Chapter 5
Additional Applications of Newton's Laws

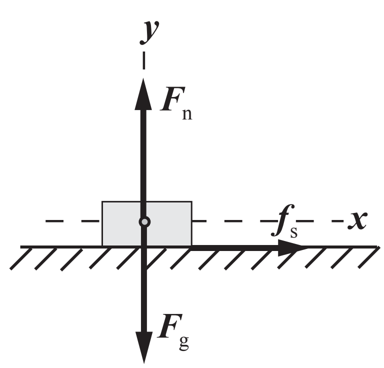
Conceptual Problems

1 • [SSM] Various objects lie on the bed of a truck that is moving along a straight horizontal road. If the truck gradually speeds up, what force acts on the objects to cause them to speed up too? Explain why some of the objects might stay stationary on the floor while others might slip backward on the floor.

Determine the Concept The forces acting on the objects are the normal and frictional forces exerted by the truck bed and the gravitational force exerted by Earth., and t The static (if the objects do not slip) frictional forces exerted by the floor of the truck bed cause them to speed up. Because the objects are speeding up (accelerating), there must be a net force acting on them. Of these forces, the only one that acts in the direction of the acceleration is the $\boxed{\text{static friction force}}$. The maximum acceleration is determined not by the mass of the objects but instead by the value of the coefficient of static friction. This will vary from object to object depending on its material and surface characteristics.

2 • Blocks made of the same material but differing in size lie on the bed of a truck that is moving along a straight horizontal road. All of the blocks will slide if the truck's acceleration is sufficiently great. How does the minimum acceleration at which a small block slips compare with the minimum acceleration at which a much heavier block slips?

Determine the Concept The forces acting on an object are the normal force exerted by the floor of the truck, the gravitational force exerted by the earth, and the friction force; also exerted by the floor of the truck. Of these forces, the only one that acts in the direction of the acceleration (chosen to be to the right) is the static friction force. Apply Newton's 2nd law to the object to determine how the critical acceleration depends on its weight.



Taking the positive x direction to be to the right, apply $\Sigma F_x = ma_x$ to the object:

$$f_s = \mu_s F_g = \mu_s mg = ma_x$$

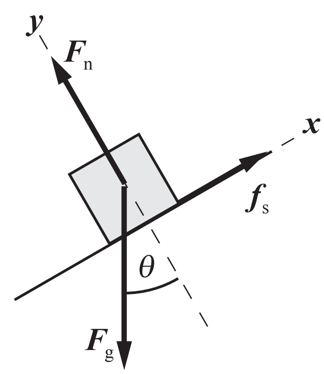
Solving for a_x yields:

$$a_x = \mu_s g$$

Because a_x is independent of m and F_g, the critical accelerations are the same.

3 • A block of mass m rests on a plane that is inclined at an angle θ with the horizontal. It follows that the coefficient of static friction between the block and plane is (*a*) $\mu_s \geq g$, (*b*) $\mu_s = \tan\theta$, (*c*) $\mu_s \leq \tan\theta$, (*d*) $\mu_s \geq \tan\theta$.

Determine the Concept The forces acting on the block are the normal force $\vec{F}_n$ exerted by the incline, the weight of the block $\vec{F}_g$ exerted by the earth, and the static friction force $\vec{f}_s$ exerted by an external agent. We can use the definition of μ_s and the conditions for equilibrium to determine the relationship between μ_s and θ.



Apply $\sum F_x = ma_x$ to the block: $\boldsymbol{f}_s - \boldsymbol{F}_g \sin\boldsymbol{\theta} = 0$

or, because $F_g = mg$,

$$\boldsymbol{f}_s - \boldsymbol{mg}\sin\boldsymbol{\theta} = 0 \quad (1)$$

Apply $\sum F_y = ma_y$ in the y direction:

$$\boldsymbol{F}_n - \boldsymbol{mg}\cos\boldsymbol{\theta} = 0 \quad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\tan\theta = \frac{f_s}{F_n}$$

Substitute for f_s ($\leq \mu_s F_n$) and simplify to obtain:

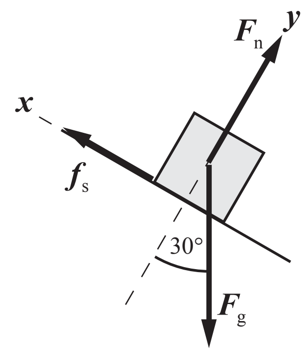
$$\tan\theta \leq \frac{\mu_s F_n}{F_n} = \mu_s$$ and $\boxed{(\boldsymbol{d})}$ is correct.

4 • A block of mass m is at rest on a plane that is inclined at an angle of 30° with the horizontal, as shown in Figure 5-56. Which of the following statements about the magnitude of the static frictional force f_s is necessarily true? (*a*) $f_s > mg$. (*b*) $f_s > mg\cos 30°$. (*c*) $f_s = mg\cos 30°$. (*d*) $f_s = mg\sin 30°$. (*e*) None of these statements is true.

Determine the Concept The block is in equilibrium under the influence of $\vec{F}_n$, $\vec{F}_g$, and $\vec{f}_s$; that is,

$$\vec{F}_n + \vec{F}_g + \vec{f}_s = \mathbf{0}$$

We can apply Newton's 2nd law in the x direction to determine the relationship between f_s and $F_g = mg$.

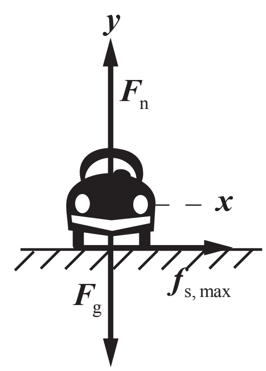


Apply $\sum F_x = 0$ to the block: $f_s - mg\sin\theta = 0$

Solve for f_s to obtain: $f_s = mg\sin\theta$ and $\boxed{(d)}$ is correct.

5 •• On an icy winter day, the coefficient of friction between the tires of a car and a roadway is reduced to one-quarter of its value on a dry day. As a result, the maximum speed $v_{\text{max dry}}$ at which the car can safely negotiate a curve of radius R is reduced. The new value for this speed is (*a*) $v_{\text{max dry}}$, (*b*) $0.71v_{\text{max dry}}$, (*c*) $0.50v_{\text{max dry}}$, (*d*) $0.25v_{\text{max dry}}$, (*e*) reduced by an unknown amount depending on the car's mass.

Picture the Problem The forces acting on the car as it rounds a curve of radius R at maximum speed are shown on the free-body diagram to the right. The centripetal force is the static friction force exerted by the roadway on the tires. We can apply Newton's 2nd law to the car to derive an expression for its maximum speed and then compare the speeds under the two friction conditions described.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_{s,\max} = m\frac{v_{\max}^2}{R}$$

and

$$\sum F_y = F_n - mg = 0$$

From the y equation we have: $F_n = mg$

Express $f_{s,\max}$ in terms of F_n in the x equation and solve for $v_{\max}$ to obtain:

$$v_{\max} = \sqrt{\mu_s gR} \quad (1)$$

When $\mu_s = \mu'_s$:

$$v'_{max} = \sqrt{\mu'_s gR} \quad (2)$$

Dividing equation (2) by equation (1) yields:

$$\frac{v'_{max}}{v_{max}} = \frac{\sqrt{\mu'_s gR}}{\sqrt{\mu_s gR}} = \sqrt{\frac{\mu'_s}{\mu_s}}$$

Solve for v'_{max} to obtain:

$$v'_{max} = \sqrt{\frac{\mu'_s}{\mu_s}} v_{max}$$

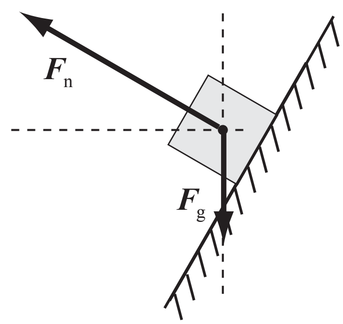
Evaluate v'_{max} for $\mu'_s = \frac{1}{4}\mu_s$:

$$v'_{max} = \sqrt{\tfrac{1}{4}} v_{max} = 0.5 v_{max} = 50\% v_{max}$$

and $\boxed{(c)}$ is correct.

6 •• If it is started properly on the frictionless inside surface of a cone (Figure 5-57), a block is capable of maintaining uniform circular motion. Draw the free-body diagram of the block and identify clearly which force (or forces or force components) is responsible for the centripetal acceleration of the block.

Determine the Concept The forces acting on the block are the normal force $\vec{F}_n$ exerted by the surface of the cone and the gravitational force $\vec{F}_g$ exerted by the earth. The horizontal component of $\vec{F}_n$ is responsible for the centripetal force on the block.



7 •• Here is an interesting experiment that you can perform at home: take a wooden block and rest it on the floor or some other flat surface. Attach a rubber band to the block and pull gently and steadily on the rubber band in the horizontal direction. At some point, the block will start moving, but it will not move smoothly. Instead, it will start moving, stop again, start moving again, stop again, and so on. Explain why the block moves this way. (The start-stop motion is sometimes called "stick-slip" motion.)

Determine the Concept As the spring is extended, the force exerted by the spring on the block increases. Once that force is greater than the maximum value of the force of static friction on the block, the block will begin to move. However, as it accelerates, it will shorten the length of the spring, decreasing the force that the spring exerts on the block. As this happens, the force of kinetic friction can then slow the block to a stop, which starts the cycle over again. One interesting application of this to the real world is the bowing of a violin string: The string under tension acts like the spring, while the bow acts as the block, so as the bow is dragged across the string, the string periodically sticks and frees itself from the bow.

8 • Viewed from an inertial reference frame, an object is seen to be moving in a circle. Which, if any, of the following statements are true. (*a*) A non-zero net force is acting acts the object. (*b*) The object cannot have a radially outward force acting on it. (*c*) At least one of the forces acting on the object must point directly toward the center of the circle.

(*a*) True. The velocity of an object moving in a circle is continually changing independently of whether the object's speed is changing. The change in the velocity vector and the acceleration vector and the net force acting on the object all point toward the center of circle. This center-pointing force is called a centripetal force.

(*b*) False. The only condition that must be satisfied in order that the object move along a circular path is that the *net* force acting on it be radially inward.

(*c*) False. The only condition that must be satisfied in order that the object move along a circular path is that the *net* force acting on it be radially inward.

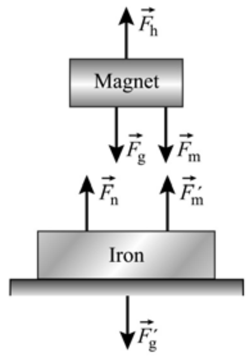
9 •• A particle is traveling in a vertical circle at constant speed. One can conclude that the magnitude of its _____ is constant. (*a*) velocity, (*b*) acceleration, (*c*) net force, (*d*) apparent weight.

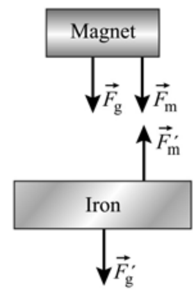
Determine the Concept A particle traveling in a vertical circle experiences a downward gravitational force plus an additional force that constrains it to move along a circular path. Because the speed of the particle is constant, the magnitude of its velocity is constant. Because the magnitude of its velocity is constant, its acceleration must be constant. Because the magnitude of its acceleration is constant, the magnitude of the net force acting on it must be constant. Therefore, $\boxed{(\boldsymbol{a})}$, $\boxed{(\boldsymbol{b})}$, and $\boxed{(\boldsymbol{c})}$ are correct.

10 •• You place a lightweight piece of iron on a table and hold a small kitchen magnet above the iron at a distance of 1.00 cm. You find that the magnet cannot lift the iron, even though there is obviously a force between the iron and the magnet. Next, again holding the magnet 1.00 cm above the iron, you drop them from arm's length, releasing them from rest simultaneously. As they fall, the magnet and the piece of iron bang into each other before hitting the floor. (*a*) Draw free-body diagrams illustrating all of the forces on the magnet and the iron for each demonstration. (*b*) Explain why the magnet and iron move closer together while they are falling, even though the magnet cannot lift the piece of iron when it is sitting on the table.

Determine the Concept We can analyze these demonstrations by drawing force diagrams for each situation. In both diagrams, h denotes ″hand″, g denotes ″gravitational″, m denotes ″magnetic″, and n denotes ″normal.″

(*a*) Demonstration 1: Demonstration 2:

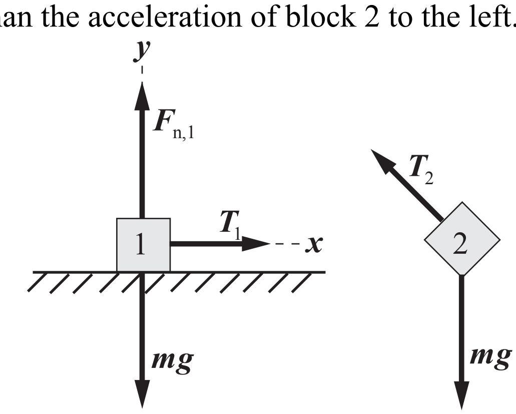




(*b*) Because the magnet doesn't lift the iron in the first demonstration, the force exerted on the iron must be less than its (the iron's) weight. This is still true when the two are falling, but the motion of the iron is not restrained by the table, and the motion of the magnet is not restrained by the hand. Looking at the second diagram, the net force pulling the magnet down is greater than its weight, implying that its acceleration is greater than g. The opposite is true for the iron: the magnetic force acts upwards, slowing it down, so its acceleration will be less than g. Because of this, the magnet will catch up to the iron piece as they fall.

11 ••• [SSM] The following question is an excellent "braintwister" invented by Boris Korsunsky. Two identical blocks are attached by a massless string running over a pulley as shown in Figure 5-58. The rope initially runs over the pulley at the rope's midpoint, and the surface that block 1 rests on is frictionless. Blocks 1 and 2 are initially at rest when block 2 is released with the string taut and horizontal. Will block 1 hit the pulley before or after block 2 hits the wall? (Assume that the initial distance from block 1 to the pulley is the same as the initial distance from block 2 to the wall.) There is a very simple solution.

Picture the Problem The following free-body diagrams show the forces acting on the two objects some time after block 2 is dropped. Note that, while $\vec{T}_1 \neq \vec{T}_2$, $T_1 = T_2$. The only force pulling block 2 to the left is the horizontal component of the tension $\vec{T}_2$. Because this force is smaller than the magnitude of the tension, the acceleration of block 1, which is identical to block 2, to the right ($T_1 = T_2$) will always be greater than the acceleration of block 2 to the left.



Because the initial distance from block 1 to the pulley is the same as the initial distance of block 2 to the wall, block 1 will hit the pulley before block 2 hits the wall.

12 •• In class, most professors do the following experiment while discussing the conditions under which air drag can be neglected while analyzing free-fall. First, a flat piece of paper and a small lead weight are dropped next to each other, and clearly the paper's acceleration is less than that of the lead weight. Then, the paper is crumpled into a small wad and the experiment repeated. Over the distance of a meter or two, it is clear the acceleration of the paper is now very close to that of the lead weight. To your dismay, the professor calls on you to explain why the paper's acceleration changed so dramatically. Repeat your explanation here!

Determine the Concept Air drag depends on the frontal area presented. Reducing it by crumpling the paper makes the force of air drag a lot less so that gravity is the most important force. The paper will thus accelerate at approximately g (until speeds are high enough for drag forces to come back into play in spite of the reduced area).

13 •• **[SSM]** Jim decides to attempt to set a record for terminal speed in skydiving. Using the knowledge he has gained from a physics course, he makes the following plans. He will be dropped from as high an altitude as possible (equipping himself with oxygen), on a warm day and go into a ″knife″ position in which his body is pointed vertically down and his hands are pointed ahead. He will outfit himself with a special sleek helmet and rounded protective clothing. Explain how each of these factors helps Jim attain the record.

Determine the Concept On a warm day the air is less dense. The air is also less dense at high altitudes. Pointing his hands results in less area being presented to air drag forces and, hence, reduces them. Rounded and sleek clothing has the same effect as pointing his hands. All are attempts to maximize his acceleration to near g for a good part of the drop by minimizing air drag forces.

14 •• You are sitting in the passenger seat in a car driving around a circular, horizontal, flat racetrack at a high speed. As you sit there, you ″feel″ a ″force″ pushing you toward the outside of the track. What is the true direction of the force acting on you, and where does it come from? (Assume that you do not slide across the seat.) Explain the *sensation* of an "outward force" on you in terms of the Newtonian perspective.

Determine the Concept In your frame of reference (the accelerating reference frame of the car), the direction of the force must point toward the center of the circular path along which you are traveling; that is, in the direction of the centripetal force that keeps you moving in a circle. The friction between you and the seat you are sitting on supplies this force. The reason you seem to be "pushed" to the outside of the curve is that your body's inertia "wants", in accordance with Newton's first law (the law of inertia), to keep it moving in a straight line–that is, tangent to the curve.

15 • **[SSM]** The mass of the moon is only about 1% of that of Earth. Therefore, the force that keeps the moon in its orbit around Earth (*a*) is much smaller than the gravitational force exerted on the moon by Earth, (*b*) is much greater than the gravitational force exerted on the moon by Earth, (*c*) is the gravitational force exerted on the moon by Earth, (*d*) cannot be answered yet, because we have not yet studied Newton's law of gravity.

Determine the Concept The centripetal force that keeps the moon in its orbit around the earth is provided by the gravitational force the earth exerts on the moon. As described by Newton's 3rd law, this force is equal in magnitude to the force the moon exerts on the earth. $\boxed{(c)}$ is correct.

16 • A block is sliding on a frictionless surface along a loop-the-loop, as in Figure 5-59. The block is moving fast enough so that it never loses contact with the track. Match the points along the track to the appropriate free-body diagrams in the figure.

Determine the Concept The only forces acting on the block are its weight and the force the surface exerts on it. Because the loop-the-loop surface is frictionless, the force it exerts on the block must be perpendicular to its surface.

At point A the weight is downward and the normal force is to the right. The normal force is the centripetal force. Free-body diagram 3 matches these forces.

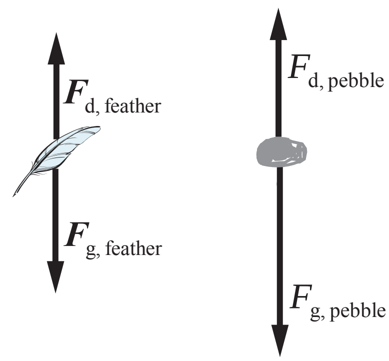
At point B the weight is downward, the normal force is upward, and the normal force is greater than the weight so that their difference is the centripetal force. Free-body diagram 4 matches these forces.

At point C the weight is downward and the normal force is to the left. The normal force is the centripetal force. Free-body diagram 5 matches these forces.

At point D both the weight and the normal forces are downward. Their sum is the centripetal force. Free-body diagram 2 matches these forces.

17 •• **[SSM]** (*a*) A pebble and a feather held at the same height above the ground are simultaneously dropped. During the first few milliseconds following release the drag force on the pebble is less than that on the feather, but later on during the fall the *opposite* is true. Explain. (*b*) In light of this result, explain how the pebble's acceleration can be so obviously larger than that of the feather. (*Hint*: *Draw a free-body diagram of each object*.)

Determine the Concept The drag force acting on the objects is given by $F_d = \frac{1}{2}CA\rho v^2$, where A is the projected surface area, v is the object's speed, ρ is the density of air, and C is a dimensionless coefficient. We'll assume that, over the height of the fall, the density of air ρ is constant. The free-body diagrams for a feather and a pebble several milliseconds into their fall are shown to the right. The forces acting on both objects are the downward gravitational force $\vec{F}_g$ and an upward drag force $\vec{F}_d$.



(*a*) The drag force on an object is proportional to some power of its speed. For a millisecond or two following release, the speeds of both the pebble and the feather are negligible, so the drag forces are negligible and they both fall with the same free-fall acceleration g. During this brief period their speeds remain equal, so the object that presents the greater area has the greater drag force. It is the feather that presents the greater area, so during this brief period the drag force on the feather is greater than that on the pebble.

A short time after the initial period the feather reaches terminal speed, after which the drag force on it remains equal to the gravitational force on it. However, the gravitational force on the pebble is much greater than that on the feather, so the pebble continues to gain speed long after the feather reaches terminal speed. As the pebble continues to gain speed, the drag force on it continues to increase. As a result, the drag force on the pebble eventually exceeds the drag force on the feather.

(*b*) The acceleration of the feather rapidly decreases because the drag force on it approaches the gravitational force on it shortly after release. However, the drag force on the pebble does not approach the gravitational force on it until much higher speeds are attained, which means the acceleration of the pebble remains high for a longer period of time.

18 •• Two pucks of masses m_1 and m_2 are lying on a frictionless table and are connected by a massless spring of force constant k. A horizontal force F_1 directed away from m_2 is then exerted on m_1. What is the magnitude of the resulting acceleration of the center of mass of the two-puck system? (*a*) F_1/m_1. (*b*) $F_1/(m_1 + m_2)$. (*c*) $(F_1 + kx)/(m_1 + m_2)$, where x is the amount the spring is stretched. (*d*) $(m_1 + m_2)F_1/m_1m_2$.

Determine the Concept The acceleration of the center of mass of a system of particles is described by $\vec{F}_{\text{net,ext}} = \sum_{\text{i}} \vec{F}_{\text{i,ext}} = M\vec{a}_{\text{cm}}$, where M is the total mass of the system.

Express the acceleration of the center of mass of the two pucks:

$$a_{\text{cm}} = \frac{F_{\text{net,ext}}}{M} = \frac{F_1}{m_1 + m_2}$$

because the spring force is an internal force. $\boxed{(\boldsymbol{b})}$ is correct.

19 •• The two pucks in Problem 18 lie unconnected on a frictionless table. A horizontal force F_1 directed away from m_2 is then exerted on m_1. How does the magnitude of the resulting acceleration of the center of mass of the two-puck system compare to the acceleration of m_1? Explain your reasoning.

Determine the Concept The acceleration of the puck whose mass is m_1 is related to the net force F_1 acting on it through Newton's 2nd law.

Because the pucks are no longer connected, the acceleration of the center of mass is:

$$\boldsymbol{a}_{\text{CM, disconnected}} = \frac{\boldsymbol{F}_1}{\boldsymbol{m}_1}$$

From Problem 18:

$$\boldsymbol{a}_{\text{CM, connected}} = \frac{\boldsymbol{F}_1}{\boldsymbol{m}_1 + \boldsymbol{m}_2}$$

Because $\dfrac{\boldsymbol{F}_1}{\boldsymbol{m}_1} > \dfrac{\boldsymbol{F}_1}{\boldsymbol{m}_1 + \boldsymbol{m}_2}$ the acceleration of m_1 is greater.

20 •• If only external forces can cause the center of mass of a system of particles to accelerate, how can a car on level ground ever accelerate? We normally think of the car's engine as supplying the force needed to accelerate the car, but is this true? Where does the external force that accelerates the car come from?

Determine the Concept There is only one force which can cause the car to move forward−the friction of the road! The car's engine causes the tires to rotate, but if the road were frictionless (as is closely approximated by icy conditions) the wheels would simply spin without the car moving anywhere. Because of friction, the car's tire pushes backwards against the road−from Newton's third law, the frictional force acting on the tire must then push it forward. This may seem odd, as we tend to think of friction as being a retarding force only, but true.

21 •• When we push on the brake pedal to slow down a car, a brake pad is pressed against the rotor so that the friction of the pad slows the wheel's rotation. However, the friction of the pad against the rotor *cannot be* the force that slows the car down, because it is an internal force (both the rotor and the wheel are parts of the car, so any forces between them are purely internal to the system). What is the external force that slows down the car? Give a detailed explanation of how this force operates.

Determine the Concept The friction of the tire against the road causes the car to slow down. This is rather subtle, as the tire is in contact with the ground without slipping at all times, and so as you push on the brakes harder, the force of static friction of the road against the tires must increase.

22 •• Give an example of each of the following. (*a*) A three-dimensional object that has no matter at its center of mass. (*b*) A solid object whose center of mass is outside of it. (*c*) A solid sphere whose center of mass does not lie at its geometrical center. (*d*) A long wooden stick whose center of mass does not lie at its middle.

(*a*) A solid spherical shell, or donut, or tire.

(*b*) A solid hemispherical shell.

(*c*) Any sphere with one side a different density than the other, or a density variation that isn't radially symmetric.

(*d*) Any stick with a non-uniform and non-symmetric density variation. A baseball bat is a good example of such a stick.

23 •• **[SSM]** When you are standing upright, your center of mass is located within the volume of your body. However, as you bend over (say to pick up a package), its location changes. Approximately where is it when you are bent over at right angles and what change in your body caused the center of mass location to change? Explain.

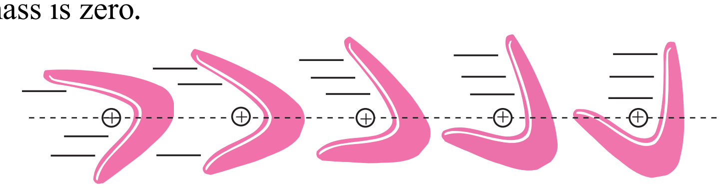
Determine the Concept Relative to the ground, your center of mass moves downward. This is because some of your mass (hips) moved backward, some of your mass (your head and shoulders) moved forward, and the top half of your body moved downward.

24 •• Early on their three-day (one-way) trip to the moon, the Apollo team (late 1960s to early 1970s) would explosively separate the lunar ship from the third-stage booster (that provided the final ″boost″) while still fairly close to Earth. During the explosion, how did the velocity of each of the two pieces of the system change? How did the velocity of the center of mass of the system change? What would be your answers if you were talking about a time a few hours after the explosion? (*Hint:* The system is still well within the gravitational field of Earth and the moon is still far enough away that its gravitational force is much less than that of Earth.)

Determine the Concept The spacecraft speed increased toward the moon. The speed of the third-stage booster decreased but the booster continued to move away from Earth and toward the moon. Right after the explosion the center of mass velocity was the same as before the explosion. A few hours after the explosion, however, the backward pull of gravity of Earth will cause the speed of the center of mass of the system to decrease because the speeds of both the lunar ship and the booster decrease.

25 •• You throw a boomerang and for a while it ″flies″ horizontally in a straight line at a constant speed, while spinning rapidly. Draw a series of pictures, as viewed vertically down from overhead, of the boomerang in different rotational positions as it moves parallel to the surface of Earth. On each picture, indicate the location of the boomerang's center of mass and connect the dots to trace the trajectory of its center of mass. What is the center of mass's acceleration during this part of the flight?

Determine the Concept The diagram shows a spinning boomerang with its center of mass at the location of the circle. As viewed from above, the center of mass moves in a straight line as the boomerang spins about it. The acceleration of the center of mass is zero.

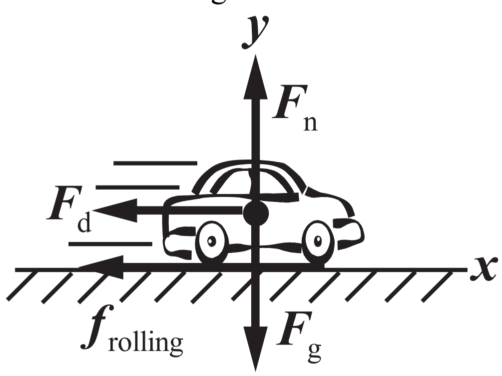


Estimation and Approximation

26 •• To determine the aerodynamic drag on a car, automotive engineers often use the ″coast-down″ method. The car is driven on a long, flat road at some convenient speed (60 mi/h is typical), shifted into neutral, and allowed to coast to a stop. The time that it takes for the speed to drop by successive 5-mi/h intervals is measured and used to compute the net force slowing the car down. (*a*) One day, a group measured that a Toyota Tercel with a mass of 1020 kg coasted down from

60.0 mi/h to 55.0 mi/h in 3.92 s. Estimate the average net force slowing the car down in this speed range. (*b*) If the coefficient of rolling friction for this car is known to be 0.020, what is the force of rolling friction that is acting to slow it down? Assuming that the only two forces acting on the car are rolling friction and aerodynamic drag, what is the average drag force acting on the car? (*c*) The drag force has the form $\frac{1}{2}C\rho Av^2$, where *A* is the cross-sectional area of the car facing into the air, *v* is the car's speed, ρ is the density of air, and *C* is a dimensionless constant of order 1. If the cross-sectional area of the car is 1.91 m^2, determine *C* from the data given. (The density of air is 1.21 kg/m^3; use 57.5 mi/h for the speed of the car in this computation.)

Picture the Problem The forces acting on the Tercel as it slows from 60 to 55 mi/h are a rolling-friction force exerted by the roadway, an air-drag force exerted by the air, the normal force exerted by the roadway, and the gravitational force exerted by the earth. The car is moving in the positive *x* direction. We can use Newton's 2nd law to calculate the average force from the rate at which the car's speed decreases and the rolling force from its definition. The drag force can be inferred from the average- and rolling-friction forces and the drag coefficient from the defining equation for the drag force.



(*a*) Apply $\sum F_x = ma_x$ to the car to relate the average force acting on it to its average velocity:

$$F_{av} = ma_{av} = m\frac{\Delta v}{\Delta t}$$

Substitute numerical values and evaluate F_{av}:

$$\boldsymbol{F}_{av} = (1020\,\text{kg})\frac{5\dfrac{\text{mi}}{\text{h}}\times 1.609\dfrac{\text{km}}{\text{mi}}\times\dfrac{1\,\text{h}}{3600\,\text{s}}\times\dfrac{1000\,\text{m}}{\text{km}}}{3.92\,\text{s}} = 581\,\text{N} = \boxed{0.58\,\text{kN}}$$

(*b*) The rolling-friction force is the product of the coefficient of rolling friction and the normal force:

$$\boldsymbol{f}_{\text{rolling}} = \boldsymbol{\mu}_{\text{rolling}}\boldsymbol{F}_{\text{n}} = \boldsymbol{\mu}_{\text{rolling}}\boldsymbol{mg}$$

Substitute numerical values and evaluate f_{rolling}:

$$f_{\text{rolling}} = (0.020)(1020\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{0.20\,\text{kN}}$$

Assuming that only two forces are acting on the car in the direction of its motion, express their relationship and solve for and evaluate the drag force:

$$\boldsymbol{F}_{\text{av}} = \boldsymbol{F}_{\text{d}} + \boldsymbol{f}_{\text{rolling}}$$

and

$$\boldsymbol{F}_{\text{d}} = \boldsymbol{F}_{\text{av}} - \boldsymbol{f}_{\text{rolling}} = 581\,\text{N} - 200\,\text{N} = \boxed{0.38\,\text{kN}}$$

(*c*) Using the definition of the drag force and its calculated value from (*b*) and the average speed of the car during this 5 mph interval, solve for *C*:

$$\boldsymbol{F}_{\text{d}} = \tfrac{1}{2}\boldsymbol{C}\boldsymbol{\rho}\boldsymbol{A}\boldsymbol{v}^2 \Rightarrow \boldsymbol{C} = \frac{2\boldsymbol{F}_{\text{d}}}{\boldsymbol{\rho}\boldsymbol{A}\boldsymbol{v}^2}$$

Substitute numerical values and evaluate *C*:

$$\boldsymbol{C} = \frac{2(381\,\text{N})}{(1.21\,\text{kg/m}^3)(1.91\,\text{m}^2)\left(57.5\frac{\text{mi}}{\text{h}}\times\frac{1.609\,\text{km}}{\text{mi}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{10^3\,\text{m}}{\text{km}}\right)^2} = \boxed{0.50}$$

27 •• [SSM] Using dimensional analysis, determine the units and dimensions of the constant *b* in the retarding force bv^n if (*a*) $n = 1$ and (*b*) $n = 2$. (*c*) Newton showed that the air resistance of a falling object with a circular cross section should be approximately $\frac{1}{2}\rho\pi r^2 v^2$, where $\rho = 1.20\ \text{kg/m}^3$, the density of air. Show that this is consistent with your dimensional analysis for part (*b*). (*d*) Find the terminal speed for a 56.0-kg skydiver; approximate his cross-sectional area as a disk of radius 0.30 m. The density of air near the surface of Earth is 1.20 kg/m^3. (*e*) The density of the atmosphere decreases with height above the surface of Earth; at a height of 8.0 km, the density is only 0.514 kg/m^3. What is the terminal velocity at this height?

Picture the Problem We can use the dimensions of force and velocity to determine the dimensions of the constant b and the dimensions of ρ, r, and v to show that, for n = 2, Newton's expression is consistent dimensionally with our result from part (b). In Parts (d) and (e), we can apply Newton's 2nd law under terminal velocity conditions to find the terminal velocity of the sky diver near the surface of the earth and at a height of 8 km. Assume that $g = 9.81\ \text{m/s}^2$ remains constant. (Note: At 8 km, $g = 9.78\ \text{m/s}^2$. However, it will not affect the result in Part (e).)

(a) Solve the drag force equation for b with $n = 1$:

$$b = \frac{F_\text{d}}{v}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[\boldsymbol{b}] = \frac{\frac{\text{ML}}{\text{T}^2}}{\frac{\text{L}}{\text{T}}} = \boxed{\frac{\text{M}}{\text{T}}}$$

and the units of b are $\boxed{\text{kg/s}}$

(b) Solve the drag force equation for b with $n = 2$:

$$b = \frac{F_\text{d}}{v^2}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[\boldsymbol{b}] = \frac{\frac{\text{ML}}{\text{T}^2}}{\left(\frac{\text{L}}{\text{T}}\right)^2} = \boxed{\frac{\text{M}}{\text{L}}}$$

and the units of b are $\boxed{\text{kg/m}}$

(c) Express the dimensions of Newton's expression:

$$[\boldsymbol{F}_\text{d}] = \left[\tfrac{1}{2}\boldsymbol{\rho\pi r}^2\boldsymbol{v}^2\right] = \left(\frac{\text{M}}{\text{L}^3}\right)(\text{L})^2\left(\frac{\text{L}}{\text{T}}\right)^2$$

$$= \boxed{\frac{\text{ML}}{\text{T}^2}}$$

From Part (b) we have:

$$[\boldsymbol{F}_\text{d}] = [\boldsymbol{bv}^2] = \left(\frac{\text{M}}{\text{L}}\right)\left(\frac{\text{L}}{\text{T}}\right)^2 = \boxed{\frac{\text{ML}}{\text{T}^2}}$$

(d) Letting the downward direction be the positive y direction, apply $\sum F_y = ma_y$ to the sky diver:

$$mg - \tfrac{1}{2}\rho\pi r^2 v_\text{t}^2 = 0 \Rightarrow \boldsymbol{v}_\text{t} = \sqrt{\frac{2\boldsymbol{mg}}{\boldsymbol{\rho\pi r}^2}}$$

Substitute numerical values and evaluate v_t:

$$v_t = \sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(1.2\,\text{kg/m}^3)(0.30\,\text{m})^2}} = \boxed{57\,\text{m/s}}$$

(*e*) Evaluate v_t at a height of 8 km:

$$v_t = \sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(0.514\,\text{kg/m}^3)(0.30\,\text{m})^2}} = \boxed{87\,\text{m/s}}$$

28 •• Estimate the terminal velocity of an average sized raindrop and a golf-ball- sized hailstone. (*Hint*: See Problems 26 and 27.)

Picture the Problem From Newton's 2nd law, the equation describing the motion of falling raindrops and large hailstones is $mg - F_d = ma$ where $F_d = \frac{1}{2}\rho\pi r^2 v^2 = bv^2$ is the drag force. Under terminal speed conditions ($a = 0$), the drag force is equal to the weight of the falling object. Take the radius of a raindrop to be 0.50 mm and the radius of a golf-ball sized hailstone to be 2.0 cm.

Express the relationship between v_t and the weight of a falling object under terminal speed:

$$bv_t^2 = mg \Rightarrow v_t = \sqrt{\frac{mg}{b}} \qquad (1)$$

Using $b = \frac{1}{2}\pi\rho r^2$, evaluate b_r:

$$b_r = \tfrac{1}{2}\pi(1.2\,\text{kg/m}^3)(0.50\times10^{-3}\,\text{m})^2 = 4.71\times10^{-7}\,\text{kg/m}$$

Evaluating b_h yields:

$$b_h = \tfrac{1}{2}\pi(1.2\,\text{kg/m}^3)(2.0\times10^{-2}\,\text{m})^2 = 7.54\times10^{-4}\,\text{kg/m}$$

Express the mass of a sphere in terms of its volume and density:

$$m = \rho V = \frac{4\pi r^3\rho}{3}$$

Using $\rho_r = 1.0\times10^3\,\text{kg/m}^3$, evaluate m_r:

$$m_r = \frac{4\pi(0.50\times10^{-3}\,\text{m})^3(1.0\times10^3\,\text{kg/m}^3)}{3} = 5.24\times10^{-7}\,\text{kg}$$

Using $\rho_h = 920\,\text{kg/m}^3$, evaluate m_h:

$$m_h = \frac{4\pi(2.0\times10^{-2}\,\text{m})^3(920\,\text{kg/m}^3)}{3} = 3.08\times10^{-2}\,\text{kg}$$

Substitute numerical values in equation (1) and evaluate $v_{t,r}$:

$$v_{t,r} = \sqrt{\frac{(5.24\times10^{-7}\text{ kg})(9.81\text{ m/s}^2)}{4.71\times10^{-7}\text{ kg/m}}} = \boxed{3.3\text{ m/s}}$$

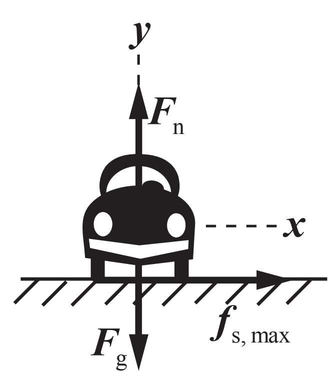
Substitute numerical values in equation (1) and evaluate $v_{t,h}$:

$$v_{t,h} = \sqrt{\frac{(3.08\times10^{-2}\text{ kg})(9.81\text{ m/s}^2)}{7.54\times10^{-4}\text{ kg/m}}} = \boxed{20\text{ m/s}}$$

29 •• Estimate the minimum coefficient of static friction needed between a car's tires and the pavement in order to complete a left turn at a city street intersection at the posted straight-ahead speed limit of 25 mph and on narrow inner-city streets. Comment on the wisdom of attempting such a turn at that speed.

Picture the Problem In order to perform this estimate, we need to determine a rough radius of curvature for the car's turn in a normal city intersection. Assuming the car goes from right-hand lane to right-hand lane, and assuming fairly normal dimensions of 40 feet for the width of the street, the center of the car's path travels along a circle of, say, 30 feet in radius. The net centripetal force is provided by the force of static friction and the acceleration of the car is equal to this net force divided by the mass of the car. Finally, we solve for the coefficient of static friction.

A diagram showing the forces acting on the car as it rounds the curve is shown to the right.



Apply $\sum F_x = ma_x$ to the car's tires:

$$f_{s,\max} = ma_x = m\frac{v^2}{r}$$

or, because $f_{s,\max} = \mu_s F_n$

$$\mu_s F_n = m\frac{v^2}{r} \Rightarrow \mu_s = \frac{mv^2}{rF_n} \qquad (1)$$

Apply $\sum F_y = ma_y$ to the car's tires:

$$F_n - F_g = 0$$

or, because $F_g = mg$,

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Substituting for F_n in equation (1) yields:

$$\mu_s = \frac{mv^2}{rmg} = \frac{v^2}{rg}$$

Substitute numerical values and evaluate μ_s:

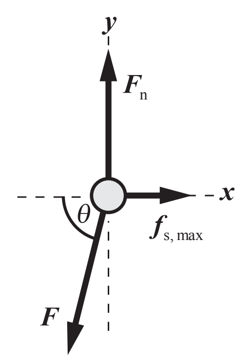
$$\mu_s = \frac{\left(25\,\frac{\text{mi}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1609\,\text{m}}{\text{mi}}\right)^2}{\left(30\,\text{ft} \times 0.3048\,\frac{\text{m}}{\text{ft}}\right)\left(9.81\,\text{m/s}^2\right)} = \boxed{1.4}$$

This is probably not such a good idea. Tires on asphalt or concrete have a maximum coefficient of static friction of about 1.

30 •• Estimate the widest stance you can take when standing on a dry, icy surface. That is, how wide can you safely place your feet and not slip into an undesired ″split?″ Let the coefficient of static friction of rubber on ice be roughly 0.25.

Picture the Problem We need to estimate the forces active at the place of each foot. Assuming a symmetrical stance, with the defining angle being the angle between each leg and the ground, θ, we can then draw a force diagram and apply Newton's 2nd law to your foot. The free-body diagram shows the normal force, exerted by the icy surface, the maximum static friction force, also exerted by the icy surface, and the force your weight exerts on your foot.

A free-body diagram showing the forces acting on one foot is shown to the right.



Apply $\sum \vec{F} = m\vec{a}$ to one of your feet:

$$\sum F_x = f_{s,\max} - F_g \cos\theta = 0$$

and

$$\sum F_y = F_n - F_g \sin\theta = 0$$

Because $F_g = mg$ and $f_{s,\,max} = \mu_s F_n$:

$$\boldsymbol{\mu}_s \boldsymbol{F}_n - \boldsymbol{mg}\cos\boldsymbol{\theta} = 0 \qquad (1)$$

and

$$\boldsymbol{F}_n = \boldsymbol{mg}\sin\boldsymbol{\theta}$$

Substituting for F_n in equation (1) yields:

$$\boldsymbol{\mu}_s \boldsymbol{mg}\sin\boldsymbol{\theta} - \boldsymbol{mg}\cos\boldsymbol{\theta} = 0$$

Solving for θ yields:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{1}{\boldsymbol{\mu}_s}\right)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{1}{0.25}\right) = 76°$$

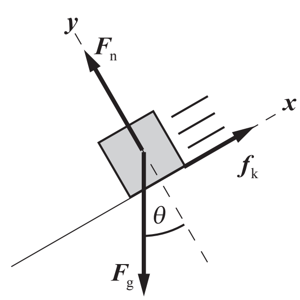
This angle corresponds to an angle between your legs of about 28°.

Friction

31 • **[SSM]** A block of mass m slides at constant speed down a plane inclined at an angle of θ with the horizontal. It follows that (*a*) $\mu_k = mg \sin\theta$, (*b*) $\mu_k = \tan\theta$, (*c*) $\mu_k = 1 - \cos\theta$, (*d*) $\mu_k = \cos\theta - \sin\theta$.

Picture the Problem The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, and $\vec{f}_k$; that is, $\vec{F}_n + m\vec{g} + \vec{f}_k = 0$. We can apply Newton's 2nd law to determine the relationship between f_k, θ, and mg.

A pictorial representation showing the forces acting on the sliding block is shown to the right.



Using its definition, express the coefficient of kinetic friction:

$$\mu_k = \frac{f_k}{F_n} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$f_k - mg\sin\theta = ma_x$$

or, because $a_x = 0$,

$$f_k = mg\sin\theta$$

Apply $\sum F_y = ma_y$ to the block:

$F_n - mg\cos\theta = ma_y$

or, because $a_y = 0$,

$F_n = mg\cos\theta$

Substitute for f_k and F_n in equation (1) and simplify to obtain:

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

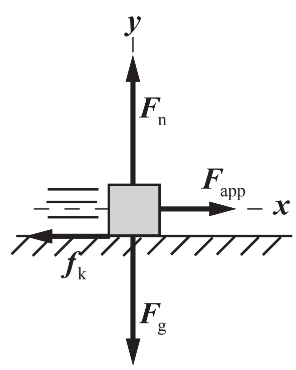
and $\boxed{(\boldsymbol{b})}$ is correct.

32 • A block of wood is pulled at constant velocity by a horizontal string across a horizontal surface with a force of 20 N. The coefficient of kinetic friction between the surfaces is 0.3. The force of friction is (*a*) impossible to determine without knowing the mass of the block, (*b*) impossible to determine without knowing the speed of the block, (*c*) 0.30 N, (*d*) 6.0 N, or (*e*) 20 N.

Picture the Problem The block is in equilibrium under the influence of $\vec{F}_n$, $\vec{F}_g$, $\vec{F}_{app}$, and $\vec{f}_k$; that is

$$\vec{F}_n + \vec{F}_g + \vec{F}_{app} + \vec{f}_k = 0$$

We can apply Newton's 2nd law to determine f_k.



Apply $\sum F_x = ma_x$ to the block:

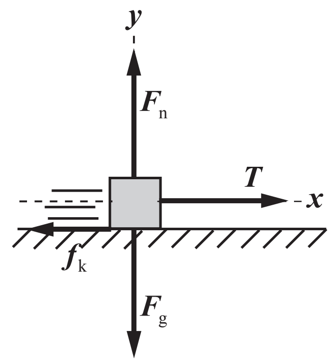
$F_{app} - f_k = ma_x$

or, because $a_x = 0$,

$f_k = F_{app} = 20\,\text{N}$ and $\boxed{(\boldsymbol{e})}$ is correct.

33 • **[SSM]** A block weighing 20-N rests on a horizontal surface. The coefficients of static and kinetic friction between the surface and the block are $\mu_s = 0.80$ and $\mu_k = 0.60$. A horizontal string is then attached to the block and a constant tension T is maintained in the string. What is the subsequent force of friction acting on the block if (*a*) $T = 15$ N or (*b*) $T = 20$ N?

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force. We can apply the definition of the maximum static friction to decide whether $f_{s,max}$ or T is greater.



Noting that $F_n = F_g$, calculate the maximum static friction force:

$$f_{s,max} = \mu_s F_n = \mu_s F_g = (0.80)(20\text{ N}) = 16\text{ N}$$

(*a*) Because $f_{s,max} > T$:

$$f = f_s = T = \boxed{15\text{ N}}$$

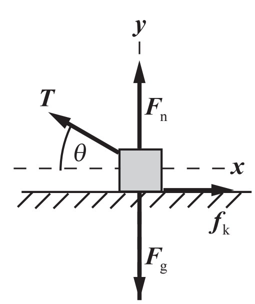
(*b*) Because $T > f_{s,max}$:

$$f = f_k = \mu_k F_n = \mu_k F_g = (0.60)(20\text{ N}) = \boxed{12\text{ N}}$$

34 • A block of mass *m* is pulled at a constant velocity across a horizontal surface by a string as shown in Figure 5-60. The magnitude of the frictional force is (*a*) $\mu_k\, mg$, (*b*) $T\cos\theta$, (*c*) $\mu_k\,(T - mg)$, (*d*) $\mu_k\, T\sin\theta$, or (*e*) $\mu_k\,(mg - T\sin\theta)$.

Picture the Problem The block is in equilibrium under the influence of the forces $\vec{T}$, $\vec{f}_k$, $\vec{F}_n$, and $\vec{F}_g$; that is $\vec{T} + \vec{f}_k + \vec{F}_g + \vec{F}_n = 0$. We can apply Newton's 2nd law to determine the relationship between T and f_k.

A free-body diagram showing the forces acting on the block is shown to the right.



Apply $\sum F_x = ma_x$ to the block:

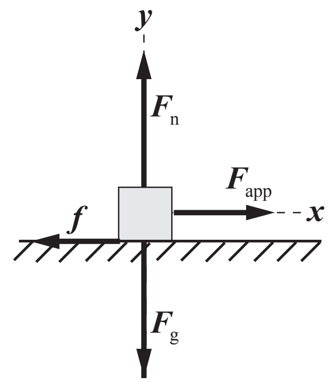
$$-T\cos\theta + f_k = ma_x$$

Because $a_x = 0$:

$$f_k = T\cos\theta \text{ and } \boxed{(b)} \text{ is correct.}$$

35 • **[SSM]** A 100-kg crate rests on a thick-pile carpet. A weary worker then pushes on the crate with a horizontal force of 500 N. The coefficients of static and kinetic friction between the crate and the carpet are 0.600 and 0.400, respectively. Find the subsequent frictional force exerted by the carpet on the crate.

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force. If it is, then the box moves and the friction force is the force of kinetic friction. If it is less, the box does not move.



The maximum static friction force is given by:

$$\boldsymbol{f}_{s,max} = \boldsymbol{\mu}_s \boldsymbol{F}_n$$

or, because $F_n = F_g = mg$,

$$\boldsymbol{f}_{s,max} = \boldsymbol{\mu}_s \boldsymbol{mg}$$

Substitute numerical values and evaluate $f_{s,max}$:

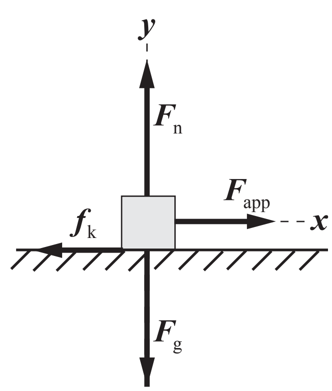
$$f_{s,max} = (0.600)(100\,\text{kg})(9.81\,\text{m/s}^2) = 589\,\text{N}$$

Because $f_{s,\,max} > F_{app}$, the box does not move and :

$$F_{app} = f_s = \boxed{500\,\text{N}}$$

36 • A box weighing 600 N is pushed along a horizontal floor at constant velocity with a force of 250 N parallel to the floor. What is the coefficient of kinetic friction between the box and the floor?

Picture the Problem Because the box is moving with constant velocity, its acceleration is zero and it is in equilibrium under the influence of $\vec{F}_{app}$, $\vec{F}_n$, $\vec{F}_g$, and $\vec{f}_k$; that is, $\vec{F}_{app} + \vec{F}_n + \vec{F}_g + \vec{f}_k = 0$. We can apply Newton's 2nd law to determine the relationship between f_k and mg.



The definition of μ_k is:

$$\mu_k = \frac{f_k}{F_n} \qquad (1)$$

Apply $\sum F_y = ma_y$ to the box:

$\boldsymbol{F}_n - \boldsymbol{F}_g = m\boldsymbol{a}_y$

or, because $a_y = 0$,

$\boldsymbol{F}_n - \boldsymbol{F}_g = 0 \Rightarrow \boldsymbol{F}_n = \boldsymbol{F}_g = m\boldsymbol{g} = 600\,\text{N}$

Apply $\sum F_x = ma_x$ to the box:

$F_{app} - f_k = ma_x$

or, because $a_x = 0$,

$F_{app} = f_k = 250\,\text{N}$

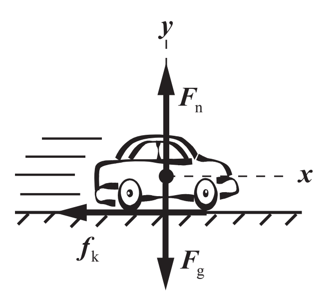
Substitute numerical values in equation (1) and evaluate μ_k:

$$\mu_k = \frac{250\,\text{N}}{600\,\text{N}} = \boxed{0.417}$$

37 • **[SSM]** The coefficient of static friction between the tires of a car and a horizontal road is 0.60. Neglecting air resistance and rolling friction, (*a*) what is the magnitude of the maximum acceleration of the car when it is braked? (*b*) What is the shortest distance in which the car can stop if it is initially traveling at 30 m/s?

Picture the Problem Assume that the car is traveling to the right and let the positive *x* direction also be to the right. We can use Newton's 2nd law of motion and the definition of μ_s to determine the maximum acceleration of the car. Once we know the car's maximum acceleration, we can use a constant-acceleration equation to determine the least stopping distance.

(*a*) A pictorial representation showing the forces acting on the car is shown to the right.



Apply $\sum F_x = ma_x$ to the car:

$$-f_{s,max} = -\mu_s F_n = ma_x \qquad (1)$$

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$\boldsymbol{F}_n - \boldsymbol{F}_g = m\boldsymbol{a}_y$

$F_n - w = ma_y = 0$

or, because $a_y = 0$ and $F_g = mg$,

$$\boldsymbol{F}_n = m\boldsymbol{g} \qquad (2)$$

Substitute for F_n in equation (1) to obtain:

$$-f_{s,max} = -\mu_s mg = ma_x$$

Solving for $a_{x,max}$ yields:

$$a_{x,max} = \mu_s g$$

Substitute numerical values and evaluate $a_{x,max}$:

$$a_{x,max} = (0.60)(9.81\,\text{m/s}^2) = 5.89\,\text{m/s}^2$$
$$= \boxed{5.9\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the stopping distance of the car to its initial velocity and its acceleration:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

or, because $v_x = 0$,

$$0 = v_{0x}^2 + 2a_x\Delta x \Rightarrow \Delta x = \frac{-v_{0x}^2}{2a_x}$$

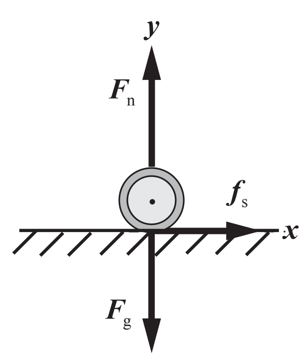
Using $a_x = -5.89$ m/s^2 because the acceleration of the car is to the left, substitute numerical values and evaluate Δx:

$$\Delta x = \frac{-(30\,\text{m/s})^2}{2(-5.89\,\text{m/s}^2)} = \boxed{76\,\text{m}}$$

38 • The force that accelerates a car along a flat road is the frictional force exerted by the road on the car's tires. (*a*) Explain why the acceleration can be greater when the wheels do not slip. (*b*) If a car is to accelerate from 0 to 90 km/h in 12 s, what is the minimum coefficient of friction needed between the road and tires? Assume that the drive wheels support exactly half the weight of the car.

Picture the Problem We can use the definition of acceleration and apply Newton's 2nd law to the horizontal and vertical components of the forces to determine the minimum coefficient of friction between the road and the tires.

(*a*) The free-body diagram shows the forces acting on the tires on the drive wheels, the tires we're assuming support half the weight of the car.



Because $\mu_s > \mu_k$, f will be greater if the wheels do not slip.

(*b*) Apply $\sum F_x = ma_x$ to the car:

$$f_s = \mu_s F_n = ma_x \qquad (1)$$

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$$F_n - \tfrac{1}{2}mg = ma_y$$

or, because $a_y = 0$ and $\boldsymbol{F}_g = \tfrac{1}{2}\boldsymbol{mg}$,

$$\Rightarrow \boldsymbol{F}_n = \tfrac{1}{2}\boldsymbol{mg}$$

Substituting for F_n in equation (1) yields:

$$\tfrac{1}{2}\mu_s mg = ma_x \Rightarrow \mu_s = \frac{2a_x}{g}$$

Substitute for a_x to obtain:

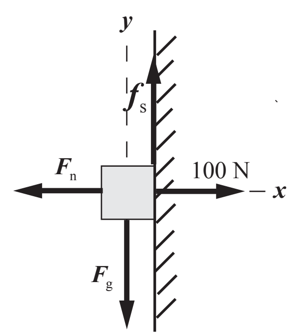
$$\mu_s = \frac{2\Delta v}{g\Delta t}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{2\left(90\,\frac{\text{km}}{\text{h}} \times \frac{10^3\ \text{m}}{\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)}{\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(12\,\text{s})} = \boxed{0.42}$$

39 •• A 5.00-kg block is held at rest against a vertical wall by a horizontal force of 100 N. (*a*) What is the frictional force exerted by the wall on the block? (*b*) What is the minimum horizontal force needed to prevent the block from falling if the static coefficient of friction between the wall and the block is 0.400?

Picture the Problem The block is in equilibrium under the influence of the forces shown on the force diagram. We can use Newton's 2nd law and the definition of μ_s to solve for f_s and F_n.



(*a*) Apply $\sum F_y = ma_y$ to the block:

$$f_s - mg = ma_y$$

or, because $a_y = 0$,

$$f_s - mg = 0 \Rightarrow \boldsymbol{f}_s = \boldsymbol{mg}$$

Substitute numerical values and evaluate f_s :

$$f_s = (5.00\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49.1\,\text{N}}$$

(*b*) Use the definition of μ_s to express F_n:

$$F_n = \frac{f_{s,\max}}{\mu_s}$$

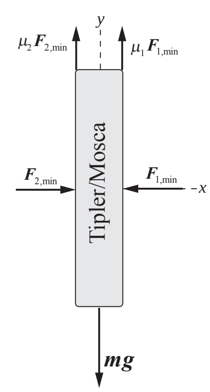
Substitute numerical values and evaluate F_n:

$$F_n = \frac{49.1\,\text{N}}{0.400} = \boxed{123\,\text{N}}$$

40 •• A tired and overloaded student is attempting to hold a large physics textbook wedged under his arm, as shown in Figure 5-61. The textbook has a mass of 3.2 kg, while the coefficient of static friction of the textbook against the student's underarm is 0.320 and the coefficient of static friction of the book against the student's shirt is 0.160. (*a*) What is the minimum horizontal force that the student must apply to the textbook to prevent it from falling? (*b*) If the student can only exert a force of 61 N, what is the acceleration of the textbook as it slides from under his arm? The coefficient of kinetic friction of arm against textbook is 0.200, while that of shirt against textbook is 0.090.

Picture the Problem We can apply Newton's 2nd law to relate the minimum force required to hold the book in place to its mass and to the coefficients of static friction. In Part (*b*), we can proceed similarly to relate the acceleration of the book to the coefficients of kinetic friction.

(*a*) The force diagram shows the forces acting on the book. The normal force is the net force the student exerts in squeezing the book. Let the horizontal direction be the *x* direction and upward the *y* direction. Note that the normal force is the same on either side of the book because it is not accelerating in the horizontal direction. The book could be accelerating downward.



Apply $\sum \vec{F} = m\vec{a}$ to the book:

$$\sum F_x = F_{2,\text{min}} - F_{1,\text{min}} = 0$$

and

$$\sum F_y = \mu_{s,1} F_{1,\text{min}} + \mu_{s,2} F_{2,\text{min}} - mg = 0$$

Noting that $F_{1,\text{min}} = F_{2,\text{min}}$, solve the *y* equation for F_{min}:

$$F_{\text{min}} = \frac{mg}{\mu_{s,1} + \mu_{s,2}}$$

Substitute numerical values and evaluate F_{min}:

$$F_{\text{min}} = \frac{(3.2\,\text{kg})(9.81\,\text{m/s}^2)}{0.320 + 0.160} = \boxed{65\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ with the book accelerating downward, to obtain:

$$\sum F_y = \mu_{k,1} F + \mu_{k,2} F - mg = ma$$

Solving for a_y yields:

$$a_y = \frac{\mu_{k,1} + \mu_{k,2}}{m}F - g$$

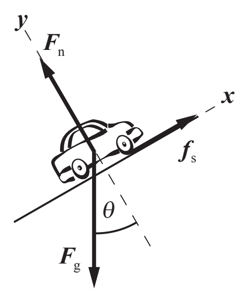
Substitute numerical values and evaluate a_y:

$$a_y = \left|\left(\frac{0.200 + 0.090}{3.2\,\text{kg}}\right)(61\,\text{N}) - 9.81\,\text{m/s}^2\right|$$
$$= \boxed{4.3\,\text{m/s}^2\text{, downward}}$$

41 •• You are racing in a rally on a snowy day when the temperature is near the freezing point. The coefficient of static friction between a car's tires and an icy road is 0.080. Your crew boss is concerned about some of the hills on the course and wants you to think about switching to studded tires. To address the issue, he wants to compare the actual hill angles on the course to see which of them your car can negotiate. (*a*) What is the angle of the steepest incline that a vehicle with four-wheel drive can climb at constant speed? (*b*) Given that the hills are icy, what is the steepest possible hill angle for the same four-wheel drive car to descend at constant speed?

Picture the Problem We can use the definition of the coefficient of static friction and Newton's 2nd law to relate the angle of the incline to the forces acting on the car.

(*a*) The free-body diagram shows the forces acting on the car when it is either moving up the hill or down the hill without acceleration. The friction force that the ground exerts on the tires is the force f_s shown acting up the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_s - F_g \sin\theta = 0 \qquad (1)$$
and
$$\sum F_y = F_n - F_g \cos\theta = 0 \qquad (2)$$

Because $F_g = mg$, equations (1) and (2) become:

$$f_s - mg\sin\theta = 0 \qquad (3)$$
and
$$F_n - mg\cos\theta = 0 \qquad (4)$$

Solving equation (3) for f_s and equation (4) for F_n yields:

$$f_s = mg\sin\theta$$
and
$$F_n = mg\cos\theta$$

Use the definition of μ_s to relate f_s and F_n:

$$\mu_s = \frac{f_s}{F_n} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

Solving for θ yields:

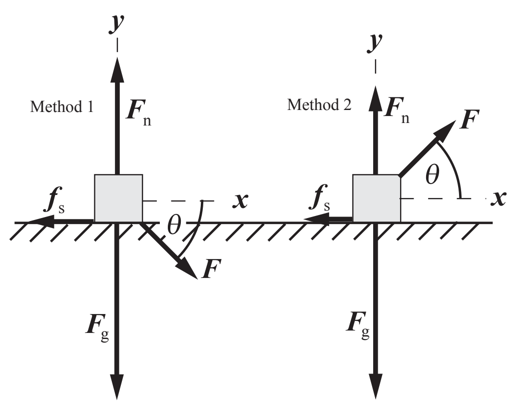
$$\boldsymbol{\theta} = \tan^{-1}(\boldsymbol{\mu}_s) = \tan^{-1}(0.080) = \boxed{4.6°}$$

(*b*) Proceed exactly as in (*a*) to obtain:

$$\boldsymbol{\theta} = \tan^{-1}(0.080) = \boxed{4.6°}$$

42 •• A 50-kg box that is resting on a level floor must be moved. The coefficient of static friction between the box and the floor is 0.60. One way to move the box is to push down on the box at an angle θ below the horizontal. Another method is to pull up on the box at an angle θ above the horizontal. (*a*) Explain why one method requires less force than the other. (*b*) Calculate the minimum force needed to move the box by each method if θ = 30º and compare the answer with the results when θ = 0°.

Picture the Problem The free-body diagrams for the two methods are shown to the right. Method 1 results in the box being pushed into the floor, increasing the normal force and the static friction force. Method 2 partially lifts the box,, reducing the normal force and the static friction force. We can apply Newton's 2nd law to obtain expressions that relate the maximum static friction force to the applied force $\vec{F}$.



(*a*) Method 2 is preferable because it reduces F_n and, therefore, f_s.

(*b*) Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta - f_x = F\cos\theta - \mu_s F_n = 0$$

Method 1: Apply $\sum F_y = ma_y$ to the block and solve for F_n:

$$F_n - mg - F\sin\theta = 0$$

and

$$F_n = mg + F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg + F\sin\theta) \quad (1)$$

Method 2: Apply $\sum F_y = ma_y$ to the forces in the y direction and solve for F_n:

$$F_n - mg + F\sin\theta = 0$$

and

$$F_n = mg - F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg - F\sin\theta) \quad (2)$$

Express the condition that must be satisfied to move the box by either method:

$$f_{s,max} = F\cos\theta \quad (3)$$

Method 1: Substitute (1) in (3) and solve for F:

$$F_1 = \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} \quad (4)$$

Method 2: Substitute (2) in (3) and solve for F:

$$F_2 = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \quad (5)$$

Substitute numerical values and evaluate equations (4) and (5) with $\theta = 30°$:

$$\boldsymbol{F}_1(30°) = \frac{(0.60)(50\text{ kg})(9.81\text{ m/s}^2)}{\cos 30° - (0.60)\sin 30°} = \boxed{0.52\text{ kN}}$$

and

$$\boldsymbol{F}_2(30°) = \frac{(0.60)(50\text{ kg})(9.81\text{ m/s}^2)}{\cos 30° + (0.60)\sin 30°} = \boxed{0.25\text{ kN}}$$

Evaluate equations (4) and (5) with $\theta = 0°$:

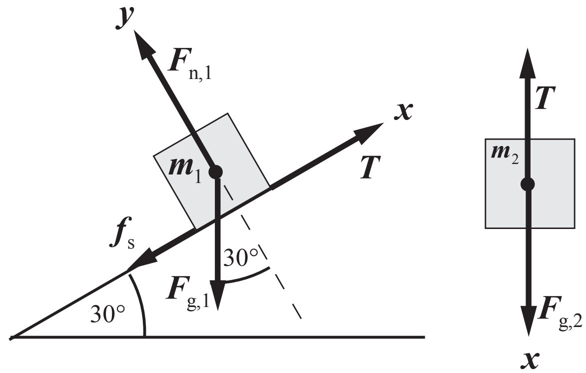
$$\boldsymbol{F}_1(0°) = \frac{(0.60)(50\text{ kg})(9.81\text{ m/s}^2)}{\cos 0° - (0.60)\sin 0°} = \boxed{0.29\text{ kN}}$$

and

$$\boldsymbol{F}_2(0°) = \frac{(0.60)(50\text{ kg})(9.81\text{ m/s}^2)}{\cos 0° + (0.60)\sin 0°} = \boxed{0.29\text{ kN}}$$

43 •• **[SSM]** A block of mass m_1 = 250 g is at rest on a plane that makes an angle of θ = 30° with the horizontal. The coefficient of kinetic friction between the block and the plane is 0.100. The block is attached to a second block of mass m_2 = 200 g that hangs freely by a string that passes over a frictionless, massless pulley (Figure 5-62). When the second block has fallen 30.0 cm, what will be its speed?

Picture the Problem Choose a coordinate system in which the +x direction is up the incline for the block whose mass is m_1 and downward for the block whose mass is m_2. We can find the speed of the system when it has moved a given distance by using a constant-acceleration equation. We'll assume that the string is massless and that it does not stretch. Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration a. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system.



Using a constant-acceleration equation, relate the speed of the system to its acceleration and displacement:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

and, because $v_{0x} = 0$,

$$v_x^2 = 2a_x\Delta x \Rightarrow v_x = \sqrt{2a_x\Delta x} \qquad (1)$$

Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\sum F_x = T - f_k - F_{g,1}\sin 30° = m_1 a_x \qquad (2)$$

and

$$\sum F_y = F_{n,1} - F_{g,1}\cos 30° = 0 \qquad (3)$$

Because $F_{g,1} = m_1 g$, equations (2) and (3) can be written as:

$$T - f_k - m_1 g\sin 30° = m_1 a_x \qquad (4)$$

and

$$F_{n,1} = m_1 g\cos 30° \qquad (5)$$

Using $f_k = \mu_k F_{n,1}$, substitute equation (5) in equation (4) to obtain:

$$T - \mu_k m_1 g\cos 30° - m_1 g\sin 30° = m_1 a_x \qquad (6)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$F_{g,2} - T = m_2 a_x$$

or, because $F_{g,2} = m_2 g$,

$$m_2 g - T = m_2 a_x \qquad (7)$$

Add equations (6) and (7) to eliminate T and solve for a_x to obtain:

$$a_x = \frac{(m_2 - \mu_k m_1\cos 30° - m_1\sin 30°)g}{m_1 + m_2}$$

Substituting for a_x in equation (1) and simplifying yields:

$$v_x = \sqrt{\frac{2[m_2 - m_1(\mu_k \cos 30° + \sin 30°)]g\Delta x}{m_1 + m_2}}$$

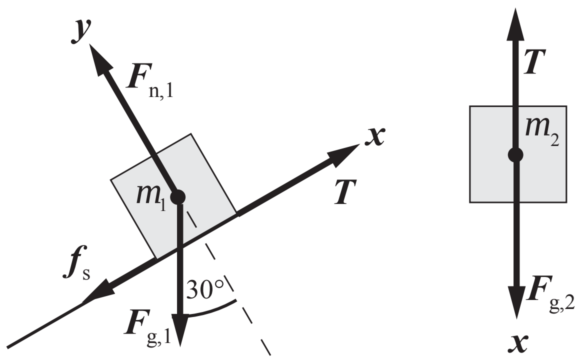
Substitute numerical values and evaluate v_x:

$$v_x = \sqrt{\frac{2[0.200\,\text{kg} - (0.250\,\text{kg})((0.100)\cos 30° + \sin 30°)](9.81\,\text{m/s}^2)(0.300\,\text{m})}{0.250\,\text{kg} + 0.200\,\text{kg}}}$$

$$= \boxed{84\,\text{m/s}}$$

44 •• In Figure 5-62 $m_1 = 4.0$ kg and the coefficient of static friction between the block and the incline is 0.40. (*a*) Find the range of possible values for m_2 for which the system will be in static equilibrium. (*b*) Find the frictional force on the 4.0-kg block if $m_1 = 1.0$ kg?

Picture the Problem Choose a coordinate system in which the +*x* direction is up the incline for the block whose mass is m_1 and downward for the block whose mass is m_2. We'll assume that the string is massless and that it does not stretch. Under the influence of the forces shown in the free-body diagrams, the blocks are in static equilibrium. While f_s can be either up or down the incline, the free-body diagram shows the situation in which motion is impending up the incline. The application of Newton's 2nd law to each block, followed by the elimination of the tension *T* and the use of the definition of f_s, will allow us to determine the range of values for m_2.



(*a*) Noting that $F_{g,1} = m_1 g$, apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\sum F_x = T \pm f_{s,max} - m_1 g \sin 30° = 0 \quad (1)$$

and

$$\sum F_y = F_{n,1} - m_1 g \cos 30° = 0 \quad (2)$$

Using $f_{s,\max} = \mu_s F_n$, substitute equation (2) in equation (1) to obtain:

$$T \pm \mu_s m_1 g \cos 30° - m_1 g \sin 30° = 0 \quad (3)$$

Noting that $\boldsymbol{F}_{g,2} = \boldsymbol{m}_2\boldsymbol{g}$, apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = 0 \qquad (4)$$

Add equations (3) and (4) to eliminate T and solve for m_2:

$$\boldsymbol{m}_2 = \boldsymbol{m}_1(\pm \boldsymbol{\mu}_s \cos 30° + \sin 30°) \qquad (5)$$

Substitute numerical values to obtain:

$$\boldsymbol{m}_2 = (4.0\,\text{kg})[\pm(0.40)\cos 30° + \sin 30°]$$

Denoting the value of m_2 with a plus sign as $m_{2,+}$ and the value of m_2 with the minus sign as $m_{2,-}$ determine the range of values of m_2 for which the system is in static equilibrium:

$$\boldsymbol{m}_{2,+} = 3.4\,\text{kg and m}_{2,-} = 0.61\,\text{kg}$$

and

$$\boxed{0.61\,\text{kg} \le \boldsymbol{m}_2 \le 3.4\,\text{kg}}$$

(*b*) With $m_2 = 1$ kg, the impending motion is down the incline and the static friction force is up the incline. Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T + f_s - m_1 g \sin 30° = 0 \qquad (6)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = 0 \qquad (7)$$

Add equations (6) and (7) and solve for f_s to obtain:

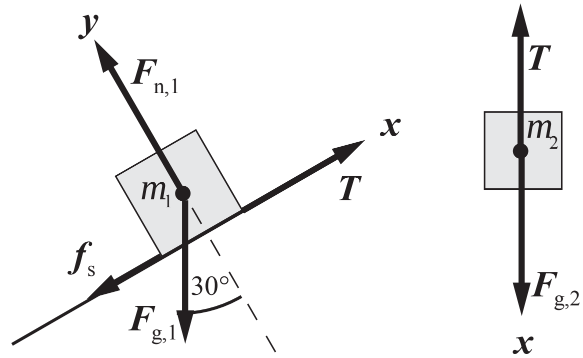
$$f_s = (m_1 \sin 30° - m_2)g$$

Substitute numerical values and evaluate f_s:

$$\boldsymbol{f}_s = [(4.0\,\text{kg})\sin 30° - 1.0\,\text{kg}](9.81\,\text{m/s}^2) = \boxed{9.8\,\text{N}}$$

45 •• In Figure 5-62, $m_1 = 4.0$ kg, $m_2 = 5.0$ kg, and the coefficient of kinetic friction between the inclined plane and the 4.0-kg block is $\mu_k = 0.24$. Find the magnitude of the acceleration of the masses and the tension in the cord.

Picture the Problem Choose a coordinate system in which the +*x* direction is up the incline for the block whose mass is m_1 and downward for the block whose mass is m_2. We'll assume that the string is massless and that it does not stretch. Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration *a*. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system. Finally, we can substitute for the tension in either of the motion equations to determine the acceleration of the masses.



Noting that $F_{g,1} = m_1 g$, apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\sum F_x = T - f_k - m_1 g \sin 30° = m_1 a_x \quad (1)$$

and

$$\sum F_y = F_{n,1} - m_1 g \cos 30° = 0 \quad (2)$$

Using $f_k = \mu_k F_n$, substitute equation (2) in equation (1) to obtain:

$$T - \mu_k m_1 g \cos 30° - m_1 g \sin 30° = m_1 a_x \quad (3)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = m_2 a_x \quad (4)$$

Add equations (3) and (4) to eliminate T and solve for a_x to obtain:

$$a_x = \frac{(m_2 - \mu_k m_1 \cos 30° - m_1 \sin 30°)g}{m_1 + m_2}$$

Substituting numerical values and evaluating a_x yields:

$$a_x = \frac{[5.0\text{ kg} - (0.24)(4.0\text{ kg})\cos 30° - (4.0\text{ kg})\sin 30°](9.81\text{ m/s}^2)}{4.0\text{ kg} + 5.0\text{ kg}} = \boxed{2.4\text{ m/s}^2}$$

Solving equation (3) for T yields:

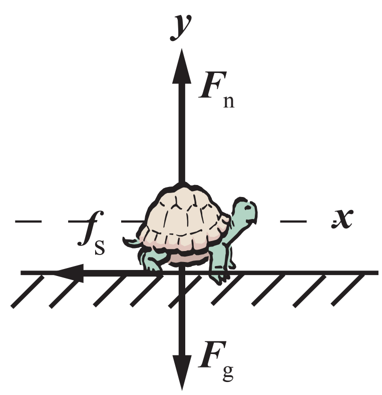
$$T = m_1 a_x + [\mu_k \cos 30° + \sin 30°] m_1 g$$

Substitute numerical values and evaluate T:

$$T = (4.0\text{ kg})(2.36\text{ m/s}^2) + [(0.24)\cos 30° + \sin 30°](4.0\text{ kg})(9.81\text{ m/s}^2) = \boxed{37\text{ N}}$$

46 •• A 12-kg turtle rests on the bed of a zookeeper's truck, which is traveling down a country road at 55 mi/h. The zookeeper spots a deer in the road, and slows to a stop in 12 s. Assuming constant acceleration, what is the minimum coefficient of static friction between the turtle and the truck bed surface is needed to prevent the turtle from sliding?

Picture the Problem We can determine the acceleration necessary for the truck and turtle by considering the displacement of both during the given time interval. The static friction force must provide the necessary acceleration for the turtle. The turtle, if it is not to slip, must have this acceleration which is produced by the static friction force acting on it



The required coefficient of static friction is given by:

$$\mu_s = \frac{f_s}{F_n} \quad (1)$$

Letting m represent the mass of the turtle, apply $\sum \vec{F} = m\vec{a}$ to the turtle:

$$\sum F_x = -f_s = ma_x \quad (2)$$

and

$$\sum F_y = F_n - F_g = ma_y \quad (3)$$

Solving equation (2) for f_s yields:

$$f_s = -ma_x$$

Because $a_y = 0$ and $F_g = mg$, equation (3) becomes:

$$F_n = F_g = mg$$

Substituting for f_s and F_n in equation (1) and simplifying yields:

$$\mu_s = \frac{-ma_x}{mg} = \frac{-a_x}{g} \quad (4)$$

The acceleration of the truck and turtle is given by:

$$a_x = \frac{\Delta v}{\Delta t} = \frac{v_{f,x} - v_{i,x}}{\Delta t}$$

or, because $v_{f,x} = 0$,

$$a_x = \frac{-v_{i,x}}{\Delta t}$$

Substitute for a_x in equation (4) to obtain:

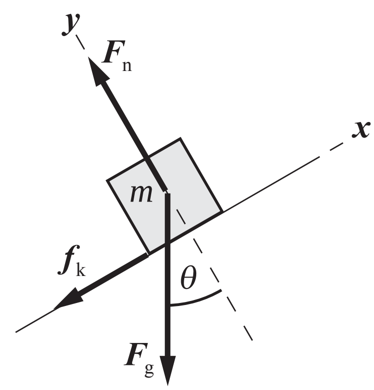
$$\mu_s = \frac{v_{i,x}}{g\Delta t}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{55\,\frac{\text{mi}}{\text{h}} \times \frac{1609\,\text{m}}{\text{mi}} \times \frac{1\,\text{h}}{3600\,\text{s}}}{\left(9.81\,\text{m/s}^2\right)\left(12\,\text{s}\right)} = \boxed{0.21}$$

47 •• **[SSM]** A 150-g block is projected up a ramp with an initial speed of 7.0 m/s. The coefficient of kinetic friction between the ramp and the block is 0.23. (*a*) If the ramp is inclined 25° with the horizontal, how far along the surface of the ramp does the block slide before coming to a stop? (*b*) The block then slides back down the ramp. What is the minimum coefficient of static friction between the block and the ramp?

Picture the Problem The force diagram shows the forces acting on the block as it slides up the ramp. Note that the block is accelerated by $\vec{f}_\text{k}$ and the x component of $\vec{F}_\text{g}$. We can use a constant-acceleration equation to express the displacement of the block up the ramp as a function of its acceleration and Newton's 2nd law to find the acceleration of the block as it slides up the ramp.



(*a*) Use a constant-acceleration equation to relate the distance the block slides up the incline to its initial speed and acceleration:

$$\boldsymbol{v}_x^2 = \boldsymbol{v}_{0x}^2 + 2\boldsymbol{a}_x\Delta\boldsymbol{x}$$

or, because $v_x = 0$,

$$0 = \boldsymbol{v}_{0x}^2 + 2\boldsymbol{a}_x\Delta\boldsymbol{x} \Rightarrow \Delta\boldsymbol{x} = \frac{-\boldsymbol{v}_{0x}^2}{2\boldsymbol{a}_x} \quad (1)$$

Apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\sum \boldsymbol{F}_x = -\boldsymbol{f}_\text{k} - \boldsymbol{F}_\text{g}\sin\boldsymbol{\theta} = \boldsymbol{ma}_x \quad (2)$$

and

$$\sum \boldsymbol{F}_y = \boldsymbol{F}_\text{n} - \boldsymbol{F}_\text{g}\cos\boldsymbol{\theta} = 0 \quad (3)$$

Substituting $\boldsymbol{f}_\text{k} = \boldsymbol{\mu}_\text{k}\boldsymbol{F}_\text{n}$ and $\boldsymbol{F}_\text{g} = \boldsymbol{mg}$ in equations (2) and (3) yields:

$$-\boldsymbol{\mu}_\text{k}\boldsymbol{F}_\text{n} - \boldsymbol{mg}\sin\boldsymbol{\theta} = \boldsymbol{ma}_x \quad (4)$$

and

$$\boldsymbol{F}_\text{n} - \boldsymbol{mg}\cos\boldsymbol{\theta} = 0 \quad (5)$$

Eliminate F_n between equations (4) and (5) to obtain:

$$-\boldsymbol{\mu}_\text{k}\boldsymbol{mg}\cos\boldsymbol{\theta} - \boldsymbol{mg}\sin\boldsymbol{\theta} = \boldsymbol{ma}_x$$

Solving for a_x yields:

$$\boldsymbol{a}_x = -\left(\boldsymbol{\mu}_\text{k}\cos\boldsymbol{\theta} + \sin\boldsymbol{\theta}\right)\boldsymbol{g}$$

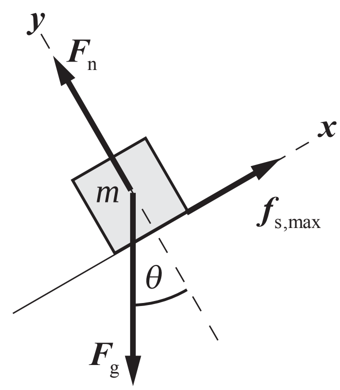
Substitute for a in equation (1) to obtain:

$$\Delta x = \frac{v_{0x}^2}{2(\mu_k \cos\theta + \sin\theta)g}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(7.0\text{ m/s})^2}{2[(0.23)\cos 25° + \sin 25°](9.81\text{ m/s}^2)} = 3.957\text{ m} = \boxed{4.0\text{ m}}$$

(*b*) At the point at which the block is instantaneously at rest, static friction becomes operative and, if the static friction coefficient is too high, the block will not resume motion, but will remain at the high point. We can determine the minimum value of the coefficient of static friction for which this occurs by considering the equality of the static friction force and the component of the weight of the block down the ramp.



Apply $\sum \vec{F} = m\vec{a}$ to the block when it is in equilibrium at the point at which it is momentarily at rest:

$$\sum F_x = f_{s,max} - F_g \sin\theta = 0 \quad (5)$$

and

$$\sum F_y = F_n - F_g \cos\theta = 0 \quad (6)$$

Solving equation (6) for F_n yields:

$$F_n = F_g \cos\theta$$

Because $f_{s,max} = \mu_s F_n$, equation (5) becomes:

$$\mu_s F_g \cos\theta - F_g \sin\theta = 0$$

or

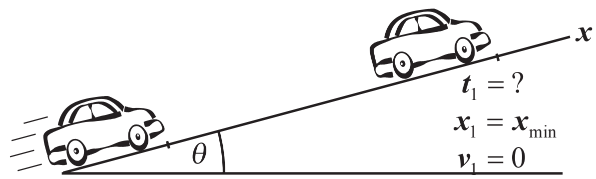
$$\mu_s \cos\theta - \sin\theta = 0 \Rightarrow \mu_s = \tan\theta$$

Substitute the numerical value of θ and evaluate μ_s:

$$\mu_s = \tan 25° = \boxed{0.47}$$

48 •• An automobile is going up a 15º grade at a speed of 30 m/s. The coefficient of static friction between the tires and the road is 0.70. (*a*) What minimum distance does it take to stop the car? (*b*) What minimum distance would it take to stop if the car were going down the grade?

Picture the Problem We can find the stopping distances by applying Newton's 2nd law to the automobile and then using a constant-acceleration equation. The friction force the road exerts on the tires and the component of the car's weight along the incline combine to provide the net force that stops the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to obtain its stopping distance.



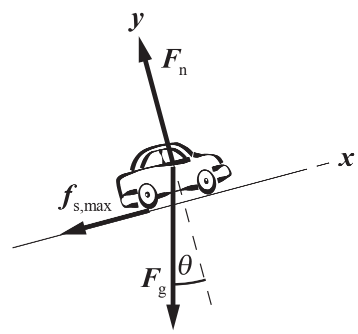
(*a*) Using a constant-acceleration equation, relate the final speed of the car to its initial speed, acceleration, and displacement; solve for its displacement:

$$v_{1x}^2 = v_{0x}^2 + 2a_{max,x}x_{min}$$

or, because $v_{1x} = 0$,

$$x_{min} = \frac{-v_{0x}^2}{2a_{max,x}} \quad (1)$$

Draw the free-body diagram for the car going up the incline:



Noting that $F_g = mg$, apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = -f_{s,max} - mg\sin\theta = ma_x \quad (2)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (3)$$

Substitute $f_{s,max} = \mu_s F_n$ and F_n from equation (3) in equation (2) and solve for $a_{max,x}$:

$$a_{max,x} = -g(\mu_s\cos\theta + \sin\theta)$$

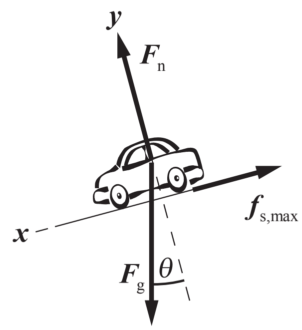
Substituting for $a_{max,x}$ in equation (1) yields:

$$x_{min} = \frac{v_{0x}^2}{2g(\mu_s\cos\theta + \sin\theta)}$$

Substitute numerical values and evaluate x_{min}:

$$x_{min} = \frac{(30\text{ m/s})^2}{2(9.81\text{ m/s}^2)((0.70)\cos 15° + \sin 15°)} = \boxed{49\text{ m}}$$

(*b*) When the car is going down the incline, the static friction force is up the incline as shown in the free-body diagram to the right. Note the change in coordinate system from Part (*a*).



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = mg\sin\theta - f_{s,max} = ma_x$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Proceed as in (*a*) to obtain:

$$a_{max,x} = g(\sin\theta - \mu_s\cos\theta)$$

Substituting for $a_{max,x}$ in equation (1) yields:

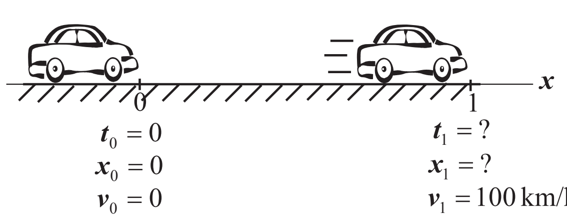
$$x_{min} = \frac{-v_{0x}^2}{2g(\sin\theta - \mu_s\cos\theta)}$$

Substitute numerical values and evaluate x_{min}:

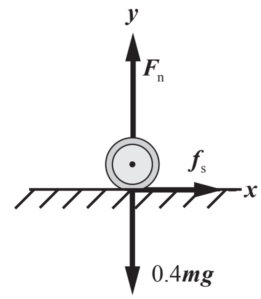
$$x_{min} = \frac{-(30\text{ m/s})^2}{2(9.81\text{ m/s}^2)(\sin 15° - (0.70)\cos 15°)} = \boxed{0.11\text{ km}}$$

49 •• A rear-wheel-drive car supports 40 percent of its weight on its two drive wheels and has a coefficient of static friction of 0.70 with a horizontal straight road. (*a*) Find the vehicle's maximum acceleration. (*b*) What is the shortest possible time in which this car can achieve a speed of 100 km/h? (Assume the engine has unlimited power.)

Picture the Problem The friction force the road exerts on the tires provides the net force that accelerates the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to calculate how long it takes it to reach 100 km/h.



(*a*) Because 40% of the car's weight is on its two drive wheels and the accelerating friction forces act just on these wheels, the free-body diagram shows just the forces acting on the drive wheels.



Apply $\sum \vec{F} = m\vec{a}$ to the drive wheels:

$$\sum F_x = f_{s,max} = ma_{max,x} \qquad (1)$$

and

$$\sum F_y = F_n - 0.4mg = 0 \qquad (2)$$

Use the definition of $f_{s,\,max}$ in equation (1) and eliminate F_n between the two equations to obtain:

$$a_{max,x} = 0.4\mu_s g$$

Substitute numerical values and evaluate $a_{max,\,x}$:

$$a_{max,x} = 0.4(0.70)(9.81\,\text{m/s}^2) = 2.747\ \text{m/s}^2 = \boxed{2.7\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the initial and final velocities of the car to its acceleration and the elapsed time; solve for the time:

$$v_{1,x} = v_{0,x} + a_{max,x}\Delta t$$

or, because $v_{0,x} = 0$ and $\Delta t = t_1$,

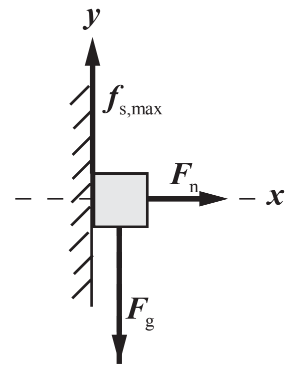
$$t_1 = \frac{v_{1,x}}{a_{max,x}}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{100\dfrac{\text{km}}{\text{h}} \times \dfrac{1\,\text{h}}{3600\,\text{s}} \times \dfrac{1000\,\text{m}}{\text{km}}}{2.747\,\text{m/s}^2} = \boxed{10\,\text{s}}$$

50 •• You and your best pal make a friendly bet that you can place a 2.0-kg box against the side of a cart, as in Figure 5-63, and that the box will not fall to the ground, even though you guarantee to use no hooks, ropes, fasteners, magnets, glue, or adhesives of any kind. When your friend accepts the bet, you begin pushing the cart in the direction shown in the figure. The coefficient of static friction between the box and the cart is 0.60. (*a*) Find the minimum acceleration for which you will win the bet. (*b*) What is the magnitude of the frictional force in this case? (*c*) Find the force of friction on the box if the acceleration is twice the minimum needed for the box not to fall. (*d*) Show that, for a box of any mass, the box will not fall if the magnitude of the forward acceleration is $a \geq g/\mu_s$, where μ_s is the coefficient of static friction.

Picture the Problem To hold the box in place, the acceleration of the cart and box must be great enough so that the static friction force acting on the box will equal the weight of the box. We can use Newton's 2nd law to determine the minimum acceleration required.



(*a*) Noting that $\boldsymbol{F}_{\text{g}} = \boldsymbol{mg}$, apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\sum \boldsymbol{F}_x = \boldsymbol{F}_{\text{n}} = \boldsymbol{ma}_{\min,x} \qquad (1)$$

and

$$\sum F_y = f_{\text{s,max}} - mg = 0 \qquad (2)$$

Substituting $\mu_{\text{s}}\boldsymbol{F}_{\text{n}}$ for $f_{\text{s, max}}$ in equation (2) yields:

$$\mu_{\text{s}}\boldsymbol{F}_{\text{n}} - \boldsymbol{mg} = 0$$

Substitute for F_{n} from equation (1) to obtain:

$$\mu_{\text{s}}\left(\boldsymbol{ma}_{\min,x}\right) - \boldsymbol{mg} = 0 \Rightarrow \boldsymbol{a}_{\min,x} = \frac{\boldsymbol{g}}{\mu_{\text{s}}}$$

Substitute numerical values and evaluate $a_{\min,x}$:

$$\boldsymbol{a}_{\min,x} = \frac{9.81\,\text{m/s}^2}{0.60} = \boxed{16\,\text{m/s}^2}$$

(*b*) From equation (2) we have:

$$\boldsymbol{f}_{\text{s,max}} = \boldsymbol{mg}$$

Substitute numerical values and evaluate $f_{\text{s, max}}$:

$$\boldsymbol{f}_{\text{s,max}} = (2.0\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{20\text{ N}}$$

(*c*) If *a* is twice that required to hold the box in place, f_{s} will still have its maximum value given by:

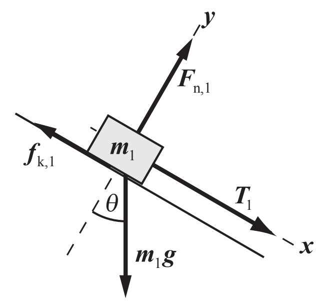
$$\boldsymbol{f}_{\text{s,max}} = \boxed{20\text{ N}}$$

(*d*) Because $\boldsymbol{a}_{\min,x} = \boldsymbol{g}/\mu_{\text{s}}$, the box will not fall if $\boldsymbol{a} \geq \boldsymbol{g}/\mu_{\text{s}}$.

51 •• Two blocks attached by a string (Figure 5-64) slide down a 10º incline. Block 1 has mass $m_1 = 0.80$ kg and block 2 has mass $m_1 = 0.25$ kg. In addition, the kinetic coefficients of friction between the blocks and the incline are 0.30 for block 1 and 0.20 for block 2. Find (*a*) the magnitude of the acceleration of the blocks, and (*b*) the tension in the string.

Picture the Problem Assume that the string is massless and does not stretch. Then the blocks have a common acceleration and the tension in the string acts on both blocks in accordance with Newton's third law of motion. Let down the incline be the positive *x* direction. Draw the free-body diagrams for each block and apply Newton's second law of motion and the definition of the kinetic friction force to each block to obtain simultaneous equations in a_x and T.

Draw the free-body diagram for the block whose mass is m_1:



Apply $\sum \vec{F} = m\vec{a}$ to the upper block:

$$\sum F_x = -f_{k,1} + T_1 + m_1 g \sin\theta = m_1 a_{1,x} \quad (1)$$

and

$$\sum F_y = F_{n,1} - m_1 g \cos\theta = 0 \quad (2)$$

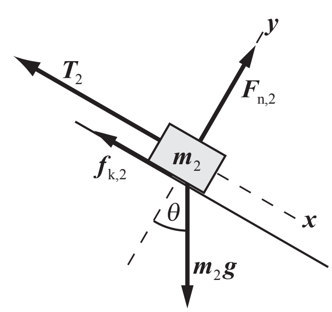
The relationship between $f_{k,1}$ and $F_{n,1}$ is:

$$f_{k,1} = \mu_{k,1} F_{n,1} \quad (3)$$

Eliminate $f_{k,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$-\mu_{k,1} m_1 g \cos\theta + T_1 + m_1 g \sin\theta = m_1 a_{1,x} \quad (4)$$

Draw the free-body diagram for the block whose mass is m_2:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = -f_{k,2} - T_2 + m_2 g \sin\theta = m_2 a_{2,x} \quad (5)$$

and

$$\sum F_y = F_{n,2} - m_2 g \cos\theta = 0 \quad (6)$$

The relationship between $f_{k,2}$ and $F_{n,2}$ is:

$$f_{k,2} = \mu_{k,2} F_{n,2} \quad (7)$$

Eliminate $f_{k,2}$ and $F_{n,2}$ between (5), (6), and (7) to obtain:

$$-\mu_{k,2} m_2 g \cos\theta - T_2 + m_2 g \sin\theta = m_1 a_{1,x} \quad (8)$$

Noting that $T_2 = T_1 = T$, add equations (4) and (8) to eliminate T, and solve for $|a_{1,x}|$:

$$|a_{1,x}| = \left|\left[\sin\theta - \frac{\mu_{k,1} m_1 + \mu_{k,2} m_2}{m_1 + m_2}\cos\theta\right] g\right|$$

Substitute numerical values and evaluate $a_{1,x}$:

$$|a_{1,x}| = \left|\left[\sin 10° - \frac{(0.20)(0.25\text{ kg}) + (0.30)(0.80\text{ kg})}{0.25\text{ kg} + 0.80\text{ kg}}\cos 10°\right](9.81\text{ m/s}^2)\right| = \boxed{0.96\text{ m/s}^2}$$

(*b*) Eliminate a_x between equations (4) and (8) and solve for $T = T_1 = T_2$ to obtain:

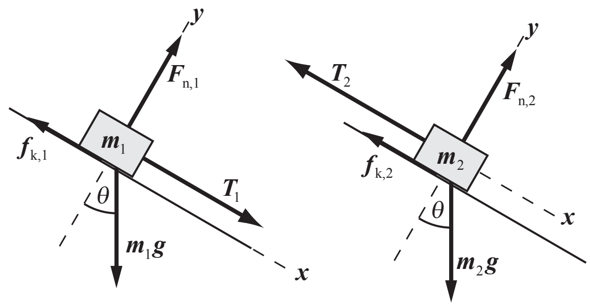
$$T = \frac{m_1 m_2 (\mu_{k,2} - \mu_{k,1}) g \cos\theta}{m_1 + m_2}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.25\text{ kg})(0.80\text{ kg})(0.30 - 0.20)(9.81\text{ m/s}^2)\cos 10°}{0.25\text{ kg} + 0.80\text{ kg}} = \boxed{0.18\text{ N}}$$

52 •• Two blocks of masses m_1 and m_2 are sliding down an incline as shown in Figure 5-64. They are connected by a massless *rod*. The coefficients of kinetic friction between the block and the surface are μ_1 for block 1 and μ_2 for block 2. (*a*) Determine the acceleration of the two blocks. (*b*) Determine the force that the rod exerts on each of the two blocks. Show that the forces are both 0 when $\mu_1 = \mu_2$ and give a simple, nonmathematical argument why this is true.

Picture the Problem The free-body diagrams show the forces acting on the two blocks as they slide down the incline. Down the incline has been chosen as the positive x direction. T is the force transmitted by the rod; it can be either tensile ($T > 0$) or compressive ($T < 0$). By applying Newton's 2nd law to these blocks, we can obtain equations in T and a_x from which we can eliminate either by solving them simultaneously. Once we have expressed T, the role of the rod will become apparent.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to block 1:

$$\sum F_x = T_1 + m_1 g \sin\theta - f_{k,1} = m_1 a_x$$

and

$$\sum F_y = F_{n,1} - m_1 g \cos\theta = 0$$

Apply $\sum \vec{F} = m\vec{a}$ to block 2:

$$\sum F_x = m_2 g \sin\theta - T_2 - f_{k,2} = m_2 a_x$$

and

$$\sum F_y = F_{n,2} - m_2 g \cos\theta = 0$$

Letting $T_1 = T_2 = T$, use the definition of the kinetic friction force to eliminate $f_{k,1}$ and $F_{n,1}$ between the equations for block 1 and $f_{k,2}$ and $F_{n,1}$ between the equations for block 2 to obtain:

$$m_1 a_x = m_1 g \sin\theta + T - \mu_1 m_1 g \cos\theta \quad (1)$$

and

$$m_2 a_x = m_2 g \sin\theta - T - \mu_2 m_2 g \cos\theta \quad (2)$$

Add equations (1) and (2) to eliminate T and solve for a_x:

$$a_x = \boxed{g\left(\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}\cos\theta\right)}$$

(*b*) Rewrite equations (1) and (2) by dividing both sides of (1) by m_1 and both sides of (2) by m_2 to obtain.

$$a_x = g\sin\theta + \frac{T}{m_1} - \mu_1 g \cos\theta \quad (3)$$

and

$$a_x = g\sin\theta - \frac{T}{m_2} - \mu_2 g \cos\theta \quad (4)$$

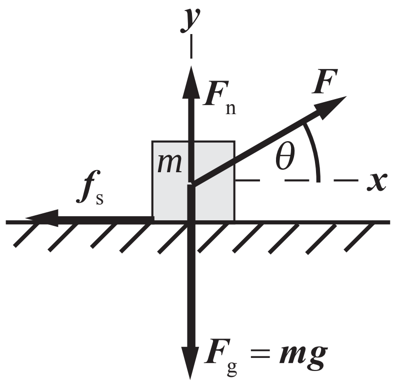
Subtracting (4) from (3) and rearranging yields:

$$T = \boxed{\left(\frac{m_1 m_2}{m_1 - m_2}\right)(\mu_1 - \mu_2) g \cos\theta}$$

If $\mu_1 = \mu_2$, $T = 0$ and the blocks move down the incline with the same acceleration of $g(\sin\theta - \mu\cos\theta)$. Inserting a stick between them can't change this; therefore, the stick must exert no force on either block.

53 •• **[SSM]** A block of mass m rests on a horizontal table (Figure 5-65). The block is pulled by a massless rope with a force $\vec{F}$ at an angle θ. The coefficient of static friction is 0.60. The minimum value of the force needed to move the block depends on the angle θ. (*a*) Discuss qualitatively how you would expect this force to depend on θ. (*b*) Compute the force for the angles $\theta = 0°$, 10°, 20°, 30°, 40°, 50°, and 60°, and make a plot of F versus θ for $mg = 400$ N. From your plot, at what angle is it most efficient to apply the force to move the block?

Picture the Problem The vertical component of $\vec{F}$ reduces the normal force; hence, the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.



(*a*) The static-frictional force opposes the motion of the object, and the maximum value of the static-frictional force is proportional to the normal force F_N. The normal force is equal to the weight minus the vertical component F_V of the force F. Keeping the magnitude F constant while increasing θ from zero results in an increase in F_V and a decrease in F_n; thus decreasing the maximum static-frictional force f_{max}. The object will begin to move if the horizontal component F_H of the force F exceeds f_{max}. An increase in θ results in a decrease in F_H. As θ increases from 0, the decrease in F_N is larger than the decrease in F_H, so the object is more and more likely to slip. However, as θ approaches 90°, F_H approaches zero and no movement will be initiated. If F is large enough and if θ increases from 0, then at some value of θ the block will start to move.

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = F\cos\theta - f_s = 0 \qquad (1)$$

and

$$\sum F_y = F_n + F\sin\theta - mg = 0 \qquad (2)$$

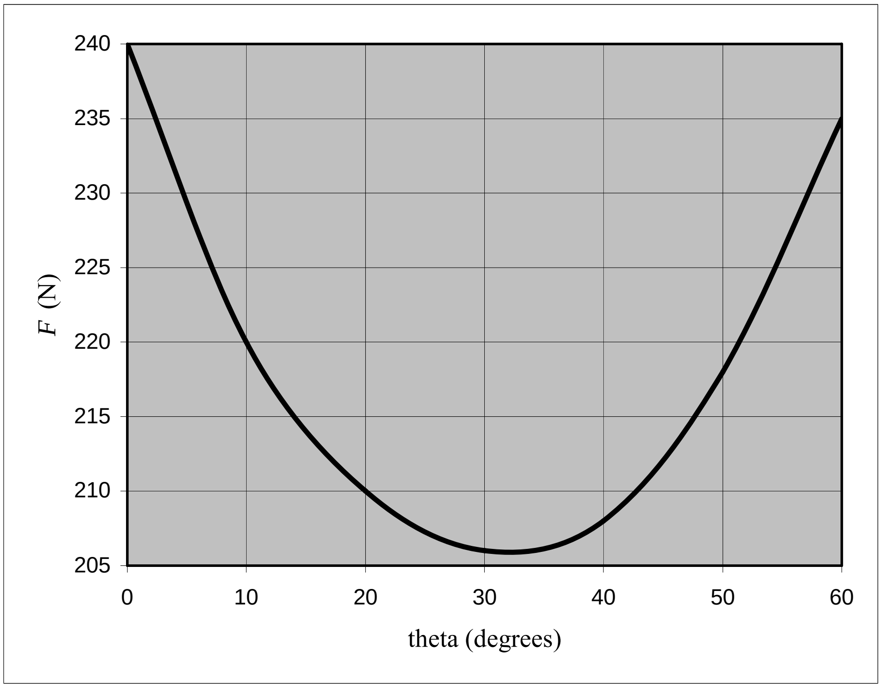
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Use this function with mg = 400 N to generate the following table:

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	220	210	206	208	218	235

The following graph of $F(\theta)$ was plotted using a spreadsheet program.

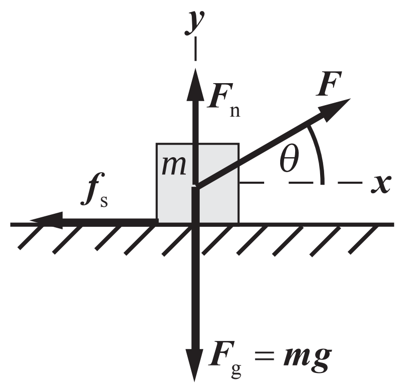


From the graph, we can see that the minimum value for F occurs when $\theta \approx 32°$.

Remarks: An alternative to manually plotting *F* as a function of *θ* or using a spreadsheet program is to use a graphing calculator to enter and graph the function.

54 •• Consider the block in Problem 53. Show that, in general, the following results hold for a block of mass m resting on a horizontal surface whose coefficient of static friction is μ_s. (*a*) If you want to apply the *minimum* possible force to move the block, you should apply it with the force pulling upward at an angle $\theta_{min} = \tan^{-1} \mu_s$, and (*b*) the minimum force necessary to start the block moving is $F_{min} = \left(\mu_s / \sqrt{1+\mu_s^2}\right) mg$. (*c*) Once the block starts moving, if you want to apply the least possible force to *keep* it moving, should you keep the angle at which you are pulling the same? increase it? decrease it?

Picture the Problem The free-body diagram shows the forces acting on the block. We can apply Newton's 2nd law, under equilibrium conditions, to relate F to θ and then set its derivative with respect to θ equal to zero to find the value of θ that minimizes F.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\sum F_y = F_n + F\sin\theta - mg = 0 \quad (2)$$

Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} \quad (3)$$

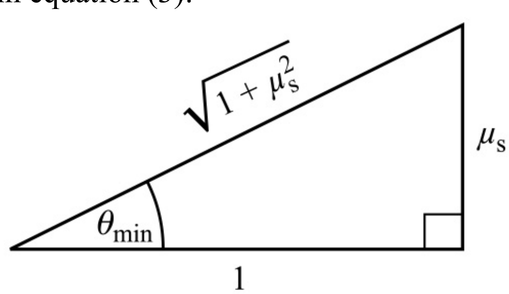
To find θ_{min}, differentiate F with respect to θ and set the derivative equal to zero for extrema of the function:

$$\begin{aligned}\frac{dF}{d\theta} &= \frac{(\cos\theta + \mu_s\sin\theta)\frac{d}{d\theta}(\mu_s mg)}{(\cos\theta + \mu_s\sin\theta)^2} - \frac{\mu_s mg\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta)}{(\cos\theta + \mu_s\sin\theta)^2} \\ &= \frac{\mu_s mg(-\sin\theta + \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0 \text{ for extrema}\end{aligned}$$

Solving for θ_{min} yields:

$$\theta_{min} = \boxed{\tan^{-1}\mu_s}$$

(b) Use the reference triangle shown below to substitute for $\cos\theta$ and $\sin\theta$ in equation (3):



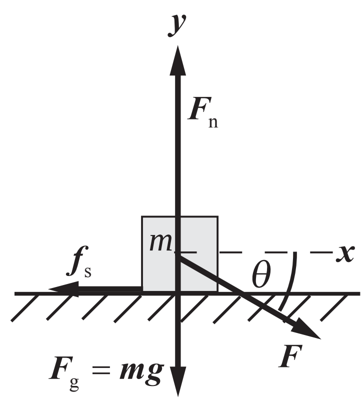
$$\begin{aligned}F_{min} &= \frac{\mu_s mg}{\dfrac{1}{\sqrt{1+\mu_s^2}} + \mu_s\dfrac{\mu_s}{\sqrt{1+\mu_s^2}}} \\ &= \frac{\mu_s mg}{\dfrac{1+\mu_s^2}{\sqrt{1+\mu_s^2}}} \\ &= \boxed{\frac{\mu_s}{\sqrt{1+\mu_s^2}}mg}\end{aligned}$$

(c) The coefficient of kinetic friction is less than the coefficient of static friction. An analysis identical to the one above shows that the minimum force one should apply to keep the block moving should be applied at an angle given by $\theta_{min} = \tan^{-1}\mu_k$. Therefore, once the block is moving, the coefficient of friction will decrease, so the angle can be decreased.

55 •• Answer the questions in Problem 54, but for a force $\vec{F}$ that pushes down on the block at an angle θ *below* the horizontal.

Picture the Problem The vertical component of $\vec{F}$ increases the normal force and the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.

(a) As θ increases from zero, F increases the normal force exerted by the surface and the static friction force. As the horizontal component of F decreases with increasing θ, one would expect F to continue to increase.



(b) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\sum F_y = F_n - F\sin\theta - mg = 0 \quad (2)$$

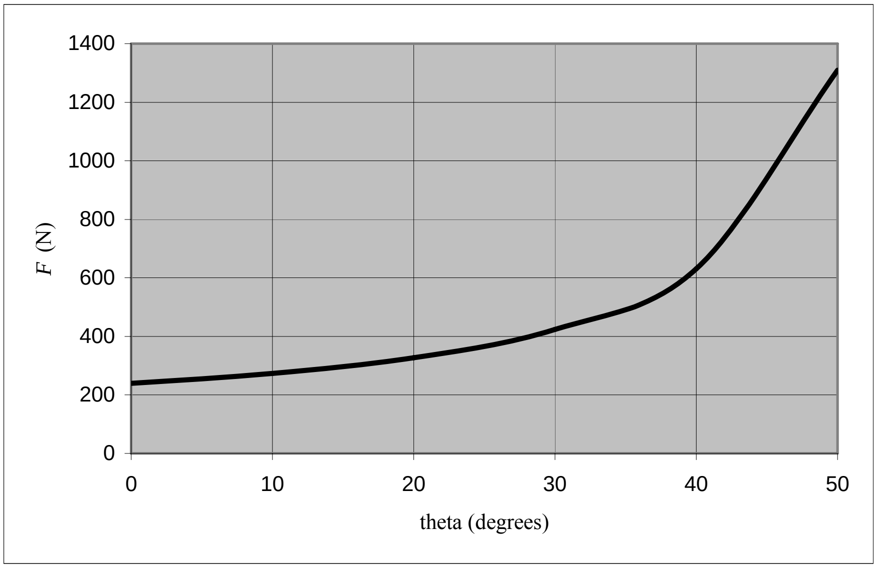
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} \quad (3)$$

Use this function with mg = 400 N to generate the table shown below.

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	273	327	424	631	1310	very large

The following graph of F as a function of θ, plotted using a spreadsheet program, confirms our prediction that F continues to increase with θ.



(a) From the graph we see that:

$$\theta_{\min} = \boxed{0^\circ}$$

(b) Evaluate equation (3) for $\theta = 0°$ to obtain:

$$F = \frac{\mu_s mg}{\cos 0^\circ - \mu_s \sin 0^\circ} = \boxed{\mu_s mg}$$

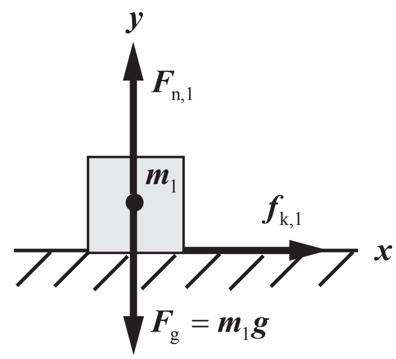
(c) You should keep the angle at 0°.

Remarks: An alternative to the use of a spreadsheet program is to use a graphing calculator to graph the function.

56 •• A 100-kg mass is pulled along a frictionless surface by a horizontal force $\vec{F}$ such that its acceleration is $a_1 = 6.00$ m/s^2 (Figure 5-66). A 20.0-kg mass slides along the top of the 100-kg mass and has an acceleration of $a_2 = 4.00$ m/s^2. (It thus slides backward relative to the 100-kg mass.) (a) What is the frictional force exerted by the 100-kg mass on the 20.0-kg mass? (b) What is the net force acting on the 100-kg mass? What is the force F? (c) After the 20.0-kg mass falls off the 100-kg mass, what is the acceleration of the 100-kg mass? (Assume that the force F does not change.)

Picture the Problem The forces acting on each of these masses are shown in the free-body diagrams below. m_1 represents the mass of the 20.0-kg mass and m_2 that of the 100-kg mass. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ (= $f_{k,2}$) act on both masses but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the various forces and the acceleration called for in this problem.

(a) Draw a free-body diagram showing the forces acting on the block whose mass is 20 kg:



Apply $\sum \vec{F} = m\vec{a}$ to this mass:

$$\sum F_x = f_{k,1} = m_1 a_{1x} \quad (1)$$

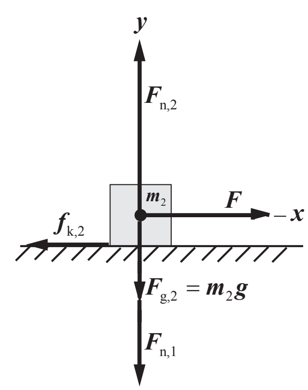
and

$$\sum F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Substitute numerical values in equation (1) and evaluate $f_{k,1}$:

$$f_{k,1} = (20.0\text{ kg})(4.00\text{ m/s}^2) = \boxed{80.0\text{ N}}$$

(b) Draw a free-body diagram showing the forces acting on the block whose mass is 100 kg:



Apply $\sum F_x = ma_x$ to the 100-kg object and evaluate F_{net}:

$$F_{\text{net}} = m_2 a_{2x} = (100\text{ kg})(6.00\text{ m/s}^2) = \boxed{600\text{ N}}$$

Express F in terms of F_{net} and $f_{k,2}$:

$$F = F_{\text{net}} + f_{k,2}$$

Substitute numerical values and evaluate F:

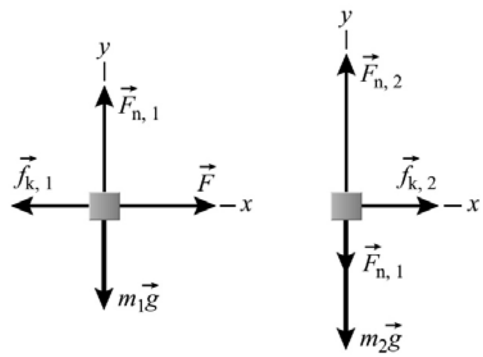
$$F = 600\text{ N} + 80.0\text{ N} = \boxed{680\text{ N}}$$

(c) When the 20.0-kg object falls off, the 680-N force acts just on the 100-kg object and its acceleration is given by Newton's 2nd law:

$$a = \frac{F_{\text{net}}}{m} = \frac{680\text{ N}}{100\text{ kg}} = \boxed{6.80\text{ m/s}^2}$$

57 •• A 60.0-kg block slides along the top of a 100-kg block. The 60.0-kg block has an acceleration of 3.0 m/s^2 when a horizontal force of 320 N is applied, as in Figure 5-67. There is no friction between the 100-kg block and a horizontal frictionless surface, but there is friction between the two blocks. (*a*) Find the coefficient of kinetic friction between the blocks. (*b*) Find the acceleration of the 100-kg block during the time that the 60.0-kg block remains in contact.

Picture the Problem The forces acting on each of these blocks are shown in the free-body diagrams to the right. m_1 represents the mass of the 60-kg block and m_2 that of the 100-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ (= $f_{k,2}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the coefficient of kinetic friction and acceleration of the 100-kg block.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 60-kg block:

$$\sum F_x = F - f_{k,1} = m_1 a_{1,x} \quad (1)$$

and

$$\sum F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Apply $\sum F_x = ma_x$ to the 100-kg block:

$$f_{k,2} = m_2 a_{2,x} \quad (3)$$

Using equation (2), express the relationship between the kinetic friction forces $\vec{f}_{k,1}$ and $\vec{f}_{k,2}$:

$$f_{k,1} = f_{k,2} = f_k = \mu_k F_{n,1} = \mu_k m_1 g \quad (4)$$

Substitute for $f_{k,1}$ in equation (1) and solve for μ_k:

$$\mu_k = \frac{F - m_1 a_1}{m_1 g}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{320\,\text{N} - (60\,\text{kg})(3.0\,\text{m/s}^2)}{(60\,\text{kg})(9.81\,\text{m/s}^2)} = 0.238$$
$$= \boxed{0.24}$$

(*b*) Substitute for $f_{k,2}$ in equation (3) and solve for a_2 to obtain:

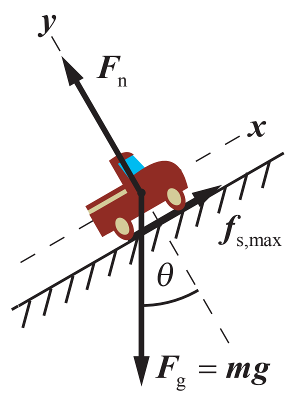
$$a_{2,x} = \frac{\mu_k m_1 g}{m_2}$$

Substitute numerical values and evaluate $a_{2,x}$:

$$a_{2,x} = \frac{(0.238)(60\,\text{kg})(9.81\,\text{m/s}^2)}{100\,\text{kg}} = \boxed{1.4\,\text{m/s}^2}$$

58 •• The coefficient of static friction between a rubber tire and the road surface is 0.85. What is the maximum acceleration of a 1000-kg four-wheel-drive truck if the road makes an angle of 12º with the horizontal and the truck is (*a*) climbing and (*b*) descending?

Picture the Problem Choose a coordinate system in which the +*x* direction is up the incline and apply Newton's 2nd law of motion. The free-body diagram shows the truck climbing the incline with maximum acceleration.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the truck when it is climbing the incline:

$$\sum F_x = f_{s,max} - mg\sin\theta = ma_x \quad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

Solve equation (2) for F_n and use the definition $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s mg\cos\theta \quad (3)$$

Substitute $f_{s,max}$ in equation (1) and solve for a_x:

$$a_x = g(\mu_s\cos\theta - \sin\theta)$$

Substitute numerical values and evaluate a_x:

$$a_x = (9.81\,\text{m/s}^2)[(0.85)\cos 12° - \sin 12°] = \boxed{6.1\,\text{m/s}^2}$$

(*b*) When the truck is descending the incline with maximum acceleration, the static friction force points down the incline; that is, its direction is reversed Apply $\sum F_x = ma_x$ to the truck under these conditions to obtain:

$$-f_{s,max} - mg\sin\theta = ma_x \quad (4)$$

Substitute for $f_{s,max}$ in equation (4) and solve for a_x:

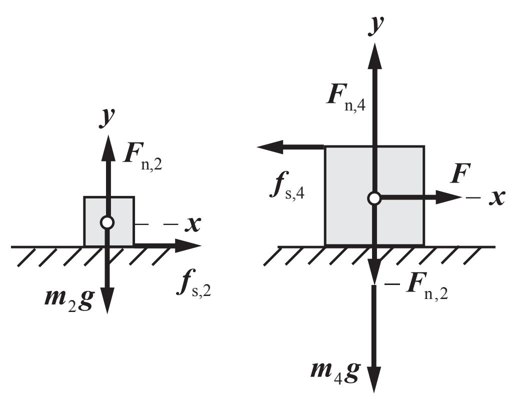
$$a_x = -g(\mu_s\cos\theta + \sin\theta)$$

Substitute numerical values and evaluate a_x:

$$a_x = (-9.81\,\mathrm{m/s^2})[(0.85)\cos 12° + \sin 12°] = \boxed{-10\,\mathrm{m/s^2}}$$

59 •• A 2.0-kg block sits on a 4.0-kg block that is on a frictionless table (Figure 5-68). The coefficients of friction between the blocks are $\mu_s = 0.30$ and $\mu_k = 0.20$. (*a*) What is the maximum horizontal force *F* that can be applied to the 4.0-kg block if the 2.0-kg block is not to slip? (*b*) If *F* has half this value, find the acceleration of each block and the force of friction acting on each block. (*c*) If *F* is twice the value found in (*a*), find the acceleration of each block..

Picture the Problem The forces acting on each of the blocks are shown in the free-body diagrams to the right. m_2 represents the mass of the 2.0-kg block and m_4 that of the 4.0-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,2}$ and the friction force $f_{s,2}$ (= $f_{s,4}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of the maximum static friction force can be used to determine the maximum force acting on the 4.0-kg block for which the 2.0-kg block does not slide.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 2.0-kg block:

$$\sum F_x = f_{s,2,max} = m_2 a_{2,max} \quad (1)$$

and

$$\sum F_y = F_{n,2} - m_2 g = 0 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the 4.0-kg block:

$$\sum F_x = F - f_{\text{s},2,\max} = m_4 a_{4,\max} \qquad (3)$$

and

$$\sum F_y = F_{\text{n},4} - F_{\text{n},2} - m_4 g = 0 \qquad (4)$$

Using equation (2), express the relationship between the static friction forces $\vec{f}_{\text{s},2,\max}$ and $\vec{f}_{\text{s},4,\max}$:

$$f_{\text{s},2,\max} = f_{s,4,\max} = \mu_\text{s} m_2 g \qquad (5)$$

Substitute for $\vec{f}_{\text{s},2,\max}$ in equation (1) and solve for $a_{2,\max}$:

$$a_{2,\max} = \mu_\text{s} g$$

Solve equation (3) for $F = F_{\max}$ and substitute for $a_{2,\max}$ and $a_{4,\max}$ to obtain:

$$F_{\max} = m_4 \mu_\text{s} g + \mu_\text{s} m_2 g = (m_4 + m_2)\mu_\text{s} g$$

Substitute numerical values and evaluate $F_{\max}$:

$$F_{\max} = (4.0\,\text{kg} + 2.0\,\text{kg})(0.30)(9.81\,\text{m/s}^2) = 17.66\,\text{N} = \boxed{18\,\text{N}}$$

(*b*) Use Newton's 2nd law to express the acceleration of the blocks moving as a unit:

$$a_x = \frac{F}{m_1 + m_2} = \frac{\frac{1}{2}F_{\max}}{m_1 + m_2}$$

Substitute numerical values and evaluate *a*:

$$a_x = \frac{\frac{1}{2}(17.66\,\text{N})}{2.0\,\text{kg} + 4.0\,\text{kg}} = 1.472\,\text{m/s}^2 = \boxed{1.5\,\text{m/s}^2}$$

Because the friction forces are an action-reaction pair, the friction force acting on each block is given by:

$$f_\text{s} = m_1 a_x$$

Substitute numerical values and evaluate f_s:

$$f_\text{s} = (2.0\,\text{kg})(1.472\,\text{m/s}^2) = \boxed{2.9\,\text{N}}$$

(*c*) If $F = 2F_{max}$, then the 2.0-kg block slips on the 4.0-kg block and the friction force (now kinetic) is given by:

$$\boldsymbol{f} = \boldsymbol{f}_k = \mu_k \boldsymbol{m}_2 \boldsymbol{g}$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 2.0-kg block to the net force acting on it and solve for $a_{2,x}$:

$$\boldsymbol{f}_k = \mu_k \boldsymbol{m}_1 \boldsymbol{g} = \boldsymbol{m}_2 \boldsymbol{a}_{2,x}$$

and

$$\boldsymbol{a}_{2,x} = \mu_k \boldsymbol{g}$$

Substitute numerical values and evaluate $a_{2,x}$:

$$a_{2,x} = (0.20)(9.81\,\text{m/s}^2) = 1.96\,\text{m/s}^2 = \boxed{2.0\,\text{m/s}^2}$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 4.0-kg block to the net force acting on it:

$$\boldsymbol{F} - \mu_k \boldsymbol{m}_2 \boldsymbol{g} = \boldsymbol{m}_4 \boldsymbol{a}_{4,x}$$

Solving for $a_{4,x}$ yields:

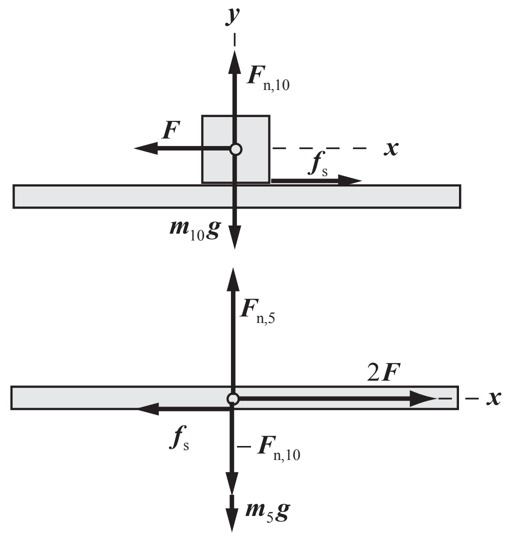
$$\boldsymbol{a}_{4,x} = \frac{\boldsymbol{F} - \mu_k \boldsymbol{m}_2 \boldsymbol{g}}{\boldsymbol{m}_4}$$

Substitute numerical values and evaluate $a_{4,x}$:

$$a_{4,x} = \frac{2(17.66\,\text{N}) - (0.20)(2.0\,\text{kg})(9.81\,\text{m/s}^2)}{4.0\,\text{kg}} = \boxed{7.8\,\text{m/s}^2}$$

60 •• A 10.0-kg block rests on a 5.0-kg bracket, as shown in Figure 5-69. The 5.0-kg bracket sits on a frictionless surface. The coefficients of friction between the 10.0-kg block and the bracket on which it rests are $\mu_s = 0.40$ and $\mu_k = 0.30$. (*a*) What is the maximum force *F* that can be applied if the 10.0-kg block is not to slide on the bracket? (*b*) What is the corresponding acceleration of the 5.0-kg bracket?

Picture the Problem The top diagram shows the forces action on the 10-kg block and the bottom diagram shows the forces acting on the 5.0-kg bracket. If the block is not to slide on the bracket, the maximum value for $\vec{F}$ must equal the maximum value of f_s. This value for $\vec{F}$ will produce the maximum acceleration of block and bracket system. We can apply Newton's 2nd law and the definition of $f_{s,max}$ to first calculate the maximum acceleration and then the maximum value of F.



Apply $\sum \vec{F} = m\vec{a}$ to the 10-kg block when it is experiencing its maximum acceleration:

$$\sum F_x = f_{s,max} - F = m_{10}a_{10,max} \qquad (1)$$

and

$$\sum F_y = F_{n,10} - m_{10}g = 0 \qquad (2)$$

Express the static friction force acting on the 10-kg block:

$$f_{s,max} = \mu_s F_{n,10} \qquad (3)$$

Eliminate $f_{s,max}$ and $F_{n,10}$ from equations (1), (2) and (3) to obtain:

$$\mu_s m_{10} g - F = m_{10} a_{10,max} \qquad (4)$$

Apply $\sum F_x = ma_x$ to the bracket to obtain:

$$2F - \mu_s m_{10} g = m_5 a_{5,max} \qquad (5)$$

Because $a_{5,max} = a_{10,max}$, denote this acceleration by a_{max}. Eliminate F from equations (4) and (5) and solve for a_{max}:

$$a_{max} = \frac{\mu_s m_{10} g}{m_5 + 2m_{10}}$$

(*b*) Substitute numerical values and evaluate a_{max}:

$$a_{max} = \frac{(0.40)(10\,\text{kg})(9.81\,\text{m/s}^2)}{5.0\,\text{kg} + 2(10\,\text{kg})} = 1.57\,\text{m/s}^2 = \boxed{1.6\,\text{m/s}^2}$$

Solve equation (4) for $F = F_{max}$:

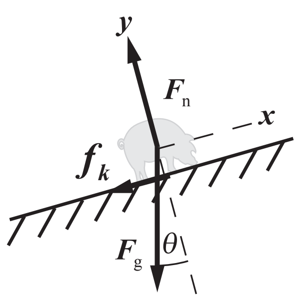
$$F = \mu_s m_{10} g - m_{10} a_{max} = m_{10}(\mu_s g - a_{max})$$

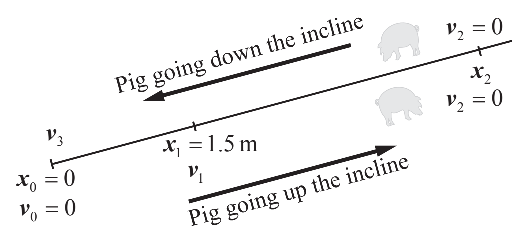
(*a*) Substitute numerical values and evaluate *F*:

$$F = (10\,\text{kg})\left[(0.40)\left(9.81\,\text{m/s}^2\right) - 1.57\,\text{m/s}^2\right] = \boxed{24\,\text{N}}$$

61 ••• You and your friends push a 75.0-kg greased pig up an aluminum slide at the county fair, starting from the low end of the slide. The coefficient of kinetic friction between the pig and the slide is 0.070. (*a*) All of you pushing together (parallel to the incline) manage to accelerate the pig from rest at the constant rate of 5.0 m/s^2 over a distance of 1.5 m, at which point you release the pig. The pig continues up the slide, reaching a maximum *vertical height* above its release point of 45 cm. What is the angle of inclination of the slide? (*b*) At the maximum height the pig turns around and begins to slip down once slide, how fast is it moving when it arrives at the low end of the slide?

Picture the Problem The free-body diagram shows the forces acting on the pig sometime after you and your friends have stopped pushing on it but before it has momentarily stopped moving up the slide. We can use a constant-acceleration equations and Newton's 2nd law to find the angle of inclination of the slide and the pig's speed when it returns to bottom of the slide. The pictorial representation assigns a coordinate system and variable names to the variables that we need to consider in solving this problem.





(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the pig:

$$\sum F_x = -f_k - F_g \sin\theta = ma_x \quad (1)$$

and

$$\sum F_y = F_n - F_g \cos\theta = 0 \quad (2)$$

Substitute $f_k = \mu_k F_n$ and $F_g = mg$ in equation (1) to obtain:

$$-\mu_k F_n - mg\sin\theta = ma_x \quad (3)$$

Solving equation (2) for F_n yields:

$$F_n = F_g \cos\theta$$

Substitute for F_n in equation (3) to obtain:

$$-\mu_k mg\cos\theta - mg\sin\theta = ma_x$$

Solving for a_x yields:

$$a_x = -g(\mu_k \cos\theta + \sin\theta) \quad (4)$$

Use a constant-acceleration equation to relate the distance d the pig slides up the ramp to its speed after you and your friends have stopped pushing, its final speed, and its acceleration:

$$v_2^2 = v_1^2 + 2a_x d$$

or, because $v_2 = 0$,

$$0 = v_1^2 + 2a_x d$$

Substituting for a_x from equation (4) yields:

$$0 = v_1^2 - 2g(\mu_k \cos\theta + \sin\theta)d \qquad (5)$$

Use a constant-acceleration equation to relate the pig's speed v_1 after being accelerated to the distance Δx over which it was accelerated:

$$v_1^2 = v_0^2 + 2a_x \Delta x$$

or, because $v_0 = 0$,

$$v_1^2 = 2a_x \Delta x$$

Substitute for v_1^2 in equation (5) to obtain:

$$0 = 2a_x \Delta x - 2g(\mu_k \cos\theta + \sin\theta)d \qquad (6)$$

Use trigonometry to relate the vertical distance h risen by the pig during its slide to the distance d it moves up the slide:

$$h = d\sin\theta \Rightarrow d = \frac{h}{\sin\theta}$$

Substituting for d in equation (6) yields:

$$0 = 2a_x \Delta x - 2g(\mu_k \cos\theta + \sin\theta)\frac{h}{\sin\theta}$$

Solve for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{\mu_k h}{\dfrac{a_x \Delta x}{g} - h}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{(0.070)(0.45\ \text{m})}{\dfrac{(5.0\ \text{m/s}^2)(1.5\ \text{m})}{9.81\ \text{m/s}^2} - 0.45\ \text{m}}\right]$$

$$= 5.719° = \boxed{5.7°}$$

(*b*) Use a constant-acceleration equation to express v_3 as a function of the pig's acceleration down the incline a_{down}, its initial speed v_2, the distance Δd it slides down the incline:

$$v_3^2 = v_2^2 + 2a_{\text{down}} \Delta d$$

or, because $v_2 = 0$,

$$v_3^2 = 2a_{\text{down}} \Delta d \Rightarrow v_3 = \sqrt{2a_{\text{down}} \Delta d} \qquad (7)$$

When the pig is sliding down the incline, the kinetic friction force shown in the free-body diagram is reversed (it points up the incline … opposing the pig's motion) and the pig's acceleration is:

$$a_{\text{down}} = g(\sin\theta - \mu_{\text{k}}\cos\theta)$$

Substitute for a_{down} in equation (7) to obtain:

$$v_3 = \sqrt{2g(\sin\theta - \mu_{\text{k}}\cos\theta)\Delta d} \quad (8)$$

Δd is the sum of the distances the pig was pushed and then slid to a momentary stop:

$$\Delta d = 1.5\text{ m} + \frac{d}{\sin\theta}$$

Substituting for Δd in equation (8) yields:

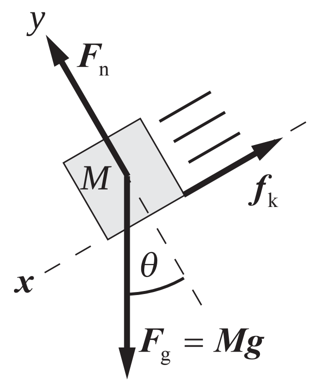
$$v_3 = \sqrt{2g(\sin\theta - \mu_{\text{k}}\cos\theta)\left(1.5\text{ m} + \frac{d}{\sin\theta}\right)}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\text{ m/s}^2)[\sin 5.719° - (0.070)\cos 5.719°]\left(1.5\text{ m} + \frac{0.45\text{ m}}{\sin 5.719°}\right)} = \boxed{1.9\text{ m/s}}$$

62 ••• A 100-kg block on an inclined plane is attached to another block of mass *m* via a string, as in Figure 5-70. The coefficients of static and kinetic friction for the block and the incline are $\mu_{\text{s}} = 0.40$ and $\mu_{\text{k}} = 0.20$ and the plane is inclined 18º with horizontal. (*a*) Determine the range of values for *m*, the mass of the hanging block, for which the 100-kg block will not move unless disturbed, but if nudged, will slide down the incline. (*b*) Determine a range of values for *m* for which the 100-kg block will not move unless nudged, but if nudged will slide up the incline.

Picture the Problem The free-body diagram shows the block sliding down the incline under the influence of a friction force, its weight, and the normal force exerted on it by the inclined surface. We can find the range of values for *m* for the two situations described in the problem statement by applying Newton's 2nd law of motion to, first, the conditions under which the block will not move or slide if pushed, and secondly, if pushed, the block will move up the incline.



(*a*) Assume that the block is sliding down the incline with a constant velocity and with no hanging weight ($m = 0$) and apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum \boldsymbol{F_x} = -\boldsymbol{f}_\text{k} + \boldsymbol{Mg}\sin\boldsymbol{\theta} = 0$$

and

$$\sum \boldsymbol{F_y} = \boldsymbol{F}_\text{n} - \boldsymbol{Mg}\cos\boldsymbol{\theta} = 0$$

Using $f_\text{k} = \mu_\text{k} F_\text{n}$, eliminate F_n between the two equations and solve for the net force acting on the block:

$$\boldsymbol{F}_\text{net} = -\boldsymbol{\mu}_\text{k}\boldsymbol{Mg}\cos\boldsymbol{\theta} + \boldsymbol{Mg}\sin\boldsymbol{\theta}$$

If the block is moving, this net force must be nonnegative and:

$$(-\mu_\text{k}\cos\theta + \sin\theta)Mg \geq 0$$

This condition requires that:

$$\mu_\text{k} \leq \tan\theta = \tan 18° = 0.325$$

Because $\mu_\text{k} = 0.2$, this condition is satisfied and:

$$m_\text{min} = 0$$

To find the maximum value, note that the maximum possible value for the tension in the rope is *mg*. For the block to move down the incline, the component of the block's weight parallel to the incline minus the frictional force must be greater than or equal to the tension in the rope:

$$Mg\sin\theta - \mu_\text{k} Mg\cos\theta \geq mg$$

Solving for m_max yields:

$$m_\text{max} \leq M(\sin\theta - \mu_\text{k}\cos\theta)$$

Substitute numerical values and evaluate m_max:

$$\boldsymbol{m}_\text{max} \leq (100\,\text{kg})[\sin 18° - (0.20)\cos 18°] = 11.9\,\text{kg}$$

The range of values for *m* is: $\boxed{0 \le \boldsymbol{m} \le 12\,\text{kg}}$

(*b*) If the block is being dragged up the incline, the frictional force will point down the incline, and:

$$Mg\sin\theta + \mu_k Mg\cos\theta < m_{\min} g$$

Solve for $m_{\min}$:

$$m_{\min} > M(\sin\theta + \mu_k\cos\theta)$$

Substitute numerical values and evaluate $m_{\min}$:

$$\boldsymbol{m}_{\min} > (100\,\text{kg})[\sin 18° + (0.20)\cos 18°] = 49.9\,\text{kg}$$

If the block is not to move unless pushed:

$$Mg\sin\theta + \mu_s Mg\cos\theta > m_{\max} g$$

Solve for $m_{\max}$:

$$m_{\max} < M(\sin\theta + \mu_s\cos\theta)$$

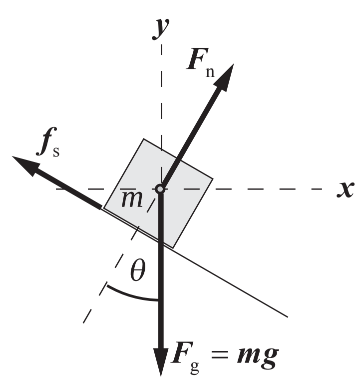
Substitute numerical values and evaluate $m_{\max}$:

$$\boldsymbol{m}_{\max} < (100\,\text{kg})[\sin 18° + (0.40)\cos 18°] = 68.9\,\text{kg}$$

The range of values for *m* is: $\boxed{50\,\text{kg} \le \boldsymbol{m} \le 69\,\text{kg}}$

63 ••• A block of mass 0.50 kg rests on the inclined surface of a wedge of mass 2.0 kg, as in Figure 5-71. The wedge is acted on by a horizontal applied force $\vec{F}$ and slides on a frictionless surface. (*a*) If the coefficient of static friction between the wedge and the block is $\mu_s = 0.80$ and the wedge is inclined 35º with the horizontal, find the maximum and minimum values of the applied force for which the block does not slip. (*b*) Repeat part (*a*) with $\mu_s = 0.40$.

Picture the Problem The free-body diagram shows the forces acting on the 0.50-kg block when the acceleration is a minimum. Note the choice of coordinate system that is consistent with the direction of $\vec{F}$. Apply Newton's 2nd law to the block and to the block-plus-incline and solve the resulting equations for $F_{\min}$ and $F_{\max}$.



(*a*) The maximum and minimum values of the applied force for which the block does not slip are related to the maximum and minimum accelerations of the block:

$$F_{max} = m_{tot}a_{x,max} \qquad (1)$$

and

$$F_{min} = m_{tot}a_{x,min} \qquad (2)$$

where m_{tot} is the combined mass of the block and incline.

Apply $\sum \vec{F} = m\vec{a}$ to the 0.50-kg block:

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = ma_x \qquad (3)$$

and

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0 \qquad (4)$$

Under minimum acceleration, $f_s = f_{s,\,max}$. Express the relationship between $f_{s,\,max}$ and F_n:

$$f_{s,max} = \mu_s F_n$$

Substitute $f_{s,\,max}$ for f_s in equation (4) and solve for F_n:

$$F_n = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

Substitute for F_n and $f_{s,\,max}$ in equation (3) and solve for $a_x = a_{x,min}$:

$$a_{x,min} = g\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}$$

Substitute for $a_{x,\,min}$ in equation (2) to obtain:

$$F_{min} = m_{tot}g\left(\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}\right)$$

Substitute numerical values and evaluate F_{min}:

$$F_{min} = \left|(2.5\text{ kg})(9.81\text{ m/s}^2)\left[\frac{\sin 35° - (0.80)\cos 35°}{\cos 35° + (0.80)\sin 35°}\right]\right| = \boxed{1.6\text{ N}}$$

To find the maximum acceleration, reverse the direction of $\vec{f}_s$ and apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = ma_x$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Proceed as above to obtain:

$$a_{x,max} = g\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}$$

Substitute for $a_{x,\,max}$ in equation (1) to obtain:

$$F_{max} = m_{tot}g\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

Substitute numerical values and evaluate $\boldsymbol{F}_{max}$:

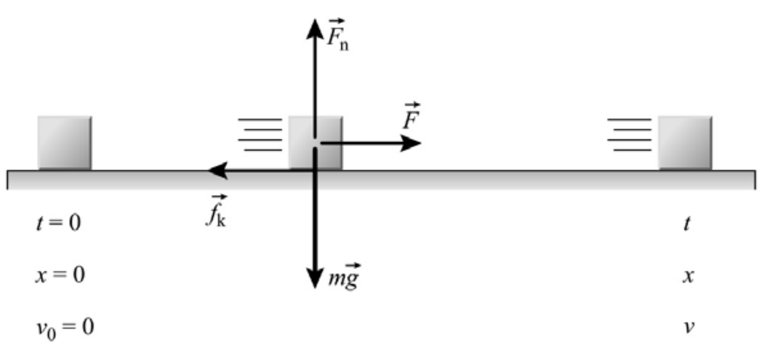
$$\boldsymbol{F}_{max} = (2.5\text{ kg})(9.81\text{ m/s}^2)\left[\frac{\sin 35° + (0.80)\cos 35°}{\cos 35° - (0.80)\sin 35°}\right] = \boxed{84\text{ N}}$$

(*b*) Repeat Part (*a*) with $\mu_s = 0.40$ to obtain:

$$\boldsymbol{F}_{min} = \boxed{5.8\text{ N}} \text{ and } F_{max} = \boxed{37\text{ N}}$$

64 ••• In your physics lab, you and your lab partners push a block of wood with a mass of 10.0 kg (starting from rest), with a constant horizontal force of 70 N across a wooden floor. In the previous week's laboratory meeting, your group determined that the coefficient of kinetic friction was not exactly constant, but instead was found to vary with the object's speed according to $\mu_k = 0.11/(1 + 2.3 \times 10^{-4} v^2)^2$. Write a **spreadsheet** program using Euler's method to calculate and graph both the speed and the position of the block as a function of time from 0 to 10 s. Compare this result to the result you would get if you assumed the coefficient of kinetic friction had a constant value of 0.11.

Picture the Problem The kinetic friction force f_k is the product of the coefficient of sliding friction μ_k and the normal force F_n the surface exerts on the sliding object. By applying Newton's 2nd law in the vertical direction, we can see that, on a horizontal surface, the normal force is the weight of the sliding object. We can apply Newton's 2nd law in the horizontal (x) direction to relate the block's acceleration to the net force acting on it. In the spreadsheet program, we'll find the acceleration of the block from this net force (which is speed dependent), calculate the increase in the block's speed from its acceleration and the elapsed time and add this increase to its speed at end of the previous time interval, determine how far it has moved in this time interval, and add this distance to its previous position to find its current position. We'll also calculate the position of the block x_2, under the assumption that $\mu_k = 0.11$, using a constant-acceleration equation.

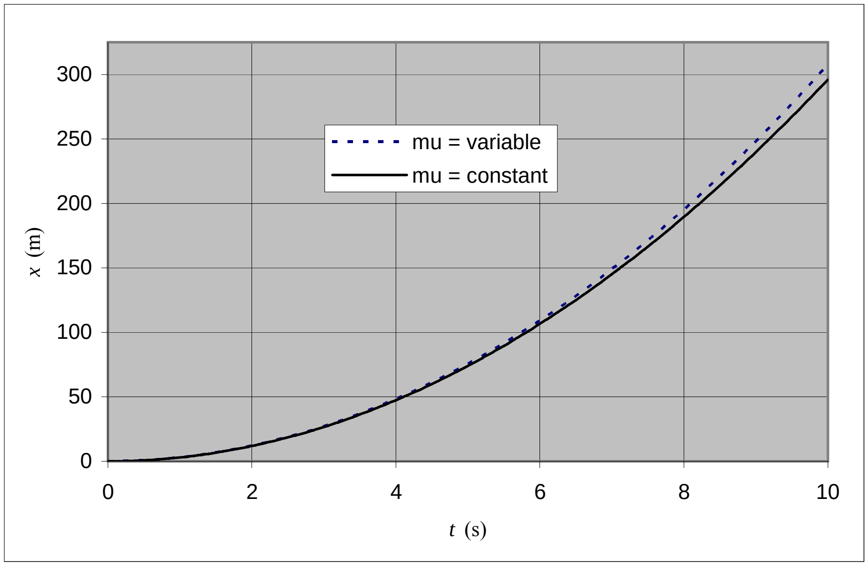


The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

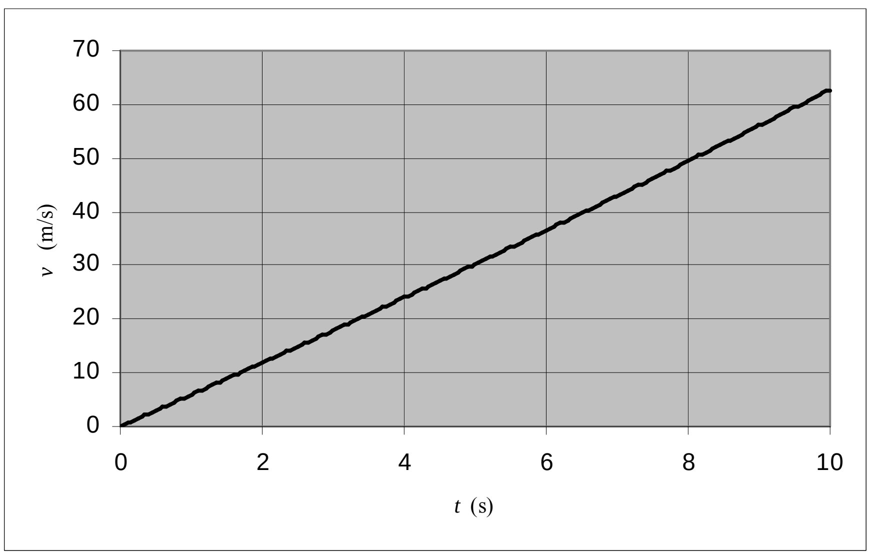
Cell	Formula/Content	Algebraic Form
C9	C8+$B$6	$t+\Delta t$
D9	D8+F9*$B$6	$v+a\Delta t$
E9	$B$5−($B$3)*($B$2)*$B$5/ (1+$B$4*D9^2)^2	$F-\dfrac{\mu_k mg}{\left(1+2.34\times10^{-4}v^2\right)^2}$
F9	E10/$B$5	F_{net}/m
G9	G9+D10*$B$6	$x+v\Delta t$
K9	0.5*5.922*I10^2	$\frac{1}{2}at^2$
L9	J10-K10	$x-x_2$

	A	B	C	D	E	F	G	H	I	J
1	g=	9.81	m/s^2							
2	Coeff1=	0.11								
3	Coeff2=	2.30E−04								
4	m=	10	kg							
5	F_{app}=	70	N							
6	Δt=	0.05	s							
7										
8							t	x	x_2	$x-x_2$
9	t	v	F_{net}	a,	x			$\mu_{variable}$	$\mu_{constant}$	
10	0.00	0.00			0.00		0.00	0.00	0.00	0.00
11	0.05	0.30	59.22	5.92	0.01		0.05	0.01	0.01	0.01
12	0.10	0.59	59.22	5.92	0.04		0.10	0.04	0.03	0.01
13	0.15	0.89	59.22	5.92	0.09		0.15	0.09	0.07	0.02
14	0.20	1.18	59.22	5.92	0.15		0.20	0.15	0.12	0.03
15	0.25	1.48	59.23	5.92	0.22		0.25	0.22	0.19	0.04
205	9.75	61.06	66.84	6.68	292.37		9.75	292.37	281.48	10.89
206	9.80	61.40	66.88	6.69	295.44		9.80	295.44	284.37	11.07
207	9.85	61.73	66.91	6.69	298.53		9.85	298.53	287.28	11.25
208	9.90	62.07	66.94	6.69	301.63		9.90	301.63	290.21	11.42
209	9.95	62.40	66.97	6.70	304.75		9.95	304.75	293.15	11.61
210	10.00	62.74	67.00	6.70	307.89		10.00	307.89	296.10	11.79

The position of the block as a function of time, for a constant coefficient of friction ($\mu_k = 0.11$) is shown as a solid line on the following graph and for a variable coefficient of friction, is shown as a dotted line. Because the coefficient of friction decreases with increasing particle speed, the particle travels slightly farther when the coefficient of friction is variable.

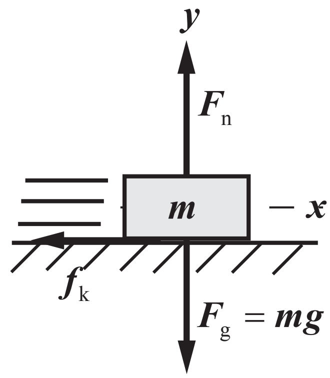


The speed of the block as a function of time, with variable coefficient of kinetic friction, is shown in the following graph.



65 ••• In order to determine the coefficient of kinetic friction of a block of wood on a horizontal table surface, you are given the following assignment: Take the block of wood and give it an initial velocity across the surface of the table. Using a stopwatch, measure the time Δt it takes for the block to come to a stop and the total displacement Δx of the block travels following the push. (*a*) Using Newton's laws and a free-body diagram of the block, show that the expression for the coefficient of kinetic friction is given by $\mu_k = 2\Delta x/[(\Delta t)^2 g]$. (*b*) If the block slides a distance of 1.37 m in 0.97 s, calculate μ_k. (*c*) What was the initial speed of the block?

Picture the Problem The free-body diagram shows the forces acting on the block as it moves to the right. The kinetic friction force will slow the block and, eventually, bring it to rest. We can relate the coefficient of kinetic friction to the stopping time and distance by applying Newton's 2nd law and then using constant-acceleration equations.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block of wood:

$$\sum F_x = -f_k = ma_x$$

and

$$\sum F_y = F_n - mg = 0$$

Using the definition of f_k, eliminate F_n between the two equations to obtain:

$$a_x = -\mu_k g \quad (1)$$

Use a constant-acceleration equation to relate the acceleration of the block to its displacement and its stopping time:

$$\Delta x = v_{0y}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \quad (2)$$

Relate the initial speed of the block, v_0, to its displacement and stopping distance:

$$\Delta x = v_{av}\Delta t = \frac{v_{0y} + v_y}{2}\Delta t$$

or, because $v = 0$,

$$\Delta x = \tfrac{1}{2}v_{0y}\Delta t \quad (3)$$

Use this result to eliminate v_{0y} in equation (2):

$$\Delta x = -\tfrac{1}{2}a_x(\Delta t)^2 \quad (4)$$

Substitute equation (1) in equation (4) and solve for μ_k:

$$\mu_k = \boxed{\frac{2\Delta x}{g(\Delta t)^2}}$$

(*b*) Substitute for $\Delta x = 1.37$ m and $\Delta t = 0.97$ s to obtain:

$$\mu_k = \frac{2(1.37\,\text{m})}{(9.81\,\text{m/s}^2)(0.97\,\text{s})^2} = \boxed{0.30}$$

(*c*) Use equation (3) to find v_0:

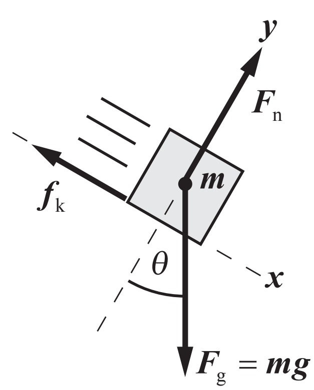
$$v_0 = \frac{2\Delta x}{\Delta t} = \frac{2(1.37\,\text{m})}{0.97\,\text{s}} = \boxed{2.8\,\text{m/s}}$$

66 ••• (*a*) A block is sliding down an inclined plane. The coefficient of kinetic friction between the block and the plane is μ_k, the angle the plane makes with the horizontal is θ and the acceleration of the plane is a_x. Show that a graph of $a_x/\cos\theta$ versus $\tan\theta$ (where a_x is the acceleration down the incline and θ is the angle the plane is inclined with the horizontal) would be a straight line with slope g and intercept $-\mu_k g$. (*b*) The following data show the acceleration of a block sliding down an inclined plane as a function of the angle θ that the plane is inclined with the horizontal

θ (degrees)	Acceleration (m/s^2)
25.0	1.69
27.0	2.10
29.0	2.41
31.0	2.89
33.0	3.18
35.0	3.49
37.0	3.79
39.0	4.15
41.0	4.33
43.0	4.72
45.0	5.11

Using a **spreadsheet** program, graph these data and fit a straight line (a Trendline in Excel parlance) to them to determine μ_k and g. What is the percentage difference between the obtained value of g and the commonly specified value of 9.81 m/s^2?

Picture the Problem The free-body diagram shows the forces acting on the block as it slides down an incline. We can apply Newton's 2nd law to these forces to obtain the acceleration of the block and then manipulate this expression algebraically to show that a graph of $a/\cos\theta$ versus $\tan\theta$ will be linear with a slope equal to the acceleration due to gravity and an intercept whose absolute value is the coefficient of kinetic friction.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block as it slides down the incline:

$$\sum F_x = mg\sin\theta - f_k = ma_x$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Substitute $\mu_k F_n$ for f_k and eliminate F_n between the two equations to obtain:

$$a_x = g(\sin\theta - \mu_k \cos\theta)$$

Divide both sides of this equation by $\cos\theta$ to obtain:

$$\frac{a_x}{\cos\theta} = \boxed{g\tan\theta - g\mu_k}$$

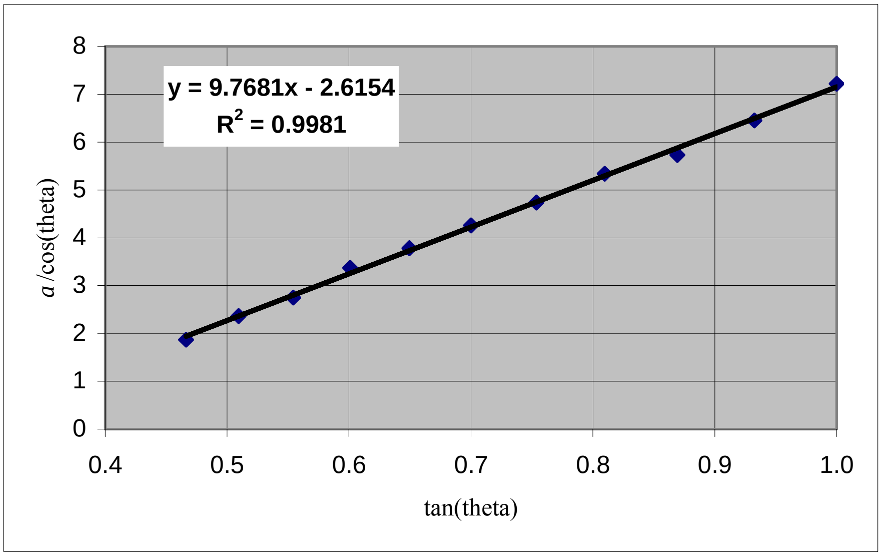
Note that this equation is of the form $y = mx + b$. Thus, if we graph $a/\cos\theta$ versus $\tan\theta$, we should get a straight line with slope g and y-intercept $-g\mu_k$.

(*b*) A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
C7	θ	
D7	a	
E7	TAN(C7*PI()/180)	$\tan\left(\theta \times \frac{\pi}{180}\right)$
F7	D7/COS(C7*PI()/180)	$\frac{a}{\cos\left(\theta \times \frac{\pi}{180}\right)}$

	C	D	E	F
6	θ	a	$\tan\theta$	$a/\cos\theta$
7	25.0	1.69	0.466	1.866
8	27.0	2.10	0.510	2.362
9	29.0	2.41	0.554	2.751
10	31.0	2.89	0.601	3.370
11	33.0	3.18	0.649	3.786
12	35.0	3.49	0.700	4.259
13	37.0	3.78	0.754	4.735
14	39.0	4.15	0.810	5.338
15	41.0	4.33	0.869	5.732
16	43.0	4.72	0.933	6.451
17	45.0	5.11	1.000	7.220

A graph of $a/\cos\theta$ versus $\tan\theta$ follows. From the curve fit (Excel's Trendline was used), $\boldsymbol{g} = \boxed{9.77\ \text{m/s}^2}$ and $\boldsymbol{\mu}_\text{k} = \dfrac{2.62\ \text{m/s}^2}{9.77\ \text{m/s}^2} = \boxed{0.268}$.



The percentage error in g from the commonly accepted value of 9.81 m/s^2 is:

$$100\left(\frac{9.81\ \text{m/s}^2 - 9.77\ \text{m/s}^2}{9.81\ \text{m/s}^2}\right) = \boxed{0.408\%}$$

Drag Forces

67 • **[SSM]** A Ping-Pong ball has a mass of 2.3 g and a terminal speed of 9.0 m/s. The drag force is of the form bv^2. What is the value of b?

Picture the Problem The ping-pong ball experiences a downward gravitational force exerted by the earth and an upward drag force exerted by the air. We can apply Newton's 2nd law to the Ping-Pong ball to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values. Let the downward direction be the $+y$ direction.

Apply $\sum F_y = ma_y$ to the Ping-Pong ball:

$$mg - bv^2 = ma_y$$

When the Ping-Pong ball reaches its terminal speed $v = v_\text{t}$ and $a_y = 0$:

$$mg - bv_\text{t}^2 = 0 \Rightarrow b = \frac{mg}{v_\text{t}^2}$$

Substitute numerical values and evaluate b:

$$\boldsymbol{b} = \frac{\left(2.3\times10^{-3}\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{\left(9.0\,\text{m/s}\right)^2} = \boxed{2.8\times10^{-4}\ \text{kg/m}}$$

68 • A small pollution particle settles toward Earth in still air. The terminal speed is 0.30 mm/s, the mass of the particle is 1.0 × 10^{-10} g and the drag force is of the form bv. What is the value of b?

Picture the Problem The pollution particle experiences a downward gravitational force exerted by the earth and an upward drag force exerted by the air. We can apply Newton's 2nd law to the particle to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values. Let the downward direction by the $+y$ direction.

Apply $\sum F_y = ma_y$ to the particle:

$$mg - bv = ma_y$$

When the particle reaches its terminal speed $v = v_t$ and $a_y = 0$:

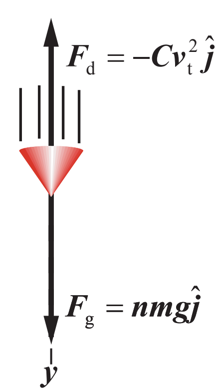
$$mg - bv_t = 0 \Rightarrow b = \frac{mg}{v_t}$$

Substitute numerical values and evaluate b:

$$b = \frac{\left(1.0\times10^{-13}\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{3.0\times10^{-4}\ \text{m/s}} = \boxed{3.3\times10^{-9}\ \text{kg/s}}$$

69 •• **[SSM]** A common classroom demonstration involves dropping basket-shaped coffee filters and measuring the time required for them to fall a given distance. A professor drops a single basket-shaped coffee filter from a height h above the floor, and records the time for the fall as Δt. How far will a stacked set of n identical filters fall during the same time interval Δt? Consider the filters to be so light that they instantaneously reach their terminal velocities. Assume a drag force that varies as the square of the speed and assume the filters are released oriented right-side up.

Picture the Problem The force diagram shows n coffee filters experiencing an upward drag force exerted by the air that varies with the square of their terminal velocity and a downward gravitational force exerted by the earth. We can use the definition of average velocity and Newton's 2nd law to establish the dependence of the distance of fall on n.



Express the distance fallen by 1 coffee filter, falling at its terminal speed, in time Δt:

$$d_{1\text{ filter}} = v_{t,1\text{ filter}}\Delta t \qquad (1)$$

Express the distance fallen by n coffee filters, falling at their terminal speed, in time Δt:

$$d_{n\text{ filters}} = v_{t,n\text{ filters}}\Delta t \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{d_{n\text{ filters}}}{d_{1\text{ filter}}} = \frac{v_{t,n\text{ filters}}\Delta t}{v_{t,1\text{ filter}}\Delta t} = \frac{v_{t,n\text{ filters}}}{v_{t,1\text{ filter}}}$$

Solving for $d_{n\text{ filters}}$ yields:

$$d_{n\text{ filters}} = \left(\frac{v_{t,n\text{ filters}}}{v_{t,1\text{ filter}}}\right)d_{1\text{ filter}} \qquad (3)$$

Apply $\sum F_y = ma_y$ to the coffee filters:

$$nmg - Cv_t^2 = ma_y$$

or, because $a_y = 0$,

$$nmg - Cv_t^2 = 0$$

Solving for v_t yields:

$$v_{t,n\text{ filters}} = \sqrt{\frac{nmg}{C}} = \sqrt{\frac{mg}{C}}\sqrt{n}$$

or, because $v_{t,1\text{ filter}} = \sqrt{\frac{mg}{C}}$,

$$v_{t,n\text{ filters}} = v_{t,1\text{ filter}}\sqrt{n}$$

Substitute for $v_{t,n\text{ filters}}$ in equation (3) to obtain:

$$d_{n\text{ filters}} = \left(\frac{v_{t,1\text{ filter}}\sqrt{n}}{v_{t,1\text{ filter}}}\right)d_{1\text{ filter}} = \boxed{\sqrt{n}\,d_{1\text{ filter}}}$$

This result tells us that n filters will fall farther, in the same amount of time, than 1 filter by a factor of $\sqrt{n}$.

70 •• A skydiver of mass 60.0 kg can slow herself to a constant speed of 90 km/h by orienting her body horizontally, looking straight down with arms and legs extended. In this position she presents the maximum cross-sectional area and thus maximize the air-drag force on her. (*a*) What is the magnitude of the drag force on the skydiver? (*b*) If the drag force is given by bv^2, what is the value of b? (*c*) At some instant she quickly flips into a ″knife″ position, orienting her body vertically with her arms straight down. Suppose this reduces the value of b by 55 percent from the value in Parts (*a*) and (*b*). What is her acceleration at the instant she achieves the ″knife″ position?

Picture the Problem The skydiver experiences a downward gravitational force exerted by the earth and an upward drag force exerted by the air. Let the upward direction be the $+y$ direction and apply Newton's 2nd law to the sky diver.

(*a*) Apply $\sum F_y = ma_y$ to the sky diver:

$$F_\mathrm{d} - mg = ma_y \quad (1)$$

or, because $a_y = 0$,

$$F_\mathrm{d} = mg \quad (2)$$

Substitute numerical values and evaluate F_d:

$$F_\mathrm{d} = (60.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 588.6\,\mathrm{N} = \boxed{589\,\mathrm{N}}$$

(*b*) Substitute $F_\mathrm{d} = bv_\mathrm{t}^2$ in equation (2) to obtain:

$$bv_\mathrm{t}^2 = mg \Rightarrow b = \frac{mg}{v_\mathrm{t}^2} = \frac{F_\mathrm{d}}{v_\mathrm{t}^2}$$

Substitute numerical values and evaluate b:

$$b = \frac{588.6\,\mathrm{N}}{\left(90\,\frac{\mathrm{km}}{\mathrm{h}} \times \frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right)^2} = 0.9418\,\mathrm{kg/m} = \boxed{0.94\,\mathrm{kg/m}}$$

(*c*) Solving equation (1) for a_y yields:

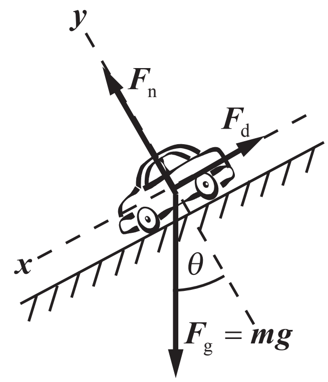
$$a_y = \left|\frac{F_\mathrm{d}}{m} - g\right|$$

Because b is reduced to 55 percent of its value in Part (*b*), so is F_d. Substitute numerical values and evaluate a_y:

$$a_y = \left|\frac{(0.55)(588.6\,\mathrm{N})}{60.0\,\mathrm{kg}} - 9.81\,\mathrm{m/s^2}\right| = \boxed{4.41\,\mathrm{m/s^2},\ \text{downward}}$$

71 •• Your team of test engineers is to release the parking brake so an 800-kg car will roll down a very long 6.0% grade in preparation for a crash test at the bottom of the incline. (On a 6.0 % grade the change in altitude is 6.0% of the horizontal distance traveled.) The total resistive force (air drag plus rolling friction) for this car has been previously established to be F_d = 100 N + (1.2 N·s^2/m^2)v^2, where m and v are the mass and speed of the car. What is the terminal speed for the car rolling down this grade?

Picture the Problem The free-body diagram shows the forces acting on the car as it descends the grade with its terminal velocity and a convenient coordinate system. The application of Newton's 2nd law with a = 0 and F_d equal to the given function will allow us to solve for the terminal velocity of the car.



Apply $\sum F_x = ma_x$ to the car: $mg\sin\theta - F_d = ma_x$

or, because $v = v_t$ and $a_x = 0$,

$mg\sin\theta - F_d = 0$

Substitute for F_d to obtain: $mg\sin\theta - 100\,\text{N} - \left(1.2\,\text{N}\cdot\text{s}^2/\text{m}^2\right)v_t^2 = 0$

Solving for v_t yields:

$$v_t = \sqrt{\frac{mg\sin\theta - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}}$$

Substitute numerical values and evaluate v_t:

$$\boldsymbol{v}_t = \sqrt{\frac{(800\,\text{kg})(9.81\,\text{m/s}^2)\sin 6.0° - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}} = \boxed{25\,\text{m/s}}$$

72 ••• Small slowly moving spherical particles experience a drag force given by Stokes' law: $F_d = 6\pi\eta rv$, where r is the radius of the particle, v is its speed, and η is the coefficient of viscosity of the fluid medium. (a) Estimate the terminal speed of a spherical pollution particle of radius 1.00×10^{-5} m and density of 2000 kg/m^3. (b) Assuming that the air is still and that η is 1.80×10^{-5} N·s/m^2, estimate the time it takes for such a particle to fall from a height of 100 m.

Picture the Problem Let the downward direction be the +y direction and apply Newton's 2nd law to the particle to obtain an equation from which we can find the particle's terminal speed.

(a) Apply $\sum F_y = ma_y$ to a pollution particle: $\boldsymbol{mg} - 6\pi\eta\,\boldsymbol{rv} = \boldsymbol{ma}_y$

or, because $a_y = 0$,

$$\boldsymbol{mg} - 6\pi\eta\,\boldsymbol{rv}_t = 0 \Rightarrow \boldsymbol{v}_t = \frac{\boldsymbol{mg}}{6\pi\eta\,\boldsymbol{r}}$$

Express the mass of a sphere in terms of its volume:

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substitute for m to obtain:

$$v_t = \frac{2r^2 \rho g}{9\eta}$$

Substitute numerical values and evaluate v_t:

$$\boldsymbol{v}_t = \frac{2(1.00\times10^{-5}\ \text{m})^2(2000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}{9(1.80\times10^{-5}\ \text{N}\cdot\text{s/m}^2)} = 2.422\ \text{cm/s} = \boxed{2.42\,\text{cm/s}}$$

(b) Use distance equals average speed times the fall time to find the time to fall 100 m at 2.42 cm/s:

$$\boldsymbol{t} = \frac{10^4\ \text{cm}}{2.422\,\text{cm/s}} = 4.128\times10^3\ \text{s} = \boxed{1.15\,\text{h}}$$

73 ••• **[SSM]** You are on an environmental chemistry internship, and in charge of a sample of air-containing pollution particles of the size and density given in Problem 72. You capture the sample in an 8.0-cm-long test tube. You then place the test tube in a centrifuge with the midpoint of the test tube 12 cm from the center of the centrifuge. You set the centrifuge to spin at 800 revolutions per minute. (a) Estimate the time you have to wait so that nearly all of the pollution particles settle to the end of the test tube. (b) Compare this to the time required for a pollution particle to fall 8.0 cm under the action of gravity and subject to the drag force given in Problem 72.

Picture the Problem The motion of the centrifuge will cause the pollution particles to migrate to the end of the test tube. We can apply Newton's 2nd law and Stokes' law to derive an expression for the terminal speed of the sedimentation particles. We can then use this terminal speed to calculate the sedimentation time. We'll use the 12 cm distance from the center of the centrifuge as the average radius of the pollution particles as they settle in the test tube. Let R represent the radius of a particle and r the radius of the particle's circular path in the centrifuge.

(a) Express the sedimentation time in terms of the sedimentation speed v_t:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{v_t} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to a pollution particle:

$$6\pi\eta R v_t = ma_c$$

Express the mass of the particle in terms of its radius R and density ρ:

$$m = \rho V = \tfrac{4}{3}\pi R^3 \rho$$

Express the acceleration of the pollution particles due to the motion of the centrifuge in terms of their orbital radius r and period T:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Substitute for m and a_c and simplify to obtain:

$$6\pi\eta R v_t = \tfrac{4}{3}\pi R^3 \rho\left(\frac{4\pi^2 r}{T^2}\right) = \frac{16\pi^3 \rho r R^3}{3T^2}$$

Solving for v_t yields:

$$v_t = \frac{8\pi^2 \rho r R^2}{9\eta T^2}$$

Substitute for v_t in equation (1) and simplify to obtain:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{\frac{8\pi^2 \rho r R^2}{9\eta T^2}} = \frac{9\eta T^2 \Delta x}{8\pi^2 \rho r R^2}$$

Substitute numerical values and evaluate $\Delta t_{\text{sediment}}$:

$$\Delta t_{\text{sediment}} = \frac{9\left(1.8\times10^{-5}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(\frac{1}{800\,\frac{\text{rev}}{\text{min}}\cdot\frac{1\,\text{min}}{60\,\text{s}}}\right)^2 (8.0\,\text{cm})}{8\pi^2\left(2000\,\frac{\text{kg}}{\text{m}^3}\right)(0.12\,\text{m})\left(10^{-5}\,\text{m}\right)^2} = 38.47\,\text{ms} = \boxed{38\,\text{ms}}$$

(*b*) In Problem 72 it was shown that the rate of fall of the particles in air is 2.42 cm/s. Find the time required to fall 8.0 cm in air under the influence of gravity:

$$\Delta \boldsymbol{t}_{\text{air}} = \frac{\Delta \boldsymbol{x}}{\boldsymbol{v}} = \frac{8.0\,\text{cm}}{2.42\,\text{cm/s}} = 3.31\,\text{s}$$

Find the ratio of the two times:

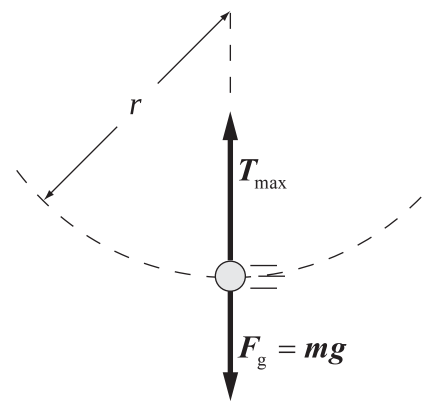
$$\frac{\Delta t_{\text{air}}}{\Delta t_{\text{sediment}}} = \frac{3.31\,\text{s}}{38.47\,\text{ms}} = 86$$

With the drag force in Problem 72 it takes about 86 times longer than it does using the centrifuge.

Motion Along a Curved Path

74 • A 0.050-kg ball at the end of a string rotates at constant speed in a vertical circle with a radius of 0.20 m. What is the maximum speed of the ball so that the tension is not to exceed 10 N?

Picture the Problem The tension in the string is a maximum when the ball is at the bottom of the vertical circle. We can use Newton's 2nd law to relate the tension in the string to the mass m of the ball, the radius r of its path, and the constant speed of the ball along its circular path.



Apply $\sum \boldsymbol{F}_{\text{radial}} = m\boldsymbol{a}_{\text{radial}}$ to the ball:

$$\boldsymbol{T}_{\text{max}} - m\boldsymbol{g} = m\frac{\boldsymbol{v}_{\text{max}}^2}{\boldsymbol{r}}$$

Solving for v_{max} yields:

$$\boldsymbol{v}_{\text{max}} = \sqrt{\left(\frac{\boldsymbol{T}_{\text{max}}}{m} - \boldsymbol{g}\right)\boldsymbol{r}}$$

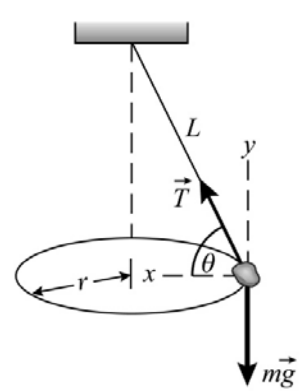
Substitute numerical values and evaluate v_{max}:

$$\boldsymbol{v}_{\text{max}} = \sqrt{\left(\frac{10\ \text{N}}{0.050\ \text{kg}} - 9.81\ \text{m/s}^2\right)(0.20\ \text{m})}$$

$$= \boxed{6.2\ \text{m/s}}$$

75 • **[SSM]** A 95-g stone is whirled in a horizontal circle on the end of an 85-cm-long string. The stone takes 1.2 s to make one complete revolution. Determine the angle that the string makes with the horizontal.

Picture the Problem The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$$\sum F_x = T\cos\theta = ma_c = m\frac{v^2}{r} \quad (1)$$

and

$$\sum F_y = T\sin\theta - mg = 0 \quad (2)$$

Use the right triangle in the diagram to relate r, L, and θ:

$$\boldsymbol{r = L\cos\theta} \quad (3)$$

Eliminate T and r between equations (1), (2) and (3) and solve for v_2:

$$v^2 = gL\cot\theta\cos\theta \quad (4)$$

Express the speed of the stone in terms of its period:

$$v = \frac{2\pi r}{t_{1\,\text{rev}}} \quad (5)$$

Eliminate v between equations (4) and (5) and solve for θ:

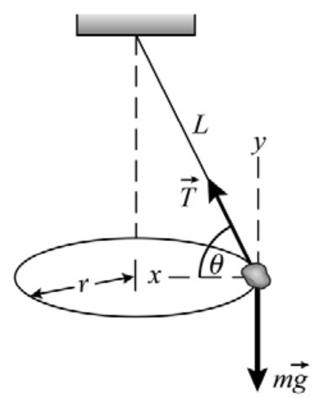
$$\theta = \sin^{-1}\left(\frac{gt_{1\,\text{rev}}^2}{4\pi^2 L}\right)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \sin^{-1}\left[\frac{(9.81\,\text{m/s}^2)(1.2\,\text{s})^2}{4\boldsymbol{\pi}^2(0.85\,\text{m})}\right] = \boxed{25°}$$

76 •• A 0.20-kg stone is whirled in a horizontal circle on the end of an 0.80-m-long string. The string makes an angle of 20º with the horizontal. Determine the speed of the stone.

Picture the Problem The only forces acting on the stone as it moves in a horizontal circle are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$$\sum F_x = T\cos\theta = ma_c = m\frac{v^2}{r} \quad (1)$$

and

$$\sum F_y = T\sin\theta - mg = 0 \quad (2)$$

Use the right triangle in the diagram to relate r, L, and θ:

$$\boldsymbol{r} = \boldsymbol{L}\cos\boldsymbol{\theta} \qquad (3)$$

Eliminate T and r between equations (1), (2), and (3) and solve for v:

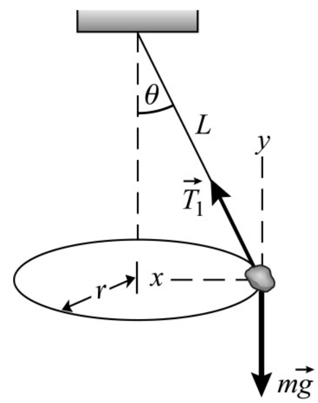
$$v = \sqrt{gL\cot\theta\cos\theta}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{(9.81\,\text{m/s}^2)(0.80\,\text{m})\cot 20°\cos 20°} = \boxed{4.5\,\text{m/s}}$$

77 •• A 0.75-kg stone attached to a string is whirled in a horizontal circle of radius 35 cm as in the tetherball Example 5-11. The string makes an angle of 30º with the vertical. (*a*) Find the speed of the stone. (*b*) Find the tension in the string.

Picture the Problem The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the vertical is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the stone:

$$\sum F_x = T\sin\theta = ma_c = m\frac{v^2}{r} \qquad (1)$$

and

$$\sum F_y = T\cos\theta - mg = 0 \qquad (2)$$

Eliminate T between equations (1) and (2) and solve for v:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{(0.35\,\text{m})(9.81\,\text{m/s}^2)\tan 30°} = \boxed{1.4\,\text{m/s}}$$

(*b*) Solving equation (2) for T yields:

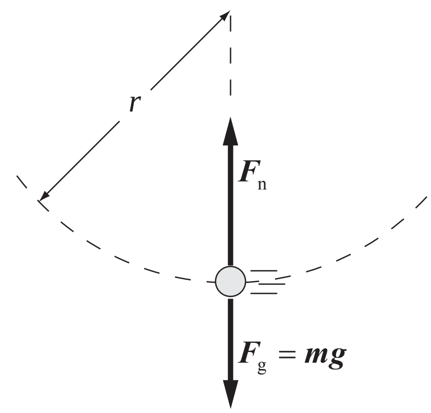
$$T = \frac{mg}{\cos\theta}$$

Substitute numerical values and evaluate T:

$$\boldsymbol{T} = \frac{(0.75\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 30°} = \boxed{8.5\,\text{N}}$$

78 •• A pilot with a mass of 50 kg comes out of a vertical dive in a circular arc such that at the bottom of the arc her upward acceleration is 3.5*g*. (*a*) How does the magnitude of the force exerted by the airplane seat on the pilot at the bottom of the arc compare to her weight? (*b*) Use Newton's laws of motion to explain why the pilot might be subject to a blackout. This means that an above normal volume of blood "pools" in her lower limbs. How would an inertial reference frame observer describe the cause of the blood pooling?

Picture the Problem The diagram shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll apply Newton's 2nd law for circular motion to determine F_n and the radius of the circular path followed by the airplane.



(*a*) Apply $\sum F_{radial} = ma_{radial}$ to the pilot:

$$F_n - mg = ma_c \Rightarrow F_n = m(g + a_c)$$

Because $a_c = 3.5g$:

$$F_n = m(g + 3.5g) = 4.5mg$$

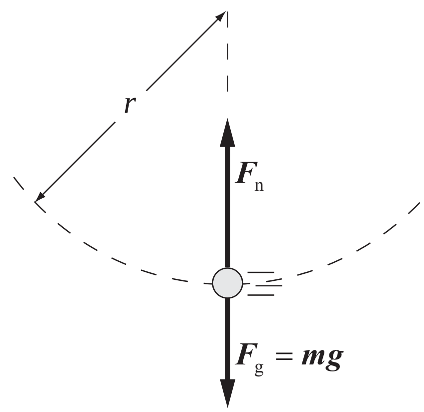
The ratio of F_n to her weight is:

$$\frac{F_n}{mg} = \frac{4.5mg}{mg} = \boxed{4.5}$$

(*b*) An observer in an inertial reference frame would see the pilot's blood continue to flow in a straight line tangent to the circle at the lowest point of the arc. The pilot accelerates upward away from this lowest point and therefore it appears, from the reference frame of the plane, as though the blood accelerates downward.

79 •• A 80.0-kg airplane pilot pulls out of a dive by following, at constant speed, the arc of a circle whose radius is 300 m. At the bottom of the circle, where his speed is 180 km/h, (*a*) what are the direction and magnitude of his acceleration? (*b*) What is the net force acting on him at the bottom of the circle? (*c*) What is the force exerted on the pilot by the airplane seat?

Picture the Problem The diagram shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll use the definitions of centripetal acceleration and centripetal force and apply Newton's 2nd law to calculate these quantities and the normal force acting on her.



(*a*) The pilot's acceleration is centripetal and given by:

$$a_c = \frac{v^2}{r}\text{, upward}$$

Substitute numerical values and evaluate a_c:

$$\boldsymbol{a}_c = \frac{\left(180\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{300\,\text{m}} = 8.333\,\text{m/s}^2 = \boxed{8.33\,\text{m/s}^2\text{, upward}}$$

(*b*) The net force acting on her at the bottom of the circle is the force responsible for her centripetal acceleration:

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}_c = (80.0\,\text{kg})(8.333\,\text{m/s}^2) = \boxed{667\,\text{N, upward}}$$

(*c*) Apply $\sum F_y = ma_y$ to the pilot:

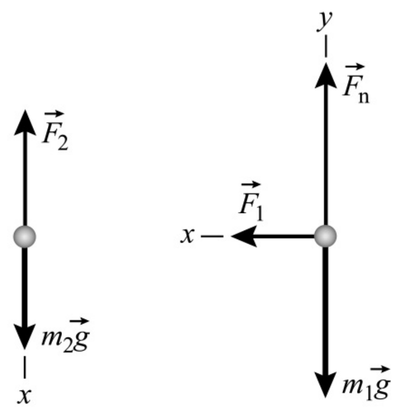
$$F_n - mg = ma_c \Rightarrow F_n = m(g + a_c)$$

Substitute numerical values and evaluate F_n:

$$F_n = (80.0\,\text{kg})(9.81\,\text{m/s}^2 + 8.33\,\text{m/s}^2) = \boxed{1.45\,\text{kN, upward}}$$

80 •• A small object of mass m_1 moves in a circular path of radius r on a frictionless horizontal tabletop (Figure 5-72). It is attached to a string that passes through a small frictionless hole in the center of the table. A second object with a mass of m_2 is attached to the other end of the string. Derive an expression for r in terms of m_1, m_2, and the time T for one revolution.

Picture the Problem Assume that the string is massless and that it does not stretch. The free-body diagrams for the two objects are shown to the right. The hole in the table changes the direction the tension in the string (which provides the centripetal force required to keep the object moving in a circular path) acts. The application of Newton's 2nd law and the definition of centripetal force will lead us to an expression for r as a function of m_1, m_2, and the time T for one revolution.



Apply $\sum F_x = ma_x$ to both objects and use the definition of centripetal acceleration to obtain:

$$m_2 g - F_2 = 0$$

and

$$F_1 = m_1 a_c = m_1 \frac{v^2}{r}$$

Because $F_1 = F_2$ we can eliminate both of them between these equations to obtain:

$$m_2 g - m_1 \frac{v^2}{r} = 0 \qquad (1)$$

Express the speed v of the object in terms of the distance it travels each revolution and the time T for one revolution:

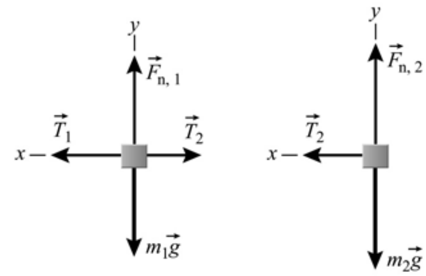
$$v = \frac{2\pi r}{T}$$

Substitute for v in equation (1) to obtain:

$$m_2 g - m_1 \frac{4\pi^2 r}{T^2} = 0 \Rightarrow r = \boxed{\frac{m_2 g T^2}{4\pi^2 m_1}}$$

81 •• **[SSM]** A block of mass m_1 is attached to a cord of length L_1, which is fixed at one end. The block moves in a horizontal circle on a frictionless tabletop. A second block of mass m_2 is attached to the first by a cord of length L_2 and also moves in a circle on the same frictionless tabletop, as shown in Figure 5-73. If the period of the motion is T, find the tension in each cord in terms of the given symbols.

Picture the Problem The free-body diagrams show the forces acting on each block. We can use Newton's 2nd law to relate these forces to each other and to the masses and accelerations of the blocks.



Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T_1 - T_2 = m_1 \frac{v_1^2}{L_1}$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$T_2 = m_2 \frac{v_2^2}{L_1 + L_2}$$

Relate the speeds of each block to their common period T and their distance from the center of the circle:

$$v_1 = \frac{2\pi L_1}{T} \text{ and } v_2 = \frac{2\pi(L_1 + L_2)}{T}$$

In the second force equation, substitute for v_2, and simplify to obtain:

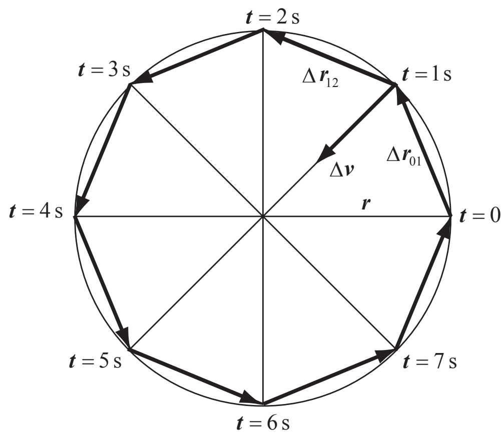
$$T_2 = \boxed{[m_2(L_1 + L_2)]\left(\frac{2\pi}{T}\right)^2}$$

Substitute for T_2 and v_1 in the first force equation to obtain:

$$T_1 = \boxed{[m_2(L_1 + L_2) + m_1 L_1]\left(\frac{2\pi}{T}\right)^2}$$

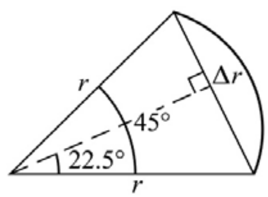
82 •• A particle moves with constant speed in a circle of radius 4.0 cm. It takes 8.0 s to complete each revolution. (*a*) Draw the path of the particle to scale, and indicate the particle's position at 1.0-s intervals. (*b*) Sketch the displacement vectors for each interval. These vectors also indicate the directions for the average-velocity vectors for each interval. (*c*) Graphically find the magnitude of the change in the average velocity $|\Delta\vec{v}|$ for two consecutive 1-s intervals. Compare $|\Delta\vec{v}|/\Delta t$, measured in this way, with the magnitude of the instantaneous acceleration computed from $a_c = v^2/r$.

Picture the Problem (*a*) and (*b*) The path of the particle and its position at 1-s intervals are shown in the following diagram. The displacement vectors are also shown. The velocity vectors for the average velocities in the first and second intervals are along $\vec{r}_{01}$ and $\vec{r}_{12}$, respectively. $\Delta\vec{v}$ points toward the center of the circle.



(*c*) Use the diagram below to show that:

$$\Delta\boldsymbol{r} = 2\boldsymbol{r}\sin 22.5°$$

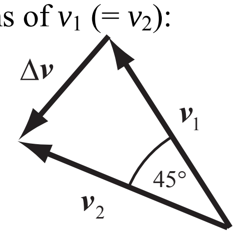


Express the magnitude of the average velocity of the particle along the chords:

$$\left|\vec{v}_{\text{av}}\right| = \frac{\Delta r}{\Delta t} = \frac{2r\sin 22.5°}{\Delta t}$$

Using the diagram below, express Δv in terms of v_1 ($= v_2$):

$$\Delta\boldsymbol{v} = 2\boldsymbol{v}_1\sin 22.5°$$



Express Δv using v_{av} as v_1:

$$\Delta\boldsymbol{v} = 2\left(\frac{2\boldsymbol{r}\sin 22.5°}{\Delta\boldsymbol{t}}\right)\sin 22.5°$$

$$= \frac{4\boldsymbol{r}\sin^2 22.5°}{\Delta\boldsymbol{t}}$$

Express $\boldsymbol{a} = \dfrac{\Delta\boldsymbol{v}}{\Delta\boldsymbol{t}}$:

$$\boldsymbol{a} = \frac{\dfrac{4\boldsymbol{r}\sin^2 22.5°}{\Delta\boldsymbol{t}}}{\Delta\boldsymbol{t}} = \frac{4\boldsymbol{r}\sin^2 22.5°}{(\Delta\boldsymbol{t})^2}$$

Substitute numerical values and evaluate a:

$$a=\frac{4(4.0\text{ cm})\sin^2 22.5°}{(1.0\text{ s})^2}=2.34\text{ cm/s}^2$$
$$=\boxed{2.3\text{ cm/s}^2}$$

The radial acceleration of the particle is given by:

$$a_c=\frac{v^2}{r}$$

Express the speed v ($= v_1 = v_2 \ldots$) of the particle along its circular path:

$$v=\frac{2\pi r}{T}$$

Substituting for v in the expression for a_c yields:

$$a_c=\frac{\left(\frac{2\pi r}{T}\right)^2}{r}=\frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

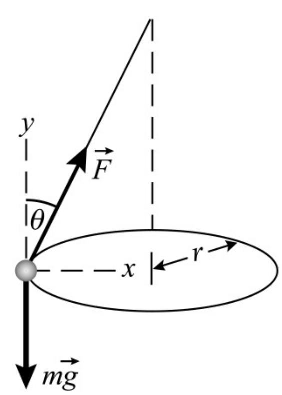
$$a_c=\frac{4\pi^2(4.0\text{ cm})}{(8.0\text{ s})^2}=2.47\text{ cm/s}^2$$
$$=\boxed{2.5\text{ cm/s}^2}$$

Compare a_c and a by taking their ratio:

$$\frac{a_c}{a}=\frac{2.47\text{ cm/s}^2}{2.34\text{ cm/s}^2}=1.06\Rightarrow a_c=\boxed{1.1a}$$

83 •• You are swinging your younger sister in a circle of radius 0.75 m, as shown in Figure 5-74. If her mass is 25 kg and you arrange it so she makes one revolution every 1.5 s, (*a*) what is the magnitude and direction of the force that must be exerted by you on her? (Assume her to be a point particle.) (*b*) What is the magnitude and direction of the force she exerts on you?

Picture the Problem The diagram to the right has the free-body diagram for the child superimposed on a pictorial representation of her motion. The force you exert on your sister is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive y direction is θ. We can infer her speed from the given information concerning the radius of her path and the period of her motion. Applying Newton's 2nd law will allow us to find both the direction and magnitude of $\vec{F}$.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the child:

$$\sum F_x = F\sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F\cos\theta - mg = 0$$

Eliminate *F* between these equations and solve for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Express *v* in terms of the radius and period of the child's motion:

$$v = \frac{2\pi r}{T}$$

Substitute for *v* in the expression for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{4\pi^2 r}{gT^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{4\pi^2(0.75\,\text{m})}{(9.81\,\text{m/s}^2)(1.5\,\text{s})^2}\right] = 53.3°$$

$$= \boxed{53°} \text{ above horizontal}$$

Solve the *y* equation for *F*:

$$F = \frac{mg}{\cos\theta}$$

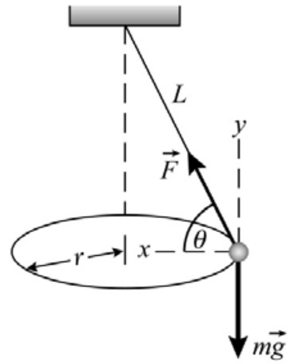
Substitute numerical values and evaluate *F*:

$$F = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 53.3°} = \boxed{0.41\,\text{kN}}$$

(*b*) The force your sister exerts on you is the reaction force to the force you exert on her. Thus its magnitude is the same as the force you exert on her (0.41 kN) and its direction is 53° below horizontal.

84 •• The string of a conical pendulum is 50.0 cm long and the mass of the bob is 0.25 kg. (*a*) Find the angle between the string and the horizontal when the tension in the string is six times the weight of the bob. (*b*) Under those conditions, what is the period of the pendulum?

Picture the Problem The diagram to the right has the free-body diagram for the bob of the conical pendulum superimposed on a pictorial representation of its motion. The tension in the string is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive *x* direction is θ. We can find θ from the *y* equation and the information provided about the tension. Then, by using the definition of the speed of the bob in its orbit and applying Newton's 2nd law as it describes circular motion, we can find the period *T* of the motion.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the pendulum bob:

$$\sum F_x = F\cos\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F\sin\theta - mg = 0$$

Using the given information that $F = 6mg$, solve the y equation for θ and simplify to obtain:

$$\boldsymbol{\theta} = \sin^{-1}\left(\frac{\boldsymbol{mg}}{\boldsymbol{F}}\right) = \sin^{-1}\left(\frac{\boldsymbol{mg}}{6\boldsymbol{mg}}\right) = \boxed{9.6^\circ}$$

(*b*) With $F = 6mg$, solve the x equation for v:

$$v = \sqrt{6rg\cos\theta}$$

Relate the period T of the motion to the speed of the bob and the radius of the circle in which it moves:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{6rg\cos\theta}}$$

From the diagram, one can see that:

$$\boldsymbol{r} = \boldsymbol{L}\cos\boldsymbol{\theta}$$

Substitute for r in the expression for the period and simplify to obtain:

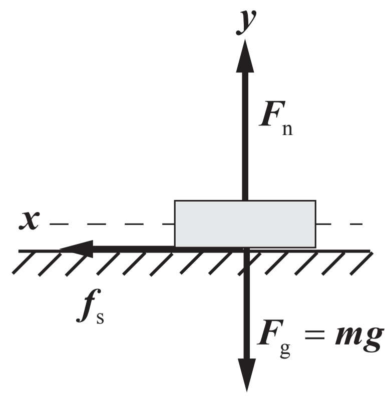
$$T = 2\pi\sqrt{\frac{L}{6g}}$$

Substitute numerical values and evaluate T:

$$\boldsymbol{T} = 2\boldsymbol{\pi}\sqrt{\frac{0.50\text{ m}}{6(9.81\text{ m/s}^2)}} = \boxed{0.58\text{ s}}$$

85 •• A 100-g coin sits on a horizontally rotating turntable. The turntable makes exactly 1.00 revolution each second. The coin is located 10 cm from the axis of rotation of the turntable. (*a*) What is the frictional force acting on the coin? (*b*) If the coin slides off the turntable when it is located more than 16.0 cm from the axis of rotation, what is the coefficient of static friction between the coin and the turntable?

Picture the Problem The static friction force f_s is responsible for keeping the coin from sliding on the turntable. Using Newton's 2nd law of motion, the definition of the period of the coin's motion, and the definition of the maximum static friction force, we can find the magnitude of the friction force and the value of the coefficient of static friction for the two surfaces.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the coin:

$$\sum F_x = f_s = m\frac{v^2}{r} \qquad (1)$$

and

$$\sum F_y = F_n - mg = 0$$

If T is the period of the coin's motion, its speed is given by:

$$v = \frac{2\pi r}{T}$$

Substitute for v in equation (1) and simplify to obtain:

$$f_s = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{4\pi^2((0.100\text{ kg})(0.10\text{ m}))}{(1.00\text{ s})^2} = \boxed{0.40\text{ N}}$$

(*b*) Determine F_n from the y equation:

$$F_n = mg$$

If the coin is on the verge of sliding at $r = 16$ cm, $f_s = f_{s,max}$. Solve for μ_s in terms of $f_{s,max}$ and F_n:

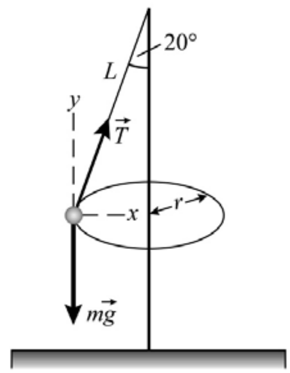
$$\mu_s = \frac{f_{s,max}}{F_n} = \frac{\frac{4\pi^2 mr}{T^2}}{mg} = \frac{4\pi^2 r}{gT^2}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{4\pi^2(0.160\text{ m})}{(9.81\text{ m/s}^2)(1.00\text{ s})^2} = \boxed{0.644}$$

86 •• A 0.25-kg tether ball is attached to a vertical pole by a 1.2-m cord. Assume the radius of the ball is negligible. If the ball moves in a horizontal circle with the cord making an angle of 20º with the vertical, (*a*) what is the tension in the cord? (*b*) What is the speed of the ball?

Picture the Problem The forces acting on the tetherball are shown superimposed on a pictorial representation of the motion. The horizontal component of $\vec{T}$ is the centripetal force. Applying Newton's 2nd law of motion and solving the resulting equations will yield both the tension in the cord and the speed of the ball.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the tetherball:

$$\sum F_x = T\sin 20° = m\frac{v^2}{r}$$

and

$$\sum F_y = T\cos 20° - mg = 0$$

Solve the *y* equation for *T*:

$$T = \frac{mg}{\cos 20°}$$

Substitute numerical values and evaluate *T*:

$$\boldsymbol{T} = \frac{(0.25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 20°} = \boxed{2.6\,\text{N}}$$

(*b*) Eliminate *T* between the force equations and solve for *v* to obtain:

$$v = \sqrt{rg\tan 20°}$$

Note from the diagram that:

$$r = L\sin 20°$$

Substitute for *r* in the expression for *v* to obtain:

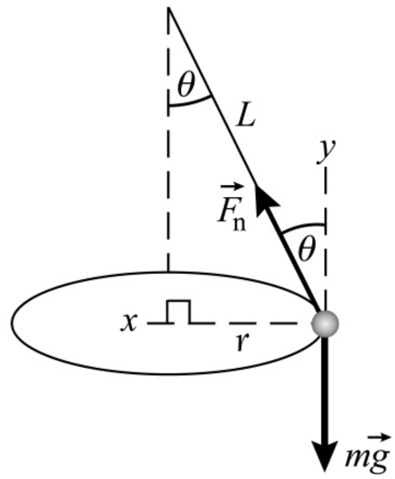
$$v = \sqrt{gL\sin 20°\tan 20°}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(9.81\,\text{m/s}^2)(1.2\,\text{m})\sin 20°\tan 20°} = \boxed{1.2\,\text{m/s}}$$

87 ••• A small bead with a mass of 100 g (Figure 5-75) slides without friction along a semicircular wire with a radius of 10 cm that rotates about a vertical axis at a rate of 2.0 revolutions per second. Find the value of θ for which the bead will remain stationary relative to the rotating wire.

Picture the Problem The semicircular wire of radius 10 cm limits the motion of the bead in the same manner as would a 10-cm string attached to the bead and fixed at the center of the semicircle. The horizontal component of the normal force the wire exerts on the bead is the centripetal force. The application of Newton's 2nd law, the definition of the speed of the bead in its orbit, and the relationship of the frequency of a circular motion to its period will yield the angle at which the bead will remain stationary relative to the rotating wire.



Apply $\sum\vec{F} = m\vec{a}$ to the bead:

$$\sum F_x = F_\text{n}\sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_\text{n}\cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

The frequency of the motion is the reciprocal of its period T. Express the speed of the bead as a function of the radius of its path and its period:

$$v = \frac{2\pi r}{T}$$

Using the diagram, relate r to L and θ:

$$r = L\sin\theta$$

Substitute for r and v in the expression for $\tan\theta$ and solve for θ:

$$\theta = \cos^{-1}\left[\frac{gT^2}{4\pi^2 L}\right]$$

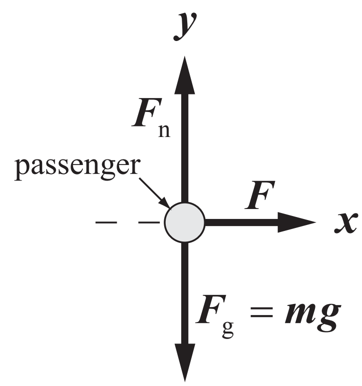
Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(9.81\,\text{m/s}^2)(0.50\,\text{s})^2}{4\pi^2(0.10\,\text{m})}\right] = \boxed{52°}$$

Centripetal Force

88 • A car speeds along the curved exit ramp of a freeway. The radius of the curve is 80.0 m. A 70.0-kg passenger holds the armrest of the car door with a 220-N force in order to keep from sliding across the front seat of the car. (Assume the exit ramp is not banked and ignore friction with the car seat.) What is the car's speed?

Picture the Problem The force $\vec{F}$ the passenger exerts on the armrest of the car door is the radial force required to maintain the passenger's speed around the curve and is related to that speed through Newton's 2nd law of motion.



Apply $\sum F_x = ma_x$ to the forces acting on the passenger:

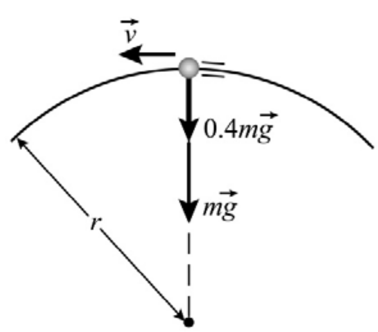
$$F = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{rF}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{(80.0\,\text{m})(220\,\text{N})}{70.0\,\text{kg}}} = \boxed{15.9\,\text{m/s}}$$

89 • **[SSM]** The radius of curvature of the track at the top of a loop-the-loop on a roller-coaster ride is 12.0 m. At the top of the loop, the force that the seat exerts on a passenger of mass m is 0.40mg. How fast is the roller-coaster car moving as it moves through the highest point of the loop.

Picture the Problem The speed of the roller coaster is embedded in the expression for its radial acceleration. The radial acceleration is determined by the net radial force acting on the passenger. We can use Newton's 2nd law to relate the net force on the passenger to the speed of the roller coaster.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the passenger:

$$\boldsymbol{mg} + 0.40\boldsymbol{mg} = \boldsymbol{m}\frac{\boldsymbol{v}^2}{\boldsymbol{r}} \Rightarrow \boldsymbol{v} = \sqrt{1.40\boldsymbol{gr}}$$

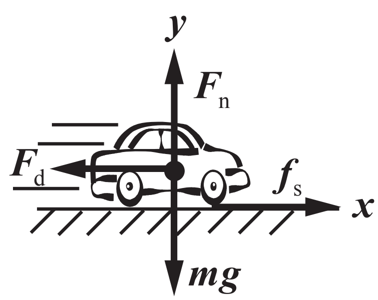
Substitute numerical values and evaluate v:

$$v = \sqrt{(1.40)(9.81\,\text{m/s}^2)(12.0\,\text{m})} = \boxed{12.8\,\text{m/s}}$$

90 •• On a runway of a decommissioned airport, a 2000-kg car travels at a constant speed of 100 km/h. At 100-km/h the air drag on the car is 2000 N. Assume that rolling friction is negligible. (*a*) What is the force of static friction exerted on the car by the surface, and what is the minimum coefficient of static friction necessary for the car to sustain this speed? (*b*) The car continues to travel at 100 km/h, but now along a path with radius of curvature *r*. For what value of *r* will the angle between the static frictional force vector and the velocity vector equal 45.0°, and for what value of *r* will it equal 83.0°? (*c*) What is the minimum coefficient of static friction necessary for the car to hold this last radius of curvature without skidding? Comment on whether you think it is realistic to expect the car to take this curve.

Picture the Problem (*a*) We can apply Newton's 2nd law to the car to find the force of static friction exerted on the car by the surface and use the definition of the coefficient of static friction to find the minimum coefficient of static friction necessary for the car to sustain its speed. In Part (*b*), we can again apply Newton's 2nd law, this time in both tangential and radial form, to find the values of *r* for the given angles.

(*a*) The forces acting on the car as it moves at a constant speed of 100 km/h are shown in the pictorial representation to the right.



Apply $\sum \vec{F} = m\vec{a}$ to the car to obtain:

$$\sum F_x = f_s - F_d = 0 \qquad (1)$$

and

$$\sum F_y = F_n - mg = 0 \qquad (2)$$

Solving equation (1) for f_s yields:

$$f_s = F_d = \boxed{2000\text{ N}}$$

Use the definition of the coefficient of static friction to obtain:

$$\mu_{s,\min} = \frac{f_s}{F_n}$$

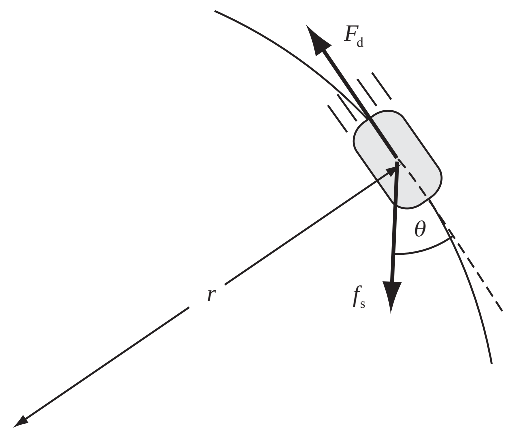
From equation (2), $F_n = mg$; hence:

$$\mu_{s,\min} = \frac{f_s}{mg}$$

Substitute numerical values and evaluate $\mu_{s,\,min}$:

$$\mu_{s,\,min} = \frac{2000\ \text{N}}{(2000\ \text{kg})(9.81\ \text{m/s}^2)} = \boxed{0.102}$$

(*b*) The horizontal forces acting on the car (shown as a top view) as it travels clockwise along a path of radius of curvature *r* are shown in the diagram to the right.



Apply $\sum \boldsymbol{F}_{radial} = \boldsymbol{m}\boldsymbol{a}_c$ to the car to obtain:

$$f_s \sin\theta = m\frac{v^2}{r} \qquad (1)$$

Appling $\sum F_{tangential} = ma_t$ to the car yields:

$$f_s \cos\theta - F_d = 0$$

or

$$f_s \cos\theta = F_d \qquad (2)$$

Divide equation (1) by equation (2):

$$\tan\theta = \frac{mv^2}{rF_d} \Rightarrow r = \frac{mv^2}{F_d \tan\theta}$$

Substitute numerical values and evaluate *r* for $\theta = 45.0°$:

$$r = \frac{(2000\ \text{kg})\left(100\frac{\text{km}}{\text{h}}\cdot\frac{10^3\ \text{m}}{\text{km}}\cdot\frac{1\ \text{h}}{3600\ \text{s}}\right)^2}{(2000\ \text{N})\tan 45.0°} = \boxed{772\ \text{m}}$$

Substitute numerical values and valuate *r* for $\theta = 83.0°$:

$$r = \frac{(2000\ \text{kg})\left(100\frac{\text{km}}{\text{h}}\cdot\frac{10^3\ \text{m}}{\text{km}}\cdot\frac{1\ \text{h}}{3600\ \text{s}}\right)^2}{(2000\ \text{N})\tan 83.0°} = 94.74\ \text{m} = \boxed{94.7\ \text{m}}$$

(*c*) The minimum coefficient of static friction necessary for the car to hold this last radius of curvature without skidding is given by:

$$\mu_{s,\,min} = \frac{\boldsymbol{f}_{s,\,min}}{\boldsymbol{mg}} = \frac{\boldsymbol{m}\frac{\boldsymbol{v}^2}{\boldsymbol{r}}}{\boldsymbol{mg}} = \frac{\boldsymbol{v}^2}{\boldsymbol{rg}}$$

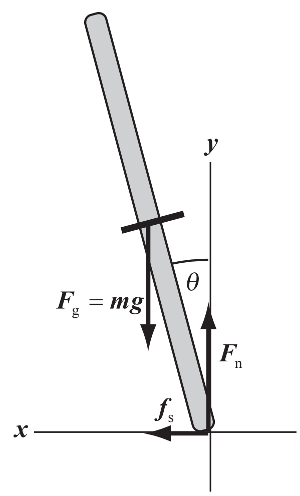
Substitute numerical values and evaluate $\mu_{\text{s, min}}$:

$$\mu_{\text{s, min}} = \frac{\left(100\,\frac{\text{km}}{\text{h}}\cdot\frac{10^3\text{ m}}{\text{km}}\cdot\frac{1\text{ h}}{3600\text{ s}}\right)^2}{(94.74\text{ m})(9.81\text{ m/s}^2)} = \boxed{0.830}$$

Although 0.830 is a rather high value for a coefficient of static friction, it is possible that a car could safely navigate this curve at the given speed.

91 •• Suppose you ride a bicycle in a 20-m-radius circle on a horizontal surface. The resultant force exerted by the surface on the bicycle (normal force plus frictional force) makes an angle of 15º with the vertical. (*a*) What is your speed? (*b*) If the frictional force on the bicycle is half its maximum possible value, what is the coefficient of static friction?

Picture the Problem The forces acting on the bicycle are shown in the force diagram. The static friction force is the centripetal force exerted by the surface on the bicycle that allows it to move in a circular path. $\vec{F}_n + \vec{f}_s$ makes an angle θ with the vertical direction. The application of Newton's 2nd law will allow us to relate this angle to the speed of the bicycle and the coefficient of static friction.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the bicycle:

$$\sum F_x = f_s = \frac{mv^2}{r}$$

and

$$\sum F_y = F_n - mg = 0$$

Relate F_n and f_s to θ:

$$\tan\theta = \frac{f_s}{F_n} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

Solving for v yields:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{(20\,\text{m})(9.81\,\text{m/s}^2)\tan 15°}=\boxed{7.3\,\text{m/s}}$$

(b) Relate f_s to μ_s and F_n:

$$f_s=\tfrac{1}{2}f_{s,\max}=\tfrac{1}{2}\mu_s mg$$

Solve for μ_s and substitute for f_s to obtain:

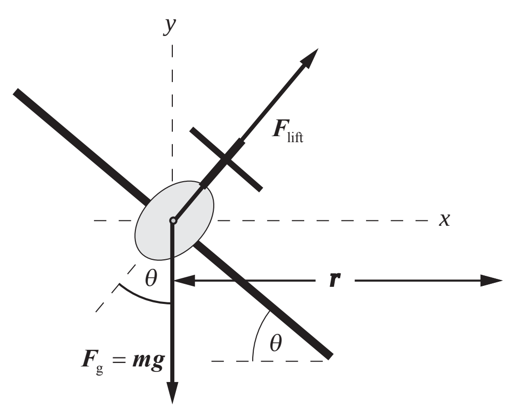
$$\mu_s=\frac{2f_s}{mg}=\frac{2v^2}{rg}$$

Substitute numerical values and evaluate μ_s

$$\mu_s=\frac{2(7.25\,\text{m/s})^2}{(20\,\text{m})(9.81\,\text{m/s}^2)}=\boxed{0.54}$$

92 •• An airplane is flying in a horizontal circle at a speed of 480 km/h. The plane is banked for this turn, its wings tilted at an angle of 40º from the horizontal (Figure 5-76). Assume that a lift force acting perpendicular to the wings acts on the aircraft as it moves through the air. What is the radius of the circle in which the plane is flying?

Picture the Problem The diagram shows the forces acting on the plane as it flies in a horizontal circle of radius r. We can apply Newton's 2nd law to the plane and eliminate the lift force in order to obtain an expression for r as a function of v and θ.



Apply $\sum\vec{F}=m\vec{a}$ to the plane:

$$\sum F_x=F_{\text{lift}}\sin\theta=m\frac{v^2}{r}$$

and

$$\sum F_y=F_{\text{lift}}\cos\theta-mg=0$$

Eliminate F_{lift} between these equations to obtain:

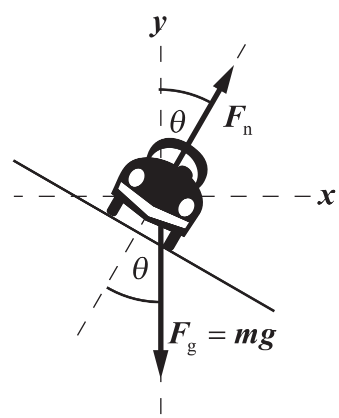
$$\tan\theta=\frac{v^2}{rg}\Rightarrow r=\frac{v^2}{g\tan\theta}$$

Substitute numerical values and evaluate r:

$$R=\frac{\left(480\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{(9.81\,\text{m/s}^2)\tan 40°}=\boxed{2.2\,\text{km}}$$

93 •• An automobile club plans to race a 750-kg car at the local racetrack. The car needs to be able to travel around several 160-m-radius curves at 90 km/h. What should the banking angle of the curves be so that the force of the pavement on the tires of the car is in the normal direction? (*Hint: What does this requirement tell you about the frictional force?*)

Picture the Problem Under the conditions described in the problem statement, the only forces acting on the car are the normal force exerted by the road and the gravitational force exerted by the earth. The horizontal component of the normal force is the centripetal force. The application of Newton's 2nd law will allow us to express θ in terms of v, r, and g.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_\text{n} \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_\text{n} \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

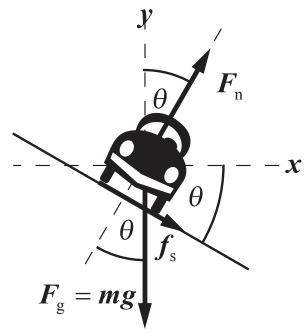
$$\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}\left\{\frac{\left(90\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{(160\,\text{m})(9.81\,\text{m/s}^2)}\right\} = \boxed{22°}$$

94 •• A curve of radius 150 m is banked at an angle of 10º. An 800-kg car negotiates the curve at 85 km/h without skidding. Neglect the effects of air drag and rolling friction. Find (*a*) the normal force exerted by the pavement on the tires, (*b*) the frictional force exerted by the pavement on the tires, (*c*) the minimum coefficient of static friction between the pavement and the tires.

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Multiply the x equation by $\sin\theta$ and the y equation by $\cos\theta$ to obtain:

$$f_s \sin\theta\cos\theta + F_n \sin^2\theta = m\frac{v^2}{r}\sin\theta$$

and

$$F_n \cos^2\theta - f_s \sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solve for F_n:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta = m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 10° + \frac{\left(85\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{150\,\text{m}}\sin 10°\right] = 8.245\,\text{kN}$$

$$= \boxed{8.3\,\text{kN}}$$

(*b*) Solve the y equation for f_s:

$$f_s = \frac{F_n\cos\theta - mg}{\sin\theta}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{(8.245\,\text{kN})\cos 10° - (800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 10°} = 1.565\,\text{kN} = \boxed{1.6\,\text{kN}}$$

(*c*) Express $\mu_{s,\min}$ in terms of f_s and F_n:

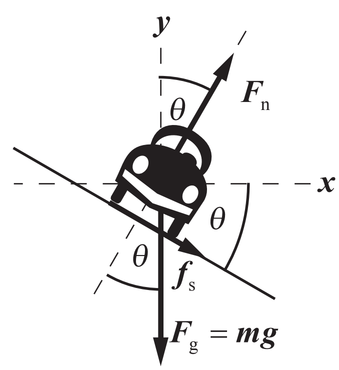
$$\mu_{s,\min} = \frac{f_s}{F_n}$$

Substitute numerical values and evaluate $\mu_{s,\min}$:

$$\mu_{s,\min} = \frac{1.565\ \text{kN}}{8.245\,\text{kN}} = \boxed{0.19}$$

95 •• On another occasion, the car in Problem 94 negotiates the curve at 38 km/h. Neglect the effects of air drag and rolling friction. Find (*a*) the normal force exerted on the tires by the pavement, and (*b*) the frictional force exerted on the tires by the pavement.

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Multiply the *x* equation by $\sin\theta$ and the *y* equation by $\cos\theta$:

$$f_s \sin\theta\cos\theta + F_n \sin^2\theta = m\frac{v^2}{r}\sin\theta$$

$$F_n \cos^2\theta - f_s \sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solving for F_n yields:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta = m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos10° + \frac{\left(38\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{150\,\text{m}}\sin10°\right] = 7.83\,\text{kN}$$

$$= \boxed{7.8\,\text{kN}}$$

(*b*) Solve the *y* equation for f_s:

$$\boldsymbol{f}_s = \frac{\boldsymbol{F}_n\cos\boldsymbol{\theta} - \boldsymbol{mg}}{\sin\boldsymbol{\theta}} = \boldsymbol{F}_n\cot\boldsymbol{\theta} - \frac{\boldsymbol{mg}}{\sin\boldsymbol{\theta}}$$

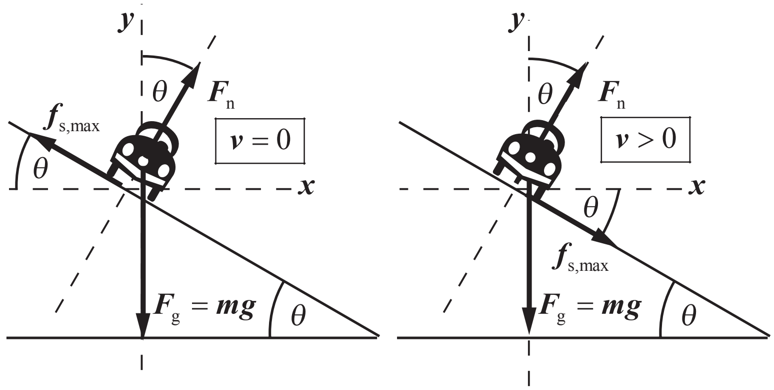
Substitute numerical values and evaluate f_s:

$$\boldsymbol{f}_s = (7.83\,\text{kN})\cot10° - \frac{(800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin10°} = \boxed{-0.78\,\text{kN}}$$

The negative sign tells us that f_s points upward along the inclined plane rather than as shown in the force diagram.

96 ••• As a civil engineer intern during one of your summers in college, you are asked to design a curved section of roadway that meets the following conditions: With ice on the road, when the coefficient of static friction between the road and rubber is 0.080, a car at rest must not slide into the ditch and a car traveling less than 60 km/h must not skid to the outside of the curve. Neglect the effects of air drag and rolling friction. What is the minimum radius of curvature of the curve and at what angle should the road be banked?

Picture the Problem The free-body diagram to the left is for the car at rest. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the moving car is to slide toward the outside of the curve.



Apply $\sum \vec{F} = m\vec{a}$ to the car that is at rest:

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0 \quad (1)$$

and

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = 0 \quad (2)$$

Substitute $f_s = f_{s,max} = \mu_s F_n$ in equation (2) and solve for the maximum allowable value of θ:

$$\boldsymbol{\theta} = \tan^{-1}(\boldsymbol{\mu}_s)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \tan^{-1}(0.080) = 4.57° = \boxed{4.6°}$$

Apply $\sum \vec{F} = m\vec{a}$ to the car that is moving with speed v:

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0 \quad (3)$$

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r} \quad (4)$$

Substitute $f_s = \mu_s F_n$ in equations (3) and (4) and simplify to obtain:

$$F_n(\cos\theta - \mu_s \sin\theta) = mg \quad (5)$$

and

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v^2}{r} \quad (6)$$

Substitute numerical values in equations (5) and (6) to obtain:

$$0.9904F_n = mg$$

and

$$0.1595F_n = m\frac{v^2}{r}$$

Eliminate F_n between these equations and solve for r:

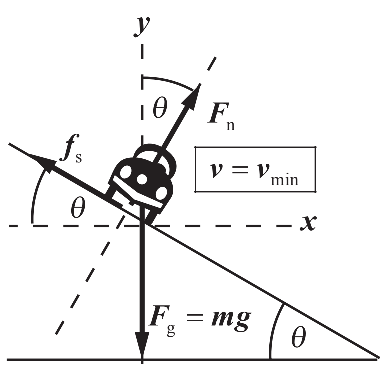
$$r = \frac{v^2}{0.1610g}$$

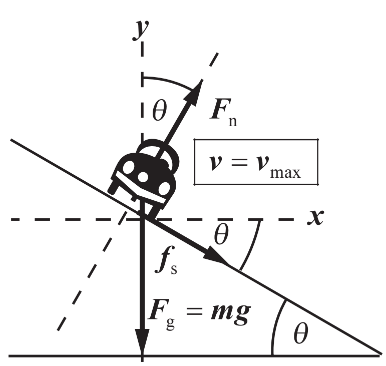
Substitute numerical values and evaluate r:

$$\boldsymbol{r} = \frac{\left(60\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{0.1610\left(9.81\,\text{m/s}^2\right)} = \boxed{0.18\,\text{km}}$$

97 ••• A curve of radius 30 m is banked so that a 950-kg car traveling at 40.0 km/h can round it even if the road is so icy that the coefficient of static friction is approximately zero. You are commissioned to tell the local police the range of speeds at which a car can travel around this curve without skidding. Neglect the effects of air drag and rolling friction. If the coefficient of static friction between the road and the tires is 0.300, what is the range of speeds you tell them?

Picture the Problem The free-body diagram to the left is for the car rounding the curve at the minimum (not sliding down the incline) speed. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the car moving with the maximum safe speed is to slide toward the outside of the curve. Application of Newton's 2nd law and the simultaneous solution of the force equations will yield v_{min} and v_{max}.





Apply $\sum \vec{F} = m\vec{a}$ to a car traveling around the curve when the coefficient of static friction is zero:

$$\sum F_x = F_n \sin\theta = m\frac{v_{\text{min}}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Divide the first of these equations by the second to obtain:

$$\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Substitute numerical values and evaluate the banking angle:

$$\theta = \tan^{-1}\left[\frac{\left(40\frac{\text{km}}{\text{h}}\times\frac{10^3\,\text{m}}{\text{km}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{(30\,\text{m})(9.81\,\text{m/s}^2)}\right] = 22.76°$$

Apply $\sum\vec{F} = m\vec{a}$ to the car traveling around the curve at minimum speed:

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = m\frac{v_{\min}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,\max} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\sin\theta - \mu_s \cos\theta) = m\frac{v_{\min}^2}{r}$$

and

$$F_n(\cos\theta + \mu_s \sin\theta) = mg$$

Evaluate these equations for θ = 22.876 and μ_s = 0.300:

$$0.1102F_n = m\frac{v_{\min}^2}{r} \text{ and } 1.038F_n = mg$$

Eliminate F_n between these two equations and solve for $v_{\min}$:

$$v_{\min} = \sqrt{0.106rg}$$

Substitute numerical values and evaluate $v_{\min}$:

$$\boldsymbol{v}_{\min} = \sqrt{0.106(30\,\text{m})(9.81\,\text{m/s}^2)}$$
$$= 5.6\,\text{m/s} \approx 20\,\text{km/h}$$

Apply $\sum\vec{F} = m\vec{a}$ to the car traveling around the curve at maximum speed:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v_{\max}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,\max} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v_{\max}^2}{r}$$

and

$$F_n(\cos\theta - \mu_s \sin\theta) = mg$$

Evaluate these equations for $\theta = 22.76°$ and $\mu_s = 0.300$:

$$0.6635F_n = m\frac{v_{max}^2}{r}$$

and

$$0.8061F_n = 0.8061mg$$

Eliminate F_n between these two equations and solve for v_{max}:

$$v_{max} = \sqrt{0.8231rg}$$

Substitute numerical values and evaluate v_{max}:

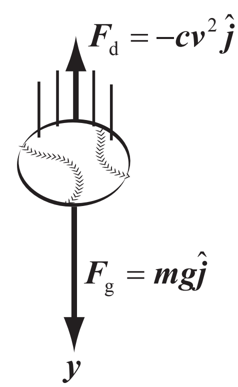
$$\boldsymbol{v}_{max} = \sqrt{(0.8231)(30\,\text{m})(9.81\,\text{m/s}^2)} = 16\,\text{m/s} \approx 56\,\text{km/h}$$

You should tell them that the safe range of speeds is $\boxed{20\,\text{km/m} \le \boldsymbol{v} \le 56\,\text{km/h}}$.

Euler's Method

98 •• You are riding in a hot air balloon when you throw a baseball straight down with an initial speed of 35.0 km/h. The baseball falls with a terminal speed of 150 km/h. Assuming air drag is proportional to the speed squared, use Euler's method (**spreadsheet**) to estimate the speed of the ball after 10.0 s. What is the uncertainty in this estimate? You drop a second baseball, this one released from rest. How long does it take for it to reach 99 percent of its terminal speed? How far does it fall during this time?

Picture the Problem The free-body diagram shows the forces acting on the baseball sometime after it has been thrown downward but before it has reached its terminal speed. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the ball and using its terminal speed to express the constant in the acceleration equation in terms of the ball's terminal speed. We can then use $v_{n+1} = v_n + a_n\Delta t$ to find the speed of the ball at any given time.



Apply Newton's 2nd law to the ball to obtain:

$$mg - bv^2 = m\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g - \frac{b}{m}v^2 \quad (1)$$

When the ball reaches its terminal speed, its acceleration is zero:

$$0 = g - \frac{b}{m}v_t^2 \Rightarrow \frac{b}{m} = \frac{g}{v_t^2}$$

Substitute in equation (1) to obtain:

$$\frac{dv}{dt} = g\left(1 - \frac{v^2}{v_t^2}\right)$$

Express the position of the ball to obtain:

$$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$$

Letting a_n be the acceleration of the ball at time t_n, express its speed when $t = t_{n+1}$:

$$v_{n+1} = v_n + a_n \Delta t$$

where $a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$ and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

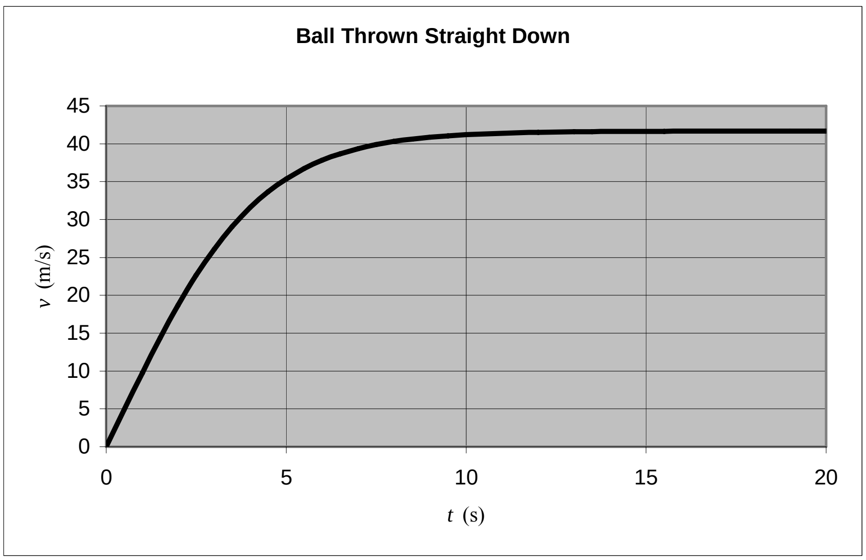
Cell	Formula/Content	Algebraic Form
A10	B9+$B$1	$t + \Delta t$
B10	B9+0.5*(C9+C10)*$B$1	$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$
C10	C9+D9*$B$1	$v_{n+1} = v_n + a_n\Delta t$
D10	$B$4*(1−C10^2/$B$5^2)	$a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$

	A	B	C	D
1	$\Delta t=$	0.5	s	
2	$x_0=$	0	m	
3	$v_0=$	9.722	m/s	
4	$a_0=$	9.81	m/s^2	
5	$v_t=$	41.67	m/s	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.0	0	9.7	9.28
10	0.5	6	14.4	8.64
11	1.0	14	18.7	7.84
12	1.5	25	22.6	6.92
28	9.5	317	41.3	0.17
29	10.0	337	41.4	0.13
30	10.5	358	41.5	0.10

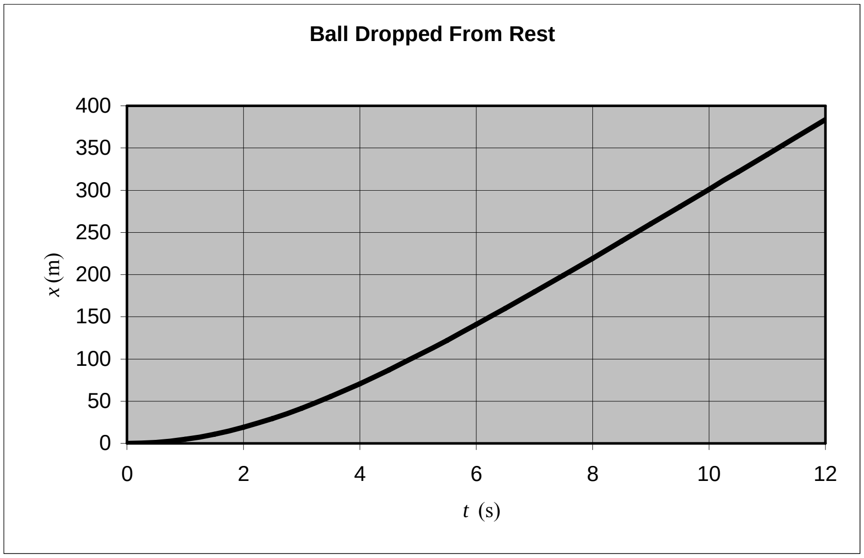
38	14.5	524	41.6	0.01
39	15.0	545	41.7	0.01
40	15.5	566	41.7	0.01
41	16.0	587	41.7	0.01
42	16.5	608	41.7	0.00

From the table we can see that the speed of the ball after 10 s is approximately $\boxed{41.4\,\text{m/s.}}$ We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at $t = 10$ s. Doing so yields $v(10\text{ s}) \approx 41.3$ m/s, a difference of about $\boxed{0.02\%.}$

The following graph shows the velocity of the ball thrown straight down as a function of time.

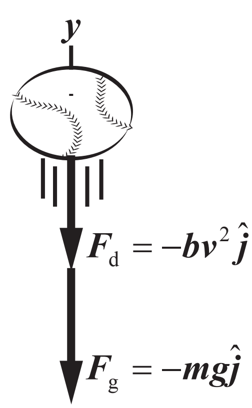


Reset Δt to 0.5 s and set $v_0 = 0$. Ninety-nine percent of 41.67 m/s is approximately 41.3 m/s. Note that the ball will reach this speed in about $\boxed{10.5\,\text{s}}$ and that the distance it travels in this time is about $\boxed{322\,\text{m.}}$ The following graph shows the distance traveled by the ball dropped from rest as a function of time.



99 •• **[SSM]** You throw a baseball straight up with an initial speed of 150 km/h. The ball's terminal speed when falling is also 150 km/h. (*a*) Use Euler's method (**spreadsheet**) to estimate its height 3.50 s after release. (*b*) What is the maximum height it reaches? (*c*) How long after release does it reach its maximum height? (*d*) How much later does it return to the ground? (*e*) Is the time the ball spends on the way up less than, the same as, or greater than the time it spends on the way down?

Picture the Problem The free-body diagram shows the forces acting on the baseball after it has left your hand. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the baseball. We can then use $v_{n+1} = v_n + a_n\Delta t$ and $x_{n+1} = x_n + v_n\Delta t$ to find the speed and position of the ball.



Apply $\sum F_y = ma_y$ to the baseball:

$$-bv|v| - mg = m\frac{dv}{dt}$$

where $|v| = v$ for the upward part of the flight of the ball and $|v| = -v$ for the downward part of the flight.

Solve for dv/dt to obtain:

$$\frac{dv}{dt} = -g - \frac{b}{m} v|v|$$

Under terminal speed conditions ($|v| = -v_t$):

$$0 = -g + \frac{b}{m} v_t^2 \text{ and } \frac{b}{m} = \frac{g}{v_t^2}$$

Substituting for b/m yields:

$$\frac{dv}{dt} = -g - \frac{g}{v_t^2} v|v| = -g\left(1 + \frac{v|v|}{v_t^2}\right)$$

Letting a_n be the acceleration of the ball at time t_n, express its position and speed when $t = t_n + 1$:

$$y_{n+1} = y_n + \tfrac{1}{2}(v_n + v_{n-1})\Delta t$$

and

$$v_{n+1} = v_n + a_n \Delta t$$

where $a_n = -g\left(1 + \frac{v_n|v_n|}{v_t^2}\right)$ and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D11	D10+$B$6	$t + \Delta t$
E10	41.7	v_0
E11	E10−$B$4*(1+E10*ABS(E10)/($B$5^2))*$B$6	$v_{n+1} = v_n + a_n \Delta t$
F10	0	y_0
F11	F10+0.5*(E10+E11)*$B$6	$y_{n+1} = y_n + \frac{1}{2}(v_n + v_{n-1})\Delta t$
G10	0	y_0
G11	$E$10*D11−0.5*$B$4*D11^2	$v_0 t - \frac{1}{2} g t^2$

	A	B	C	D	E	F	G
4	g=	9.81	m/s^2				
5	v_t=	41.7	m/s				
6	Δt=	0.1	s				
7							
8							
9				t	v	y	$y_{\text{no drag}}$
10				0.00	41.70	0.00	0.00
11				0.10	39.74	4.07	4.12
12				0.20	37.87	7.95	8.14
40				3.00	3.01	60.13	81.00
41				3.10	2.03	60.39	82.18

42				3.20	1.05	60.54	83.26
43				3.30	0.07	60.60	84.25
44				3.40	−0.91	60.55	85.14
45				3.50	−1.89	60.41	85.93
46				3.60	−2.87	60.17	86.62
78				6.80	−28.34	6.26	56.98
79				6.90	−28.86	3.41	54.44
80				7.00	−29.37	0.49	51.80
81				7.10	−29.87	−2.47	49.06

(*a*) When $t = 3.50$ s, the height of the ball is about $\boxed{60.4\,\text{m}}$.

(*b*) The maximum height reached by the ball is $\boxed{60.6\,\text{m}}$.

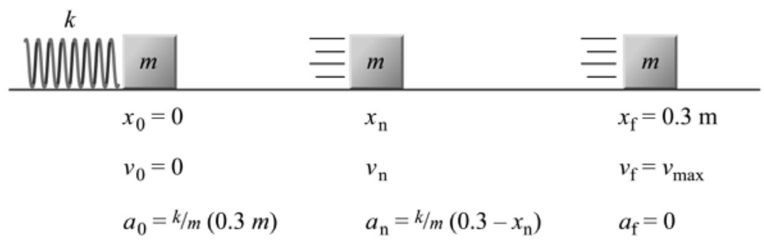
(*c*) The time the ball takes to reach its maximum height is about $\boxed{3.0\,\text{s}}$.

(*d*) The ball hits the ground at about $t = \boxed{7.0\,\text{s}}$

(*e*) Because the time the ball takes to reach its maximum height is less than half its time of flight, the time the ball spends on the way up less than the time it spends on the way down

100 ••• A 0.80-kg block on a horizontal frictionless surface is held against a massless spring, compressing it 30 cm. The force constant of the spring is 50 N/m. The block is released and the spring pushes it 30 cm. Use Euler's method (**spreadsheet**) with $\Delta t = 0.005$ s to estimate the time it takes for the spring to push the block the 30 cm. How fast is the block moving at this time? What is the uncertainty in this speed?

Picture the Problem The pictorial representation shows the block in its initial position against the compressed spring, later as the spring accelerates it to the right, and finally when it has reached its maximum speed at $x_f = 0.30$ m. In order to use Euler's method, we'll need to determine how the acceleration of the block varies with its position. We can do this by applying Newton's 2nd law to the box. We can then use $v_{n+1} = v_n + a_n\Delta t$ and $x_{n+1} = x_n + v_n\Delta t$ to find the speed and position of the block.



Apply $\sum F_x = ma_x$ to the block: $\quad \boldsymbol{k}(0.30\,\text{m} - \boldsymbol{x}_n) = \boldsymbol{ma}_n$

Solve for a_n: $\quad \boldsymbol{a}_n = \dfrac{\boldsymbol{k}}{\boldsymbol{m}}(0.30\,\text{m} - \boldsymbol{x}_n)$

Express the position and speed of the block when $t = t_n + 1$:

$$x_{n+1} = x_n + v_n \Delta t$$

and

$$v_{n+1} = v_n + a_n \Delta t$$

where $\boldsymbol{a}_n = \dfrac{\boldsymbol{k}}{\boldsymbol{m}}(0.30\,\text{m} - \boldsymbol{x}_n)$ and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A10	A9+$B$1	$t + \Delta t$
B10	B9+C10*$B$1	$x_n + v_n \Delta t$
C10	C9+D9*$B$1	$v_n + a_n \Delta t$
D10	($B$4/$B$5)*(0.30–B10)	$\dfrac{\boldsymbol{k}}{\boldsymbol{m}}(0.30 - \boldsymbol{x}_n)$

	A	B	C	D
1	Δt=	0.005	s	
2	x_0=	0	m	
3	v_0=	0	m/s	
4	k =	50	N/m	
5	m =	0.80	kg	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.000	0.00	0.00	18.75
10	0.005	0.00	0.09	18.72
11	0.010	0.00	0.19	18.69
12	0.015	0.00	0.28	18.63
45	0.180	0.25	2.41	2.85

46	0.185	0.27	2.42	2.10
47	0.190	0.28	2.43	1.34
48	0.195	0.29	2.44	0.58
49	0.200	0.30	2.44	–0.19

From the table we can see that it took about $\boxed{0.200\,\text{s}}$ for the spring to push the block 30 cm and that it was traveling about $\boxed{2.44\,\text{m/s}}$ at that time. We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at $t = 10$ s. Doing so yields $v(0.20\text{ s}) \approx 2.41$ m/s, a difference of about $\boxed{1.2\%.}$

Finding the Center of Mass

101 • Three point masses of 2.0 kg each are located on the x axis. One is at the origin, another at $x = 0.20$ m, and another at $x = 0.50$ m. Find the center of mass of the system.

Picture the Problem We can use its definition to find the center of mass of this system.

The x coordinate of the center of mass is given by:

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(2.0\,\text{kg})(0) + (2.0\,\text{kg})(0.20\,\text{m}) + (2.0\,\text{kg})(0.50\,\text{m})}{2.0\,\text{kg} + 2.0\,\text{kg} + 2.0\,\text{kg}} = 0.23\,\text{m}$$

Because the point masses all lie along the x axis:

$y_{\text{cm}} = 0$ and the center of mass of this system of particles is at $\boxed{(0.23\,\text{m}, 0)}$.

102 • On a weekend archeological dig, you discover an old club-ax that consists of a symmetrical 8.0-kg stone attached to the end of a uniform 2.5-kg stick. You measure the dimensions of the club-ax as shown in Figure 5-77. How far is the center of mass of the club-ax from the handle end of the club-ax?

Picture the Problem Let the left end of the handle be the origin of our coordinate system. We can disassemble the club-ax, find the center of mass of each piece, and then use these coordinates and the masses of the handle and stone to find the center of mass of the club-ax.

Express the center of mass of the handle plus stone system:

$$x_{cm}=\frac{m_{stick}x_{cm,stick}+m_{stone}x_{cm,stone}}{m_{stick}+m_{stone}}$$

Assume that the stone is drilled and the stick passes through it. Use symmetry considerations to locate the center of mass of the stick:

$$x_{cm,stick}=49\text{ cm}$$

Use symmetry considerations to locate the center of mass of the stone:

$$x_{cm,stone}=89\text{ cm}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm}=\frac{(2.5\text{ kg})(49\text{ cm})+(8.0\text{ kg})(89\text{ cm})}{2.5\text{ kg}+8.0\text{ kg}}=\boxed{79\text{ cm}}$$

103 • Three balls A, B, and C, with masses of 3.0 kg, 1.0 kg, and 1.0 kg, respectively, are connected by massless rods, as shown in Figure 5-78. What are the coordinates of the center of mass of this system?

Picture the Problem We can treat each of balls as though they are point objects and apply the definition of the center of mass to find (x_{cm}, y_{cm}).

The x coordinate of the center of mass is given by:

$$x_{cm}=\frac{m_Ax_A+m_Bx_B+m_Cx_C}{m_A+m_B+m_C}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm}=\frac{(3.0\text{ kg})(2.0\text{ m})+(1.0\text{ kg})(1.0\text{ m})+(1.0\text{ kg})(3.0\text{ m})}{3.0\text{ kg}+1.0\text{ kg}+1.0\text{ kg}}=2.0\text{ m}$$

The y coordinate of the center of mass is given by:

$$y_{cm}=\frac{m_Ay_A+m_By_B+m_Cy_C}{m_A+m_B+m_C}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{cm}=\frac{(3.0\text{ kg})(2.0\text{ m})+(1.0\text{ kg})(1.0\text{ m})+(1.0\text{ kg})(0)}{3.0\text{ kg}+1.0\text{ kg}+1.0\text{ kg}}=1.4\text{ m}$$

The center of mass of this system of balls is at $\boxed{(2.0\text{ m},1.4\text{ m})}$.

Alternate Solution Using Vectors

Picture the Problem We can use the vector expression for the center of mass to find (x_{cm}, y_{cm}).

The vector expression for the center of mass is:

$$M\vec{r}_{cm} = \sum_i m_i\vec{r}_i \text{ or } \vec{r}_{cm} = \frac{\sum_i m_i\vec{r}_i}{M} \qquad (1)$$

where

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

The position vectors for the objects located at A, B, and C are:

$$\vec{r}_A = \left(2.0\hat{i} + 2.0\hat{j} + 0\hat{k}\right)\text{m},$$

$$\vec{r}_B = \hat{i} + \hat{j} + 0\hat{k}\ ,$$

and

$$\vec{r}_C = \left(3.0\hat{i} + 0\hat{j} + 0\hat{k}\right)\text{m}$$

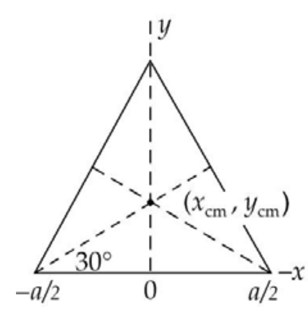
Substitute numerical values in (1) and simplify to obtain:

$$\begin{aligned}\vec{r}_{cm} &= \frac{1}{(3.0\,\text{kg} + 1.0\,\text{kg} + 1.0\,\text{kg})}\Big[(3.0\,\text{kg})\left(2.0\hat{i} + 2.0\hat{j} + 0\hat{k}\right)\text{m} + (1.0\,\text{kg})\left(\hat{i} + \hat{j} + 0\hat{k}\right)\text{m} \\ &\qquad + (1.0\,\text{kg})\left(3.0\hat{i} + 0\hat{j} + 0\hat{k}\right)\text{m}\Big] \\ &= (2.0\,\text{m})\hat{i} + (1.4\,\text{m})\hat{j} + 0\hat{k}\end{aligned}$$

The center of mass of this system of balls is at $\boxed{(2.0\,\text{m}, 1.4\,\text{m}, 0)}$.

104 • By symmetry, locate the center of mass of an equilateral triangle with edges of length *a*. The triangle has one vertex on the *y* axis and the others at (–*a*/2, 0) and (+*a*/2, 0).

Picture the Problem The figure shows an equilateral triangle with its *y*-axis vertex above the *x* axis. The bisectors of the vertex angles are also shown. We can find *x* coordinate of the center-of-mass by inspection and the *y* coordinate using trigonometry.



From symmetry considerations:

$$x_{cm} = 0$$

Express the trigonometric relationship between $a/2$, 30°, and y_{cm}:

$$\tan 30° = \frac{y_{cm}}{\frac{1}{2}a}$$

Solve for y_{cm} and simplify to obtain:

$$y_{cm} = \tfrac{1}{2}a\tan 30° = 0.29a$$

The center of mass of an equilateral triangle oriented as shown above is at $\boxed{(0, 0.29a)}$.

105 •• **[SSM]** Find the center of mass of the uniform sheet of plywood in Figure 5-79. Consider this as a system of effectively two sheets, letting one have a "negative mass" to account for the cutout. Thus, one is a square sheet of 3.0-m edge length and mass m_1 and the second is a rectangular sheet measuring 1.0 m × 2.0 m with a mass of $-m_2$. Let the coordinate origin be at the lower left corner of the sheet.

Picture the Problem Let the subscript 1 refer to the 3.0-m by 3.0-m sheet of plywood before the 2.0-m by 1.0-m piece has been cut from it. Let the subscript 2 refer to 2.0-m by 1.0-m piece that has been removed and let σ be the area density of the sheet. We can find the center-of-mass of these two regions; treating the missing region as though it had negative mass, and then finding the center-of-mass of the U-shaped region by applying its definition.

Express the coordinates of the center of mass of the sheet of plywood:

$$x_{cm} = \frac{m_1 x_{cm,1} - m_2 x_{cm,2}}{m_1 - m_2}$$

and

$$y_{cm} = \frac{m_1 y_{cm,1} - m_2 y_{cm,2}}{m_1 - m_2}$$

Use symmetry to find $x_{cm,1}$, $y_{cm,1}$, $x_{cm,2}$, and $y_{cm,2}$:

$$x_{cm,1} = 1.5\,\text{m},\ y_{cm,1} = 1.5\,\text{m}$$

and

$$x_{cm,2} = 1.5\,\text{m},\ y_{cm,2} = 2.0\,\text{m}$$

Determine m_1 and m_2:

$$m_1 = \sigma A_1 = 9\sigma\,\text{kg}$$

and

$$m_2 = \sigma A_2 = 2\sigma\,kg$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(1.5\,\text{kg})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}}$$

$$= 1.5\,\text{m}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{cm}=\frac{(9\sigma\,\text{kg})(1.5\,\text{m})-(2\sigma\,\text{kg})(2.0\,\text{m})}{9\sigma\,\text{kg}-2\sigma\,\text{kg}}=1.4\,\text{m}$$

The center of mass of the U-shaped sheet of plywood is at $\boxed{(1.5\,\text{m}, 1.4\,\text{m})}$.

106 •• A can in the shape of a symmetrical cylinder with mass *M* and height *H* is filled with water. The initial mass of the water is *M*, the same mass as the can. A small hole is punched in the bottom of the can, and the water drains out. (*a*) If the height of the water in the can is *x*, what is the height of the center of mass of the can plus the water remaining in the can? (*b*) What is the minimum height of the center of mass as the water drains out?

Picture the Problem We can use its definition to find the center of mass of the can plus water. By setting the derivative of this function equal to zero, we can find the value of *x* that corresponds to the minimum height of the center of mass of the water as it drains out and then use this extreme value to express the minimum height of the center of mass.

(*a*) Using its definition, express the location of the center of mass of the can + water:

$$x_{cm}=\frac{M\left(\frac{H}{2}\right)+m\left(\frac{x}{2}\right)}{M+m} \qquad (1)$$

Let the cross-sectional area of the cup be *A* and use the definition of density to relate the mass *m* of water remaining in the can at any given time to its depth *x*:

$$\rho=\frac{M}{AH}=\frac{m}{Ax}\Rightarrow m=\frac{x}{H}M$$

Substitute for *m* in equation (1) and simplify to obtain:

$$x_{cm}=\frac{M\left(\frac{H}{2}\right)+\left(\frac{x}{H}M\right)\left(\frac{x}{2}\right)}{M+\frac{x}{H}M}=\boxed{\frac{H}{2}\left(\frac{1+\left(\frac{x}{H}\right)^2}{1+\frac{x}{H}}\right)}$$

(*b*) Differentiate x_{cm} with respect to *x* and set the derivative equal to zero for extrema:

$$\frac{dx_{\text{cm}}}{dx}=\frac{H}{2}\frac{d}{dx}\left(\frac{1+\left(\frac{x}{H}\right)^2}{1+\frac{x}{H}}\right)=\frac{H}{2}\left\{\frac{\left(1+\frac{x}{H}\right)2\left(\frac{x}{H}\right)\left(\frac{1}{H}\right)}{\left(1+\frac{x}{H}\right)^2}-\frac{\left[1+\left(\frac{x}{H}\right)^2\right]\left(\frac{1}{H}\right)}{\left(1+\frac{x}{H}\right)^2}\right\}=0$$

Simplify this expression to obtain a quadratic equation in *x*/*H*:

$$\left(\frac{x}{H}\right)^2+2\left(\frac{x}{H}\right)-1=0$$

Solving for *x*/*H* yields:

$$x=H\left(\sqrt{2}-1\right)\approx 0.414H$$

where we've kept the positive solution because a negative value for *x*/*H* would make no sense.

Use your graphing calculator to convince yourself that the graph of x_{cm} as a function of *x* is concave upward at $x\approx 0.414H$ and that, therefore, the minimum value of x_{cm} occurs at $x\approx 0.414H$.

Evaluate x_{cm} at $x=H\left(\sqrt{2}-1\right)$ to obtain:

$$x_{\text{cm}}\Big|_{x=H\left(\sqrt{2}-1\right)}=\frac{H}{2}\left(\frac{1+\left(\frac{H\left(\sqrt{2}-1\right)}{H}\right)^2}{1+\frac{H\left(\sqrt{2}-1\right)}{H}}\right)$$

$$=\boxed{H\left(\sqrt{2}-1\right)}$$

107 •• **[SSM]** Two identical uniform rods each of length *L* are glued together so that the angle at the joint is 90°. Determine the location of the center of mass (in terms of *L*) of this configuration relative to the origin taken to be at the joint. (*Hint: You do not need the mass of the rods (why?), but you should start by assuming a mass m and see that it cancels out.*)

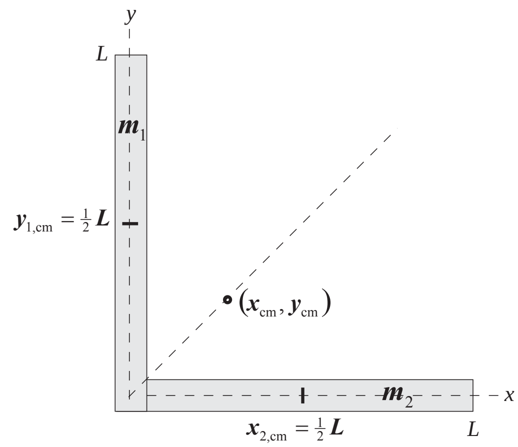
Picture the Problem A pictorial representation of the system is shown to the right. The x and y coordinates of the two rods are

$$\left(x_{1,\text{cm}}, y_{1,\text{cm}}\right) = \left(0, \tfrac{1}{2}L\right)$$

and

$$\left(x_{2,\text{cm}}, y_{2,\text{cm}}\right) = \left(\tfrac{1}{2}L, 0\right).$$

We can use the definition of the center of mass to find the coordinates $\left(x_{\text{cm}}, y_{\text{cm}}\right)$.



The x coordinate of the center of mass is given by:

$$x_{\text{cm}} = \frac{x_{1,\text{cm}}m_1 + x_{2,\text{cm}}m_2}{m_1 + m_2}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(0)m_1 + \left(\frac{1}{2}L\right)m_2}{m_1 + m_2}$$

or, because $m_1 = m_2 = m$,

$$x_{\text{cm}} = \frac{(0)m + \left(\frac{1}{2}L\right)m}{m + m} = \tfrac{1}{4}L$$

The y coordinate of the center of mass is given by:

$$y_{\text{cm}} = \frac{y_{1,\text{cm}}m_1 + y_{2,\text{cm}}m_2}{m_1 + m_2}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{\text{cm}} = \frac{\left(\frac{1}{2}L\right)m_1 + (0)m_2}{m_1 + m_2}$$

or, because $m_1 = m_2 = m$,

$$y_{\text{cm}} = \frac{\left(\frac{1}{2}L\right)m + (0)m}{m + m} = \tfrac{1}{4}L$$

The center of mass of this system is located at $\boxed{\left(\tfrac{1}{4}L, \tfrac{1}{4}L\right)}$

Remarks: Note that the center of mass is located at a distance $d = \frac{1}{2}L\cos 45°$ from the vertex on the axis of symmetry bisecting the two arms.

108 ••• Repeat the analysis of Problem 107 with a general angle θ at the joint instead of 90°. Does your answer agree with the specific 90°- angle answer in Problem 107 if you set θ equal to 90°? Does your answer give plausible results for angles of zero and 180°?

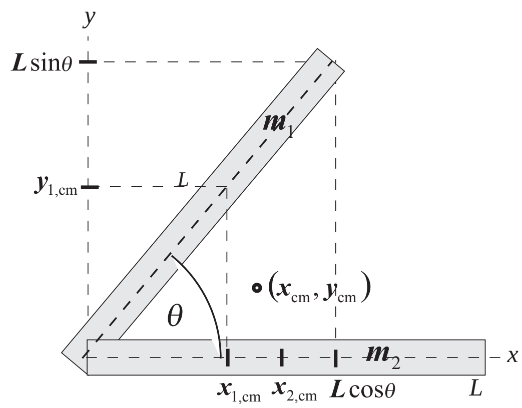
Picture the Problem The pictorial representation of the system is shown to the right. The x and y coordinates of the two rods are

$$\left(x_{1,\text{cm}}, y_{1,\text{cm}}\right) = \left(0, \tfrac{1}{2}L\right)$$

and

$$\left(x_{2,\text{cm}}, y_{2,\text{cm}}\right) = \left(\tfrac{1}{2}L, 0\right).$$

We can use the definition of the center of mass to find the coordinates $\left(x_{\text{cm}}, y_{\text{cm}}\right)$.



The x coordinate of the center of mass is given by:

$$x_{\text{cm}} = \frac{x_{1,\text{cm}}m_1 + x_{2,\text{cm}}m_2}{m_1 + m_2}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{\left(\tfrac{1}{2}L\cos\theta\right)m_1 + \left(\tfrac{1}{2}L\right)m_2}{m_1 + m_2}$$

or, because $m_1 = m_2 = m$,

$$x_{\text{cm}} = \frac{\left(\tfrac{1}{2}L\cos\theta\right)m + \left(\tfrac{1}{2}L\right)m}{m + m} = \tfrac{1}{4}L\left(1 + \cos\theta\right)$$

The y coordinate of the center of mass is given by:

$$y_{\text{cm}} = \frac{y_{1,\text{cm}}m_1 + y_{2,\text{cm}}m_2}{m_1 + m_2}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{\text{cm}} = \frac{\left(\tfrac{1}{2}L\sin\theta\right)m_1 + (0)m_2}{m_1 + m_2}$$

or, because $m_1 = m_2 = m$,

$$y_{\text{cm}} = \frac{\left(\tfrac{1}{2}L\sin\theta\right)m + (0)m}{m + m} = \tfrac{1}{4}L\sin\theta$$

The center of mass of this system of rods is located at $\boxed{\left(\tfrac{1}{4}L\left(1 + \cos\theta\right), \tfrac{1}{4}L\sin\theta\right)}$.

For $\theta = 0°$:

$$\left(\tfrac{1}{4}L\left(1 + \cos\theta\right), \tfrac{1}{4}L\sin\theta\right) = \left(\tfrac{1}{4}L\left(1 + \cos 0°\right), \tfrac{1}{4}L\sin 0°\right) = \boxed{\left(\tfrac{1}{2}L, 0\right)}$$ as expected.

For $\theta = 90°$:

$$\left(\tfrac{1}{4}L\left(1 + \cos\theta\right), \tfrac{1}{4}L\sin\theta\right) = \left(\tfrac{1}{4}L\left(1 + \cos 90°\right), \tfrac{1}{4}L\sin 90°\right) = \boxed{\left(\tfrac{1}{4}L, \tfrac{1}{4}L\right)}$$ in agreement with Problem 107.

For $\boldsymbol{\theta} = 180°$:

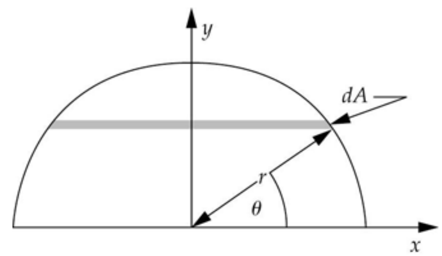
$$\left(\tfrac{1}{4}\boldsymbol{L}(1+\cos\boldsymbol{\theta}), \tfrac{1}{4}\boldsymbol{L}\sin\boldsymbol{\theta}\right) = \left(\tfrac{1}{4}\boldsymbol{L}(1+\cos 180°), \tfrac{1}{4}\boldsymbol{L}\sin 180°\right) = \boxed{(0,0)}$$ as expected.

Remarks: Note that the center of mass is located at a distance $d = \frac{1}{4}\sqrt{2}L\sqrt{1+\cos\theta}$ from the vertex on an axis that makes an angle $\boldsymbol{\phi} = \tan^{-1}\left[\dfrac{\sin\boldsymbol{\theta}}{1+\cos\boldsymbol{\theta}}\right]$ with the x axis.

*Finding the Center of Mass by Integration

109 •• Show that the center of mass of a uniform semicircular disk of radius R is at a point $4R/(3\pi)$ from the center of the circle.

Picture the Problem A semicircular disk and a surface element of area dA is shown in the diagram. Because the disk is a continuous object, we'll use $M\vec{r}_{\text{cm}} = \int \vec{r}\,dm$ and symmetry to find its center of mass.



Express the coordinates of the center of mass of the semicircular disk:

$x_{\text{cm}} = 0$ by symmetry.

$$y_{\text{cm}} = \frac{\int y\sigma\,dA}{M} \qquad (1)$$

Express y as a function of r and θ:

$$y = r\sin\theta$$

Express dA in terms of r and θ:

$$dA = r\,d\theta\,dr$$

Express M as a function of r and θ:

$$M = \sigma A_{\text{half disk}} = \tfrac{1}{2}\sigma\pi R^2$$

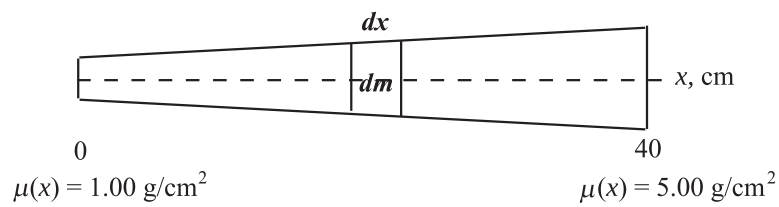
Substitute in equation (1) and evaluate y_{cm}:

$$y_{\text{cm}} = \frac{\sigma\int_0^R\int_0^\pi r^2\sin\theta\,d\theta\,dr}{M} = \frac{2\sigma}{M}\int_0^R r^2 dr$$

$$= \frac{2\sigma}{3M}R^3 = \boxed{\frac{4}{3\pi}R}$$

110 •• Find the location of the center of mass of a nonuniform rod 0.40 m in length if its density varies linearly from 1.00 g/cm at one end to 5.00 g/cm at the other end. Specify the center-of-mass location relative to the less-massive end of the rod.

Picture the Problem The pictorial representation summarizes the information we're given about the non-uniform rod. We can use the definition of the center of mass for a continuous object to find the center of mass of the non-uniform rod.



The x coordinate of the center of mass of the non-uniform rod is given by:

$$x_{\text{cm}} = \frac{\int x\,dm}{\int dm}$$

or, because $dm = \mu(x)dx$,

$$x_{\text{cm}} = \frac{\int x\mu(x)dx}{\int \mu(x)dx} \qquad (1)$$

By symmetry:

$$y_{\text{cm}} = 0$$

Use the given information regarding the linear variation in the density of the non-uniform rod to express $\mu(x)$:

$$\mu(x) = 1.00\text{ g/cm} + \left(0.10\text{ g/cm}^2\right)x$$

Substituting for $\mu(x)$ in equation (1) yields:

$$x_{\text{cm}} = \frac{\int_0^{40\text{ cm}} x\left[1.00\text{ g/cm} + \left(0.10\text{ g/cm}^2\right)\right]x\,dx}{\int_0^{40\text{ cm}} \left[1.00\text{ g/cm} + \left(0.10\text{ g/cm}^2\right)\right]x\,dx}$$

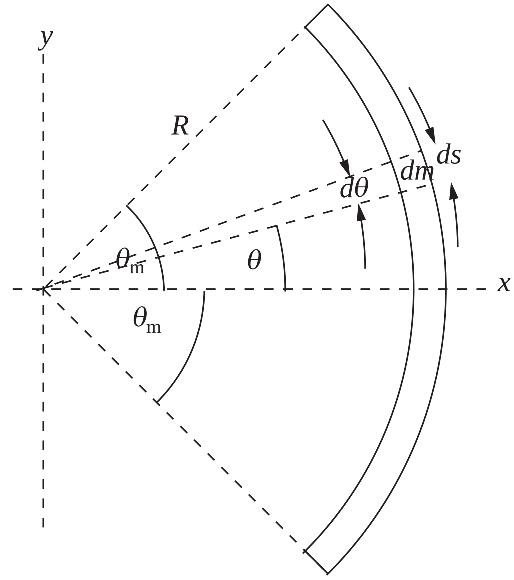
Evaluate these integrals to obtain:

$$x_{\text{cm}} = 24\text{ cm}$$

The coordinates of the center of mass of the non-uniform rod are $\boxed{(24\text{ cm}, 0)}$.

111 ••• You have a thin uniform wire bent into part of a circle that is described by a radius R and angle θ_m (see Figure 5-80). Show that the location of its center of mass is on the x axis and located a distance $x_{\text{cm}} = (R \sin \theta_m)/\theta_m$, where θ_m is expressed in radians. Double check by showing that this answer gives the physically expected limit for $\theta_m = 180°$. Verify that your answer gives you the result in the text (in the subsection Finding the Center of Mass by Integration) for the special case of $\theta_{\text{m}} = 90°$.

Picture the Problem We can use the definition of the center of mass for a continuous object to find the center of mass of the non-uniform rod.



The x coordinate of the center of mass of the thin uniform wire is given by:

$$x_{\text{cm}} = \frac{\int x\,dm}{\int dm}$$

or, because $dm = \lambda ds = \lambda R d\theta$,

$$x_{\text{cm}} = \frac{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} x\lambda R d\theta}{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} \lambda R d\theta} = \frac{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} x d\theta}{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} d\theta} \qquad (1)$$

By symmetry:

$$y_{\text{cm}} = 0$$

Because $x = R\cos\theta$, equation (1) becomes:

$$x_{\text{cm}} = \frac{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} R\cos\theta\, d\theta}{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} d\theta} = \frac{R\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} \cos\theta\, d\theta}{\int_{-\theta_{\text{m}}}^{\theta_{\text{m}}} d\theta}$$

Evaluate these integrals to obtain:

$$x_{\text{cm}} = \frac{R\sin\theta_{\text{m}}}{\theta_{\text{m}}}$$

The coordinates of the center of mass of the thin uniform wire are

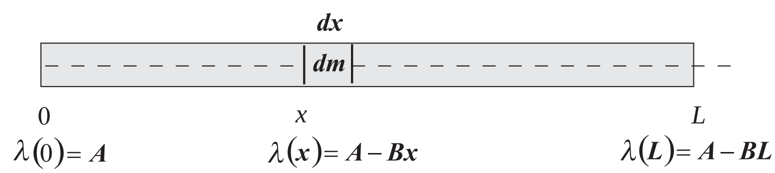
$$\boxed{\left(\frac{R\sin\theta_{\text{m}}}{\theta_{\text{m}}}, 0\right)}.$$

For $\theta_{\rm m} = 180° = \pi$ radians: $x_{\rm cm} = \dfrac{R\sin\pi}{\pi} = \boxed{0}$ as expected.

For $\theta_{\rm m} = 90° = \dfrac{\pi}{2}$ radians: $x_{\rm cm} = \dfrac{R\sin\left(\dfrac{\pi}{2}\right)}{\dfrac{\pi}{2}} = \boxed{\dfrac{2R}{\pi}}$ as expected.

112 ••• A long, thin wire of length L has a density given by $A - Bx$, where A and B are positive constants and x is the distance from the more massive end.
(a) A condition for this problem to be realistic is that $A > BL$. Explain why.
(b) Determine $x_{\rm cm}$ in terms of L, A, and B. Does your answer makes sense if $B = 0$? Explain.

Picture the Problem The pictorial representation summarizes the information we're given about the long thin wire. We can use the definition of the center of mass for a continuous object to find the center of mass of the non-uniform rod.



(a) At the end, the density has to be positive, so $A - BL > 0$ or $A > BL$.

(b) The x coordinate of the center of mass of the non-uniform rod is given by:

$$x_{\rm cm} = \frac{\int x\,dm}{\int dm} \qquad (1)$$

By symmetry:

$$y_{\rm cm} = 0$$

The density of the long thin wire decreases with distance according to:

$$dm = (A - Bx)dx$$

Substituting for dm in equation (1) yields:

$$x_{\rm cm} = \frac{\int_0^L x(A - Bx)dx}{\int_0^L (A - Bx)dx}$$

Evaluate these integrals to obtain:

$$x_{\rm cm} = \frac{L}{2}\left(\frac{1 - \dfrac{2BL}{3A}}{1 - \dfrac{BL}{2A}}\right)$$

The coordinates of the center of mass of the long thin wire are

$$\boxed{\left(\frac{\boldsymbol{L}}{2}\left(\frac{1-\dfrac{2\boldsymbol{BL}}{3A}}{1-\dfrac{\boldsymbol{BL}}{2A}}\right),0\right)}.$$

Because $\boldsymbol{A} > \boldsymbol{BL}$, both the numerator and denominator are positive. Because the denominator is always larger than the numerator, it follows that $\boldsymbol{x}_{\text{cm}} < \frac{1}{2}\boldsymbol{L}$. This makes physical sense because the mass of the rod decreases with distance and so most of it is to the left of the midpoint of the rod. Note also that if $B = 0$, our result predicts a uniform density (of A) and the center of mass is at the midpoint of the rod (as it should be).

Motion of the Center of Mass

113 • **[SSM]** Two 3.0-kg particles have velocities $\vec{v}_1 = (2.0\text{ m/s})\,\hat{\boldsymbol{i}} + (3.0\text{ m/s})\,\hat{\boldsymbol{j}}$ and $\vec{v}_2 = (4.0\text{ m/s})\,\hat{\boldsymbol{i}} - (6.0\text{ m/s})\,\hat{\boldsymbol{j}}$. Find the velocity of the center of mass of the system.

Picture the Problem The velocity of the center of mass of a system of particles is related to the total momentum of the system through $\vec{\boldsymbol{P}} = \sum_{\text{i}} m_{\text{i}}\vec{\boldsymbol{v}}_{\text{i}} = M\vec{\boldsymbol{v}}_{\text{cm}}$.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-particle system to the momenta of the individual particles:

$$\vec{\boldsymbol{v}}_{\text{cm}} = \frac{\sum_{\text{i}} m_{\text{i}}\vec{\boldsymbol{v}}_{\text{i}}}{M} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{cm}}$:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\text{cm}} &= \frac{(3.0\,\text{kg})\left[(2.0\,\text{m/s})\hat{\boldsymbol{i}} + (3.0\,\text{m/s})\hat{\boldsymbol{j}}\right] + (3.0\,\text{kg})\left[(4.0\,\text{m/s})\hat{\boldsymbol{i}} - (6.0\,\text{m/s})\hat{\boldsymbol{j}}\right]}{3.0\,\text{kg} + 3.0\,\text{kg}} \\ &= \boxed{(3.0\,\text{m/s})\hat{\boldsymbol{i}} - (1.5\,\text{m/s})\hat{\boldsymbol{j}}}\end{aligned}$$

114 • A 1500-kg car is moving westward with a speed of 20.0 m/s, and a 3000-kg truck is traveling east with a speed of 16.0 m/s. Find the velocity of the center of mass of the car-truck system.

Picture the Problem Choose a coordinate system in which east is the positive x direction and use the relationship $\vec{P} = \sum_i m_i\vec{v}_i = M\vec{v}_{cm}$ to determine the velocity of the center of mass of the system.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-vehicle system to the momenta of the individual vehicles:

$$\vec{v}_{cm} = \frac{\sum_i m_i\vec{v}_i}{M} = \frac{m_t\vec{v}_t + m_c\vec{v}_c}{m_t + m_c}$$

Express the velocity of the truck:

$$\vec{v}_t = (16.0\,\text{m/s})\hat{i}$$

Express the velocity of the car:

$$\vec{v}_c = (-20.0\,\text{m/s})\hat{i}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3000\,\text{kg})(16.0\,\text{m/s})\,\hat{i} + (1500\,\text{kg})(-20.0\,\text{m/s})\,\hat{i}}{3000\,\text{kg} + 1500\,\text{kg}} = \boxed{(4.00\,\text{m/s})\,\hat{i}}$$

115 • A force $\vec{F} = 12\text{ N }\hat{i}$ is applied to the 3.0-kg ball in Figure 5-78 in Problem 103. (No forces act on the other two balls.) What is the acceleration of the center of mass of the three-ball system?

Picture the Problem The acceleration of the center of mass of the ball is related to the net external force through Newton's 2nd law: $\vec{F}_{net,ext} = M\vec{a}_{cm}$.

Use Newton's 2nd law to express the acceleration of the ball:

$$\vec{a}_{cm} = \frac{\vec{F}_{net,ext}}{M}$$

Substitute numerical values and evaluate $\vec{a}_{cm}$:

$$\vec{a}_{cm} = \frac{(12\,\text{N})\hat{i}}{3.0\,\text{kg} + 1.0\,\text{kg} + 1.0\,\text{kg}} = \boxed{(2.4\,\text{m/s}^2)\hat{i}}$$

116 •• A block of mass m is attached to a string and suspended inside an otherwise empty box of mass M. The box rests on a scale that measures the system's weight. (a) If the string breaks, does the reading on the scale change? Explain your reasoning. (b) Assume that the string breaks and the mass m falls with constant acceleration g. Find the magnitude and direction of the acceleration of the center of mass of the box–block system. (*c*) Using the result from (*b*), determine the reading on the scale while *m* is in free-fall.

Picture the Problem Choose a coordinate system in which upward is the positive *y* direction. We can use Newton's 2nd law, $\vec{F}_{\text{net,ext}} = M\vec{a}_{\text{cm}}$, to find the acceleration of the center of mass of this two-body system.

(*a*) Yes; initially the scale reads $(M + m)g$. While the block is in free fall, the reading is Mg.

(*b*) Using Newton's 2nd law, express the acceleration of the center of mass of the system:

$$\vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{net,ext}}}{m_{\text{tot}}}$$

Substitute to obtain:

$$\vec{a}_{\text{cm}} = \boxed{-\frac{mg}{M+m}\hat{j}}$$

(*c*) Use Newton's 2nd law to express the net force acting on the scale while the block is falling:

$$F_{\text{net,ext}} = (M+m)g - (M+m)a_{\text{cm}}$$

Substitute for a_{cm} and simplify to obtain:

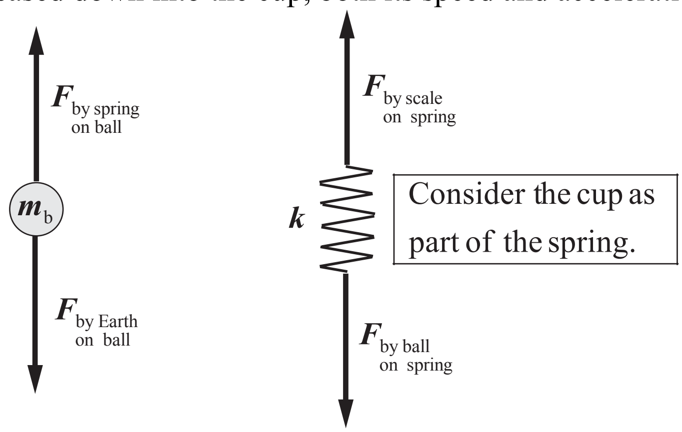
$$F_{\text{net,ext}} = (M+m)g - (M+m)\left(\frac{mg}{M+m}\right) = \boxed{Mg}$$

as expected, given our answer to (*a*).

117 •• **[SSM]** The bottom end of a massless, vertical spring of force constant *k* rests on a scale and the top end is attached to a massless cup, as in Figure 5-81. Place a ball of mass m_b gently into the cup and ease it down into an equilibrium position where it sits at rest in the cup. (*a*) Draw the separate free-body diagrams for the ball and the spring. (*b*) Show that in this situation, the spring compression *d* is given by $d = m_b g/k$. (*c*) What is the scale reading under these conditions?

Picture the Problem (*b*) We can apply Newton's 2nd law to the ball to find an expression for the spring's compression when the ball is at rest in the cup. (*c*) The scale reading is the force exerted by the spring on the scale and can be found from the application of Newton's 2nd law to the cup (considered as part of the spring).

(*a*) The free-body diagrams for the ball and spring follow. Note that, because the ball has been eased down into the cup, both its speed and acceleration are zero.



(*b*) Letting the upward direction be the positive *y* direction, apply $\sum F_y = ma_y$ to the ball when it is at rest in the cup and the spring has been compressed a distance *d*:

$$F_{\substack{\text{by spring}\\ \text{on ball}}} - F_{\substack{\text{by Earth}\\ \text{on ball}}} = m_{\text{b}}a_y$$

or, because $a_y = 0$,

$$F_{\substack{\text{by spring}\\ \text{on ball}}} - F_{\substack{\text{by Earth}\\ \text{on ball}}} = 0 \qquad (1)$$

Because $F_{\substack{\text{by spring}\\ \text{on ball}}} = kd$ and $F_{\substack{\text{by Earth}\\ \text{on ball}}} = m_{\text{b}}g$:

$$kd - m_{\text{b}}g = 0 \Rightarrow d = \boxed{\frac{m_{\text{b}}g}{k}}$$

(*c*) Apply $\sum F_y = ma_y$ to the spring:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} - F_{\substack{\text{by ball}\\ \text{on spring}}} = m_{\text{spring}}a_y \qquad (2)$$

or, because $a_y = 0$,

$$F_{\substack{\text{by scale}\\ \text{on spring}}} - F_{\substack{\text{by ball}\\ \text{on spring}}} = 0$$

Because $F_{\substack{\text{by spring}\\ \text{on ball}}} = F_{\substack{\text{by ball}\\ \text{on spring}}}$, adding equations (1) and (2) yields:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} - F_{\substack{\text{by Earth}\\ \text{on ball}}} = 0$$

Solving for $F_{\substack{\text{by scale}\\ \text{on spring}}}$ yields:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} = F_{\substack{\text{by Earth}\\ \text{on ball}}} = \boxed{m_{\text{b}}g}$$

118 ••• In the Atwood's machine in Figure 5-82 the string passes over a fixed cylinder of mass m_c. The cylinder does not rotate. Instead, the string slides on its frictionless surface. (*a*) Find the acceleration of the center of mass of the two-block-cylinder-string system. (*b*) Use Newton's second law for systems to find the force *F* exerted by the support. (*c*) Find the tension *T* in the string connecting the blocks and show that $F = m_c g + 2T$.

Picture the Problem Assume that the object whose mass is m_1 is moving downward and take that direction to be the positive direction. We'll use Newton's 2nd law for a system of particles to relate the acceleration of the center of mass to the acceleration of the individual particles.

(*a*) Relate the acceleration of the center of mass to m_1, m_2, m_c and their accelerations:

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_c\vec{a}_c$$

Because m_1 and m_2 have a common acceleration a and $a_c = 0$:

$$a_{cm} = a\frac{m_1 - m_2}{m_1 + m_2 + m_c}$$

From Problem 4-84 we have::

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Substitute to obtain:

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}g\right)\left(\frac{m_1 - m_2}{m_1 + m_2 + m_c}\right) = \boxed{\frac{(m_1 - m_2)^2}{(m_1 + m_2)(m_1 + m_2 + m_c)}g}$$

(*b*) Use Newton's 2nd law for a system of particles to obtain:

$$F - Mg = -Ma_{cm}$$

where $M = m_1 + m_2 + m_c$ and F is positive upwards.

Solve for F, substitute for a_{cm} from Part (*a*), and simplify to obtain:

$$F = Mg - Ma_{cm} = Mg - \frac{(m_1 - m_2)^2}{m_1 + m_2}g = \boxed{\left[\frac{4m_1m_2}{m_1 + m_2} + m_c\right]g}$$

(*c*) From Problem 4-84 we have:

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

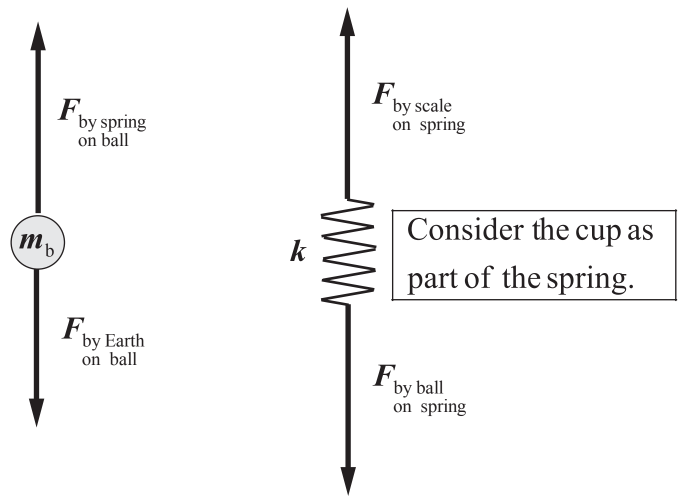
Substitute in our result from Part (*b*) and simplify to obtain:

$$F = \left[2\frac{2m_1m_2}{m_1 + m_2} + m_c\right]g = \left[2\frac{T}{g} + m_c\right]g = \boxed{2T + m_c g}$$

119 ••• Starting with the equilibrium situation in Problem 117, the whole system (scale spring, cup, and ball) is now subjected to an upward acceleration of magnitude a (for example, in an elevator). Repeat the free-body diagrams and calculations in Problem 117.

Picture the Problem Because the whole system is accelerating upward, the net upward force acting on the system must be upward. Because the spring is massless, the two forces acting on it remain equal and are oppositely directed. (*b*) We can apply Newton's 2nd law to the ball to find an expression for the spring's compression under the given conditions. (*c*) The scale reading, as in Problem 117, is the force the scale exerts on the spring and can be found from the application of Newton's 2nd law to the spring.

(*a*) The free-body diagrams for the ball and spring follow. Note that, because the system is accelerating upward, $F_{\substack{\text{by spring}\\ \text{on ball}}} > F_{\substack{\text{by Earth}\\ \text{on ball}}}$ whereas $F_{\substack{\text{by scale}\\ \text{on spring}}} = F_{\substack{\text{by ball}\\ \text{on spring}}}$.



(*b*) Letting the upward direction be the positive y direction, apply $\sum F_y = ma_y$ to the ball when the spring is compressed a distance d':

$$F_{\substack{\text{by spring}\\ \text{on ball}}} - F_{\substack{\text{by Earth}\\ \text{on ball}}} = m_\text{b} a_y \qquad (1)$$

Because $F_{\substack{\text{by spring}\\ \text{on ball}}} = k'd$, $a_y = a$, and $F_{\substack{\text{by Earth}\\ \text{on ball}}} = m_\text{b} g$:

$$kd' - m_\text{b} g = m_\text{b} a \Rightarrow d' = \boxed{\frac{m_\text{b}(g+a)}{k}}$$

(*c*) Apply $\sum F_y = ma_y$ to the spring:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} - F_{\substack{\text{by ball}\\ \text{on spring}}} = 0 \qquad (2)$$

Because $F_{\substack{\text{by spring}\\ \text{on ball}}} = F_{\substack{\text{by ball}\\ \text{on spring}}}$ and $a_y = a$, adding equations (1) and (2) yields:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} - F_{\substack{\text{by Earth}\\ \text{on ball}}} = m_\text{b} a$$

Solving for $F_{\substack{\text{by scale}\\ \text{on spring}}}$ yields:

$$F_{\substack{\text{by scale}\\ \text{on spring}}} = m_{\text{b}}a + F_{\substack{\text{by Earth}\\ \text{on ball}}}$$

Substitute for $F_{\substack{\text{by Earth}\\ \text{on ball}}}$ and simplify to obtain:

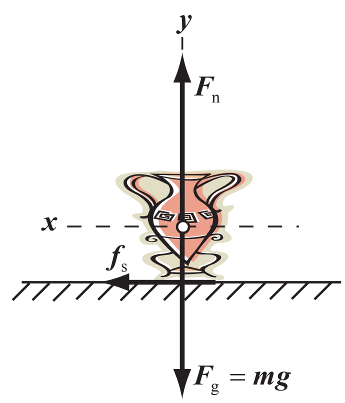
$$F_{\substack{\text{by scale}\\ \text{on spring}}} = \boxed{m_{\text{b}}(a+g)}$$

Remarks: Note that the two forces acting on the spring and the upward force acting on the ball, while still equal, are larger (because the system is accelerating upward) than they were in Problem 117.

General Problems

120 • In designing your new house in California, you are prepared for it to withstand a maximum horizontal acceleration of $0.50g$. What is the minimum coefficient of static friction between the floor and your prized Tuscan vase so that the vase does not slip on the floor under these conditions?

Picture the Problem The forces acting on the vase are the gravitational force $\vec{\boldsymbol{F}}_{\text{g}} = m\vec{\boldsymbol{g}}$ exerted by the earth, the normal force $\vec{\boldsymbol{F}}_{\text{n}}$ exerted by the floor, and the static friction force $\vec{\boldsymbol{f}}_{\text{s}}$, also exerted by the floor. We can apply Newton's 2nd law to find the minimum coefficient of static friction that will prevent the vase from slipping.



Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the vase to obtain:

$$\sum F_x = f_{\text{s}} = ma_x \qquad (1)$$

and

$$\sum F_y = F_{\text{n}} - mg = 0$$

Relate the static friction force $\vec{\boldsymbol{f}}_{\text{s}}$ to the minimum coefficient of static friction $\mu_{\text{s,min}}$ that will prevent the vase from slipping:

$$f_{\text{s}} = \mu_{\text{s,min}} F_{\text{n}}$$

or, because $F_{\text{n}} = mg$,

$$f_{\text{s}} = \mu_{\text{s,min}} mg$$

Substituting for f_{s} in equation (1) yields:

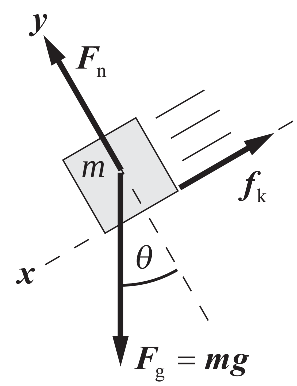
$$\mu_{\text{s,min}} mg = ma_x \Rightarrow \mu_{\text{s,min}} = \frac{a_x}{g}$$

Because $\boldsymbol{a}_x = 0.50\boldsymbol{g}$:

$$\mu_{\text{s,min}} = \frac{0.50\boldsymbol{g}}{\boldsymbol{g}} = \boxed{0.50}$$

121 • A 4.5-kg block slides down an inclined plane that makes an angle of 28° with the horizontal. Starting from rest, the block slides a distance of 2.4 m in 5.2 s. Find the coefficient of kinetic friction between the block and plane.

Picture the Problem The forces that act on the block as it slides down the incline are shown on the free-body diagram to the right. The acceleration of the block can be determined from the distance-and-time information given in the problem. The application of Newton's 2nd law to the block will lead to an expression for the coefficient of kinetic friction as a function of the block's acceleration and the angle of the incline.



Apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\sum F_x = mg\sin\theta - f_k = ma_x \quad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

Set $f_k = \mu_k F_n$ in equation (1) to obtain:

$$mg\sin\theta - \mu_k F_n = ma_x \quad (3)$$

Solve equation (2) for F_n and substitute in equation (3) to obtain:

$$mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

Solving for μ_k yields:

$$\mu_k = \frac{g\sin\theta - a_x}{g\cos\theta} \quad (4)$$

Using a constant-acceleration equation, relate the distance the block slides to its sliding time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 \Rightarrow a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for a_x in equation (4) to obtain:

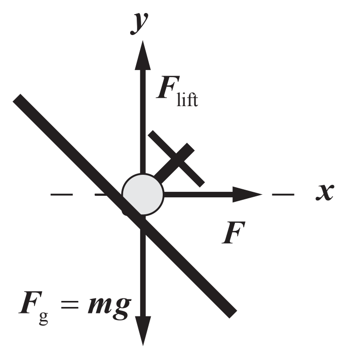
$$\mu_k = \frac{g\sin\theta - \dfrac{2\Delta x}{(\Delta t)^2}}{g\cos\theta}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(9.81\,\text{m/s}^2)\sin 28° - \dfrac{2(2.4\,\text{m})}{(5.2\,\text{s})^2}}{(9.81\,\text{m/s}^2)\cos 28°} = \boxed{0.51}$$

122 •• You plan to fly a model airplane of mass 0.400 kg that is attached to a horizontal string. The plane will travel in a horizontal circle of radius 5.70 m. (Assume the weight of the plane is balanced by the upward "lift" force of the air on the wings of the plane.) The plane will make 1.20 revolutions every 4.00 s. (*a*) Find the speed at which you must fly the plane. (*b*) Find the force exerted on your hand as you hold the string (assume the string is massless).

Picture the Problem The force exerted on your hand as you hold the string is the reaction force to the tension $\vec{F}$ in the string and, hence, has the same magnitude. The speed of the plane can be calculated from the data concerning the radius of its path and the time it takes to make one revolution. The application of Newton's 2nd law will give us the tension F in the string.



(*a*) Express the speed of the airplane in terms of the circumference of the circle in which it is flying and its period:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(5.70\,\text{m})}{\dfrac{4.00\,\text{s}}{1.20\,\text{rev}}} = \boxed{10.7\,\text{m/s}}$$

(*b*) Apply $\sum F_x = ma_x$ to the model airplane:

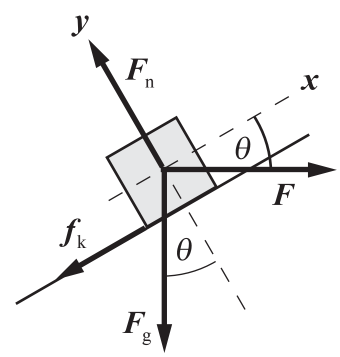
$$F = m\frac{v^2}{r} = m\frac{\left(\dfrac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate F:

$$F = \frac{4\pi^2(0.400\,\text{kg})(5.70\,\text{m})}{\left(\dfrac{4.00\,\text{s}}{1.20\,\text{rev}}\right)^2} = \boxed{8.10\,\text{N}}$$

123 •• Your moving company is to load a crate of books on a truck with the help of some planks that slope upward at 30.0º. The mass of the crate is 100 kg, and the coefficient of sliding friction between it and the planks is 0.500. You and your employees push *horizontally* with a combined net force $\vec{F}$. Once the crate has started to move, how large must F be in order to keep the crate moving at constant speed?

Picture the Problem The free-body diagram shows the forces acting on the crate of books. The kinetic friction force opposes the motion of the crate up the incline. Because the crate is moving at constant speed in a straight line, its acceleration is zero. We can determine F by applying Newton's 2nd law to the crate, substituting for f_k, eliminating the normal force, and solving for the required force.



Apply $\sum\vec{F} = m\vec{a}$ to the crate, with both a_x and a_y equal to zero, to the crate:

$$\sum F_x = F\cos\theta - f_k - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - F\sin\theta - mg\cos\theta = 0$$

Because $f_k = \mu_k F_n$, the x-equation becomes:

$$F\cos\theta - \mu_k F_n - mg\sin\theta = 0 \quad (1)$$

Solving the y-equation for F_n yields:

$$F_n = F\sin\theta + mg\cos\theta$$

Substitute for F_n in equation (1) and solve for F to obtain:

$$F = \frac{mg(\sin\theta + \mu_k\cos\theta)}{\cos\theta - \mu_k\sin\theta}$$

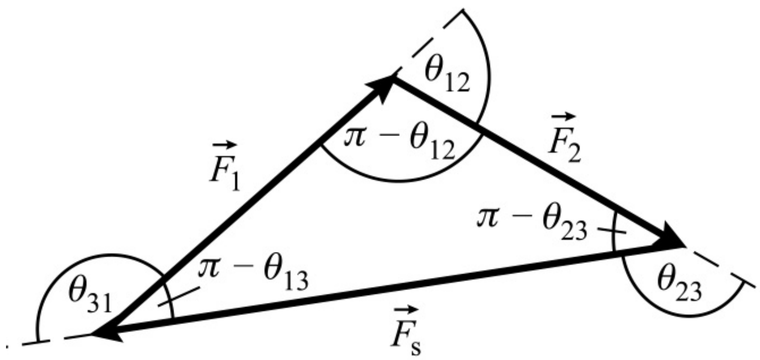
Substitute numerical values and evaluate F:

$$\boldsymbol{F} = \frac{(100\,\text{kg})(9.81\,\text{m/s}^2)(\sin 30° + (0.500)\cos 30.0°)}{\cos 30.0° - (0.500)\sin 30°} = \boxed{1.49\,\text{kN}}$$

124 •• Three forces act on an object in static equilibrium (Figure 5-83). (*a*) If F_1, F_2, and F_3 represent the magnitudes of the forces acting on the object, show that $F_1/\sin\theta_{23} = F_2/\sin\theta_{31} = F_3/\sin\theta_{12}$. (*b*) Show that $F_1^2 = F_2^2 + F_3^2 + 2F_2F_3\cos\theta_{23}$.
[

Picture the Problem The fact that the object is in static equilibrium under the influence of the three forces means that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Drawing the corresponding force triangle will allow us to relate the forces to the angles between them through the law of sines and the law of cosines.

(*a*) Using the fact that the object is in static equilibrium, redraw the force diagram connecting the forces head-to-tail:



Appling the law of sines to the triangle yields:

$$\frac{F_1}{\sin(\pi - \theta_{23})} = \frac{F_2}{\sin(\pi - \theta_{13})} = \frac{F_3}{\sin(\pi - \theta_{12})}$$

Use the trigonometric identity $\sin(\pi - \alpha) = \sin\alpha$ to obtain:

$$\boxed{\frac{F_1}{\sin\theta_{23}} = \frac{F_2}{\sin\theta_{13}} = \frac{F_3}{\sin\theta_{12}}}$$

(*b*) Appling the law of cosines to the triangle yields:

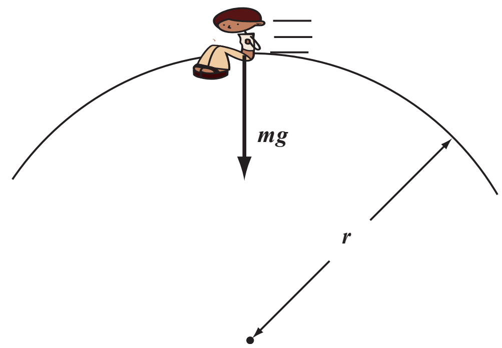
$$F_1^2 = F_2^2 + F_3^2 - 2F_2F_3\cos(\pi - \theta_{23})$$

Use the trigonometric identity $\cos(\pi - \alpha) = -\cos\alpha$ to obtain:

$$F_1^2 = \boxed{F_2^2 + F_3^2 + 2F_2F_3\cos\theta_{23}}$$

125 •• In a carnival ride, you sit on a seat in a compartment that rotates with constant speed in a vertical circle of radius 5.0 m. The ride is designed so your head always points toward the center of the circle. (*a*) If the ride completes one full circle in 2.0 s, find the direction and magnitude of your acceleration. (*b*) Find the slowest rate of rotation (in other words, the longest time T_m to complete one full circle) if the seat belt is to exert no force on you at the top of the ride.

Picture the Problem We can calculate your acceleration from your speed that, in turn, is a function of the period of the motion. To determine the longest period of the motion, we focus our attention on the situation at the very top of the ride when the seat belt is exerting no force on you. We can use Newton's 2nd law to relate the period of the motion to your acceleration and speed.



(*a*) Because the motion is at constant speed, your acceleration is entirely radial and is given by:

$$a_c = \frac{v^2}{r}$$

Express your speed as a function of the radius of the circle and the period of the motion:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the expression for a_c to obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(5.0\,\text{m})}{(2.0\,\text{s})^2} = \boxed{49\,\text{m/s}^2}$$

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to yourself when you are at the top of the circular path and the seat belt is exerting no force on you:

$$\sum F_r = mg = m\frac{v^2}{r} \Rightarrow v = \sqrt{gr}$$

Express the period of your motion as a function of the radius of the circle:

$$T_m = \frac{2\pi r}{v}$$

Substituting for v and simplifying yields:

$$T_m = \frac{2\pi r}{\sqrt{gr}} = 2\pi\sqrt{\frac{r}{g}}$$

The slowest rate of rotation is the reciprocal of T_m:

$$T_m^{-1} = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

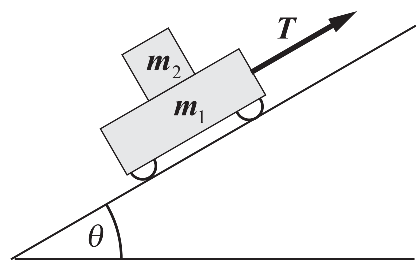
Substitute numerical values and evaluate T_m^{-1}:

$$T_m^{-1} = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{5.0\,\text{m}}} = 0.2229\,\text{s}^{-1} \times \frac{60\,\text{s}}{\text{min}} \approx \boxed{13\,\text{rev/min}}$$

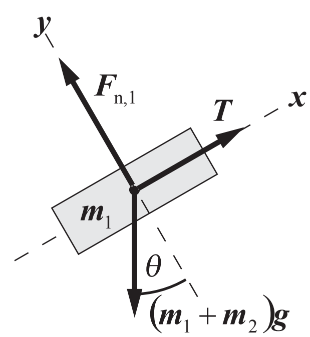
Remarks: The rider is ″weightless″ under the conditions described in Part (*b*).

126 •• A flat-topped toy cart moves on frictionless wheels, pulled by a rope under tension T. The mass of the cart is m_1. A load of mass m_2 rests on top of the cart with the coefficient of static friction μ_s between the cart and the load. The cart is pulled up a ramp that is inclined at angle θ above the horizontal. The rope is parallel to the ramp. What is the maximum tension T that can be applied without causing the load to slip?

Picture the Problem The pictorial representation to the right shows the cart and its load on the inclined plane. The load will not slip provided its maximum acceleration is not exceeded. We can find that maximum acceleration by applying Newton's 2nd law to the load. We can then apply Newton's 2nd law to the cart-plus-load system to determine the tension in the rope when the system is experiencing its maximum acceleration.



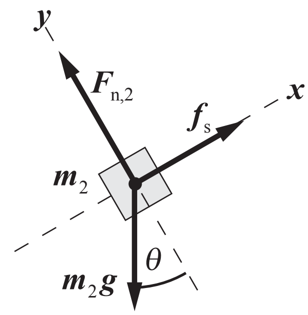
Draw the free-body diagram for the cart and its load:



Apply $\sum F_x = ma_x$ to the cart plus its load:

$$T - (m_1 + m_2)g\sin\theta = (m_1 + m_2)a_{x,\text{max}} \quad (1)$$

Draw the free-body diagram for the load of mass m_2 on top of the cart:



Apply $\sum\vec{F} = m\vec{a}$ to the load on top of the cart:

$$\sum F_x = f_{s,max} - m_2 g\sin\theta = m_2 a_{x,max}$$
and
$$\sum F_y = F_{n,2} - m_2 g\cos\theta = 0$$

Because $f_{s,max} = \mu_s F_{n,2}$, the x equation becomes:

$$\mu_s F_{n,2} - m_2 g\sin\theta = m_2 a_{x,max} \qquad (2)$$

Solving the y equation for $F_{n,2}$ yields:

$$F_{n,2} = m_2 g\cos\theta$$

Substitute for $F_{n,2}$ in equation (2) to obtain:

$$\mu_s m_2 g\cos\theta - m_2 g\sin\theta = m_2 a_{x,max}$$

Solving for $a_{x,max}$ and simplifying yields:

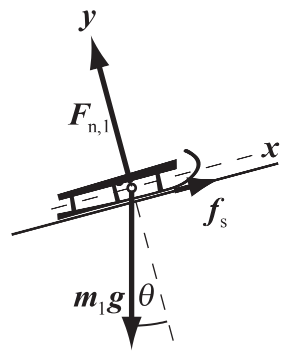
$$a_{x,max} = g(\mu_s\cos\theta - \sin\theta) \qquad (3)$$

Substitute for $a_{x,max}$ in equation (1) and solve for T to obtain:

$$T = \boxed{(m_1 + m_2)g\mu_s\cos\theta}$$

127 ••• A sled weighing 200 N that is held in place by static friction, rests on a 15° incline (Figure 5-84). The coefficient of static friction between the sled and the incline is 0.50. (*a*) What is the magnitude of the normal force on the sled? (*b*) What is the magnitude of the static frictional force on the sled? (*c*) The sled is now pulled up the incline at constant speed by a child walking up the incline ahead of the sled. The child weighs 500 N and pulls on the rope with a constant force of 100 N. The rope makes an angle of 30° with the incline and has negligible mass. What is the magnitude of the kinetic frictional force on the sled? (*d*) What is the coefficient of kinetic friction between the sled and the incline? (*e*) What is the magnitude of the force exerted on the child by the incline?

Picture the Problem The free-body diagram for the sled while it is held stationary by the static friction force is shown to the right. We can solve this problem by repeatedly applying Newton's 2nd law under the conditions specified in each part of the problem.



(*a*) Apply $\sum F_y = ma_y$ to the sled:

$$F_{n,1} - m_1 g\cos\theta = 0$$

Solve for $F_{n,1}$:

$$F_{n,1} = m_1 g\cos\theta$$

Substitute numerical values and evaluate $F_{n,1}$:

$$F_{n,1} = (200\,\text{N})\cos 15° = \boxed{0.19\,\text{kN}}$$

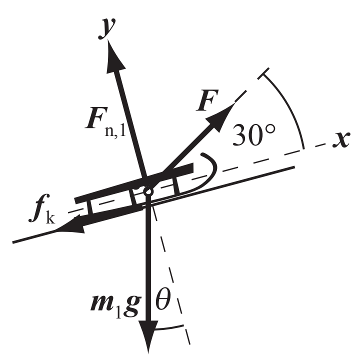
(*b*) Apply $\sum F_x = ma_x$ to the sled:

$$f_s - m_1 g\sin\theta = 0 \Rightarrow f_s = m_1 g\sin\theta$$

Substitute numerical values and evaluate f_s:

$$f_s = (200\,\text{N})\sin 15° = \boxed{52\,\text{N}}$$

(*c*) Draw the free-body diagram for the sled when it is moving up the incline at constant speed:



Apply $\sum \boldsymbol{F}_x = 0$ to the sled to obtain:

$$F\cos 30° - m_1 g\sin\theta - f_k = 0$$

Solving for f_k yields:

$$f_k = F\cos 30° - m_1 g\sin\theta$$

Substitute numerical values and evaluate f_k:

$$f_k = (100\,\text{N})\cos 30° - (200\,\text{N})\sin 15° = 34.84\,\text{N} = \boxed{35\,\text{N}}$$

(*d*) The coefficient of kinetic friction between the sled and the incline is given by:

$$\mu_k = \frac{f_k}{F_{n,1}} \quad (1)$$

Apply $\sum \boldsymbol{F}_y = 0$ to the sled to obtain:

$$F_{n,1} + F\sin 30° - m_1 g\cos\theta = 0$$

Solve for $\boldsymbol{F}_{n,1}$:

$$F_{n,1} = m_1 g\cos\theta - F\sin 30°$$

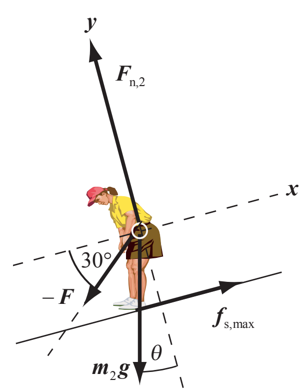
Substituting for $\boldsymbol{F}_{n,1}$ in equation (1) yields:

$$\mu_k = \frac{f_k}{m_1 g\cos\theta - F\sin 30°}$$

Substitute numerical values and evaluate $\boldsymbol{\mu_k}$:

$$\boldsymbol{\mu_k} = \frac{34.84\ \text{N}}{(200\ \text{N})\cos 15° - (100\ \text{N})\sin 30°} = \boxed{0.24}$$

(*e*) Draw the free body diagram for the child:



Express the net force F_c exerted on the child by the incline:

$$F_c = \sqrt{F_{n2}^2 + f_{s,max}^2} \quad (1)$$

Noting that the child is stationary, apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the child:

$$\sum F_x = f_{s,max} - F\cos 30° - m_2 g\sin 15° = 0$$

and

$$\sum F_y = F_{n2} - m_2 g\sin 15° - F\sin 30° = 0$$

Solve the x equation for $f_{s,max}$ and the y equation for $F_{n,2}$:

$$f_{s,max} = F\cos 30° + m_2 g\sin 15°$$

and

$$\boldsymbol{F}_{n,2} = \boldsymbol{m}_2\boldsymbol{g}\sin 15° + \boldsymbol{F}\sin 30°$$

Substitute numerical values and evaluate F_x and $F_{n,2}$:

$$f_{s,max} = (500\,\text{N})\cos 30° + (100\,\text{N})\sin 15° = 458.9\,\text{N}$$

and

$$F_{n,2} = (100\,\text{N})\sin 15° + (500\,\text{N})\sin 30° = 275.9\,\text{N}$$

Substitute numerical values in equation (1) and evaluate F:

$$F_c = \sqrt{(275.9\,\text{N})^2 + (458.9\,\text{N})^2} = \boxed{0.54\,\text{kN}}$$

128 •• In 1976, Gerard O'Neill proposed that large space stations be built for human habitation in orbit around Earth and the moon. Because prolonged free-fall has adverse medical effects, he proposed making the stations in the form of long cylinders and spinning them around the cylinder axis to provide the inhabitants with the sensation of gravity. One such O'Neill colony is to be built 5.0 miles long, with a diameter of 0.60 mi. A worker on the inside of the colony would experience a sense of ″gravity, ″ because he would be in an accelerated frame of reference due to the rotation. (*a*) Show that the "acceleration of gravity" experienced by the worker in the O'Neill colony is equal to his centripetal acceleration. (*Hint:* Consider someone ″looking in″ from outside the colony.) (*b*) If we assume that the space station is composed of several decks that are at varying distances (radii) from the axis of rotation, show that the ″acceleration of gravity″ becomes weaker the closer the worker gets to the axis. (*c*) How many revolutions per minute would this space station have to make to give an ″acceleration of gravity″ of 9.8 m/s^2 at the outermost edge of the station?

Picture the Problem Let v represent the speed of rotation of the station, and r the distance from the center of the station. Because the O'Neill colony is, presumably, in deep space, the only acceleration one would experience in it would be that due to its rotation.

(*a*) The acceleration of anyone who is standing inside the station is $a = v^2/r$. This acceleration is directed toward the axis of rotation. If someone inside the station drops an apple, the apple will not have any forces acting on it once released, but will move along a straight line at constant speed. However, from the point of view of our observer inside the station, if he views himself as unmoving, the apple is perceived to have an acceleration of v^2/r directed away from the axis of rotation (a "centrifugal" acceleration).

(*b*) Each deck must rotate the central axis with the same period T. Relate the speed of a person on a particular deck to his/her distance r from the center:

$$v = \frac{2\pi r}{T}$$

Express the "acceleration of gravity" perceived by someone a distance r from the center:

$$\boldsymbol{a}_{\mathrm{c}}=\frac{\boldsymbol{v}^2}{\boldsymbol{r}}=\frac{4\boldsymbol{\pi}^2\boldsymbol{r}}{\boldsymbol{T}^2}$$, a result that tells us that the "acceleration of gravity" decreases as r decreases.

(c) Relate the desired acceleration to the radius of Babylon 5 and its period:

$$a=\frac{4\pi^2 r}{T^2}\Rightarrow T=\sqrt{\frac{4\pi^2 r}{a}}$$

Substitute numerical values and evaluate T:

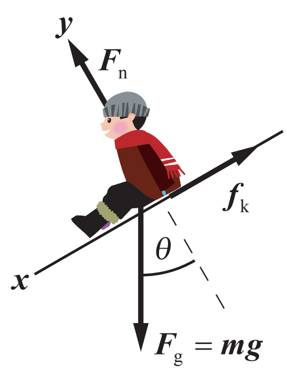
$$T=\sqrt{\frac{4\pi^2\left(0.30\,\text{mi}\times\frac{1.609\,\text{km}}{\text{mi}}\right)}{9.8\,\text{m/s}^2}}$$
$$=44\,\text{s}\approx 0.74\,\text{min}$$

Take the reciprocal of this time to find the number of revolutions per minute Babylon 5 has to make in order to provide this "earth-like" acceleration:

$$T^{-1}=\boxed{1.4\,\text{rev}/\text{min}}$$

129 •• A child of mass m slides down a slide inclined at 30º in time t_1. The coefficient of kinetic friction between her and the slide is μ_k. She finds that if she sits on a small sled (also of mass m) with frictionless runners, she slides down the same slide in time $\frac{1}{2}t_1$. Find μ_k.

Picture the Problem The following free-body diagram shows the forces acting on the child as she slides down the incline. We'll first use Newton's 2nd law to derive an expression for μ_k in terms of her acceleration and then use Newton's 2nd law to find her acceleration when riding the frictionless cart. Using a constant-acceleration equation, we'll relate these two accelerations to her descent times and solve for her acceleration when sliding. Finally, we can use this acceleration in the expression for μ_k.



Apply $\sum \vec{F} = m\vec{a}$ to the child as she slides down the incline:	$\sum F_x = mg\sin\theta - f_k = ma_{1,x}$ and $\sum F_y = F_n - mg\cos\theta = 0$	
Because $f_k = \mu_k F_n$, the x-equation can be written:	$mg\sin\theta - \mu_k F_n = ma_{1,x}$	(1)
Solving the y-equation for F_n yields:	$F_n = mg\cos\theta$	
Substitute for F_n in equation (1) to obtain:	$mg\sin\theta - \mu_k mg\cos\theta = ma_{1,x}$	
Solving for μ_k yields:	$\mu_k = \tan 30° - \dfrac{a_{1,x}}{g\cos 30°}$	(2)
Apply $\sum F_x = ma_x$ to the child as she rides the frictionless cart down the incline and solve for her acceleration $a_{2,x}$:	$mg\sin 30° = ma_{2,x}$ and $a_{2,x} = g\sin 30°$	
Letting s represent the distance she slides down the incline, use a constant-acceleration equation to relate her sliding times to her accelerations and distance traveled down the slide :	$s = v_{0x}t_1 + \frac{1}{2}a_{1,x}t_1^2$ where $v_{0x} = 0$ and $s = v_{0x}t_2 + \frac{1}{2}a_{2,x}t_2^2$ where $v_{0x} = 0$	
Equate these expressions, substitute $t_2 = \frac{1}{2}t_1$ and solve for $a_{1,x}$:	$a_{1,x} = \frac{1}{4}a_{2,x} = \frac{1}{4}g\sin 30°$	
Substitute for $a_{1,x}$ in equation (2) to obtain:	$\mu_k = \tan 30° - \dfrac{\frac{1}{4}g\sin 30°}{g\cos 30°} = \frac{3}{4}\tan 30°$	
Substitute numerical values and evaluate μ_k:	$\mu_k = \frac{3}{4}\tan 30° = \boxed{0.43}$	

130 ••• The position of a particle of mass $m = 0.80$ kg as a function of time is given by $\vec{r} = x\hat{i} + y\hat{j} = (R\sin\omega t)\hat{i} + (R\cos\omega t)\hat{j}$, where $R = 4.0$ m and $\omega = 2\pi\,\text{s}^{-1}$.

(*a*) Show that the path of this particle is a circle of radius *R*, with its center at the origin of the *xy* plane. (*b*) Compute the velocity vector. Show that $v_x/v_y = -y/x$. (*c*) Compute the acceleration vector and show that it is in directed toward the origin and has the magnitude v^2/R. (*d*) Find the magnitude and direction of the net force acting on the particle.

Picture the Problem The path of the particle is a circle if *r* is constant. Once we have shown that it is, we can calculate its value from its components and determine the particle's velocity and acceleration by differentiation. The direction of the net force acting on the particle can be determined from the direction of its acceleration.

(*a*) Express the magnitude of $\vec{r}$ in terms of its components:

$$r = \sqrt{r_x^2 + r_y^2}$$

Evaluate *r* with $r_x = R\sin\omega t$ and $r_y = R\cos\omega t$:

$$r = \sqrt{[R\sin\omega t]^2 + [R\cos\omega t]^2} = \sqrt{R^2\left(\sin^2\omega t + \cos^2\omega t\right)} = R$$

This result shows that the path of the particle is a circle of radius *R* centered at the origin.

(*b*) Differentiate $\vec{r}$ with respect to time to obtain $\vec{v}$:

$$\vec{v} = d\vec{r}/dt = [R\omega\cos\omega t]\,\hat{i} + [-R\omega\sin\omega t]\,\hat{j} = \boxed{[(8.0\pi\cos 2\pi t)\text{m/s}]\,\hat{i} - [(8.0\pi\sin 2\pi t)\text{m/s}]\,\hat{j}}$$

Express the ratio $\dfrac{v_x}{v_y}$:

$$\frac{v_x}{v_y} = \frac{8.0\pi\cos\omega t}{-8.0\pi\sin\omega t} = -\cot\omega t \quad (1)$$

Express the ratio $-\dfrac{y}{x}$:

$$-\frac{y}{x} = -\frac{R\cos\omega t}{R\sin\omega t} = -\cot\omega t \quad (2)$$

From equations (1) and (2) we have:

$$\boxed{\frac{v_x}{v_y} = -\frac{y}{x}}$$

(*c*) Differentiate $\vec{v}$ with respect to time to obtain $\vec{a}$:

$$\vec{a} = \frac{d\vec{v}}{dt} = \boxed{\left[\left(-16\pi^2\text{m/s}^2\right)\sin\omega t\right]\hat{i} + \left[\left(-16\pi^2\text{m/s}^2\right)\cos\omega t\right]\hat{j}}$$

Factor $-4.0\pi^2/\text{s}^2$ from $\vec{a}$ to obtain:

$$\vec{a} = \left(-4.0\pi^2\ \text{s}^{-2}\right)\left[(4.0\sin\omega t)\hat{i} + (4.0\cos\omega t)\hat{j}\right] = \boxed{\left(-4.0\pi^2\ \text{s}^{-2}\right)\vec{r}}$$

Because $\vec{a}$ is in the opposite direction from $\vec{r}$, it is directed toward the center of the circle in which the particle is traveling.

Find the ratio $\dfrac{v^2}{R}$:

$$a = \frac{v^2}{R} = \frac{(8.0\pi\ \text{m/s})^2}{4.0\ \text{m}} = \boxed{16\pi^2\ \text{m/s}^2}$$

(*d*) Apply $\sum\vec{F} = m\vec{a}$ to the particle:

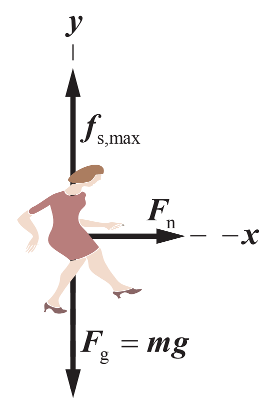
$$F_{\text{net}} = ma = (0.80\ \text{kg})(16\pi^2\ \text{m/s}^2) = \boxed{13\pi^2\ \text{N}}$$

Because the direction of $\vec{F}_{\text{net}}$ is the same as that of $\vec{a}$, $\vec{F}_{\text{net}}$ is toward the center of the circle.

131 ••• You are on an amusement park ride with your back against the wall of a spinning vertical cylinder. The floor falls away and you are held up by static friction. Assume your mass is 75 kg. (*a*) Draw a free-body diagram of yourself. (*b*) Use this diagram with Newton's laws to determine the force of friction on you. (*c*) If the radius of the cylinder is 4.0 m and the coefficient of static friction between you and the wall is 0.55. What is the minimum number of revolutions per minute necessary to prevent you from sliding down the wall? Does this answer hold only for you? Will other, more massive, patrons fall downward? Explain.

Picture the Problem The application of Newton's 2nd law and the definition of the maximum static friction force will be used to determine the period T of the motion. The reciprocal of the period will give us the minimum number of revolutions required per unit time to hold you in place.

(*a*) The free-body diagram showing the forces acting on you when you are being held in place by the maximum static friction force is shown to the right.



(*b*) Apply $\sum \vec{F} = m\vec{a}$ to yourself while you are held in place by friction:

$$\sum F_x = F_n = m\frac{v^2}{r} \qquad (1)$$

and

$$\sum F_y = f_{s,max} - mg = 0 \qquad (2)$$

Solve equation (2) for $f_{s,max}$:

$$f_{s,max} = mg$$

Substitute numerical values and evaluate $f_{s,max}$:

$$f_{s,max} = (75\text{ kg})(9.81\text{ m/s}^2) = \boxed{0.74\text{ kN}}$$

(*c*) The number of revolutions per minute *N* is the reciprocal of the period in minutes:

$$N = \frac{1}{T} \qquad (3)$$

Because $f_{s,max} = \mu_s F_n$, equation (1) can be written:

$$\frac{f_{s,max}}{\mu_s} = \frac{mg}{\mu_s} = m\frac{v^2}{r} \qquad (4)$$

Your speed is related to the period of your motion:

$$v = \frac{2\pi r}{T}$$

Substitute for *v* in equation (4) to obtain:

$$\frac{mg}{\mu_s} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 mr}{T^2}$$

Solving for *T* yields:

$$T = 2\pi\sqrt{\frac{\mu_s r}{g}}$$

Substitute for *T* in equation (3) to obtain:

$$N = \frac{1}{2\pi\sqrt{\frac{\mu_s r}{g}}} = \frac{1}{2\pi}\sqrt{\frac{g}{\mu_s r}}$$

Substitute numerical values and evaluate *N*:

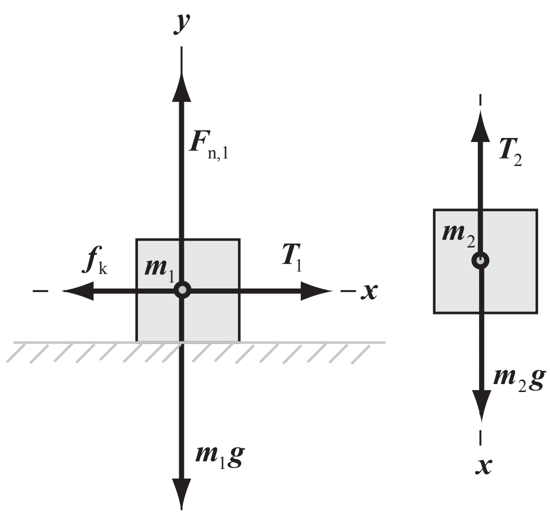
$$N = \frac{1}{2\pi}\sqrt{\frac{9.81\text{ m/s}^2}{(0.55)(4.0\text{ m})}} = 0.336\text{ rev/s}$$

$$= \boxed{20\text{ rev/min}}$$

Because your mass does not appear in the expression for *N*, this result holds for all patrons, regardless of their mass.

132 ••• An object of mass m_1 is on a horizontal table. The object is attached to a 2.5-kg object (m_2) by a light string that passes over a pulley at the edge of the table. The object of mass m_2 dangles 1.5 m above the ground (Figure 5-85). The string that connects them passes over a frictionless, massless pulley. This system is released from rest at $t = 0$ and the 2.5-kg object strikes the ground at $t = 0.82$ s. The system is now placed in its initial configuration and a 1.2-kg object is placed on top of the block of mass m_1. Released from rest, the 2.5-kg object now strikes the ground 1.3 s later. Determine the mass m_1 and the coefficient of kinetic friction between the object whose mass is m_1 and the table.

Picture the Problem The free-body diagrams below show the forces acting on the objects whose masses are m_1 and m_2. The application of Newton's 2nd law and the use of a constant-acceleration equation will allow us to find a relationship between the coefficient of kinetic friction and m_1. The repetition of this procedure with the additional object on top of the object whose mass is m_1 will lead us to a second equation that, when solved simultaneously with the former equation, leads to a quadratic equation in m_1. Finally, its solution will allow us to substitute in an expression for μ_k and determine its value.



Using a constant-acceleration equation, relate the displacement of the system in its first configuration as a function of its acceleration and fall time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_{1x}(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_{x,1}(\Delta t)^2 \Rightarrow a_{x,1} = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate $a_{x,1}$:

$$a_{x,1} = \frac{2(1.5\,\text{m})}{(0.82\,\text{s})^2} = 4.4616\,\text{m/s}^2$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_2:

$$m_2 g - T_1 = m_2 a_{x,1} \Rightarrow T_1 = m_2(g - a_{1x})$$

Substitute numerical values and evaluate T_1:

$$T_1 = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 4.4616\,\text{m/s}^2) = 13.371\,\text{N}$$

Apply $\sum \vec{F} = m\vec{a}$ to the object whose mass is m_1:

$$\sum F_x = T_1 - f_k = m_1 a_{x,1}$$

and

$$\sum F_y = F_{n,1} - m_1 g = 0$$

Using $f_k = \mu_k F_n$, eliminate F_n between the two equations to obtain:

$$T_1 - \mu_k m_1 g = m_1 a_{x,1} \qquad (1)$$

Find the acceleration $\boldsymbol{a}_{x,2}$ for the second run:

$$\boldsymbol{a}_{x,2} = \frac{2\Delta \boldsymbol{x}}{(\Delta \boldsymbol{t})^2} = \frac{2(1.5\,\text{m})}{(1.3\,\text{s})^2} = 1.775\,\text{m/s}^2$$

T_2 is given by:

$$T_2 = m_2(g - a_{x,2}) = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 1.775\,\text{m/s}^2) = 20.1\,\text{N}$$

Apply $\sum F_x = ma_x$ to the system with the 1.2-kg object in place:

$$T_2 - \mu_k(m_1 + 1.2\,\text{kg})g = (m_1 + 1.2\,\text{kg})a_{x,2} \qquad (2)$$

Solve equation (1) for μ_k:

$$\mu_k = \frac{T_1 - m_1 a_{x,1}}{m_1 g} \qquad (3)$$

Substitute for μ_k in equation (2) and simplify to obtain the quadratic equation in m_1:

$$2.686 m_1^2 + 9.940 m_1 - 16.05 = 0$$

Use your graphing calculator or the quadratic formula to obtain:

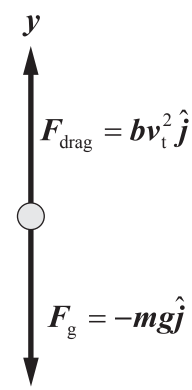
$$\boldsymbol{m}_1 = 1.215\,\text{kg} = \boxed{1.2\,\text{kg}}$$

Substitute numerical values in equation (3) and evaluate μ_k:

$$\boldsymbol{\mu}_k = \frac{13.375\,\text{N} - (1.215\,\text{kg})(4.4616\,\text{m/s}^2)}{(1.215\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.67}$$

133 ••• Sally claims flying squirrels do not really fly; they jump and use folds of skin that connect their forelegs and their back legs like a parachute to allow them to glide from tree to tree. Liz decides to test Sally's hypothesis by calculating the terminal speed of a falling outstretched flying squirrel. If the constant b in the drag force is proportional to the area of the object facing the air flow, use the results of Example 5-12 and some assumptions about the size of the squirrel to estimate its terminal (downward) speed. Is Sally's claim supported by Liz's calculation?

Picture the Problem A free-body diagram showing the forces acting on the squirrel under terminal-speed conditions is shown to the right. We'll assume that b is proportional to the squirrel's frontal area and that this area is about 0.1 that of a human being. Further, we'll assume that the mass of a squirrel is about 1.0 kg. Applying Newton's 2nd law will lead us to an expression for the squirrel's terminal speed.



Apply $\sum F_y = ma_y$ to the squirrel:

$$F_d - F_g = ma_y$$

or, because $a_y = 0$,

$$F_d - F_g = 0$$

Substituting for F_d and F_g yields:

$$bv_t^2 - mg = 0 \Rightarrow v_t = \sqrt{\frac{mg}{b}} \qquad (1)$$

Assuming that $A_{\text{squirrel}} = (0.1)A_{\text{human}}$:

$$b = (0.1)b_{\text{human}}$$

From Example 5-12:

$$b_{\text{human}} = 0.251\,\text{kg/m}$$

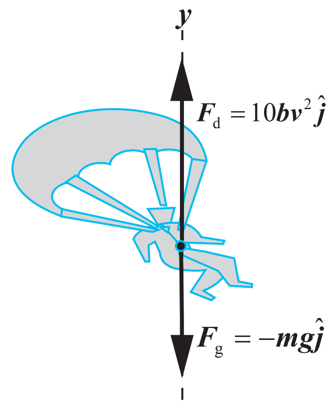
Substitute numerical values in equation (1) and evaluate v_t:

$$v_t = \sqrt{\frac{(1.0\,\text{kg})(9.81\,\text{m/s}^2)}{(0.1)(0.251\,\text{kg/m})}} \approx \boxed{20\,\text{m/s}}$$

Because the squirrel's terminal speed is approximately 80 km/h and this value is less than half that of the skydiver in Example 5-12, Sally's claim seems to be supported by Liz's calculation.

134 ••• After a parachutist jumps from an airplane (but before he pulls the rip cord to open his parachute), a downward speed of up to 180 km/h can be reached. When the parachute is finally opened, the drag force is increased by about a factor of 10, and this can create a large jolt on the jumper. Suppose this jumper falls at 180 km/h before opening his chute. (a) Determine the parachutist's acceleration when the chute is just opened, assuming his mass is 60 kg. (b) If rapid accelerations greater than 5.0g can harm the structure of the human body, is this a safe practice?

Picture the Problem The free-body diagram shows the drag force $\vec{F}_d$ exerted by the air and the gravitational force $\vec{F}_g$ exerted by the Earth acting on on the parachutist just after his chute has opened. We can apply Newton's 2nd law to the parachutist to obtain an expression for his acceleration as a function of his speed and then evaluate this expression for $v = v_t$.



(*a*) Apply $\sum F_y = ma_y$ to the parachutist immediately after the chute opens:

$$10bv^2 - mg = ma_{\text{chute open}}$$

Solving for $a_{\text{chute open}}$ yields:

$$a_{\text{chute open}} = 10\frac{b}{m}v^2 - g \qquad (1)$$

Before the chute opened:

$$bv^2 - mg = ma_y$$

Under terminal speed conditions, $a_y = 0$ and:

$$bv_t^2 - mg = 0 \Rightarrow \frac{b}{m} = \frac{g}{v_t^2}$$

Substitute for b/m in equation (1) to obtain:

$$a_{\text{chute open}} = 10\left(g\frac{1}{v_t^2}\right)v^2 - g = \left[10\left(\frac{1}{v_t^2}\right)v^2 - 1\right]g$$

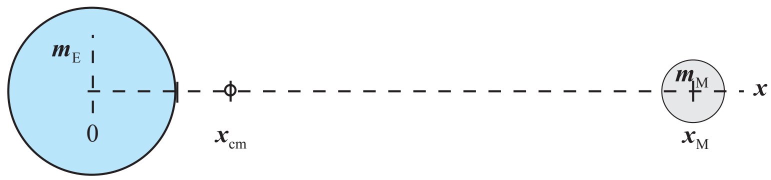
Evaluating $a_{\text{chute open}}$ for $v = v_t$ yields:

$$a_{\text{chute open}} = \left[10\left(\frac{1}{v_t^2}\right)v_t^2 - 1\right]g = \boxed{9g}$$

(*b*) Because this acceleration exceeds the safe acceleration by 4*g*, this is not a safe practice.

135 • Find the location of the center of mass of the Earth–moon system relative to the center of Earth. Is it inside or outside the surface of Earth?

Picture the Problem We can use the definition of the location of the center of mass of a system of particles to find the location of the center of mass of the Earth-moon system. The following pictorial representation is, of course, not shown to scale.



The center of mass of the Earth-moon system is given by:

$$x_{cm} = \frac{m_E x_{cm,E} + m_M x_{cm,M}}{m_E + m_M}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{\left(5.98\times10^{24}\ \text{kg}\right)(0)+\left(7.36\times10^{22}\ \text{kg}\right)\left(3.84\times10^{8}\ \text{m}\right)}{5.98\times10^{24}\ \text{kg}+7.36\times10^{22}\ \text{kg}} = \boxed{4.67\times10^{6}\ \text{m}}$$

Because the radius of the Earth is 6.37×10^6 m, the center of mass of the Earth-moon system is inside Earth.

136 •• A circular plate of radius R has a circular hole of radius $R/2$ cut out of it (Figure 5-86). Find the center of mass of the plate after the hole has been cut. (*Hint:* The plate can be modeled as two disks superimposed, with the hole modeled as a disk negative mass.)

Picture the Problem By symmetry, $x_{cm} = 0$. Let σ be the mass per unit area of the disk. The mass of the modified disk is the difference between the mass of the whole disk and the mass that has been removed.

Start with the definition of y_{cm}:

$$y_{cm} = \frac{\sum_i m_i y_i}{M - m_{hole}} = \frac{m_{disk} y_{disk} - m_{hole} y_{hole}}{M - m_{hole}}$$

Express the mass of the complete disk:

$$M = \sigma A = \sigma\pi r^2$$

Express the mass of the material removed:

$$m_{hole} = \sigma\pi\left(\frac{r}{2}\right)^2 = \tfrac{1}{4}\sigma\pi r^2 = \tfrac{1}{4}M$$

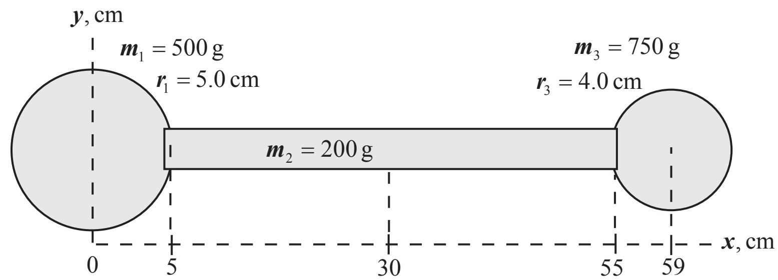
Substitute and simplify to obtain:

$$y_{cm} = \frac{M(0)-\left(\frac{1}{4}M\right)\left(-\frac{1}{2}r\right)}{M-\frac{1}{4}M} = \boxed{\tfrac{1}{6}r}$$

137 •• An unbalanced baton consists of a 50-cm-long uniform rod of mass 200 g. At one end there is a 10-cm-diameter uniform solid sphere of mass

500 g, and at the other end there is a 8.0-cm-diameter uniform solid sphere of mass 750 g. (The center-to-center distance between the spheres is 59 cm.) (*a*) Where, relative to the center of the light sphere, is the center of mass of this baton? (*b*) If this baton is tossed straight up (but spinning) so that its initial center of mass speed is 10.0 m/s, what is the velocity of the center of mass 1.5 s later? (*c*) What is the net external force on the baton while in the air? (*d*) What is the baton's acceleration 1.5 s following its release?

Picture the Problem The pictorial representation summarizes the information concerning the unbalanced baton and shows a convenient choice for a coordinate system.



(*a*) The *x* coordinate of the center of mass of the unbalanced baton is given by:

$$x_{cm} = \frac{m_1 x_{cm,1} + m_2 x_{cm,2} + m_3 x_{cm,3}}{m_1 + m_2 + m_3}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(500\text{ g})(0) + (200\text{ g})(30\text{ cm}) + (750\text{ g})(59\text{ cm})}{500\text{ g} + 200\text{ g} + 750\text{ g}} = \boxed{35\text{ cm}}$$

(*b*) Because the center of mass of the baton acts like a point particle, use a constant-acceleration equation to express is velocity as a function of time:

$$v(t) = v_{0,y} + a_y t$$

or, because $a_y = -g$,

$$v(t) = v_{0,y} - g\,t$$

Substitute numerical values and evaluate $v(1.5\text{ s})$:

$$\begin{aligned} v(1.5\text{ s}) &= 10.0\text{ m/s} - (9.81\text{ m/s}^2)(1.5\text{ s}) \\ &= -4.7\text{ m/s} \\ &= \boxed{4.7\text{ m/s downward}} \end{aligned}$$

(*c*) The net external force acting on the baton is the gravitational force (weight) exerted on it by the earth:

$$F_{net} = F_g = (m_1 + m_2 + m_3)g$$

Substitute numerical values and evaluate F_{net}:

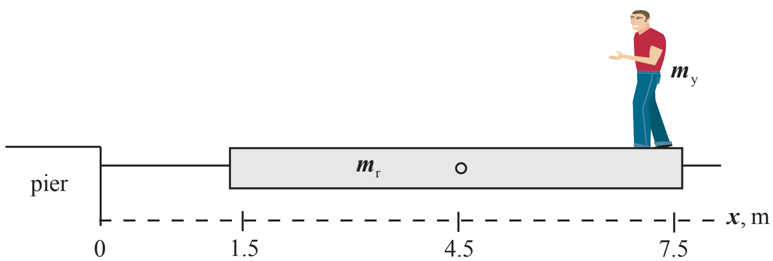
$$F_{net} = (500\text{ g} + 200\text{ g} + 750\text{ g})(9.81\text{ m/s}^2) = \boxed{14.2\text{ kN}}$$

(*d*) Again, because the center of mass acts like a point particle, its acceleration is that of a point particle in flight near the surface of the earth:

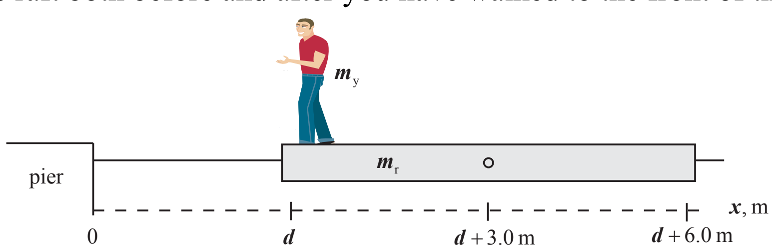
$$\boldsymbol{a}_{cm} = \boldsymbol{g} = \boxed{-9.81\text{ m/s}^2}$$

138 •• You are standing at the very rear of a 6.0-m-long, 120-kg raft that is at rest in a lake with its prow only 0.50 m from the end of the pier (Figure 5-87). Your mass is 60 kg. Neglect frictional forces between the raft and the water. (*a*) How far from the end of the pier is the center of mass of the you-raft system? (*b*) You walk to the front of the raft and then stop. How far from the end of the pier is the center of mass now? (*c*) When you are at the front of the raft, how far are you from the end of the pier?

Picture the Problem Let the origin be at the edge of the pier and the positive *x* direction be to the right as shown in the pictorial representation immediately below.



In the following pictorial representation, *d* is the distance of the end of the raft from the pier after you have walked to its front. The raft moves to the left as you move to the right; with the center of mass of the you-raft system remaining fixed (because $F_{ext,net} = 0$). The diagram shows the initial ($x_{y,i}$) and final ($x_{y,f}$) positions of yourself as well as the initial ($x_{r\_cm,i}$) and final ($x_{r\_cm,f}$) positions of the center of mass of the raft both before and after you have walked to the front of the raft.



(*a*) x_{cm} before you walk to the front of the raft is given by:

$$x_{cm} = \frac{m_y x_{y,i} + m_r x_{r\_cm,i}}{m_y + m_r}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(60\text{ kg})(7.5\text{ m}) + (120\text{ kg})(4.5\text{ m})}{60\text{ kg} + 120\text{ kg}} = \boxed{5.5\text{ m}}$$

(*b*) Express the distance of the raft from the pier after you have walked to the front of the raft:

$$d = 0.50\text{ m} + x_{f,y} \qquad (1)$$

Express x_{cm} before you walk to the front of the raft:

$$x_{cm} = \frac{m_y x_{y,i} + m_r x_{r\_cm,i}}{m_y + m_r}$$

Express x_{cm} after you have walked to the front of the raft:

$$x_{cm} = \frac{m_y x_{y,f} + m_r x_{r\_cm,f}}{m_y + m_r}$$

Because $F_{ext,net} = 0$, the center of mass remains fixed and we can equate these two expressions for x_{cm} to obtain:

$$m_y x_{y,i} + m_r x_{r\_cm,i} = m_y x_{y,f} + m_r x_{r\_cm,f}$$

Solving for $x_{y,f}$ yields:

$$x_{y,f} = x_{y,i} - \frac{m_r}{m_y}\left(x_{r\_cm,f} - x_{r\_cm,i}\right)$$

From the figure it can be seen that $x_{r\_cm,f} - x_{r\_cm,i} = x_{y,f}$. Substitute $x_{y,f}$ for $x_{r\_cm,f} - x_{r\_cm,i}$ to obtain:

$$x_{y,f} = \frac{m_y x_{y,i}}{m_y + m_r}$$

Substitute numerical values and evaluate $x_{y,f}$:

$$x_{y,f} = \frac{(60\text{ kg})(6.0\text{ m})}{60\text{ kg} + 120\text{ kg}} = 2.0\text{ m}$$

Substitute for $x_{y,f}$ in equation (1) to obtain:

$$d = 2.00\text{ m} + 0.50\text{ m} = \boxed{2.5\text{ m}}$$

(*c*) x_{cm} after you've walked to the front of the raft is given by:

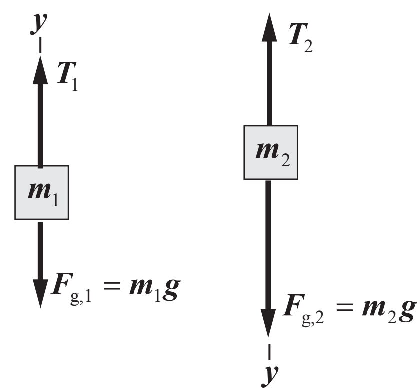
$$x_{cm} = \frac{m_y d + m_r\left(d + x_{r\_cm}\right)}{m_y + m_r}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(60\,\text{kg})(2.50\,\text{m}) + (120\,\text{kg})(2.50\,\text{m} + 3.0\,\text{m})}{60\,\text{kg} + 120\,\text{kg}} = \boxed{4.5\,\text{m}}$$

139 •• An Atwood's machine that has a frictionless massless pulley and massless strings has a 2.00-kg object hanging from one side and 4.00-kg object hanging from the other side. (*a*) What is the speed of each object 1.50 s after they are released from rest? (*b*) At that time, what is the velocity of the center of mass of the two objects? (*c*) At that time, what is the acceleration of the center of mass of the two objects?

Picture the Problem The forces acting on the 2.00-kg (m_1) and 4.00-kg (m_2) objects are shown in the free-body diagrams to the right. Note that the two objects have the same acceleration and common speeds. We can use constant-acceleration equations and Newton's 2nd law in the analysis of the motion of these objects.



(*a*) Use a constant-acceleration equation to relate the speed of the objects to their common acceleration:

$$\boldsymbol{v}(\boldsymbol{t}) = \boldsymbol{v}_{0y} + \boldsymbol{a}_y \boldsymbol{t}$$

or, because $v_{0y} = 0$,

$$\boldsymbol{v}(\boldsymbol{t}) = \boldsymbol{a}_y \boldsymbol{t} \qquad (1)$$

Apply $\sum \boldsymbol{F}_y = m\boldsymbol{a}_y$ to the object whose mass is m_1:

$$T_1 - m_1 g = m_1 a_y$$

Apply $\sum \boldsymbol{F}_y = m\boldsymbol{a}_y$ to the object whose mass is m_2:

$$m_2 g - T_2 = m_2 a_y$$

Because $T_1 = T_2$, adding these equations yields:

$$m_2 g - m_1 g = m_2 a_y + m_1 a_y$$

Solve for a_y to obtain:

$$a_y = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \qquad (2)$$

Substituting for a_y in equation (1) yields:

$$v(t)=\left(\frac{m_2-m_1}{m_2+m_1}\right)g\,t$$

Substitute numerical values and evaluate $\boldsymbol{v}(1.50\text{ s})$:

$$\boldsymbol{v}(1.50\text{ s})=\left(\frac{4.00\text{ kg}-2.00\text{ kg}}{4.00\text{ kg}+2.00\text{ kg}}\right)(9.81\text{ m/s}^2)(1.50\text{ s})=4.905\text{ m/s}=\boxed{4.91\text{ m/s}}$$

(*b*) The velocity of the center of mass is given by:

$$\vec{\boldsymbol{v}}_{\text{cm}}=\frac{\boldsymbol{m}_1\vec{\boldsymbol{v}}_1+\boldsymbol{m}_2\vec{\boldsymbol{v}}_2}{\boldsymbol{m}_1+\boldsymbol{m}_2}$$

Substitute numerical values and evaluate $\boldsymbol{v}_{\text{cm}}(1.50\text{ s})$:

$$\boldsymbol{v}_{\text{cm}}(1.50\text{ s})=\frac{(2.00\text{ kg})(4.905\text{ m/s})\hat{\boldsymbol{j}}+(4.00\text{ kg})(-4.905\text{ m/s})\hat{\boldsymbol{j}}}{2.00\text{ kg}+4.00\text{ kg}}=(-1.64\text{ m/s})\hat{\boldsymbol{j}}$$

The velocity of the center of mass is $\boxed{\text{1.64 m/s downward.}}$

(*c*) The acceleration of the center of mass is given by:

$$\vec{\boldsymbol{a}}_{\text{cm}}=\frac{\boldsymbol{m}_1\vec{\boldsymbol{a}}_1+\boldsymbol{m}_2\vec{\boldsymbol{a}}_2}{\boldsymbol{m}_1+\boldsymbol{m}_2}$$

Substituting for a_y from equation (2) and simplifying yields:

$$\vec{\boldsymbol{a}}_{\text{cm}}=\frac{\boldsymbol{m}_1\left(\frac{\boldsymbol{m}_2-\boldsymbol{m}_1}{\boldsymbol{m}_2+\boldsymbol{m}_1}\right)\boldsymbol{g}\,\hat{\boldsymbol{j}}+\boldsymbol{m}_2\left(-\frac{\boldsymbol{m}_2-\boldsymbol{m}_1}{\boldsymbol{m}_2+\boldsymbol{m}_1}\right)\boldsymbol{g}\,\hat{\boldsymbol{j}}}{\boldsymbol{m}_1+\boldsymbol{m}_2}=-\left(\frac{\boldsymbol{m}_2-\boldsymbol{m}_1}{\boldsymbol{m}_2+\boldsymbol{m}_1}\right)^2\boldsymbol{g}\,\hat{\boldsymbol{j}}$$

Substitute numerical values and evaluate a_{cm}:

$$\vec{\boldsymbol{a}}_{\text{cm}}=-\left(\frac{4.00\text{ kg}-2.00\text{ kg}}{4.00\text{ kg}+2.00\text{ kg}}\right)^2(9.81\text{ m/s}^2)\hat{\boldsymbol{j}}=(-1.09\text{ m/s}^2)\hat{\boldsymbol{j}}$$

The acceleration of the center of mass is $\boxed{1.09\text{ m/s}^2\text{ downward.}}$