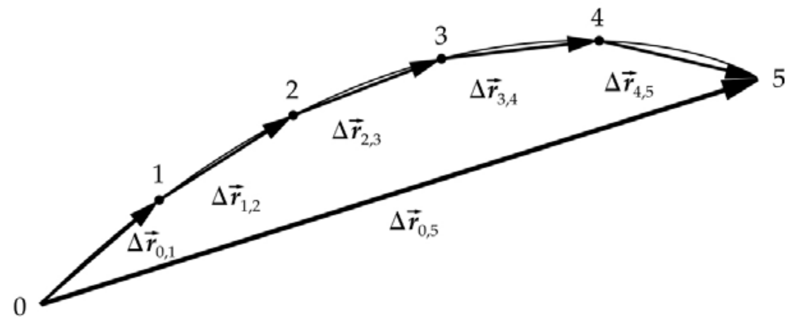
Chapter 3
Motion in Two and Three Dimensions

Conceptual Problems

1 • **[SSM]** Can the magnitude of the displacement of a particle be less than the distance traveled by the particle along its path? Can its magnitude be more than the distance traveled? Explain.

Determine the Concept The distance traveled along a path can be represented as a sequence of displacements.



Suppose we take a trip along some path and consider the trip as a sequence of many very small displacements. The net displacement is the vector sum of the very small displacements, and the total distance traveled is the sum of the magnitudes of the very small displacements. That is,

$$\textit{total distance} = \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|$$

where N is the number of very small displacements. (For this to be exactly true we have to take the limit as N goes to infinity and each displacement magnitude goes to zero.) Now, using ″the shortest distance between two points is a straight line,″ we have

$$\left|\Delta\vec{r}_{0,N}\right| \le \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|,$$

where $\left|\Delta\vec{r}_{0,N}\right|$ is the magnitude of the net displacement.

Hence, we have shown that the magnitude of the displacement of a particle is less than or equal to the distance it travels along its path.

2 • Give an example in which the distance traveled is a significant amount, yet the corresponding displacement is zero. Can the reverse be true? If so, give an example.

Determine the Concept The displacement of an object is its final position vector minus its initial position vector ($\Delta\vec{r} = \vec{r}_f - \vec{r}_i$). The displacement can be less but never more than the distance traveled. Suppose the path is one complete trip around the earth at the equator. Then, the displacement is 0 but the distance traveled is $2\pi R_e$. No, the reverse cannot be true.

3 • What is the average velocity of a batter who hits a home run (from when he hits the ball to when he touches home plate after rounding the bases)?

Determine the Concept The important distinction here is that ***average velocity*** is being requested, as opposed to ***average speed***.

The average velocity is defined as the displacement divided by the elapsed time.

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{0}{\Delta t} = 0$$

The displacement for any trip around the bases is zero. Thus we see that no matter how fast the runner travels, the average velocity is always zero at the end of each complete circuit of the bases.

What is the correct answer if we were asked for *average speed*?

The average speed is defined as the distance traveled divided by the elapsed time.

$$\boldsymbol{v}_{av} = \frac{\text{total distance}}{\Delta \boldsymbol{t}}$$

For one complete circuit of any track, the total distance traveled will be greater than zero and so the average speed is not zero.

4 • A baseball is hit so its initial velocity upon leaving the bat makes an angle of 30° above the horizontal. It leaves that bat at a height of 1.0 m above the ground and lands untouched for a single. During its flight, from just after it leaves the bat to just before it hits the ground, describe how the angle between its velocity and acceleration vectors changes. Neglect any effects due to air resistance.

Determine the Concept The angle between its velocity and acceleration vectors starts at 30° + 90° or 120° because the acceleration of the ball is straight down. At the peak of the flight of the ball the angle reduces to 90° because the ball's velocity vector is horizontal. When the ball reaches the same elevation that it started from the angle is 90° − 30° or 60°.

5 • If an object is moving toward the west at some instant, in what direction is its acceleration? (*a*) north, (*b*) east, (*c*) west, (*d*) south, (*e*) may be any direction.

Determine the Concept The *instantaneous acceleration* is the limiting value, as Δt approaches zero, of $\Delta\vec{v}/\Delta t$ and is in the same direction as $\Delta\vec{v}$.

Other than through the definition of $\vec{a}$, the instantaneous velocity and acceleration vectors are unrelated. Knowing the direction of the velocity at one instant tells one nothing about how the velocity is changing at that instant. $\boxed{(e)}$ is correct.

6 • Two astronauts are working on the lunar surface to install a new telescope. The acceleration due to gravity on the Moon is only 1.64 m/s^2. One astronaut tosses a wrench to the other astronaut but the speed of throw is excessive and the wrench goes over her colleague's head. When the wrench is at the highest point of its trajectory (*a*) its velocity and acceleration are both zero, (*b*) its velocity is zero but its acceleration is nonzero, (*c*) its velocity is nonzero but its acceleration is zero, (*d*) its velocity and acceleration are both nonzero, (*e*) insufficient information is given to choose between any of the previous choices.

Determine the Concept When the wrench reaches its maximum height, it is still moving horizontally but its acceleration is downward. $\boxed{(d)}$ is correct.

7 • The velocity of a particle is directed toward the east while the acceleration is directed toward the northwest as shown in Figure 3-27. The particle is (*a*) speeding up and turning toward the north, (*b*) speeding up and turning toward the south, (*c*) slowing down and turning toward the north, (*d*) slowing down and turning toward the south, (*e*) maintaining constant speed and turning toward the south.

Determine the Concept The change in the velocity is in the same direction as the acceleration. Choose an *x-y* coordinate system with east being the positive *x* direction and north the positive *y* direction. Given our choice of coordinate system, the *x* component of $\vec{a}$ is negative and so $\vec{v}$ will decrease. The *y* component of $\vec{a}$ is positive and so $\vec{v}$ will increase toward the north. $\boxed{(c)}$ is correct.

8 • Assume you know the position vectors of a particle at two points on its path, one earlier and one later. You also know the time it took the particle to move from one point to the other. Then you can then compute the particle's (*a*) average velocity, (*b*) average acceleration, (*c*) instantaneous velocity, (*d*) instantaneous acceleration.

Determine the Concept All you can compute is the average velocity, since no instantaneous quantities can be computed and you need two instantaneous velocities to compute the average acceleration. $\boxed{(a)}$ is correct.

9 • Consider the path of a moving particle. (*a*) How is the velocity vector related geometrically to the path of the particle? (*b*) Sketch a curved path and draw the velocity vector for the particle for several positions along the path.

Determine the Concept (*a*) The velocity vector, as a consequence of always being in the direction of motion, is tangent to the path.

(*b*) A sketch showing four velocity vectors for a particle moving along a curved path is shown to the right.

10 • The acceleration of a car is zero when it is (*a*) turning right at a constant speed, (*b*) driving up a long straight incline at constant speed, (*c*) traveling over the crest of a hill at constant speed, (*d*) bottoming out at the lowest point of a valley at constant speed, (*e*) speeding up as it descends a long straight decline.

Determine the Concept An object experiences acceleration whenever either its speed changes or it changes direction.

The acceleration of a car moving in a straight path at constant speed is zero. In the other examples, either the magnitude or the direction of the velocity vector is changing and, hence, the car is accelerated. $\boxed{(\boldsymbol{b})}$ is correct.

11 • **[SSM]** Give examples of motion in which the directions of the velocity and acceleration vectors are (*a*) opposite, (*b*) the same, and (*c*) mutually perpendicular.

Determine the Concept The velocity vector is defined by $\vec{\boldsymbol{v}} = d\vec{\boldsymbol{r}}/dt$, while the acceleration vector is defined by $\vec{\boldsymbol{a}} = d\vec{\boldsymbol{v}}/dt$.

(*a*) A car moving along a straight road while braking.

(*b*) A car moving along a straight road while speeding up.

(*c*) A particle moving around a circular track at constant speed.

12 • How is it possible for a particle moving at constant speed to be accelerating? Can a particle with constant velocity be accelerating at the same time?

Determine the Concept A particle experiences accelerated motion when either its speed or direction of motion changes.

A particle moving at constant speed in a circular path is accelerating because the direction of its velocity vector is changing.

If a particle is moving at constant velocity, it is not accelerating.

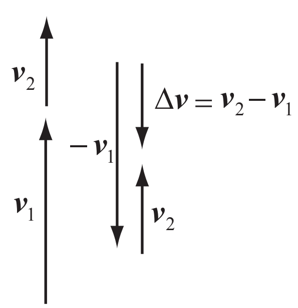
13 • **[SSM]** Imagine throwing a dart straight upward so that it sticks into the ceiling. After it leaves your hand, it steadily slows down as it rises before it sticks. (*a*) Draw the dart's velocity vector at times t_1 and t_2, where t_1 and t_2 occur after it leaves your hand but before it impacts the ceiling, and $\Delta t = t_2 - t_1$ is small. From your drawing find the direction of the change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, and thus the direction of the acceleration vector. (*b*) After it has stuck in the ceiling for a few seconds, the dart falls down to the floor. As it falls it speeds up, of course, until it hits the floor. Repeat Part (*a*) to find the direction of its acceleration vector as it falls. (*c*) Now imagine tossing the dart horizontally. What is the direction of its acceleration vector after it leaves your hand, but before it strikes the floor?

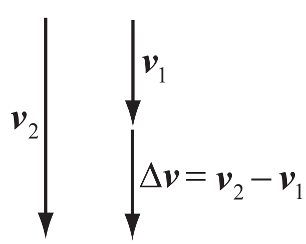
Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector,* $\Delta\vec{v}$.

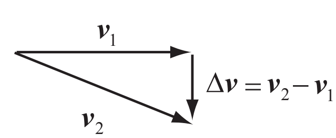
(*a*) The sketch for the dart thrown upward is shown to the right. The acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.

(*b*) The sketch for the falling dart is shown to the right. Again, the acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.

(*c*) The acceleration vector is in the direction of the *change* in the velocity vector … and hence is downward as shown to the right:

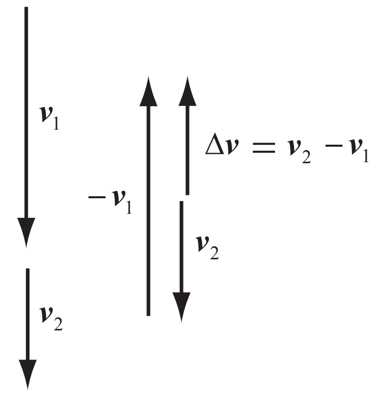






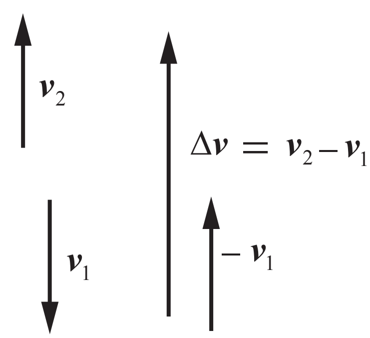
14 • As a bungee jumper approaches the lowest point in her descent, the rubber cord holding her stretches and she loses speed as she continues to move downward. Assuming that she is dropping straight down, make a motion diagram to find the direction of her acceleration vector as she slows down by drawing her velocity vectors at times t_1 and t_2, where t_1 and t_2 are two instants during the portion of her descent that she is losing speed, and $\Delta t = t_2 - t_1$ is small. From your drawing find the direction of the change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, and thus the direction of the acceleration vector.

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector*, $\Delta\vec{v}$. The drawing is shown to the right.



15 • After reaching the lowest point in her jump at time t_{low}, a bungee jumper moves upward, gaining speed for a short time until gravity again dominates her motion. Draw her velocity vectors at times t_1 and t_2, where $\Delta t = t_2 - t_1$ is small and $t_1 < t_{low} < t_2$. From your drawing find the direction of the change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, and thus the direction of the acceleration vector at time t_{low}.

Determine the Concept The acceleration vector is in the same direction as the *change in the velocity vector,* $\Delta\vec{v}$. The drawing is shown to the right.



16 • A river is 0.76 km wide. The banks are straight and parallel (Figure 3-28). The current is 4.0 km/h and is parallel to the banks. A boat has a maximum speed of 4.0 km/h in still water. The pilot of the boat wishes to go on a straight line from A to B, where the line AB is perpendicular to the banks. The pilot should (*a*) head directly across the river, (*b*) head 53º upstream from the line AB, (*c*) head 37º upstream from the line AB, (*d*) give up—the trip from A to B is not possible with a boat of this limited speed, (*e*) do none of the above.

Determine the Concept We can decide what the pilot should do by considering the speeds of the boat and of the current. The speed of the stream is equal to the maximum speed of the boat in still water. The best the boat can do is, while facing directly upstream, maintain its position relative to the bank. So, the pilot should give up. $\boxed{(\boldsymbol{d})}$ is correct.

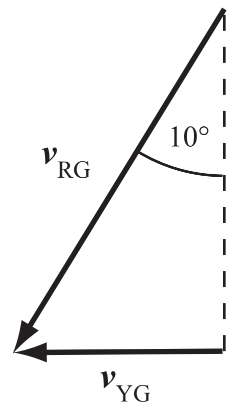
17 • [SSM] During a heavy rain, the drops are falling at a constant velocity and at an angle of 10° west of the vertical. You are walking in the rain and notice that only the top surfaces of your clothes are getting wet. In what direction are you walking? Explain.

Determine the Concept You must be walking west to make it appear to you that the rain is exactly vertical.

18 • In Problem 17, what is your walking speed if the speed of the drops relative to the ground is 5.2 m/s?

Determine the Concept Let $\vec{v}_{\mathrm{YG}}$ represent your velocity relative to the ground and $\vec{v}_{\mathrm{RG}}$ represent the velocity of the rain relative to the ground. Then your speed relative to the ground is given by

$$v_{\mathrm{YG}} = (5.2\ \mathrm{m/s})\sin 10° = \boxed{0.90\ \mathrm{m/s}}$$



19 • True or false (Ignore any effects due to air resistance):

(*a*) When a projectile is fired horizontally, it takes the same amount of time to reach the ground as an identical projectile dropped from rest from the same height.
(*b*) When a projectile is fired from a certain height at an upward angle, it takes longer to reach the ground than does an identical projectile dropped from rest from the same height.
(*c*) When a projectile is fired horizontally from a certain height, it has a higher speed upon reaching the ground than does an identical projectile dropped from rest from the same height.

(*a*) True. In the absence of air resistance, both projectiles experience the same downward acceleration. Because both projectiles have initial vertical velocities of zero, their vertical motions must be identical.

(*b*) True. When a projectile is fired from a certain height at an upward angle, its time in the air is twice the time it takes to fall from its maximum height. This distance is greater than it is when the projectile is fired horizontally from the same height.

(*c*) True. When a projectile is fired horizontally, its velocity upon reaching the ground has a horizontal component in addition to the vertical component it has when it is dropped from rest. The magnitude of this velocity is related to its horizontal and vertical components through the Pythagorean Theorem.

20 • A projectile is fired at 35º above the horizontal. Any effects due to air resistance are negligible. At the highest point in its trajectory, its speed is 20 m/s. The initial velocity had a horizontal component of (*a*) 0, (*b*) (20 m/s) cos 35º, (*c*) (20 m/s) sin 35º, (*d*) (20 m/s)/cos 35º, (*e*) 20 m/s.

Determine the Concept In the absence of air resistance, the horizontal component of the projectile's velocity is constant for the duration of its flight. At the highest point, the speed is the horizontal component of the initial velocity. The vertical component is zero at the highest point. $\boxed{(e)}$ is correct.

21 • **[SSM]** A projectile is fired at 35º above the horizontal. Any effects due to air resistance are negligible. The initial velocity of the projectile has a vertical component that is (*a*) less than 20 m/s, (*b*) greater than 20 m/s, (*c*) equal to 20 m/s, (*d*) cannot be determined from the data given.

Determine the Concept $\boxed{(a)}$ is correct. Because the initial horizontal velocity is 20 m/s, and the launch angle is less than 45 degrees, the initial vertical velocity must be less than 20 m/s.

22 • A projectile is fired at 35º above the horizontal. Any effects due to air resistance are negligible. The projectile lands at the same elevation of launch, so the initial velocity of the projectile has a vertical component that is (*a*) the same as its initial vertical component in magnitude and direction, (*b*) the same as its initial vertical component in magnitude but opposite in sign, (*c*) less than its initial vertical component in magnitude but with the same sign, (*d*) less than as its initial vertical component in magnitude but with the opposite sign.

Determine the Concept $\boxed{(b)}$ is correct. The landing speed is the same as the launch speed. Because the horizontal velocity component does not change, the vertical component of the velocity at landing must be the same magnitude but oppositely directed compared to that at launch.

23 • Figure 3-29 represents the trajectory of a projectile going from A to E. Air resistance is negligible. What is the direction of the acceleration at point B? (*a*) up and to the right, (*b*) down and to the left, (*c*) straight up, (*d*) straight down, (*e*) The acceleration of the ball is zero.

Determine the Concept $\boxed{(d)}$ is correct. In the absence of air resistance, the acceleration of the ball depends only on the *change in its velocity* and is independent of its velocity. As the ball moves along its trajectory between points A and C, the vertical component of its velocity decreases and the *change* in its velocity is a downward pointing vector. Between points C and E, the vertical component of its velocity increases and the *change* in its velocity is also a downward pointing vector. There is no change in the horizontal component of the velocity.

24 • Figure 3-29 represents the trajectory of a projectile going from A to E. Air resistance is negligible. (*a*) At which point(s) is the speed the greatest? (*b*) At which point(s) is the speed the least? (*c*) At which two points is the speed the same? Is the velocity also the same at these points?

Determine the Concept In the absence of air resistance, the horizontal component of the velocity remains constant throughout the flight. The vertical component has its maximum values at launch and impact.

(*a*) The speed is greatest at A and E.

(*b*) The speed is least at point C.

(*c*) The speed is the same at A and E. No. The horizontal components are equal at these points but the vertical components are oppositely directed.

25 • **[SSM]** True or false:

(*a*) If an object's speed is constant, then its acceleration must be zero.
(*b*) If an object's acceleration is zero, then its speed must be constant.
(*c*) If an object's acceleration is zero, its velocity must be constant.
(*d*) If an object's speed is constant, then its velocity must be constant.
(*e*) If an object's velocity is constant, then its speed must be constant.

Determine the Concept Speed is a scalar quantity, whereas acceleration, equal to the *rate of change* of velocity, is a vector quantity.

(*a*) False. Consider a ball on the end of a string. The ball can move with constant *speed* (a scalar) even though its *acceleration* (a vector) is always changing direction.

(*b*) True. From its definition, if the acceleration is zero, the velocity must be constant and so, therefore, must be the speed.

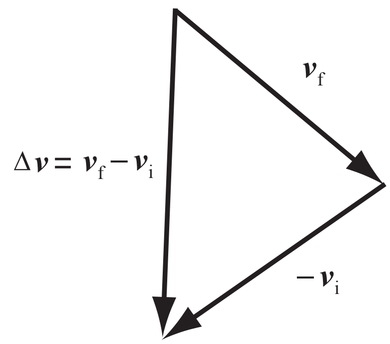
(*c*) True. An object's velocity must change in order for the object to have an acceleration other than zero.

(*d*) False. Consider an object moving at constant speed along a circular path. Its velocity changes continuously along such a path.

(*e*) True. If the velocity of an object is constant, then both its direction and magnitude (speed) must be constant.

26 • The initial and final velocities of a particle are as shown in Figure 3-30. Find the direction of the average acceleration.

Determine the Concept The average acceleration vector is defined by $\vec{a}_{av} = \Delta\vec{v}/\Delta t$. The direction of $\vec{a}_{av}$ is that of $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, as shown to the right.



27 •• The automobile path shown in Figure 3-31 is made up of straight lines and arcs of circles. The automobile starts from rest at point A. After it reaches point B, it travels at constant speed until it reaches point E. It comes to rest at point F. (*a*) At the middle of each segment (AB, BC, CD, DE, and EF), what is the direction of the velocity vector? (*b*) At which of these points does the automobile have a nonzero acceleration? In those cases, what is the direction of the acceleration? (*c*) How do the magnitudes of the acceleration compare for segments BC and DE?

Determine the Concept The velocity vector is in the same direction as *the change in the position vector* while the acceleration vector is in the same direction as *the change in the velocity vector*. Choose a coordinate system in which the y direction is north and the x direction is east.

(*a*)

Path	Direction of velocity vector
AB	north
BC	northeast
CD	east
DE	southeast
EF	south

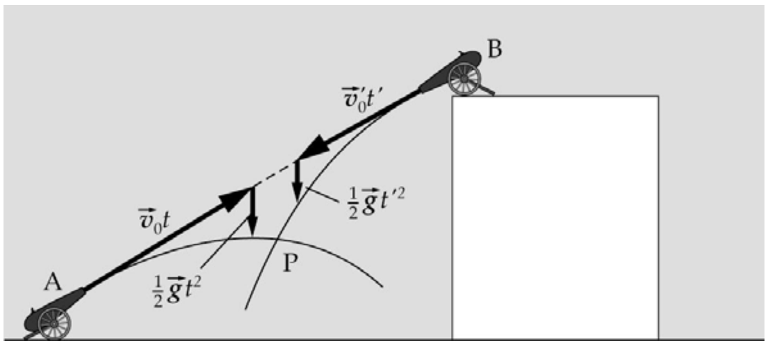
(*b*)

Path	Direction of acceleration vector
AB	north
BC	southeast
CD	0
DE	southwest
EF	north

(*c*) The magnitudes are comparable, but larger for DE because the radius of the path is smaller there.

28 •• Two cannons are pointed directly toward each other as shown in Figure 3-32. When fired, the cannonballs will follow the trajectories shown—P is the point where the trajectories cross each other. If we want the cannonballs to hit each other, should the gun crews fire cannon A first, cannon B first, or should they fire simultaneously? Ignore any effects due to air resistance.

Determine the Concept We'll assume that the cannons are identical and use a constant-acceleration equation to express the displacement of each cannonball as a function of time. Having done so, we can then establish the condition under which they will have the same vertical position at a given time and, hence, collide. The modified diagram shown below shows the displacements of both cannonballs.



Express the displacement of the cannonball from cannon A at any time t after being fired and before any collision:

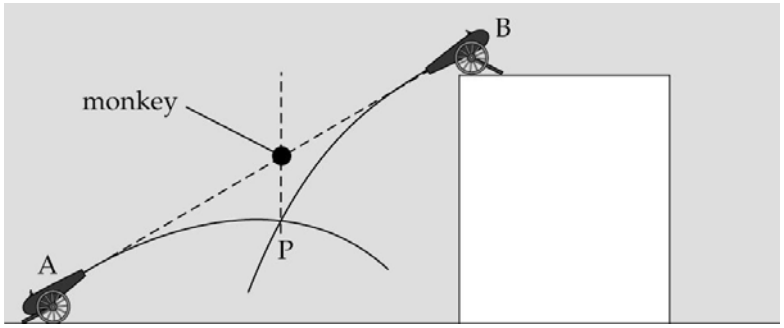
$$\Delta\vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{g}t^2$$

Express the displacement of the cannonball from cannon A at any time t' after being fired and before any collision:

$$\Delta\vec{r}\,' = \vec{v}_0't' + \tfrac{1}{2}\vec{g}t'^2$$

If the guns are fired simultaneously, $t = t'$ and the balls are the same distance $\frac{1}{2}\boldsymbol{g}t^2$ below the line of sight at all times. Therefore, they should fire the guns simultaneously.

Remarks: This is the "monkey and hunter" problem in disguise. If you imagine a monkey in the position shown below, and the two guns are fired simultaneously, and the monkey begins to fall when the guns are fired, then the monkey and the two cannonballs will all reach point P at the same time.

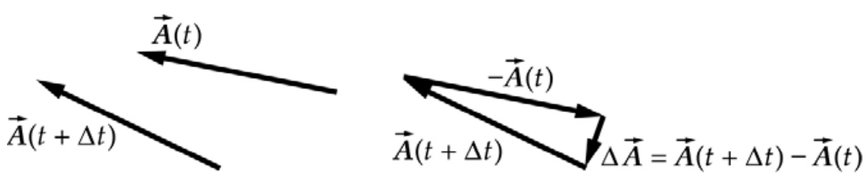


29 •• Galileo wrote the following in his ***Dialogue concerning the two world systems***: "Shut yourself up . . . in the main cabin below decks on some large ship, and . . . hang up a bottle that empties drop by drop into a wide vessel beneath it. When you have observed [this] carefully . . . have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. The droplets will fall as before into the vessel beneath without dropping towards the stern, although while the drops are in the air the ship runs many spans." Explain this quotation.

Determine the Concept The droplet leaving the bottle has the same horizontal velocity as the ship. During the time the droplet is in the air, it is also moving horizontally with the same velocity as the rest of the ship. Because of this, it falls into the vessel, which has the same horizontal velocity. Because you have the same horizontal velocity as the ship does, you see the same thing as if the ship were standing still.

30 • A man swings a stone attached to a rope in a horizontal circle at constant speed. Figure 3-33 represents the path of the rock looking down from above. (*a*) Which of the vectors could represent the velocity of the stone? (*b*) Which could represent the acceleration?

Determine the Concept (*a*) Because $\vec{A}$ and $\vec{D}$ are tangent to the path of the stone, either of them could represent the velocity of the stone.



(*b*) Let the vectors $\vec{A}(t)$ and $\vec{A}(t+\Delta t)$ be of equal length but point in slightly different directions as the stone moves around the circle. These two vectors and $\Delta\vec{A}$ are shown in the diagram above. Note that $\Delta\vec{A}$ is nearly perpendicular to $\vec{A}(t)$. For very small time intervals, $\Delta\vec{A}$ and $\vec{A}(t)$ are perpendicular to one another. Therefore, $d\vec{A}/dt$ is perpendicular to $\vec{A}$ and only the vector $\vec{E}$ could represent the acceleration of the stone.

31 •• True or false:

(*a*) An object cannot move in a circle unless it has centripetal acceleration.
(*b*) An object cannot move in a circle unless it has tangential acceleration.
(*c*) An object moving in a circle cannot have a variable speed.
(*d*) An object moving in a circle cannot have a constant velocity.

(*a*) True. An object accelerates when its velocity changes; that is, when either its speed or its direction changes. When an object moves in a circle the direction of its motion is continually changing.

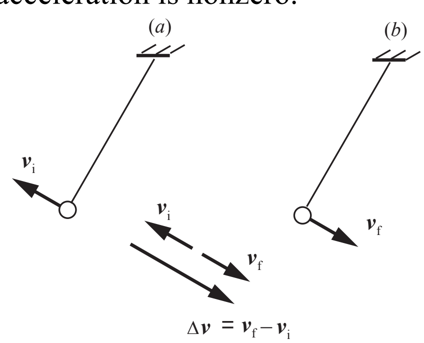
(*b*) False. An object moving in a circular path at constant speed has a tangential acceleration of zero.

(*c*) False. A good example is a rock wedged in the tread of an automobile tire. Its speed changes whenever the car's speed changes.

(*d*) True. The velocity vector of any object moving in a circle is continually changing direction.

32 •• Using a motion diagram, find the direction of the acceleration of the bob of a pendulum when the bob is at a point where it is just reversing its direction.

Picture the Problem In the diagram, (*a*) shows the pendulum just before it reverses direction and (*b*) shows the pendulum just after it has reversed its direction. The acceleration of the bob is in the direction of the *change* in the velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ and is tangent to the pendulum trajectory at the point of reversal of direction. This makes sense because, at an extremum of motion, $v = 0$, so there is no centripetal acceleration. However, because the velocity is reversing direction, the tangential acceleration is nonzero.

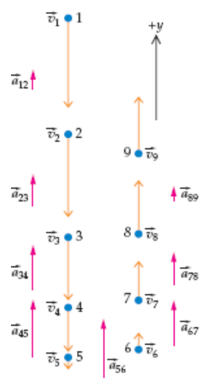


33 •• **[SSM]** During your rookie bungee jump, your friend records your fall using a camcorder. By analyzing it frame by frame, he finds that the *y*-component of your velocity is (recorded every 1/20th of a second) as follows:

t (s)	12.05	12.10	12.15	12.20	12.25	12.30	12.35	12.40	12.45
v_y (m/s)	–0.78	–0.69	–0.55	–0.35	–0.10	0.15	0.35	0.49	0.53

(*a*) Draw a motion diagram. Use it to find the direction and relative magnitude of your average acceleration for each of the eight successive 0.050 s time intervals in the table. (*b*) Comment on how the *y* component of your acceleration does or does not vary in sign and magnitude as you reverse your direction of motion.

Determine the Concept (*a*) The motion diagram shown below was constructed using the data in the table shown below the motion diagram. Note that the motion diagram has been rotated 90°. The upward direction is to the left. The column for Δv in the table to the right was calculated using $\Delta \boldsymbol{v} = \boldsymbol{v}_i - \boldsymbol{v}_{i-1}$ and the column for a was calculated using $\boldsymbol{a} = (\boldsymbol{v}_i - \boldsymbol{v}_{i-1})/\Delta \boldsymbol{t}$.



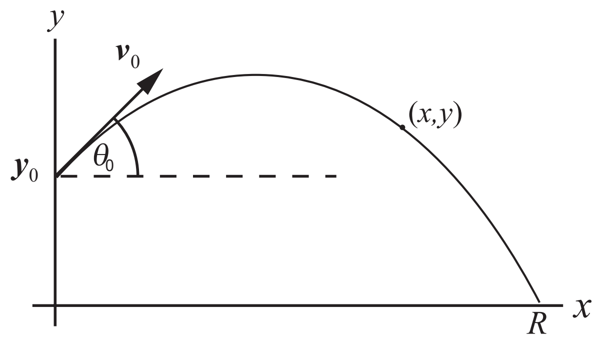
i	v	Δv	a_{ave}
	(m/s)	(m/s)	(m/s^2)
1	−0.78		
2	−0.69	0.09	1.8
3	−0.55	0.14	2.8
4	−0.35	0.20	4.0
5	−0.10	0.25	5.0
6	0.15	0.25	5.0
7	0.35	0.20	4.0
8	0.49	0.14	2.8
9	0.53	0.04	0.8

(*b*) The acceleration vector always points upward and so the sign of its *y* component does not change. The magnitude of the acceleration vector is greatest when the bungee cord has its maximum extension (your speed, the magnitude of your velocity, is least at this time and times near it) and is less than this maximum value when the bungee cord has less extension.

Estimation and Approximation

34 •• Estimate the speed in mph with which water comes out of a garden hose using your past observations of water coming out of garden hoses and your knowledge of projectile motion.

Picture the Problem Based on your experience with garden hoses, you probably know that the maximum range of the water is achieved when the hose is inclined at about 45° with the vertical. A reasonable estimate of the range of such a stream is about 4.0 m when the initial height of the stream is 1.0 m. Use constant-acceleration equations to obtain expressions for the x and y coordinates of a droplet of water in the stream and then eliminate time between these equations to obtain an equation that you can solve for v_0.



Use constant-acceleration equations to express the x and y components of a molecule of water in the stream:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
and
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $x_0 = 0$, $x = R$, $a_y = -g$, and $a_x = 0$:

$$x = v_{0x}t \qquad (1)$$
and
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad (2)$$

Express v_{0x} and v_{0y} in terms of v_0 and θ:

$v_{0x} = v_0\cos\theta_0$ and $v_{0y} = v_0\sin\theta_0$
where θ_0 is the angle the stream makes with the horizontal.

Substitute in equations (1) and (2) to obtain:

$$x = (v_0\cos\theta_0)t \qquad (3)$$
and
$$y = y_0 + (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad (4)$$

Eliminating t from equations (3) and (4) yields:

$$y = y_0 + (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$

When the stream of water hits the ground, $y = 0$ and $x = R$:

$$0 = y_0 + (\tan\theta_0)R - \frac{g}{2v_0^2\cos^2\theta_0}R^2$$

Solving this equation for v_0 yields:

$$v_0 = R\sqrt{\frac{g}{2\cos^2\theta\left[R\tan\theta + y_0\right]}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = (4.0\text{ m})\sqrt{\frac{9.81\text{ m/s}^2}{2\cos^2 45°[(4.0\text{ m})\tan 45° + 1.0\text{ m}]}} = 5.603\text{ m/s}$$

Use a conversion factor, found in Appendix A, to convert m/s to mi/h:

$$v_0 = 5.603\text{ m/s} \times \frac{1\text{ mi/h}}{0.4470\text{ m/s}} \approx \boxed{13\text{ mi/h}}$$

35 •• You won a contest to spend a day with all team during their spring training camp. You are allowed to try to hit some balls thrown by a pitcher. Estimate the acceleration during the hit of a fastball thrown by a major league pitcher when you hit the ball squarely-straight back at the pitcher. You will need to make reasonable choices for ball speeds, both just before and just after the ball is hit, and of the contact time the ball has with the bat.

Determine the Concept The magnitude of the acceleration of the ball is given by $a = |\vec{a}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right|$. Let $\vec{v}_{\text{after}}$ represent the velocity of the ball just after its collision with the bat and $\vec{v}_{\text{before}}$ its velocity just before this collision. Most major league pitchers can throw a fastball at least 90 mi/h and some occasionally throw as fast as 100 mi/h. Let's assume that the pitcher throws you an 80 mph fastball.

The magnitude of the acceleration of the ball is:

$$a = \left|\frac{\vec{v}_{\text{after}} - \vec{v}_{\text{before}}}{\Delta t}\right|.$$

Assuming that v_{after} and v_{before} are both 80 mi/h and that the ball is in contact with the bat for 1 ms:

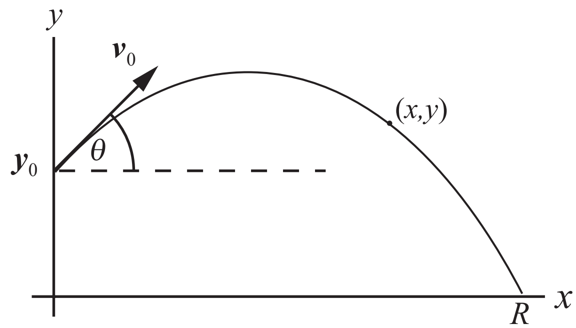
$$a = \frac{80\text{ mi/h} - (-80\text{ mi/h})}{1\text{ ms}} = \frac{160\text{ mi/h}}{1\text{ ms}}$$

Converting a to m/s^2 yields:

$$a = \frac{160\text{ mi/h}}{1\text{ ms}} \times \frac{0.447\text{ m/s}}{1\text{ mi/h}} \approx \boxed{7\times10^4\text{ m/s}^2}$$

36 •• Estimate how far you can throw a ball if you throw it (*a*) horizontally while standing on level ground, (*b*) at $\theta = 45°$ above horizontal while standing on level ground, (*c*) horizontally from the top of a building 12 m high, (*d*) at $\theta = 45°$ above horizontal from the top of a building 12 m high. Ignore any effects due to air resistance.

Picture the Problem During the flight of the ball the acceleration is constant and equal to 9.81 m/s^2 directed downward. We can find the flight time from the vertical part of the motion, and then use the horizontal part of the motion to find the horizontal distance. We'll assume that the release point of the ball is 2.0 m above your feet. A sketch of the motion that includes coordinate axes, the initial and final positions of the ball, the launch angle, and the initial velocity follows.



Obviously, how far you throw the ball will depend on how fast you can throw it. A major league baseball pitcher can throw a fastball at 90 mi/h or so. Assume that you can throw a ball at two-thirds that speed to obtain:

$$v_0 = 60\,\text{mi/h}\times\frac{0.447\,\text{m/s}}{1\,\text{mi/h}} \approx 27\,\text{m/s}$$

There is no acceleration in the x direction, so the horizontal motion is one of constant velocity. Express the horizontal position of the ball as a function of time:

$$x = v_{0x}t = (v_0\cos\theta_0)t \qquad (1)$$

Assuming that the release point of the ball is a distance y_0 above the ground, express the vertical position of the ball as a function of time:

$$\begin{aligned} y &= y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \\ &= y_0 + (v_0\sin\theta_0)t + \tfrac{1}{2}a_y t^2 \end{aligned} \qquad (2)$$

Eliminating t between equations (1) and (2) yields:

$$y = y_0 + (\tan\theta_0)x + \left(\frac{a_y}{2v_0^2\cos^2\theta_0}\right)x^2$$

(a) If you throw the ball horizontally our equation becomes:

$$y = y_0 + \left(\frac{a_y}{2v_0^2}\right)x^2$$

Substitute numerical values to obtain:

$$y = 2.0\,\text{m} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2}\right)x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 2.0\,\text{m} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2}\right)R^2$$

Solving for R yields: $\boldsymbol{R} \approx \boxed{17\,\text{m}}$

(*b*) For $\theta = 45°$ we have:

$$\boldsymbol{y} = 2.0\,\text{m} + (\tan 45°)\boldsymbol{x} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2 \cos^2 45°}\right)\boldsymbol{x}^2$$

At impact, y =0 and $x = R$:

$$0 = 2.0\,\text{m} + (\tan 45°)\boldsymbol{R} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2 \cos^2 45°}\right)\boldsymbol{R}^2$$

Use the quadratic formula or your graphing calculator to obtain: $\boldsymbol{R} = \boxed{75\,\text{m}}$

(*c*) If you throw the ball horizontally from the top of a building that is 12 m high our equation becomes:

$$\boldsymbol{y} = 14.0\,\text{m} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2}\right)\boldsymbol{x}^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 14.0\,\text{m} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2}\right)\boldsymbol{R}^2$$

Solve for R to obtain: $\boldsymbol{R} = \boxed{45\,\text{m}}$

(*d*) If you throw the ball at an angle of 45° from the top of a building that is 12 m high our equation becomes:

$$\boldsymbol{y} = 14\,\text{m} + (\tan 45°)\boldsymbol{x} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2 \cos^2 45°}\right)\boldsymbol{x}^2$$

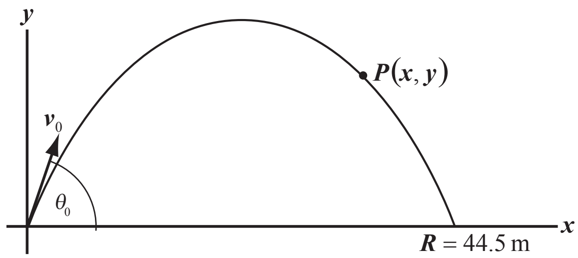
At impact, $y = 0$ and $x = R$:

$$0 = 14\,\text{m} + (\tan 45°)\boldsymbol{R} + \left(\frac{-9.81\,\text{m/s}^2}{2(27\,\text{m/s})^2 \cos^2 45°}\right)\boldsymbol{R}^2$$

Use the quadratic formula or your graphing calculator to obtain: $\boldsymbol{R} = \boxed{85\,\text{m}}$

37 •• In 1978, Geoff Capes of Great Britain threw a heavy brick a horizontal distance of 44.5 m. Find the approximate speed of the brick at the highest point of its flight, neglecting any effects due to air resistance.

Picture the Problem We'll ignore the height of Geoff's release point above the ground and assume that he launched the brick at an angle of 45°. Because the velocity of the brick at the highest point of its flight is equal to the horizontal component of its initial velocity, we can use constant-acceleration equations to relate this velocity to the brick's *x* and *y* coordinates at impact. The diagram shows an appropriate coordinate system and the brick when it is at point *P* with coordinates (*x*, *y*).



Using a constant-acceleration equation, express the *x* and *y* coordinates of the brick as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
and
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $x_0 = 0$, $y_0 = 0$, $a_y = -g$, and $a_x = 0$:

$$x = v_{0x}t$$
and
$$y = v_{0y}t - \tfrac{1}{2}gt^2$$

Eliminate *t* between these equations to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_{0x}^2}x^2$$

where we have used $\tan\boldsymbol{\theta}_0 = \dfrac{\boldsymbol{v}_{0y}}{\boldsymbol{v}_{0x}}$.

When the brick strikes the ground $y = 0$ and $x = R$:

$$0 = (\tan\theta_0)R - \frac{g}{2v_{0x}^2}R^2$$

where *R* is the range of the brick.

Solve for v_{0x} to obtain:

$$v_{0x} = \sqrt{\frac{gR}{2\tan\theta_0}}$$

Substitute numerical values and evaluate v_{0x}:

$$\boldsymbol{v}_{0x} = \sqrt{\frac{(9.81\,\text{m/s}^2)(44.5\,\text{m})}{2\tan 45°}} \approx \boxed{15\,\text{m/s}}$$

Note that, at the brick's highest point, $v_y = 0$.

Position, Displacement, Velocity and Acceleration Vectors

38 • A wall clock has a minute hand with a length of 0.50 m and an hour hand with a length of 0.25 m. Take the center of the clock as the origin, and use a Cartesian coordinate system with the positive *x* axis pointing to 3 o'clock and the positive *y* axis pointing to 12 o'clock. Using unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$, express the position vectors of the tip of the hour hand ($\vec{A}$) and the tip of the minute hand ($\vec{B}$) when the clock reads (*a*) 12:00, (*b*) 3:00, (*c*) 6:00, (*d*) 9:00.

Picture the Problem Let the +*y* direction be straight up and the +*x* direction be to the right.

(*a*) At 12:00, both hands are positioned along the +*y* axis.

The position vector for the tip of the hour hand at12:00 is: $\vec{A} = \boxed{(0.25\,\text{m})\hat{\boldsymbol{j}}}$

The position vector for the tip of the minute hand at 12:00 is: $\vec{B} = \boxed{(0.50\,\text{m})\hat{\boldsymbol{j}}}$

(*b*) At 3:00, the minute hand is positioned along the +*y* axis, while the hour hand is positioned along the +*x* axis.

The position vector for the tip of the hour hand at 3:00 is: $\vec{A} = \boxed{(0.25\,\text{m})\hat{\boldsymbol{i}}}$

The position vector for the tip of the minute hand at 3:00 is: $\vec{B} = \boxed{(0.50\,\text{m})\hat{\boldsymbol{j}}}$

(*c*) At 6:00, the minute hand is positioned along the –*y* axis, while the hour hand is positioned along the +*y* axis.

The position vector for the tip of the hour hand at 6:00 is: $\vec{A} = \boxed{-(0.25\,\text{m})\hat{\boldsymbol{j}}}$

The position vector for the tip of the minute hand at 6:00 is: $\vec{B} = \boxed{(0.50\,\text{m})\hat{\boldsymbol{j}}}$

(*d*) At 9:00, the minute hand is positioned along the +*y* axis, while the hour hand is positioned along the –*x* axis.

The position vector for the tip of the hour hand at 9:00 is: $\vec{A} = \boxed{-(0.25\,\text{m})\hat{\boldsymbol{i}}}$

The position vector for the tip of the minute hand at 9:00 is: $\vec{B} = \boxed{(0.50\,\text{m})\hat{\boldsymbol{j}}}$

39 • **[SSM]** In Problem 38, find the displacements of the tip of each hand (that is, $\Delta\vec{A}$ and $\Delta\vec{B}$) when the time advances from 3:00 P.M. to 6:00 P.M.

Picture the Problem Let the +*y* direction be straight up, the +*x* direction be to the right, and use the vectors describing the ends of the hour and minute hands in Problem 38 to find the displacements $\Delta\vec{A}$ and $\Delta\vec{B}$.

The displacement of the minute hand as time advances from 3:00 P.M. to 6:00 P.M. is given by:

$$\Delta\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_6 - \vec{\boldsymbol{B}}_3$$

From Problem 38:

$$\vec{\boldsymbol{B}}_6 = (0.50\,\text{m})\hat{\boldsymbol{j}} \text{ and } \vec{\boldsymbol{B}}_3 = (0.50\,\text{m})\hat{\boldsymbol{j}}$$

Substitute and simplify to obtain:

$$\Delta\vec{\boldsymbol{B}} = (0.50\,\text{m})\hat{\boldsymbol{j}} - (0.50\,\text{m})\hat{\boldsymbol{j}} = \boxed{0}$$

The displacement of the hour hand as time advances from 3:00 P.M. to 6:00 P.M. is given by:

$$\Delta\vec{\boldsymbol{A}} = \vec{\boldsymbol{A}}_6 - \vec{\boldsymbol{A}}_3$$

From Problem 38:

$$\vec{\boldsymbol{A}}_6 = -(0.25\,\text{m})\hat{\boldsymbol{j}} \text{ and } \vec{\boldsymbol{A}}_3 = (0.25\,\text{m})\hat{\boldsymbol{i}}$$

Substitute and simplify to obtain:

$$\Delta\vec{\boldsymbol{A}} = \boxed{-(0.25\,\text{m})\hat{\boldsymbol{j}} - (0.25\,\text{m})\hat{\boldsymbol{i}}}$$

40 • In Problem 38, write the vector that describes the displacement of a fly if it quickly goes from the tip of the minute hand to the tip of the hour hand at 3:00 P.M.

Picture the Problem Let the positive *y* direction be straight up, the positive *x* direction be to the right, and use the vectors describing the ends of the hour and minute hands in Problem 38 to find the displacement $\vec{\boldsymbol{D}}$ of the fly as it goes from the tip of the minute hand to the tip of the hour hand at 3:00 P.M.

The displacement of the fly is given by:

$$\vec{D} = \vec{A}_3 - \vec{B}_3$$

From Problem 38:

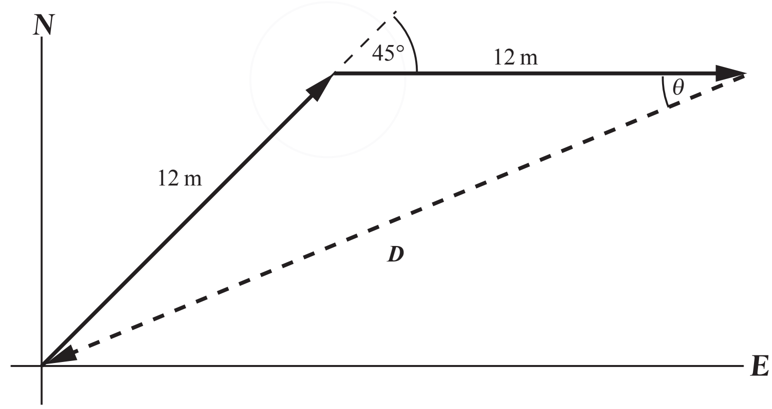
$$\vec{A}_3 = (0.25\,\text{m})\hat{i} \text{ and } \vec{B}_3 = (0.50\,\text{m})\hat{j}$$

Substitute for $\vec{A}_3$ and $\vec{B}_3$ to obtain:

$$\vec{D} = \boxed{(0.25\,\text{m})\hat{i} - (0.50\,\text{m})\hat{j}}$$

41 • A bear, awakening from winter hibernation, staggers directly northeast for 12 m and then due east for 12 m. Show each displacement graphically and graphically determine the single displacement that will take the bear back to her cave, to continue her hibernation.

Picture the Problem The single displacement for the bear to make it back to its cave is the vector $\vec{\boldsymbol{D}}$. Its magnitude D and direction θ can be determined by drawing the bear's displacement vectors to scale.

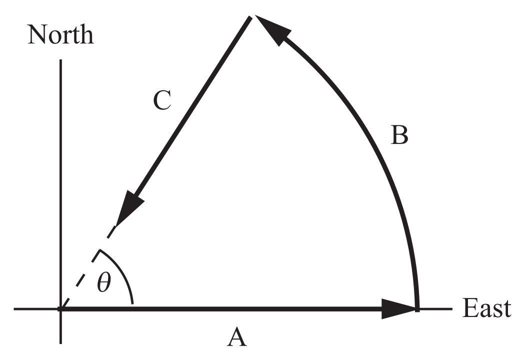


$$\boldsymbol{D} \approx \boxed{22\,\text{m}} \text{ and } \boldsymbol{\theta} \approx \boxed{23°}$$

Remarks: The direction of $\vec{\boldsymbol{D}}$ is 180° + $\boldsymbol{\theta}$ ≈ 203°

42 • A scout walks 2.4 km due East from camp, then turns left and walks 2.4 km along the arc of a circle centered at the campsite, and finally walks 1.5 km directly toward the camp. (*a*) How far is the scout from camp at the end of his walk? (*b*) In what direction is the scout's position relative to the campsite? (*c*) What is the ratio of the final magnitude of the displacement to the total distance walked?

Picture the Problem The figure shows the paths walked by the Scout. The length of path A is 2.4 km; the length of path B is 2.4 km; and the length of path C is 1.5 km:



(*a*) Express the distance from the campsite to the end of path C:

$$2.4\text{ km} - 1.5\text{ km} = \boxed{0.9\,\text{km}}$$

(*b*) Determine the angle θ subtended by the arc at the origin (campsite):

$$\theta_{\text{radians}} = \frac{\text{arc length}}{\text{radius}} = \frac{2.4\,\text{km}}{2.4\,\text{km}}$$
$$= 1\,\text{rad} = 57.3°$$

His direction from camp is 1 rad North of East.

(*c*) Express the total distance as the sum of the three parts of his walk:

$$d_{\text{tot}} = d_{\text{east}} + d_{\text{arc}} + d_{\text{toward camp}}$$

Substitute the given distances to find the total:

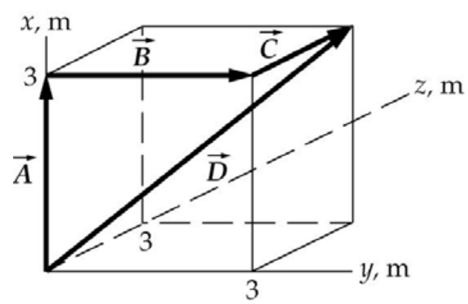
$$d_{\text{tot}} = 2.4\text{ km} + 2.4\text{ km} + 1.5\text{ km} = 6.3\text{ km}$$

Express the ratio of the magnitude of his displacement to the total distance he walked and substitute to obtain a numerical value for this ratio:

$$\frac{\text{Magnitude of his displacement}}{\text{Total distance walked}} = \frac{0.9\,\text{km}}{6.3\,\text{km}} = \boxed{\frac{1}{7}}$$

43 •• **[SSM]** The faces of a cubical storage cabinet in your garage has 3.0-m-long edges that are parallel to the *xyz* coordinate planes. The cube has one corner at the origin. A cockroach, on the hunt for crumbs of food, begins at that corner and walks along three edges until it is at the far corner. (*a*) Write the roach's displacement using the set of $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, and $\hat{\boldsymbol{k}}$ unit vectors, and (*b*) find the magnitude of its displacement.

Picture the Problem While there are several walking routes the cockroach could take to get from the origin to point C, its displacement will be the same for all of them. One possible route is shown in the figure.



(*a*) The roach's displacement $\vec{D}$ during its trip from the origin to point C is:

$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = \boxed{(3.0\,\text{m})\hat{\boldsymbol{i}} + (3.0\,\text{m})\hat{\boldsymbol{j}} + (3.0\,\text{m})\hat{\boldsymbol{k}}}$$

(*b*) The magnitude of the roach's displacement is given by:

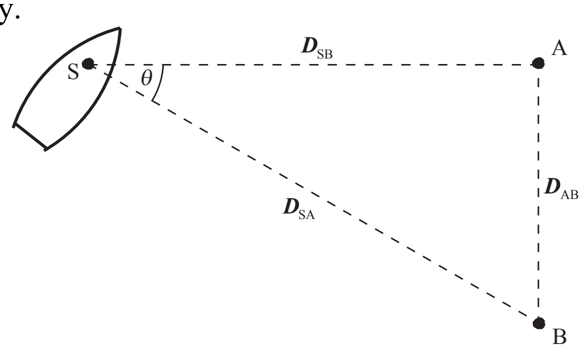
$$D = \sqrt{D_x^2 + D_y^2 + D_z^2}$$

Substitute for D_x, D_y, and D_z and evaluate D to obtain:

$$D = \sqrt{(3.0\,\text{m})^2 + (3.0\,\text{m})^2 + (3.0\,\text{m})^2} = \boxed{5.2\,\text{m}}$$

44 • You are the navigator of a ship at sea. You receive radio signals from two transmitters A and B, which are 100 km apart, one due south of the other. The direction finder shows you that transmitter A is at a heading of 30º south of east from the ship, while transmitter B is due east. Calculate the distance between your ship and transmitter B.

Picture the Problem The diagram shows the locations of the transmitters relative to the ship and defines the distances separating the transmitters from each other and from the ship. We can find the distance between the ship and transmitter B using trigonometry.



Relate the distance between A and B to the distance from the ship to A and the angle θ:

$$\tan\theta = \frac{D_{AB}}{D_{SB}} \Rightarrow D_{SB} = \frac{D_{AB}}{\tan\theta}$$

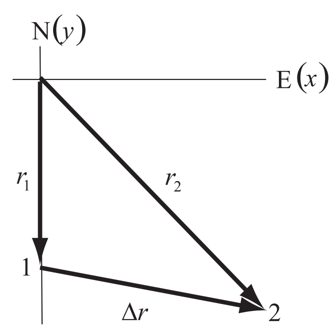
Substitute numerical values and evaluate D_{SB}:

$$\boldsymbol{D}_{SB} = \frac{100\,\text{km}}{\tan 30°} = \boxed{1.7\times10^{5}\,\text{m}}$$

Velocity and Acceleration Vectors

45 • A stationary radar operator determines that a ship is 10 km due south of him. An hour later the same ship is 20 km due southeast. If the ship moved at constant speed and always in the same direction, what was its velocity during this time?

Picture the Problem For constant speed and direction, the instantaneous velocity is identical to the average velocity. Take the origin to be the location of the stationary radar and let the +*x* direction be to the East and the +*y* direction be to the North.



Express the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \qquad (1)$$

Determine the position vectors $\vec{r}_1$ and $\vec{r}_2$:

$\vec{r}_1 = (-10\,\text{km})\hat{j}$

and

$\vec{r}_2 = (14.1\,\text{km})\hat{i} + (-14.1\,\text{km})\hat{j}$

Find the displacement vector $\Delta\vec{r}$:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (14.1\,\text{km})\hat{i} + (-4.1\,\text{km})\hat{j}$$

Substitute for $\Delta\vec{r}$ and Δt in equation (1) to find the average velocity.

$$\vec{v}_{\text{av}} = \frac{(14.1\,\text{km})\hat{i} + (-4.1\,\text{km})\hat{j}}{1.0\,\text{h}} = \boxed{(14\,\text{km/h})\hat{i} + (-4.1\,\text{km/h})\hat{j}}$$

46 • A particle's position coordinates (x, y) are (2.0 m, 3.0 m) at $t = 0$; (6.0 m, 7.0 m) at $t = 2.0$ s; and (13 m, 14 m) at $t = 5.0$ s. (*a*) Find the magnitude of the average velocity from $t = 0$ to $t = 2.0$ s. (*b*) Find the magnitude of the average velocity from t = 0 to t = 5.0 s.

Picture the Problem The average velocity is the change in position divided by the elapsed time.

(*a*) The magnitude of the average velocity is given by:

$$|\vec{v}_{\text{av}}| = \frac{|\Delta\vec{r}|}{\Delta t}$$

Find the position vectors and the displacement vector:

$\vec{r}_0 = (2.0\text{ m})\hat{i} + (3.0\text{ m})\hat{j}$

$\vec{r}_2 = (6.0\text{ m})\hat{i} + (7.0\text{ m})\hat{j}$

and

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (4.0\text{ m})\hat{i} + (4.0\text{ m})\hat{j}$

Find the magnitude of the displacement vector for the interval between $t = 0$ and $t = 2.0$ s:

$$|\Delta\vec{r}_{02}| = \sqrt{(4.0\text{ m})^2 + (4.0\text{ m})^2} = 5.66\text{ m}$$

Substitute to determine $|\vec{v}_{\text{av}}|$:

$$|\vec{v}_{\text{av}}| = \frac{5.66\,\text{m}}{2.0\,\text{s}} = \boxed{2.8\,\text{m/s}}$$

θ is given by:

$$\theta = \tan^{-1}\left(\frac{\Delta r_{y,02}}{\Delta r_{x,02}}\right) = \tan^{-1}\left(\frac{4.0\text{ m}}{4.0\text{ m}}\right) = \boxed{45°}$$

measured from the positive x axis.

(*b*) Repeat (*a*), this time using the displacement between $t = 0$ and $t = 5.0$ s to obtain:

$$\vec{r}_5 = (13\text{ m})\hat{i} + (14\text{ m})\hat{j}$$

$$\Delta\vec{r}_{05} = \vec{r}_5 - \vec{r}_0 = (11\text{ m})\hat{i} + (11\text{ m})\hat{j}.$$

$$|\Delta\vec{r}_{05}| = \sqrt{(11\text{ m})^2 + (11\text{ m})^2} = 15.6\text{ m}$$

$$|\vec{v}_{\text{av}}| = \frac{15.6\text{ m}}{5.0\text{ s}} = \boxed{3.1\text{ m/s}}.$$

and

$$\theta = \tan^{-1}\left(\frac{11\text{ m}}{11\text{ m}}\right) = \boxed{45°}$$

measured from the $+x$ axis.

47 • **[SSM]** A particle moving at a velocity of 4.0 m/s in the $+x$ direction is given an acceleration of 3.0 m/s^2 in the $+y$ direction for 2.0 s. Find the final speed of the particle.

Picture the Problem The magnitude of the velocity vector at the end of the 2 s of acceleration will give us its speed at that instant. This is a constant-acceleration problem.

Find the final velocity vector of the particle:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = v_{x0}\hat{i} + a_yt\hat{j}$$
$$= (4.0\text{ m/s})\hat{i} + (3.0\text{ m/s}^2)(2.0\text{ s})\hat{j}$$
$$= (4.0\text{ m/s})\hat{i} + (6.0\text{ m/s})\hat{j}$$

The magnitude of $\vec{v}$ is:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Substitute for v_x and v_y and evaluate $|\vec{v}|$:

$$|\vec{v}| = \sqrt{(4.0\text{ m/s})^2 + (6.0\text{ m/s})^2} = \boxed{7.2\text{ m/s}}$$

48 • Initially, a swift-moving hawk is moving due west with a speed of 30 m/s; 5.0 s later it is moving due north with a speed of 20 m/s. (*a*) What are the magnitude and direction of $\Delta\vec{v}_{\text{av}}$ during this 5.0-s interval? (*b*) What are the magnitude and direction of $\vec{a}_{\text{av}}$ during this 5.0-s interval?

Picture the Problem Choose a coordinate system in which north coincides with the positive y direction and east with the positive x direction. Expressing the hawk's velocity vectors is the first step in determining $\Delta\vec{v}$ and $\vec{a}_{\text{av}}$.

(*a*) The change in the hawk's velocity during this interval is:

$$\Delta\vec{v}_{\text{av}} = \vec{v}_{\text{N}} - \vec{v}_{\text{W}}$$

$\vec{v}_{\text{W}}$ and $\vec{v}_{\text{N}}$ are given by:

$$\vec{v}_{\text{W}} = -(30\text{ m/s})\hat{\boldsymbol{i}} \text{ and } \vec{v}_{\text{N}} = (20\text{ m/s})\hat{\boldsymbol{j}}$$

Substitute for $\vec{v}_{\text{W}}$ and $\vec{v}_{\text{N}}$ and evaluate $\Delta\vec{v}$:

$$\begin{aligned}\Delta\vec{v}_{\text{av}} &= (20\text{ m/s})\hat{\boldsymbol{j}} - \left[-(30\text{ m/s})\hat{\boldsymbol{i}}\right]\\ &= (30\text{ m/s})\hat{\boldsymbol{i}} + (20\text{ m/s})\hat{\boldsymbol{j}}\end{aligned}$$

The magnitude of $\Delta\vec{v}_{\text{av}}$ is given by:

$$|\Delta\vec{v}_{\text{av}}| = \sqrt{\Delta \boldsymbol{v}_x^2 + \Delta \boldsymbol{v}_y^2}$$

Substitute numerical values and evaluate Δv:

$$\begin{aligned}|\Delta\vec{v}_{\text{av}}| &= \sqrt{(30\text{ m/s})^2 + (20\text{ m/s})^2}\\ &= \boxed{36\text{ m/s}}\end{aligned}$$

The direction of Δv is given by:

$$\boldsymbol{\theta} = 180° - \tan^{-1}\left(\frac{\Delta \boldsymbol{v}_y}{\Delta \boldsymbol{v}_x}\right)$$

where θ is measured from the positive x axis.

Substitute numerical values and evaluate θ:

$$\begin{aligned}\theta &= 180° - \tan^{-1}\left(\frac{20\text{ m/s}}{30\text{ m/s}}\right)\\ &= 180° - 34° = 146° = \boxed{150°}\end{aligned}$$

(*b*) The hawk's average acceleration during this interval is:

$$\vec{a}_{\text{av}} = \frac{\Delta\vec{v}}{\Delta \boldsymbol{t}}$$

Substitute for $\Delta\vec{v}$ and Δt to obtain:

$$\begin{aligned}\vec{a}_{\text{av}} &= \frac{(30\text{ m/s})\hat{\boldsymbol{i}} + (20\text{ m/s})\hat{\boldsymbol{j}}}{5.0\text{ s}}\\ &= (6.0\text{ m/s}^2)\hat{\boldsymbol{i}} + (4.0\text{ m/s}^2)\hat{\boldsymbol{j}}\end{aligned}$$

The magnitude of $\vec{a}_{\text{av}}$ is given by:

$$|\vec{a}_{\text{av}}| = \sqrt{\boldsymbol{a}_x^2 + \boldsymbol{a}_y^2}$$

Substitute numerical values and evaluate $|\vec{a}_{\text{av}}|$:

$$\begin{aligned}|\vec{a}_{\text{av}}| &= \sqrt{(6.0\text{ m/s}^2) + (4.0\text{ m/s}^2)}\\ &= \boxed{7.2\text{ m/s}^2}\end{aligned}$$

The direction of $\vec{a}_{\text{av}}$ is given by:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{4.0\,\frac{\text{m}}{\text{s}^2}}{6.0\,\frac{\text{m}}{\text{s}^2}}\right) = \boxed{34°}$$

where θ is measured from the positive x axis.

49 • At $t = 0$, a particle located at the origin has a velocity of 40 m/s at $\theta = 45°$. At $t = 3.0$ s, the particle is at $x = 100$ m and $y = 80$ m and has a velocity of 30 m/s at $\theta = 50°$. Calculate (*a*) the average velocity and (*b*) the average acceleration of the particle during this 3.0-s interval.

Picture the Problem The initial and final positions and velocities of the particle are given. We can find the average velocity and average acceleration using their definitions by first calculating the given displacement and velocities using unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$.

(*a*) The average velocity of the particle is the ratio of its displacement to the elapsed time:

$$\vec{\boldsymbol{v}}_{\text{av}} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta \boldsymbol{t}}$$

The displacement of the particle during this interval of time is:

$$\Delta\vec{\boldsymbol{r}} = (100\,\text{m})\hat{\boldsymbol{i}} + (80\,\text{m})\hat{\boldsymbol{j}}$$

Substitute to find the average velocity:

$$\vec{\boldsymbol{v}}_{\text{av}} = \frac{(100\,\text{m})\hat{\boldsymbol{i}} + (80\,\text{m})\hat{\boldsymbol{j}}}{3.0\,\text{s}} = (33.3\,\text{m/s})\hat{\boldsymbol{i}} + (26.7\,\text{m/s})\hat{\boldsymbol{j}} = \boxed{(33\,\text{m/s})\hat{\boldsymbol{i}} + (27\,\text{m/s})\hat{\boldsymbol{j}}}$$

(*b*) The average acceleration is:

$$\vec{\boldsymbol{a}}_{\text{av}} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta \boldsymbol{t}} = \frac{\vec{\boldsymbol{v}}_2 - \vec{\boldsymbol{v}}_1}{\Delta \boldsymbol{t}}$$

$\vec{\boldsymbol{v}}_1$ and $\vec{\boldsymbol{v}}_2$ are given by:

$$\vec{\boldsymbol{v}}_1 = (40\,\text{m/s})\cos 45°\hat{\boldsymbol{i}} + (40\,\text{m/s})\sin 45°\hat{\boldsymbol{j}} = (28.28\,\text{m/s})\hat{\boldsymbol{i}} + (28.28\,\text{m/s})\hat{\boldsymbol{j}}$$

and

$$\vec{\boldsymbol{v}}_2 = (30\,\text{m/s})\cos 50°\hat{\boldsymbol{i}} + (30\,\text{m/s})\sin 50°\hat{\boldsymbol{j}} = (19.28\,\text{m/s})\hat{\boldsymbol{i}} + (22.98\,\text{m/s})\hat{\boldsymbol{j}}$$

Substitute for $\vec{v}_1$ and $\vec{v}_2$ and evaluate $\vec{a}_{\text{av}}$:

$$\vec{a}_{\text{av}} = \frac{(19.28\,\text{m/s})\hat{i} + (22.98\,\text{m/s})\hat{j} - \left[(28.28\,\text{m/s})\hat{i} + (28.28\,\text{m/s})\hat{j}\right]}{3.0\,\text{s}}$$
$$= \frac{(-9.00\,\text{m/s})\hat{i} + (-5.30\,\text{m/s})\hat{j}}{3.0\,\text{s}} = \boxed{(-3.0\,\text{m/s}^2)\hat{i} + (-1.8\,\text{m/s}^2)\hat{j}}$$

50 •• At time zero, a particle is at $x = 4.0$ m and $y = 3.0$ m and has velocity $\vec{v} = (2.0\text{ m/s})\hat{i} + (-9.0\text{ m/s})\hat{j}$. The acceleration of the particle is constant and is given by $\vec{a} = (4.0\text{ m/s}^2)\hat{i} + (3.0\text{ m/s}^2)\hat{j}$. (*a*) Find the velocity at $t = 2.0$ s. (*b*) Express the position vector at $t = 4.0$ s in terms of $\hat{i}$ and $\hat{j}$. In addition, give the magnitude and direction of the position vector at this time.

Picture the Problem The acceleration is constant so we can use the constant-acceleration equations in vector form to find the velocity at $t = 2.0$ s and the position vector at $t = 4.0$ s.

(*a*) The velocity of the particle, as a function of time, is given by:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

Substitute for $\vec{v}_0$ and $\vec{a}$ to find $\vec{v}(2.0\,\text{s})$:

$$\vec{v}(2.0\,\text{s}) = (2.0\,\text{m/s})\hat{i} + (-9.0\,\text{m/s})\hat{j} + \left[(4.0\,\text{m/s}^2)\hat{i} + (3.0\,\text{m/s}^2)\hat{j}\right](2.0\,\text{s})$$
$$= \boxed{(10\,\text{m/s})\hat{i} + (-3.0\,\text{m/s})\hat{j}}$$

(*b*) Express the position vector as a function of time:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Substitute numerical values and evaluate $\vec{r}$:

$$\vec{r} = (4.0\,\text{m})\hat{i} + (3.0\,\text{m})\hat{j} + \left[(2.0\,\text{m/s})\hat{i} + (-9.0\,\text{m/s})\hat{j}\right](4.0\,\text{s})$$
$$+ \tfrac{1}{2}\left[(4.0\,\text{m/s}^2)\hat{i} + (3.0\,\text{m/s}^2)\hat{j}\right](4.0\,\text{s})^2$$
$$= (44.0\,\text{m})\hat{i} + (-9.0\,\text{m})\hat{j}$$
$$= \boxed{(44\,\text{m})\hat{i} + (-9.0\,\text{m})\hat{j}}$$

The magnitude of $\vec{r}$ is given by:

$$r = \sqrt{r_x^2 + r_y^2}$$

Substitute numerical values and evaluate $r(4.0\text{ s})$:

$$\boldsymbol{r}(4.0\text{ s}) = \sqrt{(44.0\text{ m})^2 + (-9.0\text{ m})^2} = \boxed{45\text{ m}}$$

The direction of $\vec{\boldsymbol{r}}$ is given by:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{\boldsymbol{r}_y}{\boldsymbol{r}_x}\right)$$

Because $\vec{\boldsymbol{r}}$ is in the 4th quadrant, its direction measured from the $+x$ axis is:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{-9.0\text{ m}}{44.0\text{ m}}\right) = -11.6° = \boxed{-12°}$$

51 •• **[SSM]** A particle has a position vector given by $\vec{\boldsymbol{r}} = (30t)\hat{\boldsymbol{i}} + (40t - 5t^2)\hat{\boldsymbol{j}}$, where r is in meters and t is in seconds. Find the instantaneous-velocity and instantaneous-acceleration vectors as functions of time t.

Picture the Problem The velocity vector is the time-derivative of the position vector and the acceleration vector is the time-derivative of the velocity vector.

Differentiate $\vec{\boldsymbol{r}}$ with respect to time:

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{dt} = \frac{d}{dt}\left[(30t)\hat{\boldsymbol{i}} + (40t - 5t^2)\hat{\boldsymbol{j}}\right] = \boxed{30\hat{\boldsymbol{i}} + (40 - 10t)\hat{\boldsymbol{j}}}$$

where $\vec{\boldsymbol{v}}$ has units of m/s if t is in seconds.

Differentiate $\vec{\boldsymbol{v}}$ with respect to time:

$$\vec{\boldsymbol{a}} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[30\hat{\boldsymbol{i}} + (40 - 10t)\hat{\boldsymbol{j}}\right] = \boxed{(-10\text{ m/s}^2)\hat{\boldsymbol{j}}}$$

52 •• A particle has a constant acceleration of $\vec{\boldsymbol{a}} = (6.0\text{ m/s}^2)\hat{\boldsymbol{i}} + (4.0\text{ m/s}^2)\hat{\boldsymbol{j}}$. At time $t = 0$, the velocity is zero and the position vector is $\vec{\boldsymbol{r}}_0 = (10\text{ m})\hat{\boldsymbol{i}}$. (*a*) Find the velocity and position vectors as functions of time t. (*b*) Find the equation of the particle's path in the xy plane and sketch the path.

Picture the Problem We can use constant-acceleration equations in vector form to find the velocity and position vectors as functions of time t. In (*b*), we can eliminate t from the equations giving the x and y components of the particle to find an expression for y as a function of x.

(*a*) Use $\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_0 + \vec{\boldsymbol{a}}\boldsymbol{t}$ with $\vec{\boldsymbol{v}}_0 = 0$ to find $\vec{\boldsymbol{v}}$:

$$\vec{\boldsymbol{v}} = \boxed{\left[(6.0\text{ m/s}^2)\hat{\boldsymbol{i}} + (4.0\text{ m/s}^2)\hat{\boldsymbol{j}}\right]\boldsymbol{t}}$$

Use $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ with $\vec{r}_0 = (10\,\text{m})\hat{i}$ to find $\vec{r}$:

$$\vec{r} = \boxed{\left[(10\text{ m}) + (3.0\text{ m/s}^2)t^2\right]\hat{i} + \left[(2.0\text{ m/s}^2)t^2\right]\hat{j}}$$

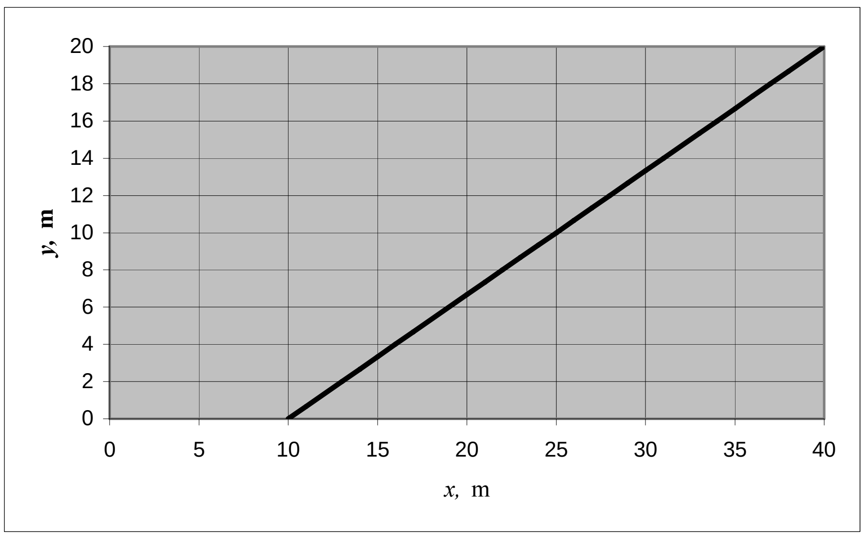
(*b*) Obtain the *x* and *y* components of the path from the vector equation in (*a*):

$$x = 10\text{ m} + (3.0\text{ m/s}^2)t^2$$
and
$$y = (2.0\text{ m/s}^2)t^2$$

Eliminate *t* from these equations and solve for *y* to obtain:

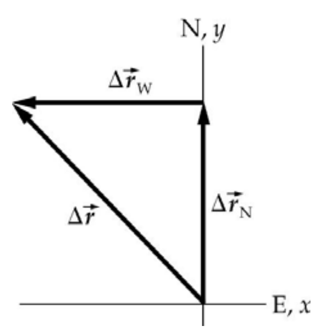
$$\boxed{y = \tfrac{2}{3}x - \tfrac{20}{3}\text{ m}}$$

The graph of $y = \frac{2}{3}x - \frac{20}{3}$ m is shown below. Note that the path in the *xy* plane is a straight line.



53 •• **[SSM]** Starting from rest at a dock, a motor boat on a lake heads north while gaining speed at a constant 3.0 m/s^2 for 20 s. The boat then heads west and continues for 10 s at the speed that it had at 20 s. (*a*) What is the average velocity of the boat during the 30-s trip? (*b*) What is the average acceleration of the boat during the 30-s trip? (*c*) What is the displacement of the boat during the 30-s trip?

Picture the Problem The displacements of the boat are shown in the figure. Let the +*x* direction be to the east and the +*y* direction be to the north. We need to determine each of the displacements in order to calculate the average velocity of the boat during the 30-s trip.



(*a*) The average velocity of the boat is given by:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \quad (1)$$

The total displacement of the boat is given by:

$$\Delta\vec{r} = \Delta\vec{r}_N + \Delta\vec{r}_W = \tfrac{1}{2}a_N(\Delta t_N)^2\hat{j} + v_W\Delta t_W(-\hat{i}) \quad (2)$$

To calculate the displacement we first have to find the speed after the first 20 s:

$$v_W = v_{N,f} = a_N\Delta t_N$$

Substitute numerical values and evaluate v_W:

$$v_W = (3.0\ \text{m/s}^2)(20\ \text{s}) = 60\ \text{m/s}$$

Substitute numerical values in equation (2) and evaluate $\Delta\vec{r}(30\ \text{s})$:

$$\Delta\vec{r}(30\ \text{s}) = \tfrac{1}{2}(3.0\ \text{m/s}^2)(20\ \text{s})^2\hat{j} - (60\ \text{m/s})(10\ \text{s})\hat{i} = (600\ \text{m})\hat{j} - (600\ \text{m})\hat{i}$$

Substitute numerical values in equation (1) to find the boat's average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(600\ \text{m})(-\hat{i}+\hat{j})}{30\ \text{s}} = \boxed{(20\ \text{m/s})(-\hat{i}+\hat{j})}$$

(*b*) The average acceleration of the boat is given by:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Substitute numerical values and evaluate $\vec{a}_{av}$:

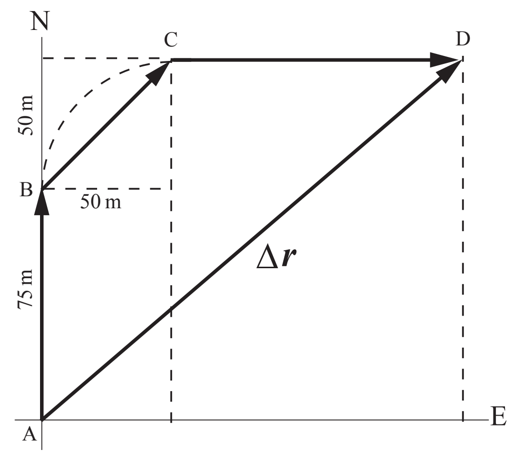
$$\vec{a}_{av} = \frac{(-60\ \text{m/s})\hat{i} - 0}{30\ \text{s}} = \boxed{(-2.0\ \text{m/s}^2)\hat{i}}$$

(*c*) The displacement of the boat from the dock at the end of the 30-s trip was one of the intermediate results we obtained in Part (*a*).

$$\Delta\vec{r} = (600\ \text{m})\hat{j} + (-600\ \text{m})\hat{i} = \boxed{(600\ \text{m})(-\hat{i}+\hat{j})}$$

54 •• Starting from rest at point A, you ride your motorcycle north to point B 75.0 m away, increasing speed at steady rate of 2.00 m/s^2. You then gradually turn toward the east along a circular path of radius 50.0 m at constant speed from B to point C until your direction of motion is due east at C. You then continue eastward, slowing at a steady rate of 1.00 m/s^2 until you come to rest at point D. (*a*) What is your average velocity and acceleration for the trip from A to D? (*b*) What is your displacement during your trip from A to C? (*c*) What distance did you travel for the entire trip from A to D?

Picture the Problem The diagram shown to the right summarizes the information given in the problem statement. Let the $+x$ direction be to the east and the $+y$ direction be to the north. To find your average velocity you'll need to find your displacement $\Delta\vec{r}$ and the total time required for you to make this trip. You can express $\Delta\vec{r}$ as the sum of your displacements $\Delta\vec{r}_{AB}$, $\Delta\vec{r}_{BC}$, and $\Delta\vec{r}_{CD}$. The total time for your trip is the sum of the times required for each of the segments. Because the acceleration is constant (but different) along each of the segments of your trip, you can use constant-acceleration equations to find each of these quantities.



(*a*) The average velocity for your trip is given by:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \quad (1)$$

The total displacement for your trip is the sum of the displacements along the three segments:

$$\Delta\vec{r} = \Delta\vec{r}_{AB} + \Delta\vec{r}_{BC} + \Delta\vec{r}_{CD} \quad (2)$$

The total time for your trip is the sum of the times for the three segments:

$$\Delta\boldsymbol{t} = \Delta\boldsymbol{t}_{AB} + \Delta\boldsymbol{t}_{BC} + \Delta\boldsymbol{t}_{CD} \quad (3)$$

The displacements of the three segments of the trip are:

$$\Delta\vec{r}_{AB} = 0\hat{\boldsymbol{i}} + (75.0\text{ m})\hat{\boldsymbol{j}}\,,$$
$$\Delta\vec{r}_{BC} = (50.0\text{ m})\hat{\boldsymbol{i}} + (50.0\text{ m})\hat{\boldsymbol{j}}\,,$$
and
$$\Delta\vec{r}_{CD} = \Delta\boldsymbol{r}_{CD}\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}}$$

In order to find Δx_{CD}, you need to find the time for the C to D segment of the trip. Use a constant-acceleration equation to express Δx_{CD}:

$$\Delta x_{CD} = v_C\Delta t_{CD} + \tfrac{1}{2}a_{CD}(\Delta t_{CD})^2 \quad (4)$$

Because $v_C = v_B$:

$$\boldsymbol{v}_C = \boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{a}_{AB}\Delta\boldsymbol{t}_{AB}$$
or, because $v_A = 0$,
$$\boldsymbol{v}_C = \boldsymbol{a}_{AB}\Delta\boldsymbol{t}_{AB} \quad (5)$$

Use a constant-acceleration equation to relate Δt_{AB} to Δy_{AB}:

$$\Delta \boldsymbol{y}_{AB} = \boldsymbol{v}_A \Delta \boldsymbol{t}_{AB} + \tfrac{1}{2}\boldsymbol{a}_{AB}(\Delta \boldsymbol{t}_{AB})^2$$

or, because $v_A = 0$,

$$\Delta \boldsymbol{y}_{AB} = \tfrac{1}{2}\boldsymbol{a}_{AB}(\Delta \boldsymbol{t}_{AB})^2 \Rightarrow \Delta \boldsymbol{t}_{AB} = \sqrt{\frac{2\Delta \boldsymbol{y}_{AB}}{\boldsymbol{a}_{AB}}}$$

Substitute numerical values and evaluate Δt_{AB}:

$$\Delta \boldsymbol{t}_{AB} = \sqrt{\frac{2(75.0\text{ m})}{2.00\text{ m/s}^2}} = 8.660\text{ s}$$

Substitute for Δt_{AB} in equation (5) to obtain:

$$\boldsymbol{v}_C = \boldsymbol{a}_{AB}\sqrt{\frac{2\Delta \boldsymbol{y}_{AB}}{\boldsymbol{a}_{AB}}} = \sqrt{2\boldsymbol{a}_{AB}\Delta \boldsymbol{y}_{AB}}$$

Substitute numerical values and evaluate v_C:

$$\boldsymbol{v}_C = \sqrt{2(2.00\text{ m/s}^2)(75.0\text{ m})} = 17.32\text{ m/s}$$

The time required for the circular segment BC is given by:

$$\Delta \boldsymbol{t}_{BC} = \frac{\tfrac{1}{2}\boldsymbol{\pi}\,\boldsymbol{r}}{\boldsymbol{v}_B}$$

Substitute numerical values and evaluate Δt_{BC}:

$$\Delta \boldsymbol{t}_{BC} = \frac{\tfrac{1}{2}\boldsymbol{\pi}(50.0\text{ m})}{17.32\text{ m/s}} = 4.535\text{ s}$$

The time to travel from C to D is given by:

$$\Delta \boldsymbol{t}_{CD} = \frac{\Delta \boldsymbol{v}_{CD}}{\boldsymbol{a}_{CD}} = \frac{\boldsymbol{v}_D - \boldsymbol{v}_C}{\boldsymbol{a}_{CD}}$$

or, because $v_D = 0$,

$$\Delta \boldsymbol{t}_{CD} = \frac{-\boldsymbol{v}_C}{\boldsymbol{a}_{CD}}$$

Substitute numerical values and evaluate Δt_{CD}:

$$\Delta t_{CD} = \frac{-17.32\text{ m/s}}{-1.00\text{ m/s}^2} = 17.32\text{ s}$$

Now we can use equation (4) to evaluate Δt_{CD}:

$$\Delta \boldsymbol{x}_{CD} = (17.32\text{ m/s})(17.32\text{ s}) + \tfrac{1}{2}(-1.00\text{ m/s}^2)(17.32\text{ s})^2 = 150\text{ m}$$

The three displacements that you need to add in order to get $\Delta\vec{\boldsymbol{r}}$ are:

$$\Delta\vec{\boldsymbol{r}}_{AB} = 0\hat{\boldsymbol{i}} + (75.0\text{ m})\hat{\boldsymbol{j}},$$

$$\Delta\vec{\boldsymbol{r}}_{BC} = (50.0\text{ m})\hat{\boldsymbol{i}} + (50.0\text{ m})\hat{\boldsymbol{j}},$$

and

$$\Delta\vec{\boldsymbol{r}}_{CD} = (150\text{ m})\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}}$$

Substitute in equation (2) to obtain:

$$\begin{aligned}\Delta\vec{r} &= 0\hat{i} + (75.0\text{ m})\hat{j} + (50.0\text{ m})\hat{i} + (50.0\text{ m})\hat{j} + (150\text{ m})\hat{i} + 0\hat{j}\\ &= (200\text{ m})\hat{i} + (125\text{ m})\hat{j}\end{aligned}$$

Substituting in equation (3) for Δt_{AB}, Δt_{BC}, and Δt_{CD} yields:

$$\Delta t = 8.660\text{ s} + 4.535\text{ s} + 17.32\text{ s} = 30.51\text{ s}$$

Use equation (1) to find $\vec{v}_{\text{av}}$:

$$\vec{v}_{\text{av}} = \frac{(200\text{ m})\hat{i} + (125\text{ m})\hat{j}}{30.51\text{ s}} = \boxed{(6.56\text{ m/s})\hat{i} + (4.10\text{ m/s})\hat{j}}$$

Because the final and initial velocities are zero, the average acceleration is zero.

(*b*) Express your displacement from A to C as the sum of the displacements from A to B and from B to C:

$$\Delta\vec{r}_{\text{AC}} = \Delta\vec{r}_{\text{AB}} + \Delta\vec{r}_{\text{BC}}$$

From (*a*) you have:

$$\Delta\vec{r}_{\text{AB}} = 0\hat{i} + (75.0\text{ m})\hat{j}$$

and

$$\Delta\vec{r}_{\text{BC}} = (50.0\text{ m})\hat{i} + (50.0\text{ m})\hat{j}$$

Substitute and simplify to obtain:

$$\begin{aligned}\Delta\vec{r}_{\text{AC}} &= 0\hat{i} + (75.0\text{ m})\hat{j} + (50.0\text{ m})\hat{i} + (50.0\text{ m})\hat{j}\\ &= \boxed{(50.0\text{ m})\hat{i} + (125\text{ m})\hat{j}}\end{aligned}$$

(*c*) Express the total distance you traveled as the sum of the distances traveled along the three segments:

$$d_{\text{tot}} = d_{\text{AB}} + \widehat{d}_{\text{BC}} + d_{\text{CD}}$$

where $\widehat{d}_{\text{BC}} = \frac{1}{2}\pi r$ and r is the radius of the circular arc.

Substituting for $\widehat{d}_{\text{BC}}$ yields:

$$d_{\text{tot}} = d_{\text{AB}} + \tfrac{1}{2}\pi r + d_{\text{CD}}$$

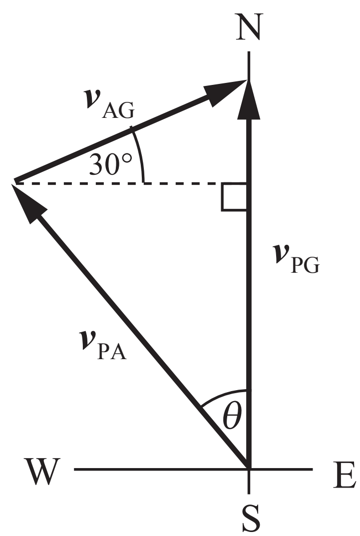
Substitute numerical values and evaluate d_{tot}:

$$d_{\text{tot}} = 75.0\text{ m} + \tfrac{1}{2}\pi(50.0\text{ m}) + 150\text{ m} = \boxed{304\text{ m}}$$

Relative Velocity

55 •• A plane flies at an airspeed of 250 km/h. A wind is blowing at 80 km/h toward the direction 60º east of north. (*a*) In what direction should the plane head in order to fly due north relative to the ground? (*b*) What is the speed of the plane relative to the ground?

Picture the Problem Choose a coordinate system in which north is the $+y$ direction and east is the $+x$ direction. Let θ be the angle between north and the direction of the plane's heading. The velocity of the plane relative to the ground, $\vec{v}_{PG}$, is the sum of the velocity of the plane relative to the air, $\vec{v}_{PA}$, and the velocity of the air relative to the ground, $\vec{v}_{AG}$. That is, $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. The pilot must head in such a direction that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.



(*a*) From the diagram one can see that:

$$v_{AG}\cos 30° = v_{PA}\sin\theta$$

Solving for θ yields:

$$\theta = \sin^{-1}\left[\frac{v_{AG}\cos 30°}{v_{PA}}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(80\text{ km/h})\cos 30°}{250\text{ km/h}}\right] = 16.1° \approx \boxed{16°\text{ west of north}}$$

(*b*) Because the plane is headed due north, add the north components of $\vec{v}_{PA}$ and $\vec{v}_{AG}$ to determine the plane's ground speed:

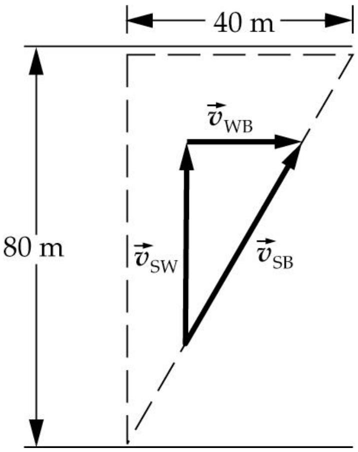
$$\left|\vec{v}_{PG}\right| = (250\text{ km/h})\cos 16.1° + (80\text{ km/h})\sin 30° = \boxed{280\text{ km/s}}$$

56 •• A swimmer *heads* directly across a river, swimming at 1.6 m/s relative to the water. She arrives at a point 40 m downstream from the point directly across the river, which is 80 m wide. (*a*) What is the speed of the river current? (*b*) What is the swimmer's speed relative to the shore? (*c*) In what direction should the swimmer head in order to arrive at the point directly opposite her starting point?

Picture the Problem Let $\vec{v}_{SB}$ represent the velocity of the swimmer relative to the shore; $\vec{v}_{SW}$ the velocity of the swimmer relative to the water; and $\vec{v}_{WB}$ the velocity of the water relative to the shore; i.e.,

$$\vec{v}_{SB} = \vec{v}_{SW} + \vec{v}_{WB}$$

The current of the river causes the swimmer to drift downstream.



(*a*) The triangles shown in the figure are similar right triangles. Set up a proportion between their sides and solve for the speed of the water relative to the bank:

$$\frac{v_{WB}}{v_{SW}} = \frac{40\,\text{m}}{80\,\text{m}}$$

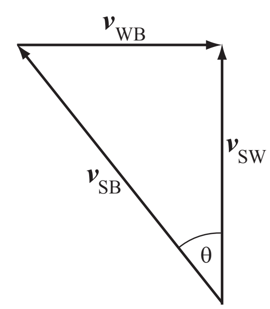
and

$$v_{WB} = \tfrac{1}{2}(1.6\,\text{m/s}) = \boxed{0.80\,\text{m/s}}$$

(*b*) Use the Pythagorean Theorem to solve for the swimmer's speed relative to the shore:

$$v_{SB} = \sqrt{v_{SW}^2 + v_{WB}^2} = \sqrt{(1.6\,\text{m/s})^2 + (0.80\,\text{m/s})^2} = \boxed{1.8\,\text{m/s}}$$

(*c*) The swimmer should head in a direction such that the upstream component of her velocity relative to the shore ($\vec{v}_{SB}$) is equal to the speed of the water relative to the shore ($\vec{v}_{WB}$):



Referring to the diagram, relate $\sin\theta$ to $\vec{v}_{WB}$ and $\vec{v}_{SB}$:

$$\sin\theta = \frac{v_{WB}}{v_{SB}} \Rightarrow \theta = \sin^{-1}\left(\frac{v_{WB}}{v_{SB}}\right)$$

Substitute numerical values and evaluate θ:

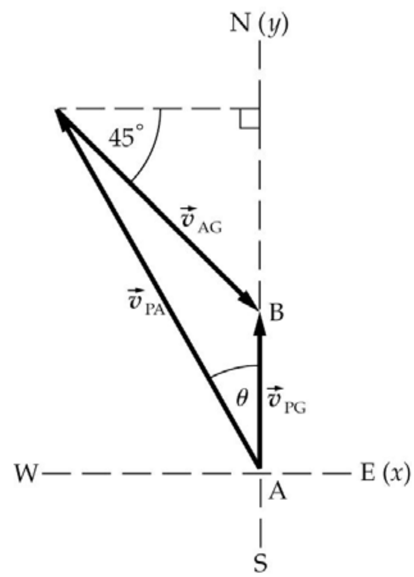
$$\theta = \sin^{-1}\left(\frac{0.80\,\text{m/s}}{1.6\,\text{m/s}}\right) = \boxed{30°}$$

57 •• **[SSM]** A small plane departs from point A heading for an airport 520 km due north at point B. The airspeed of the plane is 240 km/h and there is a steady wind of 50 km/h blowing directly toward the southeast. Determine the proper heading for the plane and the time of flight.

Picture the Problem Let the velocity of the plane relative to the ground be represented by $\vec{v}_{PG}$; the velocity of the plane relative to the air by $\vec{v}_{PA}$, and the velocity of the air relative to the ground by $\vec{v}_{AG}$. Then

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} \quad (1)$$

Choose a coordinate system with the origin at point A, the +*x* direction to the east, and the +*y* direction to the north. θ is the angle between north and the direction of the plane's heading. The pilot must head so that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.



Use the diagram to express the condition relating the eastward component of $\vec{v}_{AG}$ and the westward component of $\vec{v}_{PA}$. This must be satisfied if the plane is to stay on its northerly course. [Note: this is equivalent to equating the *x*-components of equation (1).]

$$v_{AG}\cos 45° = v_{PA}\sin\theta$$

Now solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{v_{AG}\cos 45°}{v_{PA}}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(50\text{ km/h})\cos 45°}{240\text{ km/h}}\right] = 8.47°$$
$$= \boxed{8.5°}$$

Add the north components of $\vec{v}_{PA}$ and $\vec{v}_{AG}$ to find the velocity of the plane relative to the ground:

$$v_{PG} + v_{AG}\sin 45° = v_{PA}\cos 8.47°$$

Solving for v_{PG} yields:

$$v_{PG} = v_{PA}\cos 8.47° - v_{AG}\sin 45°$$

Substitute numerical values and evaluate v_{PG} to obtain:

$$v_{PG} = (240\text{ km/h})\cos 8.47° - (50\text{ km/h})\sin 45° = 202.0\text{ km/h}$$

The time of flight is given by:

$$t_{\text{flight}} = \frac{\text{distance travelled}}{v_{PG}}$$

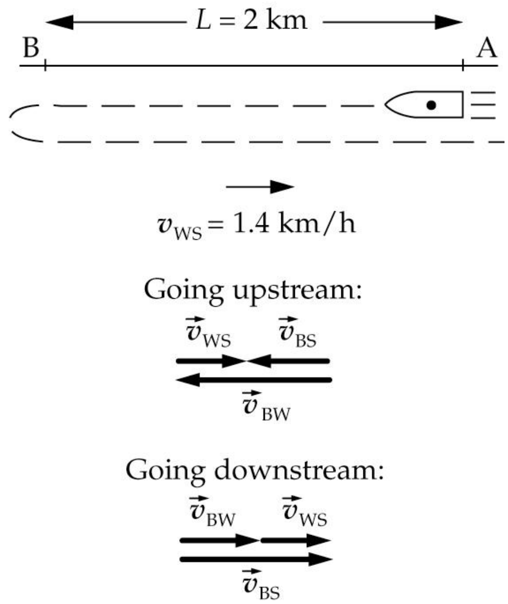
Substitute numerical values and evaluate t_{flight}:

$$t_{\text{flight}} = \frac{520\,\text{km}}{202.0\,\text{km/h}} = \boxed{2.57\,\text{h}}$$

58 •• Two boat landings are 2.0 km apart on the same bank of a stream that flows at 1.4 km/h. A motorboat makes the round trip between the two landings in 50 min. What is the speed of the boat relative to the water?

Picture the Problem Let $\vec{v}_{BS}$ be the velocity of the boat relative to the shore; $\vec{v}_{BW}$ be the velocity of the boat relative to the water; and $\vec{v}_{WS}$ represent the velocity of the water relative to the shore. Independently of whether the boat is going upstream or downstream:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$



Going upstream, the speed of the boat relative to the shore is reduced by the speed of the water relative to the shore.

Going downstream, the speed of the boat relative to the shore is increased by the same amount.

For the upstream leg of the trip:

$$v_{BS} = v_{BW} - v_{WS}$$

For the downstream leg of the trip:

$$v_{BS} = v_{BW} + v_{WS}$$

Express the total time for the trip in terms of the times for its upstream and downstream legs:

$$t_{\text{total}} = t_{\text{upstream}} + t_{\text{downstream}} = \frac{L}{v_{BW} - v_{WS}} + \frac{L}{v_{BW} + v_{WS}}$$

Multiply both sides of the equation by $(v_{BW} - v_{WS})(v_{BW} + v_{WS})$ (the product of the- denominators) and rearrange the terms to obtain:

$$v_{BW}^2 - \frac{2L}{t_{\text{total}}} v_{BW} - v_{WS}^2 = 0$$

Substituting numerical values yields:

$$v_{BW}^2 - \frac{2(2.0\text{ km})}{\frac{5}{6}\text{ h}} v_{BW} - (1.4\text{ km/h})^2 = 0$$

or

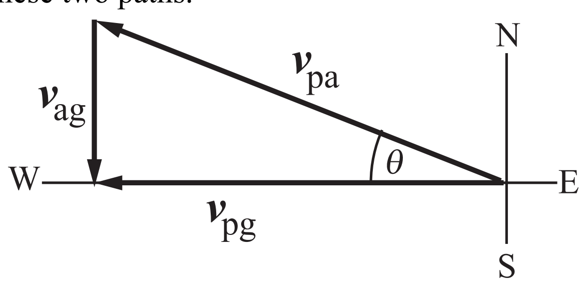
$$v_{BW}^2 - (4.80\text{ km/h})v_{BW} - 1.96\,(\text{km})^2/\text{h}^2 = 0$$

Use the quadratic formula or your graphing calculator to solve the quadratic equation for v_{BW} (Note that only the positive root is physically meaningful.):

$$v_{BW} = \boxed{5.2\text{ km/h}}$$

59 •• During a radio-controlled model-airplane competition, each plane must fly from the center of a 1.0-km-radius circle to any point on the circle and back to the center. The winner is the plane that has the shortest round-trip time. The contestants are free to fly their planes along any route as long as the plane begins at the center, travels to the circle, and then returns to the center. On the day of the race, a steady wind blows out of the north at 5.0 m/s. Your plane can maintain an air speed of 15 m/s. Should you fly your plane upwind on the first leg and downwind on the trip back, or across the wind flying east and then west? Optimize your chances by calculating the round-trip time for both routes using your knowledge of vectors and relative velocities. With this pre-race calculation, you can determine the best route and have a major advantage over the competition!

Picture the Problem Let $\vec{v}_{pg}$ be the velocity of the plane relative to the ground; $\vec{v}_{ag}$ be the velocity of the air relative to the ground; and $\vec{v}_{pa}$ the velocity of the plane relative to the air. Then, $\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$. The wind will affect the flight times differently along these two paths.



The velocity of the plane, relative to the ground, on its eastbound leg is equal to its velocity on its westbound leg. Using the diagram, express the velocity of the plane relative to the ground for both directions:

$$v_{pg} = \sqrt{v_{pa}^2 - v_{ag}^2}$$

Substitute numerical values and evaluate v_{pg}:

$$v_{pg} = \sqrt{(15\text{ m/s})^2 - (5.0\text{ m/s})^2} = 14.1\text{m/s}$$

Express the time for the east-west roundtrip in terms of the distances and velocities for the two legs:

$$t_{\text{roundtrip,EW}} = t_{\text{eastbound}} + t_{\text{westbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,eastbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,westbound}}}$$

Substitute numerical values and evaluate $t_{\text{roundtrip,EW}}$:

$$t_{\text{roundtrip,EW}} = \frac{2\times 1.0\text{ km}}{14.1\text{ m/s}} = 142\text{ s}$$

Use the distances and velocities for the two legs to express the time for the north-south roundtrip:

$$t_{\text{roundtrip,NS}} = t_{\text{northbound}} + t_{\text{southbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,northbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,southbound}}}$$

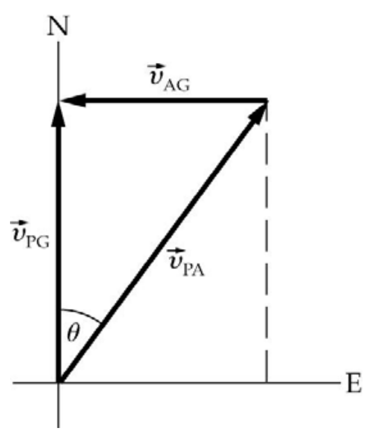
Substitute numerical values and evaluate $t_{\text{roundtrip,NS}}$:

$$t_{\text{roundtrip,NS}} = \frac{1.0\text{ km}}{(15\text{ m/s}) - (5.0\text{ m/s})} + \frac{1.0\text{ km}}{(15\text{ m/s}) + (5.0\text{ m/s})} = 150\text{ s}$$

Because $t_{\text{roundtrip,EW}} < t_{\text{roundtrip,NS}}$, you should fly your plane across the wind.

60 • You are piloting a small plane that can maintain an air speed of 150 kt (knots, or nautical miles per hour) and you want to fly due north (azimuth = 000º) relative to the ground. (*a*) If a wind of 30 kt is blowing from the east (azimuth = 090º), calculate the heading (azimuth) you must ask your co-pilot to maintain. (*b*) At that heading, what will be your groundspeed?

Picture the Problem This is a relative velocity problem. The given quantities are the direction of the velocity of the plane relative to the ground and the velocity (magnitude and direction) of the air relative to the ground. Asked for is the direction of the velocity of the air relative to the ground. Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, draw a vector addition diagram and solve for the unknown quantities.



(*a*) Referring to the diagram, relate the heading you must take to the wind speed and the speed of your plane relative to the air:

$$\boldsymbol{\theta} = \sin^{-1}\left(\frac{\boldsymbol{v}_{\mathrm{AG}}}{\boldsymbol{v}_{\mathrm{PA}}}\right)$$

Substitute numerical values and evaluate θ:

$$\boldsymbol{\theta} = \sin^{-1}\left(\frac{30\text{ kts}}{150\text{ kts}}\right) = 11.5°$$

Because this is also the angle of the plane's heading clockwise from north, it is also its azimuth or the required true heading:

$$\mathrm{Az} = (011.5°) = \boxed{(012°)}$$

(*b*) Referring to the diagram, relate your heading to your plane's speeds relative to the ground and relative to the air:

$$\cos\boldsymbol{\theta} = \frac{\boldsymbol{v}_{\mathrm{PG}}}{\boldsymbol{v}_{\mathrm{PA}}}$$

Solve for the speed of your plane relative to the ground to obtain:

$$\boldsymbol{v}_{\mathrm{PG}} = \boldsymbol{v}_{\mathrm{PA}}\cos\boldsymbol{\theta}$$

Substitute numerical values and evaluate v_{PG}:

$$\boldsymbol{v}_{\mathrm{PG}} = (150\text{ kt})\cos 11.5° = 147\text{ kt}$$
$$= \boxed{1.5\times10^{2}\text{ kt}}$$

61 •• **[SSM]** Car A is traveling east at 20 m/s toward an intersection. As car A crosses the intersection, car B starts from rest 40 m north of the intersection and moves south, steadily gaining speed at 2.0 m/s^2. Six seconds after A crosses the intersection find (*a*) the position of B relative to A, (*b*) the velocity of B relative to A, and (*c*) the acceleration of B relative to A. (*Hint: Let the unit vectors* $\hat{\boldsymbol{i}}$ *and* $\hat{\boldsymbol{j}}$ *be toward the east and north, respectively, and express your answers using* $\hat{\boldsymbol{i}}$ *and* $\hat{\boldsymbol{j}}$.)

Picture the Problem The position of B relative to A is the vector from A to B; that is,

$$\vec{r}_{\text{AB}} = \vec{r}_{\text{B}} - \vec{r}_{\text{A}}$$

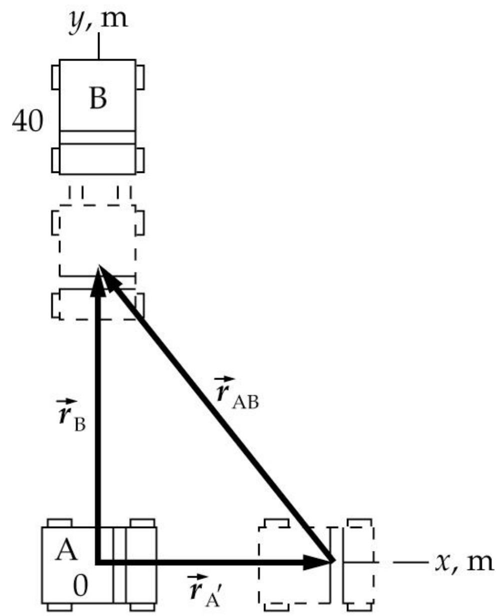
The velocity of B relative to A is

$$\vec{v}_{\text{AB}} = d\vec{r}_{\text{AB}}/dt$$

and the acceleration of B relative to A is

$$\vec{a}_{\text{AB}} = d\vec{v}_{\text{AB}}/dt$$

Choose a coordinate system with the origin at the intersection, the positive x direction to the east, and the positive y direction to the north.



(a) Find $\vec{r}_{\text{B}}$ and $\vec{r}_{\text{A}}$:

$$\vec{r}_{\text{B}} = \left[40\text{ m} - \tfrac{1}{2}\left(2.0\text{ m/s}^2\right)t^2\right]\hat{j}$$

and

$$\vec{r}_{\text{A}} = \left[\left(20\text{ m/s}\right)t\right]\hat{i}$$

Use $\vec{r}_{\text{AB}} = \vec{r}_{\text{B}} - \vec{r}_{\text{A}}$ to find $\vec{r}_{\text{AB}}$:

$$\vec{r}_{\text{AB}} = \left[\left(-20\text{ m/s}\right)t\right]\hat{i} + \left[40\text{ m} - \tfrac{1}{2}\left(2.0\text{ m/s}^2\right)t^2\right]\hat{j}$$

Evaluate $\vec{r}_{\text{AB}}$ at $t = 6.0$ s:

$$\begin{aligned}\vec{r}_{\text{AB}}(6.0\text{ s}) &= \left[\left(-20\text{ m/s}\right)\left(6.0\text{ s}\right)\right]\hat{i} + \left[40\text{ m} - \tfrac{1}{2}\left(2.0\text{ m/s}^2\right)\left(6.0\text{ s}\right)^2\right]\hat{j} \\ &= \boxed{(1.2\times10^2\text{ m})\,\hat{i} + (4.0\text{ m})\,\hat{j}}\end{aligned}$$

(b) Find $\vec{v}_{\text{AB}} = d\vec{r}_{\text{AB}}/dt$:

$$\begin{aligned}\vec{v}_{\text{AB}} &= \frac{d\vec{r}_{\text{AB}}}{dt} = \frac{d}{dt}\left[\left\{\left(-20\text{ m/s}\right)t\right\}\vec{i} + \left\{40\text{ m} - \tfrac{1}{2}\left(2.0\text{ m/s}^2\right)t^2\right\}\hat{j}\right] \\ &= (-20\text{ m/s})\,\hat{i} + (-2.0\text{ m/s}^2)\,t\,\hat{j}\end{aligned}$$

Evaluate $\vec{v}_{\text{AB}}$ at $t = 6.0$ s:

$$\vec{v}_{\text{AB}}\left(6.0\text{ s}\right) = (-20\text{ m/s})\,\hat{i} + (-2.0\text{ m/s}^2)\left(6.0\text{ s}\right)\hat{j} = \boxed{\left(-20\text{ m/s}\right)\hat{i} - \left(12\text{ m/s}\right)\hat{j}}$$

(*c*) Find $\vec{a}_{AB} = d\vec{v}_{AB}/dt$:

$$\vec{a}_{AB} = \frac{d}{dt}\left[(-20\text{ m/s})\,\hat{i} + (-2.0\text{ m/s}^2)\,t\,\hat{j}\right] = \boxed{\left(-2.0\,\text{m/s}^2\right)\hat{j}}$$

Note that $\vec{a}_{AB}$ is independent of time.

62 •• While walking between gates at an airport, you notice a child running along a moving walkway. Estimating that the child runs at a constant speed of 2.5 m/s relative to the surface of the walkway, you decide to try to determine the speed of the walkway itself. You watch the child run on the entire 21-m walkway in one direction, immediately turn around, and run back to his starting point. The entire trip taking a total elapsed time of 22 s. Given this information, what is the speed of the moving walkway relative to the airport terminal?

Picture the Problem We don't need to know which direction the child ran first, and which he ran second. Because we have the total length of the walkway, the elapsed time for the round trip journey, and the child's walking speed relative to the walkway, we are given sufficient information to determine the moving walkway's speed. The distance covered in the airport is 21 m, and this is covered in a total time of 22 s. When the child walks in the direction of the walkway, his velocity in the airport is the sum of his walking velocity, $\vec{v}_{\text{child}}$, and the walkway velocity, $\vec{v}_{\text{ww}}$. On the return trip, the velocity in the airport is the difference of these two velocities.

Express the total time for the child's run in terms of his running times with and against the moving walkway:

$$\Delta t_{\text{tot}} = \Delta t_{\text{with}} + \Delta t_{\text{against}}$$

In terms of the length L of the walkway and the speeds, relative to the airport, of the child walking with and against the moving walkway:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{with}}} + \frac{L}{v_{\text{against}}} \qquad (1)$$

The speeds of the child relative to the airport, then, in each case, are:

$$v_{\text{with}} = |\vec{v}_{\text{child}}| + |\vec{v}_{\text{ww}}| = v_{\text{child}} + v_{\text{ww}}$$

and

$$v_{\text{against}} = |\vec{v}_{\text{child}}| - |\vec{v}_{\text{ww}}| = v_{\text{child}} - v_{\text{ww}}$$

Substitute for v_{with} and v_{against} in equation (1) to obtain:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{child}} + v_{\text{ww}}} + \frac{L}{v_{\text{child}} + v_{\text{ww}}}$$

Solving this equation for v_{ww} yields:

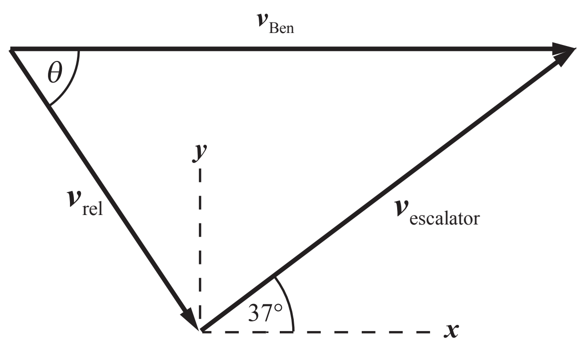
$$v_{\text{ww}} = \sqrt{v_{\text{child}}^2 - \frac{2v_{\text{child}}L}{\Delta t_{\text{tot}}}}$$

Substitute numerical values and evaluate v_{ww}:

$$v_{ww} = \sqrt{(2.5\ \text{m/s})^2 - \frac{2(2.5\ \text{m/s})(21\ \text{m})}{22\ \text{s}}} = \boxed{1.2\ \text{m/s}}$$

63 •• **[SSM]** Ben and Jack are shopping in a department store. Ben leaves Jack at the bottom of the escalator and walks east at a speed of 2.4 m/s. Jack then stands on the escalator, which is inclined at an angle of 37° above the horizontal and travels eastward and upward at a speed of 2.0 m/s. (*a*) What is the velocity of Ben relative to Jack? (*b*) At what speed should Jack walk up the escalator so that he is always directly above Ben (until he reaches the top)?

Picture the Problem The velocity of Ben relative to the velocity of Jack $\vec{v}_{\text{rel}}$ is the difference between the vectors $\vec{v}_{\text{Ben}}$ and $\vec{v}_{\text{escalator}}$. Choose the coordinate system shown and express these vectors using the unit vectors $\hat{i}$ and $\hat{j}$.



(*a*) The velocity of Ben relative to Jack is given by:

$$\vec{v}_{\text{rel}} = \vec{v}_{\text{Ben}} - \vec{v}_{\text{escalator}}$$

The velocities of the floor walker and the escalator are:

$$\vec{v}_{\text{Ben}} = (2.4\ \text{m/s})\,\hat{i}$$

and

$$\vec{v}_{\text{escalator}} = (2.0\ \text{m/s})\cos 37°\,\hat{i} + (2.0\ \text{m/s})\sin 37°\,\hat{j}$$

Substitute for $\vec{v}_{\text{Ben}}$ and $\vec{v}_{\text{escalator}}$ and simplify to obtain:

$$\begin{aligned}\vec{v}_{\text{rel}} &= (2.4\ \text{m/s})\,\hat{i} - \left[(2.0\ \text{m/s})\cos 37°\,\hat{i} + (2.0\ \text{m/s})\sin 37°\,\hat{j}\right] \\ &= (0.803\ \text{m/s})\,\hat{i} - (1.20\ \text{m/s})\,\hat{j} \\ &= \boxed{(0.80\ \text{m/s})\,\hat{i} - (1.2\ \text{m/s})\,\hat{j}}\end{aligned}$$

The magnitude and direction of $\vec{v}_{\text{rel}}$ are given by:

$$\left|\vec{v}_{\text{rel}}\right| = \sqrt{v_{\text{rel},x}^2 + v_{\text{rel},y}^2}$$

and

$$\theta = \tan^{-1}\left(\frac{v_{\text{rel},y}}{v_{\text{rel},x}}\right)$$

Substitute numerical values and evaluate $\left|\vec{v}_{\text{rel}}\right|$ and θ:

$$\left|\vec{v}_{\text{rel}}\right| = \sqrt{(0.803\ \text{m/s})^2 + (-1.20\ \text{m/s})^2} = \boxed{1.4\ \text{m/s}}$$

and

$$\theta = \tan^{-1}\left(\frac{-1.20\ \text{m/s}}{0.803\ \text{m/s}}\right) = -56.2° = \boxed{56°\ \text{below the horizontal}}$$

(*b*) If Jack walks so that he is always directly above Ben, the sum of the horizontal component of his velocity and the horizontal component of the escalator's velocity must equal the velocity of Ben:

$$\left(v_{\text{Jack}} + v_{\text{escalator}}\right)\cos 37° = v_{\text{Ben}}$$

or, solving for v_{Jack},

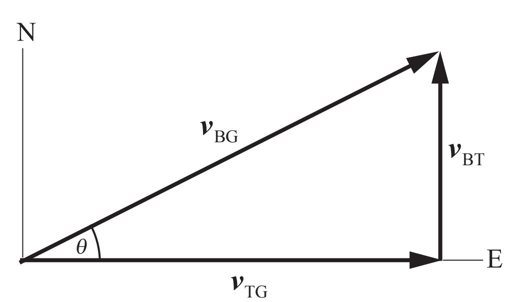
$$v_{\text{Jack}} = \frac{v_{\text{Ben}}}{\cos 37°} - v_{\text{escalator}}$$

Substitute numerical values and evaluate v_{Jack}:

$$v_{\text{Jack}} = \frac{2.4\ \text{m/s}}{\cos 37°} - 2.0\ \text{m/s} = \boxed{1.0\ \text{m/s}}$$

64 ••• A juggler traveling in a train on level track throws a ball straight up, relative to the train, with a speed of 4.90 m/s. The train has a velocity of 20.0 m/s due east. As observed by the juggler, (*a*) what is the ball's total time of flight, and (*b*) what is the displacement of the ball during its rise? According to a friend standing on the ground next to the tracks, (*c*) what is the ball's initial speed, (*d*) what is the angle of the launch, and (*e*) what is the displacement of the ball during its rise?

Picture the Problem The vector diagram shows the relationship between the velocity of the ball relative to the train $\vec{v}_{\text{BT}}$, the velocity of the train relative to the ground $\vec{v}_{\text{TG}}$, and the velocity of the ball relative to the ground $\vec{v}_{\text{BG}}$. Choose a coordinate system in which the $+x$ direction is to the east and the $+y$ direction is to the north.



(*a*) The ball's time-of-flight is twice the time it takes it to reach its maximum height:

$$t_{\text{flight}} = 2t_{\text{max height}} \qquad (1)$$

Use a constant-acceleration equation to relate the ball's speed to its initial speed and the time into its flight:

$$v_y = v_{0y} + a_y t$$

At its maximum height, $v_y = 0$:

$$0 = v_{0y} + a_y t_{\text{max height}}$$

or, because $a_y = -g$,

$$0 = v_{0y} - g t_{\text{max height}}$$

Solving for $t_{\text{max height}}$ yields:

$$t_{\text{max height}} = \frac{v_{0y}}{g}$$

Substitute for $t_{\text{max height}}$ in equation (1) to obtain:

$$t_{\text{flight}} = \frac{2v_{0y}}{g}$$

Substitute numerical values and evaluate t_{flight}:

$$t_{\text{flight}} = \frac{2(4.90\ \text{m/s})}{9.81\ \text{m/s}^2} = \boxed{1.00\ \text{s}}$$

(*b*) The displacement of the ball during its ascent, as observed by the juggler, is given by:

$$\Delta\vec{y}_{\text{BT}} = h\hat{j} = \left(v_{\text{av}} t_{\text{max height}}\right)\hat{j} \qquad (2)$$

where h is the maximum height of the ball.

Because the acceleration of the ball is constant, its average speed is given by:

$$v_{\text{av}} = \frac{v_{0x} + v_x}{2}$$

Because $v_x = 0$:

$$v_{\text{av}} = \tfrac{1}{2} v_{0x}$$

Substitute for v_{av} in equation (2) to obtain:

$$\Delta\vec{y}_{\text{BT}} = \left(\left(\tfrac{1}{2}v_{0x}\right)\left(\frac{v_{0x}}{g}\right)\right)\hat{j} = \frac{v_{0x}^2}{2g}\hat{j}$$

Substitute numerical values and evaluate $\Delta\vec{y}_{\text{BT}}$:

$$\Delta\vec{y}_{\text{BT}} = \frac{(4.90\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}\hat{j} = \boxed{(1.22\ \text{m})\hat{j}}$$

(*c*) $\vec{v}_{\text{BG}}$ is the sum of $\vec{v}_{\text{BT}}$ and $\vec{v}_{\text{TG}}$:

$$\vec{v}_{\text{BG}} = \vec{v}_{\text{BT}} + \vec{v}_{\text{TG}} \qquad (3)$$

Express $\vec{v}_{\text{BT}}$ and $\vec{v}_{\text{TG}}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$:

$$\vec{v}_{\text{BT}} = 0\hat{i} + (4.90\ \text{m/s})\hat{j}$$

and

$$\vec{v}_{\text{TG}} = (20.0\ \text{m/s})\hat{i} + 0\hat{j}$$

Substitute for $\vec{v}_{BT}$ and $\vec{v}_{TG}$ in equation (1) to obtain:

$$\vec{v}_{BG} = (20.0\ \text{m/s})\hat{i} + (4.90\ \text{m/s})\hat{j}$$

The magnitude of $\vec{v}_{BG}$ is given by:

$$v_{BG} = \sqrt{(20.0\ \text{m/s})^2 + (4.90\ \text{m/s})^2} = \boxed{21.6\ \text{m/s}}$$

(*d*) Referring to the vector diagram, express the angle of the launch θ in terms of v_{BT} and v_{TG}:

$$\theta = \tan^{-1}\left(\frac{v_{BT}}{v_{TG}}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{4.90\ \text{m/s}}{20.0\ \text{m/s}}\right) = \boxed{23.8^\circ}$$

(*e*) When the ball is at its peak it is 20.0 m east and 1.22 m above the friend observing from the ground. Hence its displacement is:

$$\vec{d} = \boxed{(20.0\ \text{m})\hat{i} + (1.22\ \text{m})\hat{j}}$$

Circular Motion and Centripetal Acceleration

65 • What is the magnitude of the acceleration of the tip of the minute hand of the clock in Problem 38? Express it as a fraction of the magnitude of free-fall acceleration g.

Picture the Problem We can use the definition of centripetal acceleration to express a_c in terms of the speed of the tip of the minute hand. We can find the tangential speed of the tip of the minute hand by using the distance it travels each revolution and the time it takes to complete each revolution.

Express the magnitude of the acceleration of the tip of the minute hand of the clock as a function of the length of the hand and the speed of its tip:

$$a_c = \frac{v^2}{R} \qquad (1)$$

Use the distance the minute hand travels every hour to express its speed:

$$v = \frac{2\pi R}{T}$$

Substituting for v in equation (1) yields:

$$a_c = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

$$\boldsymbol{a}_c = \frac{4\pi^2(0.50\,\text{m})}{(3600\,\text{s})^2} = 1.52\times10^{-6}\ \text{m/s}^2$$

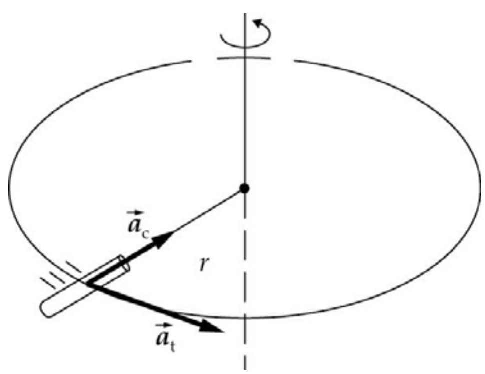
$$= \boxed{1.5\times10^{-6}\ \text{m/s}^2}$$

Express the ratio of a_c to g:

$$\frac{\boldsymbol{a}_c}{\boldsymbol{g}} = \frac{1.52\times10^{-6}\ \text{m/s}^2}{9.81\,\text{m/s}^2} = \boxed{1.55\times10^{-7}}$$

66 • You are designing a centrifuge to spins at a rate of 15,000 rev/min. (*a*) Calculate the maximum centripetal acceleration that a test-tube sample held in the centrifuge arm 15 cm from the rotation axis must withstand. (*b*) It takes 1 min, 15 s for the centrifuge to spin up to its maximum rate of revolution from rest. Calculate the *magnitude* of the tangential acceleration of the centrifuge while it is spinning up, assuming that the tangential acceleration is constant.

Picture the Problem The following diagram shows the centripetal and tangential accelerations experienced by the test tube. The tangential acceleration will be zero when the centrifuge reaches its maximum speed. The centripetal acceleration increases as the tangential speed of the centrifuge increases. We can use the definition of centripetal acceleration to express a_c in terms of the speed of the test tube. We can find the tangential speed of the test tube by using the distance it travels each revolution and the time it takes to complete each revolution. The tangential acceleration can be found from the change in the tangential speed as the centrifuge is spinning up.



(*a*) Express the maximum centripetal acceleration of the centrifuge arm as a function of the length of its arm and the speed of the test tube:

$$a_{c,\,\text{max}} = \frac{v^2}{R} \qquad (1)$$

Use the distance the test tube travels every revolution to express its speed:

$$v = \frac{2\pi R}{T}$$

Substituting for v in equation (1) yields:

$$a_{c,\,\text{max}} = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

$$\boldsymbol{a}_{\mathrm{c,\,max}} = \frac{4\boldsymbol{\pi}^2(0.15\,\mathrm{m})}{\left(\dfrac{1\,\mathrm{min}}{15000\,\mathrm{rev}} \times \dfrac{60\,\mathrm{s}}{\mathrm{min}}\right)^2} = \boxed{3.7\times10^5\ \mathrm{m/s^2}}$$

(*b*) Express the tangential acceleration in terms of the difference between the final and initial tangential speeds:

$$a_{\mathrm{t}} = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\Delta t} = \frac{\dfrac{2\pi R}{T} - 0}{\Delta t} = \frac{2\pi R}{T\Delta t}$$

Substitute numerical values and evaluate a_T:

$$\boldsymbol{a}_{\mathrm{t}} = \frac{2\boldsymbol{\pi}(0.15\,\mathrm{m})}{\left(\dfrac{1\,\mathrm{min}}{15000\,\mathrm{rev}} \times \dfrac{60\,\mathrm{s}}{\mathrm{min}}\right)(75\,\mathrm{s})} = \boxed{3.1\,\mathrm{m/s^2}}$$

67 ••• [SSM] Earth rotates on its axis once every 24 hours, so that objects on its surface that are stationary with respect to the surface execute uniform circular motion about the axis with a period of 24 hours. Consider only the effect of this rotation on the person on the surface. (Ignore Earth's orbital motion about the Sun.) (*a*) What is the speed and what is the magnitude of the acceleration of a person standing on the equator? (Express the magnitude of this acceleration as a percentage of *g*.) (*b*) What is the direction of the acceleration vector? (*c*) What is the speed and what is the magnitude of the centripetal acceleration of a person standing on the surface at 35°N latitude? (*d*) What is the angle between the direction of the acceleration of the person at 35°N latitude and the direction of the acceleration of the person at the equator if both persons are at the same longitude?

Picture the Problem The radius of Earth is 6370 km. Thus at the equator, a person undergoes circular motion with radius equal to Earth's radius, and a period of 24 h = 86400 s. At 35° N latitude, the person undergoes circular motion having radius *r* cos35° = 5220 km, and the same period.

The centripetal acceleration experienced by a person traveling with a speed *v* in a circular path of radius *r* is given by:

$$a = \frac{v^2}{r} \qquad (1)$$

The speed of the person is the distance the person travels in one revolution divided by the elapsed time (the period *T*):

$$v = \frac{2\pi r}{T}$$

Substitute for v in equation (1) to obtain:

$$a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

(*a*) Substitute numerical values and evaluate v for the person at the equator:

$$\boldsymbol{v} = \frac{2\boldsymbol{\pi}(6370\text{ km})}{86400\text{ s}} = \boxed{463\text{ m/s}}$$

Substitute numerical values and evaluate a for the person at the equator:

$$a = \frac{4\pi^2(6370\text{ km})}{(86400\text{ s})^2} = 3.369\times10^{-2}\text{ m/s}^2$$
$$= \boxed{3.37\text{ cm/s}^2}$$

The ratio of a to g is:

$$\frac{\boldsymbol{a}}{\boldsymbol{g}} = \frac{3.369\times10^{-2}\text{ m/s}^2}{9.81\text{ m/s}^2} = \boxed{0.343\%}$$

(*b*) The acceleration vector points directly at the center of Earth.

(*c*) Substitute numerical values and evaluate v for the person at 35°N latitude:

$$\boldsymbol{v} = \frac{2\boldsymbol{\pi}(5220\text{ km})}{86400\text{ s}} = \boxed{380\text{ m/s}}$$

Substitute numerical values and evaluate v for the person at 35°N latitude:

$$a = \frac{4\pi^2(5220\text{ km})}{(86400\text{ s})^2} = \boxed{2.76\text{ cm/s}^2}$$

(*d*) The plane of the person's path is parallel to the plane of the equator – the acceleration vector is in the plane – so that it is perpendicular to Earth's axis, pointing at the center of the person's revolution, rather than the center of Earth.

68 •• Determine the acceleration of the moon toward the earth, using values for its mean distance and orbital period from the Terrestrial and Astronomical Data table in this book. Assume a circular orbit. Express the acceleration as a fraction of the magnitude of free-fall acceleration *g*.

Picture the Problem We can relate the acceleration of the moon toward the earth to its orbital speed and distance from the earth. Its orbital speed can be expressed in terms of its distance from the earth and its orbital period. From the Terrestrial and Astronomical Data table, we find that the sidereal period of the moon is 27.3 d and that its mean distance from the earth is 3.84×10^8 m.

Express the centripetal acceleration of the moon:

$$a_c = \frac{v^2}{r} \quad (1)$$

Express the orbital speed of the moon:

$$v = \frac{2\pi r}{T}$$

Substituting for v in equation (1) yields:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(3.84\times10^8\ \text{m})}{\left(27.3\ \text{d}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}\right)^2} = 2.72\times10^{-3}\ \text{m/s}^2 = \boxed{2.78\times10^{-4}\ \boldsymbol{g}}$$

Remarks: Note that $\frac{a_c}{g} = \frac{\textit{radius of earth}}{\textit{distance from earth to moon}}$ (a_c is just the acceleration due to the earth's gravity evaluated at the moon's position). This is Newton's famous ″falling apple″ observation.

69 •• (*a*) What are the period and speed of the motion of a person on a carousel if the person has an acceleration magnitude of 0.80 m/s^2 when she is standing 4.0 m from the axis? (*b*) What are her acceleration magnitude and speed if she then moves in to a distance of 2.0 m from the carousel center and the carousel keeps rotating with the same period?

Picture the Problem The person riding on this carousel experiences a centripetal acceleration due to the fact that her velocity is continuously changing. Use the expression for centripetal acceleration to relate her speed to her centripetal acceleration and the relationship between distance, speed, and time to find the period of her motion.

In general, the acceleration and period of any object moving in a circular path at constant speed are given by:

$$a_c = \frac{v^2}{r} \quad (1)$$

and

$$T = \frac{2\pi r}{v} \quad (2)$$

Solving equation (1) for v yields:

$$v = \sqrt{a_c r} \quad (3)$$

Substituting for v in equation (2) yields:

$$T = \frac{2\pi r}{\sqrt{a_c r}} = 2\pi\sqrt{\frac{r}{a_c}} \quad (4)$$

(*a*) Substitute numerical values in equation (3) and evaluate *v* for a person standing 4.0 m from the axis:

$$\boldsymbol{v} = \sqrt{\left(0.80\ \text{m/s}^2\right)\left(4.0\ \text{m}\right)} = 1.79\ \text{m/s}$$
$$= \boxed{1.8\ \text{m/s}}$$

Substitute numerical values in equation (4) and evaluate *T* for a person standing 4.0 m from the axis:

$$T = 2\pi\sqrt{\frac{4.0\ \text{m}}{0.80\ \text{m/s}^2}} = 14.05\ \text{s} = \boxed{14\ \text{s}}$$

(*b*) Solve equation (2) for *v* to obtain:

$$v = \frac{2\pi r}{T}$$

For a person standing 2.0 m from the axis:

$$v = \frac{2\pi\left(2.0\ \text{m}\right)}{14.05\ \text{s}} = 0.894\ \text{m/s} = \boxed{0.89\ \text{m/s}}$$

From equation (1) we have, for her acceleration:

$$a_{\text{c}} = \frac{\left(0.894\ \text{m/s}\right)^2}{2.0\ \text{m}} = \boxed{0.40\ \text{m/s}^2}$$

70 ••• Pulsars are neutron stars that emit X rays and other radiation in such a way that we on Earth receive pulses of radiation from the pulsars at regular intervals equal to the period that they rotate. Some of these pulsars rotate with periods as short as 1 ms! The Crab Pulsar, located inside the Crab Nebula in the constellation Orion, has a period currently of length 33.085 ms. It is estimated to have an equatorial radius of 15 km, which is an average radius for a neutron star. (*a*) What is the value of the centripetal acceleration of an object on the surface and at the equator of the pulsar? (*b*) Many pulsars are observed to have periods that lengthen slightly with time, a phenomenon called "spin down." The rate of slowing of the Crab Pulsar is 3.5×10^{-13} s per second, which implies that if this rate remains constant, the Crab Pulsar will stop spinning in 9.5×10^{10} s (about 3000 years). What is the tangential acceleration of an object on the equator of this neutron star?

Picture the Problem We need to know the speed of a point on the equator where the object sits. The circular path that the object at the equator undergoes has a radius equal to that of the star, and therefore is simply the equatorial circumference – and is traversed in a time interval of one period, 33.085 ms. In (*b*) we need only consider the change in speed of an object on the surface over the course of this time period.

(*a*) The centripetal acceleration of the object is given by:

$$a_{\text{c}} = \frac{v^2}{r} \qquad (1)$$

The speed of the object is equal to related to the radius of its path and the period of its motion:

$$v = \frac{2\pi r}{T}$$

Substituting for v in equation (1) yields:

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(15\text{ km})}{(33.085\text{ ms})^2} = \boxed{5.4\times10^8\text{ m/s}^2}$$

(b) The tangential acceleration of an object on the equator of the neutron star is given by:

$$a_t = \frac{\Delta v_t}{\Delta t_{\text{spin down}}} = \frac{v_{\text{t, final}} - v_{\text{t, initial}}}{\Delta t_{\text{spin down}}}$$

Because $\boldsymbol{v}_{\text{r,final}} = 0$:

$$a_t = \frac{-v_{\text{t, initial}}}{\Delta t_{\text{spin down}}}$$

We can obtain the initial speed of the object from equation (1):

$$\boldsymbol{v}_{\text{t, initial}} = \frac{2\boldsymbol{\pi}\,\boldsymbol{r}}{\boldsymbol{T}}$$

Substitute for $\boldsymbol{v}_{\text{r,final}} = 0$ to obtain:

$$a_t = \frac{-\frac{2\pi r}{T}}{\Delta t_{\text{spin down}}} = -\frac{2\pi r}{\Delta t_{\text{spin down}}T}$$

Substitute numerical values and evaluate $\boldsymbol{a}_t$:

$$a_t = -\frac{2\pi(15\text{ km})}{(9.5\times10^{10}\text{ s})(33.085\times10^{-3}\text{ s})}$$
$$= \boxed{-3.0\times10^{-5}\text{ m/s}^2}$$

71 ••• Human blood contains plasma, platelets and blood cells. To separate the plasma from other components, centrifugation is used. Effective centrifugation requires subjecting blood to an acceleration of 2000g or more. In this situation, assume that blood is contained in test tubes that are 15 cm long and are full of blood. These tubes ride in the centrifuge tilted at an angle of 45.0° above the horizontal (See Figure 3-34.) (a) What is the distance of a sample of blood from the rotation axis of a centrifuge rotating at 3500 rpm, if it has an acceleration of 2000g? (b) If the blood at the center of the tubes revolves around the rotation axis at the radius calculated in Part (a), calculate the centripetal accelerations experienced by the blood at each end of the test tube. Express all accelerations as multiples of g.

Picture the Problem The equations that describe the centripetal acceleration and speed of objects moving in circular paths at constant speed can be used to find the distance of the sample of blood from the rotation axis as well as the accelerations experienced by the blood at each end of the test tube under the conditions described in the problem statement.

(*a*) The centripetal acceleration of the blood sample is given by:

$$a_c = \frac{v^2}{r}$$

The speed of the blood's revolution is given by:

$$v = \frac{2\pi r}{T} = 2\pi rf$$

where *f* is the frequency of revolution.

Substitute for v in the expression for a_c to obtain:

$$a_c = \frac{(2\pi rf)^2}{r} = 4\pi^2 f^2 r \qquad (1)$$

Solving for r yields:

$$r = \frac{a_c}{4\pi^2 f^2}$$

or, because $a_c = g$,

$$r = \frac{2000g}{4\pi^2 f^2}$$

Substitute numerical values and evaluate r:

$$r = \frac{2000(9.81\ \text{m/s}^2)}{4\pi^2\left(3500\,\frac{\text{rev}}{\text{min}}\times\frac{1\ \text{min}}{60\ \text{s}}\right)^2} = 0.146\ \text{m}$$

$$= \boxed{15\ \text{cm}}$$

(*b*) The center of the tube undergoes uniform circular motion with a radius of 15cm. Because the tubes are tilted at a 45 degree angle, the top surface of the blood is undergoing uniform circular motion with a radius of:

$$r_{\text{top}} = 14.6\ \text{cm} - (7.5\ \text{cm})\cos 45° = 9.30\ \text{cm}$$

The bottom surface is moving in a circle of radius:

$$\boldsymbol{r}_{\text{bottom}} = 14.6\ \text{cm} + (7.5\ \text{cm})\cos 45° = 19.90\ \text{cm}$$

The range of accelerations can be found from equation (1):

$$a_c = 4\pi^2 f^2 r$$

The minimum acceleration corresponds to r_{top}:

$$a_{\min} = 4\pi^2 f^2 r_{\text{top}}$$

Substitute numerical values and evaluate $a_{\min}$:

$$a_{\min} = 4\pi^2\left(3500\,\frac{\text{rev}}{\text{min}}\times\frac{1\ \text{min}}{60\ \text{s}}\right)^2(9.30\ \text{cm}) = 1.249\times10^4\ \text{m/s}^2 \approx \boxed{1300g}$$

The maximum acceleration corresponds to r_{bottom}:

$$a_{max} = 4\pi^2 f^2 r_{bottom}$$

Substitute numerical values and evaluate a_{max}:

$$a_{max} = 4\pi^2\left(3500\,\frac{\text{rev}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\right)^2(19.90\,\text{cm}) = 2.673\times10^4\ \text{m/s}^2 \approx \boxed{2700g}$$

The range of accelerations is 1300*g* to 2700*g*.

Projectile Motion and Projectile Range

72 • While trying out for the position of pitcher on your high school baseball team, you throw a fastball at 87 mi/h toward home plate, which is 18.4 m away. How far does the ball drop due to effects of gravity by the time it reaches home plate? (Ignore any effects due to air resistance.)

Picture the Problem Neglecting air resistance, the accelerations of the ball are constant and the horizontal and vertical motions of the ball are independent of each other. We can use the horizontal motion to determine the time-of-flight and then use this information to determine the distance the ball drops. Choose a coordinate system in which the origin is at the point of release of the ball, downward is the positive y direction, and the horizontal direction is the positive x direction.

Express the vertical displacement of the ball:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2 \qquad (1)$$

Use $v_x = \Delta x/\Delta t$ to express the time of flight:

$$\Delta t = \frac{\Delta x}{v_x}$$

Substitute for Δt in equation (1) to obtain:

$$\Delta y = \tfrac{1}{2}g\left(\frac{\Delta x}{v_x}\right)^2 = \frac{g(\Delta x)^2}{2(v_x)^2}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(18.4\,\text{m})^2}{2\left(87\,\frac{\text{mi}}{\text{h}}\cdot\frac{0.4470\,\frac{\text{m}}{\text{s}}}{1\,\frac{\text{mi}}{\text{h}}}\right)^2} = \boxed{1.1\,\text{m}}$$

73 • A projectile is launched with speed v_0 at an angle of θ_0 above the horizontal. Find an expression for the maximum height it reaches above its starting point in terms of v_0, θ_0, and g. (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the maximum height achieved by a projectile depends on the vertical component of its initial velocity.

Use a constant-acceleration equation to relate the displacement of a projectile to its initial and final speeds and its acceleration:

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y \Rightarrow \Delta \boldsymbol{y} = \frac{\boldsymbol{v}_y^2 - \boldsymbol{v}_{0y}^2}{2\boldsymbol{a}_y}$$

Because $v_y = 0$ and $a_y = -g$:

$$\Delta \boldsymbol{y} = \frac{\boldsymbol{v}_{0y}^2}{2\boldsymbol{g}}$$

The vertical component of the projectile's initial velocity is:

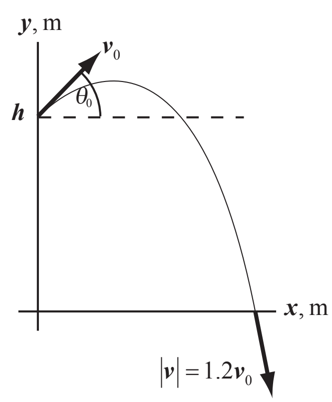
$$\boldsymbol{v}_{0y} = \boldsymbol{v}_0 \sin \boldsymbol{\theta}_0$$

Setting $\Delta \boldsymbol{y} = \boldsymbol{h}$ and substituting for v_{0y} yields:

$$h = \boxed{\frac{(v_0 \sin \theta_0)^2}{2g}}$$

74 •• A cannonball is fired with initial speed v_0 at an angle 30º above the horizontal from a height of 40 m above the ground. The projectile strikes the ground with a speed of $1.2v_0$. Find v_0. (Ignore any effects due to air resistance.)

Picture the Problem Choose the coordinate system shown to the right. Because, in the absence of air resistance, the horizontal and vertical speeds are independent of each other, we can use constant-acceleration equations to relate the impact speed of the projectile to its components.



The horizontal and vertical velocity components are:

$$\boldsymbol{v}_{0x} = \boldsymbol{v}_x = \boldsymbol{v}_0 \cos \boldsymbol{\theta}_0$$

and

$$\boldsymbol{v}_{0y} = \boldsymbol{v}_0 \sin \boldsymbol{\theta}_0$$

Using a constant-acceleration equation, relate the vertical component of the velocity to the vertical displacement of the projectile:

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $a_y = -g$ and $\Delta y = -h$,

$$\boldsymbol{v}_y^2 = (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)^2 + 2\boldsymbol{gh}$$

Express the relationship between the magnitude of a velocity vector and its components, substitute for the components, and simplify to obtain:

$$\begin{aligned}\boldsymbol{v}^2 &= \boldsymbol{v}_x^2 + \boldsymbol{v}_y^2 = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)^2 + \boldsymbol{v}_y^2 \\ &= \boldsymbol{v}_0^2(\sin^2\boldsymbol{\theta}_0 + \cos^2\boldsymbol{\theta}_0) + 2\boldsymbol{gh} \\ &= \boldsymbol{v}_0^2 + 2\boldsymbol{gh}\end{aligned}$$

Substitute for v:

$$(1.2v_0)^2 = v_0^2 + 2gh \Rightarrow \boldsymbol{v}_0 = \sqrt{\frac{2\boldsymbol{gh}}{0.44}}$$

Substitute numerical values and evaluate v_0:

$$\boldsymbol{v}_0 = \sqrt{\frac{2(9.81\,\text{m/s}^2)(40\,\text{m})}{0.44}} = \boxed{42\,\text{m/s}}$$

Remarks: Note that *v* is independent of θ_0. This will be more obvious once conservation of energy has been studied.

75 •• In Figure 3.35, what is the minimum initial speed of the dart if it is to hit the monkey before the monkey hits the ground, which is 11.2 m below the initial position of the monkey, if x is 50 m and h = 10 m? (Ignore any effects due to air resistance.)

Picture the Problem Example 3-12 shows that the dart will hit the monkey unless the dart hits the ground before reaching the monkey's line of fall. What initial speed does the dart need in order to just reach the monkey's line of fall? First, we will calculate the fall time of the monkey, and then we will calculate the horizontal component of the dart's velocity.

Relate the horizontal velocity of the dart to its launch angle and initial velocity v_0:

$$\boldsymbol{v}_x = \boldsymbol{v}_0 \cos\boldsymbol{\theta} \Rightarrow \boldsymbol{v}_0 = \frac{\boldsymbol{v}_x}{\cos\boldsymbol{\theta}}$$

Use the definition of v_x and the fact that, in the absence of air resistance, it is constant to obtain:

$$\boldsymbol{v}_x = \frac{\Delta\boldsymbol{x}}{\Delta\boldsymbol{t}}$$

Substituting in the expression for v_0 yields:

$$\boldsymbol{v}_0 = \frac{\Delta\boldsymbol{x}}{(\cos\boldsymbol{\theta})\Delta\boldsymbol{t}} \qquad (1)$$

Using a constant-acceleration equation, relate the monkey's fall distance to the fall time:

$$\Delta \boldsymbol{h} = \tfrac{1}{2}\boldsymbol{g}(\Delta \boldsymbol{t})^2 \Rightarrow \Delta \boldsymbol{t} = \sqrt{\frac{2\Delta \boldsymbol{h}}{\boldsymbol{g}}}$$

Substituting for Δt in equation (1) yields:

$$\boldsymbol{v}_0 = \frac{\Delta \boldsymbol{x}}{\cos\boldsymbol{\theta}}\sqrt{\frac{\boldsymbol{g}}{2\Delta \boldsymbol{h}}} \qquad (2)$$

Let θ be the angle the barrel of the dart gun makes with the horizontal. Then:

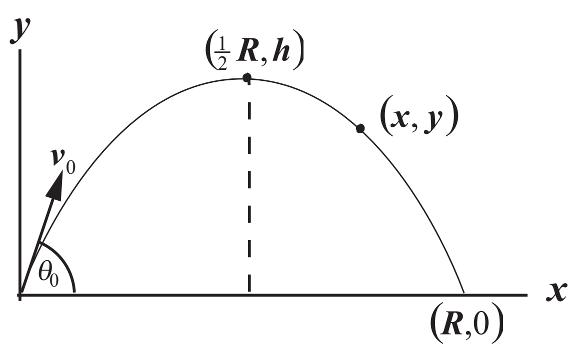
$$\theta = \tan^{-1}\left(\frac{10\,\text{m}}{50\,\text{m}}\right) = 11.3°$$

Substitute numerical values in equation (2) and evaluate v_0:

$$\boldsymbol{v}_0 = \frac{50\text{ m}}{\cos 11.3°}\sqrt{\frac{9.81\,\text{m/s}^2}{2(11.2\text{ m})}} = \boxed{34\text{ m}}$$

76 •• A projectile is launched from ground level with an initial speed of 53 m/s. Find the launch angle (the angle the initial velocity vector is above the horizontal) such that the maximum height of the projectile is equal to its horizontal range. (Ignore any effects due to air resistance.)

Picture the Problem Choose the coordinate system shown in the figure to the right. In the absence of air resistance, the projectile experiences constant acceleration in both the x and y directions. We can use the constant-acceleration equations to express the x and y coordinates of the projectile along its trajectory as functions of time. The elimination of the parameter t will yield an expression for y as a function of x that we can evaluate at (R, 0) and (R/2, h). Solving these equations simultaneously will yield an expression for θ.



Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0\cos\theta)t$$
and
$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate t from these equations to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2 \qquad (1)$$

Evaluate equation (1) at (R, 0) to obtain:

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

Evaluate equation (1) at (R/2, h) to obtain:

$$\boldsymbol{h} = \frac{(\boldsymbol{v}_0 \sin\boldsymbol{\theta})^2}{2\boldsymbol{g}}$$

Equating R and h yields:

$$\frac{2\boldsymbol{v}_0^2 \sin\boldsymbol{\theta}\cos\boldsymbol{\theta}}{\boldsymbol{g}} = \frac{(\boldsymbol{v}_0 \sin\boldsymbol{\theta})^2}{2\boldsymbol{g}}$$

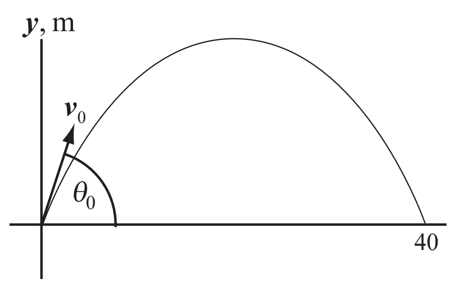
Solve for θ to obtain:

$$\boldsymbol{\theta} = \tan^{-1}(4) = \boxed{76°}$$

Note that this result is independent of v_0.

77 •• **[SSM]** A ball launched from ground level lands 2.44 s later 40.0-m away from the launch point. Find the magnitude of the initial velocity vector and the angle it is above the horizontal. (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the motion of the ball is uniformly accelerated and its horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure to the right and use constant-acceleration equations to relate the x and y components of the ball's initial velocity.



Express θ_0 in terms of v_{0x} and v_{0y}:

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{\boldsymbol{v}_{0y}}{\boldsymbol{v}_{0x}}\right) \qquad (1)$$

Use the Pythagorean relationship between the velocity and its components to express v_0:

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} \qquad (2)$$

Using a constant-acceleration equation, express the vertical speed of the projectile as a function of its initial upward speed and time into the flight:

$$v_y = v_{0y} + a_y t \Rightarrow v_{0y} = v_y - a_y t$$

Because $v_y = 0$ halfway through the flight (at maximum elevation) and $a_y = -g$:

$$v_{0y} = g t_{\text{max elevation}}$$

Substitute numerical values and evaluate v_{0y}:

$$v_{0y} = (9.81\text{ m/s}^2)(1.22\text{ s}) = 11.97\text{ m/s}$$

Because there is no acceleration in the horizontal direction, v_{0x} can be found from:

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{40.0\text{ m}}{2.44\text{ s}} = 16.39\text{ m/s}$$

Substitute for v_{0x} and v_{0y} in equation (2) and evaluate v_0:

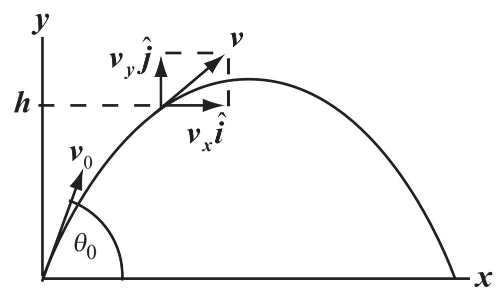
$$v_0 = \sqrt{(16.39\text{ m/s})^2 + (11.97\text{ m/s})^2} = \boxed{20.3\text{ m/s}}$$

Substitute for v_{0x} and v_{0y} in equation (1) and evaluate θ_0 :

$$\theta_0 = \tan^{-1}\left(\frac{11.97\text{ m/s}}{16.39\text{ m/s}}\right) = \boxed{36.1°}$$

78 •• Consider a ball that is launched from ground level with initial speed v_0 at an angle θ_0 above the horizontal. If we consider its speed to be v at height h above the ground, show that for a given value of h, v is independent of θ_0. (Ignore any effects due to air resistance.)

Picture the Problem In the absence of friction, the acceleration of the ball is constant and we can use the constant-acceleration equations to describe its motion. The figure shows the launch conditions and an appropriate coordinate system. The speeds v, v_x, and v_y are related by the Pythagorean Theorem.



The squares of the vertical and horizontal components of the object's velocity are:

$$\boldsymbol{v}_y^2 = \boldsymbol{v}_0^2 \sin^2\boldsymbol{\theta}_0 - 2\boldsymbol{gh}$$
and
$$\boldsymbol{v}_x^2 = \boldsymbol{v}_0^2 \cos^2\boldsymbol{\theta}_0$$

The relationship between these variables is:

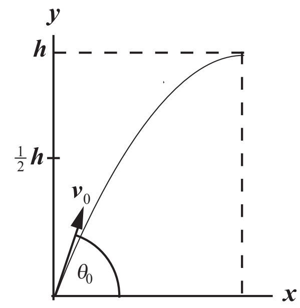
$$v^2 = v_x^2 + v_y^2$$

Substitute for v_x and v_y and simplify to obtain:

$$\begin{aligned}\boldsymbol{v}^2 &= \boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0 + \boldsymbol{v}_0^2\sin^2\boldsymbol{\theta}_0 - 2\boldsymbol{gh}\\ &= \boldsymbol{v}_0^2(\cos^2\boldsymbol{\theta}_0 + \sin^2\boldsymbol{\theta}_0) - 2\boldsymbol{gh}\\ &= \boxed{\boldsymbol{v}_0^2 - 2\boldsymbol{gh}}\end{aligned}$$

79 •• At $\frac{1}{2}$ of its maximum height, the speed of a projectile is $\frac{3}{4}$ of its initial speed. What was its launch angle? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the projectile experiences constant acceleration during its flight and we can use constant-acceleration equations to relate the speeds at half the maximum height and at the maximum height to the launch angle θ_0 of the projectile.



The angle the initial velocity makes with the horizontal is related to the initial velocity components.

$$\tan\theta_0 = \frac{v_{0y}}{v_{0x}} \quad (1)$$

Substitute $\Delta y = h$ and $v_y = 0$ in the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$ to obtain:

$$\Delta y = h \Rightarrow 0 = v_{0y}^2 - 2gh \quad (2)$$

Write the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$ for $\Delta y = \frac{1}{2}h$:

$$\Delta y = \frac{h}{2} \Rightarrow v_y^2 = v_{0y}^2 - 2g\frac{h}{2} \quad (3)$$

We are given that $v = \frac{3}{4}v_0$ when $\Delta y = \frac{1}{2}h$. Square both sides and express this using the components of the velocity. The x component of the velocity remains constant.

$$v_{0x}^2 + v_y^2 = \left(\frac{3}{4}\right)^2\left(v_{0x}^2 + v_{0y}^2\right) \quad (4)$$

where we have used $v_x = v_{0x}$.

(Equations 2, 3, and 4 constitute three equations in the four unknowns v_{0x}, v_{0y}, v_y, and h. To solve for any of these unknowns, we first need a fourth equation. However, to solve for the ratio (v_{0y}/v_{0x}) of two of the unknowns, the three equations are sufficient. That is because dividing both sides of each equation by v_{0x}^2 gives three equations and three unknowns v_y/v_{0x}, v_{0y}/v_{0x}, and h/v_{0x}^2.

Solve equation (3) for gh and substitute in equation (2):

$$v_{0y}^2 = 2\left(v_{0y}^2 - v_h^2\right) \Rightarrow v_y^2 = \frac{v_{0y}^2}{2}$$

Substitute for v_y^2 in equation (4):

$$v_{0x}^2 + \frac{1}{2}v_{0y}^2 = \left(\frac{3}{4}\right)^2\left(v_{0x}^2 + v_{0y}^2\right)$$

Divide both sides by v_{0x}^2 and solve for v_{0y}/v_{0x} to obtain:

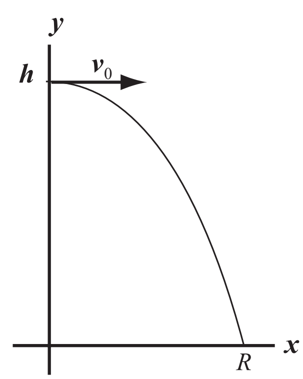
$$1 + \frac{1}{2}\frac{v_{0y}^2}{v_{0x}^2} = \frac{9}{16}\left(1 + \frac{v_{0y}^2}{v_{0x}^2}\right) \Rightarrow \frac{v_{0y}}{v_{0x}} = \sqrt{7}$$

Substitute for $\frac{v_{0y}}{v_{0x}}$ in equation (1) and evaluate θ_0:

$$\theta_0 = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\sqrt{7}\right) = \boxed{69.3°}$$

80 •• A cargo plane is flying horizontally at an altitude of 12 km with a speed of 900 km/h when a large crate falls out of the rear-loading ramp. (Ignore any effects due to air resistance.) (*a*) How long does it take the crate to hit the ground? (*b*) How far horizontally is the crate from the point where it fell off when it hits the ground? (*c*) How far is the crate from the aircraft when the crate hits the ground, assuming that the plane continues to fly with the same velocity?

Picture the Problem The horizontal speed of the crate, in the absence of air resistance, is constant and equal to the speed of the cargo plane. Choose a coordinate system in which the direction the plane is moving is the positive *x* direction and downward is the positive *y* direction and apply the constant-acceleration equations to describe the crate's displacements at any time during its flight.



(*a*) Using a constant-acceleration equation, relate the vertical displacement of the crate Δy to the time of fall Δt:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}g(\Delta t)^2$$

or, because $v_{0y} = 0$ and $\Delta y = h$,

$$h = \tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(12\times10^3\ \text{m})}{9.81\ \text{m/s}^2}} = 49.46\ \text{s} = \boxed{49\ \text{s}}$$

(*b*) The horizontal distance traveled in time Δt is:

$$R = \Delta x = v_{0x}\Delta t$$

Substitute numerical values and evaluate R:

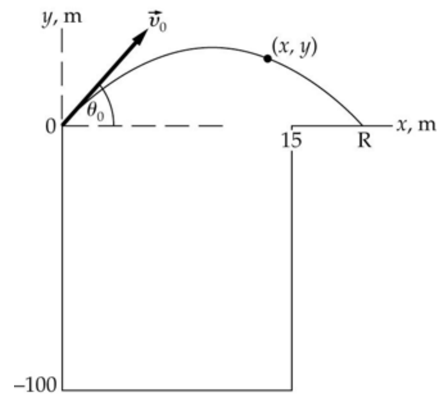
$$R = (900\ \text{km/h})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)(49.46\ \text{s})$$

$$= \boxed{12\ \text{km}}$$

(*c*) Because the velocity of the plane is constant, it will be directly over the crate when it hits the ground; that is, the distance to the aircraft will be the elevation of the aircraft. Therefore $\Delta y = \boxed{12\ \text{km}}$

81 •• **[SSM]** Wile E. Coyote (*Carnivorous hungribilous*) is chasing the Roadrunner (*Speedibus cantcatchmi*) yet again. While running down the road, they come to a deep gorge, 15.0 m straight across and 100 m deep. The Roadrunner launches himself across the gorge at a launch angle of 15º above the horizontal, and lands with 1.5 m to spare. (*a*) What was the Roadrunner's launch speed? (*b*) Wile E. Coyote launches himself across the gorge with the same initial speed, but at a different launch angle. To his horror, he is short the other lip by 0.50 m. What was his launch angle? (Assume that it was less than 15º.)

Picture the Problem In the absence of air resistance, the accelerations of both Wiley Coyote and the Roadrunner are constant and we can use constant-acceleration equations to express their coordinates at any time during their leaps across the gorge. By eliminating the parameter t between these equations, we can obtain an expression that relates their y coordinates to their x coordinates and that we can solve for their launch angles.



(*a*) Using constant-acceleration equations, express the x coordinate of the Roadrunner while it is in flight across the gorge:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0 \cos\theta_0$,

$$x = (v_0 \cos\theta_0)t \qquad (1)$$

Using constant-acceleration equations, express the y coordinate of the Roadrunner while it is in flight across the gorge:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g$ and $v_{0y} = v_0 \sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad (2)$$

Eliminate the variable t between equations (1) and (2) to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2 \qquad (3)$$

When $y = 0$, $x = R$ and equation (3) becomes:

$$0 = (\tan\boldsymbol{\theta}_0)\boldsymbol{R} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{R}^2$$

Using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, solve for v_0:

$$\boldsymbol{v}_0 = \sqrt{\frac{\boldsymbol{Rg}}{\sin 2\boldsymbol{\theta}_0}}$$

Substitute numerical values and evaluate v_0:

$$\boldsymbol{v}_0 = \sqrt{\frac{(16.5\,\text{m})(9.81\,\text{m/s}^2)}{\sin 30°}} = \boxed{18\,\text{m/s}}$$

(*b*) Letting R represent Wiley's range, solve equation (1) for his launch angle:

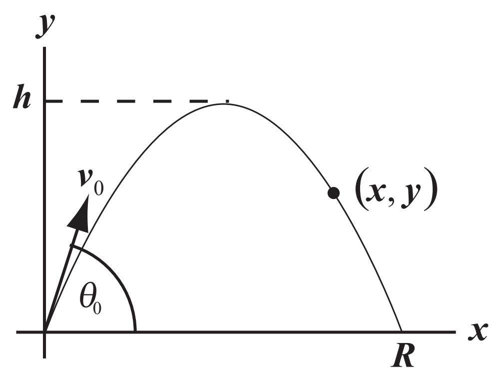
$$\theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \frac{1}{2}\sin^{-1}\left[\frac{(14.5\,\text{m})(9.81\,\text{m/s}^2)}{(18.0\,\text{m/s})^2}\right] = \boxed{13°}$$

82 •• A cannon barrel is elevated 45° above the horizontal. It fires a ball with a speed of 300 m/s. (*a*) What height does the ball reach? (*b*) How long is the ball in the air? (*c*) What is the horizontal range of the cannon ball? (Ignore any effects due to air resistance.)

Picture the Problem Because, in the absence of air resistance, the vertical and horizontal accelerations of the cannonball are constant, we can use constant-acceleration equations to express the ball's position and velocity as functions of time and acceleration. The maximum height of the ball and its time-of-flight are related to the components of its launch velocity.



(*a*) Using a constant-acceleration equation, relate h to the initial and final speeds of the cannon ball:

$$v^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $v = 0$, $a_y = -g$, and $\Delta y = h$,

$$0 = v_{0y}^2 - 2gh \Rightarrow h = \frac{v_{0y}^2}{2g}$$

The vertical component of the firing speed is given by:

$$v_{0y} = v_0 \sin\theta_0$$

Substituting for v_{0y} yields:

$$h = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Substitute numerical values and evaluate h:

$$h = \frac{(300\,\text{m/s})^2 \sin^2 45°}{2(9.81\,\text{m/s}^2)} = \boxed{2.3\,\text{km}}$$

(*b*) The total flight time of the cannon ball is given by:

$$\Delta t = t_{\text{up}} + t_{\text{dn}} = 2t_{\text{up}} = 2\frac{v_{0y}}{g}$$

$$= \frac{2v_0 \sin\theta_0}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(300\ \text{m/s})\sin 45°}{9.81\ \text{m/s}^2} = 43.2\ \text{s}$$
$$= \boxed{43\ \text{s}}$$

(*c*) Express the *x* coordinate of the cannon ball as a function of time:

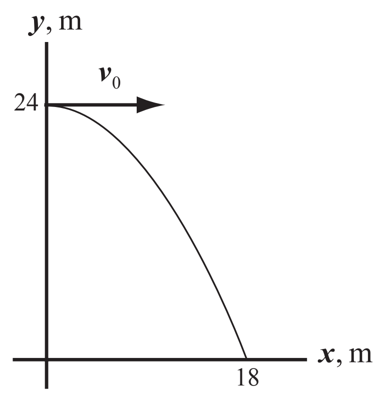
$$x = v_{0x}\Delta t = (v_0 \cos\theta_0)\Delta t$$

Evaluate *x* (= *R*) when $\Delta t = 43.2$ s:

$$x = [(300\ \text{m/s})\cos 45°](43.2\ \text{s})$$
$$= \boxed{9.2\ \text{km}}$$

83 •• **[SSM]** A stone thrown horizontally from the top of a 24-m tower hits the ground at a point 18 m from the base of the tower. (Ignore any effects due to air resistance.) (*a*) Find the speed with which the stone was thrown. (*b*) Find the speed of the stone just before it hits the ground.

Picture the Problem Choose a coordinate system in which the origin is at the base of the tower and the *x*- and *y*-axes are as shown in the figure to the right. In the absence of air resistance, the horizontal speed of the stone will remain constant during its fall and a constant-acceleration equation can be used to determine the time of fall. The final velocity of the stone will be the vector sum of its *x* and *y* components.



Because the stone is thrown horizontally:

$$v_x = v_{0x} = \frac{\Delta x}{\Delta t} \qquad (1)$$

(*a*) Using a constant-acceleration equation, express the vertical displacement of the stone as a function of the fall time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{-\frac{2\Delta y}{g}}$$

Substituting for Δt in equation (1) and simplifying yields:

$$v_x = \Delta x\sqrt{\frac{g}{-2\Delta y}}$$

Substitute numerical values and evaluate v_x:

$$v_x = (18\ \text{m})\sqrt{\frac{9.81\ \text{m/s}^2}{-2(-24\ \text{m})}} = \boxed{8.1\ \text{m}}$$

(*b*) The speed with which the stone hits the ground is related to the *x* and *y* components of its speed:

$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2} \qquad (2)$$

The *y* component of the stone's velocity at time *t* is:

$$\boldsymbol{v}_y = \boldsymbol{v}_{0y} - \boldsymbol{gt}$$

or, because $v_{0y} = 0$,

$$\boldsymbol{v}_y = -\boldsymbol{gt}$$

Substitute for v_x and v_y in equation (2) and simplify to obtain:

$$\boldsymbol{v} = \sqrt{\left(\Delta \boldsymbol{x}\sqrt{\frac{\boldsymbol{g}}{-2\Delta \boldsymbol{y}}}\right)^2 + (-\boldsymbol{gt})^2}$$

$$= \sqrt{\frac{\boldsymbol{g}(\Delta \boldsymbol{x})^2}{-2\Delta \boldsymbol{y}} + \boldsymbol{g}^2\boldsymbol{t}^2}$$

Substitute numerical values and evaluate *v*:

$$\boldsymbol{v} = \sqrt{\frac{(9.81\,\text{m/s}^2)(18\,\text{m})^2}{-2(-24\,\text{m})} + (9.81\,\text{m/s}^2)^2(2.21\,\text{s})^2} = \boxed{23\,\text{m/s}}$$

84 •• A projectile is fired into the air from the top of a 200-m cliff above a valley (Figure 3-36). Its initial velocity is 60 m/s at 60º above the horizontal. Where does the projectile land? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and its horizontal and vertical motions are independent of each other. We can use constant-acceleration equations to express the horizontal and vertical displacements of the projectile in terms of its time-of-flight.

Using a constant-acceleration equation, express the horizontal displacement of the projectile as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = v_0\cos\theta_0$ and $a_x = 0$,

$$\Delta \boldsymbol{x} = (\boldsymbol{v}_0\cos\boldsymbol{\theta}_0)\Delta \boldsymbol{t}$$

Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = v_0\sin\theta_0$ and $a_y = -g$,

$$\Delta \boldsymbol{y} = (\boldsymbol{v}_0\cos\boldsymbol{\theta}_0)\Delta \boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}(\Delta \boldsymbol{t})^2$$

Substitute numerical values to obtain the quadratic equation:

$$-200\,\text{m} = (60\text{m/s})(\sin 60°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Use the quadratic formula or your graphing calculator to obtain:

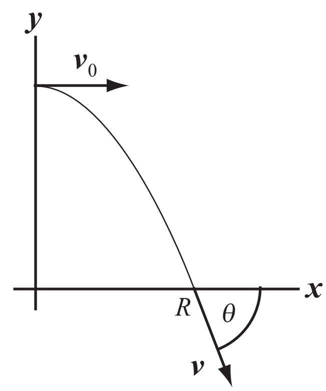
$$\Delta t = 13.6\text{ s}$$

Substitute for Δt and evaluate the horizontal distance traveled by the projectile:

$$\Delta x = (60\text{ m/s})(\cos 60°)(13.6\text{ s}) = \boxed{0.41\text{km}}$$

85 •• The range of a cannonball fired horizontally from a cliff is equal to the height of the cliff. What is the direction of the velocity vector of the projectile as it strikes the ground? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the acceleration of the cannonball is constant and its horizontal and vertical motions are independent of each other. Choose the origin of the coordinate system to be at the base of the cliff and the axes directed as shown and use constant-acceleration equations to describe both the horizontal and vertical displacements of the cannonball.



Express the direction of the velocity vector when the projectile strikes the ground:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \qquad (1)$$

Express the vertical displacement using a constant-acceleration equation:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Set $\Delta x = -\Delta y$ ($R = -h$) to obtain:

$$\Delta x = v_x\Delta t = \tfrac{1}{2}g(\Delta t)^2 \Rightarrow v_x = \frac{\Delta x}{\Delta t} = \tfrac{1}{2}g\Delta t$$

Find the y component of the projectile as it hits the ground:

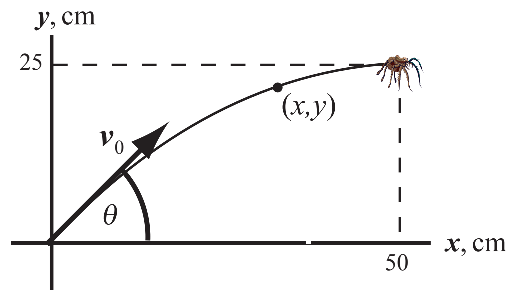
$$v_y = v_{0y} + a\Delta t = -g\Delta t = -2v_x$$

Substituting for v_y in equation (1) yields:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{-2\boldsymbol{v}_x}{\boldsymbol{v}_x}\right) = \tan^{-1}(-2) = \boxed{-63.4°}$$

86 •• An archerfish launches a droplet of water from the surface of a small lake at an angle of 60° above the horizontal. He is aiming at a juicy spider sitting on a leaf 50 cm to the east and on a branch 25 cm above the water surface. The fish is trying to knock the spider into the water so that the fish may eat the spider. (*a*) What must the speed of the water droplet be for the fish to be successful? (*b*) When it hits the spider, is the droplet rising or falling?

Picture the Problem The diagram to the right shows the trajectory of the water droplet launched by the archerfish. We can use constant-acceleration equations to derive expressions for the required speed of the droplet and for the vertical velocity of the droplet as a function of its position.



(*a*) Use a constant-acceleration equation to express the x- and y-coordinates of the droplet:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
and
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $x_0 = y_0 = 0$, $v_{0x} = v_0\cos\theta$, $v_{0y} = v_0\sin\theta$, and, in the absence of air resistance, $a_x = 0$:

$$x = (v_0 \cos\theta)t \qquad (1)$$
and
$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \qquad (2)$$

Solve equation (1) for t to obtain:

$$t = \frac{x}{v_0\cos\theta}$$

Substituting for t in equation (2) yields:

$$y = (v_0\sin\theta)\left(\frac{x}{v_0\cos\theta}\right) - \tfrac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2$$

Simplify this equation to obtain:

$$y = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

Solving for v_0 yields:

$$v_0 = \sqrt{\frac{g}{2(x\tan\theta - y)\cos^2\theta}}\,x$$

When the droplet hits the spider, its coordinates must be $x = 0.50$ m and $y = 0.25$ m. Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{\frac{9.81\,\text{m/s}^2}{2((0.50\,\text{m})\tan 60° - 0.25\,\text{m})\cos^2 60°}}(0.50\,\text{m}) = 2.822\,\text{m/s} = \boxed{2.8\,\text{m/s}}$$

(*b*) Use a constant-acceleration equation to express the *y* component of the speed of the droplet as a function of time:

$$v_y(t) = v_{0y} + a_y t$$

or, because $v_{0y} = v_0 \sin\theta$ and $a_y = -g$,

$$v_y(t) = v_0 \sin\theta - gt \qquad (3)$$

From equation (1), the time-to-the-target, $t_{\text{to target}}$, is given by:

$$t_{\text{to target}} = \frac{x_{\text{target}}}{v_x} = \frac{x_{\text{target}}}{v_{0x}} = \frac{x_{\text{target}}}{v_0 \cos\theta}$$

where x_{target} is the *x*-coordinate of the target.

Substitute in equation (3) to obtain:

$$v_y(t_{\text{to target}}) = v_0 \sin\theta - g t_{\text{to target}}$$

$$= v_0 \sin\theta - \frac{g x_{\text{target}}}{v_0 \cos\theta}$$

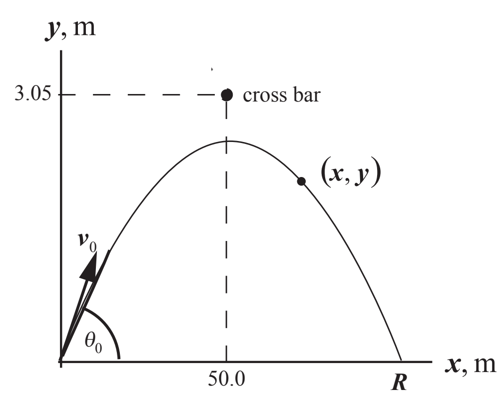
Substitute numerical values and evaluate $v_y(t_{\text{to target}})$:

$$v_y(t_{\text{to target}}) = (2.822\text{ m/s})\sin 60° - \frac{(9.81\text{ m/s}^2)(0.50\text{ m})}{(2.822\text{ m/s})\cos 60°} = -1.032\text{ m/s}$$

Because the *y*-component of the velocity of the droplet is negative at the location of the spider, the droplet is falling and moving horizontally when it hits the spider.

87 •• **[SSM]** You are trying out for the position of place-kicker on a professional football team. With the ball teed up 50.0 m from the goalposts with a crossbar 3.05 m off the ground, you kick the ball at 25.0 m/s and 30° above the horizontal. (*a*) Is the field goal attempt good? (*b*) If so, by how much does it clear the bar? If not, by how much does it go under the bar? (*c*) How far behind the plane of the goalposts does the ball land?

Picture the Problem We can use constant-acceleration equations to express the *x* and *y* coordinates of the ball along its flight path. Eliminating *t* between the equations will leave us with an equation for *y* as a function of *x* that we can use to find the height of the ball when it has reached the cross bar. We can use this same equation to find the range of the ball and, hence, how far behind the plane of the goalposts the ball lands.



(*a*) Use a constant-acceleration equation to express the *x* coordinate of the ball as a function of time:

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$x(t) = (v_0\cos\theta_0)t \Rightarrow t = \frac{x(t)}{v_0\cos\theta_0}$$

Use a constant-acceleration equation to express the *y* coordinate of the ball as a function of time:

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta_0$, and $a_y = -g$,

$$y(t) = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Substituting for *t* yields:

$$y(x) = (v_0\sin\theta_0)\frac{x(t)}{v_0\cos\theta_0} - \tfrac{1}{2}g\left(\frac{x(t)}{v_0\cos\theta_0}\right)^2$$

Simplify to obtain:

$$y(x) = (\tan\theta_0)x(t) - \frac{g}{2v_0^2\cos^2\theta_0}(x(t))^2$$

Substitute numerical values and evaluate $y(50.0\text{ m})$:

$$y(50.0\text{ m}) = (\tan 30°)(50.0\text{ m}) - \frac{9.81\text{ m/s}^2}{2(25.0\text{ m/s})^2\cos^2 30°}(50.0\text{ m})^2 = 2.71\text{ m}$$

Because 2.71 m < 3.05 m, the ball goes under the crossbar and the kick is no good.

(*b*) The ball goes under the bar by:

$$d_{\text{under}} = 3.05\text{ m} - 2.71\text{ m} = \boxed{0.34\text{ m}}$$

(*c*) The distance the ball lands behind the goalposts is given by:

$$d_{\text{behind the goal posts}} = R - 50.0\text{ m} \qquad (3)$$

Evaluate the equation derived in (*a*) for $y = 0$ and $x(t) = R$:

$$0 = (\tan\theta_0)R - \frac{g}{2v_0^2\cos^2\theta_0}R^2$$

Solving for v_0 yields:

$$R = \frac{v_0^2\sin 2\theta_0}{g}$$

Substitute for R in equation (3) to obtain:

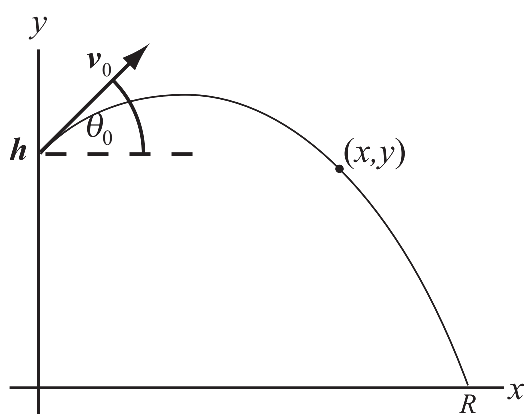
$$\boldsymbol{d}_{\substack{\text{behind the}\\ \text{goal posts}}} = \frac{\boldsymbol{v}_0^2 \sin 2\boldsymbol{\theta}_0}{\boldsymbol{g}} - 50.0\ \text{m}$$

Substitute numerical values and evaluate $\boldsymbol{d}_{\substack{\text{behind the}\\ \text{goal posts}}}$:

$$\boldsymbol{d}_{\substack{\text{behind the}\\ \text{goal posts}}} = \frac{(25.0\ \text{m/s})^2 \sin 2(30°)}{9.81\ \text{m/s}^2} - 50.0\ \text{m} = \boxed{5.2\ \text{m}}$$

88 •• The speed of an arrow fired from a compound bow is about 45.0 m/s. (*a*) A Tartar archer sits astride his horse and launches an arrow into the air, elevating the bow at an angle of 10º above the horizontal. If the arrow is 2.25 m above the ground at launch, what is the arrow's horizontal range? Assume that the ground is level, and ignore any effects due to air resistance. (*b*) Now assume that his horse is at full gallop and moving in the same direction as the direction the archer will fire the arrow. Also assume that the archer elevates the bow at the same elevation angle as in Part (*a*) and fires. If the horse's speed is 12.0 m/s, what is the arrow's horizontal range now?

Picture the Problem Choose a coordinate system in which the origin is at ground level. Let the positive *x* direction be to the right and the positive *y* direction be upward. We can apply constant-acceleration equations to obtain equations in time that relate the range to the initial horizontal speed and the height *h* to which the initial upward speed. Eliminating time from these equations will leave us with a quadratic equation in *R,* the solution to which will give us the range of the arrow. In (*b*), we'll find the launch speed and angle as viewed by an observer who is at rest on the ground and then use these results to find the arrow's range when the horse is moving at 12.0 m/s.



(*a*) Use constant-acceleration equations to express the horizontal and vertical coordinates of the arrow's motion:

$$\Delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_0 = \boldsymbol{v}_{0x}\boldsymbol{t}$$

and

$$y = h + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

where

$$\boldsymbol{v}_{0x} = \boldsymbol{v}_0 \cos\boldsymbol{\theta}_0 \text{ and } \boldsymbol{v}_{0y} = \boldsymbol{v}_0 \sin\boldsymbol{\theta}_0$$

Solve the x-component equation for time:

$$t = \frac{\Delta x}{v_{0x}} = \frac{\Delta x}{v_0 \cos\theta_0}$$

Eliminating time from the y-component equation yields:

$$y = h + v_{0y}\frac{\Delta x}{v_{0x}} - \frac{1}{2}g\left(\frac{\Delta x}{v_{0x}}\right)^2$$

When $y = 0$, $\Delta x = R$ and:

$$0 = h + (\tan\theta_0)R - \frac{g}{2v_0^2\cos^2\theta_0}R^2$$

Solve for the range R to obtain:

$$R = \frac{v_0^2}{2g}\sin 2\theta_0\left(1 + \sqrt{1 + \frac{2gh}{v_0^2\sin^2\theta_0}}\right)$$

Substitute numerical values and evaluate R:

$$R = \frac{(45.0\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 20°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(45.0\,\text{m/s})^2(\sin^2 10°)}}\right) = \boxed{82\,\text{m}}$$

(b) Express the speed of the arrow in the horizontal direction:

$$\begin{aligned} v_x &= v_{\text{arrow}} + v_{\text{archer}} \\ &= (45.0\,\text{m/s})\cos 10° + 12.0\,\text{m/s} \\ &= 56.32\,\text{m/s} \end{aligned}$$

Express the vertical speed of the arrow:

$$v_y = (45.0\,\text{m/s})\sin 10° = 7.814\,\text{m/s}$$

Express the angle of elevation from the perspective of someone on the ground:

$$\theta_0 = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \tan^{-1}\left(\frac{7.814\,\text{m/s}}{56.32\,\text{m/s}}\right) = 7.899°$$

The arrow's speed relative to the ground is given by:

$$v_0 = \sqrt{v_x^2 + v_y^2}$$

Substitute numerical values and evaluate v_0:

$$\begin{aligned} v_0 &= \sqrt{(56.32\,\text{m/s})^2 + (7.814\,\text{m/s})^2} \\ &= 56.86\,\text{m/s} \end{aligned}$$

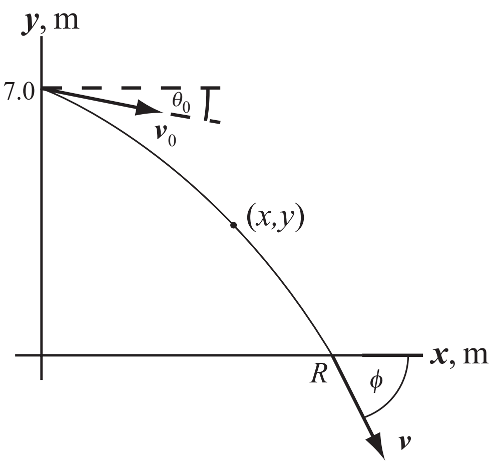
Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \frac{(56.86\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin(15.8°)\left(1+\sqrt{1+\frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(56.86\,\text{m/s})^2(\sin^2 7.899°)}}\right) = \boxed{0.10\,\text{km}}$$

Remarks: An alternative solution for part (*b*) is to solve for the range in the reference frame of the archer and then add to it the distance the frame travels, relative to the earth, during the time of flight.

89 •• The roof of a two-story house makes an angle of 30° with the horizontal. A ball rolling down the roof rolls off the edge at a speed of 5.0 m/s. The distance to the ground from that point is about two stories or 7.0 m. (*a*) How long is the ball in the air? (*b*) How far from the base of the house does it land? (*c*) What is its speed and direction just before landing?

Picture the Problem Choosing the coordinate system shown in the figure to the right, we can use a constant acceleration equation to express the y coordinate of the ball's position as a function of time. When the ball hits the ground, this position coordinate is zero and we can solve the resulting quadratic equation for the time-to-the-ground. Because, in the absence of air resistance, there is no acceleration in the horizontal direction we can find R, v, and ϕ using constant-acceleration equations.



(*a*) Use a constant-acceleration equation to relate the y coordinate of the ball to its time-in-flight:

$$\boldsymbol{y(t)} = \boldsymbol{y}_0 + \boldsymbol{v}_{0y}\boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}_y\boldsymbol{t}^2$$

or, because $v_{0y} = v_0\sin\theta_0$ and $a_y = -g$,

$$\boldsymbol{y(t)} = \boldsymbol{y}_0 + (\boldsymbol{v}_0\sin\boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2$$

$y(t_{\text{ground}}) = 0$ when the ball hits the ground and:

$$0 = \boldsymbol{y}_0 + (\boldsymbol{v}_0\sin\boldsymbol{\theta}_0)\boldsymbol{t}_{\text{ground}} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}_{\text{ground}}^2$$

Substitute numerical values to obtain:

$$0 = 7.0\ \text{m} + (-5.0\ \text{m/s})\sin 30°\, \boldsymbol{t}_{\text{ground}} - \tfrac{1}{2}(9.81\ \text{m/s}^2)\boldsymbol{t}_{\text{ground}}^2$$

Simplifying yields:

$$(4.91\ \text{m/s}^2)\boldsymbol{t}_{\text{ground}}^2 + (2.50\ \text{m/s})\boldsymbol{t}_{\text{ground}} - 7.0\ \text{m} = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$\boldsymbol{t}_{\text{ground}} = 0.966\ \text{s or } -1.48\ \text{s}$$

Because only the positive root has physical meaning:

$$\boldsymbol{t}_{\text{ground}} = \boxed{0.97\ \text{s}}$$

(*b*) The distance from the house *R* is related to the ball's horizontal speed and the time-to-ground:

$$\boldsymbol{R} = \boldsymbol{v}_{0x}\boldsymbol{t}_{\text{ground}} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)\boldsymbol{t}_{\text{ground}}$$

Substitute numerical values and evaluate *R*:

$$\boldsymbol{R} = (5.0\ \text{m/s})(\cos 30°)(0.966\ \text{s}) = \boxed{4.2\ \text{m}}$$

(*c*) The speed and direction of the ball just before landing are given by:

$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2} \qquad (1)$$

and

$$\boldsymbol{\phi} = \tan^{-1}\left(\frac{\boldsymbol{v}_y}{\boldsymbol{v}_x}\right) \qquad (2)$$

Express v_x and v_y in terms of v_0:

$$\boldsymbol{v}_x = \boldsymbol{v}_0 \cos\boldsymbol{\theta}_0$$

and

$$\boldsymbol{v}_y = \boldsymbol{v}_{0y} + \boldsymbol{a}_y\boldsymbol{t} = \boldsymbol{v}_0 \sin\boldsymbol{\theta}_0 + \boldsymbol{gt}$$

Substitute numerical values and evaluate v_x and v_y:

$$\boldsymbol{v}_x = (5.0\ \text{m/s})\cos 30° = 4.330\ \text{m/s}$$

and

$$\boldsymbol{v}_y = (5.0\ \text{m/s})\boldsymbol{s}\text{in}30° + (9.81\ \text{m/s}^2)(0.966\ \text{s}) = 11.98\ \text{m/s}$$

Substituting numerical values in equations (1) and (2) yields:

$$\boldsymbol{v} = \sqrt{(4.330\ \text{m/s})^2 + (11.98\ \text{m/s})^2} = \boxed{13\ \text{m/s}}$$

and

$$\boldsymbol{\phi} = \tan^{-1}\left(\frac{11.98\ \text{m/s}}{4.330\ \text{m/s}}\right) = \boxed{70^\circ \text{ below the horizontal}}$$

90 •• Compute $dR/d\theta_0$ from $R = (v_0^2/g)\sin(2\theta_0)$ and show that setting $dR/d\theta_0 = 0$ gives $\theta_0 = 45°$ for the maximum range.

Picture the Problem An extreme value (i.e., a maximum or a minimum) of a function is determined by setting the appropriate derivative equal to zero. Whether the extremum is a maximum or a minimum can be determined by evaluating the second derivative at the point determined by the first derivative.

Evaluate $dR/d\theta_0$:

$$\frac{dR}{d\theta_0} = \frac{v_0^2}{g}\frac{d}{d\theta_0}[\sin(2\theta_0)] = \frac{2v_0^2}{g}\cos(2\theta_0)$$

Set $dR/d\theta_0 = 0$ for extrema:

$$\frac{2v_0^2}{g}\cos(2\theta_0) = 0$$

Solve for θ_0 to obtain:

$$\theta_0 = \tfrac{1}{2}\cos^{-1}(0) = 45^\circ$$

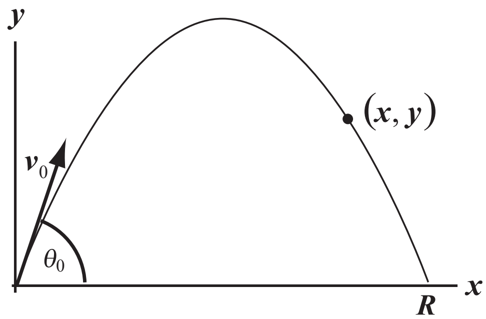
Determine whether 45° corresponds to a maximum or a minimum value of R:

$$\left.\frac{d^2R}{d\theta_0^{\,2}}\right|_{\theta_0=45^\circ} = \left[-4\left(v_0^2/g\right)\sin 2\theta_0\right]_{\theta_0=45^\circ} < 0$$

Therefore R is a maximum at $\theta_0 = 45°$.

91 •• In a science fiction short story written in the 1970s, Ben Bova described a conflict between two hypothetical colonies on the moon—one founded by the United States and the other by the USSR. In the story, colonists from each side started firing bullets at each other, only to find to their horror that their rifles had large enough muzzle velocities that the bullets went into orbit. (*a*) If the magnitude of free-fall acceleration on the moon is 1.67 m/s^2, what is the maximum range of a rifle bullet with a muzzle velocity of 900 m/s? (Assume the curvature of the surface of the moon is negligible.) (*b*) What would the muzzle velocity have to be to send the bullet into a circular orbit just above the surface of the moon?

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of a bullet in flight on the moon as a function of t. Eliminating this parameter will yield an expression for y as a function of x that we can use to find the range of the bullet. The necessity that the centripetal acceleration of an object in orbit at the surface of a body equals the acceleration due to gravity at the surface will allow us to determine the required muzzle velocity for orbital motion.



(*a*) Using a constant-acceleration equation, express the x coordinate of a bullet in flight on the moon:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0\cos\theta_0$,

$$x = (v_0 \cos\theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of a bullet in flight on the moon:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g_{\text{moon}}$ and $v_{0y} = v_0\sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}g_{\text{moon}}t^2$$

Using the x-component equation, eliminate time from the y-component equation to obtain:

$$y = (\tan\theta_0)x - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}x^2$$

When $y = 0$ and $x = R$:

$$0 = (\tan\theta_0)R - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}R^2$$

and

$$R = \frac{v_0^2}{g_{\text{moon}}}\sin 2\theta_0$$

Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \frac{(900\,\text{m/s})^2}{1.67\,\text{m/s}^2}\sin 90° = \boxed{485\,\text{km}}$$

This result is probably not very accurate because it is about 28% of the moon's radius (1740 km). This being the case, we can no longer assume that the ground is "flat" because of the curvature of the moon.

(*b*) Express the condition that the centripetal acceleration must satisfy for an object in orbit at the surface of the moon:

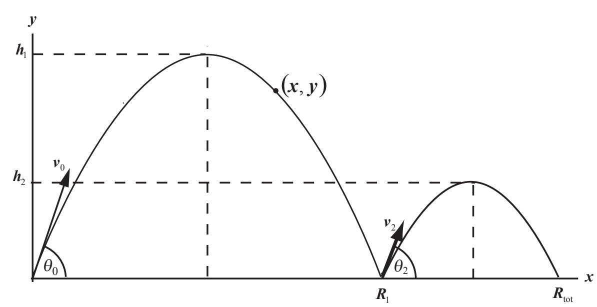
$$\boldsymbol{a}_\text{c} = \boldsymbol{g}_{\text{moon}} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}} \Rightarrow v = \sqrt{g_{\text{moon}}r}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(1.67\,\text{m/s}^2)(1.74\times 10^6\,\text{m})}$$
$$= \boxed{1.70\,\text{km/s}}$$

92 ••• On a level surface, a ball is launched from ground level at an angle of 55° above the horizontal, with an initial speed of 22 m/s. It lands on a hard surface, and bounces, reaching a peak height of 75% of the height it reached on its first arc. (Ignore any effects due to air resistance.) (*a*) What is the maximum height reached in its first parabolic arc? (*b*) How far horizontally from the launch point did it strike the ground the first time? (*c*) How far horizontally from the launch point did the ball strike the ground the second time? Assume the horizontal component of the velocity remains constant during the collision of the ball with the ground. *Note: You cannot assume the angle with which the ball leaves the ground after the first collision is the same as the initial launch angle.*

Picture the Problem The following diagram shows a convenient coordinate system to use in solving this problem as well as the relevant distances and angles. We can use constant-acceleration equations to find h_1. In (*b*) we can use the same-elevation equation to find R_1. In (*c*) we know the speed with which the tennis ball hits the ground on its first bounce – this is simply the speed with which it was launched. As it collides with the ground, it accelerates such that it leaves the surface with a speed that is reduced and is directed generally upward again. We are told that it reaches a height of 75% of its first peak height, and to assume that there is no horizontal acceleration. Again, we can use constant-acceleration equations to find θ_2 and the components of $\vec{v}_2$ and hence the range of the ball after its bounce. We can then find the total range from the sum of the ranges before and after the bounce.



(*a*) Use constant-acceleration equations to express the x and y coordinates of the tennis ball along its first parabolic trajectory as functions of time:

$$\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{v}_{0x}\boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}_x\boldsymbol{t}^2 \qquad (1)$$

and

$$\boldsymbol{y} = \boldsymbol{y}_0 + \boldsymbol{v}_{0y}\boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}_y\boldsymbol{t}^2 \qquad (2)$$

Because $x_0 = 0$, $\boldsymbol{v}_{0x} = \boldsymbol{v}_0\cos\boldsymbol{\theta}_0$, and $a_x = 0$, equation (1) becomes:

$$\boldsymbol{x} = (\boldsymbol{v}_0\cos\boldsymbol{\theta}_0)\boldsymbol{t} \qquad (3)$$

Because $y_0 = 0$, $\boldsymbol{v}_{0y} = \boldsymbol{v}_0 \sin\boldsymbol{\theta}_0$, and $a_y = -g$, equation (2) becomes:

$$\boldsymbol{y} = (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2 \qquad (4)$$

Use a constant-acceleration equation to relate the vertical velocity of the ball to time:

$$\boldsymbol{v}_y = \boldsymbol{v}_{0y} + \boldsymbol{a}_y\boldsymbol{t} = \boldsymbol{v}_0 \sin\boldsymbol{\theta}_0 - \boldsymbol{g}\boldsymbol{t}$$

When the ball is at the top of its trajectory $v_y = 0$ and:

$$0 = v_0 \sin\theta_0 - g t_{\text{top of trajectory}}$$

Solving for $t_{\text{top of trajectory}}$ yields:

$$t_{\text{top of trajectory}} = \frac{v_0 \sin\theta_0}{g}$$

Substitute in equation (4) and simplify to obtain:

$$\boldsymbol{h}_1 = (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\left(\frac{\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0}{\boldsymbol{g}}\right) - \tfrac{1}{2}\boldsymbol{g}\left(\frac{\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0}{\boldsymbol{g}}\right)^2 = \frac{\boldsymbol{v}_0^2 \sin^2\boldsymbol{\theta}_0}{2\boldsymbol{g}}$$

Substitute numerical values and evaluate h_1:

$$\boldsymbol{h}_1 = \frac{(22\ \text{m/s})^2 \sin^2 55°}{2(9.81\ \text{m/s}^2)} = 16.55\ \text{m} = \boxed{17\ \text{m}}$$

(*b*) The same-elevation range equation is:

$$\boldsymbol{R} = \frac{\boldsymbol{v}_0^2}{\boldsymbol{g}} \sin 2\boldsymbol{\theta}_0$$

Substitute numerical values and evaluate R_1:

$$\boldsymbol{R}_1 = \frac{(22\ \text{m/s})^2}{9.81\ \text{m/s}^2} \sin 2(55°) = 46.36\ \text{m} = \boxed{46\ \text{m}}$$

(*c*) The total range of the ball is the sum of the ranges R_1 and R_2:

$$\boldsymbol{R}_{\text{tot}} = \boldsymbol{R}_1 + \boldsymbol{R}_2 \qquad (5)$$

Provided we can determine the initial velocity v_{20} and rebound angle θ_2 of the rebounding ball, we can use the same-elevation equation to find R_2:

$$\boldsymbol{R}_2 = \frac{\boldsymbol{v}_{20}^2}{\boldsymbol{g}} \sin 2\boldsymbol{\theta}_2 \qquad (6)$$

where

$$\boldsymbol{v}_{20}^2 = \boldsymbol{v}_{2x}^2 + \boldsymbol{v}_{2y}^2 \qquad (7)$$

Substituting equation (6) in equation (7) yields:

$$\boldsymbol{R}_2 = \frac{\boldsymbol{v}_{2x}^2 + \boldsymbol{v}_{2y}^2}{\boldsymbol{g}} \sin 2\boldsymbol{\theta}_2 \qquad (8)$$

Use a constant-acceleration equation to relate the rebound height h_2 to the y component of the rebound velocity:

$$v_2^2 = v_{2y}^2 + 2a_y h_2$$

or, because $v_2 = 0$ and $a_y = -g$,

$$0 = v_{2y}^2 - 2gh_2$$

Solving for v_{2y} yields:

$$v_{2y}^2 = 2gh_2$$

or, because $h_2 = 0.75h_1$,

$$v_{2y}^2 = 1.5gh_1 \qquad (9)$$

Because there is no horizontal acceleration, the ball continues with the same horizontal velocity it originally had:

$$v_{2x} = v_{1x} = v_0 \cos\theta_0$$

and

$$v_{2x}^2 = v_0^2 \cos^2\theta_0 \qquad (10)$$

The rebound angle θ_2 is given by:

$$\theta_2 = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$$

Substituting for v_{2y} and v_{2x} and simplifying yields:

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{1.5gh_1}}{v_0 \cos\theta_0}\right)$$

Substitute numerical values and evaluate θ_2:

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{1.5(9.81\,\text{m/s}^2)(16.55\,\text{m})}}{(22\,\text{m/s})\cos 55°}\right) = 51.04°$$

Substitute (9) and (10) in equation (8) to obtain:

$$R_2 = \frac{v_0^2 \cos^2\theta_0 + 1.5gh_1}{g}\sin 2\theta_2$$

Substitute numerical values and evaluate R_2:

$$R_2 = \frac{(22\,\text{m/s})^2\cos^2 55° + 1.5(9.81\,\text{m/s}^2)(16.55\,\text{m})}{9.81\,\text{m/s}^2}\sin 2(51.04°) = 40.15\,\text{m}$$

Substitute numerical values for R_1 and R_2 in equation (5) and evaluate R_{tot}:

$$R_{\text{tot}} = 46.36\,\text{m} + 40.15\,\text{m} = \boxed{87\,\text{m}}$$

93 ••• In the text, we calculated the range for a projectile that lands at the same elevation from which it is fired as $R = (v_0^2 / g)\sin 2\theta_0$. A golf ball hit from an elevated tee at 45.0 m/s and an angle of 35.0° lands on a green 20.0 m below the tee (Figure 3-37). (Ignore any effects due to air resistance.) (*a*) Calculate the range using the same elevation equation $R = (v_0^2 / g)\sin 2\theta_0$ even though the ball is hit from an elevated tee. (*b*) Show that the range for the more general problem

(Figure 3-37) is given by $\boldsymbol{R}=\left(1+\sqrt{1-\frac{2\boldsymbol{g}\boldsymbol{y}}{\boldsymbol{v}_0^2\sin^2\boldsymbol{\theta}_0}}\right)\frac{\boldsymbol{v}_0^2}{2\boldsymbol{g}}\sin 2\boldsymbol{\theta}_0$, where y is the height of the tee above the green. That is, $y = -h$. (*c*) Compute the range using this formula. What is the percentage error in neglecting the elevation of the green?

Picture the Problem The initial position is on the $+y$ axis and the point of impact is on the $+x$ axis, with the x axis horizontal and the y axis vertical. We can apply constant-acceleration equations to express the horizontal and vertical coordinates of the projectile as functions of time. Solving these equations simultaneously will leave us with a quadratic equation in *R*, the solution to which is the result we are required to establish.

(*a*) The same elevation equation is:

$$\boldsymbol{R}_{\substack{\text{same}\\\text{elevation}}}=\frac{\boldsymbol{v}_0^2}{\boldsymbol{g}}\sin 2\boldsymbol{\theta}_0$$

Substitute numerical values and evaluate *R*:

$$\boldsymbol{R}_{\substack{\text{same}\\\text{elevation}}}=\frac{(45.0\text{ m/s})^2}{9.81\text{ m/s}^2}\sin 2(35.0°)=\boxed{194\text{ m}}$$

(*b*) Write the constant-acceleration equations for the horizontal and vertical components of the projectile's motion:

$$x=v_{0x}t$$

and

$$y=h+v_{0y}t+\tfrac{1}{2}(-g)t^2$$

where

$$\boldsymbol{v}_{0x}=\boldsymbol{v}_0\cos\boldsymbol{\theta}_0 \text{ and } \boldsymbol{v}_{0y}=\boldsymbol{v}_0\sin\boldsymbol{\theta}_0$$

Solve the x-component equation for t to obtain:

$$\boldsymbol{t}=\frac{\boldsymbol{x}}{\boldsymbol{v}_{0x}}=\frac{\boldsymbol{x}}{\boldsymbol{v}_0\cos\boldsymbol{\theta}_0}$$

Using the x-component equation, eliminate time from the y-component equation to obtain:

$$\boldsymbol{y}=\boldsymbol{h}+(\tan\boldsymbol{\theta}_0)\boldsymbol{x}-\frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{x}^2$$

When the projectile strikes the ground its coordinates are (R, 0) and our equation becomes:

$$0=\boldsymbol{h}+(\tan\boldsymbol{\theta}_0)\boldsymbol{R}-\frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{R}^2$$

Using the plus sign in the quadratic formula to ensure a physically meaningful root (one that is positive), solve for the range to obtain:

$$\boldsymbol{R}=\left(1+\sqrt{1+\frac{2\boldsymbol{g}\boldsymbol{h}}{\boldsymbol{v}_0^2\sin^2\boldsymbol{\theta}_0}}\right)\frac{\boldsymbol{v}_0^2}{2\boldsymbol{g}}\sin 2\boldsymbol{\theta}_0$$

Because $y = -h$:

$$\boxed{\boldsymbol{R} = \left(1 + \sqrt{1 - \frac{2\boldsymbol{gy}}{\boldsymbol{v}_0^2 \sin^2 \boldsymbol{\theta}_0}}\right) \frac{\boldsymbol{v}_0^2}{2\boldsymbol{g}} \sin 2\boldsymbol{\theta}_0}$$

(c) Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \left(1 + \sqrt{1 - \frac{2(9.81\ \text{m/s}^2)(-20.0\ \text{m})}{(45.0\ \text{m/s})^2 \sin^2 35.0°}}\right) \frac{(45.0\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} \sin 2(35.0°) = \boxed{219\ \text{m}}$$

The percent error is given by:

$$\%\ \text{error} = \left| \frac{\boldsymbol{R} - \boldsymbol{R}_{\substack{\text{same}\\ \text{elevation}}}}{\boldsymbol{R}} \right|$$

Substitute numerical values and evaluate the percent error:

$$\%\ \text{error} = \left| \frac{219\ \text{m} - 194\ \text{m}}{219\ \text{m}} \right| = \boxed{11\%}$$

94 ••• In the text, we calculated the range for a projectile that lands at the same elevation from which it is fired as $R = (v_0^2 / g) \sin 2\theta_0$ if the effects of air resistance are negligible. (a) Show that for the same conditions the change in the range for a small change in free-fall acceleration g, and the same initial speed and angle, is given by $\Delta R/R = -\Delta g/g$. (b) What would be the length of a homerun at a high altitude where g is 0.50% less than at sea level if the homerun at sea level traveled 400 ft?

Picture the Problem We can show that $\Delta R/R = -\Delta g/g$ by differentiating R with respect to g and then using a differential approximation.

(a) Differentiate the range equation with respect to g:

$$\frac{dR}{dg} = \frac{d}{dg}\left(\frac{v_0^2}{g} \sin 2\theta_0\right) = -\frac{v_0^2}{g^2} \sin 2\theta_0$$
$$= -\frac{R}{g}$$

Approximate dR/dg by $\Delta R/\Delta g$:

$$\frac{\Delta R}{\Delta g} = -\frac{R}{g}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R} = \boxed{-\frac{\Delta g}{g}}$$

That is, for small changes in gravity ($g \approx g \pm \Delta g$), the fractional change in R is linearly opposite to the fractional change in g.

(*b*) Solve the proportion derived in (*a*) for ΔR to obtain:

$$\Delta \boldsymbol{R} = -\frac{\Delta \boldsymbol{g}}{\boldsymbol{g}}\boldsymbol{R}$$

Substitute numerical values and evaluate ΔR:

$$\Delta \boldsymbol{R} = -\frac{-0.0050\boldsymbol{g}}{\boldsymbol{g}}(400\text{ ft}) = 2\text{ ft}$$

The length of the homerun at this high-altitude location would be 402 ft.

Remarks: The result in (*a*) tells us that as gravity increases, the range will decrease, and vice versa. This is as it must be because *R* is inversely proportional to *g*.

95 ••• **[SSM]** In the text, we calculated the range for a projectile that lands at the same elevation from which it is fired as $R = (v_0^2/g)\sin 2\theta_0$ if the effects of air resistance are negligible. (*a*) Show that for the same conditions the change in the range for a small change in launch speed, and the same initial angle and free-fall acceleration, is given by $\Delta R/R = 2\Delta v_0/v_0$. (*b*) Suppose a projectile's range was 200 m. Use the formula in Part (*a*) to estimate its increase in range if the launch speed were increased by 20.0%. (*c*) Compare your answer in (*b*) to the increase in range by calculating the increase in range directly from $R = (v_0^2/g)\sin 2\theta_0$. If the results for Parts (*b*) and (*c*) are different, is the estimate too large or too small?

Picture the Problem We can show that $\Delta R/R = 2\Delta v_0/v_0$ by differentiating *R* with respect to v_0 and then using a differential approximation.

(*a*) Differentiate the range equation with respect to v_0:

$$\frac{dR}{dv_0} = \frac{d}{dv_0}\left(\frac{v_0^2}{g}\sin 2\theta_0\right) = \frac{2v_0}{g}\sin 2\theta_0$$

$$= 2\frac{R}{v_0}$$

Approximate dR/dv_0 by $\Delta R/\Delta v_0$:

$$\frac{\Delta R}{\Delta v_0} = 2\frac{R}{v_0}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R} = \boxed{2\frac{\Delta v_0}{v_0}}$$

That is, for small changes in the launch velocity ($v_0 \approx v_0 \pm \Delta v_0$), the fractional change in *R* is twice the fractional change in v_0.

(*b*) Solve the proportion derived in (*a*) for ΔR to obtain:

$$\Delta R = 2R\frac{\Delta v_0}{v_0}$$

Substitute numerical values and evaluate ΔR:

$$\Delta R = 2(200\text{ m})\frac{0.20v_0}{v_0} = \boxed{80\text{ m}}$$

(*c*) The same-elevation equation is:

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \qquad (1)$$

The longer range R' resulting from a 20.0% increase in the launch speed is given by:

$$R' = \frac{(v_0 + 0.20v_0)^2}{g}\sin 2\theta_0 = \frac{(1.20v_0)^2}{g}\sin 2\theta_0 \qquad (2)$$

Divide equation (2) by equation (1) and simplify to obtain:

$$\frac{R'}{R} = \frac{\frac{(1.20v_0)^2}{g}\sin 2\theta_0}{\frac{v_0^2}{g}\sin 2\theta_0} = (1.20)^2 = 1.44$$

Solving for R' yields:

$$R' = 1.44R$$

Substitute the numerical value of R and evaluate R':

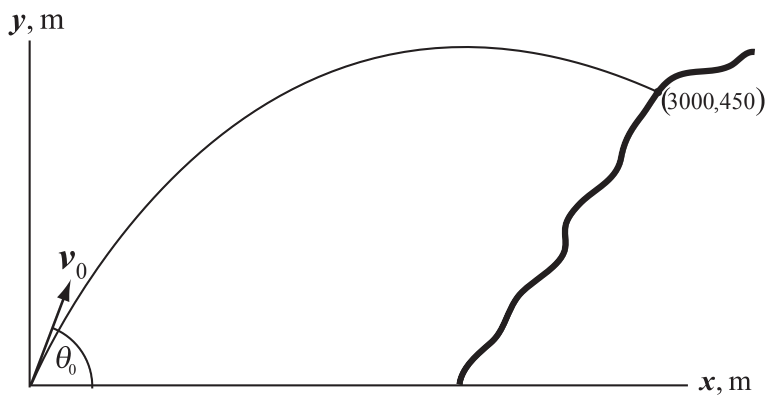
$$R' = 1.44(200\text{ m}) = \boxed{288\text{ m}}$$

The approximate solution is larger. The estimate ignores higher-order terms and they are important when the differences are not small.

Remarks: The result in (*a*) tells us that as launch velocity increases, the range will increase twice as fast, and vice versa.

96 ••• A projectile, fired with unknown initial velocity, lands 20.0 s later on the side of a hill, 3000 m away horizontally and 450 m vertically above its starting point. Neglect any effects due to air resistance. (*a*) What is the vertical component of its initial velocity? (*b*) What is the horizontal component of its initial velocity? (*c*) What was its maximum height above its launch point? (*d*) As it hit the hill, what speed did it have and what angle did its velocity make with the vertical? (Ignore air resistance.)

Picture the Problem In the absence of air resistance, the horizontal and vertical displacements of the projectile are independent of each other and describable by constant-acceleration equations. Choose the origin at the firing location and with the coordinate axes as shown in the pictorial representation and use constant-acceleration equations to relate the vertical displacement to the vertical component of the initial velocity and the horizontal velocity to the horizontal displacement and the time of flight.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of its time of flight:

$$\Delta \boldsymbol{y} = \boldsymbol{v}_{0y}\Delta \boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}_y(\Delta \boldsymbol{t})^2$$

or, because $a_y = -g$,

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 \qquad (1)$$

Solve for v_{0y}:

$$v_{0y} = \frac{\Delta y + \frac{1}{2}g(\Delta t)^2}{\Delta t}$$

Substitute numerical values and evaluate v_{0y}:

$$\boldsymbol{v}_{0y} = \frac{450\text{ m} + \frac{1}{2}(9.81\text{ m/s}^2)(20.0\text{ s})^2}{20.0\text{ s}} = 120.6\text{ m/s} = \boxed{121\text{ m/s}}$$

(*b*) The horizontal velocity remains constant, so:

$$\boldsymbol{v}_{0x} = \boldsymbol{v}_x = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \frac{3000\text{ m}}{20.0\text{ s}} = \boxed{150\text{ m/s}}$$

(*c*) From equation (1) we have, when the projectile is at its maximum height *h*:

$$\boldsymbol{h} = \boldsymbol{v}_{0y}\Delta \boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}(\Delta \boldsymbol{t})^2$$

or, because $v_{0y} = v_0\sin\theta$,

$$\boldsymbol{h} = (\boldsymbol{v}_0\sin\boldsymbol{\theta}_0)\Delta \boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}(\Delta \boldsymbol{t})^2 \qquad (2)$$

Use a constant-acceleration equation to relate the *y* component of the velocity of the projectile to its initial value and to the elapsed time:

$$\boldsymbol{v}_y = \boldsymbol{v}_{0y} + \boldsymbol{a}_y\Delta \boldsymbol{t} = \boldsymbol{v}_0\sin\boldsymbol{\theta}_0 - \boldsymbol{g}\Delta \boldsymbol{t} \quad (3)$$

The projectile is at its maximum height when its upward velocity is zero. Under this condition we have:

$$0 = \boldsymbol{v}_0\sin\boldsymbol{\theta}_0 - \boldsymbol{g}\Delta \boldsymbol{t} \Rightarrow \Delta \boldsymbol{t} = \frac{\boldsymbol{v}_0\sin\boldsymbol{\theta}_0}{\boldsymbol{g}}$$

Substitute for Δt in equation (2) to obtain:

$$\boldsymbol{h} = (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\left(\frac{\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0}{\boldsymbol{g}}\right) - \tfrac{1}{2}\boldsymbol{g}\left(\frac{\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0}{\boldsymbol{g}}\right)^2$$

Simplifying yields:

$$\boldsymbol{h} = \frac{\boldsymbol{v}_0^2 \sin^2\boldsymbol{\theta}_0}{2\boldsymbol{g}} \qquad (4)$$

$\boldsymbol{v}_0^2$ is given by:

$$\boldsymbol{v}_0^2 = \boldsymbol{v}_{0x}^2 + \boldsymbol{v}_{0y}^2$$

Substitute numerical values and evaluate $\boldsymbol{v}_0$:

$$\boldsymbol{v}_0 = \sqrt{(150\text{ m/s})^2 + (120.6\text{ m/s})^2} = 192.5\text{ m/s}$$

The horizontal displacement of the projectile is given by:

$$\Delta\boldsymbol{x} = \boldsymbol{v}_{0x}\Delta\boldsymbol{t} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)\Delta\boldsymbol{t}$$

Solving for the launch angle θ_0 yields:

$$\boldsymbol{\theta}_0 = \cos^{-1}\left(\frac{\Delta\boldsymbol{x}}{\boldsymbol{v}_0\Delta\boldsymbol{t}}\right)$$

Substitute numerical values and evaluate θ_0:

$$\boldsymbol{\theta}_0 = \cos^{-1}\left(\frac{3000\text{ m}}{(192.5\text{ m/s})(20.0\text{ s})}\right) = 38.80°$$

Substitute numerical values in equation (3) and evaluate h:

$$\boldsymbol{h} = \frac{(192.5\text{ m/s})^2 \sin^2 38.80°}{2(9.81\text{ m/s}^2)} = \boxed{742\text{ m}}$$

(*d*) Use equation (3) to evaluate $\boldsymbol{v}_y(20.0\text{ s})$:

$$\boldsymbol{v}_y(20.0\text{ s}) = 120.6\text{ m/s} - (9.81\text{ m/s}^2)(20.0\text{ s}) = -75.60\text{ m/s}$$

The speed of the projectile at impact is given by:

$$\boldsymbol{v}(20.0\text{ s}) = \sqrt{(\boldsymbol{v}_x(20.0\text{ s}))^2 + (\boldsymbol{v}_y(20.0\text{ s}))^2}$$

Substitute numerical values and evaluate $v(20.0\text{ s})$:

$$\boldsymbol{v}(20.0\text{ s}) = \sqrt{(150\text{ m/s})^2 + (-75.6\text{ m/s})^2} = \boxed{168\text{ m/s}}$$

The angle at impact is given by:

$$\boldsymbol{\theta}_{\text{impact}} = \tan^{-1}\left(\frac{\boldsymbol{v}_y(20.0\text{ s})}{\boldsymbol{v}_x(20.0\text{ s})}\right)$$

Substitute numerical values and evaluate θ_{impact}:

$$\theta_{\text{impact}} = \tan^{-1}\left(\frac{-75.60\ \text{m/s}}{150\ \text{m/s}}\right) = -26.75°$$
$$= \boxed{63.3° \text{ with the vertical}}$$

97 ••• **[SSM]** A projectile is launched over level ground at an initial elevation angle of θ. An observer standing at the launch site sees the projectile at the point of its highest elevation, and measures the angle ϕ shown in Figure 3-38. Show that $\tan\phi = \frac{1}{2}\tan\theta$. (Ignore any effects due to air resistance.)

Picture the Problem We can use trigonometry to relate the maximum height of the projectile to its range and the sighting angle at maximum elevation and the range equation to express the range as a function of the launch speed and angle. We can use a constant-acceleration equation to express the maximum height reached by the projectile in terms of its launch angle and speed. Combining these relationships will allow us to conclude that $\tan\phi = \frac{1}{2}\tan\theta$.

Referring to the figure, relate the maximum height of the projectile to its range and the sighting angle ϕ:

$$\tan\phi = \frac{h}{\frac{1}{2}R}$$

Express the range of the rocket and use the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$ to rewrite the expression as:

$$R = \frac{v_0^2}{g}\sin(2\theta) = 2\frac{v_0^2}{g}\sin\theta\cos\theta$$

Using a constant-acceleration equation, relate the maximum height h of a projectile to the vertical component of its launch speed:

$$v_y^2 = v_{0y}^2 - 2gh$$

or, because $v_y = 0$ and $v_{0y} = v_0\sin\theta$,

$$v_0^2\sin^2\theta = 2gh \Rightarrow h = \frac{v_0^2}{2g}\sin^2\theta$$

Substitute for R and h and simplify to obtain:

$$\tan\phi = \frac{2\left(\dfrac{v_0^2}{2g}\sin^2\theta\right)}{2\left(\dfrac{v_0^2}{g}\sin\theta\cos\theta\right)} = \boxed{\tfrac{1}{2}\tan\theta}$$

98 ••• A toy cannon is placed on a ramp that has a slope of angle ϕ. (*a*) If the cannonball is projected up the hill at an angle of θ_0 above the horizontal (Figure 3-39) and has a muzzle speed of v_0, show that the range R of the cannonball (as measured along the ramp) is given by $R = \dfrac{2v_0^2\cos^2\theta_0(\tan\theta_0 - \tan\phi)}{g\cos\phi}$. Neglect any effects due to air resistance.

Picture the Problem The equation of a particle's trajectory is derived in the text so we'll use it as our starting point in this derivation. We can relate the coordinates of the point of impact (x, y) to the angle ϕ and use this relationship to eliminate y from the equation for the cannonball's trajectory. We can then solve the resulting equation for x and relate the horizontal component of the point of impact to the cannonball's range.

The equation of the cannonball's trajectory is given in Equation 3-22:

$$y(x) = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Relate the x and y components of a point on the ground to the angle ϕ:

$$y(x) = (\tan\phi)x$$

Express the condition that the cannonball hits the ground:

$$(\tan\phi)x = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Solving for x yields:

$$x = \frac{2v_0^2\cos^2\theta_0(\tan\theta_0 - \tan\phi)}{g}$$

Relate the range of the cannonball's flight R to the horizontal distance x:

$$x = R\cos\phi$$

Substitute to obtain:

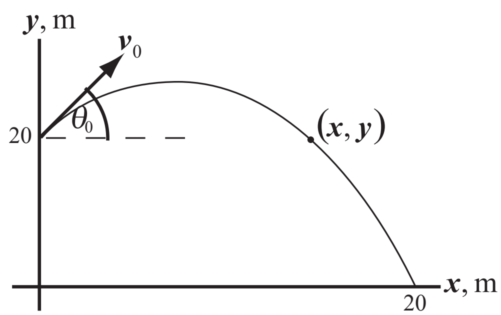
$$R\cos\phi = \frac{2v_0^2\cos^2\theta_0(\tan\theta_0 - \tan\phi)}{g}$$

Solving for R yields:

$$R = \boxed{\frac{2v_0^2\cos^2\theta_0(\tan\theta_0 - \tan\phi)}{g\cos\phi}}$$

99 ••• A rock is thrown from the top of a 20-m-high building at an angle of 53° above the horizontal. (*a*) If the horizontal range of the throw is equal to the height of the building, with what speed was the rock thrown? (*b*) How long is it in the air? (*c*) What is the velocity of the rock just before it strikes the ground? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and the horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure with the origin at the base of the building and the axes oriented as shown and apply constant- acceleration equations to relate the horizontal and vertical coordinates of the rock to the time into its flight.



(a) Using constant-acceleration equations, express the x and y coordinates of the rock as functions of the time into its flight:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
and
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $x_0 = 0$, $a_x = 0$, and $a_y = -g$:

$$x = v_{0x}t = (v_0\cos\theta_0)t \qquad (1)$$
and
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 = y_0 + (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Eliminating time between these equations yields:

$$y = y_0 + (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$

When the rock hits the ground, $x = y_0$ and:

$$0 = y_0 + (\tan\theta_0)y_0 - \frac{g}{2v_0^2\cos^2\theta_0}y_0^2$$
or
$$0 = 1 + \tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}y_0$$

Solve for v_0 to obtain:

$$v_0 = \frac{1}{\cos\theta_0}\sqrt{\frac{gy_0}{2(1+\tan\theta_0)}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{1}{\cos 53°}\sqrt{\frac{(9.81\ \text{m/s}^2)(20\ \text{m})}{2(1+\tan 53°)}} = 10.79\ \text{m/s} \approx \boxed{11\ \text{m/s}}$$

(b) Solve equation (1) for the time-to-impact to obtain:

$$t = \frac{x}{v_0\cos\theta_0}$$

Substitute numerical values and evaluate t:

$$t = \frac{20\ \text{m}}{(10.79\ \text{m/s})\cos 53°} = 3.08\ \text{s} = \boxed{3.1\text{s}}$$

(*c*) Using constant-acceleration equations, find v_y and v_x at impact:

$$v_x = v_{0x} = v_0 \cos\theta = (10.79\ \text{m/s})\cos 53° = 6.50\,\text{m/s}$$

and

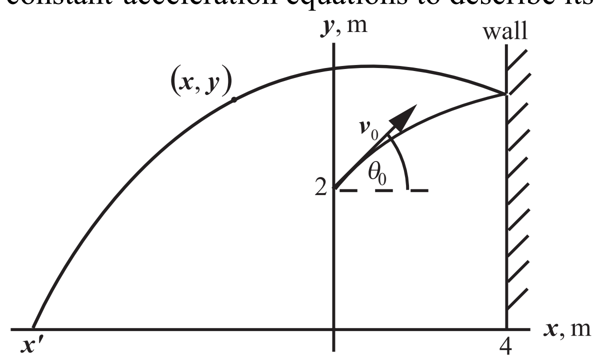
$$v_y = v_{0y} - gt = v_0 \sin\theta_0 - gt = (10.79\ \text{m/s})\sin 53° - (9.81\ \text{m/s}^2) \times (3.08\ \text{s}) = -21.6\,\text{m/s}$$

Express the velocity at impact in vector form:

$$\vec{v} = \boxed{(6.5\ \text{m/s})\hat{i} + (-22\ \text{m/s})\hat{j}}$$

100 ••• A woman throws a ball at a vertical wall 4.0 m away (Figure 3-40). The ball is 2.0 m above ground when it leaves the woman's hand with an initial velocity of 14 m/s at 45º as shown. When the ball hits the wall, the horizontal component of its velocity is reversed; the vertical component remains unchanged. (*a*) Where does the ball hit the ground? (*b*) How long was the ball in the air before it hit the wall? (*c*) Where did the ball hit the wall? (*d*) How long was the ball in the air after it left the wall? Ignore any effects due to air resistance.

Picture the Problem Choose the coordinate system shown below with the positive *x* axis to the right and the positive *y* axis upward. The solid curve represents the trajectory of the ball as it bounces from the vertical wall. Because the ball experiences constant acceleration (except during its collision with the wall), we can use constant-acceleration equations to describe its motion.



(*a*) The ball hits the ground at a point x' whose coordinate is given by:

$$x' = \Delta x - 4.0\ \text{m} \qquad (1)$$

where Δx is the displacement of the ball after it hits the wall.

Using a constant-acceleration equation, express the vertical displacement of the ball as a function of Δt:

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2} g(\Delta t)^2$$

or, because $v_{0y} = v_0 \sin\theta_0$,

$$\Delta y = (v_0 \sin\theta_0)\Delta t - \tfrac{1}{2} g(\Delta t)^2$$

Substitute numerical values to obtain:

$$\Delta y = (14\,\text{m/s})(\cos 45°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

When the ball hits the ground, $\Delta y = -2.0$ m and $\Delta t = t_{\text{flight}}$:

$$-2.0\,\text{m} = (14\,\text{m/s})(\cos 45°)t_{\text{flight}} - \tfrac{1}{2}(9.81\,\text{m/s}^2)t_{\text{flight}}^2$$

Using the quadratic formula or your graphing calculator, solve for the time of flight:

$$t_{\text{flight}} = 2.203\,\text{s}$$

The horizontal distance traveled in this time is given by:

$$\Delta x = (v_{0x}\cos\theta_0)t_{\text{flight}}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = (14\,\text{m/s})(\cos 45°)(2.203\,\text{s}) = 21.8\,\text{m}$$

Use equation (1) to find where the ball hits the ground:

$$x' = 21.8\,\text{m} - 4.0\,\text{m} = \boxed{18\text{m}}$$

(*b*) The time-to-the-wall, Δt_{wall}, is related to the displacement of the ball when it hits the wall and to the *x* component of its velocity:

$$\Delta t_{\text{wall}} = \frac{\Delta x_{\text{wall}}}{v_{0x}} = \frac{\Delta x_{\text{wall}}}{v_0\cos\theta_0}$$

Substitute numerical values and evaluate Δt_{wall}:

$$\Delta t_{\text{wall}} = \frac{4.0\,\text{m}}{(14\,\text{m/s})\cos 45°} = 0.4041\,\text{s} = \boxed{0.40\,\text{s}}$$

(*c*) First we'll find the *y* component of the ball's velocity when it hits the wall. Use a constant-acceleration equation to express $v_y(\Delta t_{\text{wall}})$:

$$v_y(\Delta t_{\text{wall}}) = v_{0y} + a_y\Delta t_{\text{wall}}$$

or, because $v_{0y} = v_0\sin\theta_0$ and $a_y = -g$,

$$v_y(\Delta t_{\text{wall}}) = v_0\sin\theta_0 - g\Delta t_{\text{wall}}$$

Substitute numerical values and evaluate $v_y(0.4041\text{ s})$:

$$v_y(0.4041\,\text{s}) = (14\,\text{m/s})\sin 45° - (9.81\,\text{m/s}^2)(0.4041\,\text{s}) = 5.935\,\text{m/s}$$

Now we need to find the height of the ball when it hit the wall. Use a constant-acceleration equation to express $y(\Delta t_{\text{wall}})$:

$$y(\Delta t_{\text{wall}}) = y_0 + v_{0y}\Delta t_{\text{wall}} + \tfrac{1}{2}a_y(\Delta t_{\text{wall}})^2$$

Because $v_{0y} = v_0 \sin\theta_0$ and $a_y = -g$:

$$y(\Delta t_{\text{wall}}) = y_0 + (v_0 \sin\theta_0)\Delta t_{\text{wall}} - \tfrac{1}{2} g (\Delta t_{\text{wall}})^2$$

Substitute numerical values and evaluate $y(0.4041\text{ s})$:

$$\begin{aligned} y(0.4041\,\text{s}) &= 2.0\,\text{m} + (14\,\text{m/s})(\sin 45°)(0.4041\,\text{s}) - \tfrac{1}{2}(9.81\,\text{m/s}^2)(0.4041\,\text{s})^2 \\ &= 5.199\,\text{m} = \boxed{5.2\,\text{m}} \end{aligned}$$

(*d*) Letting Δt_{after} represent the time the ball is in the air after its collision with the wall, express its vertical position as a function of time:

$$y(\Delta t_{\text{after}}) = y_0 + v_y(0.4041\,\text{s})\Delta t_{\text{after}} - \tfrac{1}{2} g (\Delta t_{\text{after}})^2$$

Noting that $y(\Delta t_{\text{after}}) = 0$ when the ball hits the ground, substitute numerical values to obtain:

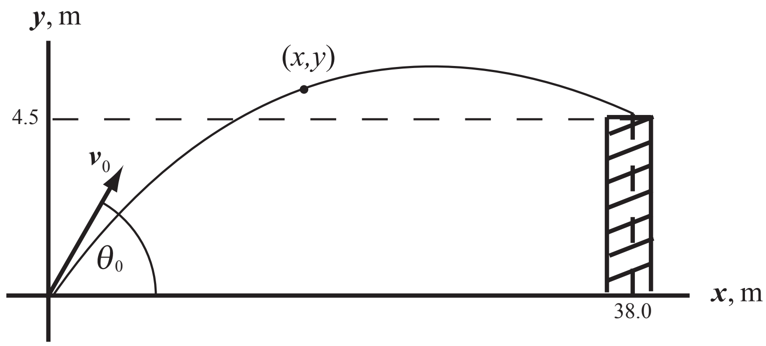
$$0 = 5.199\,\text{m} + (5.935\,\text{m/s})\Delta t_{\text{after}} - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t_{\text{after}})^2$$

Solving this equation using the quadratic formula or your graphing calculator yields:

$$\Delta t_{\text{after}} = 1.798\,\text{s} = \boxed{1.8\,\text{s}}$$

101 ••• Catapults date from thousands of years ago, and were used historically to launch everything from stones to horses. During a battle in what is now Bavaria, inventive artillerymen from the united German clans launched giant *spaetzle* from their catapults toward a Roman fortification whose walls were 8.50 m high. The catapults launched *spaetzle* projectiles from a height of 4.00 m above the ground, and 38.0 m from the walls of the fortification, at an angle of 60.0 degrees above the horizontal. (Figure 3-41.) If the projectiles were to impact the top of the wall, splattering the Roman soldiers atop the wall with pulverized pasta, (*a*) what launch speed was necessary? (*b*) How long was the spaetzle in the air? (*c*) At what speed did the projectiles hit the wall? Ignore any effects due to air resistance.

Picture the Problem The simplest solution for this problem is to consider the equation relating y and x along the projectile's trajectory that is found in equation 3-17. This equation is valid for trajectories which begin at the origin. We'll let the origin of coordinates be the point of launch (as shown in the diagram). Thus the impact point on top of the walls is y = 4.50 m, rather than 8.50 m. Our launch angle θ_0 is 60.0°, and the position of the center of the wall is 38.0 m. We can solve Equation 3-17 for the launch speed.



(*a*) Equation 3-17 is:

$$y(x) = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Solving for v_0 yields:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2[x\tan\theta_0 - y(x)]}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{38.0\text{ m}}{\cos 60.0°}\sqrt{\frac{9.81\text{ m/s}^2}{2[(38.0\text{ m})\tan 60.0° - 4.50\text{ m}]}} = 21.50\text{ m/s} = \boxed{21.5\text{ m/s}}$$

(*b*) Relate the *x* component of the spaetzle's velocity to its time-of-flight:

$$t_{\text{flight}} = \frac{\Delta x}{v_{0x}} = \frac{\Delta x}{v_0\cos\theta_0}$$

Substitute numerical values and evaluate t_{flight}:

$$t_{\text{flight}} = \frac{38.0\text{ m}}{(21.50\text{ m/s})\cos 60.0°} = 3.535\text{ s}$$

$$= \boxed{3.53\text{ s}}$$

(*c*)The speed with which the projectiles hit the wall is related to its *x* and *y* components through the Pythagorean Theorem:

$$v = \sqrt{v_x^2 + v_y^2} \qquad (1)$$

Because the acceleration in the *x* direction is zero, the *x* component of the projectile's velocity on impact equals its initial *x* component:

$$v_x = v_{0x} = v_0\cos\theta_0$$

Substitute numerical values and evaluate v_x:

$$v_x = (21.5\text{ m/s})\cos 60.0° = 10.75\text{ m/s}$$

Use a constant-acceleration equation to express the y component of the projectile's velocity at any time along its flight path:

$$v_y(t) = v_{0y} + a_y t$$

or, because $v_{0y} = v_0 \sin\theta_0$ and $a_y = -g$,

$$v_y(t) = v_0 \sin\theta_0 - gt$$

Substitute numerical values and evaluate $v_y(3.535\text{ s})$:

$$v_y(3.535\text{ s}) = (21.5\text{ m/s})\sin 60.0° - (9.81\text{ m/s}^2)(3.535\text{ s}) = -16.06\text{ m/s}$$

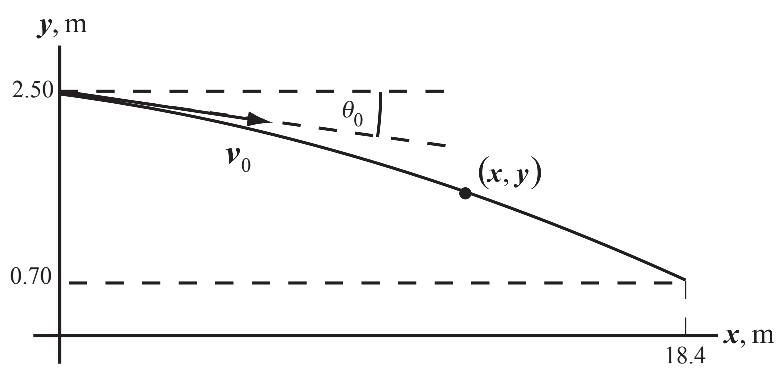
Substitute numerical values for v_x and v_y in equation (1) and evaluate v:

$$v = \sqrt{(10.75\text{ m/s})^2 + (-16.06\text{ m/s})^2} = \boxed{19.3\text{ m/s}}$$

Hitting Targets and Related Problems

102 ••• The distance from the pitcher's mound to home plate is 18.4 m. The mound is 0.20 m above the level of the field. A pitcher throws a fastball with an initial speed of 37.5 m/s. At the moment the ball leaves the pitcher's hand, it is 2.30 m above the mound. (*a*) What should the angle between $\vec{v}_0$ and the horizontal be so that the ball crosses the plate 0.70 m above ground? (Ignore any effects due to air resistance.) (*b*) With what speed does the ball cross the plate?

Picture the Problem The acceleration of the ball is constant (zero horizontally and –g vertically) and the vertical and horizontal components are independent of each other. Choose the coordinate system shown in the figure and use constant-acceleration equations to express the x and y coordinates of the ball along its trajectory as functions of time. Eliminating t between these equations will lead to an equation that can be solved for θ_0. Differentiating the x and y coordinate equations with respect to time will yield expressions for the instantaneous speed of the ball along its trajectory that you can use to find its speed as it crosses the plate.



(*a*) Express the horizontal position of the ball as a function of time:

$$x = (v_0 \cos\theta_0)t \qquad (1)$$

Express the vertical position of the ball as a function of time:

$$\boldsymbol{y} = \boldsymbol{y}_0 + (\boldsymbol{v}_0 \sin \boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2 \qquad (2)$$

Eliminate t between these equations to obtain:

$$\boldsymbol{y} = \boldsymbol{y}_0 - (\tan \boldsymbol{\theta}_0)\boldsymbol{x} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2 \cos^2 \boldsymbol{\theta}_0}\boldsymbol{x}^2$$

When the ball crosses the plate:

$$0.70\text{ m} = 2.50\text{ m} - (\tan \boldsymbol{\theta}_0)(18.4\text{ m}) - \frac{9.81\text{ m/s}^2}{2(37.5\text{ m/s})^2 \cos^2 \boldsymbol{\theta}_0}(18.4\text{ m})^2$$

Simplifying yields:

$$\frac{1}{\cos^2 \boldsymbol{\theta}_0} + 15.58 \tan \boldsymbol{\theta}_0 = 1.524$$

Use your graphing calculator to find the magnitude of θ_0:

$$\boldsymbol{\theta}_0 = 1.922° = \boxed{1.9°}$$

(*b*) The speed with which the ball crosses the plate is related to the x and y components of its speed through the Pythagorean Theorem:

$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2} \qquad (3)$$

Differentiate equation (1) with respect to time to express the ball's speed in the x direction:

$$\boldsymbol{v}_x = \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = \boldsymbol{v}_0 \cos \boldsymbol{\theta}_0$$

Differentiate equation (2) with respect to time to express the ball's speed in the y direction:

$$\boldsymbol{v}_y = \frac{\boldsymbol{dy}}{\boldsymbol{dt}} = -\boldsymbol{v}_0 \sin \boldsymbol{\theta}_0 - \boldsymbol{g}\boldsymbol{t}$$

Substituting in equation (3) and simplifying yields:

$$\boldsymbol{v} = \sqrt{(\boldsymbol{v}_0 \cos \boldsymbol{\theta}_0)^2 + (\boldsymbol{v}_0 \sin \boldsymbol{\theta}_0 \boldsymbol{g}\boldsymbol{t})^2}$$
$$= \boldsymbol{v}_0 \sqrt{\cos^2 \boldsymbol{\theta}_0 + \sin^2 \boldsymbol{\theta}_0 \boldsymbol{g}^2 \boldsymbol{t}^2}$$

From equation (1), the time required for the ball to reach home plate is:

$$\boldsymbol{t}_{\text{to plate}} = \frac{\boldsymbol{x}_{\text{to plate}}}{\boldsymbol{v}_0 \cos \boldsymbol{\theta}_0}$$

Substitute $t_{\text{to plate}}$ for t and simplify to obtain:

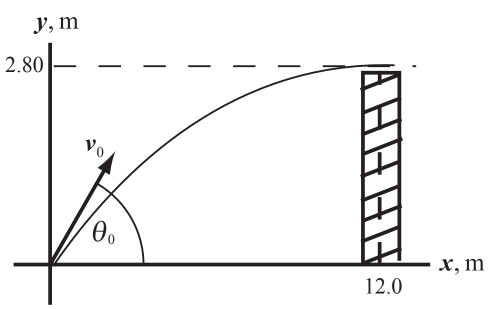
$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_0^2 \cos^2 \boldsymbol{\theta}_0 + \boldsymbol{v}_0^2 \sin^2 \boldsymbol{\theta}_0 \boldsymbol{g}^2 \left(\frac{\boldsymbol{x}_{\text{to plate}}}{\boldsymbol{v}_0 \cos \boldsymbol{\theta}_0} \right)^2} = \sqrt{\boldsymbol{v}_0^2 \cos^2 \boldsymbol{\theta}_0 + \tan^2 \boldsymbol{\theta}_0 \boldsymbol{g}^2 \boldsymbol{x}_{\text{to plate}}^2}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{(37.5\text{ m/s})^2 \cos^2 1.922° + \tan^2 1.922° (9.81\text{ m/s}^2)^2 (18.4\text{ m})^2} = \boxed{38\text{ m/s}}$$

103 ••• You are watching your friend play hockey. In the course of the game, he strikes the puck is struck in such a way that, when it is at its highest point, it just clears the surrounding 2.80-m-high Plexiglas wall that is 12.0 m away. Find (*a*) the vertical component of its initial velocity, (*b*) the time it takes to reach the wall, and (*c*) the horizontal component of its initial velocity, its initial speed and angle. Ignore any effects due to air resistance.

Picture the Problem The acceleration of the puck is constant (zero horizontally and $-g$ vertically) and the vertical and horizontal components are independent of each other. Choose a coordinate system with the origin at the point of contact with the puck and the coordinate axes as shown in the figure and use constant-acceleration equations to relate the variables v_{0y}, the time t to reach the wall, v_{0x}, v_0, and θ_0.



(*a*) Using a constant-acceleration equation for the motion in the y direction, express v_{0y} as a function of the puck's displacement Δy:

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

or, because $v_y = 0$ and $a_y = -g$,

$$0 = v_{0y}^2 - 2g\Delta y$$

Solve for v_{0y} to obtain:

$$\boldsymbol{v}_{0y} = \sqrt{2\boldsymbol{g}\Delta \boldsymbol{y}}$$

Substitute numerical values and evaluate v_{0y}:

$$v_{0y} = \sqrt{2(9.81\text{m/s}^2)(2.80\text{ m})} = 7.412\text{ m/s} = \boxed{7.41\text{m/s}}$$

(*b*) The time to reach the top of the wall is related to the initial velocity in the y direction:

$$\Delta \boldsymbol{t} = \frac{\boldsymbol{v}_{0y}}{\boldsymbol{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{7.412\,\text{m/s}}{9.81\,\text{m/s}^2} = 0.7555\,\text{s} = \boxed{0.756\text{ s}}$$

(c) Use the definition of average velocity to express v_{0x}:

$$v_{0x} = v_x = \frac{\Delta x}{\Delta t}$$

Substitute numerical values and evaluate v_{0x}:

$$v_{0x} = \frac{12.0\,\text{m}}{0.7555\,\text{s}} = 15.88\text{ m/s} = \boxed{15.9\,\text{m/s}}$$

v_0 is given by:

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{(15.88\text{ m/s})^2 + (7.412\text{ m/s})^2} = \boxed{17.5\,\text{m/s}}$$

θ_0 is related to v_{0y} and v_{0x}:

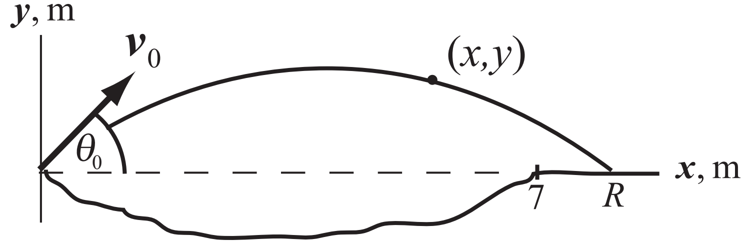
$$\theta_0 = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)$$

Substitute numerical values and evaluate θ_0 :

$$\theta_0 = \tan^{-1}\left(\frac{7.412\,\text{m/s}}{15.88\,\text{m/s}}\right) = \boxed{25.0°}$$

104 ••• Carlos is on his trail bike, approaching a creek bed that is 7.0 m wide. A ramp with an incline of 10º has been built for daring people who try to jump the creek. Carlos is traveling at his bike's maximum speed, 40 km/h. (*a*) Should Carlos attempt the jump or brake hard? (*b*) What is the minimum speed a bike must have to make this jump? Assume equal elevations on either side of the creek. Ignore any effects of air resistance.

Picture the Problem In the absence of air resistance, the acceleration of Carlos and his bike is constant and we can use constant-acceleration equations to express his x and y coordinates as functions of time. Eliminating the parameter t between these equations will yield y as a function of x … an equation we can use to decide whether he can jump the creek bed as well as to find the minimum speed required to make the jump.



(*a*) Use a constant-acceleration equation to express Carlos' horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$\boldsymbol{x} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)\boldsymbol{t}$$

Use a constant-acceleration equation to express Carlos' vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Because $y_0 = 0$, $v_{0y} = v_0\sin\theta_0$, and $a_y = -g$:

$$\boldsymbol{y} = (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2$$

Eliminate t between the x- and y-equations to obtain:

$$\boldsymbol{y} = (\tan\boldsymbol{\theta}_0)\boldsymbol{x} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{x}^2$$

Substitute $y = 0$ and $x = R$ to obtain:

$$0 = (\tan\boldsymbol{\theta}_0)\boldsymbol{R} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{R}^2$$

Solving for R yields:

$$\boldsymbol{R} = \frac{\boldsymbol{v}_0^2}{\boldsymbol{g}}\sin(2\boldsymbol{\theta}_0)$$

Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \frac{\left(40\times10^3\,\frac{\text{m}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{9.81\,\text{m/s}^2}\sin 2(10°)$$
$$= 4.3\,\text{m}$$

Because 4.3 m < 7 m, he should apply the brakes!

(*b*) Solve the equation we used in the previous step for $v_{0,\min}$:

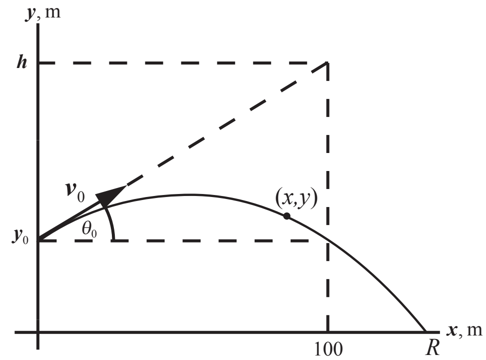
$$v_{0,\min} = \sqrt{\frac{Rg}{\sin(2\theta_0)}}$$

Letting $R = 7$ m, evaluate $v_{0,\min}$:

$$\boldsymbol{v}_{0,\min} = \sqrt{\frac{(7.0\,\text{m})(9.81\,\text{m/s}^2)}{\sin 2(10°)}}$$
$$= \boxed{14\,\text{m/s} = 50\,\text{km/h}}$$

105 ••• **[SSM]** If a bullet that leaves the muzzle of a gun at 250 m/s is to hit a target 100 m away at the level of the muzzle (1.7 m above the level ground), the gun must be aimed at a point above the target. (*a*) How far above the target is that point? (*b*) How far behind the target will the bullet strike the ground? Ignore any effects due to air resistance.

Picture the Problem In the absence of air resistance, the bullet experiences constant acceleration along its parabolic trajectory. Choose a coordinate system with the origin at the end of the barrel and the coordinate axes oriented as shown in the figure and use constant-acceleration equations to express the x and y coordinates of the bullet as functions of time along its flight path.



(*a*) Use a constant-acceleration equation to express the bullet's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$\boldsymbol{x} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)\boldsymbol{t}$$

Use a constant-acceleration equation to express the bullet's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $v_{0y} = v_0\sin\theta_0$, and $a_y = -g$,

$$\boldsymbol{y} = \boldsymbol{y}_0 + (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2$$

Eliminate t between the two equations to obtain:

$$\boldsymbol{y} = \boldsymbol{y}_0 + (\tan\boldsymbol{\theta}_0)\boldsymbol{x} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{x}^2$$

Because $y = y_0$ when the bullet hits the target:

$$0 = (\tan\boldsymbol{\theta}_0)\boldsymbol{x} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{x}^2$$

Solve for the angle above the horizontal that the rifle must be fired to hit the target:

$$\boldsymbol{\theta}_0 = \tfrac{1}{2}\sin^{-1}\left(\frac{\boldsymbol{x}\boldsymbol{g}}{\boldsymbol{v}_0^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \tfrac{1}{2}\sin^{-1}\left[\frac{(100\,\text{m})(9.81\,\text{m/s}^2)}{(250\,\text{m/s})^2}\right] = 0.450°$$

Note: A second value for θ_0, 89.6° is physically unreasonable.

Referring to the diagram, relate h to θ_0:

$$\tan\theta_0 = \frac{h}{100\,\text{m}} \Rightarrow h = (100\,\text{m})\tan\theta_0$$

Substitute numerical values and evaluate h:

$$\boldsymbol{h} = (100\,\text{m})\tan(0.450°) = \boxed{0.785\,\text{m}}$$

(b) The distance Δx behind the target where the bullet will strike the ground is given by:

$$\Delta \boldsymbol{x} = \boldsymbol{R} - 100\text{ m} \quad (1)$$

where R is the range of the bullet.

When the bullet strikes the ground, $y = 0$ and $x = R$:

$$0 = \boldsymbol{y}_0 + (\tan\boldsymbol{\theta}_0)\boldsymbol{R} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{R}^2$$

Substitute numerical values and simplify to obtain:

$$\frac{9.81\text{ m/s}^2}{2(250\text{ m/s})^2\cos^2 0.450°}\boldsymbol{R}^2 - (\tan 0.450°)\boldsymbol{R} - 1.7\text{ m} = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$\boldsymbol{R} = 206\text{ m}$$

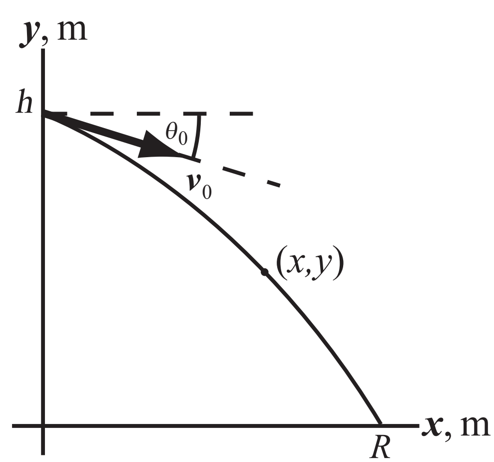
Substitute for R in equation (1) to obtain:

$$\Delta \boldsymbol{x} = 206\text{ m} - 100\text{ m} = \boxed{105\text{ m}}$$

General Problems

106 •• During a do-it-yourself roof repair project, you are on the roof of your house and accidentally drop your hammer. The hammer then slides down the roof at constant speed of 4.0 m/s. The roof makes an angle of 30º with the horizontal, and its lowest point is 10 m from the ground. (a) How long after leaving the roof does the hammer hit the ground? (b) What is the horizontal distance traveled by the hammer between the instant it leaves the roof and the instant it hits the ground? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the hammer experiences constant acceleration as it falls. Choose a coordinate system with the origin and coordinate axes as shown in the figure and use constant-acceleration equations to describe the x and y coordinates of the hammer along its trajectory. We'll use the equation describing the vertical motion to find the time of flight of the hammer and the equation describing the horizontal motion to determine its range.



(*a*) Using a constant-acceleration equation, express the x coordinate of the hammer as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$x = (v_0 \cos\theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of the hammer as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = h$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = h + (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Substituting numerical values yields:

$$\boldsymbol{y} = 10\,\text{m} + (-4.0\,\text{m/s})(\sin 30°)\boldsymbol{t} - \tfrac{1}{2}(9.81\,\text{m/s}^2)\boldsymbol{t}^2$$

When the hammer hits the ground, $y = 0$ and our equation becomes:

$$0 = 10\,\text{m} - (4.0\,\text{m/s})\sin 30°\,\boldsymbol{t}_{\text{fall}} - \tfrac{1}{2}(9.81\,\text{m/s}^2)\boldsymbol{t}_{\text{fall}}^2$$

Use the quadratic formula or your graphing calculator to solve for the time of fall:

$$\boldsymbol{t}_{\text{fall}} = 1.238\,\text{s} = \boxed{1.2\,\text{s}}$$

(*b*) Use the x-coordinate equation to express the horizontal distance traveled by the hammer as a function of its time-of-fall:

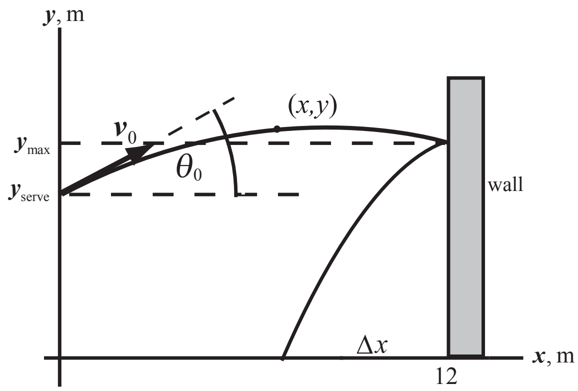
$$\boldsymbol{R} = \boldsymbol{v}_{0x}\boldsymbol{t}_{\text{fall}} = (\boldsymbol{v}_0\cos\boldsymbol{\theta}_0)\boldsymbol{t}_{\text{fall}}$$

Substitute numerical values and evaluate R:

$$\boldsymbol{R} = (4.0\,\text{m/s})(\cos 30°)(1.238\,\text{s}) = \boxed{4.3\,\text{m}}$$

107 •• A squash ball typically rebounds from a surface with 25% of the speed with which it initially struck the surface. Suppose a squash ball is served in a shallow trajectory, from a height above the ground of 45 cm, at a launch angle of 6.0° degrees above the horizontal, and at a distance of 12 m from the front wall. (*a*) If it strikes the front wall exactly at the top of its parabolic trajectory, determine how high above the floor the ball strikes the wall. (*b*) How far horizontally from the wall does it strike the floor, after rebounding? Ignore any effects due to air resistance.

Picture the Problem The following diagram shows the trajectory of the ball, a coordinate system, and the distances relevant to the solution of the problem. This problem is another variation on the projectile motion problem in which we are given the launch angle and the horizontal distance to the point at which the ball reaches its peak. At that point, the ball's velocity is solely horizontal. In order to answer the question raised in (*b*), we'll need to find the (horizontal) speed with which the ball strikes the wall. We can then use this rebound speed to find the distance Δx by applying appropriate constant-acceleration equations.



(*a*) The height above the floor at which the ball strikes the wall is given by:

$$y_{\max} = y_{\text{serve}} + h \qquad (1)$$

where h is the height above the service elevation to which the ball rises.

The distance h is related to the time Δt required for the ball to rise to its maximum height:

$$h = \tfrac{1}{2} g(\Delta t)^2 \qquad (2)$$

Δt is also related to v_{0y} and Δx_{serve}:

$$v_{0y} = g\Delta t \Rightarrow \Delta t = \frac{v_{0y}}{g} \qquad (3)$$

and

$$\Delta x_{\text{serve}} = v_{0x}\Delta t \Rightarrow \Delta t = \frac{\Delta x_{\text{serve}}}{v_{0x}} \qquad (4)$$

Substituting both (3) and (4) in equation (2) yields:

$$h = \tfrac{1}{2} g\left(\frac{v_{0y}}{g}\right)\left(\frac{\Delta x_{\text{serve}}}{v_{0x}}\right)$$

or, because $v_{0y} = v_0\sin\theta_0$ and $v_{0x} = v_0\cos\theta_0$,

$$h = \tfrac{1}{2} g\left(\frac{v_0 \sin\theta_0}{g}\right)\left(\frac{\Delta x_{\text{serve}}}{v_0 \cos\theta_0}\right)$$

Simplifying this expression yields:

$$h = \frac{\Delta x_{\text{serve}}}{2}\tan\theta_0$$

Substitute for h in equation (1) to obtain:

$$y_{\max} = y_{\text{serve}} + \frac{\Delta x_{\text{serve}}}{2}\tan\theta_0$$

Substitute numerical values and evaluate y_{max}:

$$y_{max} = 0.45\text{ m} + \frac{12\text{ m}}{2}\tan 6.0° = 1.081\text{ m}$$
$$= \boxed{1.1\text{ m}}$$

(*b*) The distance Δx the ball lands away from the wall is given by

$$\Delta x = v_{x,\text{rebound}}\Delta t$$

where Δt is the time it takes for the ball to fall to the floor.

Because the ball rebounds with 25% of the speed with which it initially struck the surface:

$$v_{x,\text{rebound}} = 0.25 v_{0x}$$

and

$$\Delta x = 0.25 v_{0x}\Delta t \qquad (5)$$

Δt is related to distance the ball falls vertically before hitting the floor:

$$y_{max} = \tfrac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2y_{max}}{g}}$$

Substitute for Δt in equation (5) to obtain:

$$\Delta x = 0.25 v_{0x}\sqrt{\frac{2y_{max}}{g}} \qquad (6)$$

To find v_{0x}, equate equations (3) and (4):

$$\frac{v_{0y}}{g} = \frac{\Delta x_{\text{serve}}}{v_{0x}} \qquad (7)$$

From the diagram, we can see that:

$$v_{0y} = v_{0x}\tan\theta_0$$

Substituting for v_{0y} in equation (7) yields:

$$\frac{v_{0x}\tan\theta_0}{g} = \frac{\Delta x_{\text{serve}}}{v_{0x}}$$

Solving for v_{0x} yields:

$$v_{0x} = \sqrt{\frac{g\Delta x_{\text{serve}}}{\tan\theta_0}}$$

Substitute for v_{0x} in equation (6) to obtain:

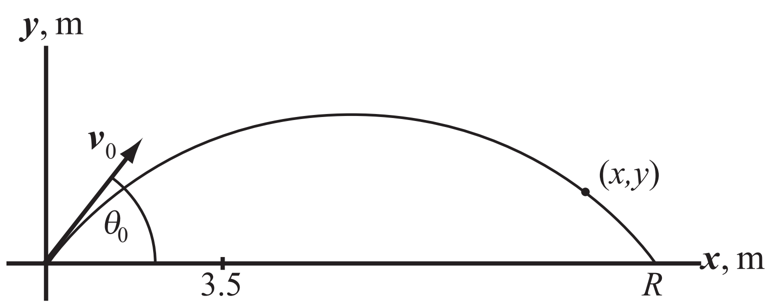
$$\Delta x = 0.25\sqrt{\frac{g\Delta x_{\text{serve}}}{\tan\theta_0}}\sqrt{\frac{2y_{max}}{g}}$$
$$= 0.25\sqrt{\frac{2\Delta x_{\text{serve}}y_{max}}{\tan\theta_0}}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = 0.25\sqrt{\frac{2(12\text{ m})(1.081\text{ m})}{\tan 6.0°}}$$
$$= \boxed{3.9\text{ m}}$$

108 •• A football quarterback throws a pass at an angle of 36.5° above the horizontal. He releases the pass 3.50 m behind the line of scrimmage. His receiver left the line of scrimmage 2.50 s earlier, goes straight downfield at a constant speed of 7.50 m/s. In order that the pass land gently in the receiver's hands without the receiver breaking stride, with what speed must the quarterback throw the pass? Assume that the ball is released at the same height it is caught and that the receiver is straight downfield from the quarterback at the time of the release. Ignore any effects due to air resistance.

Picture the Problem This problem deals with a moving target being hit by a projectile. The range required for the projectile is simply the distance that the receiver runs by the time the ball lands in his hands. We can use the equations developed in the text for both the range and time of flight of a projectile for any value of launch speed and angle, provided the launch and landing occur at the same height. The following diagram shows a convenient choice of a coordinate system as well as the trajectory of the ball. Note that the diagram includes the assumption that the ball is released at the same height it is caught.



The distance the receiver runs is given by:

$$\Delta x = v_{rec}\Delta t_{tot} \qquad (1)$$

The time-of-flight of the ball is given by:

$$\Delta t_{ball} = \Delta t_{tot} - \Delta t_{delay} \qquad (2)$$

where Δt_{delay} is the time the receiver ran before the quarterback released the ball.

The same-elevation range of the ball is:

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \qquad (3)$$

Equate equations (1) and (3) to obtain:

$$v_{rec}\Delta t_{tot} = \frac{v_0^2}{g}\sin 2\theta_0$$

Solving for Δt_{total} yields:

$$\Delta t_{tot} = \frac{v_0^2}{g v_{rec}}\sin 2\theta_0$$

The time-of-flight of the ball is given by:

$$\Delta t_{ball} = \frac{2v_0}{g}\sin\theta_0$$

Substitute for Δt_{tot} and Δt_{ball} in equation (2) to obtain:

$$\frac{2v_0}{g}\sin\theta_0 = \frac{v_0^2}{gv_{rec}}\sin 2\theta_0 - \Delta t_{delay}$$

Rearrange and simplify to obtain:

$$\frac{\sin 2\theta_0}{gv_{rec}}v_0^2 - \frac{2\sin\theta_0}{g}v_0 - \Delta t_{delay} = 0$$

Substituting numerical values yields:

$$\frac{\sin 2(36.5°)}{(9.81\,\text{m/s}^2)(7.50\,\text{m/s})}v_0^2 - \frac{2\sin 36.5°}{9.81\,\text{m/s}^2}v_0 - 2.50\,\text{s} = 0$$

or

$$\left(0.0130\,\frac{\text{s}^3}{\text{m}^2}\right)v_0^2 - \left(0.1213\,\frac{\text{s}^2}{\text{m}}\right)v_0 - 2.50\,\text{s} = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$v_0 = -9.97\,\text{m/s}$ or $19.3\,\text{m/s}$

and, because the negative throwing speed is not physical,

$$v_0 = \boxed{19.3\,\text{m/s}}$$

109 •• Suppose a test pilot is able to safely withstand an acceleration of up to 5.0 times the acceleration due to gravity (that is, remain conscious and alert enough to fly). In the course of maneuvers, he is required to fly the plane in a horizontal circle at its top speed of 1900 mi/h, (*a*) What is the radius of the smallest circle he will be able to safely fly the plane in? (*b*) How long does it take him to go halfway around this minimum-radius circle?

Picture the Problem This is a simple application of centripetal acceleration.

(*a*) Express the radius of the pilot's circular path:

$$r = \frac{v^2}{a_c}$$

or, because $a_c = 5.0g$,

$$r = \frac{v^2}{5.0g}$$

Substitute numerical values and evaluate r:

$$r = \frac{\left(1900\,\frac{\text{mi}}{\text{h}}\times 1609\,\frac{\text{m}}{\text{mi}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{5.0(9.81\,\text{m/s}^2)}$$

$$= 14.70\,\text{km} = \boxed{15\,\text{km}}$$

(*b*) The time required to reverse his direction is equal to one-half the period of his motion:

$$t_{rev} = \tfrac{1}{2}T = \frac{2\pi r}{2v} = \frac{\pi r}{v}$$

Substitute numerical values and evaluate t_{rev}:

$$t_{rev} = \frac{\pi(14.70\text{ km})}{1900\,\frac{\text{mi}}{\text{h}}\times 1609\,\frac{\text{m}}{\text{mi}}\times\frac{1\text{ h}}{3600\text{ s}}} = \boxed{54\text{ s}}$$

110 •• A particle moves in the *xy* plane with constant acceleration. At $t = 0$ the particle is at $\vec{r}_1 = (4.0\text{ m})\hat{i} + (3.0\text{ m})\hat{j}$, with velocity $\vec{v}_1$. At $t = 2.0$ s, the particle has moved to $\vec{r}_2 = (10\text{ m})\hat{i} - (2.0\text{ m})\hat{j}$ and its velocity has changed to $\vec{v}_2 = (5.0\text{ m/s})\hat{i} - (6.0\text{ m/s})\hat{j}$. (*a*) Find $\vec{v}_1$. (*b*) What is the acceleration of the particle? (*c*) What is the velocity of the particle as a function of time? (*d*) What is the position vector of the particle as a function of time?

Picture the Problem Because the acceleration is constant; we can use the constant-acceleration equations in vector form and the definitions of average velocity and average (instantaneous) acceleration to solve this problem.

(*a*) The average velocity is given by:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t} = (3.0\text{ m/s})\hat{i} + (-2.5\text{ m/s})\hat{j}$$

The average velocity can also be expressed as:

$$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2}{2} \Rightarrow \vec{v}_1 = 2\vec{v}_{av} - \vec{v}_2$$

Substitute numerical values to obtain:

$$\vec{v}_1 = \boxed{(1.0\text{ m/s})\hat{i} + (1.0\text{ m/s})\hat{j}}$$

(*b*) The acceleration of the particle is given by:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \boxed{(2.0\text{ m/s}^2)\hat{i} + (-3.5\text{ m/s}^2)\hat{j}}$$

(*c*) The velocity of the particle as a function of time is:

$$\vec{v}(t) = \vec{v}_1 + \vec{a}t = \boxed{[(1.0\text{ m/s}) + (2.0\text{ m/s}^2)t]\hat{i} + [(1.0\text{ m/s}) + (-3.5\text{ m/s}^2)t]\hat{j}}$$

(*d*) Express the position vector as a function of time:

$$\vec{r}(t) = \vec{r}_1 + \vec{v}_1 t + \tfrac{1}{2}\vec{a}t^2$$

Substitute numerical values and evaluate $\vec{r}(t)$:

$$\vec{r}(t)=\boxed{\begin{array}{r}[(4.0\text{ m})+(1.0\text{ m/s})\boldsymbol{t}+(1.0\text{ m/s}^2)\boldsymbol{t}^2]\hat{\boldsymbol{i}}\\ +[(3.0\text{ m})+(1.0\text{ m/s})\boldsymbol{t}+\left(-1.8\text{ m/s}^2\right)\boldsymbol{t}^2]\hat{\boldsymbol{j}}\end{array}}$$

111 •• **[SSM]** Plane A is headed due east, flying at an air speed of 400 mph. Directly below, at a distance of 4000 ft, plane B is headed due north, flying at an air speed of 700 mph. Find the velocity vector of plane B relative to A.

Picture the Problem The velocity of plane B relative to plane A is independent of the reference frame in which the calculation is done. The solution that follows uses the reference frame of the air. Choose a coordinate system in which the +*x* axis is to the east and the +*y* axis is to the north and write the velocity vectors for the two airplanes using unit vector notation. We can then use this vector to express the relative velocity in speed and heading form.

The velocity of plane B relative to plane A is given by:

$$\vec{v}_{\text{BA}}=\vec{v}_{\text{Ba}}-\vec{v}_{\text{Aa}} \qquad (1)$$

where the subscript a refers to the air.

Using unit vector notation, write expressions for $\vec{v}_{\text{Bg}}$ and $\vec{v}_{\text{Ag}}$:

$$\vec{v}_{\text{Ba}}=\left(700\text{ mi/h}\right)\hat{\boldsymbol{j}}$$

and

$$\vec{v}_{\text{Aa}}=\left(400\text{ mi/h}\right)\hat{\boldsymbol{i}}$$

Substitute for $\vec{v}_{\text{Ba}}$ and $\vec{v}_{\text{Aa}}$ in equation (1) to obtain:

$$\begin{aligned}\vec{v}_{\text{BA}}&=\left(700\text{ mi/h}\right)\hat{\boldsymbol{j}}-\left(400\text{ mi/h}\right)\hat{\boldsymbol{i}}\\&=-\left(400\text{ mi/h}\right)\hat{\boldsymbol{i}}+\left(700\text{ mi/h}\right)\hat{\boldsymbol{j}}\end{aligned}$$

The speed of plane B relative to plane is the magnitude of $\vec{v}_{\text{BA}}$:

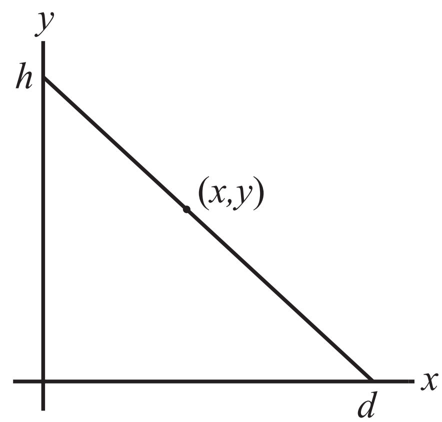
$$\begin{aligned}\left|\vec{v}_{\text{BA}}\right|&=\sqrt{\left(-400\text{ mi/h}\right)^2+\left(700\text{ mi/h}\right)^2}\\&=\boxed{806\text{ mi/h}}\end{aligned}$$

The heading of plane B relative to plane A is:

$$\begin{aligned}\boldsymbol{\theta}_{\text{BA}}&=\tan^{-1}\left(\frac{700\text{ mi/h}}{-400\text{ mi/h}}\right)\\&=\boxed{60.3^\circ\text{ north of west}}\end{aligned}$$

112 •• A diver steps off the cliffs at Acapulco, Mexico 30.0 m above the surface of the water. At that moment, he activates his rocket-powered backpack horizontally, which gives him a constant horizontal acceleration of 5.00 m/s^2 but does not affect his vertical motion. (*a*) How long does he take to reach the surface of the water? (*b*) How far out from the base of the cliff does he enter the water, assuming the cliff is vertical? (*c*) Show that his flight path is a straight line.

Picture the Problem Choose a coordinate system in which the origin is at the base of the cliff, the positive x axis is to the right, and the positive y axis is upward. Let the height of the cliff be h and the distance from the bottom of the cliff where the diver hits the water be d. Use constant-acceleration equations to obtain expressions for the x and y coordinates of the diver along his flight path, shown here as a straight line.



(*a*) Using a constant-acceleration equation, relate the vertical position of the diver to his initial position, initial speed, acceleration, and time of fall:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = h$, $v_{0y} = 0$, and $a_y = -g$,

$$y = h - \tfrac{1}{2}g t^2 \qquad (1)$$

When the diver reaches the surface of the water, $y = 0$ and:

$$0 = h - \tfrac{1}{2}g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \qquad (2)$$

Substitute numerical values and evaluate t:

$$t = \sqrt{\frac{2(30.0\ \text{m})}{9.81\ \text{m/s}^2}} = \boxed{2.47\ \text{s}}$$

(*b*) Using a constant-acceleration equation, relate the horizontal position of the diver to his initial position, initial speed, acceleration, and time of fall:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$ and $v_{0x} = 0$,

$$x = \tfrac{1}{2}a_x t^2 \qquad (3)$$

When the diver hits the water, his x coordinate is d and the time is given by equation (2):

$$d = \tfrac{1}{2}a_x \frac{2h}{g} = \frac{a_x h}{g}$$

Substitute numerical values and evaluate d:

$$d = \frac{(5.00\ \text{m/s}^2)(30.0\ \text{m})}{9.81\ \text{m/s}^2} = \boxed{15.3\ \text{m}}$$

(*c*) Solving equation (3) for t^2 yields:

$$t^2 = \frac{2x}{a_x}$$

Substitute for t in equation (1) to obtain:

$$\boldsymbol{y} = \boldsymbol{h} - \tfrac{1}{2}\boldsymbol{g}\frac{2\boldsymbol{x}}{\boldsymbol{a}_x} = \boldsymbol{h} - \frac{\boldsymbol{g}}{\boldsymbol{a}_x}\boldsymbol{x}$$

or

$$\boldsymbol{y} = -\frac{\boldsymbol{g}}{\boldsymbol{a}_x}\boldsymbol{x} + \boldsymbol{h} \qquad (4)$$

Because equation (4) is of the form $\boldsymbol{y} = \boldsymbol{mx} + \boldsymbol{b}$, the diver's trajectory into the water is a straight line.

113 •• [**SSM**] A small steel ball is projected horizontally off the top of a long flight of stairs. The initial speed of the ball is 3.0 m/s. Each step is 0.18 m high and 0.30 m wide. Which step does the ball strike first?

Picture the Problem In the absence of air resistance; the steel ball will experience constant acceleration. Choose a coordinate system with its origin at the initial position of the ball, the x direction to the right, and the y direction downward. In this coordinate system $y_0 = 0$ and $a = g$. Letting (x, y) be a point on the path of the ball, we can use constant-acceleration equations to express both x and y as functions of time and, using the geometry of the staircase, find an expression for the time of flight of the ball. Knowing its time of flight, we can find its range and identify the step it strikes first.

The angle of the steps, with respect to the horizontal, is:

$$\boldsymbol{\theta} = \tan^{-1}\left(\frac{0.18\,\text{m}}{0.30\,\text{m}}\right) = 31.0°$$

Using a constant-acceleration equation, express the x coordinate of the steel ball in its flight:

$$\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{v}_{0x}\boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}_x\boldsymbol{t}^2$$

or, because $x_{0x} = 0$ and $a_x = 0$,

$$\boldsymbol{x} = \boldsymbol{v}_{0x}\boldsymbol{t} = \boldsymbol{v}_0\boldsymbol{t} \qquad (1)$$

Using a constant-acceleration equation, express the y coordinate of the steel ball in its flight:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = 0$, and $a_y = g$,

$$y = \tfrac{1}{2}gt^2$$

The equation of the dashed line in the figure is:

$$\frac{y}{x} = \tan\theta = \frac{gt}{2v_0} \Rightarrow t = \frac{2v_0}{g}\tan\theta$$

Substitute for t in equation (1) to find the x coordinate of the landing position:

$$\boldsymbol{x} = \boldsymbol{v}_0\boldsymbol{t} = \boldsymbol{v}_0\left(\frac{2\boldsymbol{v}_0}{\boldsymbol{g}}\tan\boldsymbol{\theta}\right) = \frac{2\boldsymbol{v}_0^2}{\boldsymbol{g}}\tan\boldsymbol{\theta}$$

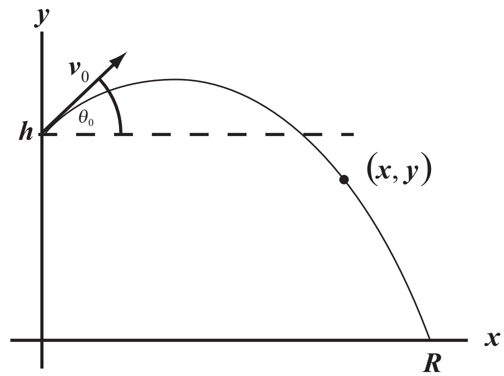
Substitute numerical values and evaluate x:

$$\boldsymbol{x} = \frac{2(3.0\,\text{m/s})^2}{9.81\,\text{m/s}^2}\tan 31° = 1.1\,\text{m}$$

The first step with $x > 1.1$ m is the 4th step.

114 •• Suppose you can throw a ball a maximum horizontal distance L when standing on level ground. How far can you throw it from the top of a building of height $h = x_0$ if you throw it at (*a*) 0º, (*b*) 30º, (*c*) 45º? Ignore any effects due to air resistance.

Picture the Problem Ignoring the influence of air resistance, the acceleration of the ball is constant once it has left your hand and we can use constant-acceleration equations to express the x and y coordinates of the ball. Elimination of t will yield an equation from which we can determine v_0. We can then use the y equation to express the time of flight of the ball and the x equation to express its range in terms of x_0, v_0, θ and the time of flight.



Use a constant-acceleration equation to express the ball's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$\boldsymbol{x} = (\boldsymbol{v}_0 \cos\boldsymbol{\theta}_0)\boldsymbol{t} \qquad (1)$$

Use a constant-acceleration equation to express the ball's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = x_0$, $v_{0y} = v_0\sin\theta_0$, and $a_y = -g$,

$$\boldsymbol{y} = \boldsymbol{x}_0 + (\boldsymbol{v}_0 \sin\boldsymbol{\theta}_0)\boldsymbol{t} - \tfrac{1}{2}\boldsymbol{g}\boldsymbol{t}^2 \qquad (2)$$

Eliminate t between equations (1) and (2) to obtain:

$$\boldsymbol{y} = \boldsymbol{x}_0 + (\tan\boldsymbol{\theta}_0)\boldsymbol{x} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{x}^2$$

For the throw while standing on level ground we have:

$$0 = (\tan\boldsymbol{\theta}_0)\boldsymbol{L} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2\cos^2\boldsymbol{\theta}_0}\boldsymbol{L}^2$$

and

$$\boldsymbol{L} = \frac{\boldsymbol{v}_0^2}{\boldsymbol{g}}\sin 2\boldsymbol{\theta}_0 = \frac{\boldsymbol{v}_0^2}{\boldsymbol{g}}\sin 2(45°) = \frac{\boldsymbol{v}_0^2}{\boldsymbol{g}}$$

Solving for v_0 yields:

$$\boldsymbol{v}_0 = \sqrt{\boldsymbol{g}\boldsymbol{L}}$$

At impact equation (2) becomes:

$$0 = x_0 + \left(\sqrt{gL}\sin\theta_0\right)t_{\text{flight}} - \tfrac{1}{2}gt_{\text{flight}}^2$$

Solve for the time of flight:

$$t_{\text{flight}} = \sqrt{\frac{L}{g}}\left(\sin\theta_0 + \sqrt{\sin^2\theta_0 + 2}\right)$$

Substitute in equation (1) to express the range of the ball when thrown from an elevation x_0 at an angle θ_0 with the horizontal:

$$R = \left(\sqrt{gL}\cos\theta_0\right)t_{\text{flight}} = \left(\sqrt{gL}\cos\theta_0\right)\sqrt{\frac{L}{g}}\left(\sin\theta_0 + \sqrt{\sin^2\theta_0 + 2}\right)$$
$$= L\cos\theta\left(\sin\theta_0 + \sqrt{\sin^2\theta_0 + 2}\right)$$

(*a*) Substitute $\theta = 0°$ and evaluate R:

$$R(0°) = \boxed{1.41L}$$

(*b*) Substitute $\theta = 30°$ and evaluate R:

$$R(30°) = \boxed{1.73L}$$

(*c*) Substitute $\theta = 45°$ and evaluate R:

$$R(45°) = \boxed{1.62L}$$

115 ••• Darlene is a stunt motorcyclist in a traveling circus. For the climax of her show, she takes off from the ramp at angle θ_0, clears a ditch of width L, and lands on an elevated ramp (height h) on the other side (Figure 3-42). (*a*) For a given height h, find the minimum necessary takeoff speed v_{min} needed to make the jump successfully. (*b*) What is v_{min} for $\theta_0 = 30°$, $L = 8.0$ m and $h = 4.0$ m? (*c*) Show that no matter what her takeoff speed, the maximum height of the platform is $h_{\text{max}} < L\tan\theta_0$. Interpret this result physically. (Neglect any effects due to air resistance and treat the rider and the bike as if they were a single particle.)

Picture the Problem Choose a coordinate system with its origin at the point where the bike becomes airborne and with the positive x direction to the right and the positive y direction upward. With this choice of coordinate system we can relate the x and y coordinates of the rider and bike (which we're treating as a single particle) using Equation 3-17.

(*a*) The path of the motorcycle is given by Equation 3-17:

$$y(x) = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

For the jump to be successful, $h < y(x)$. Solving for v_0 with $x = L$, we find that:

$$\boldsymbol{v}_{\min} > \boxed{\frac{\boldsymbol{L}}{\cos\boldsymbol{\theta}_0}\sqrt{\frac{\boldsymbol{g}}{2(\boldsymbol{L}\tan\boldsymbol{\theta}_0 - \boldsymbol{h})}}}$$

(*b*) Substitute numerical values and evaluate $v_{\min}$:

$$\boldsymbol{v}_{\min} > \frac{8.0\ \text{m}}{\cos 30°}\sqrt{\frac{9.81\ \text{m/s}^2}{2[(8.0\ \text{m})\tan 30° - 4.0\ \text{m}]}} = \boxed{26\ \text{m/s}}$$

(*c*) In order for our expression for $v_{\min}$ to be real valued; that is, to predict values for $v_{\min}$ that are physically meaningful, $L\tan\theta_0 - h_{\max} > 0$. Hence $h_{\max} < L\tan\theta_0$.

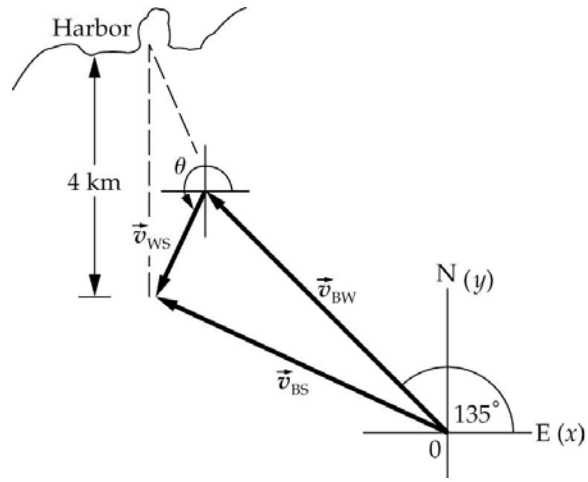
The interpretation is that the bike "falls away" from traveling on a straight-line path due to the free-fall acceleration downward. No matter what the initial speed of the bike, it must fall a little bit before reaching the other side of the ditch.

116 ••• A small boat is headed for a harbor 32 km directly northwest of its current position when it is suddenly engulfed in heavy fog. The captain maintains a compass bearing of northwest and a speed of 10 km/h relative to the water. The fog lifts 3.0 h later and the captain notes that he is now exactly 4.0 km south of the harbor. (*a*) What was the average velocity of the current during those 3.0 h? (*b*) In what direction should the boat have been heading to reach its destination along a straight course? (*c*) What would its travel time have been if it had followed a straight course?

Picture the Problem Let the origin be at the position of the boat when it was engulfed by the fog. Take the *x* and *y* directions to be east and north, respectively. Let $\vec{v}_{\text{BW}}$ be the velocity of the boat relative to the water, $\vec{v}_{\text{BS}}$ be the velocity of the boat relative to the shore, and $\vec{v}_{\text{WS}}$ be the velocity of the water with respect to the shore. Then

$$\vec{v}_{\text{BS}} = \vec{v}_{\text{BW}} + \vec{v}_{\text{WS}}.$$

θ is the angle of $\vec{v}_{\text{WS}}$ with respect to the *x* (east) direction.



(a) Find the position vector for the boat at $t = 3.0$ h:

$$\begin{aligned}\vec{r}_{\text{boat}} &= \{(32\,\text{km})(\cos 135°)\boldsymbol{t}\}\hat{\boldsymbol{i}} + \{(32\,\text{km})(\sin 135°)\boldsymbol{t} - 4.0\,\text{km}\}\hat{\boldsymbol{j}} \\ &= \{(-22.6\,\text{km})\boldsymbol{t}\}\hat{\boldsymbol{i}} + \{(22.6\,\text{km})\boldsymbol{t} - 4.0\,\text{km}\}\hat{\boldsymbol{j}}\end{aligned}$$

Find the coordinates of the boat at $t = 3.0$ h:

$$\boldsymbol{r}_x = [(10\,\text{km/h})\cos 135° + \boldsymbol{v}_{\text{WS}}\cos\boldsymbol{\theta}](3.0\,\text{h})$$

and

$$\boldsymbol{r}_y = [(10\,\text{km/h})\sin 135° + \boldsymbol{v}_{\text{WS}}\sin\boldsymbol{\theta}](3.0\,\text{h})$$

Simplify the expressions involving r_x and r_y and equate these simplified expressions to the x and y components of the position vector of the boat:

$$3\boldsymbol{v}_{\text{WS}}\cos\boldsymbol{\theta} = -1.41\,\text{km/h}$$

and

$$3\boldsymbol{v}_{\text{WS}}\sin\boldsymbol{\theta} = -2.586\,\text{km/h}$$

Divide the second of these equations by the first and solve for θ to obtain:

$$\theta = \tan^{-1}\left(\frac{-2.586\,\text{km}}{-1.41\,\text{km}}\right) = 61.4° \text{ or } 241.4°$$

Because the boat has drifted south, use $\theta = 241.4°$ to obtain:

$$\begin{aligned}\boldsymbol{v}_{\text{WS}} &= \frac{\boldsymbol{v}_x}{\cos\boldsymbol{\theta}} = \frac{-\dfrac{1.41\,\text{km/h}}{3}}{\cos(241.4°)} \\ &= \boxed{0.98\,\text{km/h at }\boldsymbol{\theta} = 241°}\end{aligned}$$

(*b*) Letting ϕ be the angle between east and the proper heading for the boat, express the components of the velocity of the boat with respect to the shore:

$$v_{\text{BS},x} = (10\text{ km/h})\cos\phi + (0.982\text{ km/h})\cos(241.3°) \qquad (1)$$

and

$$v_{\text{BS},y} = (10\text{ km/h})\sin\phi + (0.982\text{ km/h})\sin(241.3°)$$

For the boat to travel northwest: $v_{\text{BS},x} = -v_{\text{BS},y}$

Substitute the velocity components, square both sides of the equation, and simplify the expression to obtain the equations:

$$\sin\phi + \cos\phi = 0.133 \text{ and}$$
$$\sin^2\phi + \cos^2\phi + 2\sin\phi\cos\phi = 0.0177$$
or
$$1 + \sin(2\phi) = 0.0177$$

Solving for ϕ yields: $\phi = 129.6°$ or $140.4°$

Because the current pushes south, the boat must head more northerly than 135°: Using 129.6°, the correct heading is $\boxed{39.6° \text{ west of north}}$.

(*c*) Substitute for ϕ in equation (1) and evaluate $v_{\text{BS},x}$ to obtain: $v_{\text{BS},x} = -6.846\text{ km/h}$

From the diagram:

$$v_{\text{BS}} = \frac{v_{\text{BS},x}}{\cos 135°} = \frac{-6.846\text{ km/h}}{\cos 135°} = 9.681\text{ km/h}$$

To find the time to travel 32 km, divide the distance by the boat's actual speed:

$$t = \frac{32\text{ km}}{9.681\text{ km/h}} = \boxed{3.3\text{ h} \approx 3\text{ h } 18\text{ min}}$$

117 •• [**SSM**] Galileo showed that, if any effects due to air resistance are neglected, the ranges for projectiles (on a level field) whose angles of projection exceed or fall short of 45º by the same amount are equal. Prove Galileo's result.

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and the equation of a projectile for equal initial and final elevations, which was derived from the constant-acceleration equations, is applicable. We can use the equation giving the range of a projectile for equal initial and final elevations to evaluate the ranges of launches that exceed or fall short of 45° by the same amount.

Express the range of the projectile as a function of its initial speed and angle of launch:

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Letting $\theta_0 = 45° \pm \theta$ yields:

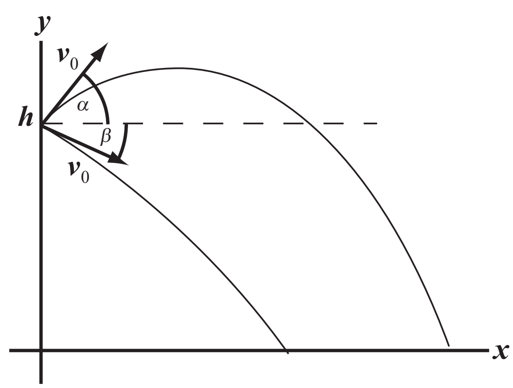
$$R = \frac{v_0^2}{g}\sin(90° \pm 2\theta) = \frac{v_0^2}{g}\cos(\pm 2\theta)$$

Because cos(–θ) = cos(+θ) (the cosine function is an *even* function):

$$\boxed{R(45° + \theta) = R(45° - \theta)}$$

118 •• Two balls are thrown with equal speeds from the top of a cliff of height h. One ball is thrown at an angle of α above the horizontal. The other ball is thrown at an angle of β below the horizontal. Show that each ball strikes the ground with the same speed, and find that speed in terms of h and the initial speed v_0. Neglect any effects due to air resistance.

Picture the Problem In the absence of air resistance, the acceleration of both balls is that due to gravity and the horizontal and vertical motions are independent of each other. Choose a coordinate system with the origin at the base of the cliff and the coordinate axes oriented as shown and use constant-acceleration equations to relate the x and y components of the ball's speed.



The speed of each ball on impact is:

$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2} \qquad (1)$$

Independently of whether a ball is thrown upward at the angle α or downward at β, the vertical motion is described by:

$$\boldsymbol{v}_y^2 = \boldsymbol{v}_{0y}^2 + 2\boldsymbol{a}_y\Delta\boldsymbol{y}$$
$$= \boldsymbol{v}_{0y}^2 - 2\boldsymbol{gh}$$

The horizontal component of the speed of each ball is:

$$\boldsymbol{v}_x = \boldsymbol{v}_{0x}$$

Substitute for v_x and v_y in equation (1) and simplify to obtain:

$$\boldsymbol{v} = \sqrt{\boldsymbol{v}_{0x}^2 + \boldsymbol{v}_{0y}^2 - 2\boldsymbol{gh}} = \boxed{\sqrt{\boldsymbol{v}_0^2 - 2\boldsymbol{gh}}}$$

119 •• In his car, a driver tosses an egg vertically from chest height so that the peak of its path is just below the ceiling of the passenger compartment, which is 65 cm above his release point. He catches the egg at the same height at which he released it. If you are a roadside observer, and measure the horizontal distance between catch and release points to be 19 m, (*a*) how fast is the car moving? (*b*) In your reference frame, at what angle above the horizontal was the egg thrown?

Picture the Problem For the first part of the question, we simply need to determine the time interval for the ball's "flight." In order to determine that, we need only consider the time required for an egg to fall 65 cm under the influence of gravity and then double that time. The path taken by the egg, according to the one observing on the side of the road, is a parabola – in fact, its path is indistinguishable from that of a projectile. We can obtain the apparent angle of launch by noting that the horizontal component of the egg's motion is simply given by the speed of the car, while the vertical component is given by the speed of the toss necessary to reach a height of 0.65 m.

(*a*) The speed of the car is given by:

$$\boldsymbol{v}_{\text{car}} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}_{\text{flight}}} \qquad (1)$$

where Δx is the distance the car moves during the time Δt_{flight} the egg is in flight.

The fall time, Δt_{fall}, for the egg to fall (from rest) and the distance Δh it falls are related according to:

$$\Delta \boldsymbol{h} = \tfrac{1}{2}\boldsymbol{g}(\Delta \boldsymbol{t}_{\text{fall}})^2 \Rightarrow \Delta \boldsymbol{t}_{\text{fall}} = \sqrt{\frac{2\Delta \boldsymbol{h}}{\text{g}}}$$

The time-of-flight is twice the egg's fall time:

$$\Delta t_{\text{flight}} = 2\Delta t_{\text{fall}}$$

Substitute for Δt_{fall} to obtain:

$$\Delta t_{\text{flight}} = 2\sqrt{\frac{2\Delta h}{\text{g}}}$$

Substituting for Δt_{flight} in equation (1) yields:

$$v_{\text{car}} = \frac{\Delta x}{2\sqrt{\frac{2\Delta h}{\text{g}}}} = \frac{\Delta x}{2}\sqrt{\frac{g}{2\Delta h}} \qquad (2)$$

Substitute numerical values and evaluate v_{car}:

$$v_{\text{car}} = \frac{19\text{ m}}{2}\sqrt{\frac{9.81\text{ m/s}^2}{2(0.65\text{ m})}} = \boxed{26\text{ m}}$$

(*b*) The angle θ_0 at which the egg was thrown is given by:

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{\boldsymbol{v}_{0y}}{\boldsymbol{v}_{0x}}\right) \qquad (3)$$

Use a constant-acceleration equation to relate the final upward speed of the egg to its initial speed and its maximum height h:

$$\boldsymbol{v}_y^2 = \boldsymbol{v}_{0y}^2 + 2\boldsymbol{a}_y\boldsymbol{h}$$

or, because $v_y = 0$ and $a_y = -g$,

$$0 = \boldsymbol{v}_{0y}^2 - 2\boldsymbol{g}\boldsymbol{h}$$

Solve for v_{0y} to obtain:

$$\boldsymbol{v}_{0y} = \sqrt{2\boldsymbol{g}\boldsymbol{h}}$$

v_{0x} is the speed of the car. Substituting in equation (2) yields:

$$\boldsymbol{v}_{\text{car}} = \boldsymbol{v}_{0x} = \frac{\Delta\boldsymbol{x}}{2}\sqrt{\frac{\boldsymbol{g}}{2\Delta\boldsymbol{h}}}$$

Substitute for v_{0x} in equation (3) to obtain:

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{\boldsymbol{v}_{0y}}{\frac{\Delta\boldsymbol{x}}{2}\sqrt{\frac{\boldsymbol{g}}{2\Delta\boldsymbol{h}}}}\right) = \tan^{-1}\left(\frac{4\Delta\boldsymbol{h}}{\Delta\boldsymbol{x}}\right)$$

Substitute numerical values and evaluate θ_0:

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{4(0.65\ \text{m})}{19\ \text{m}}\right) = \boxed{7.8^\circ}$$

120 ••• A straight line is drawn on the surface of a 16-cm-radius turntable from the center to the perimeter. A bug crawls along this line from the center outward as the turntable spins counterclockwise at a constant 45 rpm. Its walking speed relative to the turntable is a steady 3.5 cm/s. Let its initial heading be in the positive x-direction. As the bug reaches the edge of the turntable, and falls off, (still traveling at 3.5 cm/s radially, relative to the turntable, what are the x and y components of the velocity of the bug?

Picture the Problem In order to determine the total velocity vector of the bug we need to consider the two components that must be added together, namely, the bug's velocity relative to the turntable at its point of departure from the turntable, and the velocity of the point on the turntable from which the bug steps off relative to the ground.

Express the velocity of the bug relative to the ground $\vec{\boldsymbol{v}}_{\text{Bg}}$:

$$\vec{\boldsymbol{v}}_{\text{Bg}} = \vec{\boldsymbol{v}}_{\text{BT}} + \vec{\boldsymbol{v}}_{\text{Tg}} \qquad (1)$$

where $\vec{\boldsymbol{v}}_{\text{BT}}$ is the velocity of the bug relative to the turntable and $\vec{\boldsymbol{v}}_{\text{Tg}}$ is the velocity of the turntable relative to the ground.

The angle through which the turntable has rotated when the bug reaches its edge is given by:

$$\Delta\boldsymbol{\theta} = \left(\frac{360^\circ}{1\ \text{rev}}\right)\Delta\boldsymbol{t} = \left(\frac{360^\circ}{1\ \text{rev}}\right)\left(\frac{\boldsymbol{R}}{\boldsymbol{v}}\right)$$

where R is the radius of the turntable and v is the speed of the bug.

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\boldsymbol{\theta} = \left(\frac{360°}{1\,\text{rev}} \times \frac{45\,\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{16\,\text{cm}}{3.5\,\text{cm/s}}\right) = 1234°$$

The number of degrees beyond an integer multiple of revolutions is:

$$\boldsymbol{\theta} = \Delta\boldsymbol{\theta} - 360°\boldsymbol{n} = 1234° - 360°(3) = 154°$$

Now we can express the velocity of the bug relative to the turntable:

$$\vec{\boldsymbol{v}}_{\text{BT}} = (\boldsymbol{v}_{\text{B}} \cos\boldsymbol{\theta})\hat{\boldsymbol{i}} + (\boldsymbol{v}_{\text{B}} \sin\boldsymbol{\theta})\hat{\boldsymbol{j}}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{\text{BT}}$:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\text{BT}} &= ((3.5\,\text{cm/s})\cos 154°)\hat{\boldsymbol{i}} + ((3.5\,\text{cm/s})\sin 154°)\hat{\boldsymbol{j}} \\ &= (-3.146\,\text{cm/s})\hat{\boldsymbol{i}} + (1.534\,\text{cm/s})\hat{\boldsymbol{j}}\end{aligned}$$

The velocity of the point on the turntable from which the bug leaves the turntable is moving with a velocity vector pointed 90° ahead of $\vec{\boldsymbol{v}}_{\text{BT}}$; that is, tangent to the turntable:

$$\vec{\boldsymbol{v}}_{\text{Tg}} = \boldsymbol{v}_{\text{turntable}} \cos(\boldsymbol{\theta} + 90°)\hat{\boldsymbol{i}} + \boldsymbol{v}_{\text{turntable}} \sin(\boldsymbol{\theta} + 90°)\hat{\boldsymbol{j}}$$

The velocity of the turntable is given by:

$$\boldsymbol{v}_{\text{turntable}} = \frac{2\boldsymbol{\pi}\boldsymbol{R}}{\boldsymbol{T}}$$

where T is the period of the turntable's motion.

Substitute numerical values and evaluate $v_{\text{turntable}}$:

$$v_{\text{turntable}} = \frac{2\pi(16\,\text{cm/s})}{\dfrac{60\,\text{s}}{45\,\text{rev}}} = 75.40\,\text{cm/s}$$

Substitute in the expression for $\vec{\boldsymbol{v}}_{\text{Tg}}$ and simplify to obtain:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\text{Tg}} &= (75.40\,\text{cm/s})\cos(154° + 90°)\hat{\boldsymbol{i}} + (75.40\,\text{cm/s})\sin(154° + 90°)\hat{\boldsymbol{j}} \\ &= (-33.05\,\text{cm/s})\hat{\boldsymbol{i}} + (-67.77\,\text{cm/s})\hat{\boldsymbol{j}}\end{aligned}$$

Finally, substitute for $\vec{\boldsymbol{v}}_{\text{BT}}$ and $\vec{\boldsymbol{v}}_{\text{Tg}}$ in equation (1) and simplify:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\text{Bg}} &= (-3.146\,\text{cm/s})\hat{\boldsymbol{i}} + (1.534\,\text{cm/s})\hat{\boldsymbol{j}} + (-33.05\,\text{cm/s})\hat{\boldsymbol{i}} + (-67.77\,\text{cm/s})\hat{\boldsymbol{j}} \\ &= (-36\,\text{cm/s})\hat{\boldsymbol{i}} + (-66\,\text{cm/s})\hat{\boldsymbol{j}}\end{aligned}$$

The x and y components of the velocity of the bug are:

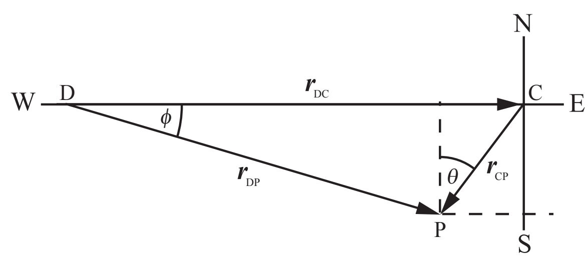
$$v_{x,\,\text{Bug}} = \boxed{-36\ \text{cm/s}}$$

and

$$v_{y,\,\text{Bug}} = \boxed{-66\ \text{cm/s}}$$

121 ••• On a windless day, a stunt pilot is flying his vintage World War I Sopwith Camel from Dubuque, Iowa, to Chicago, Illinois for an air show. Unfortunately, he is unaware that his plane's ancient magnetic compass has a serious problem in that what it records as ″north″ is in fact 16.5° east of true north. At one moment during his flight, the airport in Chicago notifies him that he is, in reality, 150 km due west of the airport. He then turns due east, according to his plane's compass, and flies for 45 minutes at 150 km/hr. At that point, he expected to see the airport and begin final descent. What is the plane's actual distance from Chicago and what should be his heading if he is to fly directly to Chicago?

Picture the Problem The error in the guidance computer aboard the airplane implies that when the pilot flies the plane at what he thinks is a heading of due East, he is in fact flying at a heading of 16.5° south of east. When he has flown at 150 km/hr for 15 minutes along that heading he is therefore somewhat west and south of Chicago. In the following diagram, D identifies the location of Dubuque, C the location of Chicago, and P the location of the plane. Choose a coordinate system in which the positive x direction is to the east and the positive y direction is to the north. Expressing the vector $\vec{r}_{\text{CP}}$ in unit vector notation will allow us to find the plane's actual distance from Chicago (the magnitude of $\vec{r}_{\text{CP}}$) and the heading of Chicago (the direction of $-\vec{r}_{\text{CP}}$.



The plane's true distance from Chicago is given by the magnitude of vector $\vec{r}_{\text{CP}}$:

$$r_{\text{CP}} = \sqrt{r_{\text{CP},x}^2 + r_{\text{CP},y}^2} \qquad (1)$$

The actual heading of Chicago relative to the plane's current position is given by:

$$\theta = \tan^{-1}\left(\frac{r_{\text{CP},x}}{r_{\text{CP},y}}\right) \qquad (2)$$

Express the relationship between the vectors $\vec{r}_{\text{DC}}$, $\vec{r}_{\text{CP}}$, and $\vec{r}_{\text{DP}}$:

$$\vec{r}_{\text{DC}} + \vec{r}_{\text{CP}} = \vec{r}_{\text{DP}}$$

or

$$\vec{r}_{\text{CP}} = \vec{r}_{\text{DP}} - \vec{r}_{\text{DC}} \qquad (3)$$

The vector $\vec{r}_{\rm DP}$ is given by:

$$\vec{r}_{\rm DP} = r_{\rm DP}\cos\phi\,\hat{i} - r_{\rm DP}\sin\phi\,\hat{j}$$

Substitute numerical values and simplify to obtain:

$$\begin{aligned}\vec{r}_{\rm DP} &= \left(150\,\frac{\rm km}{\rm h}\times 0.75\,{\rm h}\right)\cos 16.5°\,\hat{i} - \left(150\,\frac{\rm km}{\rm h}\times 0.75\,{\rm h}\right)\sin 16.5°\,\hat{j} \\ &= (107.9\,{\rm km})\hat{i} - (31.95\,{\rm km})\hat{j}\end{aligned}$$

The vector $\vec{r}_{\rm DC}$ is given by:

$$\vec{r}_{\rm DC} = (150\,{\rm km})\hat{i}$$

Substitute for $\vec{r}_{\rm DP}$ and $\vec{r}_{\rm DC}$ in equation (3) to obtain:

$$\begin{aligned}\vec{r}_{\rm CP} &= (107.9\,{\rm km})\hat{i} - (31.95\,{\rm km})\hat{j} - (150\,{\rm km})\hat{i} \\ &= (-42.10\,{\rm km})\hat{i} + (-31.95\,{\rm km})\hat{j}\end{aligned}$$

Substituting numerical values in equation (1) and evaluate $r_{\rm CP}$:

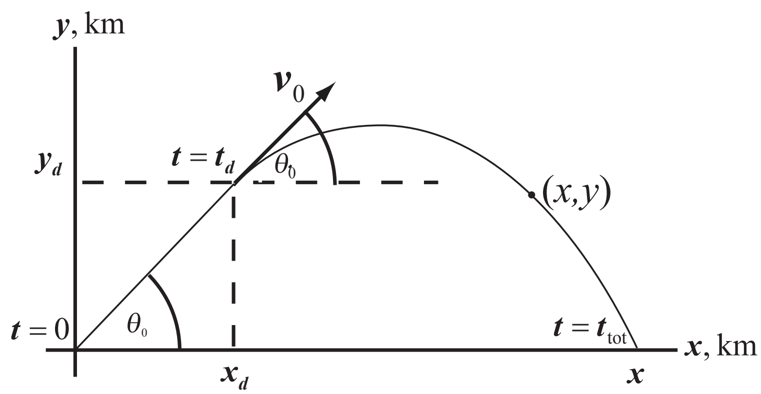
$$\begin{aligned}r_{\rm CP} &= \sqrt{(-42.10\,{\rm km})^2 + (-31.95\,{\rm km})^2} \\ &= \boxed{52.9\,{\rm km}}\end{aligned}$$

Substituting numerical values in equation (2) and evaluate θ:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{42.10\,{\rm km}}{31.95\,{\rm km}}\right) = 52.8° \\ &= \boxed{52.8°\text{ east of north}}\end{aligned}$$

122 ••• A cargo plane in flight lost a package because somebody forgot to close the rear cargo doors. You are on the team of safety experts trying to analyze what happened. From the point of takeoff, while climbing to altitude, the airplane traveled in a straight line at a constant speed of 275 mi/h at an angle of 37° above the horizontal. During this ascent, the package slid off the back ramp. You found the package in a field a distance of 7.5 km from the takeoff point. To complete the investigation you need to know exactly how long after takeoff the package slid off the back ramp of the plane. (Consider the sliding speed to be negligible.) Calculate the time at which the package fell off the back ramp. Ignore any effects due to air resistance.

Picture the Problem We can solve this problem by first considering what the projectile is doing while it is still in the cargo plane and then considering its motion as a projectile. The following diagram shows a convenient choice of coordinate system and the trajectory of the package both in the plane and when it has become a projectile. We can use constant-acceleration equations to describe both parts of its motion.



Express the x- and y-coordinates of the drop point:

$$x_d = v_{x,d} t_d \qquad (1)$$

and

$$y_d = v_{y,d} t_d \qquad (2)$$

Express the x- and y-coordinates of the package along its parabolic trajectory (after it has fallen out of the plane):

$$x(t) = x_d + v_{x,d}(t - t_d)$$

and

$$y(t) = y_d + v_{y,d}(t - t_d) - \tfrac{1}{2} g (t - t_d)^2$$

Substitute for x_d and y_d from equations (1) and (2) and simplify to obtain:

$$x(t) = v_{x,d} t_d + v_{x,d}(t - t_d) = v_{x,d} t$$

and

$$\begin{aligned} y(t) &= v_{y,d} t_d + v_{y,d}(t - t_d) - \tfrac{1}{2} g (t - t_d)^2 \\ &= v_{y,d} t - \tfrac{1}{2} g (t - t_d)^2 \end{aligned}$$

Express the x and y components of the plane's (and package's) velocity at the drop point:

$$v_{x,d} = v_{\text{plane}} \cos\theta_0$$

and

$$v_{y,d} = v_{\text{plane}} \sin\theta_0$$

$$v_{x,d} = \left(275\,\frac{\text{mi}}{\text{h}} \times 1609\,\frac{\text{m}}{\text{mi}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)\cos 37° = 98.16\ \text{m/s}$$

and

$$v_{y,d} = \left(275\,\frac{\text{mi}}{\text{h}} \times 1609\,\frac{\text{m}}{\text{mi}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)\sin 37° = 73.97\ \text{m/s}$$

At impact ($t = t_{\text{tot}}$):

$$x(t_{\text{tot}}) = x_\ell \text{ and } y(t_{\text{tot}}) = 0$$

The distance-to-impact is also given by:

$$x_\ell = v_{x,d} t_{\text{tot}} \Rightarrow t_{\text{tot}} = \frac{x_\ell}{v_{x,d}}$$

Substitute for t_{tot} in the $y(t)$ equation to obtain:

$$0 = v_{y,d} t_{tot} - \tfrac{1}{2} g (t_{tot} - t_d)^2$$

or, because $t_{tot} = x_\ell / v_{x,d}$,

$$0 = v_{y,d} \frac{x_\ell}{v_{x,d}} - \tfrac{1}{2} g \left(\frac{x_\ell}{v_{x,d}} - t_d \right)^2$$

Expand the binomial term and simplify the resulting expression to obtain the quadratic equation:

$$t_d^2 - \frac{2x_\ell}{v_{x,d}} t_d + \left[\frac{x_\ell^2}{v_{x,d}^2} - \frac{2x_\ell v_{y,d}}{g v_{x,d}} \right] = 0$$

Substitute numerical values to obtain:

$$t_d^2 - \frac{2(7.5\text{ km})}{98.16\text{ m/s}} t_d + \left[\frac{(7.5\text{ km})^2}{(98.16\text{ m/s})^2} - \frac{2(7.5\text{ km})(73.97\text{ m/s})}{(9.81\text{ m/s}^2)(98.16\text{ m/s})} \right] = 0$$

Simplification of this expression yields:

$$t_d^2 - (152.8\text{ s}) t_d + 4686\text{ s}^2 = 0$$

Use the quadratic formula or your graphing calculator to obtain:

$$t_d = 110\text{ s or } 42\text{ s}$$

Because 110 s gives too long a distance for the range of the package,

$$t_d = \boxed{42\text{ s}}$$