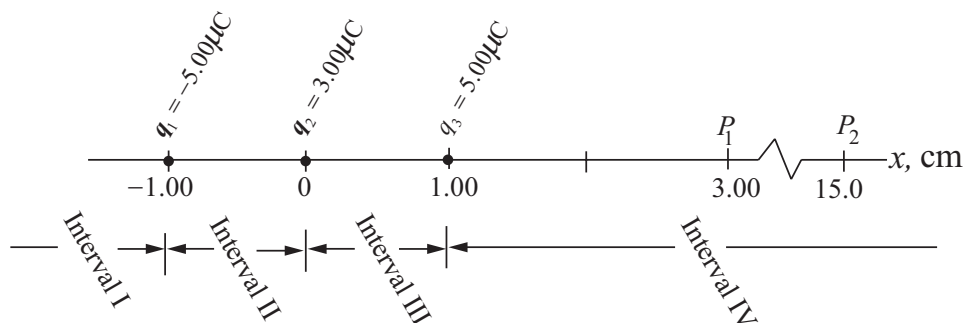
$$h = \sqrt{\frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) (1.602 \times 10^{-19} \text{ C})^2}{(1.673 \times 10^{-27} \text{ kg})(9.81 \text{ m/s}^2)}} \approx 12 \text{ cm} \approx 5 \text{ in}$$

This separation of about 5 in is greater than the height of a typical coffee cup. Thus the first proton will repel the second one out of the cup and the maximum number of protons in the cup is one.

62 •• Point charges of $-5.00 \mu\text{C}$, $+3.00 \mu\text{C}$, and $+5.00 \mu\text{C}$ are located on the x axis at $x = -1.00 \text{ cm}$, $x = 0$, and $x = +1.00 \text{ cm}$, respectively. Calculate the electric field on the x axis at $x = 3.00 \text{ cm}$ and at $x = 15.0 \text{ cm}$. Are there any points on the x axis where the magnitude of the electric field is zero? If so, where are those points?

Picture the Problem The locations of the point charges q_1 , q_2 and q_3 and the points P_1 and P_2 at which we are to calculate the electric field are shown in the diagram. From the diagram it is evident that \vec{E} along the axis has no y component. (a) We can use Coulomb's law for \vec{E} due to a point charge and the superposition principle for electric fields to find \vec{E} at points P_1 and P_2 . (b) To decide whether there are any points on the x axis where the magnitude of the electric field is zero we need to consider the intervals (identified on the following pictorial representation) I ($x < -1.00 \text{ cm}$), II ($-1.00 \text{ cm} < x < 0$), III ($0 < x < 1.00 \text{ cm}$), and IV ($1.00 \text{ cm} < x$). Throughout interval II, the three electric fields are in the same direction and so they cannot add up to zero. Throughout interval IV, the electric-field strength due to the positive charge at $x = 1.00 \text{ cm}$ is larger than the electric-field strength due to the negative charge at $x = -1.00 \text{ cm}$. The fields due

to the positive charges at $x = 0$ and $x = 1.00$ cm both point in the $+x$ direction, so the resultant field throughout this interval must also point in the $+x$ direction. Hence we can narrow our search for points on the x axis where the magnitude of the electric field is zero to intervals I and III. Setting the resultant electric-field strength on the x axis equal to zero throughout these intervals and solving the resulting quadratic equations will identify the desired points. Sketching the electric field vectors in intervals I and III will help you get the signs of the terms in the quadratic equations right.



Using Coulomb's law, and letting r_{11} , r_{21} , and r_{31} represent the distances from P_1 to point charges q_1 , q_2 , and q_3 , respectively, express the electric field at P_1 due to the three charges:

$$\begin{aligned}\vec{E}_{P_1} &= \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} \\ &= \frac{kq_1}{r_{11}^2} \hat{i} + \frac{kq_2}{r_{21}^2} \hat{i} + \frac{kq_3}{r_{31}^2} \hat{i} \\ &= k \left[\frac{q_1}{r_{11}^2} + \frac{q_2}{r_{21}^2} + \frac{q_3}{r_{31}^2} \right] \hat{i}\end{aligned}$$

Substitute numerical values and evaluate \vec{E}_{P_1} :

$$\begin{aligned}\vec{E}_{P_1} &= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[\frac{-5.00 \mu\text{C}}{(4.00 \text{ cm})^2} + \frac{3.00 \mu\text{C}}{(3.00 \text{ cm})^2} + \frac{5.00 \mu\text{C}}{(2.00 \text{ cm})^2} \right] \hat{i} \\ &= \boxed{(1.14 \times 10^8 \text{ N/C}) \hat{i}}\end{aligned}$$

Using Coulomb's law, and letting r_{12} , r_{22} , and r_{32} represent the distances from P_2 to point charges q_1 , q_2 , and q_3 , respectively, express the electric field at P_2 due to the three charges:

$$\begin{aligned}\vec{E}_{P_2} &= \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} \\ &= k \left[\frac{q_1}{r_{12}^2} + \frac{q_2}{r_{22}^2} + \frac{q_3}{r_{32}^2} \right] \hat{i}\end{aligned}$$

Substitute numerical values and evaluate \vec{E}_{P_2} :

$$\begin{aligned}\vec{E}_{P_2} &= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[\frac{-5.00 \mu\text{C}}{(16.0 \text{ cm})^2} + \frac{3.00 \mu\text{C}}{(15.0 \text{ cm})^2} + \frac{5.00 \mu\text{C}}{(14.0 \text{ cm})^2} \right] \hat{i} \\ &= \boxed{(1.74 \times 10^6 \text{ N/C}) \hat{i}}\end{aligned}$$

Applying Coulomb's law for electric fields and the superposition of fields in interval I yields:

$$\frac{5.00 \mu\text{C}}{[x - (-1.00 \text{ cm})]^2} - \frac{3.00 \mu\text{C}}{x^2} - \frac{5.00 \mu\text{C}}{(x - 1.00 \text{ cm})^2} = 0$$

where k has been divided out and x is in the interval defined by $x < 0$. The root of this equation that is in the interval $x < -1.00 \text{ cm}$ is $x = \boxed{-6.95 \text{ cm}}$.

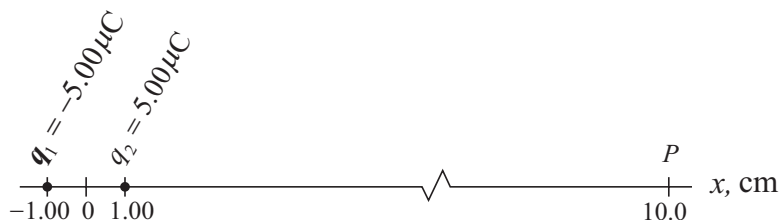
Applying Coulomb's law for electric fields and the superposition of fields in interval III yields:

$$-\frac{5.00 \mu\text{C}}{[x - (-1.00 \text{ cm})]^2} + \frac{3.00 \mu\text{C}}{x^2} - \frac{5.00 \mu\text{C}}{(1.00 \text{ cm} - x)^2} = 0$$

The root of this equation that is in the interval $0 < x < 1.00 \text{ cm}$ is $x = \boxed{0.417 \text{ cm}}$

63 •• Point charges of $-5.00 \mu\text{C}$ and $+5.00 \mu\text{C}$ are located on the x axis at $x = -1.00 \text{ cm}$ and $x = +1.00 \text{ cm}$, respectively. (a) Calculate the electric field strength at $x = 10.0 \text{ cm}$. (b) Estimate the electric field strength at $x = 10.00 \text{ cm}$ by modeling the two charges as an electric dipole located at the origin and using $E = 2kp/|x|^3$ (Equation 21-10). Compare your result with the result obtained in Part (a), and explain the reason for the difference between the two results.

Picture the Problem Let the point of interest ($x = 10.0 \text{ cm}$) be identified as point P . In Part (a) we can use Coulomb's law for \vec{E} due to a point charge and the superposition principle to find the electric field strength at P . In Part (b) we can use Equation 21-10 with $p = 2aq$ to estimate the electric field strength at P .



(a) Express the electric field at P as the sum of the fields due to the point charges located at $x = -1.00$ cm and $x = 1.00$ cm:

$$\begin{aligned}\vec{E}_P &= \vec{E}_{q_1} + \vec{E}_{q_2} \\ &= \frac{kq_1}{x_{-1.00\text{ cm} \rightarrow P}^2} \hat{i} + \frac{kq_2}{x_{1.00\text{ cm} \rightarrow P}^2} \hat{i} \\ &= k \left[\frac{q_1}{x_{-1.00\text{ cm} \rightarrow P}^2} + \frac{q_2}{x_{1.00\text{ cm} \rightarrow P}^2} \right] \hat{i}\end{aligned}$$

Substitute numerical values and evaluate \vec{E}_P :

$$\vec{E}_P = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left[\frac{-5.00 \mu\text{C}}{(11.00 \text{ cm})^2} + \frac{5.00 \mu\text{C}}{(9.00 \text{ cm})^2} \right] \hat{i} = (1.83 \times 10^6 \text{ N/C}) \hat{i}$$

and

$$E_P = \boxed{1.83 \times 10^6 \text{ N/C}}$$

(b) The electric field strength due to a dipole is given by Equation 21-10:

$$E = \frac{2kp}{|x|^3}$$

Because $p = 2qa$, where $2a$ is the separation of the charges that constitute the dipole, E is also given by:

$$E = \frac{4kqa}{|x|^3}$$

Substitute numerical values and evaluate E_P :

$$E_P = \frac{4 \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (5.00 \mu\text{C}) (1.00 \text{ cm})}{|10.0 \text{ cm}|^3} = \boxed{1.80 \times 10^6 \text{ N/C}}$$

The exact and estimated values of E_P agree to within 2%. This difference is due to the fact that the separation of the two charges of the dipole is 20% of the distance from the center of the dipole to point P .

64 •• A fixed point charge of $+2q$ is connected by strings to point charges of $+q$ and $+4q$, as shown in Figure 21-41. Find the tensions T_1 and T_2 .

Picture the Problem The electrostatic forces between the charges are responsible for the tensions in the strings. We'll assume that these are point charges and apply Coulomb's law and the principle of the superposition of forces to find the tension in each string.

Use Coulomb's law to express the net force on the charge $+q$:

$$T_1 = F_{2q} + F_{4q}$$

Substitute and simplify to obtain:

$$T_1 = \frac{kq(2q)}{d^2} + \frac{kq(4q)}{(2d)^2} = \boxed{\frac{3kq^2}{d^2}}$$

Use Coulomb's law to express the net force on the charge $+4q$:

$$T_2 = F_q + F_{2q}$$

Substitute and simplify to obtain:

$$T_2 = \frac{k(2q)(4q)}{d^2} + \frac{kq(4q)}{(2d)^2} = \boxed{\frac{9kq^2}{d^2}}$$

65 •• [SSM] A positive charge Q is to be divided into two positive point charges q_1 and q_2 . Show that, for a given separation D , the force exerted by one charge on the other is greatest if $q_1 = q_2 = \frac{1}{2}Q$.

Picture the Problem We can use Coulomb's law to express the force exerted on one charge by the other and then set the derivative of this expression equal to zero to find the distribution of the charge that maximizes this force.

Using Coulomb's law, express the force that either charge exerts on the other:

$$F = \frac{kq_1q_2}{D^2}$$

Express q_2 in terms of Q and q_1 :

$$q_2 = Q - q_1$$

Substitute for q_2 to obtain:

$$F = \frac{kq_1(Q - q_1)}{D^2}$$

Differentiate F with respect to q_1 and set this derivative equal to zero for extreme values:

$$\begin{aligned}\frac{dF}{dq_1} &= \frac{k}{D^2} \frac{d}{dq_1} [q_1(Q - q_1)] \\ &= \frac{k}{D^2} [q_1(-1) + Q - q_1] \\ &= 0 \text{ for extrema}\end{aligned}$$

Solve for q_1 to obtain:

$$q_1 = \frac{1}{2}Q \Rightarrow q_2 = Q - q_1 = \frac{1}{2}Q$$

To determine whether a maximum or a minimum exists at $q_1 = \frac{1}{2}Q$, differentiate F a second time and evaluate this derivative at $q_1 = \frac{1}{2}Q$:

$$\begin{aligned}\frac{d^2F}{dq_1^2} &= \frac{k}{D^2} \frac{d}{dq_1} [Q - 2q_1] \\ &= \frac{k}{D^2} (-2) \\ &< 0 \text{ independently of } q_1.\end{aligned}$$

$\therefore q_1 = q_2 = \frac{1}{2}Q \text{ maximizes } F.$

66 •• A point charge Q is located on the x axis at $x = 0$, and a point charge $4Q$ is located at $x = 12.0$ cm. The electric force on a point charge of $-2.00 \mu\text{C}$ is zero if that charge is placed at $x = 4.00$ cm, and is 126 N in the $+x$ direction if placed at $x = 8.00$ cm. Determine the charge Q .

Picture the Problem We can apply Coulomb's law and the superposition of forces to relate the net force acting on the charge $q = -2.00 \mu\text{C}$ to x . Because Q divides out of our equation when $F(x) = 0$, we'll substitute the data given for $x = 8.00$ cm.

Using Coulomb's law, express the net electric force on q as a function of x :

$$F(x) = -\frac{kqQ}{x^2} + \frac{kq(4Q)}{(12.0 \text{ cm} - x)^2}$$

Solving for Q yields:

$$Q = \frac{F(x)}{kq \left[-\frac{1}{x^2} + \frac{4}{(12.0 \text{ cm} - x)^2} \right]}$$

Evaluate Q for $x = 8.00$ cm:

$$Q = \frac{126 \text{ N}}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.00 \mu\text{C}) \left[-\frac{1}{(8.00 \text{ cm})^2} + \frac{4}{(4.00 \text{ cm})^2} \right]} = \boxed{2.99 \mu\text{C}}$$

67 •• Two point particles separated by 0.60 m carry a total charge of $200\ \mu\text{C}$. (a) If the two particles repel each other with a force of 80 N, what is the charge on each of the two particles? (b) If the two spheres attract each other with a force of 80 N, what are the charges on the two particles?

Picture the Problem Knowing the total charge of the two point particles, we can use Coulomb's law to find the two combinations of charge that will satisfy the condition that both are positive and hence repel each other. If the particles attract each other, then there is just one distribution of charge that will satisfy the conditions that the force is attractive and the sum of the two charges is $200\ \mu\text{C}$.

(a) Use Coulomb's law to express the repulsive electric force each particle exerts on the other:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1 :

$$q_2 = Q - q_1$$

Substitute for q_2 to obtain:

$$F = \frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$80\text{ N} = \frac{(8.988 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)[(200\ \mu\text{C})q_1 - q_1^2]}{(0.60\text{ m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-0.200\text{ mC})q_1 + 3.20 \times 10^{-3}(\text{mC})^2 = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$q_1 = 1.8 \times 10^{-5}\text{ C and } q_2 = 1.8 \times 10^{-4}\ \mu\text{C}$$

Thus the charges are:

$$\boxed{1.8 \times 10^{-5}\text{ C}} \text{ and } \boxed{1.8 \times 10^{-4}\text{ C}}$$

(b) Use Coulomb's law to express the attractive electric force each particle exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Proceed as in (a) to obtain:

$$q_1^2 + (-0.200 \text{ mC})q_1 - 3.20 \times 10^{-3} (\text{mC})^2 = 0$$

Solve this quadratic equation to obtain:

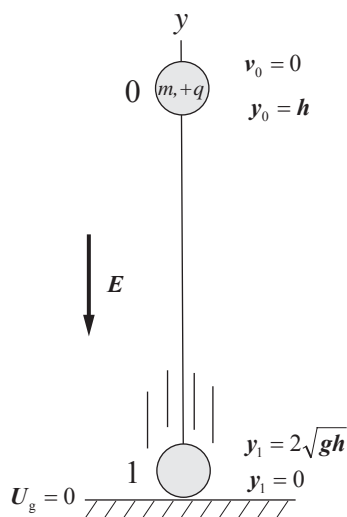
$$q_1 = -1.4 \times 10^{-5} \text{ C and } q_2 = 2.1 \times 10^{-4} \text{ C}$$

Thus the charges are:

$$\boxed{-1.4 \times 10^{-5} \text{ C}} \text{ and } \boxed{2.1 \times 10^{-4} \text{ C}}$$

68 •• A point particle that has charge $+q$ and unknown mass m is released from rest in a region that has a uniform electric field \vec{E} that is directed vertically downward. The particle hits the ground at a speed $v = 2\sqrt{gh}$, where h is the initial height of the particle. Find m in terms of E , q , and g .

Picture the Problem Choose the coordinate system shown in the diagram and let $U_g = 0$ where $y = 0$. We'll let our system include the point particle and Earth. Then the work done on the point particle by the electric field will change the energy of the system. The diagram summarizes what we know about the motion of the point particle. We can apply the work-energy theorem to our system to relate the work done by the electric field to the change in its energy.



Using the work-energy theorem, relate the work done by the electric field to the change in the energy of the system:

$$\begin{aligned} W_{\text{electric field}} &= \Delta K + \Delta U_g \\ &= K_1 - K_0 + U_{g,1} - U_{g,0} \end{aligned}$$

or, because $K_1 = U_{g,1} = 0$,

$$W_{\text{electric field}} = K_1 - U_{g,1}$$

Substitute for $W_{\text{electric field}}$, K_1 and $U_{g,1}$

and simplify to obtain:

$$\begin{aligned} qEh &= \frac{1}{2}mv_1^2 - mgh \\ &= \frac{1}{2}m(2\sqrt{gh})^2 - mgh = mgh \end{aligned}$$

Solving for m yields:

$$m = \boxed{\frac{qE}{g}}$$

69 • [SSM] A rigid 1.00-m-long rod is pivoted about its center (Figure 21-42). A charge $q_1 = 5.00 \times 10^{-7} \text{ C}$ is placed on one end of the rod, and a charge $q_2 = -q_1$ is placed a distance $d = 10.0 \text{ cm}$ directly below it. (a) What is the force exerted by q_2 on q_1 ? (b) What is the torque (measured about the rotation axis) due to that force? (c) To counterbalance the attraction between the two charges, we hang a block 25.0 cm from the pivot as shown. What value should we choose for the mass of the block? (d) We now move the block and hang it a distance of 25.0 cm from the balance point, on the same side of the balance as the charge. Keeping q_1 the same, and d the same, what value should we choose for q_2 to keep this apparatus in balance?

Picture the Problem We can use Coulomb's law, the definition of torque, and the condition for rotational equilibrium to find the electrostatic force between the two charged bodies, the torque this force produces about an axis through the center of the rod, and the mass required to maintain equilibrium when it is located either 25.0 cm to the right or to the left of the mid-point of the rod.

(a) Using Coulomb's law, express the electric force between the two charges:

$$F = \frac{kq_1q_2}{d^2}$$

Substitute numerical values and evaluate F :

$$F = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-7} \text{ C})^2}{(0.100 \text{ m})^2} = 0.2247 \text{ N} = \boxed{0.225 \text{ N}}$$

(b) Apply the definition of torque to obtain:

$$\tau = F\ell$$

Substitute numerical values and evaluate τ .

$$\begin{aligned} \tau &= (0.2247 \text{ N})(0.500 \text{ m}) \\ &= 0.1124 \text{ N} \cdot \text{m} = \boxed{0.112 \text{ N} \cdot \text{m}}, \end{aligned}$$

counterclockwise.

(c) Apply $\sum \tau_{\text{center of the rod}} = 0$ to the rod:

$$\tau - mg\ell' = 0 \Rightarrow m = \frac{\tau}{g\ell'}$$

Substitute numerical values and evaluate m :

$$\begin{aligned} m &= \frac{0.1124 \text{ N} \cdot \text{m}}{(9.81 \text{ m/s}^2)(0.250 \text{ m})} \\ &= 0.04582 \text{ kg} = \boxed{45.8 \text{ g}} \end{aligned}$$

(d) Apply $\sum \tau_{\text{center of the rod}} = 0$ to the rod:

$$-\tau + mg\ell' = 0$$

Substitute for τ :

$$-F\ell + mg\ell' = 0$$

Substitute for F :

$$-\frac{kq_1q_2'}{d^2} + mg\ell' = 0 \Rightarrow q_2' = \frac{d^2mg\ell'}{kq_1\ell}$$

where q' is the required charge.Substitute numerical values and evaluate q_2' :

$$q_2' = \frac{(0.100\text{ m})^2 (0.04582\text{ kg})(9.81\text{ m/s}^2)(0.250\text{ m})}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-7} \text{ C})(0.500\text{ m})} = \boxed{5.00 \times 10^{-7} \text{ C}}$$

70 •• Two $3.0\text{-}\mu\text{C}$ point charges are located at $x = 0, y = 2.0\text{ m}$ and at $x = 0, y = -2.0\text{ m}$. Two other point charges, each with charge equal to Q , are located at $x = 4.0\text{ m}, y = 2.0\text{ m}$ and at $x = 4.0\text{ m}, y = -2.0\text{ m}$ (Figure 21-43). The electric field at $x = 0, y = 0$ due to the presence of the four charges is $(4.0 \times 10^3 \text{ N/C})\hat{i}$. Determine Q .

Picture the Problem Let the numeral 1 refer to the point charge in the 1st quadrant and the numeral 2 to the point charge in the 4th quadrant. We can use Coulomb's law for the electric field due to a point charge and the superposition of forces to express the field at the origin and use this equation to solve for Q .

Express the electric field at the origin due to the point charges Q :

$$\begin{aligned}\vec{E}(0,0) &= \vec{E}_1 + \vec{E}_2 = \frac{kQ}{r_{1,0}^2} \hat{r}_{1,0} + \frac{kQ}{r_{2,0}^2} \hat{r}_{2,0} \\ &= \frac{kQ}{r^3} [(-4.0\text{ m})\hat{i} + (-2.0\text{ m})\hat{j}] + \frac{kQ}{r^3} [(-4.0\text{ m})\hat{i} + (2.0\text{ m})\hat{j}] = -\frac{(8.0\text{ m})kQ}{r^3} \hat{i} \\ &= E_x \hat{i}\end{aligned}$$

where r is the distance from each charge to the origin and $E_x = -\frac{(8.0\text{ m})kQ}{r^3}$.

Express r in terms of the coordinates (x, y) of the point charges:

$$r = \sqrt{x^2 + y^2}$$

Substitute for r to obtain:

$$E_x = -\frac{(8.0\text{ m})kQ}{(x^2 + y^2)^{3/2}}$$

Solving for Q yields:

$$Q = -\frac{E_x(x^2 + y^2)^{3/2}}{k(8.0\text{ m})}$$

Substitute numerical values and evaluate Q :

$$Q = -\frac{(4.0 \text{ kN/C})[(4.0 \text{ m})^2 + (2.0 \text{ m})^2]^{3/2}}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.0 \text{ m})}$$

$$= \boxed{-5.0 \mu\text{C}}$$

71 •• [SSM] Two point charges have a total charge of $200 \mu\text{C}$ and are separated by 0.600 m . (a) Find the charge of each particle if the particles repel each other with a force of 120 N . (b) Find the force on each particle if the charge on each particle is $100 \mu\text{C}$.

Picture the Problem Let the numeral 1 denote one of the small spheres and the numeral 2 the other. Knowing the total charge on the two spheres, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the spheres carry the same charge and are 0.600 m apart will yield the force each exerts on the other.

(a) Use Coulomb's law to express the repulsive force each charge exerts on the other:

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1 :

$$q_2 = Q - q_1$$

Substitute for q_2 to obtain:

$$F = \frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120 \text{ N} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)[(200 \mu\text{C})q_1 - q_1^2]}{(0.600 \text{ m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200 \mu\text{C})q_1 + 4805 (\mu\text{C})^2 = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$q_1 = 28.0 \mu\text{C} \text{ and } 172 \mu\text{C}$$

Hence the charges on the particles are:

$$\boxed{28.0 \mu\text{C}} \text{ and } \boxed{172 \mu\text{C}}$$

(b) Use Coulomb's law to express the repulsive force each charge exerts on the other when

$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

$q_1 = q_2 = 100 \mu\text{C}$:

Substitute numerical values and evaluate F :

$$F = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(100 \mu\text{C})^2}{(0.600 \text{ m})^2} = \boxed{250 \text{ N}}$$

72 •• Two point charges have a total charge of $200 \mu\text{C}$ and are separated by 0.600 m . (a) Find the charge of each particle if the particles attract each other with a force of 120 N . (b) Find the force on each particle if the charge on each particle is $100 \mu\text{C}$.

Picture the Problem Let the numeral 1 denote one of the small spheres and the numeral 2 the other. Knowing the total charge on the two point charges, we can use Coulomb's law to find the charge on each of them. A second application of Coulomb's law when the particles carry the same charge and are 0.600 m apart will yield the electric force each exerts on the other.

(a) Use Coulomb's law to express the attractive electric force each particle exerts on the other:

$$F = -\frac{kq_1q_2}{r_{1,2}^2}$$

Express q_2 in terms of the total charge and q_1 :

$$q_2 = Q - q_1$$

Substitute for q_2 to obtain:

$$F = -\frac{kq_1(Q - q_1)}{r_{1,2}^2}$$

Substitute numerical values to obtain:

$$120 \text{ N} = \frac{-(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)[(200 \mu\text{C})q_1 - q_1^2]}{(0.600 \text{ m})^2}$$

Simplify to obtain the quadratic equation:

$$q_1^2 + (-200 \mu\text{C})q_1 - 4805(\mu\text{C})^2 = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$q_1 = -21.7 \mu\text{C} \text{ and } q_2 = 222 \mu\text{C}$$

Hence the charges on the particles are:

$$\boxed{-21.7 \mu\text{C}} \text{ and } \boxed{222 \mu\text{C}}$$

(b) Use Coulomb's law to express the repulsive electric force each particle exerts on the other when $q_1 = q_2 = 100 \mu\text{C}$:

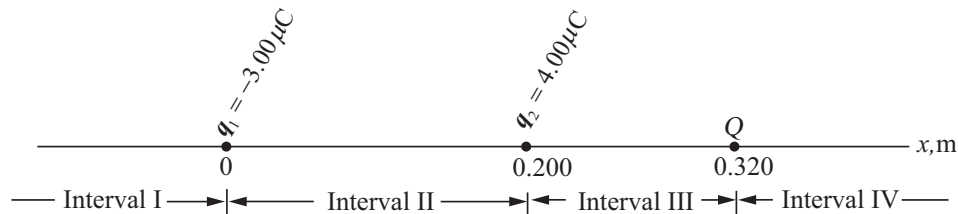
$$F = \frac{kq_1q_2}{r_{1,2}^2}$$

Substitute numerical values and evaluate F :

$$F = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(100 \mu\text{C})^2}{(0.600 \text{ m})^2} = \boxed{250 \text{ N}}$$

73 •• A point charge of $-3.00 \mu\text{C}$ is located at the origin; a point charge of $4.00 \mu\text{C}$ is located on the x axis at $x = 0.200 \text{ m}$; a third point charge Q is located on the x axis at $x = 0.320 \text{ m}$. The electric force on the $4.00\text{-}\mu\text{C}$ charge is 240 N in the $+x$ direction. (a) Determine the charge Q . (b) With this configuration of three charges, at what location(s) is the electric field zero?

Picture the Problem (a) We can use Coulomb's law for point charges and the superposition of forces to express the net electric force acting on q_2 and then solve this equation to determine the charge Q . (b) To identify the location(s) at which the electric field is zero, we'll need to systematically examine the resultant electric fields in the intervals I, II, III, and IV identified below on the pictorial representation. Drawing the electric field vectors in each of these intervals will help you get the signs of the terms in the field equations right.



(a) Use Coulomb's law to express the electric force on the particle whose charge is $4.00\text{-}\mu\text{C}$:

$$\begin{aligned}\vec{F}_2 &= \vec{F}_{1,2} + \vec{F}_{Q,2} \\ &= \frac{kq_1q_2}{r_{1,2}^2} \hat{i} + \frac{kQq_2}{r_{Q,2}^2} (-\hat{i}) \\ &= kq_2 \left[\frac{q_1}{r_{1,2}^2} - \frac{Q}{r_{Q,2}^2} \right] \hat{i} = F_2 \hat{i}\end{aligned}$$

Solving for Q yields:

$$Q = r_{Q,2}^2 \left[\frac{q_1}{r_{1,2}^2} - \frac{F_2}{kq_2} \right]$$

Substitute numerical values and evaluate Q :

$$Q = (0.120\text{m})^2 \left[\frac{-3.00\text{ }\mu\text{C}}{(0.200\text{m})^2} - \frac{240\text{ N}}{(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.00\text{ }\mu\text{C})} \right] = \boxed{-97.2\text{ }\mu\text{C}}$$

(b) Applying Coulomb's law for electric fields and the superposition of fields in interval I yields:

$$\frac{3.00\text{ }\mu\text{C}}{(x-0)^2} - \frac{4.00\text{ }\mu\text{C}}{(x-0.200\text{ m})^2} + \frac{97.2\text{ }\mu\text{C}}{(x-0.320\text{ m})^2} = 0$$

where k has been divided out and x is in the interval defined by $x < 0$. The roots of this equation are at $x = 0.170\text{ m}$ and $x = 0.220\text{ m}$. Neither of these are in interval I.

Applying Coulomb's law for electric fields and the superposition of fields in interval II yields:

$$-\frac{3.00\text{ }\mu\text{C}}{(x-0)^2} - \frac{4.00\text{ }\mu\text{C}}{(x-0.200\text{ m})^2} + \frac{97.2\text{ }\mu\text{C}}{(x-0.320\text{ m})^2} = 0$$

The roots of this equation are at $x = -0.0720\text{ m}$, $x = 0.0508\text{ m}$, $x = 0.169\text{ m}$, and $x = 0.220\text{ m}$. The second and third of these are in interval II.

Applying Coulomb's law for electric fields and the superposition of fields in interval III yields:

$$-\frac{3.00 \mu\text{C}}{(x-0)^2} + \frac{4.00 \mu\text{C}}{(x-0.200 \text{ m})^2} + \frac{97.2 \mu\text{C}}{(x-0.320 \text{ m})^2} = 0$$

The roots of this equation are at $x = -0.0649 \text{ m}$ and $x = 0.0454 \text{ m}$. Neither of these are in interval III.

Applying Coulomb's law for electric fields and the superposition of fields in interval IV yields:

$$-\frac{3.00 \mu\text{C}}{(x-0)^2} + \frac{4.00 \mu\text{C}}{(x-0.200 \text{ m})^2} - \frac{97.2 \mu\text{C}}{(x-0.320 \text{ m})^2} = 0$$

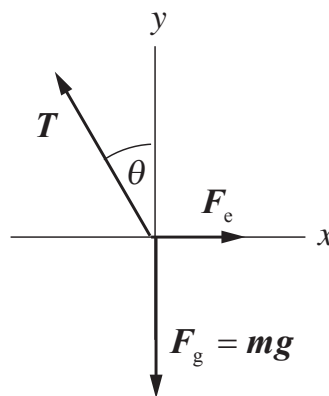
The roots of this equation are at $x = 0.170 \text{ m}$ and $x = 0.220$. Neither of these are in interval IV.

Summarizing, the electric field is zero at two locations in interval II:

$$x = \boxed{0.0508 \text{ m}} \quad \text{and} \quad x = \boxed{0.169 \text{ m}}$$

74 •• Two point particles, each of mass m and charge q , are suspended from a common point by threads of length L . Each thread makes an angle θ with the vertical as shown in Figure 21-44. (a) Show that $q = 2L \sin \theta \sqrt{(mg/k) \tan \theta}$ where k is the Coulomb constant. (b) Find q if $m = 10.0 \text{ g}$, $L = 50.0 \text{ cm}$, and $\theta = 10.0^\circ$.

Picture the Problem Each point particle is in static equilibrium under the influence of the tension \vec{T} , the gravitational force \vec{F}_g , and the electric force \vec{F}_E . We can use Coulomb's law to relate the electric force to the charge on each particle and their separation and the conditions for static equilibrium to relate these forces to the charge on each particle.



(a) Apply the conditions for static equilibrium to the point particle whose free-body diagram is shown above:

$$\sum F_x = F_E - T \sin \theta = \frac{kq^2}{r^2} - T \sin \theta = 0$$

and

$$\sum F_y = T \cos \theta - mg = 0$$

Eliminate T between these equations to obtain:

$$\tan \theta = \frac{kq^2}{mgr^2} \Rightarrow q = r \sqrt{\frac{mg \tan \theta}{k}}$$

Referring to the figure, relate the separation of the spheres r to the length of the pendulum L :

$$r = 2L \sin \theta$$

Substitute for r to obtain:

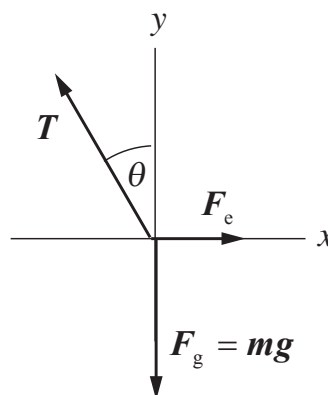
$$q = \boxed{2L \sin \theta \sqrt{\frac{mg \tan \theta}{k}}}$$

(b) Evaluate q for $m = 10.0$ g, $L = 50.0$ cm, and $\theta = 10^\circ$:

$$q = 2(0.500 \text{ m}) \sin 10.0^\circ \sqrt{\frac{(0.0100 \text{ kg})(9.81 \text{ m/s}^2) \tan 10.0^\circ}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = \boxed{0.241 \mu\text{C}}$$

75 •• Suppose that in Problem 74 $L = 1.5$ m and $m = 0.010$ kg. (a) What is the angle that each string makes with the vertical if $q = 0.75 \mu\text{C}$? (b) What is the angle that each string makes with the vertical if one particle has a charge of $0.50 \mu\text{C}$, the other has a charge of $1.0 \mu\text{C}$?

Picture the Problem Each sphere is in static equilibrium under the influence of the tension \vec{T} , the gravitational force \vec{F}_g , and the electric force \vec{F}_e . We can use Coulomb's law to relate the electric force to the charge on each sphere and their separation and the conditions for static equilibrium to relate these forces to the charge on each sphere.



(a) Apply the conditions for static equilibrium to the charged sphere:

$$\sum F_x = F_e - T \sin \theta = \frac{kq^2}{r^2} - T \sin \theta = 0$$

and

$$\sum F_y = T \cos \theta - mg = 0$$

Eliminate T between these equations to obtain:

$$\tan \theta = \frac{kq^2}{mgr^2}$$

Referring to the figure for Problem 74, relate the separation of the spheres r to the length of the pendulum L :

$$r = 2L \sin \theta$$

Substitute for r to obtain:

$$\tan \theta = \frac{kq^2}{4mgL^2 \sin^2 \theta}$$

or

$$\sin^2 \theta \tan \theta = \frac{kq^2}{4mgL^2} \quad (1)$$

Substitute numerical values and evaluate $\sin^2 \theta \tan \theta$:

$$\sin^2 \theta \tan \theta = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.75 \mu\text{C})^2}{4(0.010 \text{ kg})(9.81 \text{ m/s}^2)(1.5 \text{ m})^2} = 5.73 \times 10^{-3}$$

Because $\sin^2 \theta \tan \theta \ll 1$:

$$\sin \theta \approx \tan \theta \approx \theta$$

and

$$\theta^3 \approx 5.73 \times 10^{-3}$$

Solve for θ to obtain:

$$\theta = 0.179 \text{ rad} = \boxed{10^\circ}$$

(b) Evaluate equation (1) with replacing q^2 with $q_1 q_2$:

$$\sin^2 \theta \tan \theta = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.50 \mu\text{C})(1.0 \mu\text{C})}{4(0.010 \text{ kg})(9.81 \text{ m/s}^2)(1.5 \text{ m})^2} = 5.09 \times 10^{-3} \approx \theta^3$$

Solve for θ to obtain:

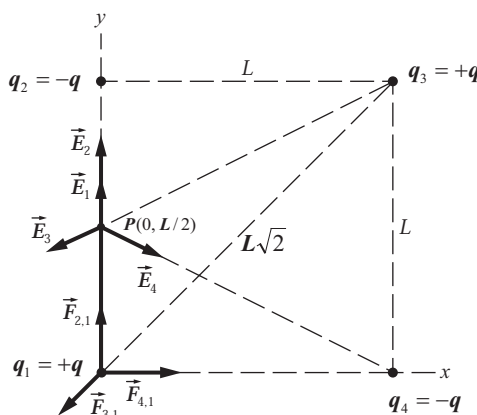
$$\theta = 0.172 \text{ rad} = \boxed{9.9^\circ}$$

76 •• Four point charges of equal magnitude are arranged at the corners of a square of side L as shown in Figure 21-45. (a) Find the magnitude and direction

of the force exerted on the charge in the lower left corner by the other three charges. (b) Show that the electric field at the midpoint of one of the sides of the square is directed along that side toward the negative charge and has a magnitude

$$E \text{ given by } E = k \frac{8q}{L^2} \left(1 - \frac{1}{5\sqrt{5}} \right).$$

Picture the Problem Let the origin be at the lower left-hand corner and designate the point charges as shown in the diagram. We can apply Coulomb's law for point charges to find the forces exerted on q_1 by q_2 , q_3 , and q_4 and superimpose these forces to find the net force exerted on q_1 . In Part (b), we'll use Coulomb's law for the electric field due to a point charge and the superposition of fields to find the electric field at point $P(0, L/2)$.



(a) Using superposition of forces, express the net force exerted on q_1 :

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

Apply Coulomb's law to express $\vec{F}_{2,1}$:

$$\begin{aligned} \vec{F}_{2,1} &= \frac{kq_2q_1}{r_{2,1}^2} \hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^3} \vec{r}_{2,1} \\ &= \frac{k(-q)q}{L^3} (-L\hat{j}) = \frac{kq^2}{L^2} \hat{j} \end{aligned}$$

Apply Coulomb's law to express $\vec{F}_{4,1}$:

$$\begin{aligned} \vec{F}_{4,1} &= \frac{kq_4q_1}{r_{4,1}^2} \hat{r}_{4,1} = \frac{kq_4q_1}{r_{4,1}^3} \vec{r}_{4,1} \\ &= \frac{k(-q)q}{L^3} (-L\hat{i}) = \frac{kq^2}{L^2} \hat{i} \end{aligned}$$

Apply Coulomb's law to express $\vec{F}_{3,1}$:

$$\begin{aligned} \vec{F}_{3,1} &= \frac{kq_3q_1}{r_{3,1}^2} \hat{r}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3} \vec{r}_{3,1} \\ &= \frac{kq^2}{2^{3/2}L^3} (-L\hat{i} - L\hat{j}) \\ &= -\frac{kq^2}{2^{3/2}L^2} (\hat{i} + \hat{j}) \end{aligned}$$

Substitute and simplify to obtain:

$$\begin{aligned}\vec{F}_1 &= \frac{kq^2}{L^2} \hat{j} - \frac{kq^2}{2^{3/2}L^2} (\hat{i} + \hat{j}) + \frac{kq^2}{L^2} \hat{i} \\ &= \frac{kq^2}{L^2} (\hat{i} + \hat{j}) - \frac{kq^2}{2^{3/2}L^2} (\hat{i} + \hat{j}) \\ &= \boxed{\frac{kq^2}{L^2} \left(1 - \frac{1}{2\sqrt{2}}\right) (\hat{i} + \hat{j})}\end{aligned}$$

(b) Using superposition of fields, express the resultant field at point P :

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \quad (1)$$

Use Coulomb's law to express \vec{E}_1 :

$$\begin{aligned}\vec{E}_1 &= \frac{kq_1}{r_{1,P}^2} \hat{r}_{1,P} = \frac{kq}{r_{1,P}^3} \left(\frac{L}{2} \hat{j}\right) \\ &= \frac{kq}{\left(\frac{L}{2}\right)^3} \left(\frac{L}{2} \hat{j}\right) = \frac{4kq}{L^2} \hat{j}\end{aligned}$$

Use Coulomb's law to express \vec{E}_2 :

$$\begin{aligned}\vec{E}_2 &= \frac{kq_2}{r_{2,P}^2} \hat{r}_{2,P} = \frac{k(-q)}{r_{2,P}^3} \left(\frac{L}{2} \hat{j}\right) \\ &= \frac{-kq}{\left(\frac{L}{2}\right)^3} \left(-\frac{L}{2} \hat{j}\right) = \frac{4kq}{L^2} \hat{j}\end{aligned}$$

Use Coulomb's law to express \vec{E}_3 :

$$\begin{aligned}\vec{E}_3 &= \frac{kq_3}{r_{3,P}^2} \hat{r}_{3,P} = \frac{kq}{r_{3,P}^3} \left(-L\hat{i} - \frac{L}{2}\hat{j}\right) \\ &= \frac{8kq}{5^{3/2}L^2} \left(-\hat{i} - \frac{1}{2}\hat{j}\right)\end{aligned}$$

Use Coulomb's law to express \vec{E}_4 :

$$\begin{aligned}\vec{E}_4 &= \frac{kq_4}{r_{4,P}^2} \hat{r}_{4,P} = \frac{k(-q)}{r_{4,P}^3} \left(-L\hat{i} + \frac{L}{2}\hat{j}\right) \\ &= \frac{8kq}{5^{3/2}L^2} \left(\hat{i} - \frac{1}{2}\hat{j}\right)\end{aligned}$$

Substitute in equation (1) and simplify to obtain:

$$\vec{E}_P = \frac{4kq}{L^2} \hat{j} + \frac{4kq}{L^2} \hat{j} + \frac{8kq}{5^{3/2}L^2} \left(-\hat{i} - \frac{1}{2}\hat{j}\right) + \frac{8kq}{5^{3/2}L^2} \left(\hat{i} - \frac{1}{2}\hat{j}\right) = \boxed{\frac{8kq}{L^2} \left(1 - \frac{\sqrt{5}}{25}\right) \hat{j}}$$

77 • [SSM] Figure 21-46 shows a dumbbell consisting of two identical small particles, each of mass m , attached to the ends of a thin (massless) rod of length a that is pivoted at its center. The particles carry charges of $+q$ and $-q$, and the dumbbell is located in a uniform electric field \vec{E} . Show that for small values of the angle θ between the direction of the dipole and the direction of the electric field, the system displays a rotational form of simple harmonic motion, and obtain an expression for the period of that motion.

Picture the Problem We can apply Newton's 2nd law in rotational form to obtain the differential equation of motion of the dipole and then use the small angle approximation $\sin\theta \approx \theta$ to show that the dipole experiences a linear restoring torque and, hence, will experience simple harmonic motion.

Apply $\sum \tau = I\alpha$ to the dipole:

$$-pE \sin \theta = I \frac{d^2 \theta}{dt^2}$$

where τ is negative because acts in such a direction as to decrease θ .

For small values of θ , $\sin\theta \approx \theta$
and:

$$-pE\theta = I \frac{d^2 \theta}{dt^2}$$

Express the moment of inertia of the dipole:

$$I = \frac{1}{2}ma^2$$

Relate the dipole moment of the dipole to its charge and the charge separation:

$$p = qa$$

Substitute for p and I to obtain:

$$\frac{1}{2}ma^2 \frac{d^2 \theta}{dt^2} = -qaE\theta$$

or

$$\boxed{\frac{d^2 \theta}{dt^2} = -\frac{2qE}{ma}\theta}$$

the differential equation for a simple harmonic oscillator with angular frequency $\omega = \sqrt{2qE/ma}$.

Express the period of a simple harmonic oscillator:

$$T = \frac{2\pi}{\omega}$$

Substitute for ω and simplify to obtain:

$$T = \boxed{2\pi \sqrt{\frac{ma}{2qE}}}$$

78 •• For the dumbbell in Problem 77, let $m = 0.0200$ kg, $a = 0.300$ m, and $\vec{E} = (600 \text{ N/C})\hat{i}$. The dumbbell is initially at rest and makes an angle of 60° with the x axis. The dumbbell is then released, and when it is momentarily aligned with the electric field, its kinetic energy is 5.00×10^{-3} J. Determine the magnitude of q .

Picture the Problem We can apply conservation of energy and the definition of the potential energy of a dipole in an electric field to relate q to the kinetic energy of the dumbbell when it is aligned with the field.

Using conservation of energy, relate the initial potential energy of the dumbbell to its kinetic energy when it is momentarily aligned with the electric field:

$$\begin{aligned}\Delta K + \Delta U &= 0 \\ \text{or, because } K_i &= 0, \\ K + \Delta U &= 0 \quad (1) \\ \text{where } K &\text{ is the kinetic energy when it is} \\ &\text{aligned with the field.}\end{aligned}$$

Express the change in the potential energy of the dumbbell as it aligns with the electric field in terms of its dipole moment, the electric field, and the angle through which it rotates:

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= -pE \cos \theta_f + pE \cos \theta_i \\ &= qaE(\cos 60^\circ - 1)\end{aligned}$$

Substitute for ΔU in equation (1) to obtain:

$$K + qaE(\cos 60^\circ - 1) = 0$$

Solving for q yields:

$$q = \frac{K}{aE(1 - \cos 60^\circ)}$$

Substitute numerical values and evaluate q :

$$\begin{aligned}q &= \frac{5.00 \times 10^{-3} \text{ J}}{(0.300 \text{ m})(600 \text{ N/C})(1 - \cos 60^\circ)} \\ &= \boxed{56 \mu\text{C}}\end{aligned}$$

79 •• [SSM] An electron (charge $-e$, mass m) and a positron (charge $+e$, mass m) revolve around their common center of mass under the influence of their attractive coulomb force. Find the speed v of each particle in terms of e , m , k , and their separation distance L .

Picture the Problem The forces the electron and the proton exert on each other constitute an action-and-reaction pair. Because the magnitudes of their charges are equal and their masses are the same, we find the speed of each particle by finding the speed of either one. We'll apply Coulomb's force law for point charges and Newton's 2nd law to relate v to e , m , k , and their separation distance L .

Apply Newton's 2nd law to the positron to obtain:

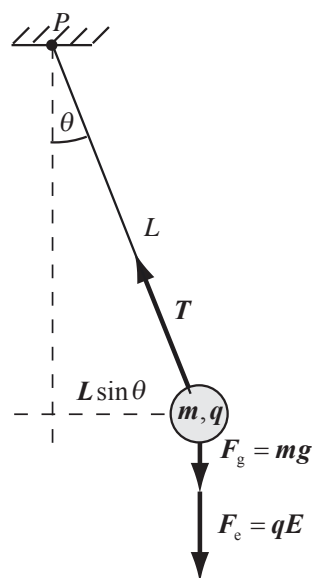
$$\frac{ke^2}{L^2} = m \frac{v^2}{\frac{1}{2}L} \Rightarrow \frac{ke^2}{L} = 2mv^2$$

Solve for v to obtain:

$$v = \sqrt{\frac{ke^2}{2mL}}$$

80 •• A simple pendulum of length 1.0 m and mass 5.0×10^{-3} kg is placed in a uniform electric field \vec{E} that is directed vertically upward. The bob has a charge of $-8.0 \mu\text{C}$. The period of the pendulum is 1.2 s. What is the magnitude and direction of \vec{E} ?

Picture the Problem Because the period of this pendulum is 60% of the period (2.006 s) of a simple pendulum with an uncharged bob, we know that the bob must be experiencing an additional downward force (in the direction of the gravitational force). Because the electric force acting on the negatively-charged bob is downward, the electric field must be upward. We can find the magnitude of the electric field by apply Newton's 2nd law in rotational form to the simple pendulum. Doing so will lead us to the equation of motion for the pendulum and from this equation we can obtain an expression relating its period to the magnitude of E . Knowing the pendulum's period in the absence of the electric field will allow us to derive an expression for E from the ratio of the two periods.



Taking clockwise torques to be negative, apply $\sum \tau_p = I_p \alpha$ to the pendulum bob to obtain:

$$\begin{aligned} -(mg + qE)L \sin \theta &= I_p \alpha \\ \text{or, because } I_p &= mL^2 \text{ and } \alpha = \frac{d^2 \theta}{dt^2}, \\ -(mg + qE)L \sin \theta &= mL^2 \frac{d^2 \theta}{dt^2} \end{aligned}$$

For small displacements from equilibrium, $\sin \theta \approx \theta$:

$$\begin{aligned} mL^2 \frac{d^2 \theta}{dt^2} &= -(mg + qE)L \theta \\ \text{or} \\ \frac{d^2 \theta}{dt^2} + \frac{mg + qE}{mL} \theta &= 0 \end{aligned}$$

This equation is equation of simple harmonic motion with:

$$\omega^2 = \frac{mg + qE}{mL}$$

The period of this motion is given by:

$$T' = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{mL}{mg + qE}}$$

In the absence of the electric field, the period of the simple pendulum would be:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Dividing the second of these equations by the first yields:

$$\begin{aligned} \frac{T}{T'} &= \frac{2\pi \sqrt{\frac{L}{g}}}{2\pi \sqrt{\frac{mL}{mg + qE}}} = \sqrt{\frac{mg + qE}{mg}} \\ &= \sqrt{1 + \frac{qE}{mg}} \end{aligned}$$

Solve for E to obtain:

$$E = \frac{mg}{q} \left[\left(\frac{T}{T'} \right)^2 - 1 \right]$$

Noting that the period of the simple pendulum in the absence of the electric field is 2.006 s, substitute numerical values and evaluate E :

$$E = \frac{(5.0 \text{ mg})(9.81 \text{ m/s}^2)}{8.0 \text{ } \mu\text{C}} \left[\left(\frac{2.006 \text{ s}}{1.2 \text{ s}} \right)^2 - 1 \right] = \boxed{1.1 \times 10^4 \text{ N/C}}, \text{ upward}$$

81 •• A point particle of mass m and charge q is constrained to move vertically inside a narrow, frictionless cylinder (Figure 21-47). At the bottom of the cylinder is a point charge Q having the same sign as q . (a) Show that the particle whose mass is m will be in equilibrium at a height $y_0 = (kqQ/mg)^{1/2}$. (b) Show that if the particle is displaced from its equilibrium position by a small amount and released, it will exhibit simple harmonic motion with angular frequency $\omega = (2g/y_0)^{1/2}$.

Picture the Problem We can use Coulomb's force law for point particles and the condition for translational equilibrium to express the equilibrium position as a function of k , q , Q , m , and g . In Part (b) we'll need to show that the displaced point charge experiences a linear restoring force and, hence, will exhibit simple harmonic motion.

(a) Apply the condition for translational equilibrium to the particle:

$$\frac{kqQ}{y_0^2} - mg = 0 \Rightarrow y_0 = \sqrt{\frac{kqQ}{mg}}$$

(b) Express the restoring force that acts on the particle when it is displaced a distance Δy from its equilibrium position:

$$F = \frac{kqQ}{(y_0 + \Delta y)^2} - \frac{kqQ}{y_0^2}$$

or, because $\Delta y \ll y_0$,

$$F \approx \frac{kqQ}{y_0^2 + 2y_0\Delta y} - \frac{kqQ}{y_0^2}$$

Simplify this expression further by writing it with a common denominator:

$$F = -\frac{2y_0\Delta y kqQ}{y_0^4 + 2y_0^3\Delta y} = -\frac{2y_0\Delta y kqQ}{y_0^4 \left(1 + 2\frac{\Delta y}{y_0}\right)} \approx -\frac{2\Delta y kqQ}{y_0^3}$$

again, because $\Delta y \ll y_0$.

From the 1st step of our solution:

$$\frac{kqQ}{y_0^2} = mg$$

Substitute for $\frac{kqQ}{y_0^2}$ and simplify to

$$F = -\frac{2mg}{y_0}\Delta y$$

obtain:

Apply Newton's 2nd law to the displaced particle to obtain:

$$m \frac{d^2 \Delta y}{dt^2} = -\frac{2mg}{y_0} \Delta y$$

or

$$\boxed{\frac{d^2 \Delta y}{dt^2} + \frac{2g}{y_0} \Delta y = 0}$$

the differential equation of simple

harmonic motion with $\omega = \boxed{\sqrt{2g/y_0}}$.

82 ••• Two neutral molecules on the x axis attract each other. Each molecule has a dipole moment \vec{p} , and these dipole moments are on the $+x$ axis and are separated by a distance d . Derive an expression for the force of attraction in terms of p and d .

Picture the Problem We can relate the force of attraction that each molecule exerts on the other to the potential energy function of either molecule using $F = -dU/dx$. We can relate U to the electric field at either molecule due to the presence of the other through $U = -pE$. Finally, the electric field at either molecule is given by $E = 2kp/x^3$.

Express the force of attraction between the dipoles in terms of the spatial derivative of the potential energy function of p_1 :

$$F = -\frac{dU_1}{dx} \quad (1)$$

Express the potential energy of the dipole p_1 :

$$U_1 = -p_1 E_1$$

where E_1 is the field at p_1 due to p_2 .

Express the electric field strength at p_1 due to p_2 :

$$E_1 = \frac{2kp_2}{x^3}$$

where x is the separation of the dipoles.

Substitute for E_1 to obtain:

$$U_1 = -\frac{2kp_1 p_2}{x^3}$$

Substitute in equation (1) and differentiate with respect to x :

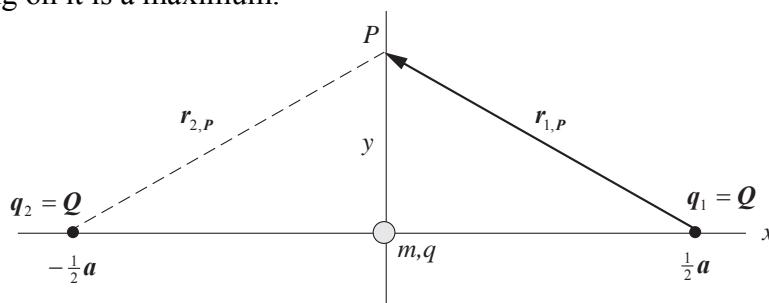
$$F = -\frac{d}{dx} \left[-\frac{2kp_1 p_2}{x^3} \right] = \frac{6kp_1 p_2}{x^4}$$

Evaluate F for $p_1 = p_2 = p$ and $x = d$ to obtain:

$$F = \boxed{\frac{6kp^2}{d^4}}$$

83 •• Two equal positive point charges Q are on the x axis at $x = \frac{1}{2}a$ and $x = -\frac{1}{2}a$. (a) Obtain an expression for the electric field on the y axis as a function of y . (b) A bead of mass M , which has a charge q , moves along the y axis on a thin frictionless taut thread. Find the electric force that acts on the bead as a function of y and determine the sign of q such that this force always points away from the origin. (c) The bead is initially at rest at the origin. If it is given a slight nudge in the $+y$ direction, how fast will the bead be traveling the instant the net force on it is a maximum? (Assume any effects due to gravity are negligible.)

Picture the Problem (a) We can use Coulomb's law for the electric field due to a point charge and superposition of fields to find the electric field at any point on the y axis. (b) Using the definition of electric field will yield an expression for the electric force that acts on the bead. In Part (c) we can set $dF_y/dy = 0$ to find the value of y that maximizes F_y and then use the work-kinetic energy theorem (integrating F_y from 0 to this extreme value will yield an expression for the work done on the bead during this displacement) to find the speed of the bead when the force acting on it is a maximum.



(a) Use Coulomb's law for the electric field due to a point charge and superposition of fields, to express the field at point P on the y axis:

$$\begin{aligned}\vec{E}_y &= \vec{E}_1 + \vec{E}_2 \\ &= \frac{kq_1}{r_{1,P}^2} \hat{r}_{1,P} + \frac{kq_2}{r_{2,P}^2} \hat{r}_{2,P}\end{aligned}$$

The unit vectors $\hat{r}_{1,P}$ and $\hat{r}_{2,P}$ are given by:

$$\hat{r}_{1,P} = \frac{\vec{r}_{1,P}}{r_{1,P}} = \frac{-\frac{1}{2}a\hat{i} + y\hat{j}}{r_{1,P}}$$

and

$$\hat{r}_{2,P} = \frac{\vec{r}_{2,P}}{r_{2,P}} = \frac{\frac{1}{2}a\hat{i} + y\hat{j}}{r_{2,P}}$$

Substituting for q_1 , q_2 , $\hat{r}_{1,P}$, $\hat{r}_{2,P}$ and $r_{1,P} = r_{2,P} = [y^2 + (\frac{1}{2}a)^2]^{1/2}$ yields:

$$\begin{aligned}\vec{E}_y &= \frac{kQ}{r_{1,P}^2} \left(-\frac{1}{2} \hat{a}\hat{i} + y\hat{j} \right) + \frac{kQ}{r_{2,P}^2} \left(\frac{1}{2} \hat{a}\hat{i} + y\hat{j} \right) = \frac{kQ}{r_{1,P}^3} \left(-\frac{1}{2} \hat{a}\hat{i} + y\hat{j} \right) + \frac{kQ}{r_{2,P}^3} \left(\frac{1}{2} \hat{a}\hat{i} + y\hat{j} \right) \\ &= \frac{kQ}{[y^2 + (\frac{1}{2}a)^2]^{3/2}} (y\hat{j}) + \frac{kQ}{[y^2 + (\frac{1}{2}a)^2]^{3/2}} (y\hat{j}) = \boxed{\frac{2kQy}{[y^2 + \frac{1}{4}a^2]^{3/2}} \hat{j}}\end{aligned}$$

(b) Relate the electric force on the bead to its charge and the electric field:

$$\vec{F} = q\vec{E}_y = \boxed{\frac{2kqQy}{[y^2 + \frac{1}{4}a^2]^{3/2}} \hat{j}}$$

where q must be positive if \vec{F} always points away from the origin.

(c) Apply the work-kinetic energy theorem to the bead as it moves from the origin to the location at which F_y is a maximum to obtain:

$$W_{\text{net}_y} = \Delta K = K_f - K_i$$

or, because $K_i = 0$,

$$W_{\text{done by } F_y} = \frac{1}{2} M v_f^2$$

Solving for v_f yields:

$$v_f = \sqrt{\frac{2W_{\text{done by } F_y}}{M}}$$

The work done on the bead by F_y as the bead moves from the origin to y_{max} (the y coordinate of the point at which F_y is a maximum) is given by:

$$W_{\text{done by } F_y} = \int_0^{y_{\text{max}}} \frac{2kqQy}{[y^2 + (\frac{1}{2}a)^2]^{3/2}} dy$$

Substituting for $W_{\text{done by } F_y}$ and simplifying yields:

$$v_f = \sqrt{\frac{4kqQ}{m} \int_0^{y_{\text{max}}} \frac{y}{[y^2 + (\frac{1}{2}a)^2]^{3/2}} dy}$$

The condition that F_y is a maximum is:

$$\frac{dF_y}{dy} = \frac{d}{dy} \left[\frac{y}{[y^2 + (\frac{1}{2}a)^2]^{3/2}} \right] = 0$$

Carrying out the details of the differentiation and solving for the critical value y_{max} yields:

$y_{\text{max}} = \frac{a}{2\sqrt{2}}$ where we've ignored the negative value because the bead is given a nudge in the $+y$ direction.

Substitute for y_{\max} in the expression for v_f to obtain:

$$v_f = \sqrt{\frac{4kqQ}{M} \int_0^{\frac{a}{2\sqrt{2}}} \frac{y}{\left[y^2 + \left(\frac{1}{2}a\right)^2\right]^{3/2}} dy}$$

Evaluating the integral yields:

$$\int_0^{\frac{a}{2\sqrt{2}}} \frac{y}{\left[y^2 + \left(\frac{1}{2}a\right)^2\right]^{3/2}} dy \approx \frac{0.367}{a}$$

Substitute for $\int_0^{\frac{a}{2\sqrt{2}}} \frac{y}{\left[y^2 + \left(\frac{1}{2}a\right)^2\right]^{3/2}} dy$

$$v_f = \sqrt{\frac{4kqQ}{M} \left(\frac{0.367}{a}\right)} = \boxed{1.21 \sqrt{\frac{kqQ}{aM}}}$$

and simplify to obtain:

84 •• A gold nucleus is 100 fm (1 fm = 10^{-15} m) from a proton, which initially is at rest. When the proton is released, it speeds away because of the repulsion that it experiences due to the charge on the gold nucleus. What is the proton's speed a large distance (assume to be infinity) from the gold nucleus? (Assume the gold nucleus remains stationary.)

Picture the Problem The work done by the electric field of the gold nucleus changes the kinetic energy of the proton. We can apply the work-kinetic energy theorem to derive an expression for the speed of the proton as a function of its distance from the gold nucleus. Because the repulsive Coulomb force \vec{F}_e varies with distance, we'll have to evaluate $\int \vec{F}_e \cdot d\vec{r}$ in order to find the work done on the proton by this force.

Apply the work-kinetic energy theorem to the proton to obtain:

$$W_{\text{net}} = \int_{r_0}^{\infty} \vec{F}_e \cdot d\vec{r} = \Delta K$$

$$\text{or, because } \int_{r_0}^{\infty} \vec{F}_e \cdot d\vec{r} = \int_{r_0}^{\infty} \frac{ke(79e)}{r^2} dr \text{ and}$$

$$K_i = 0,$$

$$79ke^2 \int_{r_0}^{\infty} \frac{dr}{r^2} = \frac{1}{2} m_p v_f^2$$

Evaluating the integral yields:

$$-79ke^2 \left[\frac{1}{r} \right]_{r_0}^{\infty} = \frac{79ke^2}{r_0} = \frac{1}{2} m_p v_f^2$$

Solve for v_f and simplify to obtain:

$$v_f = \sqrt{\frac{158ke^2}{m_p r_0}} = e \sqrt{\frac{158k}{m_p r_0}}$$

Substitute numerical values and evaluate v_f :

$$v_f = (1.602 \times 10^{-19} \text{ C}) \sqrt{\frac{158 \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right)}{(1.673 \times 10^{-27} \text{ kg})(1.00 \times 10^{-13} \text{ m})}} = \boxed{1.48 \times 10^7 \text{ m/s}}$$

85 •• [SSM] During a famous experiment in 1919, Ernest Rutherford shot doubly ionized helium nuclei (also known as alpha particles) at a gold foil. He discovered that virtually all of the mass of an atom resides in an extremely compact nucleus. Suppose that during such an experiment, an alpha particle far from the foil has an initial kinetic energy of 5.0 MeV. If the alpha particle is aimed directly at the gold nucleus, and the only force acting on it is the electric force of repulsion exerted on it by the gold nucleus, how close will it approach the gold nucleus before turning back? That is, what is the minimum center-to-center separation of the alpha particle and the gold nucleus?

Picture the Problem The work done by the electric field of the gold nucleus changes the kinetic energy of the alpha particle—eventually bringing it to rest. We can apply the work-kinetic energy theorem to derive an expression for the distance of closest approach. Because the repulsive Coulomb force \vec{F}_e varies with distance, we'll have to evaluate $\int \vec{F}_e \cdot d\vec{r}$ in order to find the work done on the alpha particles by this force.

Apply the work-kinetic energy theorem to the alpha particle to obtain:

$$W_{\text{net}} = \int_{\infty}^{r_{\min}} \vec{F}_e \cdot d\vec{r} = \Delta K$$

or, because

$$\int_{\infty}^{r_{\min}} \vec{F}_e \cdot d\vec{r} = - \int_{\infty}^{r_{\min}} \frac{k(2e)(79e)}{r^2} dr \text{ and}$$

$$K_f = 0,$$

$$-158ke^2 \int_{\infty}^{r_{\min}} \frac{dr}{r^2} = -K_i$$

Evaluating the integral yields:

$$-158ke^2 \left[\frac{1}{r} \right]_{\infty}^{r_{\min}} = -\frac{158ke^2}{r_{\min}} = -K_i$$

Solve for r_{\min} and simplify to obtain:

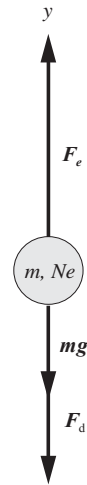
$$r_{\min} = \frac{158ke^2}{K_i}$$

Substitute numerical values and evaluate r_{\min} :

$$r_{\min} = \frac{158 \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (1.602 \times 10^{-19} \text{C})^2}{5.0 \text{ MeV} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}}} = \boxed{4.6 \times 10^{-14} \text{ m}}$$

86 •• During the Millikan experiment used to determine the charge on the electron, a charged polystyrene microsphere is released in still air in a known vertical electric field. The charged microsphere will accelerate in the direction of the net force until it reaches terminal speed. The charge on the microsphere is determined by measuring the terminal speed. During one such experiment, the microsphere has radius of $r = 5.50 \times 10^{-7} \text{ m}$, and the field has a magnitude $E = 6.00 \times 10^4 \text{ N/C}$. The magnitude of the drag force on the sphere is given by $F_D = 6\pi\eta r v$, where v is the speed of the sphere and η is the viscosity of air ($\eta = 1.8 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$). Polystyrene has density $1.05 \times 10^3 \text{ kg/m}^3$. (a) If the electric field is pointing down and the polystyrene microsphere is rising with a terminal speed of $1.16 \times 10^{-4} \text{ m/s}$, what is the charge on the sphere? (b) How many excess electrons are on the sphere? (c) If the direction of the electric field is reversed but its magnitude remains the same, what is the new terminal speed?

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass m and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force \vec{F}_e , its weight $m\vec{g}$, and the drag force \vec{F}_d . We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges N on the sphere to its mass and, using Stokes' law, find its terminal speed.



(a) Apply Newton's 2nd law to the microsphere to obtain:

$$\begin{aligned} F_e - mg - F_d &= ma_y \\ \text{or, because } a_y &= 0, \\ F_e - mg - F_{d,\text{terminal}} &= 0 \end{aligned}$$

Substitute for F_e , m , and $F_{d,\text{terminal}}$ to obtain:

$$\begin{aligned} qE - \rho Vg - 6\pi\eta r v_t &= 0 \\ \text{or, because } q &= Ne, \\ NeE - \frac{4}{3}\pi r^3 \rho g - 6\pi\eta r v_t &= 0 \end{aligned}$$

Solve for Ne to obtain:

$$Ne = \frac{\frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_t}{E} \quad (1)$$

Substitute numerical values and evaluate $\frac{4}{3}\pi r^3 \rho g$:

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi (5.50 \times 10^{-7} \text{ m})^3 (1.05 \times 10^3 \text{ kg/m}^3) (9.81 \text{ m/s}^2) = 7.18 \times 10^{-15} \text{ N}$$

Substitute numerical values and evaluate $6\pi\eta r v_t$:

$$6\pi\eta r v_t = 6\pi (1.8 \times 10^{-5} \text{ Pa} \cdot \text{s}) (5.50 \times 10^{-7} \text{ m}) (1.16 \times 10^{-4} \text{ m/s}) = 2.16 \times 10^{-14} \text{ N}$$

Substitute numerical values in equation (1) and evaluate Ne :

$$\begin{aligned} Ne &= \frac{7.18 \times 10^{-15} \text{ N} + 2.16 \times 10^{-14} \text{ N}}{6.00 \times 10^4 \text{ N/C}} \\ &= \boxed{4.8 \times 10^{-19} \text{ C}} \end{aligned}$$

(b) Divide the result in (a) by e to obtain:

$$N = \frac{4.80 \times 10^{-19} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = \boxed{3}$$

(c) With the field pointing upward, the electric force is downward and the application of $\sum F_y = ma_y$ to the bead yields:

$$\begin{aligned} F_{d,\text{terminal}} - F_e - mg &= 0 \\ \text{or} \\ 6\pi\eta r v_t - NeE - \frac{4}{3}\pi r^3 \rho g &= 0 \end{aligned}$$

Solve for v_t to obtain:

$$v_t = \frac{NeE + \frac{4}{3}\pi r^3 \rho g}{6\pi\eta r}$$

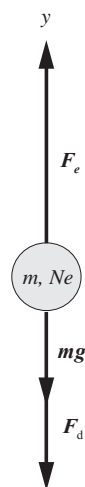
Substitute numerical values and evaluate v_t :

$$\begin{aligned} v_t &= \frac{3(1.602 \times 10^{-19} \text{ C}) \left(6.00 \times 10^4 \frac{\text{N}}{\text{C}} \right) + \frac{4}{3}\pi (5.50 \times 10^{-7} \text{ m})^3 \left(1.05 \times 10^3 \frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{6\pi (1.8 \times 10^{-5} \text{ Pa} \cdot \text{s}) (5.50 \times 10^{-7} \text{ m})} \\ &= \boxed{0.19 \text{ mm/s}} \end{aligned}$$

87 •• [SSM] In Problem 86, there is a description of the Millikan experiment used to determine the charge on the electron. During the experiment, a switch is used to reverse the direction of the electric field without changing its

magnitude, so that one can measure the terminal speed of the microsphere both as it is moving upward and as it is moving downward. Let v_u represent the terminal speed when the particle is moving up, and v_d the terminal speed when moving down. (a) If we let $u = v_u + v_d$, show that $q = 3\pi\eta ru/E$, where q is the microsphere's net charge. For the purpose of determining q , what advantage does measuring both v_u and v_d have over measuring only one terminal speed? (b) Because charge is quantized, u can only change by steps of magnitude N , where N is an integer. Using the data from Problem 86, calculate Δu .

Picture the Problem The free body diagram shows the forces acting on the microsphere of mass m and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force \vec{F}_e , its weight $m\vec{g}$, and the drag force \vec{F}_d . We can apply Newton's 2nd law, under terminal-speed conditions, to relate the number of excess charges N on the sphere to its mass and, using Stokes' law, to its terminal speed.



(a) Apply Newton's 2nd law to the microsphere when the electric field is downward:

$$\begin{aligned} F_e - mg - F_d &= ma_y \\ \text{or, because } a_y &= 0, \\ F_e - mg - F_{d,\text{terminal}} &= 0 \end{aligned}$$

Substitute for F_e and $F_{d,\text{terminal}}$ to obtain:

$$\begin{aligned} qE - mg - 6\pi\eta r v_u &= 0 \\ \text{or, because } q &= Ne, \\ NeE - mg - 6\pi\eta r v_u &= 0 \end{aligned}$$

Solve for v_u to obtain:

$$v_u = \frac{NeE - mg}{6\pi\eta r} \quad (1)$$

With the field pointing upward, the electric force is downward and the application of Newton's 2nd law to the microsphere yields:

$$\begin{aligned} F_{d,\text{terminal}} - F_e - mg &= 0 \\ \text{or} \\ 6\pi\eta r v_d - NeE - mg &= 0 \end{aligned}$$

Solve for v_d to obtain:

$$v_d = \frac{NeE + mg}{6\pi\eta r} \quad (2)$$

Add equations (1) and (2) and simplify to obtain:

$$u = v_u + v_d = \frac{NeE - mg}{6\pi\eta r} + \frac{NeE + mg}{6\pi\eta r} = \frac{NeE}{3\pi\eta r} = \boxed{\frac{qE}{3\pi\eta r}}$$

This has the advantage that you don't need to know the mass of the microsphere.

(b) Letting Δu represent the change in the terminal speed of the microsphere due to a gain (or loss) of one electron we have:

$$\Delta u = v_{N+1} - v_N$$

Noting that Δv will be the same whether the microsphere is moving upward or downward, express its terminal speed when it is moving upward with N electronic charges on it:

$$v_N = \frac{NeE - mg}{6\pi\eta r}$$

Express its terminal speed upward when it has $N + 1$ electronic charges:

$$v_{N+1} = \frac{(N+1)eE - mg}{6\pi\eta r}$$

Substitute and simplify to obtain:

$$\begin{aligned}\Delta u &= \frac{(N+1)eE - mg}{6\pi\eta r} - \frac{NeE - mg}{6\pi\eta r} \\ &= \frac{eE}{6\pi\eta r}\end{aligned}$$

Substitute numerical values and evaluate Δu :

$$\begin{aligned}\Delta u &= \frac{(1.602 \times 10^{-19} \text{ C})(6.00 \times 10^4 \text{ N/C})}{6\pi(1.8 \times 10^{-5} \text{ Pa} \cdot \text{m})(5.50 \times 10^{-7} \text{ m})} \\ &= \boxed{52 \mu\text{m/s}}\end{aligned}$$

