Chapter 4
Newton's Laws

Conceptual Problems

1 • While on a very smooth level transcontinental plane flight, your coffee cup sits motionless on your tray. Are there forces acting on the cup? If so, how do they differ from the forces that would be acting on the cup if it sat on your kitchen table at home?

Determine the Concept Yes, there are forces acting on it. They are the normal force of the table and the gravitational pull of Earth (weight). Since the cup is not accelerating relative to the ground, they are the same as being motionless at home (which is also a not accelerating relative to the ground).

2 • Suppose you are passing another car on a highway and determine, from your reference frame, that the car you pass appears to have an acceleration $\vec{a}$ to the west, in spite of the fact its driver is maintaining a constant speed and direction. Is your frame an inertial one? If not, which way is your frame accelerating compared to the (apparent) acceleration of the other car?

Determine the Concept You are in a non-inertial frame that is accelerating to the east, opposite the other car's apparent acceleration.

3 • [**SSM**] You are riding in a limousine that has opaque windows which do not allow you to see outside. The car can accelerate by speeding up, slowing down, or turning. Equipped with just a small heavy object on the end of a string, how can you use it to determine if the limousine is changing either speed or direction? Can you determine the limousine's velocity?

Determine the Concept The sum of the external forces on the object is always proportional to its acceleration relative to an inertial reference frame. Any reference frame that maintains a zero acceleration relative to an inertial reference frame is itself an inertial reference frame, and vice versa. The ground is an inertial reference frame. If the limo does not accelerate (that is if it does not change direction or speed) relative to the ground, the pendulum will dangle straight down so that the net force on the bob is zero (no acceleration). In that case, the limo is an inertial reference frame. Just with this apparatus and not looking outside you cannot tell the limo's velocity; all you know is that it is constant.

4 •• If only a single nonzero force acts on an object, must the object have an acceleration relative to all inertial reference frames? Is it possible for such an object to have zero velocity in some inertial reference frame and not in another? If so, give a specific example.

Determine the Concept An object accelerates when a *net* force acts on it. The fact that an object is accelerating tells us nothing about its velocity other than that it is always changing.

Yes, the object must have an acceleration relative to the inertial frame of reference. According to Newton's 1st and 2nd laws, an object must accelerate, relative to any inertial reference frame, in the direction of the net force. If there is "only a single nonzero force," then this force is the net force.

Yes, the object's velocity may be momentarily zero. During the period in which the force is acting, the object may be momentarily at rest, but its velocity cannot remain zero because it must continue to accelerate. Thus, its velocity is always changing.

5   ••   A baseball is acted upon by a single known force. From this information alone, can you tell in which direction the baseball is moving? Explain.

Determine the Concept No. Predicting the direction of the subsequent motion correctly requires knowledge of the initial velocity as well as the acceleration. While the acceleration can be obtained from the net force through Newton's 2nd law, the velocity can only be obtained by integrating the acceleration.

6   ••   A truck moves away from you at constant velocity, as observed by you at rest on the surface of the Earth. It follows that (*a*) no forces act on the truck, (*b*) a constant force acts on the truck in the direction of its velocity, (*c*) the net force acting on the truck is zero, (*d*) the net force acting on the truck its weight.

Determine the Concept An object in an inertial reference frame accelerates if there is a *net* force acting on it. Because the object is moving at constant velocity, the net force acting on it is zero. $\boxed{(c)}$ is correct.

7   ••   Several space probes have been launched that are now far out in space (for example, Pioneer 10 in the 1970s is well beyond our solar system limits) and they are still moving away from the Sun and its planets. How is its mass changing? Which of the known fundamental forces continue to act on it? Does it have a net force on it?

Determine the Concept The mass of the probe is constant. However, the solar system will exert attract the probe with a gravitational. As the distance between the probe and the solar system becomes larger the magnitude of the gravitational force becomes smaller. There is a net force on the probe because no other forces act on it.

8   ••   Astronauts in apparent weightlessness during their stay on the International Space Station must carefully monitor their mass because such conditions are known to cause serious medical problems. Give an example of how you might design equipment to measure the mass of an astronaut in Earth orbit.

Determine the Concept You could use a calibrated spring (a spring with a known stiffness constant) to pull on each astronaut and measure their resulting acceleration. Then you could use Newton's second law to calculate their mass.

9   ••   **[SSM]**   You are riding in an elevator. Describe two situations in which your apparent weight is greater than your true weight.

Determine the Concept Your apparent weight is the reading of a scale. If the acceleration of the elevator (and you) is directed upward, the normal force exerted by the scale on you is greater than your weight. You could be moving down but slowing or moving up and speeding up. In both cases your acceleration is upward.

10   ••   Suppose you are in a train moving at constant velocity relative to the ground. You toss a ball to your friend several seats in front of you. Use Newton's second law to explain why you cannot use your observations of the tossed ball to determine the train's velocity relative to the ground.

Determine the Concept Because you are moving with constant velocity, your frame of reference is an inertial reference frame. In an inertial reference frame there are no fictitious forces. Thus moving or not moving, the ball will follow the same trajectory in your reference frame. The net force on the ball is the same, so its acceleration is the same.

11   ••   Explain why, of the fundamental interactions, the gravitational interaction is the main concern in our everyday lives. One other on this list also plays an increasingly significant role in our rapidly advancing technology. Which one is that? Why are the others not obviously important?

Determine the Concept The strong nuclear force act only over the dimensions of a nucleus and the weak nuclear force is weak. The most significant force in our everyday world is gravity. It literally keeps us on or near the ground. The other most common force is the electromagnetic force. It provides "the glue" to hold solid together and make them rigid. It is of great importance in electric circuits.

12   ••   Give an example of an object that has three forces acting on it, and (*a*) accelerates, (*b*) moves at constant (non-zero) velocity, and (*c*) remains at rest.

Determine the Concept

(*a*) Any object for which the vector sum of the three forces doesn't add to zero. For example, a sled on a frictionless surface pulled horizontally. Normal force plus weight plus the pull don't add to zero, so it accelerates.

(*b*) Pulling a fish vertically upward at constant velocity while it is still in the water. The forces acting on the fish are the pull, the weight of the fish, and water drag forces. These forces add up to zero.

(*c*) The three forces need to add vectorially to zero. An example is a picture hung by two wires.

13 •• **[SSM]** Suppose a block of mass m_1 rests on a block of mass m_2 and the combination rests on a table as shown in Figure 4-33. Tell the name of the force and its category (contact versus action-at-a-distance) for each of the following forces; (*a*) force exerted by m_1 on m_2, (*b*) force exerted by m_2 on m_1, (*c*) force exerted by m_2 on the table, (*d*) force exerted by the table on m_2, (*e*) force exerted by the earth on m_2. Which, if any, of these forces constitute a Newton's third law pair of forces?

Determine the Concept

(*a*) The force exerted by m_1 on m_2.   Normal force, contact type.

(*b*) The force exerted by m_2 on m_1.   Normal force, contact type.

(*c*) The force exerted by m_2 on the table.   Normal force, contact type.

(*d*) The force exerted by the table on m_2.   Normal force, contact type.

(*e*) The force exerted by the earth on m_2.   Gravitational force, action-at-a-distance type.

The Newton's 3rd law force pairs are the two normal forces between the two blocks and the normal force between the table and the bottom block. The gravitational force has a 3rd law force pair, that acts on Earth and, so, is not in the question set.

14 •• You yank a fish you have just caught on your line upward from rest into your boat. Draw a free-body diagram of the fish after it has left the water and as it gains speed as it rises. In addition, tell the type (tension, spring, gravity, normal, friction, etc.) and category (contact versus action-at-a-distance) for each force on your diagram. Which, if any, pairs of the forces on your diagram constitute a Newton's third law pair? Can you tell the relative magnitudes of the forces from the information given? Explain.

Determine the Concept A free-body diagram showing the forces acting on the fish is shown to the right. A table summarizing the type and category of the forces is shown below.

fish →

$F_{\text{string on fish}}$
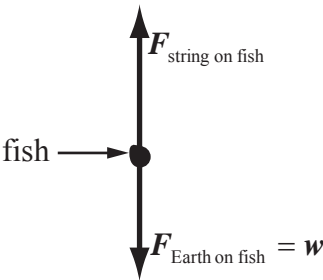
$F_{\text{Earth on fish}} = w$

Force	Type	Category
$\vec{F}_{\text{string on fish}}$	Tension	Contact
$\vec{F}_{\text{Earth on fish}}$	Gravity	Action-at-a-distance

Because the fish accelerates upward, the tension force must be greater in magnitude than the gravitational force acting on the fish.

15 •   If you gently set a fancy plate on the table, it will not break. However if you drop it from a height, it might very well break. Discuss the forces that act on the plate (as it contacts the table) in both these situations. What is different about the second situation that causes the plate to break?

Determine the Concept When the plate is sitting on the floor, the normal force F_n acting upward on it is exerted by the floor and is the same size as the gravitational force F_g on the plate. Hence, the plate does not accelerate. However, to slow the plate down as it hits the floor requires that $F_n > F_g$ (or $F_n \gg F_g$ if the floor is hard and the plate slows quickly). A large normal force exerted on delicate china can easily break it.

16 ••   For each of the following forces, give what produces it, what object it acts on, its direction, and the reaction force. (*a*) The force you exert on your briefcase to hold it at rest vertically. (*b*) The normal force on the soles of your feet as you stand barefooted on a horizontal wood floor. (*c*) The gravitational force on you as you stand on a horizontal floor. (*d*) The horizontal force exerted on a baseball by a bat as the ball is hit straight up the middle towards center field for a single.

Determine the Concept

(*a*) The force you exert on your briefcase to hold it at rest vertically.

You produce this force. It acts on the briefcase. It acts upward. The reaction force is the force the briefcase exerts on your hand.

(b) The normal force on the soles of your feet as you stand barefooted on a horizontal wood floor.	The floor produces this force. It acts on your feet. It acts upward. The reaction force is the force your feet exert on the floor.
(c) The gravitational force on you as you stand on a horizontal floor.	The earth produces this force. It acts on you. It acts downward. The reaction force is the gravitational force you exert on the earth.
(d) The horizontal force exerted on a baseball by a bat as the ball is hit straight up the middle towards center field for a single.	The bat produces this force. It acts on the ball. It acts horizontally. The reaction force is the force the ball exerts on the bat.

17 •• For each case, identify the force (including its direction) that causes the acceleration. (a) A sprinter at the very start of the race. (b) A hockey puck skidding freely but slowly coming to rest on the ice. (c) A long fly ball at the top of its arc. (d) A bungee jumper at the very bottom of her descent.

Determine the Concept

(a) A sprinter at the very start of the race:	The block's force on the sprinter accelerates the sprinter forward when the sprinter pushes backward on the blocks.
(b) A hockey puck skidding freely but slowly coming to rest on the ice:	The frictional force by the ice on the puck causes the puck to slow down. This force is directed opposite to the velocity of the puck.
(c) A long fly ball at the top of its arc:	The downward gravitation force by Earth on the ball.
(d) A bungee jumper at the very bottom of her descent:	The force exerted by the stretched bungee cord accelerates the jumper upward. Its direction is upward.

18 • True or false:

(a) If two external forces that are both equal in magnitude and opposite in direction act on the same object, the two forces can never be a Newton's third law pair.
(b) The two forces of a Newton's third law pair are equal only if the objects involved are not accelerating.

(*a*) True. By definition, third law pairs cannot act on the same object.

(*b*) False. Action and reaction forces are equal independently of any motion of the involved objects.

19   ••   An 80-kg man on ice skates is pushing his 40-kg son, also on skates, with a force of 100 N. Together, they move across the ice steadily gaining speed. (*a*) The force exerted by the boy on his father is (1) 200 N, (2) 100 N, (3) 50 N, or (4) 40 N. (*b*) How do the magnitudes of the two accelerations compare? (*c*) How do the directions of the two accelerations compare?

Determine the Concept

(*a*) (2) These forces are a Newton 3rd law force pair, and so the force exerted by the boy on his father is 100 N.

(*b*) Because the father and son move together, their accelerations will be the same.

(*c*) The directions of their acceleration are the same.

20   ••   A girl holds a stone in her hand and can move it up or down or keep it still. True or false: (*a*) The force exerted by her hand on the rock is always the same magnitude as the weight of the stone. (*b*) The force exerted by her hand on the rock is the reaction force to the pull of gravity on the stone. (*c*) The force exerted by her hand is always the same size the force her hand feels from the stone but in the opposite direction. (*d*) If the girl moves her hand down at a constant speed, then her upward force on the stone is less than the weight of the stone. (*e*) If the girl moves her hand downward but slows the stone to rest, the force of the stone on the girl's hand is the same magnitude as the pull of gravity on the stone.

(*a*) False. If the rock is accelerating, the force the girl exerts must be greater than the weight of the stone.

(*b*) False. The reaction force to the pull of gravity is the force the rock exerts on the rock.

(*c*) True. These forces constitute a Newton's third law pair.

(*d*) False. If she moves the stone downward at a constant speed, the net force acting on the stone must be zero.

(*e*) False. If she is slowing the stone, it is experiencing acceleration and the net force acting on it can not be zero.

21 •• **[SSM]** A 2.5-kg object hangs at rest from a string attached to the ceiling. (*a*) Draw a free body diagram of the object, indicate the reaction force to each force drawn, and tell what object the reaction force acts on. (*b*) Draw a free body diagram of the string, indicate the reaction force to each force drawn, and tell what object each reaction force acts on. Do not neglect the mass of the string.

Determine the Concept The force diagrams will need to include forces exerted by the ceiling, on the string, on the object, and forces exerted by the earth.

(a)

T_{string}

2.5 kg

$F_{\text{g, object}}$

(b)

$F_{\text{by ceiling}}$
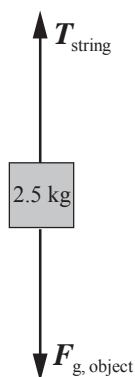
$F_{\text{g, string}}$

$F_{\text{g, object}}$

(a) The forces acting on the 2.5-kg object are the gravitational force $\vec{F}_{\text{g, object}}$ and the tension $\vec{T}_{\text{string}}$ in the string. The reaction to $\vec{T}_{\text{string}}$ is the force the object exerts downward on the string. The reaction to $\vec{F}_{\text{g, object}}$ is the force the object exerts upward on the earth.
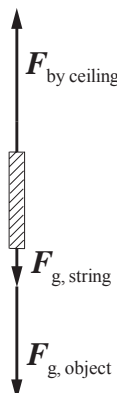
(b) The forces acting on the string are its weight $\vec{F}_{\text{g, string}}$, the weight of the object $\vec{F}_{\text{g, object}}$, and $\vec{F}_{\text{by ceiling}}$, the force exerted by the ceiling. The reaction to $\vec{F}_{\text{g, string}}$ is the force the string exerts upward on the earth. The reaction to $\vec{F}_{\text{by ceiling}}$ is a downward force the string exerts on the ceiling. The reaction to $\vec{F}_{\text{g, object}}$ is the force the object exerts upward on the earth.

22 •• (*a*) Which of the free-body diagrams in Figure 4-34. represents a block sliding down a frictionless inclined surface? (*b*) For the correct figure, label the forces and tell which are contact forces and which are action-at-a-distance forces. (*c*) For each force in the correct figure, identify the reaction force, the object it acts on and its direction.

Determine the Concept Identify the objects in the block's environment that are exerting forces on the block and then decide in what directions those forces must be acting if the block is sliding *down* the inclined plane.

(a) Free-body diagram (***c***) is correct.

(*b*) Because the incline is frictionless, the force $\vec{F}_n$ the incline exerts on the block must be normal to the surface and is a contact force. The second object capable of exerting a force on the block is the earth and its force; the gravitational force $\vec{F}_g$ acting on the block acts directly downward and is an action-at-a-distance force. The magnitude of the normal force is less than that of the weight because it supports only a portion of the weight.
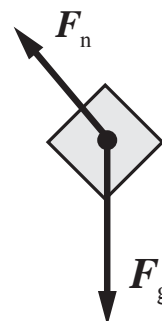
(*c*) The reaction to the normal force is the force the block exerts perpendicularly on the surface of the incline. The reaction to the gravitational force is the upward force the block exerts on the Earth.

23   ••   A box is held against a compressed, horizontal spring that is attached to a wall. The horizontal floor beneath the box is frictionless. Draw the free body diagram of the box in the following cases. (*a*) The box is held at rest against the compressed spring. (*b*) The force holding the box against the spring no longer exists, but the box is still in contact with the spring. (*c*) When the box no longer has contact with the spring.

Determine the Concept In the following free-body diagrams we'll assume that the box is initially pushed to the left to compress the spring.

(*a*) Note that, in the free-body diagram to the right, that
$$\left|\vec{F}_n\right| = \left|\vec{F}_g\right| \text{ and } \left|\vec{F}_{\text{by spring}}\right| = \left|\vec{F}_{\text{by hand}}\right|.$$
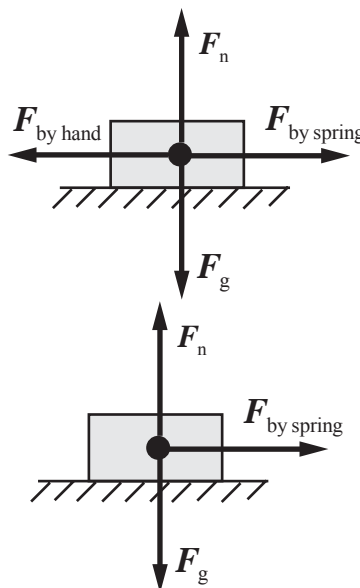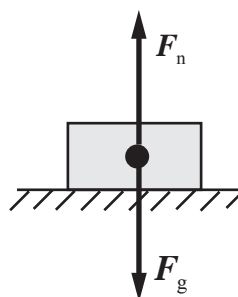
(*b*) Note that while $\vec{F}_n = \left|\vec{F}_g\right|$,

$\vec{F}_{\text{by spring}}$ is now the net force acting on the box. As the spring decompresses, $\vec{F}_{\text{by spring}}$ will become smaller.

(*c*) When the box separates from the spring, the force exerted by the spring on the box goes to zero. Note that it is still true that

$$\vec{F}_n = \left| \vec{F}_g \right|.$$

24 ••   Imagine yourself seated on a wheeled desk chair at your desk. Consider any friction forces between the chair and the floor to be negligible. However, the friction forces between the desk and the floor are not negligible. When sitting at rest, you decide you need another cup of coffee. You push horizontally against the desk, and the chair rolls backward away from the desk. (*a*) Draw your free-body diagram of yourself during the push and clearly indicate which force was responsible for your acceleration. (*b*) What is the reaction force to the force that caused your acceleration? (*c*) Draw the free-body diagram of the desk and explain why it did not accelerate. Does this violate Newton's third law? Explain.

Determine the Concept In the following free-body diagrams we'll assume that the desk is to the left and that your motion is to the right.

(*a*) Newton's third law accounts for this as follows. When you push with your hands against the desk, the desk pushes back on your hands with a force of the same magnitude but opposite direction.   This   force accelerates you backward.

(*b*) The reaction force to the force that caused your acceleration is the force that you exerted on the desk.

(*c*) When you pushed on the desk, you did not apply sufficient force to overcome   the   force   of   friction between the desk and the floor.   In terms of forces on the desk, you applied a force, and the floor applied a friction force that, when added as vectors,   cancelled.   The   desk, therefore,   did   not   accelerate   and Newton's third law is not violated.
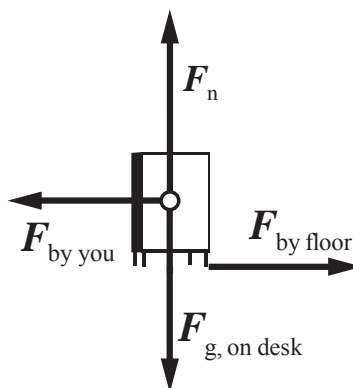
25 ••• The same (net) horizontal force F is applied for a fixed time interval Δt to each of two objects, having masses m_1 and m_2, that sit on a flat, frictionless surface. (Let $m_1 > m_2$.) (*a*) Assuming the two objects are initially at rest, what is the ratio of their accelerations during the time interval in terms of F, m_1 and m_2? (*b*) What is the ratio of their speeds v_1 and v_2 at the end of the time interval? (*c*) How far apart are the two objects (and which is ahead) the end of the time interval?

Picture the Problem We can apply Newton's 2nd Law to find the ratios of the accelerations and speeds of the two objects and constant-acceleration equations to express the separation of the objects as a function of the elapsed time.

(*a*) Use Newton's 2nd Law to express the accelerations of the two objects:

$$a_1 = \frac{F}{m_1} \text{ and } a_2 = \frac{F}{m_2}$$

Dividing the first of these equations by the second and simplifying yields:

$$\frac{a_1}{a_2} = \frac{\dfrac{F}{m_1}}{\dfrac{F}{m_2}} = \boxed{\frac{m_2}{m_1}}$$

(*b*) Because both objects started from rest, their speeds after time Δt has elapsed are:

$$v_1 = a_1 \Delta t \text{ and } v_2 = a_2 \Delta t$$

Dividing the first of these equations by the second and simplifying yields:

$$\frac{v_1}{v_2} = \frac{a_1 \Delta t}{a_2 \Delta t} = \frac{a_1}{a_2} = \boxed{\frac{m_2}{m_1}}$$

(*c*) The separation of the two objects at the end of the time interval is given by:

$$\Delta x = \Delta x_2 - \Delta x_1 \qquad (1)$$

Using a constant acceleration equation, express the distances traveled by the two objects when time Δt has elapsed:

$$\Delta x_1 = \tfrac{1}{2} a_1 (\Delta t)^2$$
and
$$\Delta x_2 = \tfrac{1}{2} a_2 (\Delta t)^2$$

Substitute for Δx_1 and Δx_2 in equation (1) and simplify to obtain:

$$\Delta x = \tfrac{1}{2} a_2 (\Delta t)^2 - \tfrac{1}{2} a_1 (\Delta t)^2$$

$$= \boxed{\tfrac{1}{2} F \left(\frac{1}{m_2} - \frac{1}{m_1} \right) (\Delta t)^2}$$

and, because $m_1 > m_2$, the object whose mass is m_2 is ahead.

Estimation and Approximation

26 ••   Most cars have four springs attaching the body to the frame, one at each wheel position. Devise an experimental method of estimating the force constant of one of the springs using your known weight and the weights of several of your friends. Use the method to estimate the force constant of your car's springs.

Picture the Problem Suppose you put in 800 lbs (or about 3600 N) of weight and the car sags several inches (or 6.00 cm). Then each spring supports about 900 N and we can use the definition of the force constant k to determine its value.

The force constant is the ratio of the compressing (or stretching) force to the compression (or stretch):

$$k = \frac{F_x}{\Delta x}$$

Substitute numerical values and evaluate k:

$$k = \frac{\frac{1}{4}(3600 \text{ N})}{6.00 \text{ cm}} \approx \boxed{150 \text{ N/cm}}$$

27 ••   **[SSM]**   Estimate the force exerted on the goalie's glove by the puck when he catches a hard slap shot for a save.

Picture the Problem Suppose the goalie's glove slows the puck from 60 m/s to zero as it recoils a distance of 10 cm. Further, assume that the puck's mass is 200 g. Because the force the puck exerts on the goalie's glove and the force the goalie's glove exerts on the puck are action-and-reaction forces, they are equal in magnitude. Hence, if we use a constant-acceleration equation to find the puck's acceleration and Newton's 2nd law to find the force the glove exerts on the puck, we'll have the magnitude of the force exerted on the goalie's glove.

Apply Newton's 2nd law to the puck as it is slowed by the goalie's glove to express the magnitude of the force the glove exerts on the puck:

$$F_{\text{glove on puck}} = \left| m_{\text{puck}} a_{\text{puck}} \right| \qquad (1)$$

Use a constant-acceleration equation to relate the initial and final speeds of the puck to its acceleration and stopping distance:

$$v^2 = v_0^2 + 2a_{\text{puck}} (\Delta x)_{\text{puck}}$$

Solving for a_{puck} yields:

$$a_{\text{puck}} = \frac{v^2 - v_0^2}{2(\Delta x)_{\text{puck}}}$$

Substitute for a_{puck} in equation (1) to obtain:

$$F_{glove\,on\,puck} = \left| \frac{m_{puck}\left(v^2 - v_0^2\right)}{2(\Delta x)_{puck}} \right|$$

Substitute numerical values and evaluate $F_{glove\,on\,puck}$:

$$F_{glove\,on\,puck} = \left| \frac{(0.200\text{ kg})\left(0 - (60\text{ m/s})^2\right)}{2(0.10\text{ m})} \right|$$

$$\approx \boxed{3.6\text{ kN}}$$

Remarks: The force on the puck is about 1800 times its weight.

28 •••   A baseball player slides into second base during a steal attempt. Assuming reasonable values for the length of the slide, the speed of the player at the beginning of the slide, and the speed of the player at the end of the slide, estimate the average force of friction acting on the player.

Picture the Problem Let's assume that the player's mass is 100 kg, that he gets going fairly quickly down the base path, and that his speed is 8.0 m/s when he begins his slide. Further, let's assume that he approaches the base at the end of the slide at 3.0 m/s. From these speeds, and the length of the slide, we can use Newton's 2nd law and a constant-acceleration equation to find the force due to friction (which causes the slowing down).

Apply Newton's 2nd law to the sliding runner:

$$\sum F_x = F_{friction} = ma \qquad (1)$$

Using a constant-acceleration equation, relate the runner's initial and final speeds to his acceleration and the length of his slide:

$$v_f^2 = v_i^2 + 2a\Delta x \Rightarrow a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

Substituting for a in equation (1) yields:

$$F_{friction} = m\frac{v_f^2 - v_i^2}{2\Delta x}$$

Assuming the player slides 2 m, substitute numerical values and evaluate $F_{friction}$:

$$F_{friction} = (100\text{ kg})\frac{(3.0\text{ m/s})^2 - (8.0\text{ m/s})^2}{2(2\text{ m})}$$

$$\approx \boxed{-2\text{ kN}}$$

where the minus sign indicates that the force of friction opposes the runner's motion.

29 ••   A race car skidding out of control manages to slow down to 90 km/h before crashing head on into a brick wall. Fortunately, the driver is wearing a seat belt. Using reasonable values for the mass of the driver and the stopping distance,

estimate the average force exerted on the driver by the seat belt, including its direction. Neglect any effects of frictional forces on the driver by the seat.

Picture the Problem Assume a crush distance of about 1 m at 90 km/h (25 m/s) and a driver's mass of 55 kg. We can use a constant-acceleration equation (the definition of average acceleration) to find the acceleration of the driver and Newton's 2nd law to find the force exerted on the driver by the seat belt.

Apply Newton's 2nd law to the driver as she is brought to rest by her seat belt:

$$F_{\text{seat belt on driverl}} = m_{\text{driver}}a_{\text{driver}} \qquad (1)$$

Use a constant-acceleration equation to relate the initial and final speeds of the driver to her acceleration and stopping distance:

$$v^2 = v_0^2 + 2a_{\text{driver}}(\Delta x)_{\text{driver}}$$

Solving for a_{driver} yields:

$$a_{\text{driver}} = \frac{v^2 - v_0^2}{2(\Delta x)_{\text{driver}}}$$

Substitute for a_{driver} in equation (1) to obtain:

$$F_{\text{seat belt on driverl}} = m_{\text{driver}}\frac{v^2 - v_0^2}{2(\Delta x)_{\text{driver}}}$$

Substitute numerical values and evaluate $F_{\text{seat belt on driver}}$:

$$F_{\text{seat belt on driver}} = (55\text{ kg})\frac{0 - (25\text{ m/s})^2}{2(1.0\text{ m})}$$

$$\approx \boxed{-17\text{ kN}}$$

where the minus sign indicates that the force exerted by the seat belt is in the opposite direction from the driver's motion.

Remarks: The average force on the ball is about 32 times her weight.

Newton's First and Second Laws: Mass, Inertia, and Force

30 •   A particle of mass m is traveling at an initial speed $v_0 = 25.0$ m/s. Suddenly a constant force of 15.0 N acts on it, bringing it to a stop in a distance of 62.5 m. (*a*) What is the direction of the force? (*b*) Determine the time it takes for the particle to come to a stop. (*c*) What is its mass?

Picture the Problem The acceleration of the particle, its stopping time, and its mass can be found using constant-acceleration equations and Newton's 2nd law. A convenient coordinate system is shown in the following diagram.
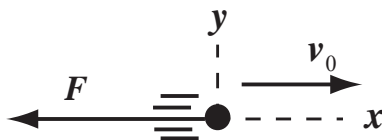
(*a*) Because the constant force slows the particle, we can conclude that, as shown in the diagram, its direction is opposite the direction of the particle's motion.

(*b*) Use a constant-acceleration equation to relate the initial and final velocities of the particle to its acceleration and stopping time:	$v_x = v_{0x} + a_x \Delta t$ or, because $v_x = 0$, $0 = v_{0x} + a_x \Delta t \Rightarrow \Delta t = -\dfrac{v_{0x}}{a_x}$ (1)
Use a constant-acceleration equation to relate the initial and final velocities of the particle to its acceleration and stopping distance:	$v_x^2 = v_{0x}^2 + 2a_x \Delta x$ or, because $v_x = 0$, $0 = v_{0x}^2 + 2a_x \Delta x \Rightarrow a_x = \dfrac{-v_{0x}^2}{2\Delta x}$
Substituting for a_x in equation (1) yields:	$\Delta t = \dfrac{2\Delta x}{v_{0x}}$
Substitute numerical values and evaluate Δt:	$\Delta t = \dfrac{2(62.5\,\text{m})}{25.0\,\text{m/s}} = \boxed{5.00\,\text{s}}$
(*c*) Apply Newton's 2nd law to the particle to obtain:	$\sum F_x = -F_{\text{net}} = ma_x$
Solving for *m* yields:	$m = \dfrac{-F_{\text{net}}}{a_x}$ (2)
Because the force is constant, you can use a constant-acceleration equation to relate the particle's initial and final speeds, acceleration, and stopping distance:	$v_x^2 = v_{0x}^2 + 2a_x \Delta x$ or, because $v_x = 0$, $0 = v_{0x}^2 + 2a_x \Delta x \Rightarrow a_x = \dfrac{-v_{0x}^2}{2\Delta x}$
Substitute for a_x in equation (2) to obtain:	$m = \dfrac{2\Delta x F_{\text{net}}}{v_{0x}^2}$
Substitute numerical values and evaluate *m*:	$m = \dfrac{2(62.5\,\text{m})(15.0\,\text{N})}{(25.0\,\text{m/s})^2} = \boxed{3.00\,\text{kg}}$

31   •   An object has an acceleration of 3.0 m/s^2 when the only force acting on it is F_0. (*a*) What is its acceleration when this force is doubled? (*b*) A second object has an acceleration of 9.0 m/s^2 under the influence of the force F_0. What is the ratio of the mass of the second object to that of the first object? (*c*) If the two objects are glued together to form a composite object, what acceleration will the force F_0 acting on the composite object produce?

Picture the Problem The acceleration of the object is related to its mass and the *net* force acting on it by $F_{net} = F_0 = ma$.

(*a*) Use Newton's 2nd law of motion to relate the acceleration of the object to the net force acting on it:

$$a = \frac{F_{net}}{m}$$

When $F_{net} = 2F_0$:

$$a = \frac{2F_0}{m} = 2a_0$$

Substitute numerical values and evaluate *a*:

$$a = 2(3.0\,\text{m/s}^2) = \boxed{6.0\,\text{m/s}^2}$$

(*b*) Let the subscripts 1 and 2 distinguish the two objects. The ratio of the two masses is found from Newton's 2nd law:

$$\frac{m_2}{m_1} = \frac{F_0/a_2}{F_0/a_1} = \frac{a_1}{a_2} = \frac{3.0\,\text{m/s}^2}{9.0\,\text{m/s}^2} = \boxed{\tfrac{1}{3}}$$

(*c*) The acceleration of the composite object is the net force divided by the total mass $m = m_1 + m_2$ of the composite object:

$$a = \frac{F_{net}}{m} = \frac{F_0}{m_1 + m_2} = \frac{F_0/m_1}{1 + m_2/m_1}$$

$$= \frac{a_1}{1 + \frac{1}{3}} = \tfrac{3}{4}a_1$$

Substitute for a_1 and evaluate *a*:

$$a = \tfrac{3}{4}(3.0\,\text{m/s}^2) = \boxed{2.3\,\text{m/s}^2}$$

32   •   A tugboat tows a ship with a constant force F_1. The increase in the ship's speed during a 10-s interval is 4.0 km/h. When a second tugboat applies an additional constant force F_2 in the same direction, the speed increases by 16 km/h during a 10-s interval. How do the magnitudes of the two tugboat forces compare? (Neglect the effects of water resistance and air resistance.)

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it by $F_{net} = ma$. Let *m* be the mass of the ship, a_1 be the acceleration of the ship when the net force acting on it is F_1, and a_2 be its acceleration when the net force is $F_1 + F_2$.

Using Newton's 2nd law, express the net force acting on the ship when its acceleration is a_1:	$F_1 = ma_1$

Express the net force acting on the ship when its acceleration is a_2:	$F_1 + F_2 = ma_2$

Divide the second of these equations by the first and solve for the ratio F_2/F_1:	$\dfrac{F_1 + F_2}{F_1} = \dfrac{ma_2}{ma_1} \Rightarrow \dfrac{F_2}{F_1} = \dfrac{a_2}{a_1} - 1$

Substitute for the accelerations to determine the ratio of the accelerating forces and solve for F_2 to obtain:	$\dfrac{F_2}{F_1} = \dfrac{\dfrac{16\,\text{km/h}}{10\,\text{s}}}{\dfrac{4.0\,\text{km/h}}{10\,\text{s}}} - 1 = 3 \Rightarrow F_2 = \boxed{3F_1}$

33 • A single constant force of 12 N acts on a particle of mass m. The particle starts from rest and travels in a straight line a distance of 18 m in 6.0 s. Find m.

Picture the Problem The mass of the particle is related to its acceleration and the *net* force acting on it by Newton's 2nd law of motion. Because the force is constant, we can use constant-acceleration formulas to calculate the acceleration. Choose a coordinate system in which the positive x direction is the direction of motion of the particle.

The mass is related to the net force and the acceleration by Newton's 2nd law:	$m = \dfrac{\Sigma \vec{F}}{\vec{a}} = \dfrac{F_x}{a_x}$	(1)

Because the force is constant, the acceleration is constant. Use a constant-acceleration equation to relate the displacement of the particle to it's acceleration:	$\Delta x = v_{0x}t + \frac{1}{2}a_x(\Delta t)^2$ or, because $v_{0x} = 0$, $\Delta x = \frac{1}{2}a_x(\Delta t)^2 \Rightarrow a_x = \dfrac{2\Delta x}{(\Delta t)^2}$

Substitute for a_x in equation (1) to obtain:	$m = \dfrac{F_x(\Delta t)^2}{2\Delta x}$

Substitute numerical values and evaluate m:	$m = \dfrac{(12\,\text{N})(6.0\,\text{s})^2}{2(18\,\text{m})} = \boxed{12\,\text{kg}}$

34 • A net force of $(6.0\,\text{N})\hat{i} - (3.0\,\text{N})\hat{j}$ acts on an object of mass 1.5 kg. Find the acceleration $\vec{a}$.

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{a} = \vec{F}_{net}/m$.

Apply Newton's 2nd law to the object to obtain:

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Substitute numerical values and evaluate $\vec{a}$:

$$\vec{a} = \frac{(6.0\,\text{N})\hat{i} - (3.0\,\text{N})\hat{j}}{1.5\,\text{kg}}$$

$$= \boxed{(4.0\,\text{m/s}^2)\hat{i} - (2.0\,\text{m/s}^2)\hat{j}}$$

35 •• **[SSM]**   A bullet of mass 1.80×10^{-3} kg moving at 500 m/s impacts a tree stump and penetrates 6.00 cm into the wood before coming to rest. (*a*) Assuming that the acceleration of the bullet is constant, find the force (including direction) exerted by the wood on the bullet. (*b*) If the same force acted on the bullet and it had the same speed but half the mass, how far would it penetrate into the wood?

Picture the Problem Choose a coordinate system in which the +*x* direction is in the direction of the motion of the bullet and use Newton's 2nd law and a constant-acceleration equation to express the relationship between $F_{stopping}$ and the mass of the bullet and its displacement as it is brought to rest in the block of wood.

(*a*) Apply Newton's 2nd law to the bullet to obtain:

$$\sum F_x = F_{stopping} = ma_x \qquad (1)$$

Use a constant-acceleration equation to relate the bullet's initial and final speeds, acceleration, and stopping distance:

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$
or, because $v_{fx} = 0$,
$$0 = v_{ix}^2 + 2a_x\Delta x \Rightarrow a_x = \frac{-v_{ix}^2}{2\Delta x}$$

Substitute for a_x in equation (1) to obtain:

$$F_{stopping} = -m\frac{v_{ix}^2}{2\Delta x} \qquad (2)$$

Substitute numerical values and evaluate $F_{stopping}$:

$$F_{stopping} = -(1.80\times10^{-3}\ \text{kg})\frac{(500\ \text{m/s})^2}{2(6.00\ \text{cm})}$$

$$= \boxed{-3.8\ \text{kN}}$$

where the minus sign indicates that $F_{stopping}$ opposes the motion of the bullet.

(*b*) Solving equation (2) for Δx yields:

$$\Delta x = -m\frac{v_{ix}^2}{2F_{stopping}} \qquad (3)$$

For $m = m'$ and $\Delta x = \Delta x'$:

$$\Delta x' = -m' \frac{v_{ix}^2}{2F_{stopping}}$$

Evaluate this expression for $m' = \frac{1}{2}m$ to obtain:

$$\Delta x' = -m \frac{v_{ix}^2}{4F_{stopping}} \qquad (4)$$

Dividing equation (4) by equation (3) yields:

$$\frac{\Delta x'}{\Delta x} = \frac{-m \dfrac{v_{ix}^2}{4F_{stopping}}}{-m \dfrac{v_{ix}^2}{2F_{stopping}}} = \frac{1}{2}$$

or

$$\Delta x' = \frac{1}{2}\Delta x$$

Substitute numerical values and evaluate $\Delta x'$:

$$\Delta x' = \frac{1}{2}(6.00\,\text{cm}) = \boxed{3.00\,\text{cm}}$$

36 •• A cart on a horizontal, linear track has a fan attached to it. The cart is positioned at one end of the track, and the fan is turned on. Starting from rest, the cart takes 4.55 s to travel a distance of 1.50 m. The mass of the cart plus fan is 355 g. Assume that the cart travels with constant acceleration. (*a*) What is the net force exerted on the cart-fan combination? (*b*) Mass is added to the cart until the total mass of the cart-fan combination is 722 g, and the experiment is repeated. How long does it take for the cart to travel 1.50 m now? Ignore the effects due to friction.

Picture the Problem Choose the coordinate system shown in the diagram to the right. The force $\vec{F}$ acting on the cart-fan combination is the consequence of the fan blowing air to the left. We can use Newton's 2nd law and a constant-acceleration equation to express the relationship between $\vec{F}$ and the mass of the cart-fan combination and the distance it travels in a given interval of time.
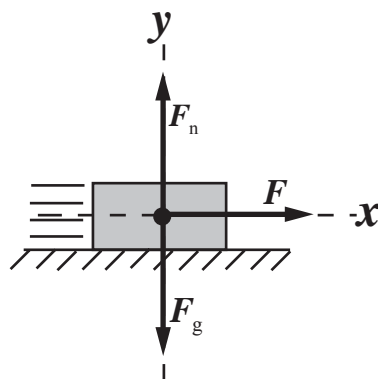
(*a*) Apply Newton's 2nd law to the cart-fan combination to obtain:

$$\sum F_x = F = ma_x \qquad (1)$$

Using a constant-acceleration equation, relate the distance the cart-fan combination travels to its initial speed, acceleration, and the elapsed time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 \Rightarrow a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for a_x in equation (1) to obtain:

$$F = m\frac{2\Delta x}{(\Delta t)^2} \qquad\qquad (2)$$

Substitute numerical values and evaluate F:

$$F = (0.355\,\text{kg})\frac{2(1.50\,\text{m})}{(4.55\,\text{s})^2}$$

$$= 0.05144\,\text{N} = \boxed{0.0514\,\text{N}}$$

(b) Solve equation (2) for Δt to obtain:

$$\Delta t = \sqrt{\frac{2m\Delta x}{F}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(0.722\,\text{kg})(1.50\,\text{m})}{0.05144\,\text{N}}} = \boxed{6.49\,\text{s}}$$

37 ••   A horizontal force F_0 causes an acceleration of 3.0 m/s² when it acts on an object of mass m sliding on a frictionless surface. Find the acceleration of the same object in the circumstances shown in Figure 4-35a and 4-35b.

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{F}_{net} = m\vec{a}$. Let m be the mass of the object and choose a coordinate system in which the direction of $2F_0$ in (b) is the +x and the direction of the left-most F_0 in (a) is the +y direction. Because both force and acceleration are vector quantities, find the resultant force in each case and then find the resultant acceleration.

(a) Apply $\sum\vec{F} = m\vec{a}$ to the object to obtain:

$$\vec{a} = \frac{\vec{F}_{net}}{m} \qquad\qquad (1)$$

Express the net force acting on the object:

$$\vec{F}_{net} = F_x\hat{i} + F_y\hat{j} = F_0\hat{i} + F_0\hat{j}$$

The magnitude and direction of this net force are given by:

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{2}F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{F_0}{F_0}\right) = 45°$$

Substitute for F_{net} in equation (1) and simplify to obtain:

$$a = \frac{\sqrt{2}F_0}{m} = \sqrt{2}a_0$$

Substitute the numerical value of a_0 and evaluate a:

$$a = \sqrt{2}\left(3.0\,\text{m/s}^2\right)$$
$$= \boxed{4.2\,\text{m/s}^2 \, @\, 45° \text{ from each force.}}$$

(b) Proceeding as in (a), express the net force acting on the object:

$$\vec{F}_{net} = F_x\hat{i} + F_y\hat{j} = \left(-F_0\sin 45°\right)\hat{i} + \left(2F_0 + F_0\cos 45°\right)\hat{j}$$

The magnitude and direction of this net force are given by:

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(-F_0\sin 45°\right)^2 + \left(2F_0 + F_0\cos 45°\right)^2} = 2.80F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{2F_0 + F_0\cos 45°}{-F_0\sin 45°}\right) = -75.4°$$
$$= 14.6° \text{ from } 2\vec{F}_0$$

Substitute for F_{net} in equation (1) and simplify to obtain:

$$a = 2.80\frac{F_0}{m} = 2.80\frac{ma_0}{m} = 2.80a_0$$

Substitute the numerical value of a_0 and evaluate a:

$$a = 2.80\left(3.0\,\text{m/s}^2\right)$$
$$= \boxed{8.4\,\text{m/s}^2 \, @\, 15° \text{ from } 2\vec{F}_0}$$

38 ••   Al and Bert stand in the middle of a large frozen lake (frictionless surface). Al pushes on Bert with a force of 20 N for 1.5 s. Bert's mass is 100 kg. Assume that both are at rest before Al pushes Bert. (a) What is the speed that Bert reaches as he is pushed away from Al? (b) What speed does Al reach if his mass is 80 kg?

Picture the Problem The speed of either Al or Bert can be obtained from their accelerations; in turn, they can be obtained from Newtons 2nd law applied to each person. The free-body diagrams to the right show the forces acting on Al and Bert. The forces that Al and Bert exert on each other are action-and-reaction forces.
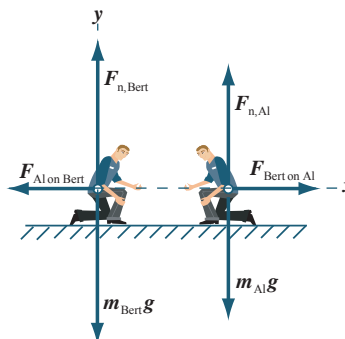
(a) Apply $\sum F_x = ma_x$ to Bert:

$$-F_{\text{Al on Bert}} = m_{\text{Bert}}a_{\text{Bert}} \Rightarrow a_{\text{Bert}} = \frac{-F_{\text{Al on Bert}}}{m_{\text{Bert}}}$$

Substitute numerical values and evaluate a_{Bert}:

$$a_{\text{Bert}} = \frac{-20\,\text{N}}{100\,\text{kg}} = -0.200\,\text{m/s}^2$$

Using a constant-acceleration equation, relate Bert's speed to his initial speed, speed after 1.5 s, and acceleration:

$$v_x = v_{0x} + a_{\text{Bert,x}}\Delta t$$

Substitute numerical values and evaluate Bert's speed at the end of 1.5 s:

$$v_x = 0 + \left(-0.200\,\text{m/s}^2\right)(1.5\,\text{s})$$
$$= \boxed{-0.30\,\text{m/s}}$$

(b) From Newton's 3$^{\text{rd}}$ law, an equal but oppositely directed force acts on Al while he pushes Bert. Because the ice is frictionless, Al speeds off in the opposite direction. Apply $\sum F_x = ma_x$ to Al:

$$\sum F_{x,\text{Al}} = F_{\text{Bert on Al},x} = m_{\text{Al}}a_{\text{Al},x}$$

Solving for Al's acceleration yields:

$$a_{\text{Al,x}} = \frac{F_{\text{Bert on Al},x}}{m_{\text{Al}}}$$

Substitute numerical values and evaluate $a_{\text{Al,x}}$:

$$a_{\text{Al,x}} = \frac{20\,\text{N}}{80\,\text{kg}} = 0.250\,\text{m/s}^2$$

Using a constant-acceleration equation, relate Al's speed to his initial speed, speed after 1.5 s, and acceleration:

$$v_x = v_{0x} + a_{\text{Al,x}}\Delta t$$

Substitute numerical values and evaluate Al's speed at the end of 1.5 s:

$$v_x(1.5\,\text{s}) = 0 + \left(0.250\,\text{m/s}^2\right)(1.5\,\text{s})$$
$$= \boxed{0.38\,\text{m/s}}$$

39 ••   If you push a block whose mass is m_1 across a frictionless floor with a force of a given magnitude, the block has an acceleration of 12 m/s^2. If you push on a different block whose mass is m_2 with a force of magnitude F_0, its acceleration is 3.0 m/s^2. (Both forces are applied horizontally.) (a) What acceleration will a horizontal force of magnitude F_0 give to a block with mass $m_2 - m_1$? (b) What acceleration will a horizontal force of magnitude F_0 give to a block with mass $m_2 + m_1$?

Picture the Problem The free-body diagrams show the forces acting on the two blocks. We can apply Newton's second law to the forces acting on the blocks and eliminate F to obtain a relationship between the masses. Additional applications of Newton's 2nd law to the sum and difference of the masses will lead us to values for the accelerations of these combinations of mass.
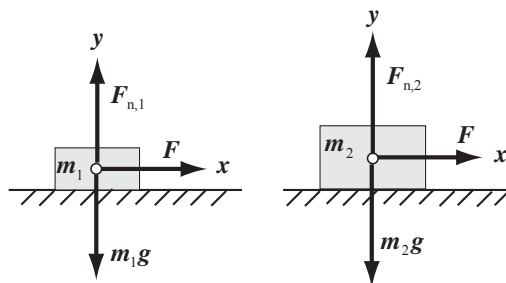
(a) Apply $\sum F_x = ma_x$ to the two blocks:

$$\sum F_{1,x} = F = m_1 a_{1,x}$$
and
$$\sum F_{2,x} = F = m_2 a_{2,x}$$

Eliminate F between the two equations and solve for m_2:

$$m_2 = \frac{a_{1,x}}{a_{2,x}} m_1$$

Substitute numerical values to obtain:

$$m_2 = \frac{12\,\text{m/s}^2}{3.0\,\text{m/s}^2} m_1 = 4.0\,m_1$$

Express the acceleration of an object whose mass is $m_2 - m_1$ when the net force acting on it is F:

$$a_x = \frac{F}{m_2 - m_1} = \frac{F}{4m_1 - m_1} = \frac{F}{3m_1} = \tfrac{1}{3} a_{1,x}$$

Substitute numerical values and evaluate a_x:

$$a_x = \tfrac{1}{3}\left(12\,\text{m/s}^2\right) = \boxed{4.0\,\text{m/s}^2}$$

(b) Express the acceleration of an object whose mass is $m_2 + m_1$ when the net force acting on it is F:

$$a_x = \frac{F}{m_2 + m_1} = \frac{F}{4m_1 + m_1} = \frac{F}{5m_1} = \tfrac{1}{5} a_{1,x}$$

Substitute numerical values and evaluate a_x:

$$a_x = \tfrac{1}{5}\left(12\,\text{m/s}^2\right) = \boxed{2.4\,\text{m/s}^2}$$

40 •• To drag a 75.0-kg log along the ground at constant velocity, your tractor has to pull it with a horizontal force of 250 N. (a) Draw the free body diagram of the log. (b) Use Newton's laws to determine the force of friction on the log. (c) What is the normal force of the ground on the log? (d) What horizontal force must you exert if you want to give the log an acceleration of 2.00 m/s^2 assuming the force of friction does not change. Redraw the log's free body diagram for this situation.

Picture the Problem Because the velocity is constant, the net force acting on the log must be zero. Choose a coordinate system in which the positive x direction is the direction of motion of the log and apply Newton's 2nd law to the log.

(*a*) The free-body diagram shows the forces acting on the log when it is being dragged in the $+x$ direction at constant velocity.
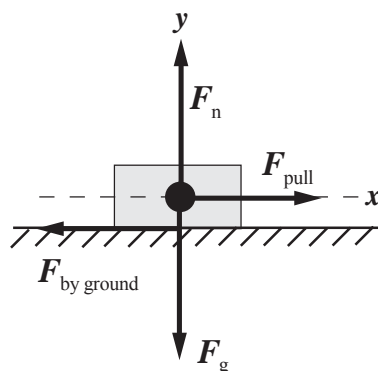
(*b*) Apply $\sum F_x = ma_x$ to the log when it is moving at constant speed:

$\sum F_x = F_{pull} - F_{by\ ground} = ma_x = 0$

or

$F_{by\ ground} = F_{pull}$

Substitute for F_{pull} and evaluate the force of friction $F_{by\ ground}$:

$F_{by\ ground} = F_{pull} = \boxed{250\,N}$

(*c*) Apply $\sum F_y = ma_y$ to the log to obtain:

$\sum F_y = F_n - F_g = ma_y = 0$

or

$F_n = F_g$

Because the gravitational force is given by $F_g = mg$:

$F_n = mg$

Substitute numerical values and evaluate F_n:

$F_n = (75.0\,kg)(9.81\,m/s^2) = \boxed{736\,N}$

(*d*) The free-body diagram shows the forces acting on the log when it is accelerating in the positive x direction.
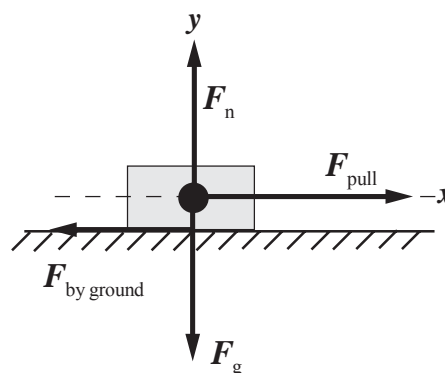
Apply $\sum F_x = ma_x$ to the log when it is accelerating to the right:

$\sum F_x = F_{pull} - F_{by\ ground} = ma_x$

Solving for F_{pull} yields:

$$F_{\text{pull}} = ma_x + F_{\text{by ground}}$$

Substitute numerical values and evaluate F_{pull}:

$$F_{\text{pull}} = (75.0\,\text{kg})(2.00\,\text{m/s}^2) + 250\,\text{N}$$
$$= \boxed{400\,\text{N}}$$

41 •• A 4.0-kg object is subjected to two constant forces, $\vec{F}_1 = (2.0\,\text{N})\hat{i} + (-3.0\,\text{N})\hat{j}$ and $\vec{F}_2 = (4.0\,\text{N})\hat{i} - (11\,\text{N})\hat{j}$. The object is at rest at the origin at time $t = 0$. (*a*) What is the object's acceleration? (*b*) What is its velocity at time $t = 3.0$ s? (*c*) Where is the object at time $t = 3.0$ s?

Picture the Problem The acceleration can be found from Newton's 2nd law. Because both forces are constant, the net force and the acceleration are constant; hence, we can use the constant-acceleration equations to answer questions concerning the motion of the object at various times.

(*a*) Apply Newton's 2nd law to the 4.0-kg object to obtain:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{\vec{F}_1 + \vec{F}_2}{m}$$

Substitute numerical values and simplify to evaluate $\vec{a}$:

$$\vec{a} = \frac{(2.0\,\text{N})\hat{i} + (-3.0\,\text{N})\hat{j} + (4.0\,\text{N})\hat{i} + (-11\,\text{N})\hat{j}}{4.0\,\text{kg}} = \frac{(6.0\,\text{N})\hat{i} + (-14\,\text{N})\hat{j}}{4.0\,\text{kg}}$$
$$= \boxed{(1.5\,\text{m/s}^2)\hat{i} + (-3.5\,\text{m/s}^2)\hat{j}}$$

(*b*) Using a constant-acceleration equation, express the velocity of the object as a function of time:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

Substitute numerical values and evaluate $\vec{v}(3.0\,\text{s})$:

$$\vec{v}(3.0\,\text{s}) = [(1.5\,\text{m/s}^2)\hat{i} + (-3.5\,\text{m/s}^2)\hat{j}](3.0\,\text{s}) = (4.5\,\text{m/s})\hat{i} + (-10.5\,\text{m/s})\hat{j}$$
$$= \boxed{(4.5\,\text{m/s})\hat{i} + (-11\,\text{m/s})\hat{j}}$$

(*c*) Express the position of the object in terms of its average velocity:

$$\vec{r} = \vec{v}_{\text{av}}t = \frac{\vec{v}_0 + \vec{v}}{2}t = \tfrac{1}{2}\vec{v}t$$

Substitute for $\vec{v}$ and evaluate this expression at $t = 3.0$ s:

$$\vec{r}(3.0\,\text{s}) = \tfrac{1}{2}[(4.5\,\text{m/s})\hat{i} + (-10.5\,\text{m/s})\hat{j}](3.0\,\text{s}) = (6.75\,\text{m})\hat{i} + (-15.8\,\text{m})\hat{j}$$
$$= \boxed{(6.8\,\text{m})\hat{i} + (-16\,\text{m})\hat{j}}$$

Mass and Weight

42 • On the moon, the acceleration due to gravity is only about 1/6 of that on Earth. An astronaut, whose weight on earth is 600 N, travels to the lunar surface. His mass, as measured on the moon, will be (*a*) 600 kg, (*b*) 100 kg, (*c*) 61.2 kg, (*d*) 9.81 kg, (*e*) 360 kg.

Picture the Problem The mass of the astronaut is independent of gravitational fields and will be the same on the moon or, for that matter, out in deep space.

Express the mass of the astronaut in terms of his weight on earth and the gravitational field at the surface of the earth:

$$m = \frac{w_{\text{earth}}}{g_{\text{earth}}} = \frac{600\,\text{N}}{9.81\,\text{N/kg}} = 61.2\,\text{kg}$$

and $\boxed{(c)}$ is correct.

43 • Find the weight of a 54-kg student in (*a*) newtons and (*b*) pounds.

Picture the Problem The weight of an object is related to its mass and the gravitational field through $F_g = mg$.

(*a*) The weight of the student is:

$$w = mg = (54\,\text{kg})(9.81\,\text{N/kg})$$
$$= 530\,\text{N}$$
$$= \boxed{5.3 \times 10^2\,\text{N}}$$

(*b*) Convert newtons to pounds:

$$w = \frac{530\,\text{N}}{4.45\,\text{N/lb}} = 119\,\text{lb} \approx \boxed{1.2 \times 10^2\,\text{lb}}$$

44 • Find the mass of a 165-lb engineer in kilograms.

Picture the Problem The mass of an object is related to its weight and the gravitational field.

Convert the weight of the man into newtons:

$$165\,\text{lb} = (165\,\text{lb})(4.45\,\text{N/lb}) = 734\,\text{N}$$

Calculate the mass of the man from his weight and the gravitational field:

$$m = \frac{w}{g} = \frac{734\,\text{N}}{9.81\,\text{N/kg}} = \boxed{74.8\,\text{kg}}$$

45 •• [SSM] To train astronauts to work on the moon, where the acceleration due to gravity is only about 1/6 of that on earth, NASA submerges them in a tank of water. If an astronaut, who is carrying a backpack, air conditioning unit, oxygen supply, and other equipment, has a total mass of 250 kg, determine the following quantities. (*a*) her weight on Earth, (*b*) her weight on

the moon, (*c*) the required upward buoyancy force of the water during her training for the moon's environment on Earth.

Picture the Problem We can use the relationship between weight (gravitational force) and mass, together with the given information about the acceleration due to gravity on the moon, to find the astronaut's weight on Earth and on the moon.

(*a*) Her weight on Earth is the product of her mass and the gravitational field at the surface of the earth:

$$w_{earth} = mg$$

Substitute numerical values and evaluate *w*:

$$w_{earth} = (250\,\text{kg})(9.81\,\text{m/s}^2) = 2.453\,\text{kN}$$
$$= \boxed{2.45\,\text{kN}}$$

(*b*) Her weight on the moon is the product of her mass and the gravitational field at the surface of the moon:

$$w_{moon} = mg_{moon} = \tfrac{1}{6}mg = \tfrac{1}{6}w_{earth}$$

Substitute for her weight on Earth and evaluate her weight on the moon:

$$w_{moon} = \tfrac{1}{6}(2.453\,\text{kN}) = \boxed{409\,\text{N}}$$

(*c*) The required upward buoyancy force of the water equals her weight of earth:

$$w_{buoyancy} = w_{earth} = \boxed{2.45\,\text{kN}}$$

46 •• It is the year 2075 and space travel is common. A physics professor brings his favorite teaching demonstration with him to the moon. The apparatus consists of a very smooth horizontal (frictionless) table and an object to slide on it. On Earth when the professor attaches a spring (spring constant 50 N/m) to the object and pulls horizontally so the spring stretches 2.0 cm, the object accelerates at 1.5 m/s². (*a*) Draw the free body diagram of the object and use it and Newton's laws to determine its mass. (*b*) What would the block's acceleration be under identical conditions on the moon?

Picture the Problem The forces acting on the object are the normal force exerted by the table, the gravitational force exerted by the earth, and the force exerted by the stretched spring.
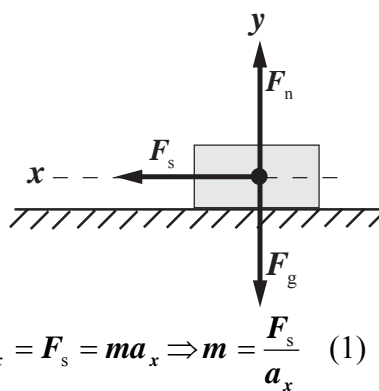
(a) The free-body diagram shown to the right assumes that the spring has been stretched to the right. Hence the force that the spring exerts on the object is to the left. Note that the $+x$ direction has been chosen to be in the same direction as the force exerted by the spring.

Apply $\sum F_x = ma_x$ to the object to obtain:

$$\sum F_x = F_s = ma_x \Rightarrow m = \frac{F_s}{a_x} \quad (1)$$

The force exerted by the spring on the object is given by:

$F_s = k\Delta x$

where Δx is the amount by which the spring has been stretched or compressed and k is the force constant.

Substituting for F_s in equation (1) yields:

$$m = \frac{k\Delta x}{a_x}$$

Substitute numerical values and evaluate m:

$$m = \frac{(50 \text{ N/m})(2.0 \text{ cm})}{1.5 \text{ m/s}^2} = \boxed{0.67 \text{ kg}}$$

(b) Because the object's mass is the same on the moon as on the Earth and the force exerted by the spring is the same, its acceleration on the moon would be the same as on the Earth.

Free-Body Diagrams: Static Equilibrium

47 •   A traffic light (mass 35.0 kg) is supported by two wires as in Figure 4-36. (a) Draw the light's free body diagram and use it to answer the following question qualitatively: Is the tension in wire 2 greater than or less than the tension in wire 1? (b) Prove your answer by applying Newton's laws and solving for the two tensions.

Picture the Problem Because the traffic light is not accelerating, the *net* force acting on it must be zero; i.e., $\vec{T}_1 + \vec{T}_2 + \vec{F}_g = 0$.

Construct a free-body diagram showing the forces acting on the support point:
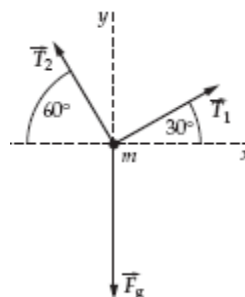
Apply $\sum F_x = ma_x$ to the support point:

$$T_1 \cos 30° - T_2 \cos 60° = ma_x = 0$$

Solve for T_2 in terms of T_1:

$$T_2 = \frac{\cos 30°}{\cos 60°} T_1 = 1.73 T_1$$

Thus T_2 is greater than T_1.

48 •   A lamp with a mass of 42.6 kg is hanging from wires as shown in Figure 4-37. The ring has negligible mass. The tension T_1 in the vertical wire is (*a*) 209 N, (*b*) 418 N, (*c*) 570 N, (*d*) 360 N, (*e*) 730 N.

Picture the Problem From the figure, it is clear that T_1 supports the full weight of the lamp. Draw a free-body diagram showing the forces acting on the lamp and apply $\sum F_y = 0$.
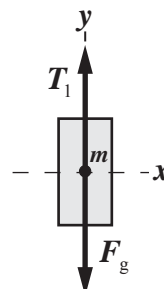
Apply $\sum F_y = 0$ to the lamp to obtain:

$$T_1 - F_g = 0$$

Solve for T_1 and substitute for F_g to obtain:

$$T_1 = F_g = mg$$

Substitute numerical values and evaluate T_1:

$$T_1 = (42.6\,\text{kg})(9.81\,\text{m/s}^2) = 418\,\text{N}$$

and $\boxed{(b)}$ is correct.

49 ••   **[SSM]**   In Figure 4-38*a*, a 0.500-kg block is suspended at the midpoint of a 1.25-m-long string. The ends of the string are attached to the ceiling at points separated by 1.00 m. (*a*) What angle does the string make with the ceiling? (*b*) What is the tension in the string? (*c*) The 0.500-kg block is removed and two 0.250-kg blocks are attached to the string such that the lengths of the three string segments are equal (Figure 4-38*b*). What is the tension in each segment of the string?

Picture the Problem The free-body diagrams for parts (*a*), (*b*), and (*c*) are shown below. In both cases, the block is in equilibrium under the influence of the forces and we can use Newton's 2nd law of motion and geometry and trigonometry to obtain relationships between θ and the tensions.
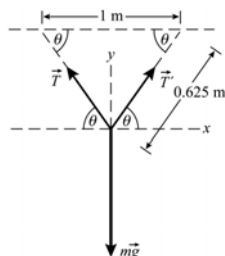
(*a*) and (*b*)   (*c*)
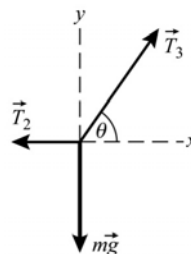
(*a*) Referring to the free-body diagram for part (*a*), use trigonometry to determine θ:

$$\theta = \cos^{-1}\left(\frac{0.50\,\text{m}}{0.625\,\text{m}}\right) = 36.9° = \boxed{37°}$$

(*b*) Noting that $T = T'$, apply $\sum F_y = ma_y$ to the 0.500-kg block and solve for the tension T:

$2T\sin\theta - mg = 0$ because $a = 0$

and

$$T = \frac{mg}{2\sin\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.500\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin36.9°} = \boxed{4.1\,\text{N}}$$

(*c*) The length of each segment is:

$$\frac{1.25\,\text{m}}{3} = 0.41667\,\text{m}$$

Find the distance *d*:
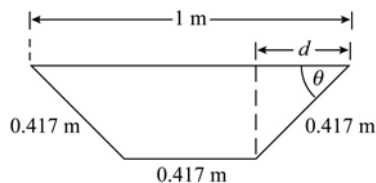
$$d = \frac{1.00\,\text{m} - 0.41667\,\text{m}}{2}$$

$$= 0.29167\,\text{m}$$

Express θ in terms of *d* and solve for its value:

$$\theta = \cos^{-1}\left(\frac{d}{0.417\,\text{m}}\right)$$

$$= \cos^{-1}\left(\frac{0.2917\,\text{m}}{0.4167\,\text{m}}\right) = 45.57°$$

Apply $\sum F_y = ma_y$ to the 0.250-kg block:

$$T_3\sin\theta - mg = 0 \Rightarrow T_3 = \frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T_3:	$T_3 = \dfrac{(0.250\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 45.57°} = 3.434\,\text{N}$
	$= \boxed{3.4\,\text{N}}$

Apply $\sum F_x = ma_x$ to the 0.250-kg block and solve for the tension T_2:

$T_3 \cos\theta - T_2 = 0$ since $a = 0$.
and
$T_2 = T_3 \cos\theta$

Substitute numerical values and evaluate T_2:

$T_2 = (3.434\,\text{N})\cos 45.57° = \boxed{2.4\,\text{N}}$

By symmetry:

$T_1 = T_3 = \boxed{3.4\,\text{N}}$

50 ••   A ball weighing 100 N is shown suspended from a system of cords (Figure 4-39). What are the tensions in the horizontal and angled cords?

Picture the Problem The suspended body is in equilibrium under the influence of the forces $\vec{T}_{\text{hor}}, \vec{T}_{45}$, and $\vec{F}_g$. That is, $\vec{T}_{\text{hor}} + \vec{T}_{45} + \vec{F}_g = 0$. Draw the free-body diagram of the forces acting on the knot just above the 100-N body. Choose a coordinate system with the positive x direction to the right and the positive y direction upward. Apply the conditions for translational equilibrium to determine the tension in the horizontal cord.
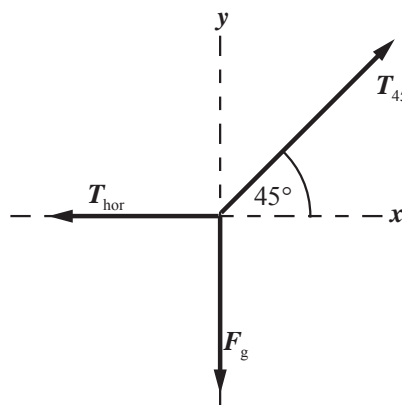
Apply $\sum F_y = ma_y$ to the knot:

$\sum F_y = T_{45}\sin 45° - F_g = ma_y = 0$

Solving for T_{45} yields:

$$T_{45} = \frac{F_g}{\sin 45°} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the knot:

$\sum F_x = T_{45}\cos 45° - T_{\text{hor}} = ma_a = 0$

Solve for T_{hor} to obtain:

$T_{\text{hor}} = T_{45}\cos 45°$

Substituting for T_{45} yields:

$T_{\text{hor}} = \dfrac{F_g}{\sin 45°}\cos 45° = F_g = \boxed{100\,\text{N}}$

Substituting for F_g in equation (1) yields:

$T_{45} = \dfrac{100\,\text{N}}{\sin 45°} = \boxed{141\,\text{N}}$

51 •• **[SSM]**   A 10-kg object on a frictionless table is subjected to two horizontal forces, $\vec{F}_1$ and $\vec{F}_2$, with magnitudes $F_1 = 20$ N and $F_2 = 30$ N, as shown in Figure 4-40. Find the third force $\vec{F}_3$ that must be applied so that the object is in static equilibrium.

Picture the Problem The acceleration of *any* object is directly proportional to the *net* force acting on it. Choose a coordinate system in which the positive x direction is the same as that of $\vec{F}_1$ and the positive y direction is to the right. Add the two forces to determine the net force and then use Newton's 2nd law to find the acceleration of the object. If $\vec{F}_3$ brings the system into equilibrium, it must be true that $\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$.

Express $\vec{F}_3$ in terms of $\vec{F}_1$ and $\vec{F}_2$: $\qquad\qquad \vec{F}_3 = -\vec{F}_1 - \vec{F}_2$ $\qquad\qquad$ (1)

Express $\vec{F}_1$ and $\vec{F}_2$ in unit vector notation:

$$\vec{F}_1 = (20\,\text{N})\hat{i}$$

and

$$\vec{F}_2 = \{(-30\,\text{N})\sin 30°\}\hat{i} + \{(30\,\text{N})\cos 30°\}\hat{j} = (-15\,\text{N})\hat{i} + (26\,\text{N})\hat{j}$$

Substitute for $\vec{F}_1$ and $\vec{F}_2$ in equation (1) and simplify to obtain:

$$\vec{F}_3 = -(20\,\text{N})\hat{i} - \left[(-15\,\text{N})\hat{i} + (26\,\text{N})\hat{j}\right] = \boxed{(-5.0\,\text{N})\hat{i} + (-26\,\text{N})\hat{j}}$$

52 ••   For the systems to be in equilibrium in Figure 4-41*a*, Figure 4-41*b*, and Figure 4-41*c* find the unknown tensions and masses.

Picture the Problem The free-body diagrams for the systems in equilibrium are shown below. Apply the conditions for translational equilibrium to find the unknown tensions.

(*a*)   (*b*)   (*c*)
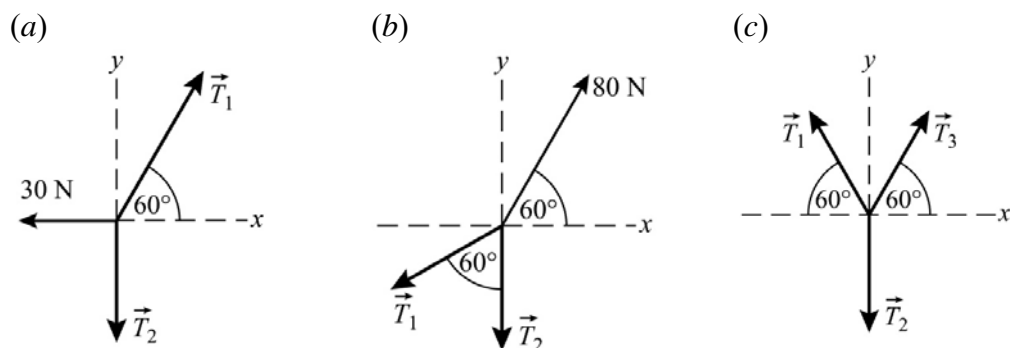
(*a*) Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the knot above the suspended mass to obtain:

$\sum F_x = T_1 \cos 60° - 30\,\text{N} = 0$ (1)

and

$\sum F_y = T_1 \sin 60° - T_2 = 0$ (2)

Solving equation (1) for T_1 yields:

$$T_1 = \frac{30\,\text{N}}{\cos 60°} = \boxed{60\,\text{N}}$$

Solving equation (2) for T_2 yields:

$$T_2 = T_1 \sin 60° = (60\,\text{N})\sin 60°$$
$$= 51.96\,\text{N} = \boxed{52\,\text{N}}$$

Because T_2 is the weight of the object whose mass is m:

$$T_2 = F_g = mg \Rightarrow m = \frac{T_2}{g}$$

Substitute numerical values and evaluate m:

$$m = \frac{51.96\,\text{N}}{9.81\,\text{m/s}^2} = \boxed{5.3\,\text{kg}}$$

(*b*) Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the knot above the suspended mass to obtain:

$\sum F_x = (80\,\text{N})\cos 60° - T_1 \sin 60° = 0$

and

$\sum F_y = (80\,\text{N})\sin 60° - T_2 - T_1 \cos 60° = 0$

Solving the first of these equations for T_1 yields:

$$T_1 = \frac{(80\,\text{N})\cos 60°}{\sin 60°} = 46.19\,\text{N}$$
$$= \boxed{46\,\text{N}}$$

Solving the second of these equations for T_2 yields:

$$T_2 = (80\,\text{N})\sin 60° - T_1 \cos 60°$$

Substitute numerical values and evaluate T_2:

$$T_2 = (80\,\text{N})\sin 60° - (46.19\,\text{N})\cos 60°$$
$$= 46.19\,\text{N} = \boxed{46\,\text{N}}$$

Because T_2 is the weight of the object whose mass is m:

$$T_2 = F_g = mg \Rightarrow m = \frac{T_2}{g}$$

Substitute numerical values and evaluate m:

$$m = \frac{46.19\,\text{N}}{9.81\,\text{m/s}^2} = \boxed{4.7\,\text{kg}}$$

(*c*) Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the knot above the suspended mass to obtain:

$\sum F_x = -T_1 \cos 60° + T_3 \cos 60° = 0$

and

$\sum F_y = T_1 \sin 60° + T_3 \sin 60° - F_g = 0$

| Solving the first of these equations for T_1 yields: | $T_1 = T_3$ |

| Solving the second of these equations for T_1 yields: | $T_1 = T_3 = \dfrac{F_g}{2\sin 60°} = \dfrac{mg}{2\sin 60°}$ |

Substitute numerical values and evaluate T_1 and T_3:

$$T_1 = T_3 = \frac{(6.0\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin 60°} = 33.98\,\text{N}$$

$$= \boxed{34\,\text{N}}$$

Because $T_2 = F_g$:

$$T_2 = (6.0\,\text{kg})(9.81\,\text{m/s}^2) = 58.9\,\text{N}$$

$$= \boxed{60\,\text{N}}$$

Because the effect of the pulley is to change the direction T_1 acts, T_1 is the weight of the object whose mass is m:

$$T_1 = mg \Rightarrow m = \frac{T_1}{g}$$

Substitute numerical values and evaluate m:

$$m = \frac{33.98\,\text{N}}{9.81\,\text{m/s}^2} = \boxed{3.5\,\text{kg}}$$

53 •• Your car is stuck in a mud hole. You are alone, but you have a long, strong rope. Having studied physics, you tie the rope tautly to a telephone pole and pull on it sideways, as shown in Figure 4-42. (*a*) Find the force exerted by the rope on the car when the angle θ is 3.00° and you are pulling with a force of 400 N but the car does not move. (*b*) How strong must the rope be if it takes a force of 600 N to move the car when θ is 4.00°?

Picture the Problem Construct the free-body diagram for that point in the rope at which you exert the force $\vec{F}$ and choose the coordinate system shown in the free-body diagram. We can apply Newton's 2nd law to the rope to relate the tension to F.
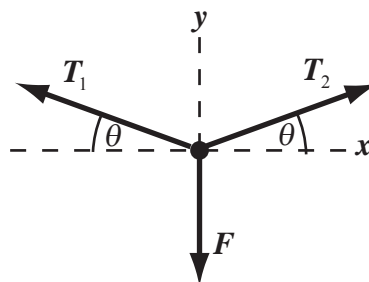
(*a*) Noting that $T_1 = T_2 = T$ and that the car's acceleration is zero, apply $\sum F_y = ma_y$ to the car:

$$2T\sin\theta - F = ma_y = 0 \Rightarrow T = \frac{F}{2\sin\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{400\,\text{N}}{2\sin 3.00°} = \boxed{3.82\,\text{kN}}$$

(b) Proceed as in Part (a) to obtain:

$$T = \frac{600\,\text{N}}{2\sin 4.00°} = \boxed{4.30\,\text{kN}}$$

54 ••• Balloon arches are often seen at festivals or celebrations; they are made by attaching helium-filled balloons to a rope that is fixed to the ground at each end. The lift from the balloons raises the structure into the arch shape. Figure 4-43a shows the geometry of such a structure: N balloons are attached at equally spaced intervals along a massless rope of length L, which is attached to two supports at its ends. Each balloon provides a lift force F. The horizontal and vertical coordinates of the point on the rope where the ith balloon is attached are x_i and y_i, and T_i is the tension in the ith segment. (Note segment 0 is the segment between the point of attachment and the first balloon, and segment N is the segment between the last balloon and the other point of attachment). (a) Figure 4-43b shows a free-body diagram for the ith balloon. From this diagram, show that the horizontal component of the force T_i (call it T_H) is the same for all the string segments. (b) By considering the vertical component of the forces, use Newton's laws to derive the following relationship between the tension in the ith and $(i-1)$th segments: $T_{i-1}\sin\theta_{i-1} - T_i\sin\theta_i = F$ (b) Show that $\tan\theta_0 = -\tan\theta_{N+1} = NF/2T_H$. (c) From the diagram and the two expressions above, show that

$\tan\theta_i = (N-2i)F/2T_H$ and that $x_i = \dfrac{L}{N+1}\sum_{j=0}^{i-1}\cos\theta_j$, $y_i = \dfrac{L}{N+1}\sum_{j=0}^{i-1}\sin\theta_j$.

Picture the Problem In Part (a) we can apply Newton's 2$^{\text{nd}}$ law to obtain the given expression for F. In (b) we can use a symmetry argument to find an expression for $\tan\theta_0$. In (c) we can use our results obtained in (a) and (b) to express x_i and y_i.

(a) Apply $\sum F_y = 0$ to the balloon:

$$F + T_i\sin\theta_i - T_{i-1}\sin\theta_{i-1} = 0$$

Solving for F yields:

$$F = \boxed{T_{i-1}\sin\theta_{i-1} - T_i\sin\theta_i}$$

(*b*) By symmetry, each support must balance half of the force acting on the entire arch. Therefore, the vertical component of the force on the support must be *NF*/2. The horizontal component of the tension must be T_H. Express $\tan\theta_0$ in terms of *NF*/2 and T_H:

$$\tan\theta_0 = \frac{NF/2}{T_H} = \frac{NF}{2T_H}$$

By symmetry, $\theta_{N+1} = -\theta_0$. Therefore, because the tangent function is odd:

$$\boxed{\tan\theta_0 = -\tan\theta_{N+1} = \frac{NF}{2T_H}}$$

(*c*) Using $T_H = T_i\cos\theta_i = T_{i-1}\cos\theta_{i-1}$, divide both sides of the result in (*a*) by T_H and simplify to obtain:

$$\frac{F}{T_H} = \frac{T_{i-1}\sin\theta_{i-1}}{T_{i-1}\cos\theta_{i-1}} - \frac{T_i\sin\theta_i}{T_i\cos\theta_i}$$
$$= \tan\theta_{i-1} - \tan\theta_i$$

Using this result, express $\tan\theta_1$:

$$\tan\theta_1 = \tan\theta_0 - \frac{F}{T_H}$$

Substitute for $\tan\theta_0$ from (*a*):

$$\tan\theta_1 = \frac{NF}{2T_H} - \frac{F}{T_H} = (N-2)\frac{F}{2T_H}$$

Generalize this result to obtain:

$$\boxed{\tan\theta_i = (N-2i)\frac{F}{2T_H}}$$

Express the length of rope between two balloons:

$$\ell_{\text{between balloons}} = \frac{L}{N+1}$$

Express the horizontal coordinate of the point on the rope where the *i*th balloon is attached, x_i, in terms of x_{i-1} and the length of rope between two balloons:

$$x_i = x_{i-1} + \frac{L}{N+1}\cos\theta_{i-1}$$

Sum over all the coordinates to obtain:

$$\boxed{x_i = \frac{L}{N+1}\sum_{j=0}^{i-1}\cos\theta_j}$$

Proceed similarly for the vertical coordinates to obtain:

$$\boxed{y_i = \frac{L}{N+1}\sum_{j=0}^{i-1}\sin\theta_j}$$

55 ••• (*a*) Consider a numerical solution to Problem 54. Write a **spreadsheet** program to make a graph of the shape of a balloon arch. Use the following parameters: $N = 10$ balloons each providing a lift force $F = 1.0$ N and each attached to a rope length $L = 11$ m, with a horizontal component of tension $T_H = 10$ N. How far apart are the two points of attachment? How high is the arch at its highest point? (*b*) Note that we haven't specified the spacing between the supports—it is determined by the other parameters. Vary T_H while keeping the other parameters the same until you create an arch that has a spacing of 8.0 m between the supports. What is T_H then? As you increase T_H, the arch should get flatter and more spread out. Does your spreadsheet model show this?

Picture the Problem
(*a*) A spreadsheet program is shown below. The formulas used to calculate the quantities in the columns are as follows:

| Cell	Content/Formula	Algebraic Form
C9	($B$2-2*B9)/(2*$B$4)	$(N-2\mathrm{i})\dfrac{F}{2T_H}$
D9	SIN(ATAN(C9))	$\sin\!\left(\tan^{-1}\theta_i\right)$
E9	COS(ATAN(C9))	$\cos\!\left(\tan^{-1}\theta_i\right)$
F9	0.000	0
G9	0.000	0
F10	F9+$B$1/($B$2+1)*E9	$x_{i-1}+\dfrac{L}{N+1}\cos\theta_{i-1}$
G10	G9+$B$1/($B$2+1)*D9	$y_{i-1}+\dfrac{L}{N+1}\cos\theta_{i-1}$

	A	B	C	D	E	F	G
1	$L =$	11	m				
2	$N =$	10					
3	$F =$	1	N				
4	$T_H=$	3.72	N				
5							
6							
7							
8		i	$\tan\theta_i$	$\sin\theta_i$	$\cos\theta_i$	x_i	y_i
9		0	1.344	0.802	0.597	0.000	0.000
10		1	1.075	0.732	0.681	0.543	0.729
11		2	0.806	0.628	0.778	1.162	1.395
12		3	0.538	0.474	0.881	1.869	1.966
13		4	0.269	0.260	0.966	2.670	2.396
14		5	0.000	0.000	1.000	3.548	2.632
15		6	−0.269	−0.260	0.966	4.457	2.632
16		7	−0.538	−0.474	0.881	5.335	2.396

17		8	−0.806	−0.628	0.778	6.136	1.966
18		9	−1.075	−0.732	0.681	6.843	1.395
19		10	−1.344	−0.802	0.597	7.462	0.729
20		11				8.005	0.000

(*b*) A horizontal component of tension 3.72 N gives a spacing of 8 m. At this spacing, the arch is 2.63 m high, tall enough for someone to walk through.
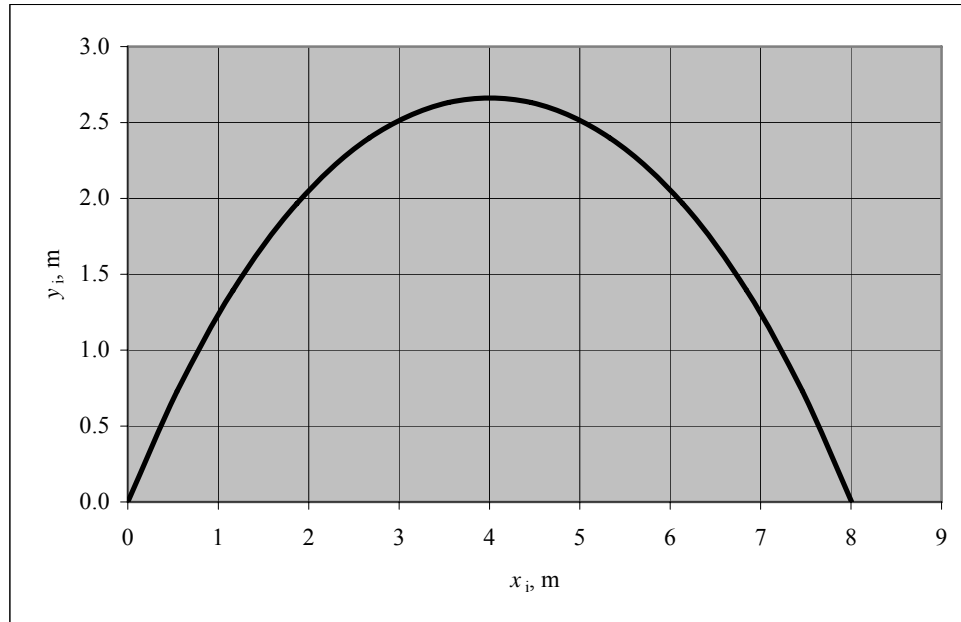

The spreadsheet graph shown below shows that changing the horizontal component of tension to 5 N broadens the arch and decreases its height as predicted by our mathematical model.
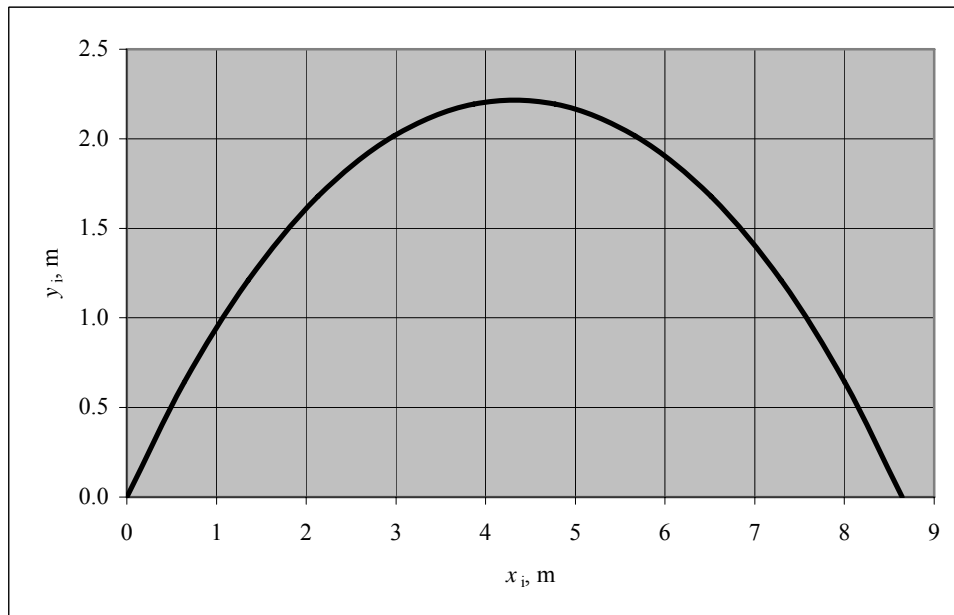
Free-Body Diagrams: Inclined Planes and the Normal Force

56 •   A large box whose mass is 20.0 kg rests on a frictionless floor. A mover pushes on the box with a force of 250 N at an angle 35.0° below the horizontal. Draw the box's free body diagram and use it to determine the acceleration of the box.

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it across the frictionless floor. We can apply Newton's 2nd law to the box to find its acceleration.
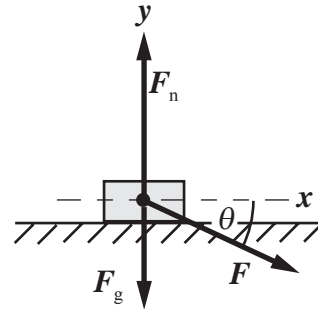
Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta = ma_x \Rightarrow a_x = \frac{F\cos\theta}{m}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{(250\,\text{N})\cos 35.0°}{20.0\,\text{kg}} = \boxed{10.2\,\text{m/s}^2}$$

57 •   A 20-kg box rests on a frictionless ramp with a 15.0° slope. The mover pulls on a rope attached to the box to pull it up the incline (Figure 4-44). If the rope makes an angle of 40.0° with the horizontal, what is the smallest force F the mover will have to exert to move the box up the ramp?

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it up the frictionless incline. We can apply Newton's 2nd law to the box to determine the smallest force that will move it up the incline at constant speed.
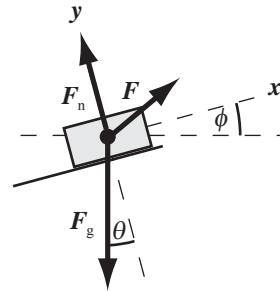
Letting $F_{\min} = F$, apply $\sum F_x = ma_x$ to the box as it moves up the incline with constant speed:

$$F_{\min}\cos(\phi - \theta) - F_g\sin\theta = 0$$

Solve for $F_{\min}$ to obtain:

$$F_{\min} = \frac{F_g\sin\theta}{\cos(\phi - \theta)}$$

Because $F_g = mg$:
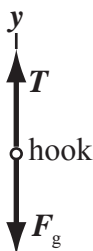
$$F_{min} = \frac{mg\sin\theta}{\cos(\phi-\theta)}$$

Substitute numerical values and evaluate F_{min}:

$$F_{min} = \frac{(20.0\,\text{kg})(9.81\,\text{m/s}^2)\sin15°}{\cos(40.0°-15.0°)}$$

$$= \boxed{56.0\,\text{N}}$$

58 •   In Figure 4-45, the objects are attached to spring scales calibrated in newtons. Give the readings of the balance(s) in each case, assuming that both the scales and the strings are massless.

Picture the Problem The balance(s) indicate the tension in the string(s). Draw free-body diagrams for each of these systems and apply the condition(s) for equilibrium.

(a)

(b)

(c)
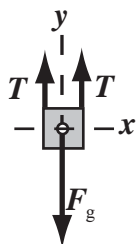
(d)

(a) Apply $\sum F_y = 0$ to the hook to obtain:

$\sum F_y = T - F_g = 0$
or, because $F_g = mg$,
$T = mg$

Substitute numerical values and evaluate T:

$T = (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98\,\text{N}}$

(b) Apply $\sum F_x = 0$ to the balance to obtain:

$\sum F_x = T - T' = 0$
or, because $T' = mg$,
$T = T' = mg$

Substitute numerical values and evaluate T and T':

$$T = T' = (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98\,\text{N}}$$

(c) Apply $\sum F_y = 0$ to the suspended object to obtain:

$$\sum F_y = 2T - F_g = 0$$
or, because $F_g = mg$,
$$2T - mg = 0 \Rightarrow T = \tfrac{1}{2}mg$$

Substitute numerical values and evaluate T:

$$T = \tfrac{1}{2}(10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49\,\text{N}}$$

(d) Apply $\sum F_x = 0$ to the balance to obtain:

$$\sum F_x = T - T' = 0$$
or, because $T' = mg$,
$$T = T' = mg$$

Substitute numerical values and evaluate T and T':

$$T = T' = (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98\,\text{N}}$$

Remarks: Note that (a), (b), and (d) give the same answers … a rather surprising result until one has learned to draw FBDs and apply the conditions for translational equilibrium.

59 ••   A box is held in position on a frictionless incline by a cable (Figure 4-46). (a) If $\theta = 60°$ and $m = 50$ kg, find the tension in the cable and the normal force exerted by the incline. (b) Find the tension as a function of θ and m, and check your result for plausibility in the special cases when $\theta = 0°$ and $\theta = 90°$.

Picture the Problem Because the box is held in place (is in equilibrium) by the forces acting on it, we know that
$$\vec{T} + \vec{F}_n + \vec{F}_g = 0$$
Choose a coordinate system in which the positive x direction is in the direction of $\vec{T}$ and the positive y direction is in the direction of $\vec{F}_n$. Apply Newton's 2nd law to the block to obtain expressions for $\vec{T}$ and $\vec{F}_n$.
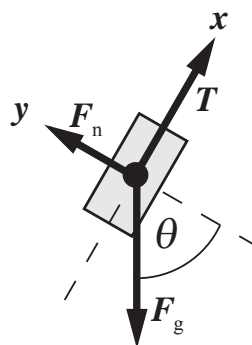
(a) Apply $\sum F_x = 0$ to the box:

$$T - F_g \sin\theta = 0$$
or, because $F_g = mg$,
$$T - mg\sin\theta = 0 \Rightarrow T = mg\sin\theta \quad (1)$$

Substitute numerical values and evaluate T:	$T = (50\,\text{kg})(9.81\,\text{m/s}^2)\sin 60°$
	$= \boxed{0.42\,\text{kN}}$

Apply $\sum F_y = 0$ to the box:

$F_n - F_g \cos\theta = 0$

or, because $F_g = mg$,

$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$

Substitute numerical values and evaluate F_n:

$F_n = (50\,\text{kg})(9.81\,\text{m/s}^2)\cos 60°$

$= \boxed{0.25\,\text{kN}}$

(b) The tension as a function of θ and m is given by equation (1):

$T = \boxed{mg\sin\theta}$

For $\theta = 90°$:

$T_{90°} = mg\sin 90° = \boxed{mg}$ ... a result we know to be correct.

For $\theta = 0°$:

$T_{0°} = mg\sin 0° = \boxed{0}$ ... a result we know to be correct.

60 •• A horizontal force of 100 N pushes a 12-kg block up a frictionless incline that makes an angle of 25° with the horizontal. (a) What is the normal force that the incline exerts on the block? (b) What is the acceleration of the block?

Picture the Problem Draw a free-body diagram for the box. Choose a coordinate system in which the positive x-axis is parallel to the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the block. Apply Newton's 2nd law of motion to find the normal force F_n that the incline exerts on the block and the acceleration a of the block.
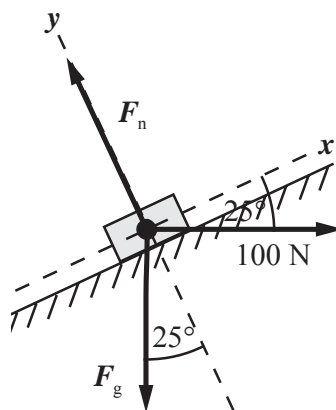
(a) Apply $\sum F_y = ma_y$ to the block:

$F_n - F_g \cos 25° - (100\,\text{N})\sin 25° = 0$

or, because $F_g = mg$,

$F_n - mg\cos 25° - (100\,\text{N})\sin 25° = 0$

Solving for F_n yields:

$F_n = mg\cos 25° + (100\,\text{N})\sin 25°$

Substitute numerical values and evaluate F_n:

$$F_n = (12\,\text{kg})(9.81\,\text{m/s}^2)\cos 25°$$
$$+ (100\,\text{N})\sin 25°$$
$$= \boxed{1.5 \times 10^2\ \text{N}}$$

(b) Apply $\sum F_x = ma_x$ to the block:

$$(100\,\text{N})\cos 25° - F_g \sin 25° = ma$$
or, because $F_g = mg$,
$$(100\,\text{N})\cos 25° - mg \sin 25° = ma$$

Solve for a to obtain:

$$a = \frac{(100\,\text{N})\cos 25°}{m} - g \sin 25°$$

Substitute numerical values and evaluate a:

$$a = \frac{(100\,\text{N})\cos 25°}{12\,\text{kg}} - (9.81\,\text{m/s}^2)\sin 25°$$
$$= \boxed{3.4\,\text{m/s}^2}$$

61 •• **[SSM]** A 65-kg student weighs himself by standing on a scale mounted on a skateboard that is rolling down an incline, as shown in Figure 4-47. Assume there is no friction so that the force exerted by the incline on the skateboard is normal to the incline. What is the reading on the scale if $\theta = 30°$?

Picture the Problem The scale reading (the boy's apparent weight) is the force the scale exerts on the boy. Draw a free-body diagram for the boy, choosing a coordinate system in which the positive x-axis is parallel to and down the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the boy. Apply Newton's 2nd law of motion in the y direction.
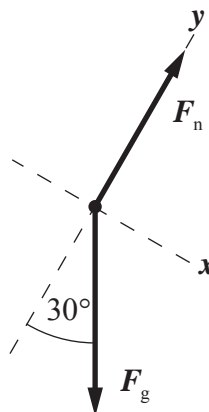
Apply $\sum F_y = ma_y$ to the boy to find F_n. Remember that there is no acceleration in the y direction:

$$F_n - F_g \cos 30° = 0$$
or, because $F_g = mg$,
$$F_n - mg \cos 30° = 0$$

Solving for F_n yields:

$$F_n = mg \cos 30°$$

Substitute numerical values and evaluate F_n:

$$F_n = (65\,\text{kg})(9.81\,\text{m/s}^2)\cos 30°$$
$$= \boxed{0.55\,\text{kN}}$$

62 ••   A block of mass m slides across a frictionless floor and then up a
frictionless ramp (Figure 4-48). The angle of the ramp is θ and the speed of the
block before it starts up the ramp is v_0. The block will slide up to some maximum
height h above the floor before stopping. Show that h is independent of θ by
deriving an expression for h in terms of v_0 and g.

Picture the Problem The free-body
diagram for the block sliding up the
incline is shown to the right. Applying
Newton's 2nd law to the forces acting in
the x direction will lead us to an
expression for a_x. Using this expression
in a constant-acceleration equation will
allow us to express h as a function of v_0
and g.
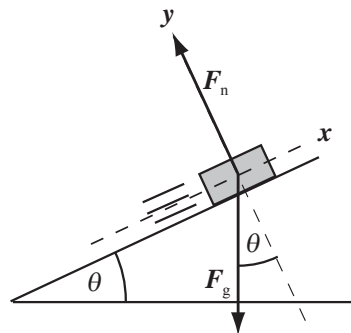
Relate the height h is related to the
distance Δx traveled up the incline:

$$h = \Delta x \sin \theta \qquad (1)$$

Using a constant-acceleration
equation, relate the final speed of the
block to its initial speed, acceleration,
and distance traveled:

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$
or, because $v_x = 0$,
$$0 = v_{0x}^2 + 2a_x \Delta x \Rightarrow \Delta x = \frac{-v_{0x}^2}{2a_x}$$

Substituting for Δx in equation (1)
yields:

$$h = \frac{-v_{0x}^2}{2a_x} \sin \theta \qquad (2)$$

Apply $\sum F_x = ma_x$ to the block and
solve for its acceleration:

$$-F_g \sin \theta = ma_x$$

Because $F_g = mg$:

$$-mg \sin \theta = ma_x \Rightarrow a_x = -g \sin \theta$$

Substitute for a_x in equation (2) and
simplify to obtain:

$$h = \left(\frac{v_{0x}^2}{2g \sin \theta} \right) \sin \theta = \boxed{\frac{v_{0x}^2}{2g}}$$

which is independent of the ramp's
angle θ.

Free-Body Diagrams: Elevators

63 •   **[SSM]**   (*a*) Draw the free body diagram (with accurate relative force
magnitudes) for an object that is hung by a rope from the ceiling of an elevator
that is ascending but slowing. (*b*) Repeat Part (*a*) but for the situation in which the
elevator is descending and speeding up. (*c*) Can you tell the difference between

the two diagrams? Comment. Explain why the diagram does not tell anything about the object's velocity.

Picture the Problem
(*a*) The free body diagram for an object that is hung by a rope from the ceiling of an ascending elevator that is slowing down is shown to the right. Note that because $F_g > T$, the net force acting on the object is downward; as it must be if the object is slowing down as it is moving upward.

(*b*) The free body diagram for an object that is hung by a rope from the ceiling of an elevator that is descending and speeding up is shown to the right. Note that because $F_g > T$, the net force acting on the object is downward; as it must be if the object is speeding up as it descends.

(*c*) No. In both cases the acceleration is downward. You can only tell the direction of the acceleration, not the direction of the velocity.

64 • A person in an elevator is holding a 10.0-kg block by a cord rated to withstand a tension of 150 N. Shortly after the elevator starts to ascend, the cord breaks. What was the minimum acceleration of the elevator when the cord broke?

Picture the Problem The free-body diagram shows the forces acting on the 10.0-kg block as the elevator accelerates upward. Apply Newton's 2nd law to the block to find the minimum acceleration of the elevator required to break the cord.

Apply $\sum F_y = ma_y$ to the block:

$T - F_g = ma_y$
or, because $F_g = mg$,
$T - mg = ma_y$

Solve for a_y to determine the minimum breaking acceleration:

$a_y = \dfrac{T - mg}{m} = \dfrac{T}{m} - g$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{150\,\text{N}}{10.0\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{5.19\,\text{m/s}^2}$$

65 •• A 2.0-kg block hangs from a spring scale calibrated in newtons that is attached to the ceiling of an elevator (Figure 4-49).What does the scale read when (*a*) the elevator is ascending with a constant speed of 30 m/s, (*b*) the elevator is descending with a constant speed of 30 m/s, (*c*) the elevator is ascending at 20 m/s and gaining speed at a rate of 3.0 m/s²? (*d*) Suppose that from $t = 0$ to $t = 5.0$ s, the elevator ascends at a constant speed of 10 m/s. Its speed is then steadily reduced to zero during the next 4.0 s, so that it is at rest at $t = 9.0$ s. Describe the reading of the scale during the interval $0 < t < 9.0$ s.

Picture the Problem The free-body diagram shows the forces acting on the 2-kg block as the elevator ascends at a constant velocity. Because the acceleration of the elevator is zero, the block is in equilibrium under the influence of $\vec{T}$ and $m\vec{g}$. Apply Newton's 2$^{\text{nd}}$ law of motion to the block to determine the scale reading.

(*a*) Apply $\sum F_y = ma_y$ to the block to obtain:

$$T - F_g = ma_y$$
or, because $F_g = mg$,
$$T - mg = ma_y \qquad (1)$$

For motion with constant velocity, $a_y = 0$ and:

$$T - mg = 0 \Rightarrow T = mg$$

Substitute numerical values and evaluate T:

$$T = (2.0\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{20\,\text{N}}$$

(*b*) As in part (*a*), for constant velocity, $a_y = 0$. Hence:

$$T - mg = 0$$
and
$$T = (2.0\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{20\,\text{N}}$$

(*c*) Solve equation (1) for T and simplify to obtain:

$$T = mg + ma_y = m(g + a_y) \qquad (2)$$

Because the elevator is ascending and its speed is increasing, we have $a_y = 3.0$ m/s². Substitute numerical values and evaluate T:

$$T = (2.0\,\text{kg})(9.81\,\text{m/s}^2 + 3.0\,\text{m/s}^2)$$
$$= \boxed{26\,\text{N}}$$

(d) During the interval $0 < t < 5.0$ s, $a_y = 0$. Hence:

$$T_{0 \to 5.0\,s} = \boxed{20\,\text{N}}$$

Using its definition, calculate a for $5.0\,\text{s} < t < 9.0$ s:

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 10\,\text{m/s}}{4.0\,\text{s}} = -2.5\,\text{m/s}^2$$

Substitute in equation (2) and evaluate T:

$$T_{5s \to 9s} = (2.0\,\text{kg})(9.81\,\text{m/s}^2 - 2.5\,\text{m/s}^2)$$

$$= \boxed{15\,\text{N}}$$

Free-Body Diagrams: Several Objects and Newton's Third Law

66 •• Two boxes of mass m_1 and m_2 connected by a massless string are being pulled along a horizontal frictionless surface, as shown in Figure 4-50. (a) Draw the free body diagram of both boxes separately and show that $T_1/T_2 = m_1/(m_1 + m_2)$. (b) Is this result plausible? Explain. Does your answer make sense both in the limit that $m_2/m_1 \gg 1$ and in the limit that $m_2/m_1 \ll 1$? Explain.

Picture the Problem Draw a free-body diagram for each box and apply Newton's 2nd law. Solve the resulting simultaneous equations for the ratio of T_1 to T_2.

(a) The free-body diagrams for the two boxes are shown below:
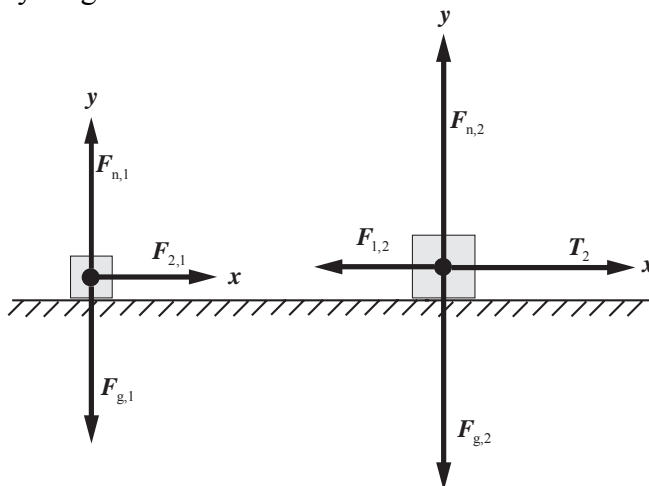
Apply $\sum F_x = ma_x$ to the box on the left:

$F_{2,1} = m_1 a_{1x}$
or, because $F_{2,1} = T_1$,
$T_1 = m_1 a_{1x}$

Apply $\sum F_x = ma_x$ to the box on the right:

$T_2 - F_{1,2} = m_2 a_{2x}$
or, because $F_{1,2} = T_1$,
$T_2 - T_1 = m_2 a_{2x}$

Because the boxes have the same acceleration, we can divide the second equation by the first to obtain:

$$\frac{T_1}{T_2 - T_1} = \frac{m_1}{m_2} \Rightarrow \frac{T_1}{T_2} = \boxed{\frac{m_1}{m_1 + m_2}}$$

(*b*) Divide the numerator and denominator of the expression inside the parentheses by m_1 to obtain:

$$\frac{T_1}{T_2} = \frac{1}{1 + \dfrac{m_2}{m_1}}$$

For $m_2/m_1 \gg 1$:

$$\frac{T_1}{T_2} \rightarrow \boxed{0}, \text{ as expected.}$$

For $m_2/m_1 \ll 1$:

$$\frac{T_1}{T_2} \rightarrow \boxed{T_2}, \text{ as expected.}$$

67 •• A box of mass $m_2 = 3.5$ kg rests on a frictionless horizontal shelf and is attached by strings to boxes of masses $m_1 = 1.5$ kg and $m_3 = 2.5$ kg as shown in Figure 4-51. Both pulleys are frictionless and massless. The system is released from rest. After it is released, find (*a*) the acceleration of each of the boxes and (*b*) the tension in each string.

Picture the Problem Call the common acceleration of the boxes *a*. Assume that box 1 moves upward, box 2 to the right, and box 3 downward and take this direction to be the positive *x* direction. Draw free-body diagrams for each of the boxes, apply Newton's 2nd law of motion, and solve the resulting equations simultaneously. Note that $T_1 = T_1'$ and $T_2 = T_2'$.
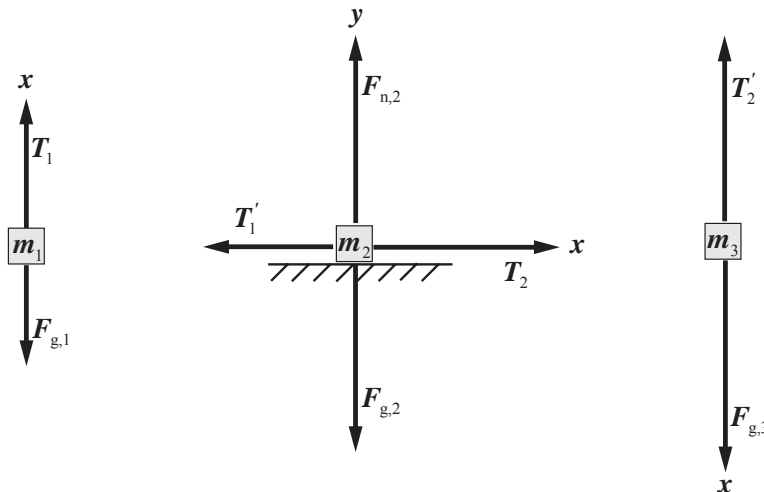
(*a*) Apply $\sum F_x = ma_x$ to the box whose mass is m_1:

$$T_1 - F_{g,1} = m_1 a_x$$
or, because $F_{g,1} = m_1 g$,
$$T_1 - m_1 g = m_1 a_x \qquad (1)$$

|	
Apply $\sum F_x = ma_x$ to the box whose mass is m_2:	$T_2 - T_1' = m_2 a_x$ or, because $T_1 = T_1'$, $T_2 - T_1 = m_2 a$ (2)
Noting that $T_2 = T_2'$, apply $\sum F_x = ma_x$ to the box whose mass is m_3:	$F_{g,3} - T_2 = m_3 a_x$ or, because $F_{g,3} = m_3 g$, $m_3 g - T_2 = m_3 a_x$ (3)
Add equations (1), (2), and (3) to obtain:	$m_3 g - m_1 g = m_1 a_x + m_2 a_x + m_3 a_x$
Solving for a_x yields:	$a_x = \dfrac{(m_3 - m_1)g}{m_1 + m_2 + m_3}$
Substitute numerical values and evaluate a_x:	$a_x = \dfrac{(2.5\,\text{kg} - 1.5\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\,\text{kg} + 3.5\,\text{kg} + 2.5\,\text{kg}}$ $= 1.31\,\text{m/s}^2$ $= \boxed{1.3\,\text{m/s}^2}$
(b) From equation (1) we have:	$T_1 = m_1 a_x + m_1 g = m_1(a_x + g)$
Substitute numerical values and evaluate T_1:	$T_1 = (1.5\,\text{kg})(1.31\,\text{m/s}^2 + 9.81\,\text{m/s}^2)$ $= \boxed{17\,\text{N}}$
From equation (3) we have:	$T_2 = m_3 g - m_3 a_x = m_3(g - a_x)$
Substitute numerical values and evaluate T_2:	$T_2 = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 1.31\,\text{m/s}^2)$ $= \boxed{21\,\text{N}}$

68 ••   Two blocks are in contact on a frictionless horizontal surface. The blocks are accelerated by a single horizontal force $\vec{F}$ applied to one of them (Figure 4-52). Find the acceleration and the contact force of block 1 on block 2 (a) in terms of F, m_1, and m_2, and (b) for the specific values $F = 3.2$ N, $m_1 = 2.0$ kg, and $m_2 = 6.0$ kg.

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{F}_{2,1}$ be the contact force exerted by the block whose mass is m_2 on the block whose mass is m_1 and $\vec{F}_{1,2}$ be the force exerted by the block whose mass is m_1 on the block

whose mass is m_2. These forces are equal and opposite so $\vec{F}_{2,1} = -\vec{F}_{1,2}$. The free-body diagrams for the blocks are shown below. Apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.
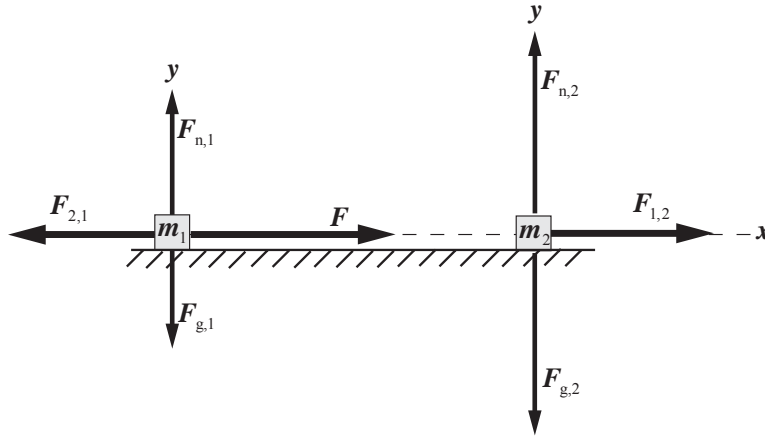
(a) Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$F - F_{2,1} = m_1 a_{1x}$

or, because the blocks have a common acceleration a_x,

$$F - F_{2,1} = m_1 a_x \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$F_{1,2} = m_2 a_x \qquad (2)$$

Adding equations (1) and (2) yields:

$F = m_1 a_x + m_2 a_x = (m_1 + m_2) a_x$

Solve for a_x to obtain:

$$a_x = \boxed{\frac{F}{m_1 + m_2}}$$

Substitute for a in equation (1) to obtain:

$$F_{1,2} = m_2 \left(\frac{F}{m_1 + m_2} \right) = \boxed{\frac{F m_2}{m_1 + m_2}}$$

(b) Substitute numerical values in the equations derived in Part (a) and evaluate a_x and $F_{1,2}$:

$$a_x = \frac{3.2\,\text{N}}{2.0\,\text{kg} + 6.0\,\text{kg}} = \boxed{0.40\,\text{m/s}^2}$$

and

$$F_{1,2} = \frac{(3.2\,\text{N})(6.0\,\text{kg})}{2.0\,\text{kg} + 6.0\,\text{kg}} = \boxed{2.4\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force F had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

69 •• Repeat Problem 68, but with the two blocks interchanged. Are your answers the same as in Problem 68? Explain.

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{F}_{2,1}$ be the contact force exerted by the block whose mass is m_2 on the block whose mass is m_1 and $\vec{F}_{1,2}$ be the force exerted by the block whose mass is m_1 on the block whose mass is m_2. These forces are equal and opposite so $\vec{F}_{2,1} = -\vec{F}_{1,2}$. The free-body diagrams for the blocks are shown. We can apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.
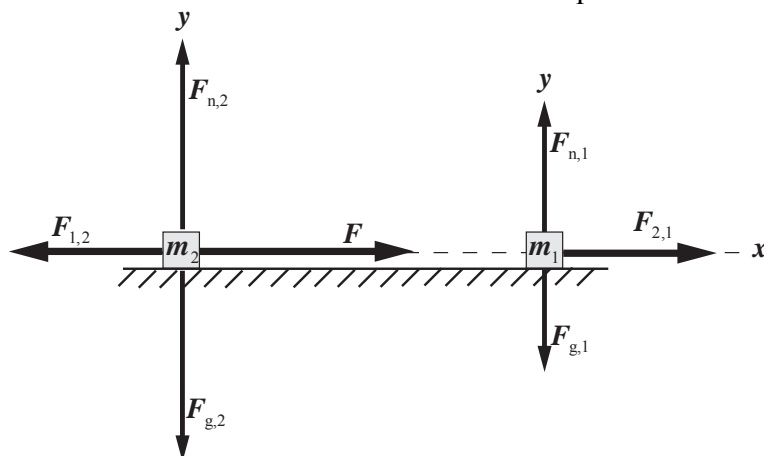
(*a*) Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$F - F_{1,2} = m_2 a_{2x}$
or, because the blocks have a common acceleration a_x,
$$F - F_{1,2} = m_2 a_x \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$F_{2,1} = m_1 a_x \qquad (2)$$

Add these equations to eliminate $F_{2,1}$ and $F_{1,2}$:

$F - F_{1,2} + F_{2,1} = m_2 a_x + m_1 a_x$
or, because $F_{1,2} = F_{2,1}$,
$$F = (m_2 + m_1) a_x$$

Solve for a_x to obtain:

$$a_x = \boxed{\dfrac{F}{m_1 + m_2}}$$

Substituting for a into equation (2) yields:

$$F_{2,1} = m_1 \left(\dfrac{F}{m_1 + m_2} \right) = \boxed{\dfrac{F m_1}{m_1 + m_2}}$$

(b) Substitute numerical values in the equations derived in part (a) and evaluate a_x and $F_{2,1}$:

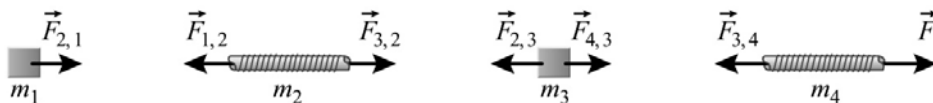$$a = \frac{3.2\,\text{N}}{2.0\,\text{kg} + 6.0\,\text{kg}} = \boxed{0.40\,\text{m/s}^2}$$

and

$$F_{2,1} = \frac{(3.2\,\text{N})(2.0\,\text{kg})}{2.0\,\text{kg} + 6.0\,\text{kg}} = \boxed{0.80\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force F had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

70   ••   Two 100-kg boxes are dragged along a horizontal frictionless surface at a constant acceleration of 1.00 m/s², as shown in Figure 4-53. Each rope has a mass of 1.00 kg. Find the force F and the tension in the ropes at points A, B, and C.

Picture the Problem The free-body diagrams for the boxes and the ropes are below. Because the vertical forces have no bearing on the problem they have not been included. Let the numeral 1 denote the 100-kg box to the left, the numeral 2 the rope connecting the boxes, the numeral 3 the box to the right and the numeral 4 the rope to which the force $\vec{F}$ is applied. $\vec{F}_{3,4}$ is the tension force exerted by the box whose mass is m_3 on the rope whose mass is m_4, $\vec{F}_{4,3}$ is the tension force exerted by the rope whose mass is m_4 on the box whose mass is m_3, $\vec{F}_{2,3}$ is the tension force exerted by the rope whose mass is m_2 on the box whose mass is m_3, $\vec{F}_{3,2}$ is the tension force exerted by the box whose mass is m_3 on the rope whose mass is m_2, $\vec{F}_{1,2}$ is the tension force exerted by the box whose mass is m_1 on the rope whose mass is m_2, and $\vec{F}_{2,1}$ is the tension force exerted by the rope whose mass is m_2 on the box whose mass is m_1. The equal and opposite pairs of forces are $\vec{F}_{2,1} = -\vec{F}_{1,2}$, $\vec{F}_{3,2} = -\vec{F}_{2,3}$, and $\vec{F}_{4,3} = -\vec{F}_{3,4}$. We can apply Newton's 2nd law to each box and rope separately and use the fact that their accelerations are equal.

$$\vec{F}_{2,1} \quad \vec{F}_{1,2} \qquad \vec{F}_{3,2} \qquad \vec{F}_{2,3}\; \vec{F}_{4,3} \qquad \vec{F}_{3,4} \qquad \vec{F}$$

$$m_1 \qquad\qquad m_2 \qquad\qquad m_3 \qquad\qquad m_4$$

Apply $\sum \vec{F} = m\vec{a}$ to the box whose mass is m_1:

$$F_{2,1} = m_1 a_1 = m_1 a \qquad\qquad (1)$$

Apply $\sum \vec{F} = m\vec{a}$ to the rope whose mass is m_2:

$$F_{3,2} - F_{1,2} = m_2 a_2 = m_2 a \qquad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the box whose mass is m_3:

$$F_{4,3} - F_{2,3} = m_3 a_3 = m_3 a \qquad (3)$$

Apply $\sum \vec{F} = m\vec{a}$ to the rope whose mass is m_4:

$$F - F_{3,4} = m_4 a_4 = m_4 a$$

Add these equations to eliminate $F_{2,1}$, $F_{1,2}$, $F_{3,2}$, $F_{2,3}$, $F_{4,3}$, and $F_{3,4}$ and solve for F:

$$F = (m_1 + m_2 + m_3 + m_4) a$$

Substitute numerical values and evaluate F:

$$F = (202\,\text{kg})(1.00\,\text{m/s}^2) = \boxed{202\,\text{N}}$$

Use equation (1) to find the tension at point A:

$$F_{2,1} = (100\,\text{kg})(1.00\,\text{m/s}^2) = \boxed{100\,\text{N}}$$

Use equation (2) to express the tension at point B:

$$F_{3,2} = F_{1,2} + m_2 a$$

Substitute numerical values and evaluate $F_{3,2}$:

$$F_{3,2} = 100\,\text{N} + (1.00\,\text{kg})(1.00\,\text{m/s}^2)$$
$$= \boxed{101\,\text{N}}$$

Use equation (3) to express the tension at point C:

$$F_{4,3} = F_{2,3} + m_3 a$$

Substitute numerical values and evaluate $F_{4,3}$:

$$F_{4,3} = 101\,\text{N} + (100\,\text{kg})(1.00\,\text{m/s}^2)$$
$$= \boxed{201\,\text{N}}$$

71 •• **[SSM]** A block of mass m is being lifted vertically by a rope of mass M and length L. The rope is being pulled upward by a force applied at its top end, and the rope and block are accelerating upward with acceleration a. The distribution of mass in the rope is uniform. Show that the tension in the rope at a distance x (where $x < L$) above the block is $(a + g)[m + (x/L)M]$.

Picture the Problem Because the distribution of mass in the rope is uniform, we can express the mass m' of a length x of the rope in terms of the total mass of the rope M and its length L. We can then express the total mass that the rope must support at a distance x above the block and use Newton's 2nd law to find the tension as a function of x.
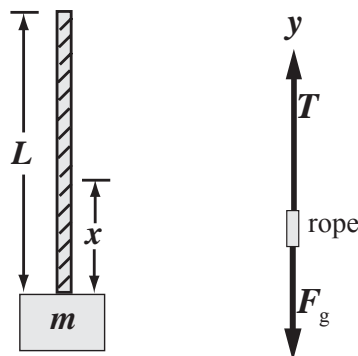
Set up a proportion expressing the mass m' of a length x of the rope as a function of M and L and solve for m':

$$\frac{m'}{x} = \frac{M}{L} \Rightarrow m' = \frac{M}{L}x$$

Express the total mass that the rope must support at a distance x above the block:

$$m + m' = m + \frac{M}{L}x$$

Apply $\sum F_y = ma_y$ to the block and a length x of the rope:

$$T - F_g = T - (m + m')g = (m + m')a_y$$

Substituting for $m + m'$ yields:

$$T - \left(m + \frac{M}{L}x\right)g = \left(m + \frac{M}{L}x\right)a_y$$

Solve for T and simplify to obtain:

$$\boxed{T = (a_y + g)\left(m + \frac{M}{L}x\right)}$$

72 •• A chain consists of 5 links, each having a mass of 0.10 kg. The chain is being pulled upward by a force applied by your hand to its top link, giving the chain an upward acceleration of 2.5 m/s^2. Find (*a*) the force F exerted on the top of the chain, (*b*) the net force on each link, and (*c*) the force each link exerts on the link below it.

Picture the Problem Choose a coordinate system with the positive y direction upward and denote the top link with the numeral 1, the second with the numeral 2, etc. The free-body diagrams show the forces acting on links 1 and 2. We can apply Newton's 2nd law to each link to obtain a system of simultaneous equations that we can solve for the force each link exerts on the link below it. Note that the net force on each link is the product of its mass and acceleration.
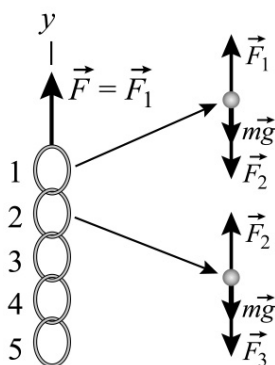
(a) Apply $\sum F_y = ma_y$ to the top link and solve for F:

$F - 5mg = 5ma$
and
$F = 5m(g + a)$

Substitute numerical values and evaluate F:

$F = 5(0.10\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2)$

$= \boxed{6.2\,\text{N}}$

(b) Apply $\sum F_y = ma_y$ to a single link:

$F_{1\text{link}} = m_{1\text{link}}a = (0.10\,\text{kg})(2.5\,\text{m/s}^2)$

$= \boxed{0.25\,\text{N}}$

(c) Apply $\sum F_y = ma_y$ to the 1st through 5th links to obtain:

$F - F_2 - mg = ma$,   (1)
$F_2 - F_3 - mg = ma$,   (2)
$F_3 - F_4 - mg = ma$,   (3)
$F_4 - F_5 - mg = ma$, and   (4)
$F_5 - mg = ma$ (5)

Add equations (2) through (5) to obtain:

$F_2 - 4mg = 4ma$

Solve for F_2 to obtain:

$F_2 = 4mg + 4ma = 4m(a + g)$

Substitute numerical values and evaluate F_2:

$F_2 = 4(0.10\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2)$

$= \boxed{4.9\,\text{N}}$

Substitute for F_2 to find F_3, and then substitute for F_3 to find F_4:

$F_3 = \boxed{3.7\,\text{N}}$ and $F_4 = \boxed{2.5\,\text{N}}$

Solve equation (5) for F_5:

$F_5 = m(g + a)$

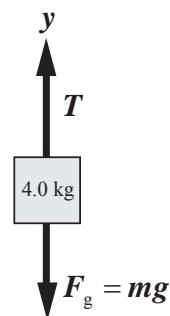Substitute numerical values and evaluate F_5:

$$F_5 = (0.10\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2)$$
$$= \boxed{1.2\,\text{N}}$$

73 •• **[SSM]** A 40.0-kg object supported by a vertical rope is initially at rest. The object is then accelerated upward from rest so that it attains a speed of 3.50 m/s in 0.700 s. (*a*) Draw the object's free body diagram with the relative lengths of the vectors showing the relative magnitudes of the forces. (*b*) Use the free body diagram and Newton's laws to determine the tension in the rope.

Picture the Problem A *net* force is required to accelerate the object. In this problem the net force is the difference between $\vec{T}$ and $\vec{F}_g(=m\vec{g})$.

(*a*) The free-body diagram of the object is shown to the right. A coordinate system has been chosen in which the upward direction is positive. The magnitude of $\vec{T}$ is approximately 1.5 times the length of $\vec{F}_g$.

(*b*) Apply $\sum F = ma_y$ to the object to obtain:

$$T - mg = ma_y$$

Solving for *T* yields:

$$T = ma + mg = m(a_y + g)$$

Using its definition, substitute for a_y to obtain:
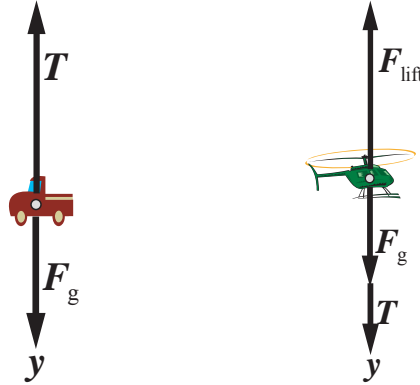
$$T = m\left(\frac{\Delta v_y}{\Delta t} + g\right)$$

Substitute numerical values and evaluate *T*:

$$T = (40.0\,\text{kg})\left(\frac{3.50\,\text{m/s}}{0.700\,\text{s}} + 9.81\,\text{m/s}^2\right)$$
$$= \boxed{592\,\text{N}}$$

74 •• A 15 000-kg helicopter is lowering a 4000-kg truck by a cable of fixed length. The truck, helicopter, and cable are descending at 15.0 m/s and must be slowed to 5.00 m/s in the next 50.0 m of descent to prevent damaging the truck. Assume a constant rate of slowing. (*a*) Draw the free body diagram of the truck. (*b*) Determine the tension in the cable. (*c*) Determine the lift force of the helicopter blades.

Picture the Problem A net force in the upward direction is required to slow the truck's descent. This net force is the difference between $\vec{T}$ and $\vec{F}_{\text{g}}$. We can use Newton's 2$^{\text{nd}}$ law and a constant-acceleration equation to find the tension in the cable that supports the truck.

(*a*) Free-body diagrams showing the forces acting on the truck and on the helicopter are shown to the right. A coordinate system in which the downward direction is positive has been chosen. Note that, because it is hovering, the helicopter is in equilibrium under the influence of the forces $\vec{F}_{\text{lift}}$, $\vec{F}_{\text{g}}$, and $\vec{T}$.

(*b*) Apply $\sum F_y = ma_y$ to the truck to obtain:

$$F_{\text{g}} - T = m_{\text{t}}a_y$$
or, because $F_{\text{g}} = m_{\text{t}}g$,
$$m_{\text{t}}g - T = m_{\text{t}}a_y$$

Solve for the tension in the lower portion of the cable:

$$T = m_{\text{t}}g - m_{\text{t}}a_y = m_{\text{t}}\left(g - a_y\right) \quad (1)$$

Use a constant-acceleration equation to relate the truck's initial and final speeds to its displacement and acceleration:

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y \Rightarrow a_y = \frac{v_y^2 - v_{0y}^2}{2\Delta y}$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{\left(5.00\,\text{m/s}\right)^2 - \left(15.0\,\text{m/s}\right)^2}{2\left(50.0\,\text{m}\right)}$$
$$= -2.000\,\text{m/s}^2$$

Substitute numerical values in equation (1) and evaluate T:

$$T = \left(4000\,\text{kg}\right)\left(9.81\,\text{m/s}^2 + 2.000\,\text{m/s}^2\right)$$
$$= 47.24\,\text{kN}$$
$$= \boxed{47.2\,\text{kN}}$$

(*c*) Apply $\sum F_y = ma_y$ to the helicopter to obtain:

$$m_{\text{h}}g + T - F_{\text{lift}} = m_{\text{h}}a_y$$

Solving for F_{lift} yields:

$$F_{\text{lift}} = m_{\text{h}}g + T - m_{\text{h}}a_y = m_{\text{h}}\left(g - a_y\right) + T$$

Substitute for T from Part (a):

$$F_{\text{lift}} = m_h\left(g - a_y\right) + m_t\left(g - a_y\right)$$
$$= \left(m_t + m_h\right)\left(g - a_y\right)$$

Substitute numerical values and evaluate F_{lift}:

$$F_{\text{lift}} = \left(4000\ \text{kg} + 15000\ \text{kg}\right)\left(9.81\ \text{m/s}^2 + 2.00\ \text{m/s}^2\right) = \boxed{224\ \text{kN}}$$

75 •• Two objects are connected by a massless string, as shown in Figure 4-54. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string for (a) in terms of θ, m_1 and m_2, and (b) $\theta = 30°$ and $m_1 = m_2 = 5.0$ kg.

Picture the Problem Because the string does not stretch or become slack, the two objects must have the same speed and therefore the magnitude of the acceleration is the same for each object. Choose a coordinate system in which up the incline is the positive x direction for the object of mass m_1 and downward is the positive x direction for the object of mass m_2. This idealized pulley acts like a piece of polished pipe; i.e., its only function is to change the direction the tension in the massless string acts. Draw a free-body diagram for each of the two objects, apply Newton's 2nd law to both objects, and solve the resulting equations simultaneously.

(a) The free-body diagrams for the two objects are shown to the right:
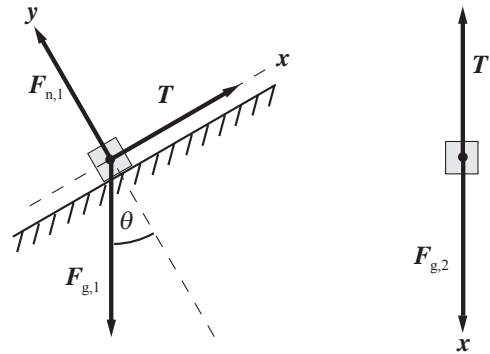
Apply $\sum F_x = ma_x$ to the object whose mass is m_1:

$$T - F_{g,1}\sin\theta = m_1 a_{1x}$$

or, because $F_{g,1} = m_1 g$ and the two objects have a common acceleration a,

$$T - m_1 g\sin\theta = m_1 a \qquad (1)$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_2 to obtain:

$$m_2 g - T = m_2 a_x \qquad (2)$$

Adding equations (1) and (2) yields:

$$m_2 g - m_1 g\sin\theta = m_1 a_x + m_2 a_x$$

Solve for a_x to obtain:

$$a_x = \boxed{\frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}} \qquad (3)$$

Substitute for a in either of the equations containing the tension and solve for T to obtain:

$$T = \boxed{\frac{gm_1m_2(1+\sin\theta)}{m_1 + m_2}} \qquad (4)$$

(b) Substitute numerical values in equation (3) and evaluate a_x:

$$a_x = \frac{(9.81\,\text{m/s}^2)(5.0\,\text{kg} - (5.0\,\text{kg})\sin 30°)}{5.0\,\text{kg} + 5.0\,\text{kg}}$$

$$= \boxed{2.5\,\text{m/s}^2}$$

Substitute numerical values in equation (4) and evaluate T:

$$T = \frac{(9.81\,\text{m/s}^2)(5.0\,\text{kg})^2(1+\sin 30°)}{5.0\,\text{kg} + 5.0\,\text{kg}}$$

$$= \boxed{37\,\text{N}}$$

**76** •• During a stage production of *Peter Pan*, the 50-kg actress playing Peter has to fly in vertically (descend). To be in time with the music, she must be lowered, starting from rest, a distance of 3.2 m in 2.2 s at constant acceleration. Backstage, a smooth surface sloped at 50° supports a counterweight of mass m, as shown in Figure 4-55. Show the calculations that the stage manager must perform to find (a) the mass of the counterweight that must be used and (b) the tension in the wire.

Picture the Problem The magnitudes of the accelerations of Peter and the counterweight are the same. Choose a coordinate system in which up the incline is the $+x$ direction for the counterweight and downward is the $+x$ direction for Peter. The pulley changes the direction the tension in the rope acts. Let Peter's mass be m_P. Ignoring the mass of the rope, draw free-body diagrams for the counterweight and Peter, apply Newton's 2nd law to each of them, and solve the resulting equations simultaneously.

(a) Using a constant-acceleration equation, relate Peter's displacement to her acceleration and descent time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 \implies a_x = \frac{2\Delta x}{(\Delta t)^2}$$

The free-body diagram for the
counterweight is shown to the right:
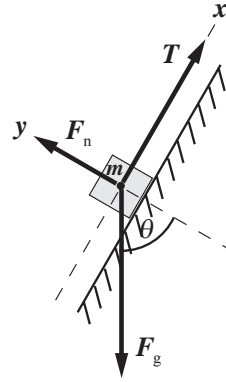
Apply $\sum F_x = ma_x$ to the
counterweight:

$$T - mg\sin 50° = ma_x$$

The free-body diagram for Peter is
shown to the right:

Noting that $T' = T$, apply
$\sum F_x = ma_x$ to Peter:

$$m_P g - T = m_P a_x$$

Adding the two equations and
solving for m yields:

$$m = \frac{m_P(g - a_x)}{a_x + g\sin 50°}$$

Substituting for a_x yields:

$$m = \frac{m_P\left(g - \dfrac{2\Delta x}{(\Delta t)^2}\right)}{\dfrac{2\Delta x}{(\Delta t)^2} + g\sin 50°}$$

Substitute numerical values and
evaluate m:

$$m = \frac{(50\,\text{kg})\left(9.81\,\text{m/s}^2 - \dfrac{2(3.2\,\text{m})}{(2.2\,\text{s})^2}\right)}{\dfrac{2(3.2\,\text{m})}{(2.2\,\text{s})^2} + 9.81\,\text{m/s}^2\sin 50°}$$

$$= 48.0\,\text{kg} = \boxed{48\,\text{kg}}$$

(b) Substitute for m in the force equation for the counterweight and solve for T to obtain:

$$T = m\left(a_x + g\sin 50°\right)$$

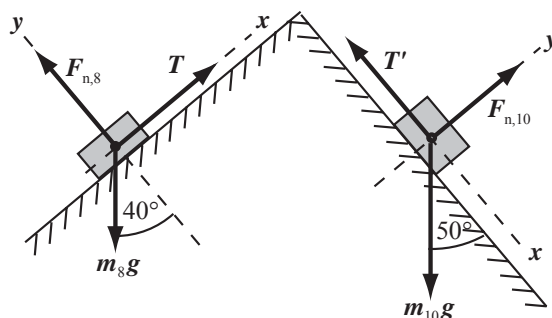Substitute numerical values and evaluate T:

$$T = \left(48.0\,\text{kg}\right)\left[1.32\,\text{m/s}^2 + \left(9.81\,\text{m/s}^2\right)\sin 50°\right] = \boxed{0.42\,\text{kN}}$$

77 ••   An 8.0-kg block and a 10-kg block, connected by a rope that passes over a frictionless peg, slide on frictionless inclines (Figure 4-56). (a) Find the acceleration of the blocks and the tension in the rope. (b) The two blocks are replaced by two others of masses m_1 and m_2 such that there is no acceleration. Find whatever information you can about the masses of these two new blocks.

Picture the Problem The magnitude of the accelerations of the two blocks are the same. Choose a coordinate system in which up the incline is the $+x$ direction for the 8.0-kg object and downward is the $+x$ direction for the 10-kg object. The peg changes the direction the tension in the rope acts. Draw free-body diagrams for each object, apply Newton's 2$^{\text{nd}}$ law of motion to both of them, and solve the resulting equations simultaneously.

(a) Apply $\sum F_x = ma_x$ to the 8.0-kg block:

$$T - m_8 g\sin 40° = m_8 a_x \qquad (1)$$

Noting that $T = T'$, apply $\sum F_x = ma_x$ to the 10-kg block:

$$m_{10} g\sin 50° - T = m_{10} a_x \qquad (2)$$

Add equations (1) and (2) to obtain:

$$m_{10} g\sin 50° - m_8 g\sin 40°$$
$$= \left(m_{10} + m_8\right)a_x$$

Solving for a_x yields:

$$a_x = \frac{g\left(m_{10}\sin 50° - m_8\sin 40°\right)}{m_8 + m_{10}}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{\left(9.81\,\text{m/s}^2\right)\left((10\,\text{kg})\sin 50° - (8.0\,\text{kg})\sin 40°\right)}{8.0\,\text{kg} + 10\,\text{kg}} = 1.37\,\text{m/s}^2 = \boxed{1.4\,\text{m/s}^2}$$

Solving the first of the two force equations for T yields:

$$T = m_8\left(g\sin 40° + a_x\right)$$

Substitute numerical values and evaluate T:

$$T = (8.0\,\text{kg})\left[\left(9.81\,\text{m/s}^2\right)\sin 40° + 1.37\,\text{m/s}^2\right] = \boxed{61\,\text{N}}$$

(b) Because the system is in equilibrium, $a_x = 0$, and equations (1) and (2) become:

$$T - m_1 g \sin 40° = 0 \qquad\qquad (3)$$
and
$$m_2 g \sin 50° - T = 0 \qquad\qquad (4)$$

Adding equations (3) and (4) yields:

$$m_2 g \sin 50° - m_1 g \sin 40° = 0$$

Solve for and evaluate the ratio m_1/m_2 to obtain:

$$\frac{m_1}{m_2} = \frac{\sin 50°}{\sin 40°} = \boxed{1.19}$$

78 •• A heavy rope of length 5.0 m and mass 4.0 kg lies on a frictionless horizontal table. One end is attached to a 6.0-kg block. The other end of the rope is pulled by a constant horizontal 100-N force. (a) What is the acceleration of the system? (b) Give the tension in the rope as a function of position along the rope.

Picture the Problem The pictorial representations shown below summarize the information given in this problem. While the mass of the rope is distributed over its length, the rope and the 6.0-kg block have a common acceleration. Choose a coordinate system in which the direction of the 100-N force is the positive x direction. Because the surface is horizontal and frictionless, the only force that influences our solution is the 100-N force.
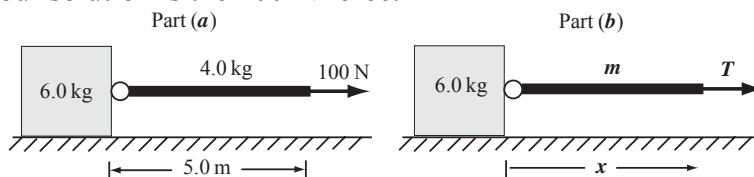
(a) Apply $\sum F_x = ma_x$ to the system shown for Part (a):

$$100\,\text{N} = (m_1 + m_2)a_x$$

Solve for a_x to obtain:

$$a_x = \frac{100\,\text{N}}{m_1 + m_2}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{100\,\text{N}}{4.0\,\text{kg} + 6.0\,\text{kg}} = \boxed{10\,\text{m/s}^2}$$

(*b*) Let *m* represent the mass of a length *x* of the rope. Assuming that the mass of the rope is uniformly distributed along its length:

$$\frac{m}{x} = \frac{m_2}{L_{\text{rope}}} = \frac{4.0\,\text{kg}}{5.0\,\text{m}} \Rightarrow m = \left(\frac{4.0\,\text{kg}}{5.0\,\text{m}}\right)x$$

Apply $\sum F_x = ma_x$ to the block in Part (*b*) to obtain:

$$T = (m_1 + m)a_x$$

Substituting for *m* yields:

$$T = \left(m_1 + \left(\frac{4.0\,\text{kg}}{5.0\,\text{m}}\right)x\right)a_x$$

Substitute numerical values and evaluate *T*:

$$T = \left[6.0\,\text{kg} + \left(\frac{4.0\,\text{kg}}{5.0\,\text{m}}\right)x\right](10\,\text{m/s}^2)$$

$$= \boxed{60\,\text{N} + (8.0\,\text{N/m})x}$$

79 •• **[SSM]** A 60-kg housepainter stands on a 15-kg aluminum platform. The platform is attached to a rope that passes through an overhead pulley, which allows the painter to raise herself and the platform (Figure 4-57). (*a*) To accelerate herself and the platform at a rate of 0.80 m/s², with what force *F* must she pull down on the rope? (*b*) When her speed reaches 1.0 m/s, she pulls in such a way that she and the platform go up at a constant speed. What force is she exerting on the rope now? (Ignore the mass of the rope.)

Picture the Problem Choose a coordinate system in which the upward direction is the positive *y* direction. Note that $\vec{F}$ is the force exerted by the painter on the rope and that $\vec{T}$ is the resulting tension in the rope. Hence the net upward force on the painter-plus-platform is $2\vec{T}$.

(*a*) Letting $m_{\text{tot}} = m_{\text{frame}} + m_{\text{painter}}$, apply $\sum F_y = ma_y$ to the frame-plus-painter:

$$2T - m_{\text{tot}}g = m_{\text{tot}}a_y$$

Solving for *T* yields:

$$T = \frac{m_{\text{tot}}(a_y + g)}{2}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(75\,\text{kg})(0.80\,\text{m/s}^2 + 9.81\,\text{m/s}^2)}{2}$$

$$= 398\,\text{N}$$

Because $F = T$: $F = 398\,\text{N} = \boxed{0.40\,\text{kN}}$

(b) Apply $\sum F_y = 0$ to obtain: $2T - m_{\text{tot}}g = 0 \Rightarrow T = \tfrac{1}{2}m_{\text{tot}}g$

Substitute numerical values and $T = \tfrac{1}{2}(75\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{0.37\,\text{kN}}$
evaluate T:

80 ••• Figure 4-58 shows a 20-kg block sliding on a 10-kg block. All surfaces are frictionless and the pulley is massless and frictionless. Find the acceleration of each block and the tension in the string that connects the blocks.

Picture the Problem Choose a coordinate system in which up the incline is the $+x$ direction and draw free-body diagrams for each block. Noting that $\vec{a}_{20} = -\vec{a}_{10}$, apply Newton's 2$^{\text{nd}}$ law to each block and solve the resulting equations simultaneously.
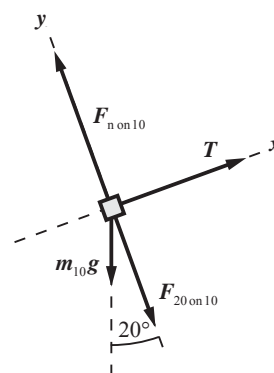
Draw a free-body diagram for the 20-kg block:
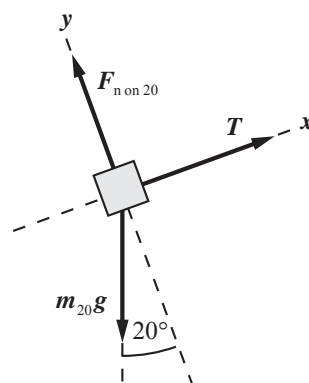
Apply $\sum F_x = ma_x$ to the block to obtain: $T - m_{20}g\sin 20° = m_{20}a_{20,x}$ (1)

Draw a free-body diagram for the 10-kg block. Because all the surfaces, including the surfaces between the blocks, are frictionless, the force the 20-kg block exerts on the 10-kg block must be normal to their surfaces as shown to the right.

Apply $\sum F_x = ma_x$ to the block to obtain: $T - m_{10}g\sin 20° = m_{10}a_{10,x}$ (2)

Because the blocks are connected by a taut string:	$a_{20,x} = -a_{10,x}$

Substituting for $a_{20,x}$ in equation (1) yields:	$T - m_{20}g \sin 20° = -m_{20}a_{10,x}$ (3)

Eliminate T between equations (2) and (3) to obtain:	$a_{10,x} = \left(\dfrac{m_{20} - m_{10}}{m_{20} + m_{10}} \right) g \sin 20°$

Substitute numerical values and evaluate $a_{10,x}$:	$a_{10,x} = \left(\dfrac{20\,\text{kg} - 10\,\text{kg}}{20\,\text{kg} + 10\,\text{kg}} \right) (9.81\,\text{m/s}^2) \sin 20°$ $= \boxed{1.1\,\text{m/s}^2}$

Because $a_{20,x} = -a_{10,x}$:	$a_{20,x} = \boxed{-1.1\,\text{m/s}^2}$

|	
Substitute for either of the accelerations in the force equations and solve for T:	$T = \boxed{45\,\text{N}}$

81 ••• A 20-kg block with a pulley attached slides along a frictionless ledge. It is connected by a massless string to a 5.0-kg block via the arrangement shown in Figure 4-59. Find (*a*) the acceleration of each block and (*b*) the tension in the connecting string.

Picture the Problem Choose a coordinate system in which the $+x$ direction is to the right and draw free-body diagrams for each block. Because of the pulley, the string exerts a force of $2T$. Apply Newton's 2$^{\text{nd}}$ law of motion to both blocks and solve the resulting equations simultaneously.

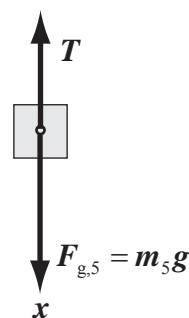|	
(*a*) Noting the effect of the pulley, express the distance the 20-kg block moves in a time Δt:	$\Delta x_{20} = \frac{1}{2}\Delta x_5 = \frac{1}{2}(10\,\text{cm}) = \boxed{5.0\,\text{cm}}$

(*b*) Draw a free-body diagram for the 20-kg block:	
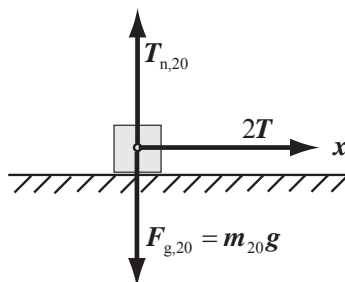
Apply $\sum F_x = ma_x$ to the block to obtain:	$2T = m_{20}a_{20,x}$

Draw a free-body diagram for the
5.0-kg block:

$$\uparrow T$$

$$F_{g,5} = m_5 g$$

$$x$$

Apply $\sum F_x = ma_x$ to the block to
obtain:

$$m_5 g - T = m_5 a_{5,x}$$

Using a constant-acceleration
equation, relate the displacement of
the 5.0-kg block to its acceleration
and the time during which it is
accelerated:

$$\Delta x_5 = \tfrac{1}{2} a_{5,x} (\Delta t)^2$$

Using a constant-acceleration
equation, relate the displacement of
the 20-kg block to its acceleration
and acceleration time:

$$\Delta x_{20} = \tfrac{1}{2} a_{20,x} (\Delta t)^2$$

Divide the first of these equations by
the second to obtain:

$$\frac{\Delta x_5}{\Delta x_{20}} = \frac{\tfrac{1}{2} a_{5,x} (\Delta t)^2}{\tfrac{1}{2} a_{20,x} (\Delta t)^2} = \frac{a_{5,x}}{a_{20,x}}$$

Use the result of Part (*a*) to obtain:

$$a_{5,x} = 2a_{20,x}$$

Let $a_{20,x} = a$. Then $a_{5,x} = 2a$ and the
force equations become:

$$2T = m_{20} a$$
and
$$m_5 g - T = m_5 (2a)$$

Eliminate *T* between the two
equations to obtain:

$$a = a_{20,x} = \frac{m_5 g}{2m_5 + \tfrac{1}{2} m_{20}}$$

Substitute numerical values and
evaluate $a_{20,x}$ and $a_{5,x}$:

$$a_{20,x} = \frac{(5.0\,\text{kg})(9.81\,\text{m/s}^2)}{2(5.0\,\text{kg}) + \tfrac{1}{2}(20\,\text{kg})} = 2.45\,\text{m/s}^2$$

$$= \boxed{2.5\,\text{m/s}^2}$$

and

$$a_5 = 2(2.45\,\text{m/s}^2) = \boxed{4.9\,\text{m/s}^2}$$

Substitute for either of the accelerations in either of the force equations and solve for T:

$$T = \boxed{25\,\text{N}}$$

82 •• The apparatus in Figure 4-60 is called an *Atwood's machine* and is used to measure the free-fall acceleration g by measuring the acceleration of the two blocks connected by a string over a pulley. Assume a massless, frictionless pulley and a massless string. (*a*) Draw a free-body diagram of each block. (*b*) Use the free-body diagrams and Newton's laws to show that the magnitude of the acceleration of either block and the tension in the string are $a = (m_1 - m_2)g/(m_1 + m_2)$ and $T = 2m_1 m_2 g/(m_1 + m_2)$. (*c*) Do these expressions give plausible results if $m_1 = m_2$, in the limit that $m_1 \gg m_2$, and in the limit that $m_1 \ll m_2$? Explain.

Picture the Problem Assume that $m_1 > m_2$. Choose a coordinate system in which the $+y$ direction is downward for the block whose mass is m_1 and upward for the block whose mass is m_2 and draw free-body diagrams for each block. Apply Newton's 2nd law to both blocks and solve the resulting equations simultaneously.

(*a*) The free-body diagrams for the two blocks are shown to the right:
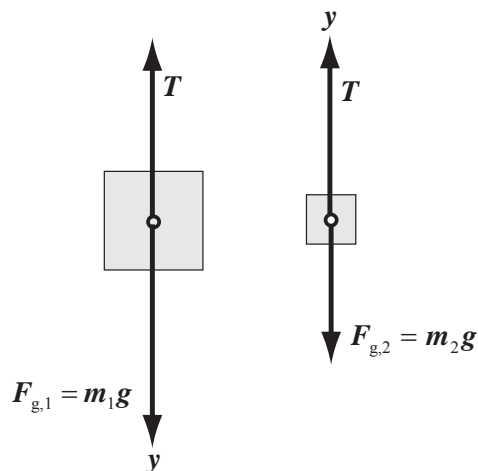
(*b*) Apply $\sum F_y = ma_y$ to the block whose mass is m_2:

$$T - m_2 g = m_2 a_{2y} \tag{1}$$

Apply $\sum F_y = ma_y$ to the block whose mass is m_1:

$$m_1 g - T = m_1 a_{1y} \tag{2}$$

Because the blocks are connected by a taut string, they have the same acceleration. Let a represent their common acceleration. Then equations (1) and (2) become:

$$T - m_2 g = m_2 a$$
and
$$m_1 g - T = m_1 a$$

Adding these equations eliminates T and yields:

$$m_1 g - m_2 g = m_1 a + m_2 a$$

Solve for a to obtain:

$$a = \boxed{\frac{m_1 - m_2}{m_1 + m_2}g} \qquad (3)$$

Substituting for a in either of the force equations and solving for T yields:

$$T = \boxed{\frac{2m_1 m_2 g}{m_1 + m_2}} \qquad (4)$$

(c) For $m_1 = m_2$, equations (3) and (4) become:

$$a = \frac{0}{m_1 + m_2}g = 0 \text{ and } T = mg \text{ ... as expected.}$$

Divide the numerators and denominators if equations (3) and (4) by m_1 to obtain:

$$a = \left(\frac{1 - \dfrac{m_2}{m_1}}{1 + \dfrac{m_2}{m_1}}\right)g \text{ and } T = \left(\frac{2m_2}{1 + \dfrac{m_2}{m_1}}\right)g$$

For $m_1 \gg m_2$ these equations become:

$$a = g \text{ and } T = 2m_2 g \text{ ... as expected.}$$

Divide the numerators and denominators if equations (3) and (4) by m_2 to obtain:

$$a = \left(\frac{\dfrac{m_1}{m_2} - 1}{\dfrac{m_1}{m_2} + 1}\right)g \text{ and } T = \frac{2m_1 g}{\dfrac{m_1}{m_2} + 1}$$

For $m_1 \ll m_2$ these equations become:

$$a = -g \text{ and } T = 2m_1 g \text{ ... as expected.}$$

83 •• If one of the masses of the Atwood's machine in Figure 4-60 is 1.2 kg, what should be the other mass so that the displacement of either mass during the first second following release is 0.30 m? Assume a massless, frictionless pulley and a massless string.

Picture the Problem The acceleration can be found from the given displacement during the first second. The ratio of the two masses can then be found from the acceleration using the first of the two equations derived in Problem 82 relating the acceleration of the Atwood's machine to its masses.

From Problem 82 we have:

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

Solving for m_1 yields:

$$m_1 = m_2 \left(\frac{g + a}{g - a}\right) \qquad (1)$$

Using a constant-acceleration equation, relate the displacement of the masses to their acceleration and solve for the acceleration:

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$\Delta y = \tfrac{1}{2}a_y(\Delta t)^2 \Rightarrow a_y = \frac{2\Delta y}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

$$a_y = \frac{2(0.30\,\text{m})}{(1.0\,\text{s})^2} = 0.600\,\text{m/s}^2$$

Substitute numerical values in equation (1) to obtain:

$$m_1 = m_2\left(\frac{9.81\,\text{m/s}^2 + 0.600\,\text{m/s}^2}{9.81\,\text{m/s}^2 - 0.600\,\text{m/s}^2}\right)$$

$$= 1.13m_2$$

Find the second value for m_2 for $m_1 = 1.2$ kg:

$$m_{2\text{nd mass}} = \boxed{1.4\,\text{kg or } 1.1\,\text{kg}}$$

84 ••• The acceleration of gravity g can be determined by measuring the time t it takes for a mass m_2 in an Atwood's machine described in Problem 82 to fall a distance L, starting from rest. (*a*) Using the results of Problem 82 (Note the acceleration is constant.), find an expression for g in terms of m_1, m_2, L, and t. (*b*) Show that a small error in the time measurement dt, will lead to an error in g by an amount dg given by $dg/g = -2dt/t$. (*c*) Assume that the only significant uncertainty in the experimental measurements is the time of fall. If $L = 3.00$ m and m_1 is 1.00 kg, find the value of m_2 such that g can be measured with an accuracy of $\pm 5\%$ with a time measurement that is accurate to ± 0.1 s.

Picture the Problem Use a constant-acceleration equation to relate the displacement of the descending (or rising) mass as a function of its acceleration and then use one of the results from Problem 82 to relate a to g. Differentiation of our expression for g will allow us to relate uncertainty in the time measurement to uncertainty in the measured value for g … and to the values of m_2 that would yield an experimental value for g that is good to within $\pm 5\%$.

(*a*) From Problem 82 we have:

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

Solving for g yields:

$$g = a\left(\frac{m_1 + m_2}{m_1 - m_2}\right) \qquad (1)$$

Using a constant-acceleration equation, express the displacement, L, as a function of t:

$$\Delta y = L = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $\Delta t = t$,

$$L = \tfrac{1}{2}a_y t^2 \Rightarrow a_y = \frac{2L}{t^2} \qquad (2)$$

Substitute for a_y in equation (1) to obtain:

$$g = \boxed{\frac{2L}{t^2}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)}$$

(b) Evaluate dg/dt to obtain:

$$\frac{dg}{dt} = -4Lt^{-3}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)$$

$$= \frac{-2}{t}\left[\frac{2L}{t^2}\right]\left(\frac{m_1 + m_2}{m_1 - m_2}\right) = \frac{-2g}{t}$$

Divide both sides of this expression by g and multiply both sides by dt to separate the variables:

$$\frac{dg}{g} = \boxed{-2\frac{dt}{t}}$$

(c) Because $\dfrac{dg}{g} = \pm 0.05$:

$$t = \left|\frac{-2dt}{dg/g}\right| = \left|\frac{-2(\pm 0.1\,\text{s})}{\pm 0.05}\right| = 4\,\text{s}$$

Substitute numerical values in equation (2) and evaluate a:

$$a_y = \frac{2(3.00\,\text{m})}{(4\,\text{s})^2} = 0.375\,\text{m/s}^2$$

Solve equation (1) for m_2 to obtain:

$$m_2 = \frac{g - a}{g + a}m_1$$

Evaluate m_2 with $m_1 = 1.00$ kg:

$$m_2 = \frac{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}(1.00\,\text{kg})$$

$$\approx 0.9\,\text{kg}$$

Solve equation (1) for m_1 to obtain:

$$m_1 = m_2\left(\frac{g + a_y}{g - a_y}\right)$$

Substitute numerical values to obtain:

$$m_1 = (0.926\,\text{kg})\left(\frac{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2}\right)$$

$$\approx 1\,\text{kg}$$

Because the masses are interchangeable:

$$m_2 = \boxed{0.9\,\text{kg or }1\,\text{kg}}$$

General Problems

85 •• A pebble of mass m rests on the block of mass m_2 of the ideal Atwood's machine in Figure 4-60. Find the force exerted by the pebble on the block of mass m_2.

Picture the Problem We can apply Newton's 2nd law to the pebble and use the expression for the acceleration derived in Problem 82 to find the force $\vec{F}_{\text{by } m_2}$ exerted by the block of mass m_2 on the pebble. The forces $\vec{F}_{\text{on } m_2}$ exerted by the pebble on the block of mass m_2 and $\vec{F}_{\text{by } m_2}$ exerted by the block on the pebble constitute a third law pair and are equal in magnitude.

(*a*) The upward direction has been chosen as the +*y* direction in the free-body diagram for the pebble shown to the right. The forces acting on the pebble are the force exerted by the object whose mass is m_2 ($\vec{F}_{\text{by } m_2}$) and the gravitational force ($\vec{F}_g = m\vec{g}$) exerted by the Earth.
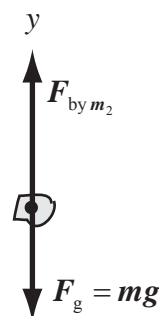
(*b*) Apply $\Sigma F_y = ma_y$ to the pebble to obtain:

$$F_{\text{by } m_2} - mg = (m + m_2)a$$

or, solving for $F_{\text{by } m_2}$,

$$F_{\text{by } m_2} = (m + m_2)a + mg$$

Because $\vec{F}_{\text{by } m_2}$ and $\vec{F}_{\text{on } m_2}$ constitute a third law pair:

$$F_{\text{on } m_2} = (m + m_2)a + mg \qquad (1)$$

From Problem 82, the acceleration of the blocks is given by:

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

With a pebble of mass *m* resting on the block of mass m_2, the expression for the acceleration becomes:

$$a = \frac{m_1 - (m + m_2)}{m_1 + (m + m_2)} g$$

Substituting for *a* in equation (1) and simplifying yields:

$$F_{\text{on } m_2} = \frac{(m + m_2)[m_1 - (m + m_2)]}{m_1 + (m + m_2)} g + mg = \boxed{\left(\frac{m^2 + m_1^2 + m_2^2}{m + m_1 + m_2}\right) g}$$

86 •• A simple accelerometer can be made by suspending a small massive object from a string attached to a fixed point on an accelerating object. Suppose such an accelerometer is attached to point *P* on the ceiling of an automobile traveling in a straight line on a flat surface at constant acceleration. Due to the acceleration, the string will make an angle θ with the vertical. (*a*) Show that the magnitude of the acceleration *a* is related to the angle θ by $a = g \tan \theta$. (*b*) Suppose the automobile brakes steadily to rest from 50 km/h over a distance of 60 m. What angle will the string make with the vertical? Will the suspended

object be positioned below and ahead (or below and behind) point P during the braking?

Picture the Problem The free-body diagram shown below and to the left shows the forces acting on an object suspended from the ceiling of a car that is gaining speed to the right. Choose the coordinate system shown and use Newton's 2nd law and constant- acceleration equations to describe the influence of the forces acting on the suspended object on its motion. The forces acting on the object are the gravitational force $m\vec{g}$ and the tension in the string $\vec{T}$. The free-body diagram shown to the right shows the forces acting on an object suspended from the ceiling of a car that is braking while moving to the right.
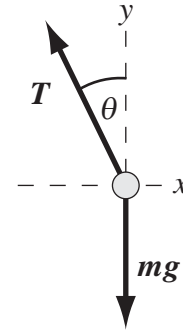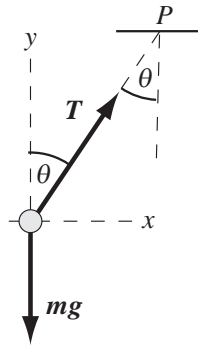
(a) Apply $\sum \vec{F} = m\vec{a}$ to the object:

$$\sum F_x = T \sin\theta = ma_x$$
and
$$\sum T_y = T \cos\theta - mg = 0$$

Take the ratio of these two equations to eliminate T and m:

$$\frac{T \sin\theta}{T \cos\theta} = \frac{ma_x}{mg}$$

or

$$\tan\theta = \frac{a_x}{g} \Rightarrow a_x = \boxed{g \tan\theta}$$

(b) Solve the equation derived in (a) for θ:

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) \qquad (2)$$

Using a constant-acceleration equation, express the velocity of the car in terms of its acceleration and solve for the acceleration:

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$
or, because $v_x = 0$,
$$0 = v_{0x}^2 + 2a_x \Delta x \Rightarrow a_x = \frac{-v_{0x}^2}{2\Delta x}$$

Substitute for a_x in equation (2) and simplify to obtain:

$$\theta = \tan^{-1}\left(\frac{-v_{0x}^2}{\dfrac{2\Delta x}{g}}\right) = \tan^{-1}\left(\frac{-v_{0x}^2}{2g\Delta x}\right)$$
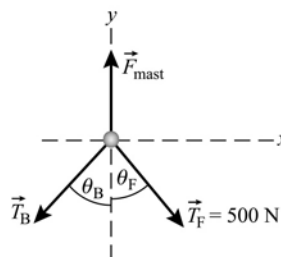
Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{-\left(50\,\dfrac{km}{h} \times \dfrac{1\,h}{3600\,s}\right)^2}{2\left(9.81\,\dfrac{m}{s^2}\right)(60\,m)}\right)$$

$$= \boxed{9.3°}$$

The suspended object will be positioned below and ahead (by 9.3°) of point P during the braking.

87   ••   **[SSM]**   The mast of a sailboat is supported at bow and stern by stainless steel wires, the forestay and backstay, anchored 10 m apart (Figure 4-61). The 12.0-m-long mast weighs 800 N and stands vertically on the deck of the boat. The mast is positioned 3.60 m behind where the forestay is attached. The tension in the forestay is 500 N. Find the tension in the backstay and the force that the mast exerts on the deck.

Picture the Problem The free-body diagram shows the forces acting at the top of the mast. Choose the coordinate system shown and use Newton's 2nd and 3rd laws of motion to analyze the forces acting on the deck of the sailboat.

Apply $\sum F_x = ma_x$ to the top of the mast:

$$T_F \sin\theta_F - T_B \sin\theta_B = 0$$

Find the angles that the forestay and backstay make with the vertical:

$$\theta_F = \tan^{-1}\left(\frac{3.60\,m}{12.0\,m}\right) = 16.7°$$

and

$$\theta_B = \tan^{-1}\left(\frac{6.40\,m}{12.0\,m}\right) = 28.1°$$

Solving the x-direction equation for T_B yields:

$$T_B = T_F \frac{\sin\theta_F}{\sin\theta_B}$$

Substitute numerical values and evaluate T_B:

$$T_B = (500\,\text{N})\frac{\sin 16.7°}{\sin 28.1°} = \boxed{305\,\text{N}}$$

Apply $\Sigma F_y = 0$ to the mast:

$$\sum F_y = F_{\text{mast}} - T_F \cos \theta_F - T_B \cos \theta_B = 0$$

Solve for F_{mast} to obtain:

$$F_{\text{mast}} = T_F \cos \theta_F + T_B \cos \theta_B$$

Substitute numerical values and evaluate F_{mast}:

$$F_{\text{mast}} = (500\,\text{N})\cos 16.7° + (305\,\text{N})\cos 28.1° = 748\,\text{N}$$

The force that the mast exerts on the deck is the sum of its weight and the downward forces exerted on it by the forestay and backstay:

$$F_{\text{mast on the deck}} = 748\,\text{N} + 800\,\text{N}$$
$$= \boxed{1.55\,\text{kN}}$$

88 •• A 50-kg block is suspended from a uniform chain that is hanging from the ceiling. The mass of the chain itself is 20 kg, and the length of the chain is 1.5 m. Determine the tension in the chain (a) at the point where the chain is attached to the block, (b) midway up the chain, and (c) at the point where the chain is attached to the ceiling.

Picture the Problem Let m be the mass of the block and M be the mass of the chain. The free-body diagrams shown below display the forces acting at the locations identified in the problem. We can apply Newton's 2$^{\text{nd}}$ law with $a_y = 0$ to each of the segments of the chain to determine the tensions.
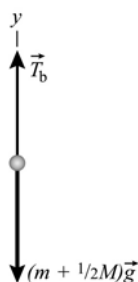
(a)   (b)   (c)
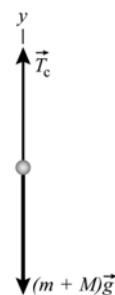
(a) Apply $\sum F_y = ma_y$ to the block and solve for T_a:

$T_a - mg = ma_y$
or, because $a_y = 0$,
$T_a = mg$

Substitute numerical values and evaluate T_a:

$T_a = (50\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{0.49\,\text{kN}}$

(b) Apply $\sum F_y = ma_y$ to the block and half the chain and solve for T_b:

$$T_b - \left(m + \frac{M}{2}\right)g = ma_y$$

or, because $a_y = 0$,

$$T_b = \left(m + \frac{M}{2}\right)g$$

Substitute numerical values and evaluate T_b:

$$T_b = (50\,\text{kg} + 10\,\text{kg})(9.81\,\text{m/s}^2)$$
$$= \boxed{0.59\,\text{kN}}$$

(c) Apply $\sum F_y = ma_y$ to the block and chain and solve for T_c:

$$T_c - (m + M)g = ma_y$$

or, because $a_y = 0$,

$$T_c = (m + M)g$$

Substitute numerical values and evaluate T_c:

$$T_c = (50\,\text{kg} + 20\,\text{kg})(9.81\,\text{m/s}^2)$$
$$= \boxed{0.69\,\text{kN}}$$

89 •• The speed of the head of a redheaded woodpecker reaches 3.5 m/s before impact with the tree. If the mass of the head is 0.060 kg and the average force on the head during impact is 6.0 N, find (a) the acceleration of the head (assuming constant acceleration), (b) the depth of penetration into the tree, and (c) the time it takes for the head to come to a stop.

Picture the Problem (a) Choose a coordinate system in which the $+x$ direction is the direction in which the woodpecker's head is moving and apply Newton's 2nd law to the woodpecker's head to find its acceleration. (b) Because we've assumed constant acceleration, we can use a constant-acceleration equation to find the depth of penetration in the tree. In Part (c), we can the constant-acceleration equation $v_x = v_{0x} + a_x \Delta t$ to find the time it takes for the head to come to a stop.

(a) Apply $\sum F_x = ma_x$ to the woodpecker's head to obtain:

$$F_{\text{on the head}} = ma_x \Rightarrow a_x = \frac{F_{\text{on the head}}}{m}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{-6.0\,\text{N}}{0.060\,\text{kg}} = \boxed{-0.10\,\text{km/s}^2}$$

(b) Use a constant-acceleration equation to relate the depth of penetration to the initial speed of the head and its acceleration:

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x \Rightarrow \Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x}$$

Because $v_x = 0$:

$$\Delta x = \frac{-v_{0x}^2}{2a_x}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{-(3.5\,\text{m/s})^2}{2(-0.10\,\text{km/s}^2)} = \boxed{6.1\,\text{cm}}$$

(c) Apply a constant-acceleration equation to relate the stopping time of the head to its initial and final speeds and to its acceleration:

$$v_x = v_{0x} + a_x \Delta t \Rightarrow \Delta t = \frac{v_x - v_{0x}}{a_x}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{0 - 3.5\,\text{m/s}}{-0.10\,\text{km/s}^2} = \boxed{35\,\text{ms}}$$

90 •• A frictionless surface is inclined at an angle of 30.0° to the horizontal. A 270-g block on the ramp is attached to a 75.0-g block using a pulley, as shown in Figure 4-62. (a) Draw two free-body diagrams, one for the 270-g block and the other for the 75.0-g block. (b) Find the tension in the string and the acceleration of the 270-g block. (c) The 270-g block is released from rest. How long does it take for it to slide a distance of 1.00 m along the surface? Will it slide up the incline, or down the incline?

Picture the Problem The application of Newton's 2$^{\text{nd}}$ law to the block and the hanging weight will lead to simultaneous equations in their common acceleration a and the tension T in the cord that connects them. Once we know the acceleration of this system, we can use a constant-acceleration equation to predict how long it takes the block to travel 1.00 m from rest. Note that the magnitudes of $\vec{T}$ and $\vec{T}'$ are equal.

(a) The free-body diagrams are shown to the right. m_{270} represents the mass of the 270-g block and m_{75} the mass of the 75.0-g block.
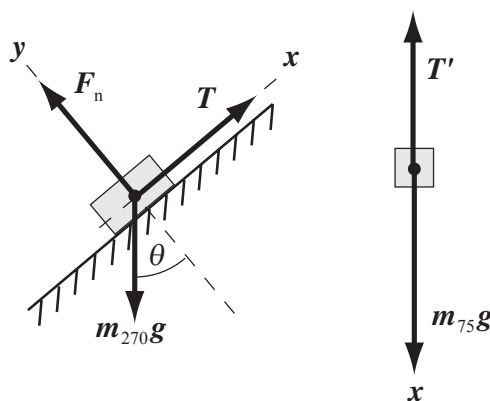
(b) Apply $\sum F_x = ma_x$ to the block and the suspended mass:

$$T - m_{270}g\sin\theta = m_{270}a_{1x}$$
and
$$m_{75}g - T = m_{75}a_{2x}$$

Letting a represent the common acceleration of the two objects, eliminate T between the two equations and solve a:

$$a = \frac{m_{75} - m_{270}\sin\theta}{m_{270} + m_{75}}g$$

Substitute numerical values and evaluate a:

$$a = \frac{0.0750\,\text{kg} - (0.270\,\text{kg})\sin 30°}{0.0750\,\text{kg} + 0.270\,\text{kg}}(9.81\,\text{m/s}^2) = -1.706\,\text{m/s}^2 = \boxed{-1.71\,\text{m/s}^2}$$

where the minus sign indicates that the acceleration is down the incline.

Substitute for a in either of the force equations to obtain:

$$T = \boxed{0.864\,\text{N}}$$

(c) Using a constant-acceleration equation, relate the displacement of the block down the incline to its initial speed and acceleration:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
or, because $v_{0x} = 0$,
$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta x}{a_x}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(-1.00\,\text{m})}{-1.706\,\text{m/s}^2}} = \boxed{1.08\,\text{s}}$$

Because the block is released from rest and its acceleration is negative, it will slide down the incline.

91 •• A box of mass m_1 is pulled along a frictionless horizontal surface by a horizontal force F that is applied to the end of a rope (see Figure 4-63). Neglect any sag of the rope. (a) Find the acceleration of the rope and block, assuming them to be one object. (b) What is the net force acting on the rope? (c) Find the tension in the rope at the point where it is attached to the block.

Picture the Problem Note that, while the mass of the rope is distributed over its length, the rope and the block have a common acceleration. Because the surface is horizontal and smooth, the only force that influences our solution is $\vec{F}$. The figure misrepresents the situation in that each segment of the rope experiences a gravitational force; the combined effect of which is that the rope must sag.

(a) Apply $\vec{a} = \vec{F}_{\text{net}} / m_{\text{tot}}$ to the rope-block system to obtain:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

(b) Apply $\sum \vec{F} = m\vec{a}$ to the rope, substitute the acceleration of the system obtained in (a), and simplify to obtain:

$$F_{net} = m_2 a = m_2 \left(\frac{F}{m_1 + m_2} \right)$$

$$= \boxed{\frac{m_2}{m_1 + m_2} F}$$

(c) Apply $\sum \vec{F} = m\vec{a}$ to the block, substitute the acceleration of the system obtained in (a), and simplify to obtain:

$$T = m_1 a = m_1 \left(\frac{F}{m_1 + m_2} \right)$$

$$= \boxed{\frac{m_1}{m_1 + m_2} F}$$

92 •• A 2.0-kg block rests on a frictionless wedge that has a 60° incline and an acceleration $\vec{a}$ to the right such that the mass remains stationary relative to the wedge (Figure 4-64). (a) Draw the free body diagram of the block and use it to determine the magnitude of the acceleration. (b) What would happen if the wedge were given an acceleration larger than this value? Smaller than this value?

Picture the Problem We can apply Newton's 2nd law to the 2-0-kg block to determine the magnitude of its acceleration.

(a) The free-body diagram of the 2.0-kg block is shown to the right. Because the surface of the wedge is frictionless, the force it exerts on the block must be normal to its surface. The forces acting on the block are the normal force $\vec{F}_n$ exerted by the frictionless surface and the gravitational force $\vec{F}_g$. The direction of the acceleration of the wedge has been chosen as the $+x$ direction.
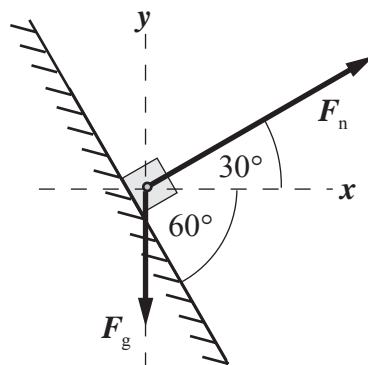
(a) Apply $\sum F_y = ma_y$ to the block to obtain:

$$F_n \sin 30° - F_g = ma_y$$

or, because $a_y = 0$ and $F_g = mg$,

$$F_n \sin 30° = mg \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$F_n \cos 30° = ma_x \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{a_x}{g} = \cot 30° \Rightarrow a_x = g \cot 30°$$

Substitute numerical values and evaluate a_x:

$$a_x = \left(9.81 \, \text{m/s}^2 \right) \cot 30° = \boxed{17 \, \text{m/s}^2}$$

(*b*) An acceleration of the wedge greater than $g\cot 30°$ would require that the normal force exerted on the body by the wedge be greater than that given in Part (*a*); that is, $F_n > mg/\sin 30°$. Under this condition, there would be a net force in the y direction and the block would accelerate up the wedge.

93 ••• **[SSM]** The masses attached to each side of an ideal Atwood's machine consist of a stack of five washers, each of mass m, as shown in Figure 4-65. The tension in the string is T_0. When one of the washers is removed from the left side, the remaining washers accelerate and the tension decreases by 0.300 N. (*a*) Find m. (*b*) Find the new tension and the acceleration of each mass when a second washer is removed from the left side.

Picture the Problem Because the system is initially in equilibrium, it follows that $T_0 = 5mg$. When one washer is moved from the left side to the right side, the remaining washers on the left side will accelerate upward (and those on the right side downward) in response to the net force that results. The free-body diagrams show the forces under this unbalanced condition. Applying Newton's 2nd law to each collection of washers will allow us to determine both the acceleration of the system and the mass of a single washer.
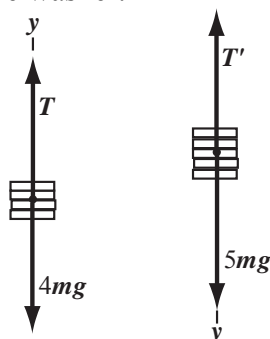
(*a*) Apply $\sum F_y = ma_y$ to the rising washers:

$$T - 4mg = (4m)a_y \qquad (1)$$

Noting that $T = T'$, apply $\sum F_y = ma_y$ to the descending masses:

$$5mg - T = (5m)a_y \qquad (2)$$

Eliminate T between these equations to obtain:

$$a_y = \tfrac{1}{9}g$$

Use this acceleration in equation (1) or equation (2) to obtain:

$$T = \frac{40}{9}mg$$

Expressing the difference ΔT between T_0 and T yields:

$$\Delta T = 5mg - \frac{40}{9}mg \Rightarrow m = \frac{\tfrac{9}{5}\Delta T}{g}$$

Substitute numerical values and evaluate m:	$m = \dfrac{\frac{9}{5}(0.300\,\text{N})}{9.81\,\text{m/s}^2} = \boxed{55.0\,\text{g}}$

(*b*) Proceed as in (*a*) to obtain: $T - 3mg = 3ma_y$ and $5mg - T = 5ma_y$

Add these equations to eliminate T and solve for a_y to obtain:	$a_y = \frac{1}{4}g$

Substitute numerical values and evaluate a_y:	$a_y = \frac{1}{4}(9.81\,\text{m/s}^2) = \boxed{2.45\,\text{m/s}^2}$

| Eliminate a_y in either of the motion equations and solve for T to obtain:	$T = \frac{15}{4}mg$

Substitute numerical values and evaluate T:	$T = \frac{15}{4}(0.05505\,\text{kg})(9.81\,\text{m/s}^2)$
	$= \boxed{2.03\,\text{N}}$

94 ••   Consider the ideal Atwood's machine in Figure 4-65. When N washers are transferred from the left side to the right side, the right side drops 47.1 cm in 0.40 s. Find N.

Picture the Problem The free-body diagram represents the Atwood's machine with N washers moved from the left side to the right side. Application of Newton's 2$^{\text{nd}}$ law to each collection of washers will result in two equations that can be solved simultaneously to relate N, a, and g. The acceleration can then be found from the given data.
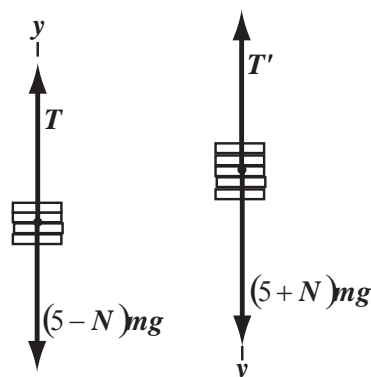
Apply $\sum F_y = ma_y$ to the rising washers:	$T - (5 - N)mg = (5 - N)ma_y$

Noting that $T = T'$, apply $\sum F_y = ma_y$ to the descending washers:	$(5 + N)mg - T = (5 + N)ma_y$

Add these equations to eliminate T:	$(5 + N)mg - (5 - N)mg$
	$= (5 - N)ma_y + (5 + N)ma_y$

Solving for N yields:

$$N = \frac{5a_y}{g}$$

Using a constant-acceleration equation, relate the distance the washers fell to their time of fall:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$\Delta y = \tfrac{1}{2}a_y(\Delta t)^2 \Rightarrow a_y = \frac{2\Delta y}{(\Delta t)^2}$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{2(0.471\,\text{m})}{(0.40\,\text{s})^2} = 5.89\,\text{m/s}^2$$

Substitute in the expression for N:

$$N = 5\left(\frac{5.89\,\text{m/s}^2}{9.81\,\text{m/s}^2}\right) = \boxed{3}$$

95 ••   Blocks of mass m and $2m$ are on a horizontal frictionless surface (Figure 4-66). The blocks are connected by a string. In addition, forces F_1 and F_2 are applied to the blocks as shown. (*a*) If the forces shown are constant, find the tension in the connecting string. (*b*) If the forces vary with time as $F_1 = Ct$ and $F_2 = 2Ct$, where $C = 5.00$ N/s and t is time, find the time t_0 at which the tension in the string equals 10.0 N.

Picture the Problem Draw the free-body diagram for the block of mass m and apply Newton's 2$^\text{nd}$ law to obtain the acceleration of the system and then the tension in the rope connecting the two blocks.
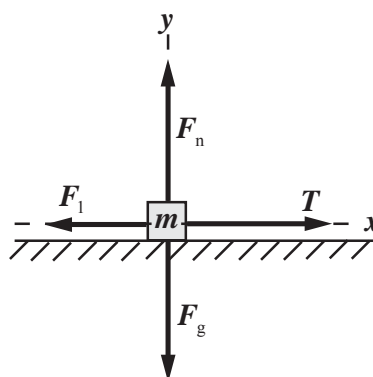
(*a*) Apply $\sum F_x = ma_x$ to the block of mass m:

$$T - F_1 = ma_x \qquad\qquad (1)$$

Apply $\sum F_x = ma_x$ to both blocks:

$$F_2 - F_1 = (m + 2m)a = 3ma_x$$

Solving for a yields:

$$a_x = \frac{F_2 - F_1}{3m}$$

Substitute for a_x in the equation (1) to obtain:

$$T - F_1 = m\left(\frac{F_2 - F_1}{3m}\right)$$

Solving for T yields:

$$T = \boxed{\tfrac{1}{3}(F_2 + 2F_1)}$$

(b) Substitute for F_1 and F_2 in the equation derived in Part (a):

$$T = \tfrac{1}{3}(2Ct + 2(Ct)) = \tfrac{4}{3}Ct \Rightarrow t = \frac{3T}{4C}$$

Substitute numerical values and evaluate $t = t_0$:

$$t_0 = \frac{3(10.0\ \text{N})}{4(5.00\ \text{N/s})} = \boxed{1.50\ \text{s}}$$

97 ••   Elvis Presley, has supposedly been sighted numerous times after he passed away on August 16, 1977. The following is a chart of what Elvis' weight would be if he were sighted on the surface of other objects in our solar system. Use the chart to determine: (a) Elvis' mass on earth, (b) Elvis' mass on Pluto, and (c) the acceleration due to gravity on Mars. (d) Compare the free-fall acceleration on Pluto to the free-fall acceleration on the moon.

planet	Elvis's weight (N)
Mercury	431
Venus	1031
Earth	1133
Mars	431
Jupiter	2880
Saturn	1222
Pluto	58
Moon	191

Picture the Problem Elvis' mass is the ratio of his weight on a given planet to the acceleration of gravity on that planet. In (b), (c), and (d) we can use the relationship between the gravitational force (weight) acting on an object, its mass (independent of location), and the local value of the free-fall acceleration.

(a) Elvis' mass on earth is given by:

$$m = \frac{w_{\text{earth}}}{g_{\text{earth}}}$$

Substitute numerical values and evaluate m:

$$m = \frac{1131\ \text{N}}{9.81\ \text{m/s}^2} = 115.3\ \text{kg} = \boxed{115\ \text{kg}}$$

(b) Because his mass is independent of his location, Elvis' mass on Pluto is 115 kg.

(*c*) The free-fall acceleration on Mars is the ratio of the weight of an object on Mars to the mass of the object:

$$g_{Mars} = \frac{w_{on\ Mars}}{m}$$

Substitute numerical values for Elvis and evaluate g_{Mars}:

$$g_{Mars} = \frac{431\ N}{115.3\ kg} = \boxed{3.74\ m/s^2}$$

(*d*) The free-fall acceleration on Pluto is given by:

$$g_{Pluto} = \frac{w_{on\ Pluto}}{m} \qquad (1)$$

The free-fall acceleration on the moon is given by:

$$g_{Moon} = \frac{w_{on\ Moon}}{m} \qquad (2)$$

Divide equation (1) by equation (2) and simplify to obtain:

$$\frac{g_{Pluto}}{g_{Moon}} = \frac{\dfrac{w_{on\ Pluto}}{m}}{\dfrac{w_{on\ Moon}}{m}} = \frac{w_{on\ Pluto}}{w_{on\ Moon}}$$

Substituting numerical values yields:

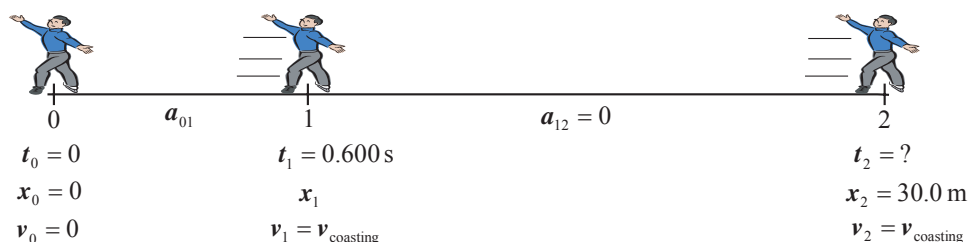$$\frac{g_{Pluto}}{g_{Moon}} = \frac{58\ N}{191\ N} = 0.30$$

or

$$g_{Pluto} = \boxed{0.30 g_{Moon}}$$

97 ••• As a prank, your friends have kidnapped you in your sleep, and transported you out onto the ice covering a local pond. When you wake up you are 30.0 m from the nearest shore. The ice is so slippery (i.e. frictionless) that you can not seem to get yourself moving. You realize that you can use Newton's third law to your advantage, and choose to throw the heaviest thing you have, one boot, in order to get yourself moving. Take your weight to be 595 N. (*a*) What direction should you throw your boot so that you will most quickly reach the shore? (*b*) If you throw your 1.20-kg boot with an average force of 420 N, and the throw takes 0.600 s (the time interval over which you apply the force), what is the magnitude of the average force that the boot exerts on you? (Assume constant acceleration.) (*c*) How long does it take you to reach shore, including the short time in which you were throwing the boot?

Picture the Problem The diagram shown below summarizes the information about your trip to the shore and will be helpful in solving Part (*c*) of the problem.

$$a_{01}$$

$$a_{12} = 0$$

0	1	2
$t_0 = 0$	$t_1 = 0.600\,\text{s}$	$t_2 = ?$
$x_0 = 0$	x_1	$x_2 = 30.0\,\text{m}$
$v_0 = 0$	$v_1 = v_{\text{coasting}}$	$v_2 = v_{\text{coasting}}$

(*a*) You should throw your boot in the direction away from the closest shore.

(*b*) The magnitude of the average force you exert on the boot equals the magnitude of the average force the boot exerts on you:

$$F_{\text{av, on you}} = \boxed{420\,\text{N}}$$

(*c*) The time required for you to reach the shore is the sum of your travel time while accelerating and your travel time while coasting:

$$\Delta t_{\text{total}} = \Delta t_{01} + \Delta t_{12}$$

or, because $\Delta t_{01} = 0.600$ s,

$$\Delta t_{\text{total}} = 0.600\,\text{s} + \Delta t_{12} \qquad (1)$$

Use a constant-acceleration equation to relate your displacement Δx_{01} to your acceleration time Δt_{01}:

$$\Delta x_{01} = \tfrac{1}{2} a_{01} \left(\Delta t_{01} \right)^2 \qquad (2)$$

Apply Newton's 2$^{\text{nd}}$ law to express your acceleration during this time interval:

$$a_{01} = \frac{F_{\text{net}}}{m} = \frac{F_{\text{av}}}{w/g} = \frac{F_{\text{av}}g}{w}$$

Substitute numerical values and evaluate a_{01}:

$$a_{01} = \frac{(420\,\text{N})(9.81\,\text{m/s}^2)}{595\,\text{N}} = 6.925\,\text{m/s}^2$$

Substitute numerical values in equation (2) and evaluate Δx_{01}:

$$\Delta x_{01} = \tfrac{1}{2}(6.925\,\text{m/s}^2)(0.600\,\text{s})^2$$
$$= 1.246\,\text{m}$$

Your coasting time is the ratio of your displacement while coasting to your speed while coasting:

$$\Delta t_{12} = \frac{\Delta x_{12}}{v_1}$$

or, because $\Delta x_{12} = 30.0\,\text{m} - \Delta x_{01}$,

$$\Delta t_{12} = \frac{30.0\,\text{m} - \Delta x_{01}}{v_1} \qquad (3)$$

Use a constant-acceleration equation to relate your terminal speed (your speed after the interval of acceleration) to your acceleration and displacement during this interval:

$$v_1 = v_0 + a_{01}\Delta t_{01}$$
or, because $v_0 = 0$,
$$v_1 = a_{01}\Delta t_{01}$$

Substitute for v_1 in equation (3) to obtain:

$$\Delta t_{12} = \frac{30.0\,\text{m} - \Delta x_{01}}{a_{01}\Delta t_{01}}$$

Substituting for $\Delta t_{\text{coasting}}$ in equation (1) yields:

$$\Delta t_{\text{total}} = 0.600\,\text{s} + \frac{30.0\,\text{m} - \Delta x_{01}}{a_{01}\Delta t_{01}}$$

Substitute numerical values and evaluate Δt_{total}:

$$\Delta t_{\text{total}} = 0.600\,\text{s} + \frac{30.0\,\text{m} - 1.246\,\text{m}}{(6.925\,\text{m/s}^2)(0.600\,\text{s})} = \boxed{7.52\,\text{s}}$$

98 ••• The pulley of an ideal Atwood's machine is given an upward acceleration a, as shown in Figure 4-67. Find the acceleration of each mass and the tension in the string that connects them.

Picture the Problem Because a constant-upward acceleration has the same effect as an increase in the acceleration due to gravity, we can use the result of Problem 82 (for the tension) with a replaced by $a + g$. The application of Newton's 2$^{\text{nd}}$ law to the object whose mass is m_2 will connect the acceleration of this body to tension from Problem 82.
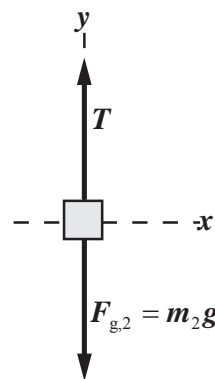
In Problem 82 it is given that, when the support pulley is not accelerating, the tension in the rope and the acceleration of the masses are related according to:

$$T = \boxed{\frac{2m_1 m_2}{m_1 + m_2}\,g}$$

Replace a with $a + g$:

$$T = \frac{2m_1 m_2}{m_1 + m_2}(a + g)$$

Apply $\sum F_y = ma_y$ to the object whose mass is m_2:

$$T - m_2 g = m_2 a_2 \Rightarrow a_2 = \frac{T - m_2 g}{m_2}$$

Substitute for T and simplify to obtain:

$$a_2 = \boxed{\frac{(m_1 - m_2)g + 2m_1a}{m_1 + m_2}}$$

The expression for a_1 is the same as for a_2 with all subscripts interchanged (note that a positive value for a_1 represents acceleration upward):

$$a_1 = \boxed{\frac{(m_2 - m_1)g + 2m_2a}{m_1 + m_2}}$$

99 ••• You are working for an automotive magazine and putting a certain new automobile (mass 650 kg) through its paces. While accelerating from rest, its onboard computer records its speed as a function of time as follows:

$$v \text{ (m/s): } \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50$$
$$t \text{ (s): } \quad 0 \quad 1.8 \quad 2.8 \quad 3.6 \quad 4.9 \quad 6.5$$

(a) Using a **spreadsheet**, find the average acceleration over each 1.8-s time interval, and graph the velocity versus time and acceleration versus time for this car. (b) Where on the graph of velocity versus time is the net force on the car highest and lowest? Explain your reasoning. (c) What is the average net force on the car over the whole trip? (d) From the graph of velocity versus time, estimate the total distance covered by the car.

Picture the Problem

(a) A spreadsheet program is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
C1	650	m
D5	(B5−B4)/2	$\frac{1}{2}\Delta t$
E5	(C5−C4)/D5	$\dfrac{\Delta v}{\Delta t}$
F5	$D$1*E5	ma
F10	(F5+F6+F7+F8+F9)/5	F_{ave}

	A	B	C	D	E	F
1			$m=$	650	kg	
2						
3		t (s)	v (m/s)	t_{midpt} (s)	a (m/s^2)	$F = ma$ (N)
4		0.0	0			
5		1.8	10	0.90	5.56	3611
6		2.8	20	2.30	10.00	6500
7		3.6	30	3.20	12.50	8125
8		4.9	40	4.25	7.69	5000

9		6.5	50	5.70	6.25	4063
10						5460
11						

A graph of velocity as a function of time follows:
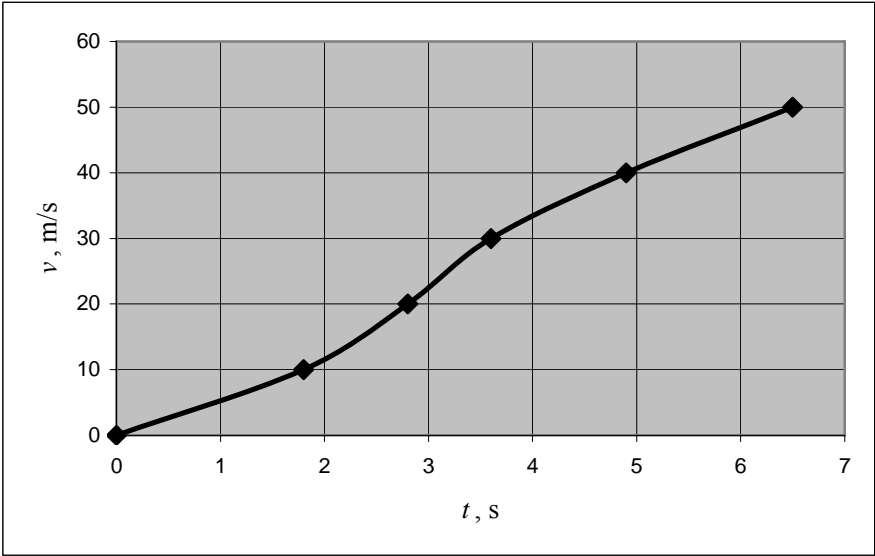
A graph of acceleration as a function of time is shown below:
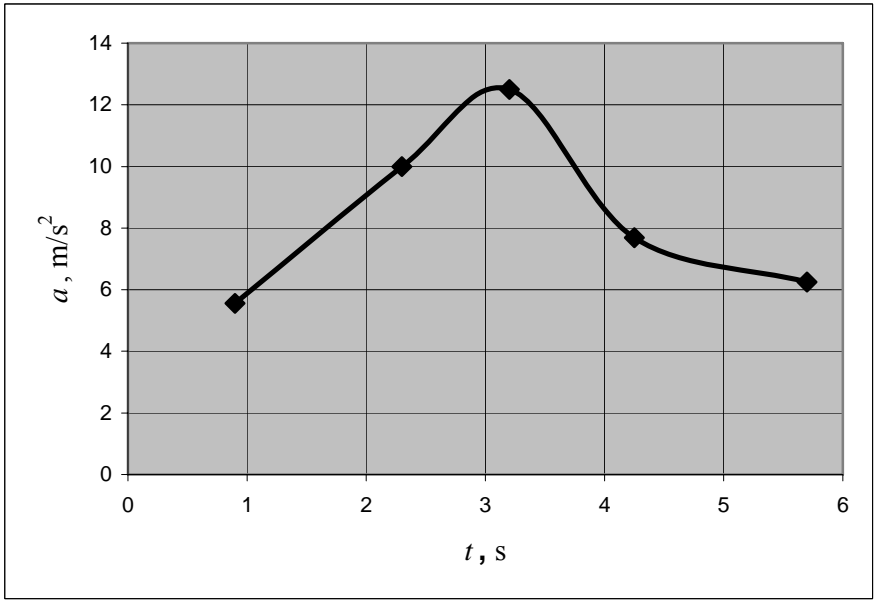
(b) Because the net force is lowest where the acceleration is lowest, we can see from the graph of velocity versus time that its slope (the acceleration) is smallest in the interval from 0 to 1.8 s. Because the net force is highest where the acceleration is highest, we can see from the graph of velocity versus time that its slope (the acceleration) is greatest in the interval from 2.8 s to 3.6 s.

(*c*) From the table we see that:

$$F_{ave} \approx \boxed{5500 \text{ N}}$$

(*d*) The distance covered by the car is the area under the graph of velocity versus time. Because the graph of speed versus time is approximately linear, we can estimate the total distance covered by the car by finding the area of the triangular region under it.

$$A_{triangle} = \tfrac{1}{2}bh = \tfrac{1}{2}(6.5 \text{ s})(50 \text{ m/s})$$

$$\approx \boxed{160 \text{ m}}$$