Chapter 2
Motion in One Dimension

Conceptual Problems

1 • What is the average velocity over the ″round trip″ of an object that is launched straight up from the ground and falls straight back down to the ground?

Determine the Concept The "average velocity" is being requested as opposed to "average speed."

The average velocity is defined as the change in position or displacement divided by the change in time.

$$v_{\text{av}} = \frac{\Delta y}{\Delta t}$$

The change in position for any "round trip" is zero by definition. So the ***average velocity*** for any round trip must also be zero.

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{y}}{\Delta \boldsymbol{t}} = \frac{0}{\Delta \boldsymbol{t}} = \boxed{0}$$

2 • An object thrown straight up falls back and is caught at the same place it is launched from. Its time of flight is T, its maximum height is H. Neglect air resistance. The correct expression for its average speed for the entire flight is (a) H/T, (b) 0, (c) $H/(2T)$, and (d) $2H/T$.

Determine the Concept The important concept here is that "average speed" is being requested as opposed to "average velocity".

Under all circumstances, including **constant acceleration**, the definition of the average speed is the ratio of the total distance traveled ($H + H$) to the elapsed time, in this case $2H/T$. $\boxed{(\boldsymbol{d})}$ is correct.

Remarks: **Because this motion involves a round trip, if the question asked for "average velocity," the answer would be zero.**

3 • Using the information in the previous question, what is its average speed just for the first half of the trip? What is its average velocity for the second half of the trip? (Answer in terms of H and T.)

Determine the Concept Under all circumstances, including ***constant acceleration***, the definition of the average speed is the ratio of the total distance traveled to the elapsed time. The average velocity, on the other hand, is the ratio of the displacement to the elapsed time.

The average speed for the first half of the trip is the height to which the object rises divided by one-half its time of flight:

$$v_{\text{av 1st half}} = \frac{H}{\frac{1}{2}T} = \boxed{\frac{2H}{T}}$$

The average velocity for the second half of the trip is the distance the object falls divided by one-half its time of flight:

$$vel_{\text{av 2nd half}} = \frac{-H}{\frac{1}{2}T} = \boxed{-\frac{2H}{T}}$$

Remarks: We could also say that the average velocity for the second half of the trip is –2*H*/*T*.

4 • Give an everyday example of one-dimensional motion where (*a*) the velocity is westward and the acceleration is eastward, and (*b*) the velocity is northward and the acceleration is northward.

Determine the Concept The important concept here is that $a = dv/dt$, where a is the acceleration and v is the velocity. Thus, the acceleration is positive if dv is positive; the acceleration is negative if dv is negative.

(*a*) An example of one-dimensional motion where the velocity is westward and acceleration is eastward is a car traveling westward and slowing down.

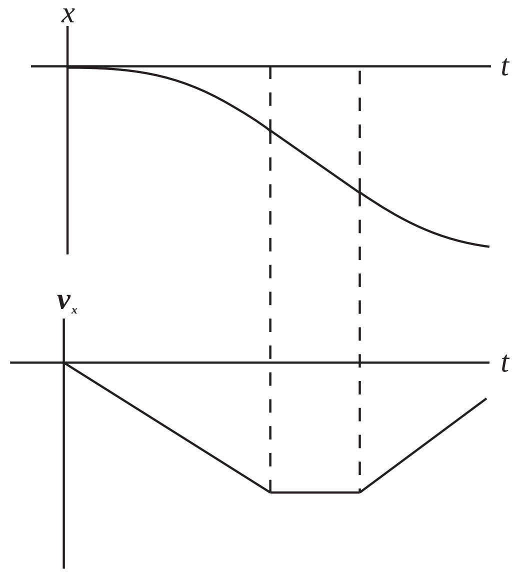
(*b*)An example of one-dimensional motion where the velocity is northward and the acceleration is northward is a car traveling northward and speeding up.

5 • **[SSM]** Stand in the center of a large room. Call the direction to your right ″positive,″ and the direction to your left ″negative.″ Walk across the room along a straight line, using a constant acceleration to quickly reach a steady speed along a straight line in the negative direction. After reaching this steady speed, keep your velocity negative but make your acceleration positive. (*a*) Describe how your speed varied as you walked. (*b*) Sketch a graph of x versus t for your motion. Assume you started at $x = 0$. (*c*) Directly under the graph of Part (*b*), sketch a graph of v_x versus t.

Determine the Concept The important concept is that when both the acceleration and the velocity are in the same direction, the speed increases. On the other hand, when the acceleration and the velocity are in opposite directions, the speed decreases.

(*a*) Your speed increased from zero, stayed constant for a while, and then decreased.

(*b*) A graph of your position as a function of time is shown to the right. Note that the slope starts out equal to zero, becomes more negative as the speed increases, remains constant while your speed is constant, and becomes less negative as your speed decreases.



(*c*) The graph of $v(t)$ consists of a straight line with negative slope (your acceleration is constant and negative) starting at (0,0), then a flat line for a while (your acceleration is zero), and finally an approximately straight line with a positive slope heading to $v = 0$.

6 • True/false: The displacement *always* equals the product of the average velocity and the time interval. Explain your choice.

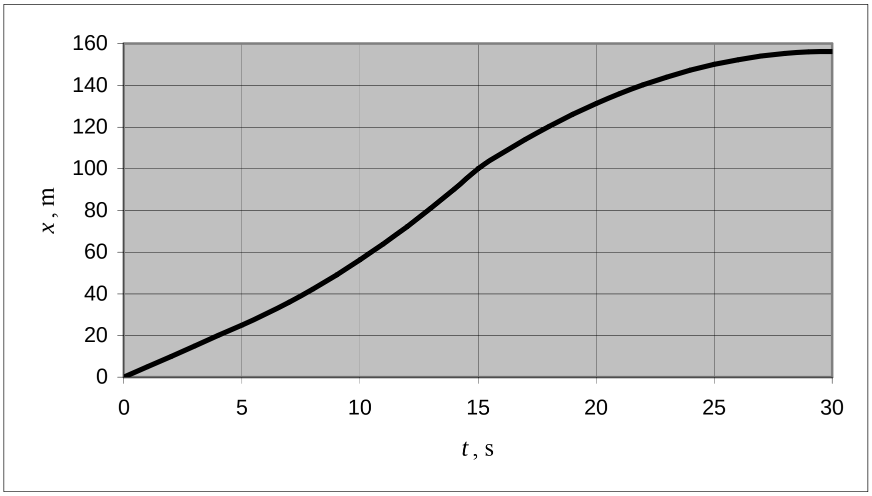
Determine the Concept True. We can use the definition of average velocity to express the displacement Δx as $\Delta x = v_{av}\Delta t$. Note that, if the acceleration is constant, the average velocity is also given by $v_{av} = (v_i + v_f)/2$.

7 • Is the statement ″for an object's velocity to remain constant, its acceleration must remain zero″ true or false? Explain your choice.

Determine the Concept True. Acceleration is the slope of the velocity versus time curve, $a = dv/dt$; while velocity is the slope of the position versus time curve, $v = dx/dt$. The speed of an object is the magnitude of its velocity. Zero acceleration implies that the velocity is constant. If the velocity is constant (including zero), the speed must also be constant.

8 • Draw careful graphs of the position and velocity and acceleration over the time period $0 \lesssim t \lesssim 30$ s for a cart that, in succession, has the following motion. The cart is moving at the constant speed of 5.0 m/s in the $+x$ direction. It passes by the origin at $t = 0.0$ s. It continues on at 5.0 m/s for 5.0 s, after which it gains speed at the constant rate of 0.50 m/s each second for 10.0 s. After gaining speed for 10.0 s, the cart loses speed at the constant rate of 0.50 m/s for the next 15.0 s.

Determine the Concept Velocity is the slope of the position versus time curve and acceleration is the slope of the velocity versus time curve. The following graphs were plotted using a spreadsheet program.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x , m
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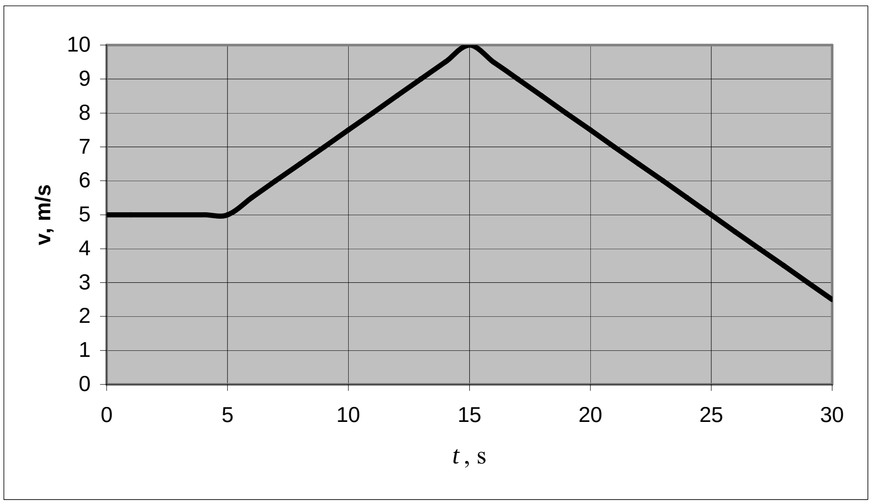
t , s


10
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8
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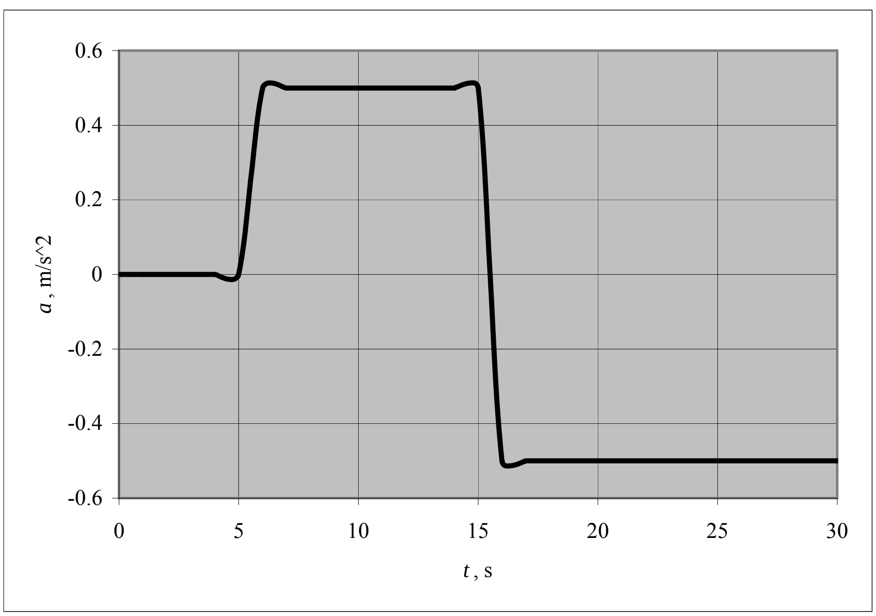

0.6
0.4
0.2
0
-0.2
-0.4
-0.6
a , m/s^2
0
5
10
15
20
25
30
t , s

9 • True/false; Average velocity *always* equals one-half the sum of the initial and final velocities. Explain your choice.

Determine the Concept False. The average velocity is defined (for any acceleration) as the change in position (the displacement) divided by the change in time $v_{av} = \Delta x/\Delta t$. It is always valid. If the acceleration remains constant the average velocity is also given by

$$v_{av} = \frac{v_i + v_f}{2}$$

Consider an engine piston moving up and down as an example of non-constant velocity. For one complete cycle, $v_f = v_i$ and $x_i = x_f$ so $v_{av} = \Delta x/\Delta t$ is zero. The formula involving the mean of v_f and v_i cannot be applied because the acceleration is not constant, and yields an incorrect nonzero value of v_i.

10 • Identical twin brothers standing on a horizontal bridge each throw a rock straight down into the water below. They throw rocks at exactly the same time, but one hits the water before the other. How can this be? Explain what they did differently. Ignore any effects due to air resistance.

Determine the Concept This can occur if the rocks have different initial speeds. Ignoring air resistance, the acceleration is constant. Choose a coordinate system in which the origin is at the point of release and upward is the positive direction. From the constant-acceleration equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ we see that the only way two objects can have the same acceleration ($-g$ in this case) and cover the same distance, $\Delta y = y - y_0$, in different times would be if the initial velocities of the two rocks were different. Actually, the answer would be the same whether or not the acceleration is constant. It is just easier to see for the special case of constant acceleration.

11 •• **[SSM]** Dr. Josiah S. Carberry stands at the top of the Sears Tower in Chicago. Wanting to emulate Galileo, and ignoring the safety of the pedestrians below, he drops a bowling ball from the top of the tower. One second later, he drops a second bowling ball. While the balls are in the air, does their separation (*a*) increase over time, (*b*) decrease, (*c*) stay the same? Ignore any effects due to air resistance.

Determine the Concept Neglecting air resistance, the balls are in free fall, each with the same free-fall acceleration, which is a constant.

At the time the second ball is released, the first ball is already moving. Thus, during any time interval their velocities will increase by exactly the same amount. What can be said about the speeds of the two balls? *The first ball will always be moving faster than the second ball.* This being the case, what happens to the separation of the two balls while they are both falling? *Their separation increases*. $\boxed{(\boldsymbol{a})}$ is correct.

12 •• Which of the position-versus-time curves in Figure 2-28 best shows the motion of an object (*a*) with positive acceleration, (*b*) with constant positive velocity, (*c*) that is always at rest, and (*d*) with negative acceleration? (There may be more than one correct answer for each part of the problem.)

Determine the Concept The slope of an $x(t)$ curve at any point in time represents the speed at that instant. The way the slope changes as time increases gives the sign of the acceleration. If the slope becomes less negative or more positive as time increases (as you move to the right on the time axis), then the acceleration is positive. If the slope becomes less positive or more negative, then the acceleration is negative. The slope of the slope of an $x(t)$ curve at any point in time represents the acceleration at that instant.

(*a*) The correct answer is $\boxed{(\boldsymbol{d})}$. The slope of curve (*d*) is positive and increasing. Therefore the velocity and acceleration are positive. We need more information to conclude that *a* is constant.

(*b*) The correct answer is $\boxed{(\boldsymbol{b})}$. The slope of curve (*b*) is positive and constant. Therefore the velocity is positive and constant.

(*c*) The correct answer is $\boxed{(\boldsymbol{e})}$. The slope of curve (*e*) is zero. Therefore, the velocity and acceleration are zero.

(*d*) The correct answers are $\boxed{(\boldsymbol{a})\text{ and }(\boldsymbol{c})}$. The slope of curve (*a*) is negative and becomes more negative as time increases. Therefore the velocity is negative and the acceleration is negative. The slope of curve (*c*) is positive and decreasing. Therefore the velocity is positive and the acceleration is negative.

13 •• **[SSM]** Which of the velocity-versus-time curves in figure 2-29 best describes the motion of an object (*a*) with constant positive acceleration, (*b*) with positive acceleration that is decreasing with time, (*c*) with positive acceleration that is increasing with time, and (*d*) with no acceleration? (There may be more than one correct answer for each part of the problem.)

Determine the Concept The slope of a $v(t)$ curve at any point in time represents the acceleration at that instant.

(*a*) The correct answer is $\boxed{(\boldsymbol{b})}$. The slope of curve (*b*) is constant and positive. Therefore the acceleration is constant and positive.

(*b*) The correct answer is $\boxed{(\boldsymbol{c})}$. The slope of curve (*c*) is positive and decreasing with time. Therefore the acceleration is positive and decreasing with time.

(*c*) The correct answer is $\boxed{(\boldsymbol{d})}$. The slope of curve (*d*) is positive and increasing with time. Therefore the acceleration is positive and increasing with time.

(*d*) The correct answer is $\boxed{(\boldsymbol{e})}$. The slope of curve (*e*) is zero. Therefore the velocity is constant and the acceleration is zero.

14 •• The diagram in Figure 2-30 tracks the location of an object moving in a straight line along the *x* axis. Assume that the object is at the origin at $t = 0$. Of the five times shown, which time (or times) represents when the object is (*a*) farthest from the origin, (*b*) at rest for an instant, (*c*) moving away from the origin?

Determine the Concept Because this graph is of distance-versus-time we can use its displacement from the time axis to draw conclusions about how far the object is from the origin. We can also use the instantaneous slope of the graph to decide whether the object is at rest and whether it is moving toward or away from the origin.

(*a*) The correct answer is $\boxed{\text{B}.}$ Because the object's initial position is at $x = 0$, point B represents the instant that the object is farthest from $x = 0$.

(*b*) The correct answers are $\boxed{\text{B and D.}}$ Because the slope of the graph is zero at points B and D, the velocity of the object is zero and it is at rest at these points.

(*c*) The correct answer is $\boxed{\text{A.}}$ Because the slope of the graph is positive at point A, the velocity of the object is positive and it is moving away from the origin.

15 •• **[SSM]** An object moves along a straight line. Its position versus time graph is shown in Figure 2-30. At which time or times is its (*a*) speed at a minimum, (*b*) acceleration positive, and (*c*) velocity negative?

Determine the Concept Because this graph is of distance-versus-time we can use its instantaneous slope to describe the object's speed, velocity, and acceleration.

(*a*) The minimum speed is zero at $\boxed{\text{B, D, and E}.}$ In the one-dimensional motion shown in the figure, the velocity is a minimum when the slope of a position-versus-time plot goes to zero (i.e., the curve becomes horizontal). At these points, the slope of the position-versus-time curve is zero; therefore, the speed is zero.

(*b*) The acceleration is positive at points $\boxed{\text{A and D.}}$ Because the slope of the graph is increasing at these points, the velocity of the object is increasing and its acceleration is positive.

(c) The velocity is negative at point $\boxed{\text{C.}}$ Because the slope of the graph is negative at point C, the velocity of the object is negative.

16 •• For each of the four graphs of x versus t in Figure 2-31 answer the following questions. (a) Is the velocity at time t_2 greater than, less than, or equal to the velocity at time t_1? (b) Is the speed at time t_2 greater than, less than, or equal to the speed at time t_1?

Determine the Concept In one-dimensional motion, the velocity is the slope of a position-versus-time plot and can be either positive or negative. On the other hand, the speed is the magnitude of the velocity and can only be positive. We'll use v to denote velocity and the word "speed" for how fast the object is moving.

(a)
curve a: $v(t_2) < v(t_1)$
curve b: $v(t_2) = v(t_1)$
curve c: $v(t_2) > v(t_1)$
curve d: $v(t_2) < v(t_1)$

(b)
curve a: $\text{speed}(t_2) < \text{speed}(t_1)$
curve b: $\text{speed}(t_2) = \text{speed}(t_1)$
curve c: $\text{speed}(t_2) < \text{speed}(t_1)$
curve d: $\text{speed}(t_2) > \text{speed}(t_1)$

17 •• True/false: Explain your reasoning for each answer. If the answer is true, give an example.

(a) If the acceleration of an object is always zero, then it cannot be moving.
(b) If the acceleration of an object is always zero, then its x-versus-t curve must be a straight line.
(c) If the acceleration of an object is nonzero at an instant, it may be momentarily at rest at that instant.

(a) False. An object moving in a straight line with constant speed has zero acceleration.

(b) True. If the acceleration of the object is zero, then its speed must be constant. The graph of x-versus-t for an object moving with constant speed is a straight line.

(c) True. A ball thrown upward is momentarily at rest when it is at the top of its trajectory. Its acceleration, however, is non-zero at this instant. Its value is the same as it was just before it came to rest and after it has started its descent.

18 •• A hard-thrown tennis ball is moving horizontally when it bangs into a vertical concrete wall at perpendicular incidence. The ball rebounds straight back off the wall. Neglect any effects due to gravity for the small time interval described here. Assume that towards the wall is the $+x$ direction. What are the directions of its velocity and acceleration (a) just before hitting the wall, (b) at maximum impact, and (c) just after leaving the wall.

Determine the Concept The tennis ball will be moving with constant velocity immediately before and after its collision with the concrete wall. It will be accelerated during the duration of its collision with the wall.

(*a*) Just before hitting the wall the velocity of the ball is to the right and, because its velocity is constant, its acceleration is zero.

(*b*) At maximum impact, the ball is reversing direction and its velocity is zero. Its acceleration is to the left.

(*c*) Just after leaving the wall, the velocity of the ball is to the left and constant. Because its velocity is constant, its acceleration is zero.

19 •• **[SSM]** A ball is thrown straight up. Neglect any effects due to air resistance. (*a*) What is the velocity of the ball at the top of its flight? (*b*) What is its acceleration at that point? (*c*) What is different about the velocity and acceleration at the top of the flight if instead the ball impacts a horizontal ceiling very hard and then returns.

Determine the Concept In the absence of air resistance, the ball will experience a constant acceleration. In the graph that follows, a coordinate system was chosen in which the origin is at the point of release and the upward direction is positive. The graph shows the velocity of a ball that has been thrown straight upward with an initial speed of 30 m/s as a function of time.

(*a*) $\boldsymbol{v}_{\text{top of flight}} = \boxed{0}$

(*b*) Note that the acceleration of the ball is the same at every point of its trajectory, including the point at which $v = 0$ (at the top of its flight).
Hence $\boldsymbol{a}_{\text{top of flight}} = \boxed{-\boldsymbol{g}}$

(c) If the ball impacts a horizontal ceiling very hard and then returns, its velocity at the top of its flight is still zero and its acceleration is still downward but greater than g in magnitude.

20 •• An object that is launched straight up from the ground, reaches a maximum height H, and falls straight back down to the ground, hitting it T seconds after launch. Neglect any effects due to air resistance. (*a*) Express the average speed for the entire trip as a function of H and T. (*b*) Express the average speed for the same interval of time as a function of the initial launch speed v_0.

Picture the Problem The average speed is being requested as opposed to average velocity. We can use the definition of average speed as distance traveled divided by the elapsed time and expression for the average speed of an object when it is experiencing constant acceleration to express v_{av} in terms of v_0.

(*a*) The average speed is defined as the total distance traveled divided by the change in time:

$$v_{\text{av}} = \frac{\text{total distance traveled}}{\text{total time}}$$

Substitute for the total distance traveled and the total time and simplify to obtain:

$$v_{\text{av}} = \frac{H+H}{T} = \boxed{\frac{2H}{T}}$$

(*b*) Express the average speed for the upward flight of the object:

$$v_{\text{av,up}} = \frac{v_0 + 0}{2} = \frac{H}{\frac{1}{2}T} \Rightarrow H = \tfrac{1}{4}v_0 T$$

Express the average speed for the downward flight of the object:

$$v_{\text{av,down}} = \frac{0 + v_0}{2} = \frac{H}{\frac{1}{2}T} \Rightarrow H = \tfrac{1}{4}v_0 T$$

Substitute in our expression for v_{av} in (*a*) and simplify to obtain:

$$v_{\text{av}} = \frac{\frac{1}{4}v_0 T + \frac{1}{4}v_0 T}{T} = \boxed{\tfrac{1}{2}v_0}$$

Because $v_0 \neq 0$, the average speed is not zero.

Remarks: 1) Because this motion involves a roundtrip, if the question asked for ″average velocity″, the answer would be zero. 2) Another easy way to obtain this result is take the absolute value of the velocity of the object to obtain a graph of its speed as a function of time. A simple geometric argument leads to the result we obtained above.

21 •• A small lead ball is thrown directly upward. Neglect any effects due to air resistance. True or false: (*a*) The magnitude of its acceleration decreases on the way up. (*b*) The direction of its acceleration on its way down is opposite to the direction of its acceleration on its way up. (*c*) The direction of its velocity on its way down is opposite to the direction of its velocity on its way up.

Determine the Concept Acceleration is the slope of the velocity-versus-time curve, $a = dv/dt$, while velocity is the slope of the position-versus-time curve, $v = dx/dt$.

(*a*) False. The velocity of the ball decreases at a steady rate. This means that the acceleration of the ball is constant.

(*b*) False. Again, zero acceleration implies that the velocity remains constant. This means that the *x*-versus-*t* curve has a constant slope (i.e., a straight line). Note: This does not necessarily mean a zero-slope line.

(*c*) True. On the way up the velocity vector points upward and on the way down it points downward.

22 •• At $t = 0$, object A is dropped from the roof of a building. At the same instant, object B is dropped from a window 10 m below the roof. Air resistance is negligible. During the descent of B to the ground, the distance between the two objects (*a*) is proportional to *t*, (*b*) is proportional to t^2, (*c*) decreases, (*d*) remains 10 m throughout.

Determine the Concept Both objects experience the same constant acceleration. Choose a coordinate system in which downward is the positive direction and use a constant-acceleration equation to express the position of each object as a function of time.

Using constant-acceleration equations, express the positions of both objects as functions of time:

$$x_A = x_{0,A} + v_0 t + \tfrac{1}{2} g t^2$$

and

$$x_B = x_{0,B} + v_0 t + \tfrac{1}{2} g t^2$$

where $v_0 = 0$.

Express the separation of the two objects by evaluating $x_B - x_A$:

$$x_B - x_A = x_{0,B} - x_{0.A} = 10\,\text{m}$$

$\boxed{(\boldsymbol{d})}$ is correct.

23 •• You are driving a Porsche that accelerates uniformly from 80.5 km/h (50 mi/h) at $t = 0.00$ to 113 km/h (70 mi/h) at $t = 9.00$ s. (*a*) Which graph in Figure 2-32 best describes the velocity of your car? (*b*) Sketch a position-versus-time graph showing the location of your car during these nine seconds, assuming we let its position *x* be zero at $t = 0$.

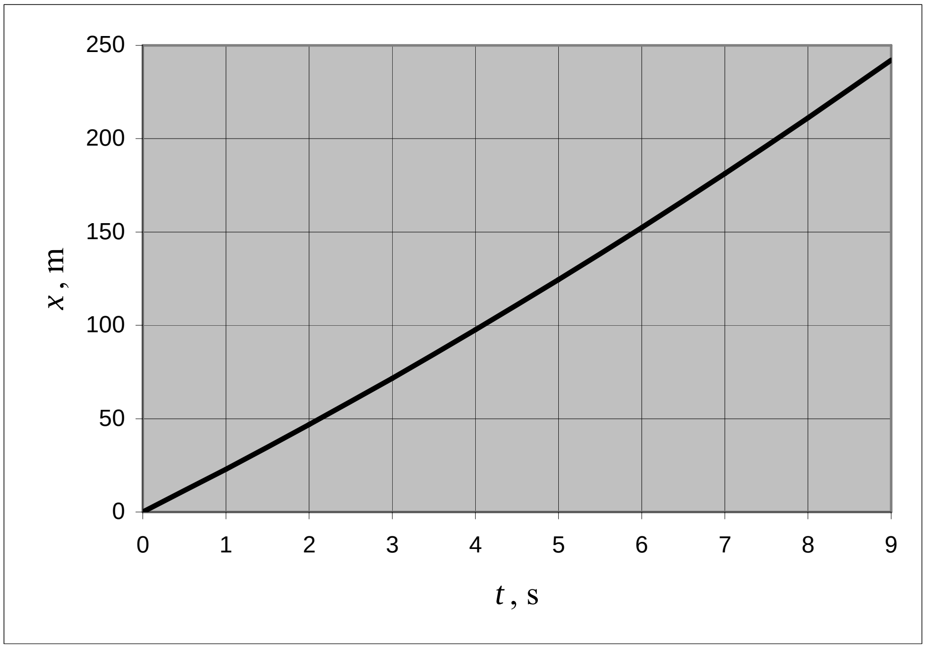
Determine the Concept Because the Porsche accelerates uniformly, we need to look for a graph that represents constant acceleration.

(*a*) Because the Porsche has a constant acceleration that is positive (the velocity is increasing), we must look for a velocity-versus-time curve with a positive constant slope and a nonzero intercept. Such a graph is shown in $\boxed{(\boldsymbol{c}).}$

(*b*) Use the data given in the problem statement to determine that the acceleration of the Porsche is 1.00 m/s^2 and that its initial speed is 22.4 m/s. The equation describing the position of the car as a function of time is

$$\boldsymbol{x} = \left(22.4\ \text{m/s}^2\right)\boldsymbol{t} + \tfrac{1}{2}\left(1.00\ \text{m/s}^2\right)\boldsymbol{t}^2.$$

The following graph of this equation was plotted using a spreadsheet program.



24 •• A small heavy object is dropped from rest and falls a distance D in a time T. After it has fallen for a time $2T$, what will be its (a) fall distance from its initial location in terms of D, (b) its speed in terms of D and t, and (c) its acceleration? (Neglect air resistance.)

Picture the Problem In the absence of air resistance, the object experiences constant acceleration. Choose a coordinate system in which the downward direction is positive and use the constant-acceleration equation to describe its motion.

(a) Relate the distance D that the object, released from rest, falls in time t:

$$x(t) = D = \tfrac{1}{2}gt^2 \qquad (1)$$

Evaluate $x(2t)$ to obtain:

$$x(2t) = \tfrac{1}{2}g(2t)^2 = 2gt^2 \qquad (2)$$

Dividing equation (2) by equation (1) and simplifying yields:

$$\frac{x(2t)}{D} = \frac{2gt^2}{\frac{1}{2}gt^2} = 4 \Rightarrow x(2t) = \boxed{4D}$$

(b) Express the speed of the object as a function of time:

$$v = v_0 + gt$$

or, because $v_0 = 0$,

$$v = gt \qquad (3)$$

Solving equation (1) for g yields:

$$g = \frac{2x}{t^2}$$

Substitute for g in equation (3) to obtain:

$$v = \frac{2x}{t^2}t = \frac{2x}{t} \qquad (4)$$

Evaluating equation (4) at time $2t$ and simplifying yields:

$$v(2t) = \frac{2x(2t)}{2t} = \boxed{\frac{4D}{t}}$$

(c) The acceleration of the object is independent of time (that is, it is constant) and is equal to $\boxed{g.}$

25 •• In a race, at an instant when two horses are running right next to each other and in the same direction (the $+x$ direction). Horse A's instantaneous velocity and acceleration are +10 m/s and +2.0 m/s^2 respectively, and horse B's instantaneous velocity and acceleration are +12 m/s and –1.0 m/s^2 respectively. Which horse is passing the other at this instant? Explain.

Determine the Concept The information about the horses' accelerations is irrelevant to the determination of which horse is passing the other at this instant. The horse running with the greater instantaneous velocity will by passing the slower horse. Hence $\boxed{\text{B is passing A.}}$ The accelerations are relevant to the determination of which horse will be in the lead at some later time.

26 •• True or false: (a) The equation $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ is always valid for all particle motion in one dimension. (b) If the velocity at a given instant is zero, the acceleration at that instant must also be zero. (c) The equation $\Delta x = v_{av}\Delta t$ holds for all particle motion in one dimension.

Determine the Concept As long as the acceleration remains constant the following constant-acceleration equations hold. If the acceleration is not constant, they do not, in general, give correct results except by coincidence.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2a\Delta x \qquad v_{av} = \frac{v_i + v_f}{2}$$

(a) False. This statement is true if and only if the acceleration is constant.

(b) False. Consider a rock thrown straight up into the air. At the "top" of its flight, the velocity is zero but it is changing (otherwise the velocity would remain zero and the rock would hover); therefore the acceleration is not zero.

(c) True. The definition of average velocity, $v_{av} = \Delta x/\Delta t$, requires that this always be true.

27 •• If an object is moving in a straight line at constant acceleration, its instantaneous velocity halfway through any time interval is (*a*) greater than its average velocity, (*b*) less than its average velocity, (*c*) equal to its average velocity, (*d*) half its average velocity, (*e*) twice its average velocity.

Determine the Concept Because the acceleration of the object is constant, the constant-acceleration equations can be used to describe its motion. The special expression for average velocity for constant acceleration is $v_{av} = \frac{v_i + v_f}{2}$. $\boxed{(c)}$ is correct.

28 •• A turtle, seeing his owner put some fresh lettuce on the opposite side of his terrarium, begins to accelerate (at a constant rate) from rest at time $t = 0$, heading directly toward the food. Let t_1 be the time at which the turtle has covered half the distance to his lunch. Derive an expression for the ratio of t_2 to t_1, where t_2 is the time at which the turtle reaches the lettuce.

Picture the Problem We are asked, essentially, to determine the time t_2, at which a displacement, Δx, is twice what it was at an earlier time, t_1. The turtle is crawling with constant acceleration so we can use the constant-acceleration equation $\Delta x = v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$ to describe the turtle's displacement as a function of time.

Express the displacement Δt of the turtle at the end of a time interval Δt:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2$$

For the two time intervals:

$$\Delta x_1 = \tfrac{1}{2}a_x t_1^2 \text{ and } \Delta x_2 = \tfrac{1}{2}a_x t_2^2$$

Express the ratio of Δx_2 to Δx_1 to obtain:

$$\frac{\Delta x_2}{\Delta x_1} = \frac{\frac{1}{2}a_x t_2^2}{\frac{1}{2}a_x t_1^2} = \frac{t_2^2}{t_1^2} \quad (1)$$

We're given that:

$$\frac{\Delta x_2}{\Delta x_1} = 2$$

Substitute in equation (1) and simplify to obtain:

$$\frac{t_2^2}{t_1^2} = 2 \Rightarrow \frac{t_2}{t_1} = \boxed{\sqrt{2}}$$

29 •• **[SSM]** The positions of two cars in parallel lanes of a straight stretch of highway are plotted as functions of time in the Figure 2-33.Take positive values of *x* as being to the right of the origin. Qualitatively answer the following: (*a*) Are the two cars ever side by side? If so, indicate that time (those times) on the axis. (*b*) Are they always traveling in the same direction, or are they moving in opposite directions for some of the time? If so, when? (*c*) Are they ever traveling at the same velocity? If so, when? (*d*) When are the two cars the farthest apart?

Determine the Concept Given the positions of the two cars as a function of time, we can use the intersections of the curves and their slopes to answer these questions.

(*a*) Yes, when the graphs intersect
(*b*) Yes, when the slopes of the curves have opposite signs
(*c*) Yes, when the curves have the same slope,
(*d*) The two cars are farthest apart at the instant the two curves are farthest apart in the *x* direction.

30 •• A car driving at constant velocity passes the origin at time $t = 0$. At the instant the car passes, a truck, at rest at the origin, begins to accelerate uniformly from rest. Figure 2-34 shows a qualitative plot of the velocities of truck and car as functions of time. Compare their displacements (from the origin), velocities, and accelerations at the instant that their curves intersect.

Determine the Concept The graph is a plot of velocity versus time. Thus, where the two curves cross, the truck and car are, at that instant, moving with equal velocities. The slope of a velocity versus time curve is equal to the instantaneous acceleration – thus, since the curve that represents the truck's velocity has a positive slope, and the car's curve has zero slope, the truck is accelerating at a higher rate than the car. Finally, the displacements of the two cars are determined by calculating the areas under the curves. In this instance, the curve representing the truck's velocity as a function of time encloses a triangular area that is exactly half that of the curve representing the car's velocity. Thus, at the instant represented by point P, the truck has gone half as far as has the car.

31 •• Josie is out for a morning jog, and during the course of her run on a straight track, has a velocity that depends upon time as shown in Figure 2-35. That is, she begins at rest, and ends at rest, peaking at a maximum velocity v_{max} at an arbitrary time t_{max}. A second runner, Reginald, runs throughout the time interval $t = 0$ to $t = t_{\mathrm{f}}$ at a constant speed v_{R}, so that each has the same displacement during the time interval. Note: t_{f} is NOT twice t_{max}, but represents an arbitrary time. What is the relationship between v_{R} and v_{max}?

Determine the Concept In this problem we are presented with curves representing the velocity as a function of time for both runners. The area under each curve represents the displacement for each runner and we are told that Josie and Reginald each have the same displacement during the time interval of length t_f. Since this is the case, we can find the relationship between v_{R} and v_{max} by equating the areas under the two curves.

Express the condition on the displacement of the two runners: $\Delta x_R = \Delta x_J$ (1)

Reginald runs at a constant velocity v for the whole of the time interval. Express his displacement Δx_R: $\Delta x_R = v_R t_f$

Josie has a different velocity profile, one which results in a triangle of height v_{max} and length t_f. Express her displacement Δx_J: $\Delta x_J = \frac{1}{2} v_{max} t_f$

Substitute for Δx_R and Δx_J in equation (1) and simplify to obtain: $v_R t_f = \frac{1}{2} v_{max} t_f \Rightarrow v_R = \boxed{\frac{1}{2} v_{max}}$

32 •• Which graph (or graphs), if any, of v versus t in Figure 2-36 best describes the motion of a particle with (*a*) positive velocity and increasing speed, (*b*) positive velocity and zero acceleration, (*c*) constant non-zero acceleration, and (*d*) a speed decrease?

Determine the Concept The velocity of the particle is positive if the curve is above the $v = 0$ line (the t axis), and the acceleration is positive if the curve has a positive slope. The speed of the particle is the magnitude of its velocity.

(*a*) Graph $\boxed{(c)}$ describes the motion of a particle with positive velocity and increasing speed because $v(t)$ is above the t axis and has a positive slope.

(*b*) Graph $\boxed{(a)}$ describes the motion of a particle with positive velocity and zero acceleration because $v(t)$ is above the t axis and its slope is zero.

(*c*) Graphs $\boxed{(c), (d) \text{ and } (e)}$ describe the motion of a particle with constant non-zero acceleration because $v(t)$ is linear and has a non-zero slope.

(*d*) Graph $\boxed{(e)}$ describes the motion of a particle with a speed decrease because it shows the speed of the particle decreasing with time.

33 •• Which graph (or graphs), if any, of v_x versus t in Figure 2-36 best describes the motion of a particle with (*a*) negative velocity and increasing speed, (*b*) negative velocity and zero acceleration, (*c*) variable acceleration, and (*d*) increasing speed?

Determine the Concept The velocity of the particle is positive if the curve is above the $v = 0$ line (the t axis), and the acceleration is positive if the curve has a positive slope. The speed of the particle is the magnitude of its velocity.

(*a*) Graph $\boxed{(\boldsymbol{d})}$ describes the motion of a particle with negative velocity and increasing speed because $v(t)$ is below the t axis and has a negative slope.

(*b*) Graph $\boxed{(\boldsymbol{b})}$ describes the motion of a particle with negative velocity and zero acceleration because $v(t)$ is below the t axis and its slope is zero.

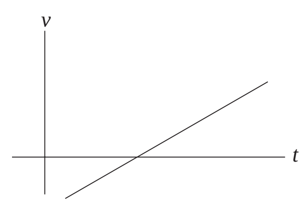
(*c*) None of these graphs describe the motion of a particle with a variable acceleration because $v(t)$ is linear.

(*d*) Graphs $\boxed{(\boldsymbol{c})\text{and}\,(\boldsymbol{d})}$ describe the motion of a particle with an increasing speed because they show the speed of the particle increasing with time.

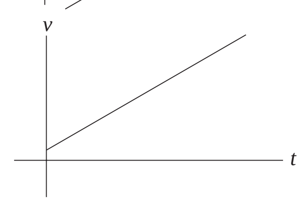
34 •• Sketch a v-versus-t curve for each of the following conditions: (*a*) Acceleration is constant but not zero. (*b*) Velocity and acceleration are both positive. (*c*) Velocity and acceleration are both negative. (*d*) Velocity is positive and acceleration is negative. (*e*) Velocity is negative and acceleration is positive. (*f*) Velocity is momentarily zero but the acceleration is not zero.

Determine the Concept Acceleration is the slope of a velocity-versus-time curve.

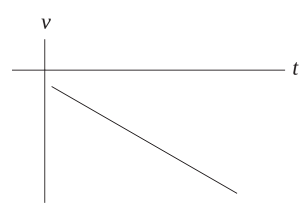
(*a*) Acceleration is constant but not zero.



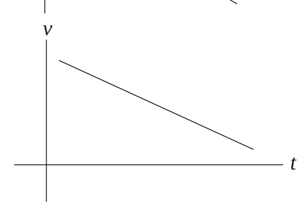
(*b*) Velocity and acceleration are both positive.



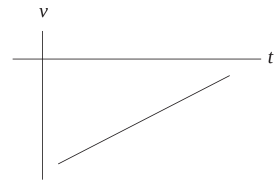
(*c*) Velocity and acceleration are both negative.



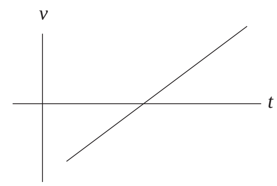
(*d*) Velocity is positive and acceleration is negative.



(*e*) Velocity is negative and acceleration is positive.



(*f*) Velocity is momentarily zero at the intercept with the *t* axis but the acceleration is not zero.



35 •• Figure 2-37 shows nine graphs of position, velocity, and acceleration for objects in motion along a straight line. Indicate the graphs that meet the following conditions: (*a*) Velocity is constant, (*b*) velocity reverses its direction, (*c*) acceleration is constant, and (*d*) acceleration is not constant. (*e*) Which graphs of position, velocity, and acceleration are mutually consistent?

Determine the Concept Velocity is the slope and acceleration is the slope of the slope of a position-versus-time curve. Acceleration is the slope of a velocity-versus-time curve.

(*a*) Graphs $\boxed{(\boldsymbol{a}), (\boldsymbol{f}), \text{and } (\boldsymbol{i})}$ describe motion at constant velocity. For constant velocity, *x* versus *t* must be a straight line; *v*-versus-*t* must be a horizontal straight line; and *a* versus *t* must be a straight horizontal line at $a = 0$.

(*b*) Graphs $\boxed{(\boldsymbol{c}) \text{and } (\boldsymbol{d})}$ describe motion in which the velocity reverses its direction. For velocity to reverse its direction *x*-versus-*t* must have a slope that changes sign and *v* versus *t* must cross the time axis. The acceleration cannot remain zero at all times.

(*c*) Graphs $\boxed{(\boldsymbol{a}), (\boldsymbol{d}), (\boldsymbol{e}), (\boldsymbol{f}), (\boldsymbol{h}), \text{and } (\boldsymbol{i})}$ describe motion with constant acceleration. For constant acceleration, *x* versus *t* must be a straight line or a parabola, *v* versus *t* must be a straight line, and *a* versus *t* must be a horizontal straight line.

(*d*) Graphs $\boxed{(\boldsymbol{b}), (\boldsymbol{c}), \text{ and } (\boldsymbol{g})}$ describe motion with non-constant acceleration. For non-constant acceleration, *x* versus *t* must not be a straight line or a parabola; *v* versus *t* must not be a straight line, or *a* versus *t* must not be a horizontal straight line.

(*e*) The following pairs of graphs are mutually consistent: $\boxed{(\boldsymbol{a}) \text{ and } (\boldsymbol{i}),}$ $\boxed{(\boldsymbol{d}) \text{ and } (\boldsymbol{h}),}$ and $\boxed{(\boldsymbol{f}) \text{ and } (\boldsymbol{i}).}$ For two graphs to be mutually consistent, the curves must be consistent with the definitions of velocity and acceleration.

Estimation and Approximation

36 • While engrossed in thought about the scintillating lecture just delivered by your physics professor you mistakenly walk directly into the wall (rather than through the open lecture hall door). Estimate the magnitude of your average acceleration as you rapidly come to a halt.

Picture the Problem The speed of one's walk varies from person to person, but 1.0 m/s is reasonable. We also need to estimate a distance within which you would stop in such a case. We'll assume a fairly short stopping distance of 1.5 cm. We'll also assume (unrealistically) that you experience constant acceleration and choose a coordinate system in which the direction you are walking is the $+x$ direction.

Using a constant-acceleration equation, relate your final speed to your initial speed, acceleration, and displacement while stopping:

$$v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a_x\Delta x \Rightarrow a_x = \frac{v_\mathrm{f}^2 - v_\mathrm{i}^2}{2\Delta x}$$

Substitute numerical values and evaluate the magnitude of your acceleration:

$$a_x = \left|\frac{(0)^2 - \left(1.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(1.5\times10^{-2}\,\mathrm{m}\right)}\right| = \boxed{33\ \mathrm{m/s^2}}$$

37 • **[SSM]** Occasionally, people can survive falling large distances if the surface they land on is soft enough. During a traverse of the Eiger's infamous Nordvand, mountaineer Carlos Ragone's rock anchor gave way and he plummeted 500 feet to land in snow. Amazingly, he suffered only a few bruises and a wrenched shoulder. Assuming that his impact left a hole in the snow 4.0 ft deep, estimate his average acceleration as he slowed to a stop (that is, while he was impacting the snow).

Picture the Problem In the absence of air resistance, Carlos' acceleration is constant. Because all the motion is downward, let's use a coordinate system in which downward is the positive direction and the origin is at the point at which the fall began.

Using a constant-acceleration equation, relate Carlos' final velocity v_2 to his velocity v_1 just before his impact, his stopping acceleration a_s upon impact, and his stopping distance Δy:

$$v_2^2 = v_1^2 + 2a_s\Delta y \Rightarrow a_s = \frac{v_2^2 - v_1^2}{2\Delta y}$$

or, because $v_2 = 0$,

$$a_s = -\frac{v_1^2}{2\Delta y} \qquad (1)$$

Using a constant-acceleration equation, relate Carlos' speed just before impact to his acceleration during free-fall and the distance he fell h:

$$v_1^2 = v_0^2 + 2a_{\text{free-fall}}h$$

or, because $v_0 = 0$ and $a_{\text{free-fall}} = g$,

$$v_1^2 = 2gh$$

Substituting for $\boldsymbol{v}_1^2$ in equation (1) yields:

$$a_s = -\frac{2gh}{2\Delta y}$$

Substitute numerical values and evaluate a_s:

$$\boldsymbol{a} = -\frac{2\left(9.81\ \text{m/s}^2\right)\left(500\ \text{ft}\right)}{2\left(4.0\ \text{ft}\right)} = \boxed{-1.2\times10^3\ \text{m/s}^2}$$

Remarks: The magnitude of this acceleration is about 125*g*!

38 •• When we solve free-fall problems near the earth, it's important to remember that air resistance may play a significant role. If its effects are significant, we may get answers that are wrong by orders of magnitude if we ignore it. How can we tell when it is valid to ignore the affects of air resistance? One way is to realize that air resistance increases with increasing speed. Thus, as an object falls and its speed *increases,* its downward acceleration *decreases.* Under these circumstances, the object's speed will approach, as a limit, a value called its *terminal speed.* This terminal speed depends upon such things as the mass and cross-sectional area of the body. Upon reaching its terminal speed, its acceleration is zero. For a "typical" skydiver falling through the air, a typical the terminal speed is about 50 m/s (roughly 120 mph). At half its terminal speed, the skydiver's acceleration will be about $\frac{3}{4}\boldsymbol{g}$. Let's take half the terminal speed as a reasonable "upper bound" beyond which we shouldn't use our constant acceleration free-fall relationships. Assuming the skydiver started from rest, (*a*) estimate how far, and for how long, the skydiver falls before we can no longer neglect air resistance. (*b*) Repeat the analysis for a ping-pong ball, which has a terminal speed of about 5.0 m/s. (*c*) What can you conclude by comparing your answers for Parts (*a*) and (*b*)?

Picture the Problem Because we're assuming that the accelerations of the skydiver and a ping-pong ball are constant to one-half their terminal velocities, we can use constant-acceleration equations to find the times required for them to reach their ″upper-bound″ velocities and their distances of fall. Let's use a coordinate system in which downward is the positive *y* direction.

(*a*) Using a constant-acceleration equation, relate the upper-bound velocity to the free-fall acceleration and the time required to reach this velocity:

$$v_{\text{upper bound}} = v_0 + g\Delta t$$

or, because $v_0 = 0$,

$$v_{\text{upper bound}} = g\Delta t \Rightarrow \Delta t = \frac{v_{\text{upper bound}}}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{25\,\text{m/s}}{9.81\,\text{m/s}^2} = 2.55\text{ s} \approx \boxed{2.6\,\text{s}}$$

Using a constant-acceleration equation, relate the skydiver's distance of fall to the elapsed time Δt:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because *v*0 = 0 and $a = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values and evaluate Δy:

$$\Delta \boldsymbol{y} = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(2.55\,\text{s})^2 \approx \boxed{32\,\text{m}}$$

(*b*) Proceed as in (*a*) with $v_{\text{upper bound}} = 5.0\text{ m/s}$ to obtain:

$$\Delta \boldsymbol{t} = \frac{\tfrac{1}{2}(50\,\text{m/s})}{9.81\,\text{m/s}^2} = 0.255\text{ s} \approx \boxed{0.26\,\text{s}}$$

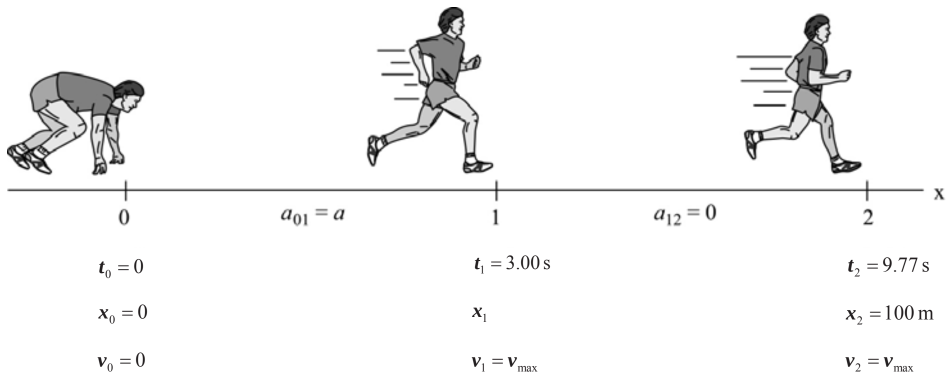
and

$$\Delta \boldsymbol{y} = \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)(0.255\text{ s})^2 \approx \boxed{32\text{ cm}}$$

(*c*) The analysis of the motion of a ping-pong ball requires the inclusion of air resistance for almost any situation, whereas the analysis of the motion of the sky diver doesn't require it until the fall distances and times are considerably longer.

39 •• On June 14, 2005 Asafa Powell of the Jamaica set a world's record for the 100-m dash with a time $t = 9.77$ s. Assuming he reached his maximum speed in 3.00 s, and then maintained that speed until the finish, estimate his acceleration during the first 3.00 s.

Picture the Problem This is a constant-acceleration problem. Choose a coordinate system in which the direction Powell is running is the positive *x* direction. During the first 3 s of the race his acceleration is positive and during the rest of the race it is zero. The pictorial representation summarizes what we know about Powell's race.



Express the total distance covered by Powell in terms of the distances covered in the two phases of his race:

$$100\,\mathrm{m} = \Delta x_{01} + \Delta x_{12} \qquad (1)$$

Express the distance he runs getting to his maximum velocity:

$$\Delta x_{01} = v_0 \Delta t_{01} + \tfrac{1}{2} a_{01} (\Delta t_{01})^2 = \tfrac{1}{2} a (3\,\mathrm{s})^2$$

The distance covered during the rest of the race at the constant maximum velocity is given by:

$$\begin{aligned}\Delta \boldsymbol{x}_{12} &= \boldsymbol{v}_{max} \Delta \boldsymbol{t}_{12} + \tfrac{1}{2} \boldsymbol{a}_{12} (\Delta \boldsymbol{t}_{12})^2 \\ &= (\boldsymbol{a} \Delta \boldsymbol{t}_{01}) \Delta \boldsymbol{t}_{12} \\ &= \boldsymbol{a} (3.00\,\mathrm{s})(6.77\,\mathrm{s})\end{aligned}$$

Substitute for these displacements in equation (1) to obtain:

$$100\,\mathrm{m} = \tfrac{1}{2} \boldsymbol{a} (3.00\,\mathrm{s})^2 + \boldsymbol{a} (3.00\,\mathrm{s})(6.77\,\mathrm{s})$$

Solving for a yields:

$$\boldsymbol{a} = \frac{100\,\mathrm{m}}{\tfrac{1}{2}(3.00\,\mathrm{s})^2 + (3.00\,\mathrm{s})(6.77\,\mathrm{s})} = \boxed{4.03\,\mathrm{m/s^2}}$$

40 •• The photograph in Figure 2-38 is a short-time exposure (1/30 s) of a juggler with two tennis balls in the air. (*a*) The tennis ball near the top of its trajectory is less blurred than the lower one. Why is that? (*b*) Estimate the speed of the ball that he is just releasing from his right hand. (*c*) Determine how high the ball should have gone above the launch point and compare it to an estimate from the picture. (*Hint: You have a built-in distance scale if you assume some reasonable value for the height of the juggler*.)

Determine the Concept This is a constant-acceleration problem with $a = -g$ if we take upward to be the positive direction. At the maximum height the ball will reach, its speed will be near zero and when the ball has just been tossed in the air its speed is near its maximum value.

(*a*) Because the ball is moving slowly its blur is relatively short (i.e., there is less blurring).

(*b*) The average speed of the ball is given by:

$$v_{av} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

Estimating how far the ball has traveled in 1/30 s yields:

$$v_{av} = \frac{2\text{ ball diameters}}{\frac{1}{30}\text{ s}}$$

The diameter of a tennis ball is approximately 5 cm:

$$v_{av} = \frac{2\,(5\text{ cm})}{\frac{1}{30}\text{ s}} \approx \boxed{3\text{ m/s}}$$

(*c*) Use a constant-acceleration equation to relate the initial and final speeds of the ball to its maximum height *h*:

$$v^2 = v_0^2 + 2a_y h \Rightarrow h = \frac{v^2 - v_0^2}{2a_y}$$

or, because $v = 0$ and $a_y = g$,

$$h = -\frac{v_0^2}{2g}$$

Substitute numerical values and evaluate *h*:

$$h = -\frac{(3\text{ m/s})^2}{2(-9.81\text{ m/s}^2)} \approx \boxed{0.5\text{ m}}$$

If we assume that the juggler is approximately 6 ft tall, then our calculated value for *h* seems to be a good approximation to the height shown in the photograph.

41 •• A rough rule of thumb for determining the distance between you and a lightning strike is to start counting the seconds that elapse ("one-Mississippi, two-Mississippi, …") until you hear the thunder (sound emitted by the lightning as it rapidly heats the air around it). Assuming the speed of sound is about 750 mi/h, (*a*) estimate how far away is a lightning strike if you counted about 5 s until you heard the thunder. (*b*) Estimate the uncertainty in the distance to the strike in Part (*a*). Be sure to explain your assumptions and reasoning. (*Hint: The speed of sound depends on the air temperature and your counting is far from exact!*)

Picture the Problem We can use the relationship between distance, speed, and time to estimate the distance to the lightning strike.

(*a*) Relate the distance Δd to the lightning strike to the speed of sound in air *v* and the elapsed time Δt:

$$\Delta d = v\Delta t$$

Substitute numerical values and evaluate Δd:

$$\Delta \boldsymbol{d} = \left(750\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{0.3048\ \frac{\text{m}}{\text{s}}}{0.6818\ \frac{\text{mi}}{\text{h}}}\right)(5\ \text{s})$$

$$\approx \boxed{1.7\ \text{km}} \approx \boxed{1\ \text{mi}}$$

(*b*) You are probably lucky if the uncertainty in your time estimate is less than 1 s (±20%), so the uncertainty in the distance estimate is about 20% of 1.7 km or approximately 300 m. This is probably much greater than the error made by assuming v is constant.

Speed, Displacement, and Velocity

42 • (*a*) An electron in a television tube travels the 16-cm distance from the grid to the screen at an average speed of 4.0×10^7 m/s. How long does the trip take? (*b*) An electron in a current-carrying wire travels at an average speed of 4.0×10^{-5} m/s. How long does it take to travel 16 cm?

Picture the Problem Think of the electron as traveling in a straight line at constant speed and use the definition of average speed.

(*a*) Using its definition, express the average speed of the electron:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of flight}} = \frac{\Delta s}{\Delta t}$$

Solve for and evaluate the time of flight:

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\ \text{m}}{4.0 \times 10^{7}\ \text{m/s}}$$

$$= 4.0 \times 10^{-9}\ \text{s} = \boxed{4.0\ \text{ns}}$$

(*b*) Calculate the time of flight for an electron in a 16-cm long current carrying wire similarly.

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\ \text{m}}{4.0 \times 10^{-5}\ \text{m/s}}$$

$$= 4.0 \times 10^{3}\ \text{s} \times \frac{1\ \text{min}}{60\ \text{s}} = \boxed{67\ \text{min}}$$

43 • **[SSM]** A runner runs 2.5 km, in a straight line, in 9.0 min and then takes 30 min to walk back to the starting point. (*a*) What is the runner's average velocity for the first 9.0 min? (*b*) What is the average velocity for the time spent walking? (*c*) What is the average velocity for the whole trip? (*d*) What is the average speed for the whole trip?

Picture the Problem In this problem the runner is traveling in a straight line but not at constant speed - first she runs, then she walks. Let's choose a coordinate system in which her initial direction of motion is taken as the positive x direction.

(*a*) Using the definition of average velocity, calculate the average velocity for the first 9 min:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \frac{2.5\text{ km}}{9.0\text{ min}} = \boxed{0.28\text{ km/min}}$$

(*b*) Using the definition of average velocity, calculate her average speed for the 30 min spent walking:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{-2.5\text{ km}}{30\text{ min}}$$
$$= \boxed{-83\text{ m/min}}$$

(*c*) Express her average velocity for the whole trip:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}_{\text{round trip}}}{\Delta \boldsymbol{t}} = \frac{0}{\Delta \boldsymbol{t}} = \boxed{0}$$

(*d*) Finally, express her average speed for the whole trip:

$$\text{speed}_{\text{av}} = \frac{\text{distance traveled}}{\text{elapsed time}}$$
$$= \frac{2(2.5\text{ km})}{30\text{ min} + 9.0\text{ min}}$$
$$= \boxed{0.13\text{ km/min}}$$

44 • A car travels in a straight line with an average velocity of 80 km/h for 2.5 h and then with an average velocity of 40 km/h for 1.5 h. (*a*) What is the total displacement for the 4.0-h trip? (*b*) What is the average velocity for the total trip?

Picture the Problem The car is traveling in a straight line but not at constant speed. Let the direction of motion be the positive x direction.

(*a*) The total displacement of the car for the entire trip is the sum of the displacements for the two legs of the trip:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2$$

Find the displacement for each leg of the trip:

$$\Delta x_1 = v_{av,1}\Delta t_1 = (80\text{ km/h})(2.5\text{ h})$$
$$= 200\,\text{km}$$

and

$$\Delta x_2 = v_{av,2}\Delta t_2 = (40\text{ km/h})(1.5\text{ h})$$
$$= 60.0\,\text{km}$$

Add the individual displacements to get the total displacement:

$$\Delta \boldsymbol{x}_{\text{total}} = \Delta \boldsymbol{x}_1 + \Delta \boldsymbol{x}_2 = 200\,\text{km} + 60.0\,\text{km}$$
$$= \boxed{2.6\times10^5\,\text{m}}$$

(*b*) As long as the car continues to move in the same direction, the average velocity for the total trip is given by:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}_{\text{total}}}{\Delta \boldsymbol{t}_{\text{total}}}$$

Substitute numerical values and evaluate v_{av}:

$$\boldsymbol{v}_{\text{av}} = \frac{2.6\times10^5\ \text{m}}{2.5\ \text{h}+1.5\ \text{h}} = \boxed{65\,\text{km/h}}$$

45 • One busy air route across the Atlantic Ocean is about 5500 km. The now-retired Concord, a supersonic jet capable of flying at twice the speed of sound was used to travel such routes. (*a*) Roughly how long did it take for a one-way flight? (Use 343 m/s for the speed of sound.) (*b*) Compare this time to the time taken by a subsonic jet flying at 0.90 times the speed of sound.

Picture the Problem However unlikely it may seem, imagine that both jets are flying in a straight line at constant speed and use the definition of average speed to find the flight times.

(*a*) The time of flight is the ratio of the distance traveled to the speed of the supersonic jet.

$$t_{\text{supersonic}} = \frac{s_{\text{Atlantic}}}{v_{\text{supersonic}}} = \frac{5500\ \text{km}}{2(343\,\text{m/s})(3600\,\text{s/h})} = 2.23\,\text{h} = \boxed{2.2\ \text{h}}$$

(*b*) Express the ratio of the time for the trip at supersonic speed to the time for the trip at subsonic speed and simplify to obtain:

$$\frac{t_{\text{supersonic}}}{t_{\text{subsonic}}} = \frac{\dfrac{s_{\text{Atlantic}}}{v_{\text{supersonic}}}}{\dfrac{s_{\text{Atlantic}}}{v_{\text{subsonic}}}} = \frac{v_{\text{subsonic}}}{v_{\text{supersonic}}}$$

Substitute numerical values and evaluate the ratio of the flight times:

$$\frac{t_{\text{supersonic}}}{t_{\text{subsonic}}} = \frac{(0.90)(343\,\text{m/s})}{(2)(343\,\text{m/s})} = \boxed{0.45}$$

46 •• The speed of light, designated by the universally recognized symbol *c*, has a value, to two significant figures, of 3.0×10^8 m/s. (*a*) How long does it take for light to travel from the sun to the earth, a distance of 1.5×10^{11} m? (*b*) How long does it take light to travel from the Moon to Earth, a distance of 3.8×10^8 m?

Picture the Problem In free space, light travels in a straight line at constant speed, *c*.

(*a*) Using the definition of average speed, express the time Δt required for light to travel from the sun to the earth:

$$\Delta \boldsymbol{t} = \frac{\Delta \boldsymbol{s}}{\text{average speed}}$$

where Δs is the distance from the sun to the earth.

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{1.5\times10^{11}\ \text{m}}{3.0\times10^{8}\ \text{m/s}} = 5.0\times10^{2}\ \text{s}$$

$$\approx \boxed{8.3\,\text{min}}$$

(*b*) Proceed as in (*a*) this time using the Moon-Earth distance:

$$\Delta \boldsymbol{t} = \frac{3.8\times10^{8}\ \text{m}}{3.0\times10^{8}\ \text{m/s}} \approx \boxed{1.3\,\text{s}}$$

47 • **[SSM]** Proxima Centauri, the closest star to us besides our own sun, is 4.1×10^{13} km from Earth. From Zorg, a planet orbiting this star, a Zorgian places an order at Tony's Pizza in Hoboken, New Jersey, communicating via light signals. Tony's fastest delivery craft travels at $1.00 \times 10^{-4}c$ (see Problem 46). (*a*) How long does it take Gregor's order to reach Tony's Pizza? (*b*) How long does Gregor wait between sending the signal and receiving the pizza? If Tony's has a "1000-years-or-it's-free" delivery policy, does Gregor have to pay for the pizza?

Picture the Problem In free space, light travels in a straight line at constant speed, *c*. We can use the definition of average speed to find the elapsed times called for in this problem.

(*a*) Using the definition of average speed (equal here to the assumed constant speed of light), solve for the time Δt required to travel the distance to Proxima Centauri:

$$\Delta \boldsymbol{t} = \frac{\text{distance traveled}}{\text{speed of light}}$$

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{4.1\times10^{16}\ \text{m}}{2.998\times10^{8}\ \text{m/s}} = 1.37\times10^{8}\ \text{s}$$

$$= \boxed{4.3\,\text{y}}$$

(*b*) Traveling at $10^{-4}c$, the delivery time (t_{total}) will be the sum of the time for the order to reach Hoboken and the time for the pizza to be delivered to Proxima Centauri:

$$\Delta \boldsymbol{t}_{\text{total}} = \Delta \boldsymbol{t}_{\substack{\text{order to be}\\ \text{sent to Hoboken}}} + \Delta \boldsymbol{t}_{\substack{\text{order to}\\ \text{be delivered}}} = 4.33\,\boldsymbol{y} + \frac{4.1\times10^{13}\ \text{km}}{\left(1.00\times10^{-4}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}$$

$$= 4.33\,\text{y} + 4.33\times10^{6}\ \text{y} \approx 4.3\times10^{6}\ \text{y}$$

Because 4.3×10^6 y >> 1000 y, Gregor does not have to pay.

48 • A car making a 100-km journey travels 40 km/h for the first 50 km. How fast must it go during the second 50 km to average 50 km/h?

Picture the Problem The time for the second 50 km is equal to the time for the entire journey less the time for the first 50 km. We can use this time to determine the average speed for the second 50 km interval from the definition of average speed.

Using the definition of average speed, find the time required for the total journey:

$$\Delta t_{\text{total}} = \frac{\Delta x}{v_{\text{av}}} = \frac{100\,\text{km}}{50\,\text{km/h}} = 2.0\,\text{h}$$

Find the time required for the first 50 km:

$$\Delta t_{\text{1st 50 km}} = \frac{50\,\text{km}}{40\,\text{km/h}} = 1.25\,\text{h}$$

Find the time remaining to travel the last 50 km:

$$\Delta t_{\text{2nd 50 km}} = t_{\text{total}} - t_{\text{1st 50 km}} = 2.0\,\text{h} - 1.25\,\text{h} = 0.75\,\text{h}$$

Finally, use the time remaining to travel the last 50 km to determine the average speed over this distance:

$$\boldsymbol{v}_{\text{av, 2nd 50 km}} = \frac{\Delta \boldsymbol{x}_{\text{2nd 50 km}}}{\Delta \boldsymbol{t}_{\text{2nd 50 km}}} = \frac{50\,\text{km}}{0.75\,\text{h}} = \boxed{67\,\text{km/h}}$$

49 •• Late in ice hockey games, the team that is losing sometimes "pulls" their goalkeeper off the ice to add an additional offensive player and increase their chances of scoring. In such cases, the goalie on the opposing team might have an opportunity to score into the unguarded net that is 55.0 m away. Suppose you are the goaltender for your university team and are in just such a situation. You launch a shot (in hopes of getting your first career goal) on the frictionless ice. You hear a disappointing "clang" as the puck strikes a goalpost (instead of going in!) exactly 2.50 s later. In this case, how fast did the puck travel? You should assume 343 m/s for the speed of sound.

Picture the Problem The distance over which both the puck and the sound from the puck hitting the goalpost must travel is 55.0 m. The time between the shot being released and the sound reaching the goalie's ear can be found by expressing the total elapsed time as the sum of the elapsed times for the shot to travel 55.0 m and for the sound to travel back to you.

The total elapsed time as the sum of the elapsed times for the shot and for the sound to travel back to you :

$$\Delta \boldsymbol{t}_{\text{total}} = \Delta \boldsymbol{t}_{\text{shot}} + \Delta \boldsymbol{t}_{\text{sound}}$$

Express the time for the shot to travel to the other net a distance Δx away:

$$\Delta t_{\text{shot}} = \frac{\Delta x}{v_{\text{shot}}}$$

Express the time for the sound to travel a distance Δx back to you:

$$\Delta t_{\text{sound}} = \frac{\Delta x}{v_{\text{sound}}}$$

Substitute in the expression for Δt_{total} to obtain:

$$\Delta t_{\text{total}} = \frac{\Delta x}{v_{\text{shot}}} + \frac{\Delta x}{v_{\text{sound}}}$$

Solving this equation for v_{shot} yields:

$$v_{\text{shot}} = \frac{v_{\text{sound}}\Delta x}{v_{\text{sound}}\Delta t_{\text{total}} - \Delta x}$$

Substitute numerical values and evaluate v_{shot}:

$$v_{\text{shot}} = \frac{(343\text{ m/s})(55.0\text{ m})}{(343\text{ m/s})(2.50\text{ s}) - 55.0\text{ m}} = \boxed{23.5\text{ m/s}}$$

50 •• Cosmonaut Andrei, your co-worker at the International Space Station, cosmonaut Andrei, tosses a banana at you with a speed of 15 m/s. At exactly the same instant, you fling a scoop of ice cream at Andrei along exactly the same path. The collision between banana and ice cream produces a banana split 7.2 m from your location 1.2 s after the banana and ice cream were launched. (*a*) How fast did you toss the ice cream? (*b*) How far were you from Andrei when you tossed the ice cream? (Neglect any effects due to gravity.)

Picture the Problem Let the subscript b refer to the banana and the subscript ic refer to the ice cream. Then the distance covered by the ice cream before collision is given by $\Delta x_{\text{ic}} = v_{\text{ic}}\Delta t$ and the distance covered by the banana is $\Delta x_{\text{b}} = v_{\text{b}}\Delta t$. The distance between you and Andrei is then the sum of these distances: $\Delta x_{\text{tot}} = \Delta x_{\text{ic}} + \Delta x_{\text{b}}$.

(*a*) The speed of the ice cream is given by:

$$v_{\text{ic}} = \frac{\Delta x_{\text{ic}}}{\Delta t}$$

where Δt is the time-to-collision.

Substitute numerical values and evaluate v_{ic}:

$$v_{\text{ic}} = \frac{7.2\text{ m}}{1.2\text{ s}} = \boxed{6.0\text{ m/s}}$$

(*b*) Express the distance between yourself and Andrei as the sum of the distances the ice cream and the banana travel:

$$\Delta x_{\text{total}} = \Delta x_{\text{ic}} + \Delta x_{\text{b}}$$

Because $\Delta x_{\text{b}} = v_{\text{b}}\Delta t$:

$$\Delta x_{\text{total}} = \Delta x_{\text{ic}} + v_{\text{b}}\Delta t$$

Substitute numerical values and evaluate Δx_{total}:

$$\Delta x_{\text{total}} = 7.2\text{ m} + (15\text{ m/s})(1.2\text{ s}) = \boxed{25\text{ m}}$$

51 •• Figure 2-39 shows the position of a particle as a function of time. Find the average velocities for the time intervals *a*, *b*, *c*, and *d* indicated in the figure.

Picture the Problem The average velocity in a time interval is defined as the displacement divided by the time elapsed; that is $v_{\text{av}} = \Delta x / \Delta t$.

(*a*) $\Delta x_a = 0$

$$v_{\text{av}} = \boxed{0}$$

(*b*) $\Delta x_b = 1$ m and $\Delta t_b = 3$ s

$$\boldsymbol{v}_{\text{av}} = \boxed{0.3\text{ m/s}}$$

(*c*) $\Delta x_c = -6$ m and $\Delta t_c = 3$ s

$$\boldsymbol{v}_{\text{av}} = \boxed{-2\text{ m/s}}$$

(*d*) $\Delta x_d = 3$ m and $\Delta t_d = 3$ s

$$\boldsymbol{v}_{\text{av}} = \boxed{1\text{ m/s}}$$

52 •• It has been found that, on average, galaxies are moving away from the Earth at a speed that is proportional to their distance from the earth. This discovery is known as Hubble's law, named for its discoverer, astrophysicist Sir Edwin Hubble. He found that the recessional speed *v* of a galaxy a distance *r* from Earth is given by $v = Hr$, where $H = 1.58 \times 10^{-18}\ \text{s}^{-1}$ is called the Hubble constant. What are the expected recessional speeds of galaxies (*a*) 5.00×10^{22} m from Earth and (*b*) 2.00×10^{25} m from Earth? (*c*) If the galaxies at each of these distances had traveled at their expected recessional speeds, how long ago would they have been at our location?

Picture the Problem In free space, light travels in a straight line at constant speed *c*. We can use Hubble's law to find the speed of the two planets.

(*a*) Using Hubble's law, calculate the speed of the first galaxy:

$$\boldsymbol{v}_{\text{a}} = (5.00\times10^{22}\text{ m})(1.58\times10^{-18}\text{ s}^{-1}) = \boxed{7.90\times10^{4}\text{ m/s}}$$

(*b*) Using Hubble's law, calculate the speed of the second galaxy:

$$\boldsymbol{v}_{\text{b}} = (2.00\times10^{25}\text{ m})(1.58\times10^{-18}\text{ s}^{-1}) = \boxed{3.16\times10^{7}\text{ m/s}}$$

(*c*) Using the relationship between distance, speed, and time for both galaxies, express how long ago Δt they were both located at the same place as Earth:

$$\Delta t = \frac{r}{v} = \frac{r}{rH} = \frac{1}{H}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = 6.33\times10^{17}\text{ s} \approx \boxed{20\times10^{9}\text{ y}}$$

53 •• **[SSM]** The cheetah can run as fast as 113 km/h, the falcon can fly as fast as 161 km/h, and the sailfish can swim as fast as 105 km/h. The three of them run a relay with each covering a distance *L* at maximum speed. What is the average speed of this relay team for the entire relay? Compare this average speed with the numerical average of the three individual speeds. Explain carefully why the average speed of the relay team is *not* equal to the numerical average of the three individual speeds.

Picture the Problem We can find the average speed of the relay team from the definition of average speed.

Using its definition, relate the average speed to the total distance traveled and the elapsed time:

$$\boldsymbol{v}_{\text{av}} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

Express the time required for each animal to travel a distance L:

$$\boldsymbol{t}_{\text{cheetah}} = \frac{\boldsymbol{L}}{\boldsymbol{v}_{\text{cheetah}}},\ \boldsymbol{t}_{\text{falcon}} = \frac{\boldsymbol{L}}{\boldsymbol{v}_{\text{falcon}}}$$

and

$$\boldsymbol{t}_{\text{sailfish}} = \frac{\boldsymbol{L}}{\boldsymbol{v}_{\text{sailfish}}}$$

Express the total time Δt:

$$\Delta t = L\left(\frac{1}{v_{\text{cheetah}}} + \frac{1}{v_{\text{falcon}}} + \frac{1}{v_{\text{sailfish}}}\right)$$

Use the total distance traveled by the relay team and the elapsed time to calculate the average speed:

$$\boldsymbol{v}_{\text{av}} = \frac{3\boldsymbol{L}}{\boldsymbol{L}\left(\frac{1}{113\,\text{km/h}} + \frac{1}{161\,\text{km/h}} + \frac{1}{105\,\text{km/h}}\right)} = 122.03\,\text{km/h} = \boxed{122\,\text{km/h}}$$

Calculating the average of the three speeds yields:

$$\text{Average}_{\text{three speeds}} = \frac{113\,\text{km/h} + 161\,\text{km/h} + 105\,\text{km/h}}{3} = 126.33\,\text{km/h} = 126\,\text{km/h}$$

$$= \boxed{1.04\boldsymbol{v}_{\text{av}}}$$

Because we've ignored the time intervals during which members of this relay team get up to their running speeds, their accelerations are zero and the average speed of the relay team is *not* equal to (it is less than) the numerical average of the three individual speeds.

54 •• Two cars are traveling along a straight road. Car A maintains a constant speed of 80 km/h and car B maintains a constant speed of 110 km/h. At $t = 0$, car B is 45 km behind car A. (*a*) How much farther will car A travel before car B overtakes it? (*b*) How much ahead of A will B be 30 s after it overtakes A?

Picture the Problem Let the position of car A at $t = 0$ be the origin of our coordinate system. Then we can use a constant-acceleration equation to express the positions of both cars as functions of time and equate these expressions to determine the time at which car A is overtaken by car B.

(*a*) Car B overtakes car A when their *x* coordinates are the same:

$$x_A(t) = x_B(t) \qquad (1)$$

Using a constant-acceleration equation, express the position of car A as a function of time:

$$x_A(t) = x_{0A} + v_A t$$

where x_{0A} is the position of car A at $t = 0$.

Because we've let car A be at the origin at $t = 0$:

$$x_A(t) = v_A t \qquad (2)$$

Using a constant-acceleration equation, express the position of car B as a function of time:

$$x_B(t) = x_{0B} + v_B t$$

where x_{0B} is the position of car B at $t = 0$.

Substitute for $x_A(t)$ and $x_B(t)$ in equation (1) to obtain:

$$v_A t = x_{0B} + v_B t \Rightarrow t = \frac{x_{0B}}{v_A - v_B}$$

Substitute numerical values and evaluate the time t at which car B overtakes car A:

$$t = \frac{-45\ \text{km}}{80\ \text{km/h} - 110\ \text{km/h}} = 1.50\ \text{h}$$

Now we can evaluate equation (2) at $t = 1.50$ h to obtain:

$$x_A(1.50\ \text{h}) = \left(80\ \frac{\text{km}}{\text{h}}\right)(1.50\ \text{h}) = 120\ \text{km}$$

$$= \boxed{1.2 \times 10^5\ \text{m}}$$

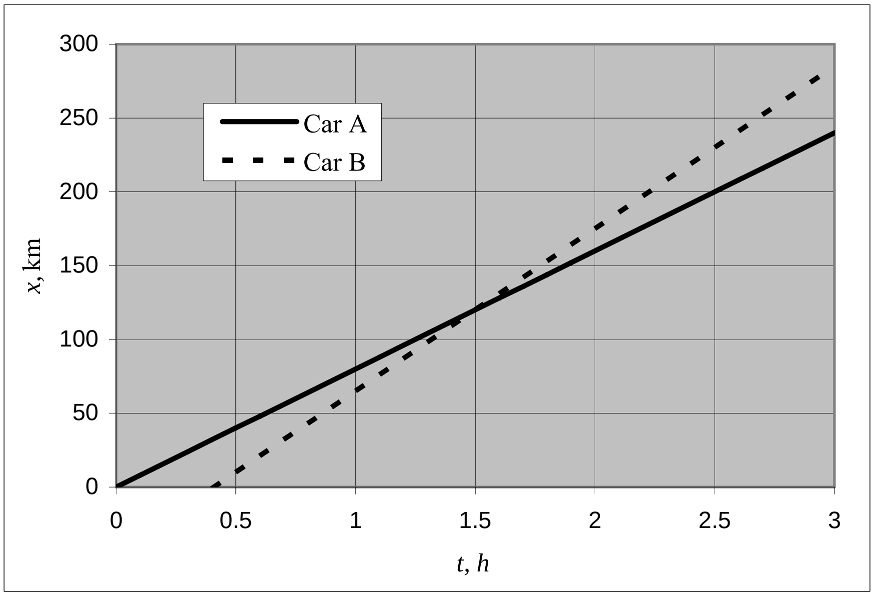
(*b*) The separation of the cars as a function of time is given by:

$$\Delta x(t) = x_B(t) - x_A(t) = x_{0B} - v_B t - v_A t$$

Substitute numerical values and evaluate $\Delta x(1.50\text{ h} + 30\text{ s}) = \Delta x(1.508\text{ h})$ to obtain:

$$\Delta \boldsymbol{x}(1.508\,\text{h}) = -45\,\text{km} + \left(110\,\frac{\text{km}}{\text{h}} - 80\,\frac{\text{km}}{\text{h}}\right)(1.508\,\text{h}) = \boxed{0.24\,\text{km}}$$

Remarks: One can use a graphing calculator or a spreadsheet program to solve this problem. A spreadsheet program was used to plot the following graph:



Note that this graph confirms our result that the cars are at the same location at t = 1.5 h.

55 •• [SSM] A car traveling at a constant speed of 20 m/s passes an intersection at time $t = 0$. A second car traveling at a constant speed of 30 m/s in the same direction passes the same intersection 5.0 s later. (*a*) Sketch the position functions $x_1(t)$ and $x_2(t)$ for the two cars for the interval $0 \le t \le 20$ s. (*b*) Determine when the second car will overtake the first. (*c*) How far from the intersection will the two cars be when they pull even? (*d*) Where is the first car when the second car passes the intersection?

Picture the Problem One way to solve this problem is by using a graphing calculator to plot the positions of each car as a function of time. Plotting these positions as functions of time allows us to visualize the motion of the two cars relative to the (fixed) ground. More importantly, it allows us to see the motion of the two cars relative to each other. We can, for example, tell how far apart the cars are at any given time by determining the length of a vertical line segment from one curve to the other.

(*a*) Letting the origin of our coordinate system be at the intersection, the position of the slower car, $x_1(t)$, is given by:

$x_1(t) = 20t$
where x_1 is in meters if t is in seconds.

Because the faster car is also moving at a constant speed, we know that the position of this car is given by a function of the form:

$x_2(t) = 30t + b$

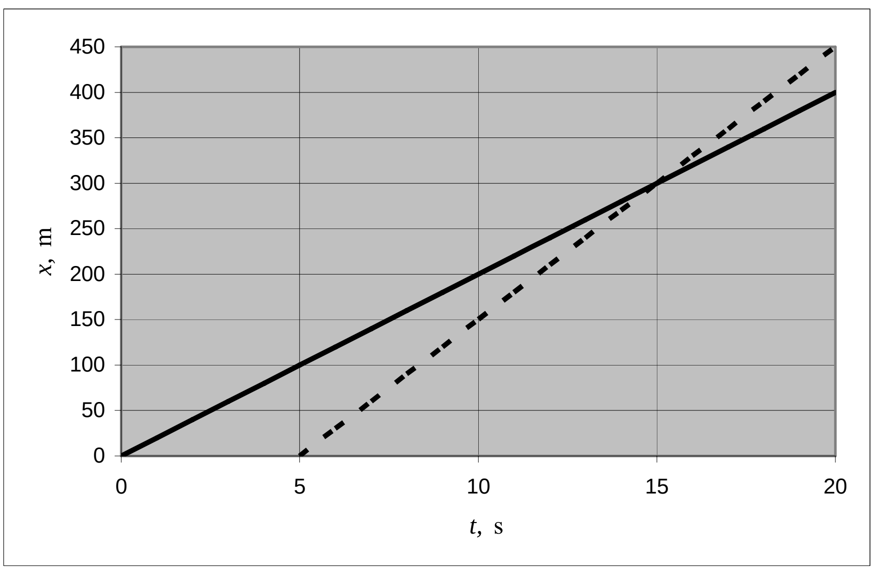
We know that when $t = 5.0$ s, this second car is at the intersection (that is, $x_2(5.0\text{ s}) = 0$). Using this information, you can convince yourself that:

$b = -150$ m

Thus, the position of the faster car is given by:

$$x_2(t) = 30t - 150$$

One can use a graphing calculator, graphing paper, or a spreadsheet to obtain the following graphs of $x_1(t)$ (the solid line) and $x_2(t)$ (the dashed line):



(*b*) Use the time coordinate of the intersection of the two lines to determine the time at which the second car overtakes the first:

From the intersection of the two lines, one can see that the second car will "overtake" (catch up to) the first car at $\boxed{\boldsymbol{t} = 15\text{ s.}}$

(c) Use the position coordinate of the intersection of the two lines to determine the distance from the intersection at which the second car catches up to the first car:

From the intersection of the two lines, one can see that the distance from the intersection is $\boxed{300\text{ m.}}$

(d) Draw a vertical line from $t = 5$ s to the solid line and then read the position coordinate of the intersection of the vertical line and the solid line to determine the position of the first car when the second car went through the intersection. From the graph, when the second car passes the intersection, the first car was $\boxed{100\text{ m ahead.}}$

56 •• Bats use echolocation to determine their distance from objects they cannot easily see in the dark. The time between the emission of high-frequency sound pulse (a click) and the detection of its echo is used to determine such distances. A bat, flying at a constant speed of 19.5 m/s in a straight line toward a vertical cave wall, makes a single clicking noise and hears the echo 0.15 s later. Assuming that she continued flying at her original speed, how close was she to the wall when she received the echo? Assume a speed of 343 m/s for the speed of sound.

Picture the Problem The sound emitted by the bat travels at v_{sound} and during the time interval Δx_{total} during which the sound travels to the wall and back to the bat, the bat travels a distance of $\Delta x_{\text{bat}} = v_{\text{bat}}\Delta t_{\text{total}}$, where v_{bat} is the bat's flying speed. The distance that the sound travels is denoted by $\Delta x_{\text{total}} = v_{\text{sound}}\Delta t_{\text{total}}$. We are interested in knowing how far the sound travels between the time of its reflection and the time of the bat's receiving the echo. This we denote Δx_{away} and can find by using the conditions that $\Delta \boldsymbol{x}_{\text{total}} = \Delta \boldsymbol{x}_{\text{toward}} + \Delta \boldsymbol{x}_{\text{away}}$ and $\Delta \boldsymbol{x}_{\text{toward}} = \Delta \boldsymbol{x}_{\text{bat}} + \Delta \boldsymbol{x}_{\text{away}}$.

Express the total distance traveled by the sound as the sum of the distance traveled toward the wall of the cave Δx_{toward} and away from the wall (toward the bat) after reflection Δx_{away}:

$$\Delta \boldsymbol{x}_{\text{total}} = \Delta \boldsymbol{x}_{\text{toward}} + \Delta \boldsymbol{x}_{\text{away}} \qquad (1)$$

The total distance traveled by the sound toward the wall is also given by:

$$\Delta x_{\text{toward}} = \Delta x_{\text{bat}} + \Delta x_{\text{away}}$$

Substitute for Δx_{toward} in equation (1) to obtain:

$$\begin{aligned}\Delta x_{\text{total}} &= \Delta x_{\text{bat}} + \Delta x_{\text{away}} + \Delta x_{\text{away}} \\ &= \Delta x_{\text{bat}} + 2\Delta x_{\text{away}}\end{aligned}$$

Solving for Δx_{away} yields:

$$\Delta x_{\text{away}} = \frac{\Delta x_{\text{total}} - \Delta x_{\text{bat}}}{2}$$

Substitute for Δx_{total} and Δx_{bat} and simplify to obtain:

$$\Delta \boldsymbol{x}_{\text{away}} = \frac{\boldsymbol{v}_{\text{sound}}\Delta \boldsymbol{t}_{\text{total}} - \boldsymbol{v}_{\text{bat}}\Delta \boldsymbol{t}_{\text{total}}}{2} = \frac{(\boldsymbol{v}_{\text{sound}} - \boldsymbol{v}_{\text{bat}})\Delta \boldsymbol{t}_{\text{total}}}{2}$$

Substitute numerical values and evaluate Δx_{away} :

$$\Delta \boldsymbol{x}_{\text{away}} = \frac{(343\text{ m/s} - 19.5\text{ m/s})(0.15\text{ s})}{2} = \boxed{24\text{ m}}$$

57 ••• A submarine can use *sonar* (sound traveling through water) to determine its distance from other objects. The time between the emission of a sound pulse (a "ping") and the detection of its echo can be used to determine such distances. Alternatively, by measuring the time between *successive* echo receptions of a *regularly timed set* of pings, the submarine's *speed* may be determined by comparing the time between echoes to the time between pings. Assume you are the sonar operator in a submarine traveling at a constant velocity underwater. Your boat is in the eastern Mediterranean Sea, where the speed of sound is known to be 1522 m/s. If you send out pings every 2.000 s, and your apparatus receives echoes reflected from an undersea cliff every 1.980 s, how fast is your submarine traveling?

Picture the Problem Both the pulses sent out by the submarine and the pulses returning from the sea-wall are traveling at 1522 m/s. As such, we can quickly determine the distance in water between two successive echo (or emitted) pulses of sound, which were emitted with a time interval between them of $\Delta t_{\text{emitted}}$. The actual distance in the seawater between the echoed pulses is given by $\Delta x = v_{\text{sound}}\Delta t_{\text{emitted}}$. Our goal is to find the time between successive pulses received by the submarine, $\Delta t_{\text{received}}$. We start our "clock", as it were, when the submarine passes one of two successive pulses that approach it, separated by the above distance Δx. After passing the first pulse, the next sound pulse moves toward the submarine at v_{sound} and the submarine moves toward the pulse at speed v_{sub}. The distance between successive pulses Δx may be divided into Δx_{sub} and Δx_{sound}, which are equal to $v_{\text{sub}}\Delta t_{\text{received}}$ and $v_{\text{sub}}\Delta t_{\text{received}}$, respectively.

The distance between successive pulses is given by:

$$\Delta \boldsymbol{x} = \Delta \boldsymbol{x}_{\text{sub}} + \Delta \boldsymbol{x}_{\text{received}}$$

Substituting for all three terms in this equation yields:

$$\boldsymbol{v}_{\text{sound}}\Delta \boldsymbol{t}_{\text{emitted}} = \boldsymbol{v}_{\text{sub}}\Delta \boldsymbol{t}_{\text{received}} + \boldsymbol{v}_{\text{sound}}\Delta \boldsymbol{t}_{\text{received}}$$

Solve for v_{sub} to obtain:

$$v_{\text{sub}} = \frac{v_{\text{sound}}\left(\Delta t_{\text{emitted}} - \Delta t_{\text{received}}\right)}{\Delta t_{\text{received}}}$$

Substitute numerical values and evaluate v_{sub}:

$$\boldsymbol{v}_{\text{sub}} = \frac{(1522\text{ m/s})(2.000\text{ s} - 1.980\text{ s})}{2.000\text{ s}} = \boxed{15\text{ m/s}}$$

Acceleration

58 • A sports car accelerates in third gear from 48.3 km/h (about 30 mi/h) to 80.5 km/h (about 50 mi/h) in 3.70 s. (*a*) What is the average acceleration of this car in m/s^2? (*b*) If the car maintained this acceleration, how fast would it be moving one second later?

Picture the Problem In Part (*a*), we can apply the definition of average acceleration to find a_{av}. In Part (*b*), we can find the change in the car's velocity in one second and add this change to its velocity at the beginning of the interval to find its speed one second later.

(*a*) The definition of average acceleration is:

$$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate a_{av}:

$$\boldsymbol{a}_{\text{av}} = \frac{80.5\text{ km/h} - 48.3\text{ km/h}}{3.70\text{ s}} = 8.70\frac{\text{km}}{\text{h}\cdot\text{s}}$$

Convert a_{av} to m/s^2:

$$a_{\text{av}} = \left(8.70\times10^3\frac{\text{m}}{\text{h}\cdot\text{s}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \boxed{2.42\text{ m/s}^2}$$

(*b*) Express the speed of the car at the end of 4.7 s:

$$\boldsymbol{v}(4.7\text{ s}) = \boldsymbol{v}(3.70\text{ s}) + \Delta\boldsymbol{v}_{1\text{ s}} = 80.5\text{ km/h} + \Delta\boldsymbol{v}_{1\text{ s}}$$

Find the change in the speed of the car in 1.00 s:

$$\Delta\boldsymbol{v} = \boldsymbol{a}_{\text{av}}\Delta\boldsymbol{t} = \left(8.70\frac{\text{km}}{\text{h}\cdot\text{s}}\right)(1.00\text{ s}) = 8.70\text{ km/h}$$

Substitute and evaluate $v(4.7\text{ s})$:

$$v(4.7\text{ s}) = 80.5\text{ km/h} + 8.7\text{ km/h} = \boxed{89.2\text{ km/h}}$$

59 • **[SSM]** An object is moving along the x axis. At $t = 5.0$ s, the object is at $x = +3.0$ m and has a velocity of +5.0 m/s. At $t = 8.0$ s, it is at $x = +9.0$ m and its velocity is –1.0 m/s. Find its average acceleration during the time interval 5.0 s ≤ t ≤ 8.0 s.

Picture the Problem We can find the change in velocity and the elapsed time from the given information and then use the definition of average acceleration.

The average acceleration is defined as the change in velocity divided by the change in time:

$$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{t}}$$

Substitute numerical values and evaluate a_{av}:

$$\boldsymbol{a}_{\text{av}} = \frac{(-1.0\ \text{m/s}) - (5.0\ \text{m/s})}{(8.0\ \text{s}) - (5.0\ \text{s})} = \boxed{-2.0\ \text{m/s}^2}$$

60 •• A particle moves along the x axis with velocity $v_x = (8.0\ \text{m/s}^2)\,t - 7.0$ m/s. (a) Find the average acceleration for two different one-second intervals, one beginning at $t = 3.0$ s and the other beginning at $t = 4.0$ s. (b) Sketch v_x versus t over the interval $0 < t < 10$ s. (c) How do the instantaneous accelerations at the middle of each of the two time intervals specified in Part (a) compare to the average accelerations found in Part (a)? Explain.

Picture the Problem The important concept here is the difference between average acceleration and instantaneous acceleration.

(a) The average acceleration is defined as the change in velocity divided by the change in time:

$$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{t}}$$

Determine v_x at $t = 3.0$ s, $t = 4.0$ s, and $t = 5.0$ s:

$$\boldsymbol{v}_x(3.0\ \text{s}) = (8.0\ \text{m/s}^2)(3.0\ \text{s}) - 7.0\ \text{m/s} = 17\ \text{m/s}$$

$$\boldsymbol{v}_x(4.0\ \text{s}) = (8.0\ \text{m/s}^2)(4.0\ \text{s}) - 7.0\ \text{m/s} = 25\ \text{m/s}$$

$$\boldsymbol{v}_x(5.0\ \text{s}) = (8.0\ \text{m/s}^2)(5.0\ \text{s}) - 7.0\ \text{m/s} = 33\ \text{m/s}$$

Find a_{av} for the two 1-s intervals:

$$\boldsymbol{a}_{\text{av, 3.0 s to 4.0 s}} = \frac{25\ \text{m/s} - 17\ \text{m/s}}{1.0\ \text{s}} = 8.0\ \text{m/s}^2$$

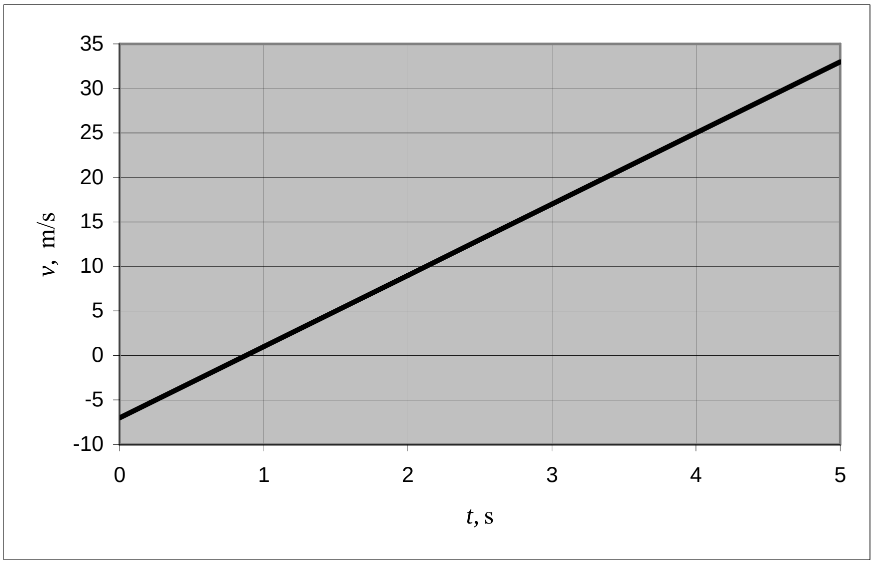
and

$$\boldsymbol{a}_{\text{av, 4.0 s to 5.0 s}} = \frac{33\ \text{m/s} - 25\ \text{m/s}}{1.0\ \text{s}} = 8.0\ \text{m/s}^2$$

The instantaneous acceleration is defined as the time derivative of the velocity or the slope of the velocity- versus-time curve:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left[\left(8.0\ \text{m/s}^2\right)t - 7.0\ \text{m/s}\right]$$
$$= \boxed{8.0\ \text{m/s}^2}$$

(*b*) The given function and a spreadsheet program were used to plot the following graph of *v*-versus-*t*:



(*c*) Because the particle's speed varies linearly with time, these accelerations are the same.

61 •• **[SSM]** The position of a certain particle depends on time according to the equation $x(t) = t^2 - 5.0t + 1.0$, where *x* is in meters if *t* is in seconds. (*a*) Find the displacement and average velocity for the interval $3.0\ \text{s} \le t \le 4.0\ \text{s}$. (*b*) Find the general formula for the displacement for the time interval from *t* to $t + \Delta t$. (*c*) Use the limiting process to obtain the instantaneous velocity for any time *t*.

Picture the Problem We can closely approximate the instantaneous velocity by the average velocity in the limit as the time interval of the average becomes small. This is important because all we can ever obtain from any measurement is the average velocity, v_{av}, which we use to approximate the instantaneous velocity *v*.

(*a*) The displacement of the particle during the interval $3.0\ \text{s} \le t \le 4.0\ \text{s}$ is given by:

$$\Delta x = x(4.0\ \text{s}) - x(3.0\ \text{s}) \qquad (1)$$

The average velocity is given by:

$$v_{av} = \frac{\Delta x}{\Delta t} \qquad (2)$$

Find x(4.0 s) and x(3.0 s):

$x(4.0\text{ s}) = (4.0)^2 - 5(4.0) + 1 = -3.0\text{ m}$
and
$x(3.0\text{ s}) = (3.0)^2 - 5(3.0) + 1 = -5.0\text{ m}$

Substitute numerical values in equation (1) and evaluate Δx:

$$\Delta \mathbf{x} = (-3.0\text{ m}) - (-5.0\text{ m}) = \boxed{2.0\text{ m}}$$

Substitute numerical values in equation (2) and evaluate v_{av}:

$$\boldsymbol{v}_{av} = \frac{2.0\text{ m}}{1.0\text{ s}} = \boxed{2.0\text{ m/s}}$$

(b) Find $x(t + \Delta t)$:

$$\begin{aligned} x(t+\Delta t) &= (t+\Delta t)^2 - 5(t+\Delta t) + 1 \\ &= (t^2 + 2t\Delta t + (\Delta t)^2) \\ &\qquad - 5(t+\Delta t) + 1 \end{aligned}$$

Express $x(t + \Delta t) - x(t) = \Delta x$:

$$\Delta x = \boxed{(2t-5)\Delta t + (\Delta t)^2}$$

where Δx is in meters if t is in seconds.

(c) From (b) find $\Delta x/\Delta t$ as $\Delta t \to 0$:

$$\begin{aligned} \frac{\Delta x}{\Delta t} &= \frac{(2t-5)\Delta t + (\Delta t)^2}{\Delta t} \\ &= 2t - 5 + \Delta t \end{aligned}$$

and

$$\boldsymbol{v} = \lim_{\Delta t \to 0}(\Delta \mathbf{x}/\Delta \boldsymbol{t}) = \boxed{2\boldsymbol{t} - 5}$$

where v is in m/s if t is in seconds.

Alternatively, we can take the derivative of $x(t)$ with respect to time to obtain the instantaneous velocity.

$$\begin{aligned} v(t) &= \frac{dx(t)}{dt} = \frac{d}{dt}\left(at^2 + bt + 1\right) \\ &= 2at + b \\ &= \boxed{2t - 5} \end{aligned}$$

62 •• The position of an object as a function of time is given by $x = At^2 - Bt + C$, where $A = 8.0\text{ m/s}^2$, $B = 6.0$ m/s, and $C = 4.0$ m. Find the instantaneous velocity and acceleration as functions of time.

Picture the Problem The instantaneous velocity is dx/dt and the acceleration is dv/dt.

Using the definitions of instantaneous velocity and acceleration, determine v and a:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[At^2 - Bt + C\right] = 2At - B$$

and

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[2At - B\right] = 2A$$

Substitute numerical values for A and B and evaluate v and a:

$$\boldsymbol{v} = 2\left(8.0\ \text{m/s}^2\right)\boldsymbol{t} - 6.0\ \text{m/s} = \boxed{\left(16.0\ \text{m/s}^2\right)\boldsymbol{t} - 6.0\ \text{m/s}}$$

and

$$\boldsymbol{a} = 2\left(8.0\ \text{m/s}^2\right) = \boxed{16.0\ \text{m/s}^2}$$

63 ••• The one-dimensional motion of a particle is plotted in Figure 2-40. (*a*) What is the average acceleration in each of the intervals *AB*, *BC*, and *CE*? (*b*) How far is the particle from its starting point after 10 s? (*c*) Sketch the displacement of the particle as a function of time; label the instants A, B, C, D, and E on your graph. (*d*) At what time is the particle traveling most slowly?

Picture the Problem We can use the definition of average acceleration ($a_{av} = \Delta v/\Delta t$) to find a_{av} for the three intervals of constant acceleration shown on the graph.

(*a*) Using the definition of average acceleration, find a_{av} for the interval AB:

$$a_{av\ AB} = \frac{15.0\ \text{m/s} - 5.0\ \text{m/s}}{3.0\ \text{s}} = \boxed{3.3\ \text{m/s}^2}$$

Find a_{av} for the interval BC:

$$a_{av\ BC} = \frac{15.0\ \text{m/s} - 15.0\ \text{m/s}}{3.0\ \text{s}} = \boxed{0}$$

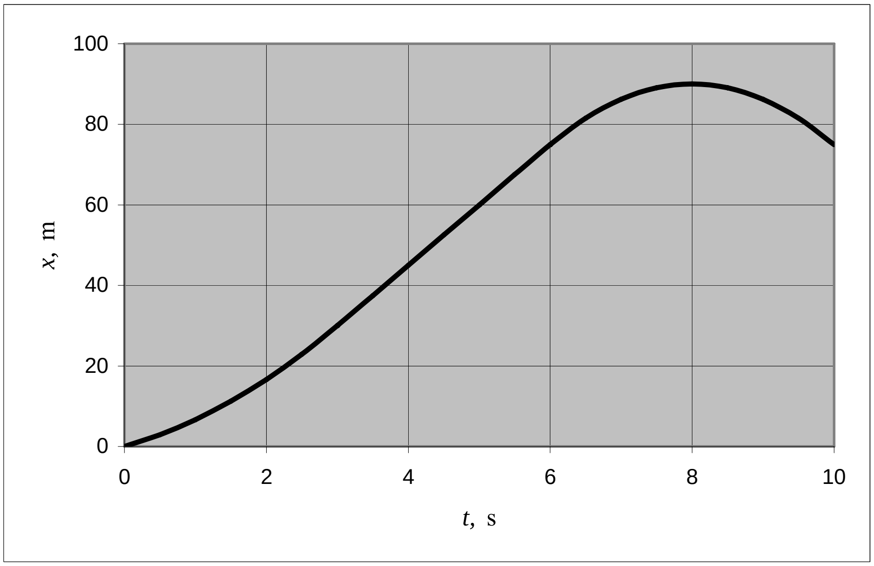
Find a_{av} for the interval CE:

$$a_{av\ CE} = \frac{-15.0\ \text{m/s} - 15.0\ \text{m/s}}{4.0\ \text{s}} = \boxed{-7.5\ \text{m/s}^2}$$

(*b*) Use the formulas for the areas of trapezoids and triangles to find the area under the graph of v as a function of t.

$$\begin{aligned}\Delta\boldsymbol{x} &= \left(\Delta\boldsymbol{x}\right)_{A\to B} + \left(\Delta\boldsymbol{x}\right)_{B\to C} + \left(\Delta\boldsymbol{x}\right)_{C\to D} + \left(\Delta\boldsymbol{x}\right)_{D\to E} \\ &= \tfrac{1}{2}\left(5.0\ \text{m/s} + 15.0\ \text{m/s}\right)\left(3.0\ \text{s}\right) + (15.0\ \text{m/s})(3.0\ \text{s}) + \tfrac{1}{2}(15.0\ \text{m/s})(2.0\ \text{s}) \\ &\qquad + \tfrac{1}{2}(-15.0\ \text{m/s})(2.0\ \text{s}) \\ &= \boxed{75\ \text{m}}\end{aligned}$$

(*c*) The graph of displacement, *x*, as a function of time, *t*, is shown in the following figure. In the region from B to C the velocity is constant so the *x*- versus-*t* curve is a straight line.



(d) Reading directly from the figure, we can find the time when the particle is moving the slowest. At point D, $t = 8$ s, the graph crosses the time axis; therefore $v = 0$.

Constant Acceleration and Free-Fall

64 • An object projected vertically upward with initial speed v_0 attains a maximum height h above its launch point. Another object projected up with initial speed $2v_0$ from the same height will attain a maximum height of (a) $4h$, (b) $3h$, (c) $2h$, (d) h. (Air resistance is negligible.)

Picture the Problem Because the acceleration is constant ($-g$) we can use a constant-acceleration equation to find the height of the projectile.

Using a constant-acceleration equation, express the height of the object as a function of its initial speed, the acceleration due to gravity, and its displacement:

$$v^2 = v_0^2 - 2g\Delta y$$

or, because $v(h) = 0$,

$$0 = v_0^2 - 2gh \Rightarrow h = \frac{v_0^2}{2g}$$

Express the ratio of the maximum height of the second object to that of the first object and simplify to obtain:

$$\frac{h_{\text{2nd object}}}{h_{\text{1st object}}} = \frac{\dfrac{(2v_0)^2}{2g}}{\dfrac{(v_0)^2}{2g}} = 4$$

Solving for $h_{\text{2nd object}}$ yields:

$$h_{\text{2nd object}} = 4h \Rightarrow (a) \text{ is correct.}$$

65 •• A car traveling along the x axis starts from rest at $x = 50$ m and accelerates at a constant rate of $8.0\ \text{m/s}^2$. (a) How fast is it going after 10 s?

(*b*) How far has it gone after 10 s? (*c*) What is its average velocity for the interval $0 \le t \le 10$ s?

Picture the Problem Because the acceleration of the car is constant we can use constant-acceleration equations to describe its motion.

(*a*) Using a constant-acceleration equation, relate the velocity to the acceleration and the time:

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{a}\boldsymbol{t}$$

Substitute numerical values and evaluate *v*:

$$\boldsymbol{v} = 0 + \left(8.0\,\frac{\text{m}}{\text{s}^2}\right)(10\,\text{s}) = \boxed{80\,\text{m/s}}$$

(*b*) Using a constant-acceleration equation, relate the displacement to the acceleration and the time:

$$\Delta x = x - x_0 = v_0 t + \frac{a}{2}t^2$$

Substitute numerical values and evaluate Δx:

$$\Delta\boldsymbol{x} = \frac{1}{2}\left(8.0\,\frac{\text{m}}{\text{s}^2}\right)(10\,\text{s})^2 = \boxed{0.40\,\text{km}}$$

(*c*) Use the definition of v_{av}:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta\boldsymbol{x}}{\Delta\boldsymbol{t}} = \frac{400\,\text{m}}{10\,\text{s}} = \boxed{40\,\text{m/s}}$$

Remarks: Because the area under a velocity-versus-time graph is the displacement of the object, we could solve this problem graphically.

66 • An object traveling along the *x* axis with an initial velocity of +5.0 m/s has a constant acceleration of +2.0 m/s^2. When its speed is 15 m/s, how far has it traveled?

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the speed of the object to its acceleration and displacement:

$$v^2 = v_0^2 + 2\,a\,\Delta x \Rightarrow \Delta\boldsymbol{x} = \frac{\boldsymbol{v}^2 - \boldsymbol{v}_0^2}{2\boldsymbol{a}}$$

Substitute numerical values and evaluate Δx:

$$\Delta\boldsymbol{x} = \frac{\left(15^2 - 5.0^2\right)\text{m}^2/\text{s}^2}{2\left(2.0\,\text{m/s}^2\right)} = \boxed{50\,\text{m}}$$

67 • **[SSM]** An object traveling along the *x* axis at constant acceleration has a velocity of +10 m/s when it is at *x* = 6.0 m and of +15 m/s when it is at *x* = 10 m. What is its acceleration?

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x \Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{\left(15^2 - 10^2\right)\text{m}^2/\text{s}^2}{2(10\,\text{m} - 6.0\,\text{m})} = \boxed{16\,\text{m/s}^2}$$

68 • The speed of an object traveling along the x axis increases at the constant rate of +4.0 m/s each second. At t = 0.0 s, its velocity is +1.0 m/s and its position is +7.0 m. How fast is it moving when its position is +8.0 m, and how much time has elapsed from the start at t = 0.0 s?

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x \Rightarrow \boldsymbol{v} = \sqrt{\boldsymbol{v}_0^2 + 2\boldsymbol{a}\,\Delta\boldsymbol{x}}$$

Substitute numerical values and evaluate v to obtain:

$$\boldsymbol{v} = \sqrt{(1.0\,\text{m/s})^2 + 2\left(4.0\,\text{m/s}^2\right)(8.0\,\text{m} - 7.0\,\text{m})} = \boxed{3.0\,\text{m/s}}$$

From the definition of average acceleration we have:

$$\Delta\boldsymbol{t} = \frac{\Delta\boldsymbol{v}}{\boldsymbol{a}_{\text{av}}}$$

Substitute numerical values and evaluate Δt:

$$\Delta\boldsymbol{t} = \frac{3.0\,\text{m/s} - 1.0\,\text{m/s}}{4.0\,\text{m/s}^2} = \boxed{0.50\,\text{s}}$$

69 •• A ball is launched directly upward from ground level with an initial speed of 20 m/s. (Air resistance is negligible.) (*a*) How long is the ball in the air? (*b*) What is the greatest height reached by the ball? (*c*) How many seconds after launch is the ball 15 m above the release point?

Picture the Problem In the absence of air resistance, the ball experiences constant acceleration. Choose a coordinate system with the origin at the point of release and the positive direction upward.

(*a*) Using a constant-acceleration equation, relate the displacement of the ball to the acceleration and the time:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

Setting $\Delta y = 0$ (the displacement for a round trip), solve for the time required for the ball to return to its starting position:

$$0 = \boldsymbol{v}_0 \boldsymbol{t}_{\text{roundtrip}} + \tfrac{1}{2}\boldsymbol{a}\boldsymbol{t}^2_{\text{roundtrip}}$$

and

$$\boldsymbol{t}_{\text{round trip}} = \frac{2\boldsymbol{v}_0}{\boldsymbol{g}}$$

Substitute numerical values and evaluate $t_{\text{round trip}}$:

$$t_{\text{round trip}} = \frac{2(20\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{4.1\,\text{s}}$$

(*b*) Using a constant-acceleration equation, relate the final speed of the ball to its initial speed, the acceleration, and its displacement:

$$v^2_{\text{top}} = v_0^2 + 2a\Delta y$$

or, because $v_{\text{top}} = 0$ and $a = -g$,

$$0 = v_0^2 + 2(-g)H \Rightarrow \boldsymbol{H} = \frac{\boldsymbol{v}_0^2}{2\boldsymbol{g}}$$

Substitute numerical values and evaluate *H*:

$$\boldsymbol{H} = \frac{(20\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{20\,\text{m}}$$

(*c*) Using the same constant-acceleration equation with which we began part (*a*), express the displacement as a function of time:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

Substitute numerical values to obtain:

$$15\,\text{m} = (20\,\text{m/s})t - \left(\frac{9.81\,\text{m/s}^2}{2}\right)t^2$$

Use your graphing calculator or the quadratic formula to solve this equation for the times at which the displacement of the ball is 15 m:

The solutions are $\boldsymbol{t} = \boxed{0.99\,\text{s}}$ (this corresponds to passing 15 m on the way up) and $\boldsymbol{t} = \boxed{3.1\,\text{s}}$ (this corresponds to passing 15 m on the way down).

70 •• In the Blackhawk landslide in California, a mass of rock and mud fell 460 m down a mountain and then traveled 8.00 km across a level plain. It has been theorized that the rock and mud moved on a cushion of water vapor. Assume that the mass dropped with the free-fall acceleration and then slid horizontally, losing speed at a constant rate. (*a*) How long did the mud take to drop the 460 m? (*b*) How fast was it traveling when it reached the bottom? (*c*) How long did the mud take to slide the 8.00 km horizontally?

Picture the Problem This is a multipart constant-acceleration problem using two different constant accelerations. We'll choose a coordinate system in which downward is the positive direction and apply constant-acceleration equations to find the required times.

(*a*) Using a constant-acceleration equation, relate the time for the slide to the distance of fall and the acceleration:

$$\Delta y = y - y_0 = h - 0 = v_0 t_1 + \tfrac{1}{2} a t_1^2$$

or, because $v_0 = 0$,

$$h = \tfrac{1}{2} a t_1^2 \Rightarrow t_1 = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate t_1:

$$\boldsymbol{t}_1 = \sqrt{\frac{2(460\,\text{m})}{9.81\,\text{m/s}^2}} = 9.684\,\text{s} = \boxed{9.68\,\text{s}}$$

(*b*) Using a constant-acceleration equation, relate the velocity at the bottom of the mountain to the acceleration and time:

$$v_1 = v_0 + a_1 t_1$$

or, because $v_0 = 0$ and $a_1 = g$,

$$v_1 = g t_1$$

Substitute numerical values and evaluate v_1:

$$\boldsymbol{v}_1 = (9.81\,\text{m/s}^2)(9.684\,\text{s}) = \boxed{95.0\,\text{m/s}}$$

(*c*) Using a constant-acceleration equation, relate the time required to stop the mass of rock and mud to its average speed and the distance it slides:

$$\Delta t = \frac{\Delta x}{v_{\text{av}}}$$

Because the acceleration is constant:

$$v_{\text{av}} = \frac{v_1 + v_{\text{f}}}{2} = \frac{v_1 + 0}{2} = \frac{v_1}{2}$$

Substitute for v_{av} to obtain:

$$\Delta t = \frac{2\Delta x}{v_1}$$

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{2(8000\,\text{m})}{95.0\,\text{m/s}} = \boxed{168\,\text{s}}$$

71 •• **[SSM]** A load of bricks is lifted by a crane at a steady velocity of 5.0 m/s when one brick falls off 6.0 m above the ground. (*a*) Sketch the position of the brick *y*(*t*) versus time from the moment it leaves the pallet until it hits the ground. (*b*) What is the greatest height the brick reaches above the ground? (*c*) How long does it take to reach the ground? (*d*) What is its speed just before it hits the ground?

Picture the Problem In the absence of air resistance, the brick experiences constant acceleration and we can use constant-acceleration equations to describe its motion. Constant acceleration implies a parabolic position-versus-time curve.

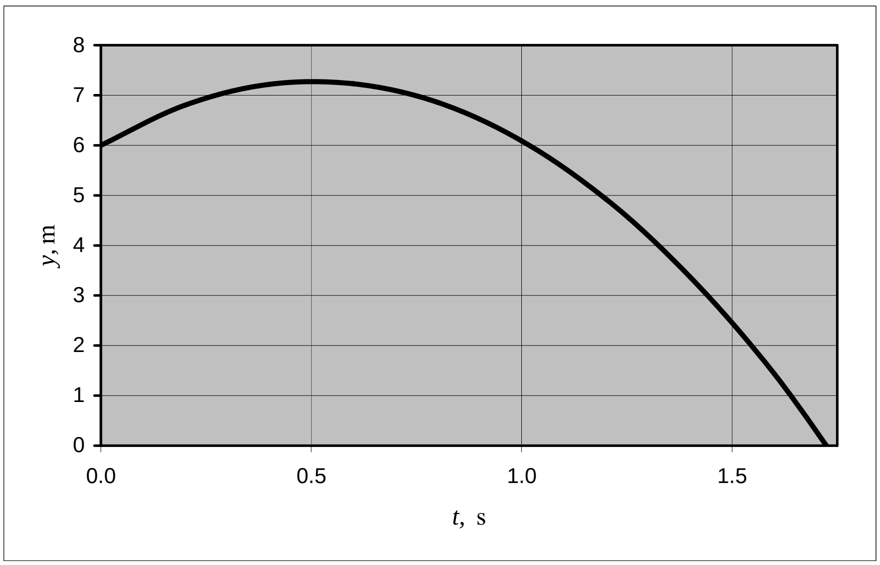
(a) Using a constant-acceleration equation, relate the position of the brick to its initial position, initial velocity, acceleration, and time into its fall:

$$y = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$

Substitute numerical values to obtain:

$$y = 6.0\,\text{m} + (5.0\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2 \qquad (1)$$

The following graph of $y = 6.0\,\text{m} + (5.0\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$ was plotted using a spreadsheet program:



(b) Relate the greatest height reached by the brick to its height when it falls off the load and the additional height it rises $\Delta y_{\max}$:

$$h = y_0 + \Delta y_{\max} \qquad (2)$$

Using a constant-acceleration equation, relate the height reached by the brick to its acceleration and initial velocity:

$$v_{\text{top}}^2 = v_0^2 + 2(-g)\Delta y_{\max}$$

or, because $v_{\text{top}} = 0$,

$$0 = v_0^2 + 2(-g)\Delta y_{\max} \Rightarrow \Delta y_{\max} = \frac{v_0^2}{2g}$$

Substitute numerical values and evaluate $\Delta y_{\max}$:

$$\Delta y_{\max} = \frac{(5.0\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 1.3\,\text{m}$$

Substitute numerical values in equation (2) and evaluate h:

$$h = 6.0\,\text{m} + 1.3\,\text{m} = \boxed{7.3\,\text{m}}$$

Note that the graph shown above confirms this result.

(c) Setting $y = 0$ in equation (1) yields:

$$0 = 6.0\,\text{m} + (5.0\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$$

Use the quadratic equation or your graphing calculator to obtain:

$\boldsymbol{t} = \boxed{1.7\,\text{s}}$ and $t = -0.71$ s. Note that the second solution is nonphysical.

(d) Using a constant-acceleration equation, relate the speed of the brick on impact to its acceleration and displacement:

$$v^2 = v_0^2 + 2gh$$

or, because $v_0 = 0$,

$$v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{2(9.81\,\text{m/s}^2)(7.3\,\text{m})} = \boxed{12\,\text{m/s}}$$

72 •• A bolt comes loose from underneath an elevator that is moving upward at a speed of 6.0 m/s. The bolt reaches the bottom of the elevator shaft in 3.0 s. (a) How high above the bottom of the shaft was the elevator when the bolt came loose? (b) What is the speed of the bolt when it hits the bottom of the shaft?

Picture the Problem In the absence of air resistance, the acceleration of the bolt is constant. Choose a coordinate system in which upward is positive and the origin is at the bottom of the shaft ($y = 0$).

(a) Using a constant-acceleration equation, relate the position of the bolt to its initial position, initial velocity, and fall time:

$$y_{\text{bottom}} = 0 = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$

Solve for the position of the bolt when it came loose:

$$y_0 = -v_0 t + \tfrac{1}{2} g t^2$$

Substitute numerical values and evaluate y_0:

$$\boldsymbol{y}_0 = -\left(6.0\,\frac{\text{m}}{\text{s}}\right)(3.0\,\text{s}) + \tfrac{1}{2}\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(3.0\,\text{s})^2 = \boxed{26\,\text{m}}$$

(b) Using a constant-acceleration equation, relate the speed of the bolt to its initial speed, acceleration, and fall time:

$$v = v_0 + at$$

Substitute numerical values and evaluate $|v|$:

$$\boldsymbol{v} = 6.0\,\frac{\text{m}}{\text{s}} - \left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(3.0\text{ s}) = -23\text{ m/s}$$

and

$$|\boldsymbol{v}| = \boxed{23\text{ m/s}}$$

73 •• An object is dropped from rest at a height of 120 m. Find the distance it falls during its final second in the air.

Picture the Problem In the absence of air resistance, the object's acceleration is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release. In this coordinate system, $a = g$ and $y = 120$ m at the bottom of the fall.

Express the distance fallen in the last second in terms of the object's position at impact and its position 1 s before impact:

$$\Delta y_{\text{last second}} = 120\text{ m} - y_{1\text{ s before impact}} \qquad (1)$$

Using a constant-acceleration equation, relate the object's position upon impact to its initial position, initial velocity, and fall time:

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

or, because $y_0 = 0$ and $v_0 = 0$,

$$y = \tfrac{1}{2} g t_{\text{fall}}^2 \Rightarrow t_{\text{fall}} = \sqrt{\frac{2y}{g}}$$

Substitute numerical values and evaluate t_{fall}:

$$t_{\text{fall}} = \sqrt{\frac{2(120\text{ m})}{9.81\text{ m/s}^2}} = 4.95\text{ s}$$

We know that, one second before impact, the object has fallen for 3.95 s. Using the same constant-acceleration equation, calculate the object's position 3.95 s into its fall:

$$y(3.95\text{ s}) = \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)(3.95\text{ s})^2 = 76.4\text{ m}$$

Substitute in equation (1) to obtain:

$$\Delta \boldsymbol{y}_{\text{last second}} = 120\text{ m} - 76.4\text{ m} = \boxed{44\text{ m}}$$

74 •• An object is released from rest at a height h. During the final second of its fall, it traverses a distance of 38 m. Determine h.

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system with the origin at the point of release and downward as the positive direction.

Using a constant-acceleration equation, relate the height h of the object to its initial and final velocities and acceleration:

$$v_f^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$, $a = g$, and $\Delta y = h$,

$$v_f^2 = 2gh \Rightarrow h = \frac{v_f^2}{2g} \quad (1)$$

Using the definition of average velocity, find the average velocity of the object during its final second of fall:

$$v_{av} = \frac{v_{f\text{-}1s} + v_f}{2} = \frac{\Delta y}{\Delta t} = \frac{38\,\text{m}}{1\,\text{s}} = 38\,\text{m/s}$$

Express the sum of the final velocity and the velocity 1 s before impact:

$$v_{f\text{-}1s} + v_f = 2(38\,\text{m/s}) = 76\,\text{m/s}$$

From the definition of acceleration, we know that the change in velocity of the object, during 1 s of fall, is 9.81 m/s:

$$\Delta \boldsymbol{v} = \boldsymbol{v}_f - \boldsymbol{v}_{f\text{-}1s} = 9.81\ \text{m/s}$$

Add the equations that express the sum and difference of $\boldsymbol{v}_{f\text{-}1s}$ and $\boldsymbol{v}_f$ to obtain:

$$\boldsymbol{v}_{f\text{-}1s} + \boldsymbol{v}_f + \boldsymbol{v}_f - \boldsymbol{v}_{f\text{-}1s} = 76\,\text{m/s} + 9.81\ \text{m/s}$$

Solving for $\boldsymbol{v}_f$ yields:

$$\boldsymbol{v}_f = \frac{76\,\text{m/s} + 9.81\,\text{m/s}}{2} = 43\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate h:

$$\boldsymbol{h} = \frac{(43\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{94\,\text{m}}$$

75 •• **[SSM]** A stone is thrown vertically downward from the top of a 200-m cliff. During the last half second of its flight, the stone travels a distance of 45 m. Find the initial speed of the stone.

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system with the origin at the bottom of the trajectory and the upward direction positive. Let $v_{f\text{-}1/2}$ be the speed one-half second before impact and v_f the speed at impact.

Using a constant-acceleration equation, express the final speed of the stone in terms of its initial speed, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2a\Delta y \Rightarrow v_0 = \sqrt{v_f^2 + 2g\Delta y} \quad (1)$$

Find the average speed in the last half second:

$$v_{av} = \frac{v_{f\text{-}1/2} + v_f}{2} = \frac{\Delta x_{\text{last half second}}}{\Delta t} = \frac{45\,\text{m}}{0.5\,\text{s}} = 90\,\text{m/s}$$

and

$$v_{f\text{-}1/2} + v_f = 2(90\,\text{m/s}) = 180\,\text{m/s}$$

Using a constant-acceleration equation, express the change in speed of the stone in the last half second in terms of the acceleration and the elapsed time and solve for the change in its speed:

$$\Delta v = v_f - v_{f\text{-}1/2} = g\Delta t = \left(9.81\,\text{m/s}^2\right)(0.5\,\text{s}) = 4.91\,\text{m/s}$$

Add the equations that express the sum and difference of $v_{f-½}$ and v_f and solve for v_f:

$$v_f = \frac{180\,\text{m/s} + 4.91\,\text{m/s}}{2} = 92.5\,\text{m/s}$$

Substitute numerical values in equation (1) and evaluate v_0:

$$\boldsymbol{v}_0 = \sqrt{(92.5\text{ m/s})^2 + 2\left(9.81\text{ m/s}^2\right)(-200\text{m})} = \boxed{68\,\text{m/s}}$$

Remarks: The stone may be thrown either up or down from the cliff and the results after it passes the cliff on the way down are the same.

76 •• An object is released from rest at a height *h*. It travels 0.4*h* during the first second of its descent. Determine the average velocity of the object during its entire descent.

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system in which downward is the positive direction and the object starts from rest. Apply constant-acceleration equations to find the average velocity of the object during its descent.

Express the average velocity of the falling object in terms of its initial and final velocities:

$$v_{av} = \frac{v_0 + v_f}{2}$$

Using a constant-acceleration equation, express the displacement of the object during the 1st second in terms of its acceleration and the elapsed time:

$$\Delta y_{\text{1st second}} = \frac{gt^2}{2} = 4.91\,\text{m} = 0.4\,h$$

Solving for the displacement h yields:

$h = 12.3$ m

Using a constant-acceleration equation, express the final velocity of the object in terms of its initial velocity, acceleration, and displacement:

$v_f^2 = v_0^2 + 2g\Delta y$

or, because $v_0 = 0$,

$v_f = \sqrt{2g\Delta y}$

Substitute numerical values and evaluate the final velocity of the object:

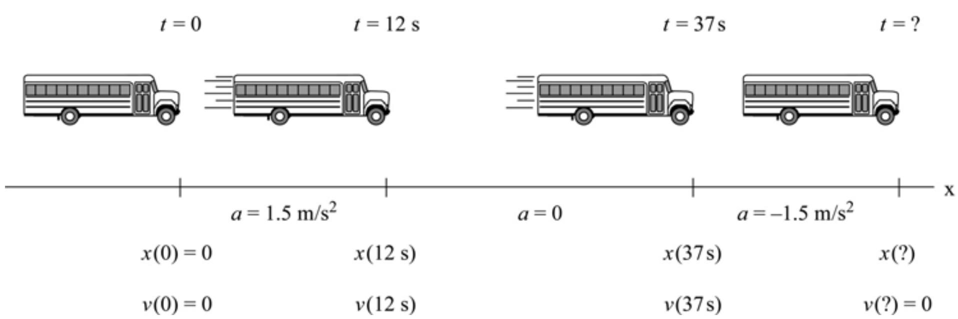
$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(12.3\,\text{m})} = 15.5\,\text{m/s}$$

Substitute in the equation for the average velocity to obtain:

$$\boldsymbol{v}_{av} = \frac{0 + 15.5\,\text{m/s}}{2} = \boxed{7.77\,\text{m/s}}$$

77 •• A bus accelerates from rest at 1.5 m/s^2 for 12 s. It then travels at constant velocity for 25 s, after which it slows to a stop with an acceleration of 1.5 m/s^2. (*a*) What is the total distance that the bus travels? (*b*) What is its average velocity?

Picture the Problem This is a three-part constant-acceleration problem. The bus starts from rest and accelerates for a given period of time, and then it travels at a constant velocity for another period of time, and, finally, decelerates uniformly to a stop. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the total displacement of the bus during the three intervals of time:

$$\Delta x_{total} = \Delta x(0 \to 12\,\text{s}) + \Delta x(12\,\text{s} \to 37\,\text{s}) + \Delta x(37\,\text{s} \to \text{end})$$

Using a constant-acceleration equation, express the displacement of the bus during its first 12 s of motion in terms of its initial velocity, acceleration, and the elapsed time; solve for its displacement:

$\Delta x(0 \to 12\,\text{s}) = v_0 t + \frac{1}{2}at^2$

or, because $v_0 = 0$,

$\Delta x(0 \to 12\,\text{s}) = \frac{1}{2}at^2 = 108\,\text{m}$

Using a constant-acceleration equation, express the velocity of the bus after 12 seconds in terms of its initial velocity, acceleration, and the elapsed time; solve for its velocity at the end of 12 s:

$$v_{12\,\mathrm{s}} = v_0 + a_{0\to 12\,\mathrm{s}}\Delta t = \left(1.5\,\mathrm{m/s^2}\right)(12\,\mathrm{s})$$
$$= 18\,\mathrm{m/s}$$

During the next 25 s, the bus moves with a constant velocity. Using the definition of average velocity, express the displacement of the bus during this interval in terms of its average (constant) velocity and the elapsed time:

$$\Delta x(12\,\mathrm{s} \to 37\,\mathrm{s}) = v_{12\,\mathrm{s}}\Delta t = (18\,\mathrm{m/s})(25\,\mathrm{s})$$
$$= 450\,\mathrm{m}$$

Because the bus slows down at the same rate that its velocity increased during the first 12 s of motion, we can conclude that its displacement during this braking period is the same as during its acceleration period and the time to brake to a stop is equal to the time that was required for the bus to accelerate to its cruising speed of 18 m/s. Hence:

$$\Delta x(37\,\mathrm{s} \to 49\mathrm{s}) = 108\,\mathrm{m}$$

Add the displacements to find the distance the bus traveled:

$$\Delta x_{\mathrm{total}} = 108\,\mathrm{m} + 450\,\mathrm{m} + 108\,\mathrm{m}$$
$$= 666\,\mathrm{m} = \boxed{0.67\,\mathrm{km}}$$

(*b*) Use the definition of average velocity to calculate the average velocity of the bus during this trip:

$$\boldsymbol{v}_{\mathrm{av}} = \frac{\Delta \boldsymbol{x}_{\mathrm{total}}}{\Delta \boldsymbol{t}} = \frac{666\,\mathrm{m}}{49\,\mathrm{s}} = \boxed{14\,\mathrm{m/s}}$$

Remarks: One can also solve this problem graphically. Recall that the area under a velocity as a function-of-time graph equals the displacement of the moving object.

78 •• Al and Bert are jogging side-by-side on a trail in the woods at a speed of 0.75 m/s. Suddenly Al sees the end of the trail 35 m ahead and decides to speed up to reach it. He accelerates at a constant rate of 0.50 m/s^2, while Bert continues on at a constant speed. (*a*) How long does it take Al to reach the end of the trail? (*b*) Once he reaches the end of the trail, he immediately turns around and heads back along the trail with a constant speed of 0.85 m/s. How long does it take him to meet up with Bert? (*c*) How far are they from the end of the trail when they meet?

Picture the Problem Because the accelerations of both Al and Bert are constant, constant-acceleration equations can be used to describe their motions. Choose the origin of the coordinate system to be where Al decides to begin his sprint.

(*a*) Using a constant-acceleration equation, relate Al's initial velocity, his acceleration, and the time to reach the end of the trail to his displacement in reaching the end of the trail:

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

Substitute numerical values to obtain:

$$35\,\text{m} = (0.75\,\text{m/s})\boldsymbol{t} + \tfrac{1}{2}(0.50\,\text{m/s}^2)\boldsymbol{t}^2$$

Use your graphing calculator or the quadratic formula to solve for the time required for Al to reach the end of the trail:

$$\boldsymbol{t} = 10.43\,\text{s} = \boxed{10\,\text{s}}$$

(*b*) Using constant-acceleration equations, express the positions of Bert and Al as functions of time. At the instant Al turns around at the end of the trail, $t = 0$. Also, $x = 0$ at a point 35 m from the end of the trail:

$$\boldsymbol{x}_{\text{Bert}} = \boldsymbol{x}_{\text{Bert,0}} + (0.75\ \text{m/s})\boldsymbol{t}$$

and

$$x_{\text{Al}} = x_{\text{Al,0}} - (0.85\,\text{m/s})t = 35\,\text{m} - (0.85\,\text{m/s})t$$

Calculate Bert's position at $t = 0$. At that time he has been running for 10.4 s:

$$\boldsymbol{x}_{\text{Bert,0}} = (0.75\,\text{m/s})(10.43\,\text{s}) = 7.823\,\text{m}$$

Because Bert and Al will be at the same location when they meet, equate their position functions and solve for t:

$$7.823\,\text{m} + (0.75\,\text{m/s})\boldsymbol{t} = 35\,\text{m} - (0.85\,\text{m/s})\boldsymbol{t}$$

and

$$\boldsymbol{t} = 16.99\ \text{s}$$

To determine the elapsed time from when Al began his accelerated run, we need to add 10.43 s to this time:

$$\boldsymbol{t}_{\text{start}} = 16.99\,\text{s} + 10.43\,\text{s} = 27.42\,\text{s} = \boxed{27\,\text{s}}$$

(*c*) Express Bert's distance from the end of the trail when he and Al meet:

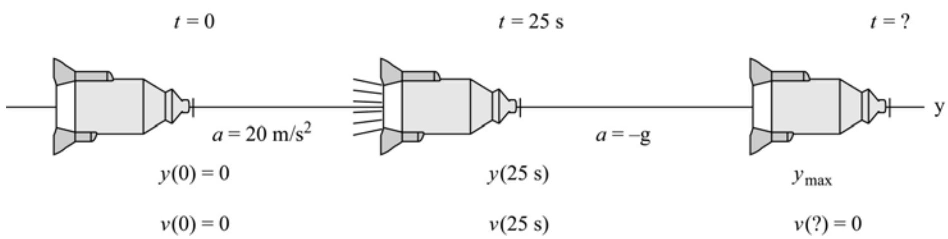
$$\boldsymbol{d}_{\text{end of trail}} = 35\,\text{m} - \boldsymbol{x}_{\text{Bert,0}} - \boldsymbol{d}_{\text{Bert runs until he meets Al}}$$

Substitute numerical values and evaluate $d_{\text{end of trail}}$:

$$d_{\text{end of trail}} = 35\,\text{m} - 7.823\,\text{m} - (16.99\,\text{s})(0.75\,\text{m/s}) = 14.43\,\text{m} = \boxed{14\,\text{m}}$$

79 •• You have designed a rocket to be used to sample the local atmosphere for pollution. It is fired vertically with a constant upward acceleration of 20 m/s^2. After 25 s, the engine shuts off and the rocket continues rising (in freefall) for a while. (Neglect any effects due to air resistance.) The rocket eventually stops rising and then falls back to the ground. You want to get a sample of air that is 20 km above the ground. (*a*) Did you reach your height goal? If not, what would you change so that the rocket reaches 20 km? (*b*) Determine the total time the rocket is in the air. (*c*) Find the speed of the rocket just before it hits the ground.

Picture the Problem This is a two-part constant-acceleration problem. Choose a coordinate system in which the upward direction is positive and to the right. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the highest point the rocket reaches, *h*, as the sum of its displacements during the first two stages of its flight:

$$h = \Delta y_{\text{1st stage}} + \Delta y_{\text{2nd stage}} \qquad (1)$$

Using a constant-acceleration equation, express the altitude reached in the first stage in terms of the rocket's initial velocity, acceleration, and burn time; solve for the first stage altitude:

$$\Delta y_{\text{1st stage}} = v_{0y}t + \tfrac{1}{2}a_{\text{1st stage}}t^2_{\text{1st stage}}$$

or, because $y_{0y} = 0$,

$$\Delta y_{\text{1st stage}} = \tfrac{1}{2}a_{\text{1st stage}}t^2_{\text{1st stage}}$$

Using a constant-acceleration equation, express the speed of the rocket at the end of its first stage in terms of its initial speed, acceleration, and displacement:

$$v_{\text{1st stage}} = v_{0y} + a_{\text{1st stage}}t_{\text{1st stage}}$$

or, because $y_{0y} = 0$,

$$v_{\text{1st stage}} = a_{\text{1st stage}}t_{\text{1st stage}}$$

Using a constant-acceleration equation, express the final speed of the rocket during the remainder of its climb in terms of its shut-off speed, free-fall acceleration, and displacement:

$$v_{\text{highest point}}^2 = v_{\text{shutoff}}^2 + 2a_{\text{2nd stage}}\Delta y_{\text{2nd stage}}$$

or, because $v_{\text{highest point}} = 0$ and $a_{\text{2nd stage}} = -g$,

$$0 = v_{\text{shutoff}}^2 - 2g\Delta y_{\text{2nd stage}}$$

Solving for $\Delta y_{\text{2nd stage}}$ yields:

$$\Delta y_{\text{2nd stage}} = \frac{v_{\text{shutoff}}^2}{2g}$$

Using a constant-acceleration equation, express the speed of the rocket at the end of its first stage in terms is its initial speed, acceleration, and displacement:

$$v_{\text{1st stage}} = v_0 + a_{\text{1st stage}}t_{\text{1st stage}}$$

or, because $v_0 = 0$ and $v_{\text{1st stage}} = v_{\text{shutoff}}$,

$$v_{\text{shutoff}} = a_{\text{1st stage}}t_{\text{1st stage}}$$

Substituting for v_{shutoff} yields:

$$\Delta y_{\text{2nd stage}} = \frac{\left(a_{\text{1st stage}}t_{\text{1st stage}}\right)^2}{2g}$$

Substitute for $\Delta y_{\text{1st stage}}$ and $\Delta y_{\text{2nd stage}}$ in equation (1) and simplify to obtain:

$$h = \tfrac{1}{2}a_{\text{1st stage}}t_{\text{1st stage}}^2 + \frac{\left(a_{\text{1st stage}}t_{\text{1st stage}}\right)^2}{2g}$$

$$= \left(\frac{1}{2} + \frac{a_{\text{1st stage}}}{2g}\right)a_{\text{1st stage}}t_{\text{1st stage}}^2$$

Substitute numerical values and evaluate *h*:

$$h = \left(\frac{1}{2} + \frac{20\,\frac{\text{m}}{\text{s}^2}}{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)}\right)\left(20\,\frac{\text{m}}{\text{s}^2}\right)(25\text{ s})^2$$

$$\approx 19\text{ km}$$

You did not achieve your goal. To go higher, you can increase the acceleration value or the time of acceleration.

(*b*) Express the total time the rocket is in the air in terms of the three segments of its flight:

$$\Delta t_{\text{total}} = \Delta t_{\text{powered climb}} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}}$$

$$= 25\text{ s} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}}$$

Express $\Delta t_{\text{2nd segment}}$ in terms of the rocket's displacement and average speed during the second segment:

$$\Delta t_{\text{2nd segment}} = \frac{\Delta y_{\text{2nd segment}}}{v_{\text{av, 2nd segment}}}$$

Substituting for $\Delta y_{\text{2nd segment}}$ and simplifying yields:

$$\Delta t_{\text{2nd segment}} = \frac{\dfrac{\left(a_{\text{1st stage}} t_{\text{1st stage}}\right)^2}{2g}}{v_{\text{av, 2nd segment}}} = \frac{\left(a_{\text{1st stage}} t_{\text{1st stage}}\right)^2}{2g v_{\text{av, 2nd segment}}}$$

Using a constant-acceleration equation, relate the fall distance to the descent time:

$$\Delta y_{\text{descent}} = v_0 t + \tfrac{1}{2} g \left(\Delta t_{\text{descent}}\right)^2$$

or, because $v_0 = 0$,

$$\Delta y_{\text{descent}} = \tfrac{1}{2} g \left(\Delta t_{\text{descent}}\right)^2$$

Solving for $\Delta t_{\text{descent}}$ yields:

$$\Delta t_{\text{descent}} = \sqrt{\frac{2\Delta y_{\text{descent}}}{g}}$$

Substitute for $\Delta t_{\text{2nd segment}}$ and $\Delta t_{\text{descent}}$ in the expression for Δt_{total} to obtain:

$$\Delta t_{\text{total}} = 25\,\text{s} + \frac{\left(a_{\text{1st stage}} \Delta t_{\text{1st stage}}\right)^2}{2g v_{\text{av, 2nd segment}}} + \sqrt{\frac{2\Delta y_{\text{descent}}}{g}}$$

Noting that, because the acceleration is constant, $v_{\text{av, 2nd segment}}$ is the average of the initial and final speeds during the second stage, substitute numerical values and evaluate Δt_{total} :

$$\Delta t_{\text{total}} = 25\,\text{s} + \frac{\left(\left(20\,\dfrac{\text{m}}{\text{s}^2}\right)(25\,\text{s})\right)^2}{2\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)\left(\dfrac{0 + 500\,\dfrac{\text{m}}{\text{s}}}{2}\right)} + \sqrt{\frac{2\left(19\times10^3\ \text{m}\right)}{9.81\,\dfrac{\text{m}}{\text{s}^2}}} \approx \boxed{1.4\times10^2\ \text{s}}$$

(c) Using a constant-acceleration equation, express the impact velocity of the rocket in terms of its initial downward velocity, acceleration under free-fall, and time of descent; solve for its impact velocity:

$$v_{\text{impact}} = v_0 + g\Delta t_{\text{descent}}$$

and, because $v_0 = 0$,

$$v_{\text{impact}} = g\Delta t_{\text{descent}}$$

Substituting for $\Delta t_{\text{descent}}$ yields:

$$v_{\text{impact}} = g\sqrt{\frac{2\Delta y_{\text{descent}}}{g}} = \sqrt{2g\Delta y_{\text{descent}}}$$

Substitute numerical values and evaluate v_{impact}:

$$v_{\text{impact}} = \sqrt{2(9.81\,\text{m/s}^2)(19\,\text{km})} = \boxed{6.1\times10^2\,\text{m/s}}$$

80 •• A flowerpot falls from the ledge of an apartment building. A person in an apartment below, coincidentally in possession of a high-speed high-precision timing system, notices that it takes 0.20 s for the pot to fall past his 4.0-m high window. How far above the top of the window is the ledge from which the pot fell? (Air resistance is negligible.)

Picture the Problem In the absence of air resistance, the acceleration of the flowerpot is constant. Choose a coordinate system in which downward is positive and the origin is at the point from which the flowerpot fell. Let t = time when the pot is at the top of the window, and $t + \Delta t$ the time when the pot is at the bottom of the window. To find the distance from the ledge to the top of the window, first find the time t_{top} that it takes the pot to fall to the top of the window.

Using a constant-acceleration equation, express the distance y below the ledge from which the pot fell as a function of time:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

or, because $a = g$ and $v_0 = y_0 = 0$,

$$y = \tfrac{1}{2}gt^2$$

Express the position of the pot as it reaches the top of the window:

$$y_{\text{top}} = \tfrac{1}{2}gt_{\text{top}}^2 \qquad (1)$$

Express the position of the pot as it reaches the bottom of the window:

$$y_{\text{bottom}} = \tfrac{1}{2}g\left(t_{\text{top}} + \Delta t_{\text{window}}\right)^2$$

where $\Delta t_{\text{window}} = t_{\text{top}} - t_{\text{bottom}}$

Subtract y_{bottom} from y_{top} to obtain an expression for the displacement Δy_{window} of the pot as it passes the window:

$$\Delta y_{\text{window}} = \tfrac{1}{2}g\left[\left(t_{\text{top}} + \Delta t_{\text{window}}\right)^2 - t_{\text{top}}^2\right] = \tfrac{1}{2}g\left[2t_{\text{top}}\Delta t_{\text{window}} + \left(\Delta t_{\text{window}}\right)^2\right]$$

Solving for t_{top} yields:

$$t_{\text{top}} = \frac{\dfrac{2\Delta y_{\text{window}}}{g} - \left(\Delta t_{\text{window}}\right)^2}{2\Delta t_{\text{window}}}$$

Substitute for t_{top} in equation (1) to obtain:

$$y_{\text{top}} = \tfrac{1}{2}g\left(\frac{\dfrac{2\Delta y_{\text{window}}}{g} - \left(\Delta t_{\text{window}}\right)^2}{2\Delta t_{\text{window}}}\right)^2$$

Substitute numerical values and evaluate y_{top}:

$$y_{top} = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(\frac{\dfrac{2(4.0\,\text{m})}{9.81\,\text{m/s}^2} - (0.20\,\text{s})^2}{2(0.20\,\text{s})}\right)^2 = \boxed{18\,\text{m}}$$

81 •• **[SSM]** In a classroom demonstration, a glider moves along an inclined air track with constant acceleration. It is projected from the low end of the track with an initial velocity. After 8.00 s have elapsed, it is 100 cm from the low end and is moving along the track at a velocity of –15 cm/s. Find the initial velocity and the acceleration.

Picture the Problem The acceleration of the glider on the air track is constant. Its average acceleration is equal to the instantaneous (constant) acceleration. Choose a coordinate system in which the initial direction of the glider's motion is the positive direction.

Using the definition of acceleration, express the average acceleration of the glider in terms of the glider's velocity change and the elapsed time:

$$a = a_{av} = \frac{\Delta v}{\Delta t}$$

Using a constant-acceleration equation, express the average velocity of the glider in terms of the displacement of the glider and the elapsed time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{v_0 + v}{2} \Rightarrow v_0 = \frac{2\Delta x}{\Delta t} - v$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{2(100\,\text{cm})}{8.00\,\text{s}} - (-15\,\text{cm/s}) = \boxed{40\,\text{cm/s}}$$

The average acceleration of the glider is:

$$a = \frac{-15\,\text{cm/s} - (40\,\text{cm/s})}{8.00\,\text{s}} = \boxed{-6.9\,\text{cm/s}^2}$$

82 •• A rock dropped from a cliff covers one-third of its total distance to the ground in the last second of its fall. Air resistance is negligible. How high is the cliff?

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and its motion can be described using the constant-acceleration equations. Choose a coordinate system in which the downward direction is positive and let the height of the cliff, which equals the displacement of the rock, be represented by *h*.

Using a constant-acceleration equation, express the height *h* of the cliff in terms of the initial velocity of the rock, acceleration, and time of fall:	$\Delta y = v_0 t + \frac{1}{2}at^2$ or, because $v_0 = 0$, $a = g$, and $\Delta y = h$, $h = \frac{1}{2}gt^2$
Using this equation, express the displacement of the rock during the first two-thirds of its fall:	$\frac{2}{3}h = \frac{1}{2}gt^2$ (1)
Using the same equation, express the displacement of the rock during its complete fall in terms of the time required for it to fall this distance:	$h = \frac{1}{2}g(t + 1\,\text{s})^2$ (2)
Substitute equation (2) in equation (1) to obtain:	$t^2 - (4\text{ s})t - 2\text{ s}^2 = 0$
Use the quadratic formula or your graphing calculator to solve for the positive root:	$t = 4.45\,\text{s}$
Evaluate $\Delta t = t + 1$ s:	$\Delta t = 4.45\text{ s} + 1\text{ s} = 5.45\text{ s}$
Substitute numerical values in equation (2) and evaluate *h*:	$\boldsymbol{h} = \frac{1}{2}(9.81\,\text{m/s}^2)(5.45\,\text{s})^2 = \boxed{146\,\text{m}}$

83 •• **[SSM]** A typical automobile under hard braking loses speed at a rate of about 7.0 m/s^2; the typical reaction time to engage the brakes is 0.50 s. A local school board sets the speed limit in a school zone such that all cars should be able to stop in 4.0 m. (*a*) What maximum speed does this imply for an automobile in this zone? (*b*) What fraction of the 4.0 m is due to the reaction time?

Picture the Problem Assume that the acceleration of the car is constant. The total distance the car travels while stopping is the sum of the distances it travels during the driver's reaction time and the time it travels while braking. Choose a coordinate system in which the positive direction is the direction of motion of the automobile and apply a constant-acceleration equation to obtain a quadratic equation in the car's initial speed v_0.

(*a*) Using a constant-acceleration equation, relate the velocity of the car to its initial velocity, acceleration, and displacement during braking:

$$v^2 = v_0^2 + 2a\Delta x_{\text{brk}}$$

or, because the final velocity is zero,

$$0 = v_0^2 + 2a\Delta x_{\text{brk}} \Rightarrow \Delta x_{\text{brk}} = -\frac{v_0^2}{2a}$$

Express the total distance traveled by the car as the sum of the distance traveled during the reaction time and the distance traveled while slowing down:

$$\Delta x_{\text{tot}} = \Delta x_{\text{react}} + \Delta x_{\text{brk}} = v_0\Delta t_{\text{react}} - \frac{v_0^2}{2a}$$

Rearrange this quadratic equation to obtain:

$$v_0^2 - 2a\Delta t_{\text{react}}v_0 + 2a\Delta x_{\text{tot}} = 0$$

Substitute numerical values and simplify to obtain:

$$v_0^2 + (7.0\,\text{m/s})v_0 - 56\,\text{m}^2/\text{s}^2 = 0$$

Use your graphing calculator or the quadratic formula to solve the quadratic equation for its positive root:

$$\boldsymbol{v}_0 = 4.76\,\text{m/s}$$

Convert this speed to mi/h:

$$\boldsymbol{v}_0 = (4.76\,\text{m/s})\left(\frac{1\,\text{mi/h}}{0.4470\,\text{m/s}}\right) = \boxed{11\,\text{mi/h}}$$

(*b*) Find the reaction-time distance:

$$\Delta \boldsymbol{x}_{\text{react}} = \boldsymbol{v}_0\Delta \boldsymbol{t}_{\text{react}} = (4.76\,\text{m/s})(0.50\,\text{s}) = 2.38\,\text{m}$$

Express and evaluate the ratio of the reaction distance to the total distance:

$$\frac{\Delta \boldsymbol{x}_{\text{react}}}{\Delta \boldsymbol{x}_{\text{tot}}} = \frac{2.38\,\text{m}}{4.0\,\text{m}} = \boxed{0.60}$$

84 •• Two trains face each other on adjacent tracks. They are initially at rest, and their front ends are 40 m apart. The train on the left accelerates rightward at 1.0 m/s^2. The train on the right accelerates leftward at 1.3 m/s^2. (*a*) How far does the train on the left travel before the front ends of the trains pass? (*b*) If the trains are each 150 m in length, how long after the start are they completely past one another, assuming their accelerations are constant?

Picture the Problem Assume that the accelerations of the trains are constant. Choose a coordinate system in which the direction of the motion of the train on the left is the positive direction. Take $x_0 = 0$ as the position of the train on the left at $t = 0$ and note that the acceleration of the train on the right is negative.

(*a*) Using a constant-acceleration equation, express the position of the front of the train on the left as a function of time:

$$x_L = \tfrac{1}{2}a_L t^2 \qquad (1)$$

Using a constant-acceleration equation, express the position of the front of the train on the right as a function of time:

$$x_R = 40\,\text{m} + \tfrac{1}{2}a_R t^2$$

Equate x_L and x_R to obtain:

$$\tfrac{1}{2}a_L t^2 = 40\,\text{m} + \tfrac{1}{2}a_R t^2 \Rightarrow t = \sqrt{\frac{80\,\text{m}}{a_L - a_R}}$$

Substituting for t in equation (1) yields:

$$\boldsymbol{x}_L = \frac{1}{2}\boldsymbol{a}_L\left(\frac{80\,\text{m}}{\boldsymbol{a}_L - \boldsymbol{a}_R}\right) = \frac{1}{2}\left(\frac{80\,\text{m}}{1 - \dfrac{\boldsymbol{a}_R}{\boldsymbol{a}_L}}\right)$$

Substitute numerical values and evaluate x_L:

$$\boldsymbol{x}_L = \tfrac{1}{2}\left(\frac{80\,\text{m}}{1 - \dfrac{1.0\,\text{m/s}^2}{1.3\,\text{m/s}^2}}\right) = \boxed{17\,\text{m}}$$

(*b*) Let the rear of the left train be at the origin at $t = 0$. Then the initial location of the rear end of the train on the right is at $x = 340$ m (150 m + 40 m + 150 m).

Using a constant-acceleration equation, express the position of the rear of the train on the left as a function of time:

$$x_L = \tfrac{1}{2}a_L t^2$$

Using a constant-acceleration equation, express the position of the rear of the train on the right as a function of time:

$$x_R = 340\,\text{m} + \tfrac{1}{2}a_R t^2$$

The trains will be completely past each other when:

$$x_L = x_R \Rightarrow \tfrac{1}{2}a_L t^2 = 340\,\text{m} + \tfrac{1}{2}a_R t^2$$

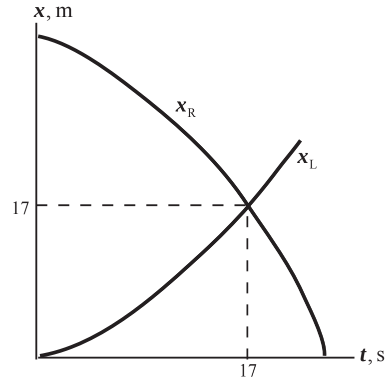
Solving for t yields:

$$t = \sqrt{\frac{680\,\text{m}}{a_L - a_R}}$$

Substitute numerical values and evaluate t:

$$\boldsymbol{t} = \sqrt{\frac{680\,\text{m}}{1.0\,\text{m/s}^2 - \left(-1.3\,\text{m/s}^2\right)}} = \boxed{17\,\text{s}}$$

Remarks: One can also solve this problem by graphing x_L and x_R as functions of t. The coordinates of the intersection of the two curves give one the time-to-passing and the distance traveled by the train on the left for Part (*a*).



85 •• Two stones are dropped from the edge of a 60-m cliff, the second stone 1.6 s after the first. How far below the top of the cliff is the second stone when the separation between the two stones is 36 m?

Picture the Problem In the absence of air resistance, the acceleration of the stones is constant. Choose a coordinate system in which the downward direction is positive and the origin is at the point of release of the stones.

Using constant-acceleration equations, relate the positions of the two stones to their initial positions, accelerations, and time-of-fall:

$$x_1 = \tfrac{1}{2}gt^2$$

and

$$x_2 = \tfrac{1}{2}g(t - 1.6\,\text{s})^2$$

Express the difference between x_1 and x_2:

$$x_1 - x_2 = 36\ \text{m}$$

Substitute for x_1 and x_2 to obtain:

$$36\,\text{m} = \tfrac{1}{2}gt^2 - \tfrac{1}{2}g(t - 1.6\,\text{s})^2$$

Solve this equation for the time t at which the stones will be separated by 36 m:

$$t = 3.09\ \text{s}$$

Substitute this result in the expression for x_2 and solve for x_2:

$$\mathbf{x}_2 = \tfrac{1}{2}(9.81\,\text{m/s}^2)(3.09\,\text{s} - 1.6\,\text{s})^2 = \boxed{11\,\text{m}}$$

86 •• A motorcycle officer hidden at an intersection observes a car driven by an oblivious driver who ignores a stop sign and continues through the intersection at constant speed. The police officer takes off in pursuit 2.0 s after the car has passed the stop sign. She accelerates at 4.2 m/s^2 until her speed is 110 km/h, and then continues at this speed until she catches the car. At that instant, the car is 1.4 km from the intersection. (*a*) How long did it take for the officer to catch up to the car? (*b*) How fast was the car traveling?

Picture the Problem The acceleration of the police officer's car is positive and constant and the acceleration of the speeder's car is zero. Choose a coordinate system such that the direction of motion of the two vehicles is the positive direction and the origin is at the stop sign.

(*a*) The time traveled by the car is given by:

$$t_{car} = 2.0\,\text{s} + t_1 + t_2 \qquad (1)$$

where t_1 is the time during which the motorcycle was accelerating and t_2 is the time during which the motorcycle moved with constant speed.

Convert 110 km/h into m/s:

$$v_1 = 110\,\frac{\text{km}}{\text{h}} \times \frac{10^3\,\text{m}}{\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 30.56\,\frac{\text{m}}{\text{s}}$$

Express and evaluate t_1:

$$t_1 = \frac{v_1}{a_{motorcycle}} = \frac{30.56\,\text{m/s}}{4.2\,\text{m/s}^2} = 7.276\,\text{s}$$

Express and evaluate d_1:

$$d_1 = \tfrac{1}{2}v_1 t_1 = \tfrac{1}{2}(30.56\,\text{m/s})(7.276\,\text{s}) = 111.2\,\text{m}$$

Determine d_2:

$$d_2 = d_{caught} - d_1 = 1400\,\text{m} - 111\,\text{m} = 1289\,\text{m}$$

Express and evaluate t_2:

$$t_2 = \frac{d_2}{v_1} = \frac{1289\,\text{m}}{30.56\,\text{m/s}} = 42.18\,\text{s}$$

Substitute in equation (1) and evaluate t_{car}:

$$t_{car} = 2.0\,\text{s} + 7.3\,\text{s} + 42.2\,\text{s} = 51.5\,\text{s} = \boxed{52\,\text{s}}$$

(*b*) The speed of the car when it was overtaken is the ratio of the distance it traveled to the elapsed time:

$$v_{car} = \frac{d_{caught}}{t_{car}}$$

Substitute numerical values and evaluate v_{car}:

$$v_{car} = \left(\frac{1400\,\text{m}}{51.5\,\text{s}}\right)\left(\frac{1\,\text{mi/h}}{0.447\,\text{m/s}}\right) = \boxed{61\,\text{mi/h}}$$

87 •• At $t = 0$, a stone is dropped from the top of a cliff above a lake. Another stone is thrown downward 1.6 s later from the same point with an initial speed of 32 m/s. Both stones hit the water at the same instant. Find the height of the cliff.

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release of the stone and apply constant-acceleration equations.

Using a constant-acceleration equation, express the height of the cliff in terms of the initial position of the stones, acceleration due to gravity, and time for the first stone to hit the water:

$$h = \tfrac{1}{2}gt_1^2$$

Express the displacement of the second stone when it hits the water in terms of its initial velocity, acceleration, and time required for it to hit the water.

$$d_2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$

where $t_2 = t_1 - 1.6$ s.

Because the stones will travel the same distances before hitting the water, equate h and d_2 to obtain:

$$\tfrac{1}{2}gt_1^2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$

or, because $t_2 = t_1 - 1.6$ s,

$$\tfrac{1}{2}gt_1^2 = v_{02}t_2 + \tfrac{1}{2}g(t_1 - 1.6\,\text{s})^2$$

Substitute numerical values to obtain:

$$\tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)\boldsymbol{t}_1^2 = (32\ \text{m/s})(\boldsymbol{t}_1 - 1.6\ \text{s}) + \tfrac{1}{2}\left(9.81\ \text{m/s}^2\right)(\boldsymbol{t}_1 - 1.6\,\text{s})^2$$

Solving this quadratic equation for t_1 yields:

$$\boldsymbol{t}_1 = 2.370\,\text{s}$$

Substitute for t_1 and evaluate h:

$$\boldsymbol{h} = \tfrac{1}{2}(9.81\,\text{m/s}^2)(2.370\,\text{s})^2 = \boxed{28\,\text{m}}$$

88 •• A passenger train is traveling at 29 m/s when the engineer sees a freight train 360 m ahead of his train traveling in the same direction on the same track. The freight train is moving at a speed of 6.0 m/s. (*a*) If the reaction time of the engineer is 0.40 s, what is the minimum (constant) rate at which the passenger train must lose speed if a collision is to be avoided? (*b*) If the engineer's reaction time is 0.80 s and the train loses speed at the minimum rate described in Part (*a*), at what rate is the passenger train approaching the freight train when the two collide? (*c*) How far will the passenger train have traveled in the time between the sighting of the freight train and the collision?

Picture the Problem Assume that the acceleration of the passenger train is constant. Let $x_p = 0$ be the location of the passenger train engine at the moment of sighting the freight train's end; let $t = 0$ be the instant the passenger train begins to slow (0.40 s after the passenger train engineer sees the freight train ahead).

Choose a coordinate system in which the direction of motion of the trains is the positive direction and use constant-acceleration equations to express the positions of the trains in terms of their initial positions, speeds, accelerations, and elapsed time.

(*a*) Using constant-acceleration equations, write expressions for the positions of the front of the passenger train and the rear of the freight train, x_p and x_f, respectively:

$$\boldsymbol{x}_{\mathrm{p}} = (29\,\mathrm{m/s})(\boldsymbol{t} + 0.40\,\mathrm{s}) - \tfrac{1}{2}\boldsymbol{a}\boldsymbol{t}^2 \quad (1)$$

and

$$\boldsymbol{x}_{\mathrm{f}} = (360\,\mathrm{m}) + (6.0\,\mathrm{m/s})(\boldsymbol{t} + 0.40\,\mathrm{s})$$

where x_p and x_f are in meters if t is in seconds.

Equate x_f and x_p to obtain an equation for t:

$$\tfrac{1}{2}at^2 - (23\,\mathrm{m/s})t + 350.8\,\mathrm{m} = 0$$

Find the discriminant $\left(D = \mathrm{B}^2 - 4\mathrm{AC}\right)$ of this equation:

$$D = (23\,\mathrm{m/s})^2 - 4\left(\frac{a}{2}\right)(350.8\,\mathrm{m})$$

The equation must have real roots if it is to describe a collision. The necessary condition for real roots is that the discriminant be greater than or equal to zero:

If $(23\ \mathrm{m/s})^2 - a\ (701.6\ \mathrm{m}) \geq 0$, then $\boldsymbol{a} \leq \boxed{0.75\ \mathrm{m/s^2}}$

(*b*) Differentiate equation (1) to express the speed of the passenger train as a function of time:

$$\boldsymbol{v}_{\mathrm{p}}(\boldsymbol{t}) = \frac{\boldsymbol{dx}_{\mathrm{p}}(\boldsymbol{t})}{\boldsymbol{dt}} = 29\,\mathrm{m/s} - \boldsymbol{at} \qquad (2)$$

Repeat the previous steps with $a = 0.75\ \mathrm{m/s^2}$ and a 0.80 s reaction time. The quadratic equation that guarantees real roots with the longer reaction time is:

$$\left(0.375\,\mathrm{m/s^2}\right)\boldsymbol{t}^2 - (23\,\mathrm{m/s})\boldsymbol{t} + 341.6\mathrm{m} = 0$$

Use the quadratic formula or your graphing calculator to find the collision times:

$t = 25.23$ s and $t = 36.10$ s

Because, when $t = 36.10$ s, the trains have already collided, this root is not a meaningful solution to our problem.

Note: In the graph shown below, you will see why we keep only the smaller of the two solutions.

Evaluate equation (2) for $t = 25.23$ s and $a = 0.75\ \mathrm{m/s^2}$:

$$\boldsymbol{v}_{\mathrm{p}}(\boldsymbol{t}) = 29\,\mathrm{m/s} - \left(0.75\,\mathrm{m/s^2}\right)(25.23\,\mathrm{s}) = \boxed{10\,\mathrm{m/s}}$$

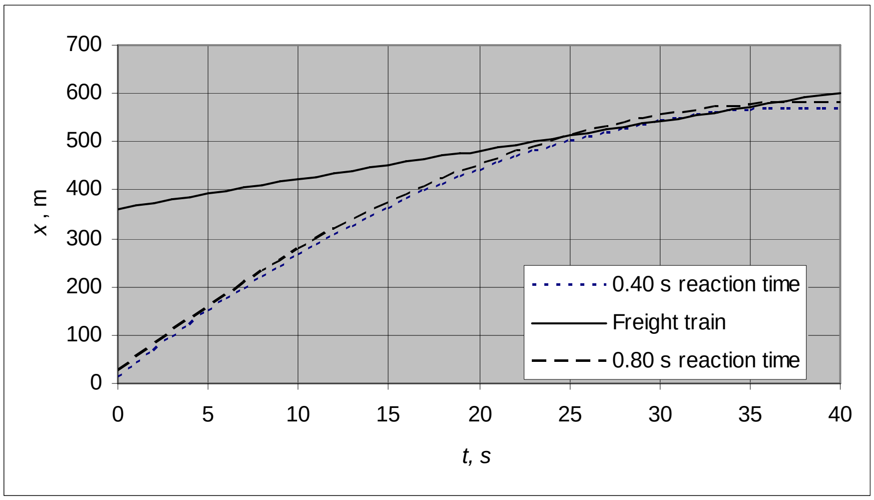
(*c*) The position of the passenger train as a function of time is given by:

$$x_{\mathrm{p}}(t)=(29\,\mathrm{m/s})(t+0.80\,\mathrm{s})-\tfrac{1}{2}\left(0.75\,\mathrm{m/s^2}\right)t^2$$

Evaluate $x_{\mathrm{p}}(25.23\,\mathrm{s})$ to obtain:

$$x_{\mathrm{p}}(26\,\mathrm{s})=(29\,\mathrm{m/s})(25.23\,\mathrm{s}+0.80\,\mathrm{s})-\tfrac{1}{2}\left(0.75\,\mathrm{m/s^2}\right)(25.23\,\mathrm{s})^2=\boxed{0.52\,\mathrm{km}}$$

The following graph shows the positions of the trains as a function of time. The solid straight line is for the constant velocity freight train; the dashed curves are for the passenger train, with reaction times of 0.40 s for the lower curve and 0.80 s for the upper curve.



Remarks: A collision occurs the first time the curve for the passenger train crosses the curve for the freight train. The smaller of the two solutions will always give the time of the collision.

89 •• The click beetle can project itself vertically with an acceleration of about 400*g* (an order of magnitude more than a human could survive!). The beetle jumps by ″unfolding″ its 0.60-cm long legs. (*a*) How high can the click beetle jump? (*b*) How long is the beetle in the air? (Assume constant acceleration while in contact with the ground and neglect air resistance.)

Picture the Problem Choose a coordinate system in which the upward direction is positive. We can use a constant-acceleration equation to find the beetle's velocity as its feet lose contact with the ground and then use this velocity to calculate the height of its jump.

(*a*) Using a constant-acceleration equation, relate the beetle's maximum height to its launch velocity, velocity at the top of its trajectory, and acceleration once it is airborne; solve for its maximum height:

$$v_{\text{highest point}}^2 = v_{\text{launch}}^2 + 2a\Delta y_{\text{free fall}} = v_{\text{launch}}^2 + 2(-g)h$$

or, because $v_{\text{highest point}} = 0$,

$$0 = v_{\text{launch}}^2 + 2(-g)h \Rightarrow h = \frac{v_{\text{launch}}^2}{2g} \quad (1)$$

Now, in order to determine the beetle's launch velocity, relate its time of contact with the ground to its acceleration and push-off distance:

$$v_{\text{launch}}^2 = v_0^2 + 2a\Delta y_{\text{launch}}$$

or, because $v_0 = 0$,

$$v_{\text{launch}}^2 = 2a\Delta y_{\text{launch}}$$

Substituting for v_{launch}^2 in equation (1) yields:

$$h = \frac{2a\Delta y_{\text{launch}}}{2g}$$

Substitute numerical values in equation (1) to find the height to which the beetle can jump:

$$h = \frac{2(400)(9.81\,\text{m/s}^2)(0.60\times10^{-2}\,\text{m})}{2(9.81\,\text{m/s}^2)} = \boxed{2.4\,\text{m}}$$

(*b*) Using a constant-acceleration equation, relate the velocity of the beetle at its maximum height to its launch velocity, free-fall acceleration while in the air, and time-to-maximum height:

$$v = v_0 + at$$

or

$$v_{\text{max. height}} = v_{\text{launch}} - gt_{\text{max. height}}$$

and, because $v_{\text{max height}} = 0$,

$$0 = v_{\text{launch}} - gt_{\text{max. height}} \Rightarrow t_{\text{max height}} = \frac{v_{\text{launch}}}{g}$$

For zero displacement and constant acceleration, the time-of-flight is twice the time-to-maximum height:

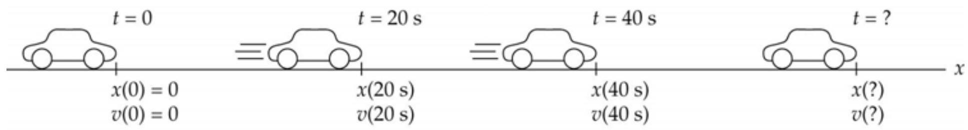
$$t_{\text{flight}} = 2t_{\text{max. height}} = \frac{2v_{\text{launch}}}{g}$$

Substitute numerical values and evaluate t_{flight}:

$$t_{\text{flight}} = \frac{2(6.9\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{1.4\,\text{s}}$$

90 •• An automobile accelerates from rest at 2.0 m/s^2 for 20 s. The speed is then held constant for 20 s, after which there is an acceleration of –3.0 m/s^2 until the automobile stops. What is the total distance traveled?

Picture the Problem This is a multipart constant-acceleration problem using three different constant accelerations (+2.0 m/s^2 for 20 s, then zero for 20 s, and then –3.0 m/s^2 until the automobile stops). The final velocity is zero. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



Add up all the displacements to get the total:

$$\Delta x_{03} = \Delta x_{01} + \Delta x_{12} + \Delta x_{23} \qquad (1)$$

Using constant-acceleration formulas, find the first displacement:

$$\Delta x_{01} = v_0 t_1 + \tfrac{1}{2} a_{01} t_1^2$$

or, because $v_0 = 0$,

$$\Delta x_{01} = \tfrac{1}{2} a_{01} t_1^2$$

The speed is constant for the second displacement. The second displacement is given by:

$$\Delta x_{12} = v_1 (t_2 - t_1)$$

or, because $v_1 = v_0 + a_{01} t_1 = 0 + a_{01} t_1$,

$$\Delta x_{12} = a_{01} t_1 (t_2 - t_1)$$

The displacement during the braking interval is related to v_3, v_2, and a_{23}:

$$v_3^2 = v_2^2 + 2a_{23} \Delta x_{23}$$

and, because $v_2 = v_1 = a_{01} t_1$ and $v_3 = 0$,

$$\Delta x_{23} = \frac{-(a_{01} t_1)^2}{2a_{23}}$$

Substituting for Δx_{01}, Δx_{12}, and Δx_{23} in equation (1) yields:

$$\Delta x_{03} = \tfrac{1}{2} a_{01} t_1^2 + a_{01} t_1 (t_2 - t_1) - \frac{(a_{01} t_1)^2}{2a_{23}}$$

Substitute numerical values and evaluate Δx_{23} to obtain:

$$\Delta x_{23} = -\frac{[(2.0\ \text{m/s})(20\ \text{s})]^2}{2(-3.0\ \text{m/s}^2)} = 267\text{m}$$

Substitute numerical values and evaluate Δx_{03}:

$$\Delta x_{03} = \tfrac{1}{2}(2.0\ \text{m/s})(20\ \text{s})^2 + (2.0\ \text{m/s})(20\ \text{s})(40\ \text{s} - 20\ \text{s}) - \frac{[(2.0\ \text{m/s})(20\ \text{s})]^2}{2(-3.0\ \text{m/s}^2)}$$

$$= \boxed{1.5\ \text{km}}$$

Remarks: Because the area under the curve of a velocity-versus-time graph equals the displacement of the object experiencing the acceleration, we could solve this problem by plotting the velocity as a function of time and finding the area bounded by it and the time axis.

91 •• **[SSM]** Consider measuring the free-fall motion of a particle (neglect air resistance). Before the advent of computer-driven data-logging software, these experiments typically employed a wax-coated tape placed vertically next to the path of a dropped electrically conductive object. A spark generator would cause an arc to jump between two vertical wires through the falling object and through the tape, thereby marking the tape at fixed time intervals Δt. Show that the change in height during successive time intervals for an object falling from rest follows *Galileo's Rule of Odd Numbers*: $\Delta y_{21} = 3\Delta y_{10}$, $\Delta y_{32} = 5\Delta y_{10}$, . . . , where Δy_{10} is the change in y during the first interval of duration Δt, Δy_{21} is the change in y during the second interval of duration Δt, etc.

Picture the Problem In the absence of air resistance, the particle experiences constant acceleration and we can use constant-acceleration equations to describe its position as a function of time. Choose a coordinate system in which downward is positive, the particle starts from rest ($v_o = 0$), and the starting height is zero ($y_0 = 0$).

Using a constant-acceleration equation, relate the position of the falling particle to the acceleration and the time. Evaluate the y-position at successive equal time intervals Δt, $2\Delta t$, $3\Delta t$, etc:

$$y_1 = \frac{-g}{2}(\Delta t)^2 = \frac{-g}{2}(\Delta t)^2$$

$$y_2 = \frac{-g}{2}(2\Delta t)^2 = \frac{-g}{2}(4)(\Delta t)^2$$

$$y_3 = \frac{-g}{2}(3\Delta t)^2 = \frac{-g}{2}(9)(\Delta t)^2$$

$$y_4 = \frac{-g}{2}(4\Delta t)^2 = \frac{-g}{2}(16)(\Delta t)^2$$

etc.

Evaluate the changes in those positions in each time interval:

$$\Delta y_{10} = y_1 - 0 = \left(\frac{-g}{2}\right)(\Delta t)^2$$

$$\Delta y_{21} = y_2 - y_1 = 3\left(\frac{-g}{2}\right)(\Delta t)^2 = 3\Delta y_{10}$$

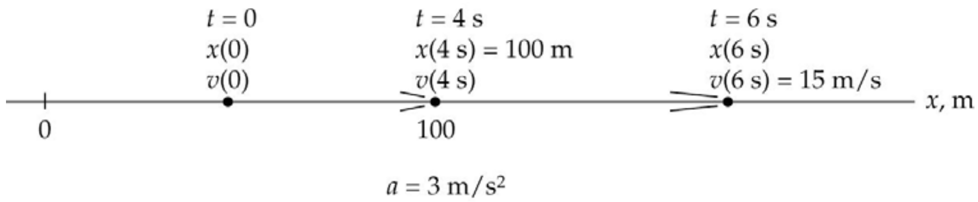
$$\Delta y_{32} = y_3 - y_2 = 5\left(\frac{-g}{2}\right)(\Delta t)^2 = 5\Delta y_{10}$$

$$\Delta y_{43} = y_4 - y_3 = 7\left(\frac{-g}{2}\right)(\Delta t)^2 = 7\Delta y_{10}$$

etc.

92 •• Starting from rest, a particle travels along the x axis with a constant acceleration of $+3.0\ \text{m/s}^2$. At a time 4.0 s following its start, it is at $x = 100$ m. At a time 6.0 s later it has a velocity of +15 m/s. Find its position at this later time.

Picture the Problem Because the particle moves with a constant acceleration we can use the constant-acceleration equations to describe its motion. A pictorial representation will help us organize the information in the problem and develop our solution strategy.



Using a constant-acceleration equation, express the position $x(t)$ of the particle in terms of its initial speed v_0, initial position x_0, and acceleration a:

$$x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad (1)$$

Using the information that when $t = 4.0$ s, $x = 100$ m, obtain an equation in x_0 and v_0:

$$x(4.0\,\text{s}) = 100\,\text{m} = x_0 + v_0(4.0\,\text{s}) + \tfrac{1}{2}(3.0\,\text{m/s}^2)(4.0\,\text{s})^2$$

Simplify this equation to obtain:

$$x_0 + (4.0\,\text{s})v_0 = 76\,\text{m} \quad (2)$$

Using the information that when $t = 6.0$ s, $v = 15$ m/s, obtain a second equation in x_0 and v_0:

$$v(6.0\,\text{s}) = 15\,\text{m/s} = v_0 + (3.0\,\text{m/s}^2)(6.0\,\text{s})$$

Solving for v_0 yields: $v_0 = -3.0$ m/s

Substitute this value for v_0 in equation (2) to obtain:

$$x_0 + (4.0\,\text{s})(-3.0\,\text{m/s}) = 76\,\text{m}$$

Solving for x_0 yields: $x_0 = 88$ m

Substitute for x_0, v_0, and a in equation (1) to obtain the general expression for the position $x(t)$ of the particle as a function of time:

$$x(t) = 88\,\text{m/s} + (-3.0\,\text{m/s})t + \tfrac{1}{2}(3.0\,\text{m/s}^2)t^2$$

Evaluating $x(6.0$ s) yields:

$$x(6.0\,\text{s}) = 88\,\text{m} + (-3.0\,\text{m/s})(6.0\,\text{s}) + \tfrac{1}{2}(3.0\,\text{m/s}^2)(6.0\,\text{s})^2 = \boxed{0.12\,\text{km}}$$

93 •• **[SSM]** If it were possible for a spacecraft to maintain a constant acceleration indefinitely, trips to the planets of the Solar System could be undertaken in days or weeks, while voyages to the nearer stars would only take a few years. (*a*) Using data from the tables at the back of the book, find the time it would take for a one-way trip from Earth to Mars (at Mars' closest approach to Earth). Assume that the spacecraft starts from rest, travels along a straight line, accelerates halfway at 1g, flips around, and decelerates at 1g for the rest of the trip. (*b*) Repeat the calculation for a 4.1×10^{13}-km trip to Proxima Centauri, our nearest stellar neighbor outside of the sun. (See Problem 47.)

Picture the Problem Note: No material body can travel at speeds faster than light. When one is dealing with problems of this sort, the kinematic formulae for displacement, velocity and acceleration are no longer valid, and one must invoke the special theory of relativity to answer questions such as these. For now, ignore such subtleties. Although the formulas you are using (i.e., the constant-acceleration equations) are not quite correct, your answer to Part (*b*) will be wrong by about 1%.

(*a*) Let $t_{1/2}$ represent the time it takes to reach the halfway point. Then the total trip time is:

$$t = 2\,t_{1/2} \qquad (1)$$

Use a constant- acceleration equation to relate the half-distance to Mars Δx to the initial speed, acceleration, and half-trip time $t_{1/2}$:

$$\Delta x = v_0 t + \tfrac{1}{2} a t_{1/2}^2$$

Because $v_0 = 0$ and $a = g$:

$$t_{1/2} = \sqrt{\frac{2\Delta x}{a}}$$

Substitute in equation (1) to obtain:

$$t = 2\sqrt{\frac{2\Delta x}{a}} \qquad (2)$$

The distance from Earth to Mars at closest approach is 7.8×10^{10} m. Substitute numerical values and evaluate t :

$$\boldsymbol{t}_{\text{round trip}} = 2\sqrt{\frac{2\left(3.9\times10^{10}\ \text{m}\right)}{9.81\,\text{m/s}^2}} = 18\times10^4\ \text{s}$$

$$\approx \boxed{2.1\,\text{d}}$$

(*b*) From Problem 47 we have:

$$\boldsymbol{d}_{\text{Proxima Centauri}} = 4.1\times10^{13}\ \text{km}$$

Substitute numerical values in equation (2) to obtain:

$$\boldsymbol{t}_{\text{round trip}} = 2\sqrt{\frac{2\left(4.1\times10^{13}\ \text{km}\right)}{9.81\,\text{m/s}^2}} = 18\times10^7\ \text{s}$$

$$\approx \boxed{5.8\ \text{y}}$$

Remarks: Our result in Part (*a*) seems remarkably short, considering how far Mars is and how low the acceleration is.

94 • The Stratosphere Tower in Las Vegas is 1137 ft high. It takes 1 min, 20 s to ascend from the ground floor to the top of the tower using the high-speed elevator. The elevator starts and ends at rest. Assume that it maintains a constant upward acceleration until it reaches its maximum speed, and then maintains a constant acceleration of equal magnitude until it comes to a stop. Find the magnitude of the acceleration of the elevator. Express this acceleration magnitude as a multiple of g (the acceleration due to gravity).

Picture the Problem Because the elevator accelerates uniformly for half the distance and uniformly decelerates for the second half, we can use constant-acceleration equations to describe its motion

Let $t_{1/2}$ = 40 s be the time it takes to reach the halfway mark. Use the constant-acceleration equation that relates the acceleration to the known variables to obtain:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2}at^2 \Rightarrow a = \frac{2\,\Delta y}{t_{1/2}^2}$$

Substitute numerical values and evaluate a:

$$\boldsymbol{a} = \frac{2\left(\tfrac{1}{2}\right)(1137\,\text{ft})(1\,\text{m}/3.281\,\text{ft})}{(40\,\text{s})^2} = 0.22\,\text{m/s}^2$$

$$= \boxed{0.022\boldsymbol{g}}$$

95 •• A train pulls away from a station with a constant acceleration of 0.40 m/s^2. A passenger arrives at a point next to the track 6.0 s after the end of the train has passed the very same point. What is the slowest constant speed at which she can run and still catch the train? On a single graph plot the position versus time curves for both the train and the passenger.

Picture the Problem Because the acceleration is constant, we can describe the motions of the train using constant-acceleration equations. Find expressions for the distances traveled, separately, by the train and the passenger. When are they equal? Note that the train is accelerating and the passenger runs at a constant minimum velocity (zero acceleration) such that she can just catch the train.

1. Using the subscripts "train" and "p" to refer to the train and the passenger and the subscript "c" to identify "critical" conditions, express the position of the train and the passenger:

$$x_{\text{train,c}}(t_{\text{c}}) = \frac{a_{\text{train}}}{2}t_{\text{c}}^2$$

and

$$x_{\text{p,c}}(t_{\text{c}}) = v_{\text{p,c}}(t_{\text{c}} - \Delta t)$$

Express the critical conditions that must be satisfied if the passenger is to catch the train:

$$v_{\text{train,c}} = v_{\text{p,c}}$$

and

$$x_{\text{train,c}} = x_{\text{p,c}}$$

2. Express the train's average velocity.

$$v_{\text{av}}(0\text{ to }t_{\text{c}}) = \frac{0 + v_{\text{train,c}}}{2} = \frac{v_{\text{train,c}}}{2}$$

3. Using the definition of average velocity, express v_{av} in terms of $x_{p,c}$ and t_c.

$$v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{0 + x_{p,c}}{0 + t_c} = \frac{x_{p,c}}{t_c}$$

4. Combine steps 2 and 3 and solve for $x_{p,c}$.

$$x_{p,c} = \frac{v_{train,c} t_c}{2}$$

5. Combine steps 1 and 4 and solve for t_c.

$$v_{p,c}(t_c - \Delta t) = \frac{v_{train,c} t_c}{2}$$

or

$$t_c - \Delta t = \frac{t_c}{2}$$

and

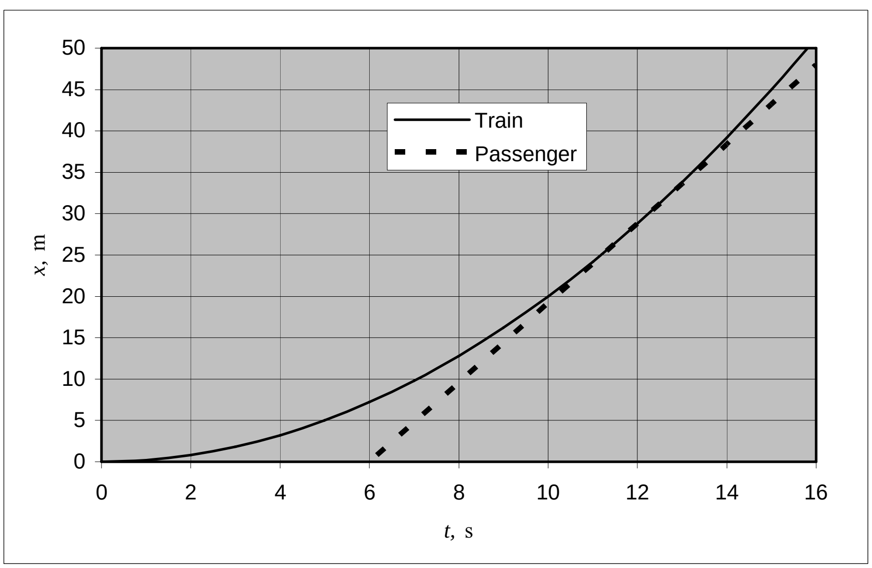
$t_c = 2\ \Delta t = 2\ (6\text{ s}) = 12\text{ s}$

6. Finally, combine steps 1 and 5 and solve for $v_{train,\,c}$.

$$\boldsymbol{v}_{p,c} = \boldsymbol{v}_{train,c} = \boldsymbol{a}_{train}\boldsymbol{t}_c = (0.40\text{ m/s}^2)(12\text{ s})$$

$$= \boxed{4.8\text{ m/s}}$$

The following graph shows the location of both the passenger and the train as a function of time. The parabolic solid curve is the graph of $x_{train}(t)$ for the accelerating train. The straight dashed line is passenger's position $x_p(t)$ if she arrives at $\Delta t = 6.0$ s after the train departs. When the passenger catches the train, our graph shows that her speed and that of the train must be equal ($v_{train,c} = v_{p,c}$). Do you see why?



96 ••• Ball A is dropped from the top of a building of height h at the same instant that ball B is thrown vertically upward from the ground. When the balls collide, they are moving in opposite directions, and the speed of A is twice the speed of B. At what fraction of the height of the building does the collision occur?

Picture the Problem Both balls experience constant acceleration once they are in flight. Choose a coordinate system with the origin at the ground and the upward direction positive. When the balls collide they are at the same height above the ground.

Using constant-acceleration equations, express the positions of both balls as functions of time. At the ground $y = 0$.

$$y_A = h - \tfrac{1}{2}gt^2$$
and
$$y_B = v_0 t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B$$
and
$$v_A = -2v_B$$

Express the velocities of both balls as functions of time:

$$v_A = -gt \quad \text{and} \quad v_B = v_0 - gt$$

Substituting the position and velocity functions into the conditions at collision gives:

$$h - \tfrac{1}{2}gt_c^2 = v_0 t_c - \tfrac{1}{2}gt_c^2$$
and
$$-gt_c = -2(v_0 - gt_c)$$
where t_c is the time of collision.

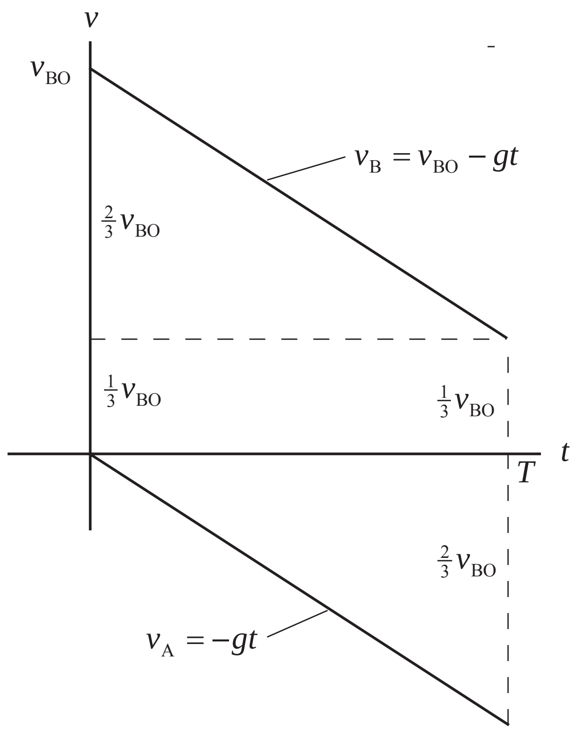
We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$$t_c = \sqrt{\frac{2h}{3g}} \quad \text{and} \quad v_0 = \sqrt{\frac{3gh}{2}}$$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$$y_A = h - \tfrac{1}{2}g\left(\frac{2h}{3g}\right) = \boxed{\frac{2h}{3}}$$

Remarks: We can also solve this problem graphically by plotting velocity-versus-time for both balls. Because ball A starts from rest its velocity is given by $v_A = -gt$. Ball B initially moves with an unknown velocity v_{B0} and its velocity is given by $v_B = v_{B0} - gt$. The graphs of these equations are shown below with T representing the time at which they collide.



The height of the building is the sum of the sum of the distances traveled by the balls. Each of these distances is equal to the magnitude of the area "under" the corresponding *v*-versus-*t* curve. Thus, the height of the building equals the area of the parallelogram, which is $v_{B0}T$. The distance that A falls is the area of the lower triangle, which is $\frac{1}{3}v_{BO}T$. Therefore, the ratio of the distance fallen by A to the height of the building is 1/3, so the collision takes place at 2/3 the height of the building.

97 ••• Solve Problem 96 if the collision occurs when the balls are moving in the same direction and the speed of A is 4 times that of B.

Picture the Problem Both balls are moving with constant acceleration. Take the origin of the coordinate system to be at the ground and the upward direction to be positive. When the balls collide they are at the same height above the ground. The velocities at collision are related by $v_A = 4v_B$.

Using constant-acceleration equations, express the positions of both balls as functions of time:

$$y_A = h - \tfrac{1}{2}gt^2 \text{ and } y_B = v_0t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B \text{ and } v_A = 4v_B$$

Express the velocities of both balls as functions of time:

$v_A = -gt$ and $v_B = v_0 - gt$

Substitute the position and velocity functions into the conditions at collision to obtain:

$h - \frac{1}{2}gt_c^2 = v_0 t_c - \frac{1}{2}gt_c^2$

and

$-gt_c = 4(v_0 - gt_c)$

where t_c is the time of collision.

We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$t_c = \sqrt{\dfrac{4h}{3g}}$ and $v_0 = \sqrt{\dfrac{3gh}{4}}$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$y_A = h - \frac{1}{2}g\left(\dfrac{4h}{3g}\right) = \boxed{\dfrac{h}{3}}$

98 ••• Starting at one station, a subway train accelerates from rest at a constant rate of 1.00 m/s^2 for half the distance to the next station, then slows down at the same rate for the second half of the journey. The total distance between stations is 900 m. (*a*) Sketch a graph of the velocity v_x as a function of time over the full journey. (*b*) Sketch a graph of the position as a function of time over the full journey. Place appropriate numerical values on both axes.

Picture the Problem The problem describes two intervals of constant acceleration; one when the train's velocity is increasing, and a second when it is decreasing.

(*a*) Using a constant-acceleration equation, relate the half-distance Δx between stations to the initial speed v_0, the acceleration a of the train, and the time-to-midpoint Δt:

$\Delta x = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$

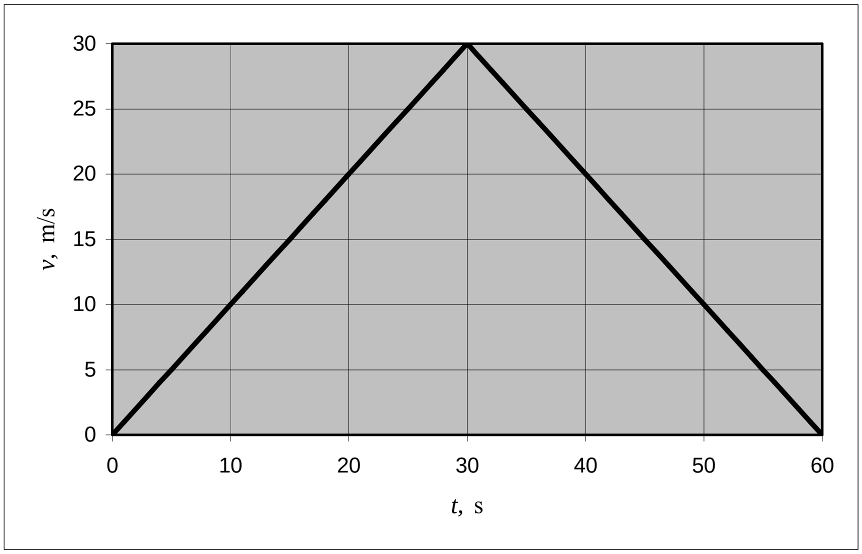
or, because $v_0 = 0$,

$\Delta x = \frac{1}{2}a(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\dfrac{2\Delta x}{a}}$

Substitute numerical values and evaluate the time-to-midpoint Δt:

$\Delta \boldsymbol{t} = \sqrt{\dfrac{2(450\,\text{m})}{1.00\,\text{m/s}^2}} = 30.0\,\text{s}$

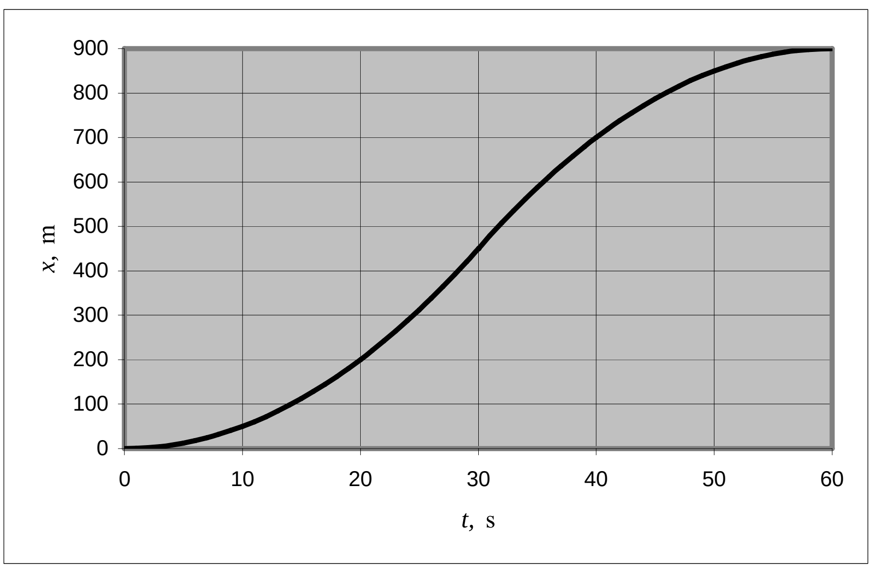
Because the train accelerates uniformly and from rest, the first part of its velocity graph will be linear, pass through the origin, and last for 30 s. Because it slows down uniformly and at the same rate for the second half of its journey, this part of its graph will also be linear but with a negative slope. A graph of v as a function of t follows.



(*b*) The graph of *x* as a function of *t* is obtained from the graph of *v* as a function of *t* by finding the area under the velocity curve. Looking at the velocity graph, note that when the train has been in motion for 10 s, it will have traveled a distance of

$$\tfrac{1}{2}(10\,\text{s})(10\,\text{m/s}) = 50\,\text{m}$$

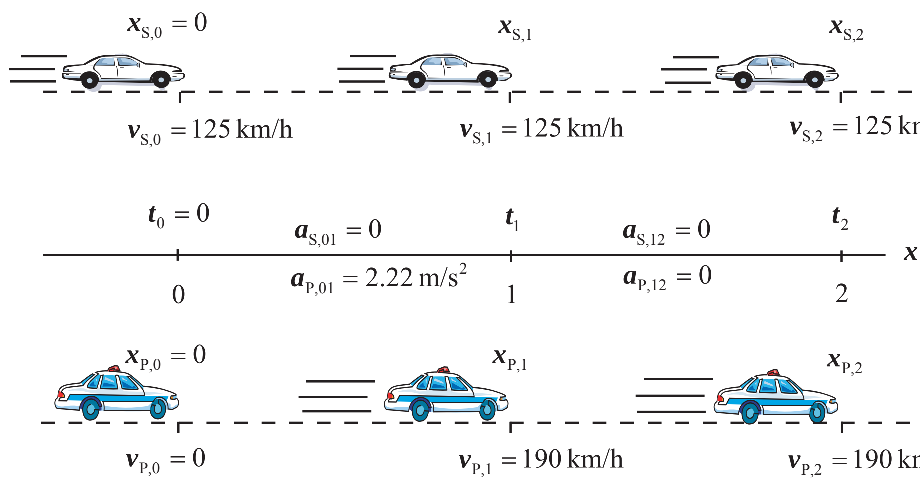
and that this distance is plotted above 10 s on the following graph.



Selecting additional points from the velocity graph and calculating the areas under the curve will confirm the graph of *x* as a function of *t*.

99 ••• **[SSM]** A speeder traveling at a constant speed of 125 km/h races past a billboard. A patrol car pursues from rest with constant acceleration of (8.0 km/h)/s until it reaches its maximum speed of 190 km/h, which it maintains until it catches up with the speeder. (*a*) How long does it take the patrol car to catch the speeder if it starts moving just as the speeder passes? (*b*) How far does each car travel? (*c*) Sketch $x(t)$ for each car.

Picture the Problem This is a two-stage constant-acceleration problem. Choose a coordinate system in which the direction of the motion of the cars is the positive direction. The pictorial representation summarizes what we know about the motion of the speeder's car and the patrol car.



Convert the speeds of the vehicles and the acceleration of the police car into SI units:

$$8.0\frac{\text{km}}{\text{h}\cdot\text{s}} = 8.0\frac{\text{km}}{\text{h}\cdot\text{s}}\times\frac{1\,\text{h}}{3600\,\text{s}} = 2.2\,\text{m/s}^2,$$

$$125\frac{\text{km}}{\text{h}} = 125\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}} = 34.7\,\text{m/s},$$

and

$$190\frac{\text{km}}{\text{h}} = 190\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}} = 52.8\,\text{m/s}$$

(*a*) Express the condition that determines when the police car catches the speeder; that is, that their displacements will be the same:

$$\Delta x_{P,02} = \Delta x_{S,02}$$

Using a constant-acceleration equation, relate the displacement of the patrol car to its displacement while accelerating and its displacement once it reaches its maximum velocity:

$$\begin{aligned}\Delta x_{P,02} &= \Delta x_{P,01} + \Delta x_{P,12}\\ &= \Delta x_{P,01} + v_{P,1}(t_2 - t_1)\end{aligned}$$

Using a constant-acceleration equation, relate the displacement of the speeder to its constant velocity and the time it takes the patrol car to catch it:

$$\Delta x_{S,02} = v_{S,02}\Delta t_{02} = (34.7\text{ m/s})t_2$$

Calculate the time during which the police car is speeding up:

$$\Delta t_{P,01} = \frac{\Delta v_{P,01}}{a_{P,01}} = \frac{v_{P,1} - v_{P,0}}{a_{P,01}} = \frac{52.8\text{ m/s} - 0}{2.2\text{ m/s}^2} = 24\text{ s}$$

Express the displacement of the patrol car:

$$\Delta x_{P,01} = v_{P,0}\Delta t_{P,01} + \tfrac{1}{2}a_{P,01}\Delta t_{P,01}^2 = 0 + \tfrac{1}{2}(2.2\text{ m/s}^2)(24\text{ s})^2 = 630\text{ m}$$

Equate the displacements of the two vehicles:

$$\Delta x_{P,02} = \Delta x_{P,01} + \Delta x_{P,12} = \Delta x_{P,01} + v_{P,1}(t_2 - t_1) = 630\text{ m} + (52.8\text{ m/s})(t_2 - 24\text{ s})$$

Substitute for $\Delta x_{P,02}$ to obtain:

$$(34.7\text{ m/s})\,t_2 = 630\text{ m} + (52.8\text{ m/s})(t_2 - 24\text{ s})$$

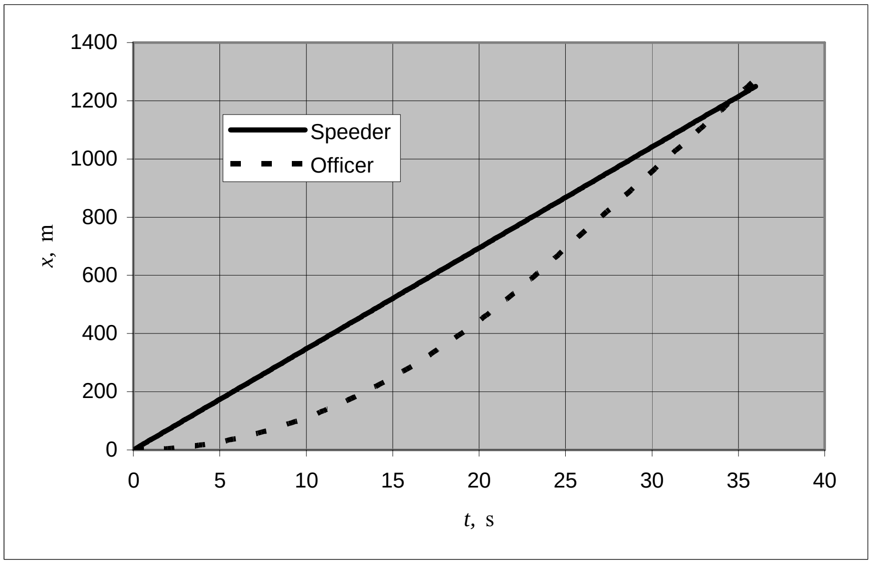
Solving for the time to catch up yields:

$$\boldsymbol{t}_2 = 35.19\text{ s} = \boxed{35\text{ s}}$$

(*b*) The distance traveled is the displacement, $\Delta x_{02,S}$, of the speeder during the catch:

$$\Delta \boldsymbol{x}_{S,02} = \boldsymbol{v}_{S,02}\Delta \boldsymbol{t}_{02} = (35\text{ m/s})(35.19\text{ s}) = \boxed{1.2\text{ km}}$$

(*c*) The graphs of x_S and x_P follow. The straight line (solid) represents $x_S(t)$ and the parabola (dashed) represents $x_P(t)$.



100 ••• When the patrol car in Problem 99 (traveling at 190 km/h), is 100 m behind the speeder (traveling at 125 km/h), the speeder sees the police car and slams on his brakes, locking the wheels. (*a*) Assuming that each car can brake at 6.0 m/s^2 and that the driver of the police car brakes instantly as she sees the brake lights of the speeder (reaction time = 0.0 s), show that the cars collide. (*b*) At what time after the speeder applies his brakes do the two cars collide? (*c*) Discuss how reaction time affects this problem.

Picture the Problem The accelerations of both cars are constant and we can use constant-acceleration equations to describe their motions. Choose a coordinate system in which the direction of motion of the cars is the positive direction, and the origin is at the initial position of the police car.

(*a*) The collision will ***not*** occur if, during braking, the displacements of the two cars differ by less than 100 m:

$$\Delta x_P - \Delta x_S < 100\text{ m}$$

Using a constant-acceleration equation, relate the speeder's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_s^2 = v_{0,s}^2 + 2a_s\Delta x_s$$

or, because $v_S = 0$,

$$\Delta x_s = \frac{-v_{0,s}^2}{2a_s}$$

Substitute numerical values and evaluate Δx_S:

$$\Delta x_S = \frac{-(34.7\text{ m/s})^2}{2(-6.0\text{ m/s}^2)} = 100\text{ m}$$

Using a constant-acceleration equation, relate the patrol car's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_{\mathrm{p}}^2 = v_{0,\mathrm{p}}^2 + 2a_{\mathrm{p}}\Delta x_{\mathrm{p}}$$

or, assuming $v_{\mathrm{p}} = 0$,

$$\Delta x_{\mathrm{p}} = \frac{-v_{0,\mathrm{p}}^2}{2a_{\mathrm{p}}}$$

Substitute numerical values and evaluate Δx_{P}:

$$\Delta \boldsymbol{x}_{\mathrm{P}} = \frac{-(52.8\,\mathrm{m/s})^2}{2(-6.0\,\mathrm{m/s^2})} = 232\,\mathrm{m}$$

Finally, substitute these displacements into the inequality that determines whether a collision occurs:

232 m – 100 m = 132 m
Because this difference is greater than 100 m, $\boxed{\text{the cars collide}}$.

(*b*) Using constant-acceleration equations, relate the positions of both vehicles to their initial positions, initial velocities, accelerations, and time in motion:

$$\boldsymbol{x}_{\mathrm{S}} = 100\,\mathrm{m} + (34.7\,\mathrm{m/s})\boldsymbol{t} - (3.0\,\mathrm{m/s^2})\boldsymbol{t}^2$$

and

$$\boldsymbol{x}_{\mathrm{P}} = (52.8\,\mathrm{m/s})\boldsymbol{t} - (3.0\,\mathrm{m/s^2})\boldsymbol{t}^2$$

Equate x_{S} and x_{P} to obtain:

$$100\,\mathrm{m} + (34.7\,\mathrm{m/s})\boldsymbol{t} - (3.0\,\mathrm{m/s^2})\boldsymbol{t}^2 = (52.8\,\mathrm{m/s})\boldsymbol{t} - (3.0\,\mathrm{m/s^2})\boldsymbol{t}^2$$

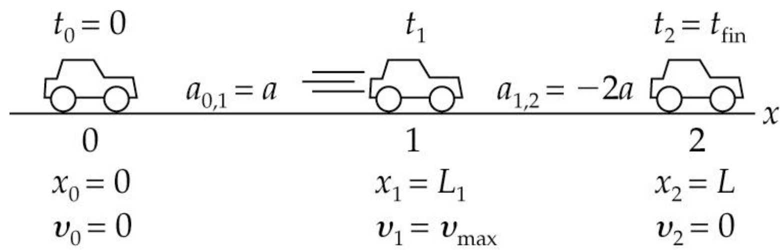
Solving this equation for t yields:

$$\boldsymbol{t} = \boxed{5.5\,\mathrm{s}}$$

(*c*) If you take the reaction time into account, the collision will occur sooner and be more severe.

101 ••• Leadfoot Lou enters the "Rest-to-Rest" auto competition, in which each contestant's car begins and ends at rest, covering a fixed distance L in as short a time as possible. The intention is to demonstrate driving skills, and to find which car is the best at the *total combination* of speeding up and slowing down. The course is designed so that maximum speeds of the cars are never reached. (*a*) If Lou's car maintains an acceleration (magnitude) of a during speedup, and maintains a deceleration (magnitude) of $2a$ during braking, at what fraction of L should Lou move his foot from the gas pedal to the brake? (*b*) What fraction of the total time for the trip has elapsed at that point? (c) What is the fastest speed Lou's car ever reaches? Neglect Lou's reaction time, and answer in terms of a and L.

Picture the Problem Lou's acceleration is constant during both parts of his trip. Let t_1 be the time when the brake is applied; L_1 the distance traveled from $t = 0$ to $t = t_1$. Let t_{fin} be the time when Lou's car comes to rest at a distance L from the starting line. A pictorial representation will help organize the given information and plan the solution.



(*a*) Express the total length, *L*, of the course in terms of the distance over which Lou will be accelerating, Δx_{01}, and the distance over which he will be braking, Δx_{12}:

$$L = \Delta x_{01} + \Delta x_{12} \quad (1)$$

Express the final velocity over the first portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_1^2 = v_0^2 + 2a_{01}\Delta x_{01}$$

or, because $v_0 = 0$, $\Delta x_{01} = L_1$, and $a_{01} = a$,

$$\Delta x_{01} = L_1 = \frac{v_1^2}{2a} = \frac{v_{max}^2}{2a}$$

Express the final velocity over the second portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_2^2 = v_1^2 + 2a_{12}\Delta x_{12}$$

or, because $v_2 = 0$ and $a_{12} = -2a$,

$$\Delta x_{12} = \frac{v_1^2}{4a} = \frac{L_1}{2}$$

Substitute for Δx_{01} and Δx_{12} to obtain:

$$L = \Delta x_{01} + \Delta x_{12} = L_1 + \tfrac{1}{2}L_1 = \tfrac{3}{2}L_1$$

and

$$L_1 = \boxed{\tfrac{2}{3}L}$$

(*b*) Using the fact that the acceleration was constant during both legs of the trip, express Lou's average velocity over each leg:

$$v_{av,01} = v_{av,12} = \frac{v_{max}}{2}$$

Express the time for Lou to reach his maximum velocity as a function of L_1 and his maximum velocity:

$$\Delta t_{01} = \frac{\Delta x_{01}}{v_{av,01}} = \frac{2L_1}{v_{max}}$$

and

$$\Delta t_{01} \propto L_1 = \tfrac{2}{3}L$$

Having just shown that the time required for the first segment of the trip is proportional to the length of the segment, use this result to express Δt_{01} (= t_1) in terms t_{fin}:

$t = \boxed{\frac{2}{3}t_{fin}}$

(*c*) Express Lou's displacement during the speeding up portion of his trip:

$\Delta x_{01} = \frac{1}{2}at_1^2$

The time required for Lou to make this speeding portion of his trip is given by:

$t_1 = \frac{v_1}{a}$

Express Lou's displacement during the slowing down portion of his trip:

$\Delta x_{12} = v_1t_1 - \frac{1}{2}(2a)t_2^2 = v_1t_1 - at_2^2$

where t_2 is his slowing down time.

Substitute in equation (1) to obtain:

$$L = \frac{1}{2}at_1^2 + v_1t_1 - at_2^2 \qquad (2)$$

Using a constant-acceleration equation, relate Lou's slowing down time for this portion of his trip to his initial and final speeds:

$0 = v_1 - 2at_2 \Rightarrow t_2 = \frac{v_1}{2a}$

Substituting for t_1 and t_2 in equation (2) yields:

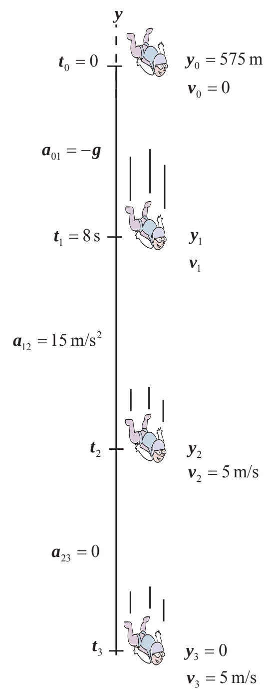
$$L = \frac{1}{2}a\left(\frac{v_1}{a}\right)^2 + v_1\left(\frac{v_1}{a}\right) - a\left(\frac{v_1}{2a}\right)^2$$

Solve for v_1 to obtain:

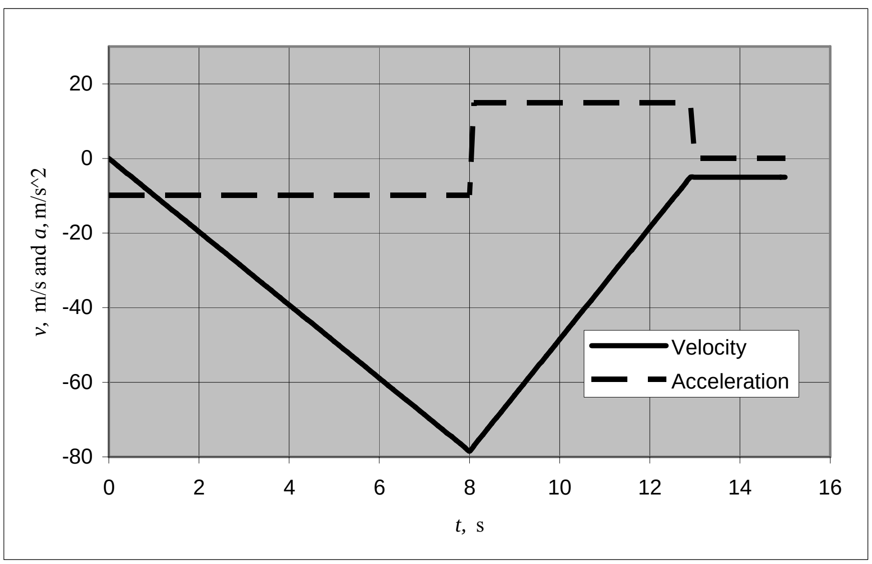
$$v_1 = \boxed{\sqrt{\frac{4aL}{3}}}$$

102 ••• A physics professor, equipped with a rocket backpack, steps out of a helicopter at an altitude of 575 m with zero initial velocity. (Neglect air resistance.) For 8.0 s, she falls freely. At that time she fires her rockets and slows her rate of descent at 15 m/s^2 until her rate of descent reaches 5.0 m/s. At this point she adjusts her rocket engine controls to maintain that rate of descent until she reaches the ground. (*a*) On a single graph, sketch her acceleration and velocity as functions of time. (Take upward to be positive.) (*b*) What is her speed at the end of the first 8.0 s? (*c*) What is the duration of her slowing down period? (*d*) How far does she travel while slowing down? (*e*) How much time is required for the entire trip from the helicopter to the ground? (*f*) What is her average velocity for the entire trip?

Picture the Problem There are three intervals of constant acceleration described in this problem. Choose a coordinate system in which the upward direction is positive. The pictorial representation summarizes the information we are given. We can apply constant-acceleration equations to find her speeds and distances traveled at the end of each phase of her descent.



(a) The graphs of $a(t)$ (dashed lines) and $v(t)$ (solid lines) follow.



(b) Use a constant- acceleration equation to express her speed in terms of her acceleration and the elapsed time:

$$v_1 = v_0 + a_{01}t_1$$

or, because $a_{01} = -g$ and $v_0 = 0$,

$$v_1 = -gt_1$$

Substitute numerical values and evaluate v_1:

$$\boldsymbol{v}_1 = \left(-9.81\text{ m/s}^2\right)\left(8.0\text{ s}\right) = -78.5\text{ m/s}$$

and her speed is $\boxed{79\text{ m/s}}$

(*c*) As in (*b*), use a constant-acceleration equation to express her speed in terms of her acceleration and the elapsed time:

$$v_2 = v_1 + a_{12}\Delta t_{12} \Rightarrow \Delta \boldsymbol{t}_{12} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\boldsymbol{a}_{12}}$$

Substitute numerical values and evaluate Δt_{12}:

$$\Delta \boldsymbol{t}_{12} = \frac{-5.0\,\text{m/s} - (-78.5\,\text{m/s})}{15\,\text{m/s}^2} = \boxed{4.9\,\text{s}}$$

(*d*) Find her average speed as she slows from 78.5 m/s to 5 m/s:

$$\boldsymbol{v}_{\text{av}} = \frac{\boldsymbol{v}_1 + \boldsymbol{v}_2}{2} = \frac{78.5\,\text{m/s} + 5.0\,\text{m/s}}{2} = 41.7\,\text{m/s}$$

Use this value to calculate how far she travels in 4.90 s:

$$\Delta \boldsymbol{y}_{12} = \boldsymbol{v}_{\text{av}}\Delta \boldsymbol{t}_{12} = (41.7\text{ m/s})(4.90\text{ s}) = 204.3\,\text{m}$$

She travels approximately 204 m while slowing down.

(*e*) Express the total time in terms of the times for each segment of her descent:

$$\Delta t_{\text{total}} = \Delta t_{01} + \Delta t_{12} + \Delta t_{23}$$

We know the times for the intervals from 0 to 1 and 1 to 2 so we only need to determine the time for the interval from 2 to 3. We can calculate Δt_{23} from her displacement and constant velocity during that segment of her descent.

$$\Delta \boldsymbol{y}_{23} = \Delta \boldsymbol{y}_{\text{total}} - \Delta \boldsymbol{y}_{01} - \Delta \boldsymbol{y}_{12} = 575\,\text{m} - \left(\frac{78.5\,\text{m/s}}{2}\right)(8.0\,\text{s}) - 204.3\,\text{m} = 56.7\,\text{m}$$

Add the times to get the total time:

$$\Delta t_{\text{total}} = \Delta t_{01} + \Delta t_{12} + \Delta t_{23} = 8.0\,\text{s} + 4.9\,\text{s} + \frac{56.7\,\text{m}}{5.0\,\text{m/s}} = 24.24\,\text{s} = \boxed{24\,\text{s}}$$

(*f*) Using its definition, calculate her average velocity:

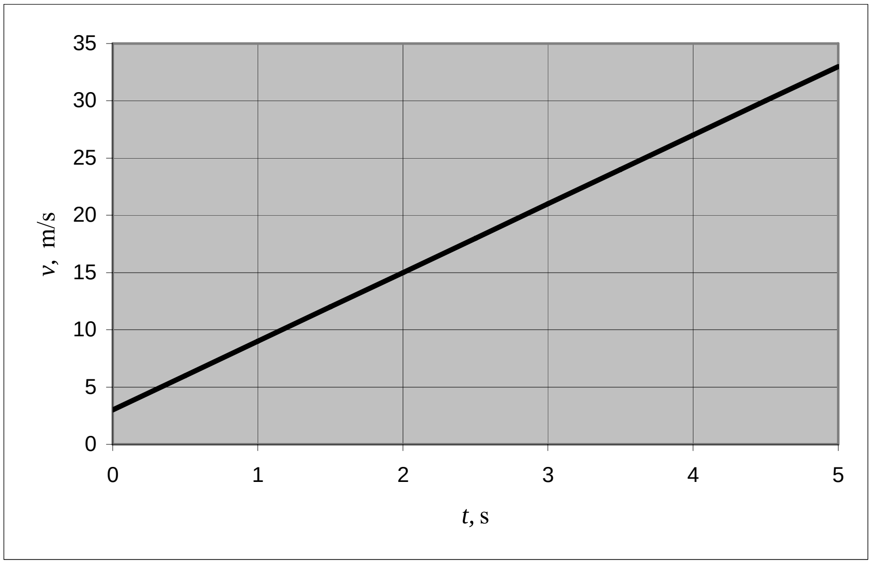
$$v_{\text{av}} = \frac{\Delta y}{\Delta t_{\text{total}}} = \frac{-575\,\text{m}}{24.24\,\text{s}} = \boxed{-24\,\text{m/s}}$$

Integration of the Equations of Motion

103 • **[SSM]** The velocity of a particle is given by $v_x(t) = (6.0\ \text{m/s}^2)t + (3.0\ \text{m/s})$. (*a*) Sketch *v* versus *t* and find the area under the curve for the interval $t = 0$ to $t = 5.0$ s. (*b*) Find the position function $x(t)$. Use it to calculate the displacement during the interval $t = 0$ to $t = 5.0$ s.

Picture the Problem The integral of a function is equal to the "area" between the curve for that function and the independent-variable axis.

(*a*) The following graph was plotted using a spreadsheet program:



The distance is found by determining the area under the curve. There are approximately 36 blocks each having an area of

(5.0 m/s)(0.5 s) = 2.5 m.

$$A_{\substack{\text{under}\\\text{curve}}} = (36\ \text{blocks})(2.5\ \text{m/block}) = \boxed{90\ \text{m}}$$

You can confirm this result by using the formula for the area of a trapezoid:

$$\boldsymbol{A} = \left(\frac{33\ \text{m/s} + 3\ \text{m/s}}{2}\right)(5.0\ \text{s} - 0\ \text{s}) = 90\ \text{m}$$

(*b*) To find the position function $x(t)$, we integrate the velocity function $v(t)$ over the time interval in question:

$$x(t) = \int_0^t v(t)\text{d}t = \int_0^t \left[(6.0\ \text{m/s}^2)t + (3.0\ \text{m/s})\right] dt$$

and

$$x(t) = \boxed{(3.0\ \text{m/s}^2)t^2 + (3.0\ \text{m/s})t}$$

Now evaluate $x(t)$ at 0 s and 5.0 s respectively and subtract to obtain Δx:

$$\Delta \boldsymbol{x} = \boldsymbol{x}(5.0\,\text{s}) - \boldsymbol{x}(0\,\text{s}) = 90\,\text{m} - 0\,\text{m}$$
$$= \boxed{90\,\text{m}}$$

104 • Figure 2-41 shows the velocity of a particle versus time. (*a*) What is the magnitude in meters represented by the area of the shaded box? (*b*) Estimate the displacement of the particle for the two 1-s intervals, one beginning at $t = 1.0$ s and the other at $t = 2.0$ s. (*c*) Estimate the average velocity for the interval $1.0\text{ s} \le t \le 3.0$ s. (*d*) The equation of the curve is $v_x = (0.50\text{ m/s}^3)t^2$. Find the displacement of the particle for the interval $1.0\text{ s} \le t \le 3.0$ s by integration and compare this answer with your answer for Part (*b*). Is the average velocity equal to the mean of the initial and final velocities for this case?

Picture the Problem The integral of $v(t)$ over a time interval is the displacement (change in position) during that time interval. The integral of a function is equivalent to the "area" between the curve for that function and the independent-variable axis. Count the grid boxes.

(*a*) Find the area of the shaded gridbox:

$$\text{Area} = (1\,\text{m/s})(1\,\text{s}) = \boxed{1\,\text{m per box}}$$

(*b*) Find the approximate area under the curve for $1.0\text{ s} \le t \le 2.0$ s:

$$\Delta x_{1.0\text{ s to }2.0\text{ s}} = \boxed{1.2\,\text{m}}$$

Find the approximate area under the curve for $2.0\text{ s} \le t \le 3.0$ s:

$$\Delta x_{2.0\text{ s to }3.0\text{ s}} = \boxed{3.2\,\text{m}}$$

(*c*) Sum the displacements to obtain the total in the interval $1.0\text{ s} \le t \le 3.0$ s:

$$\Delta x_{1.0\text{ s to }3.0\text{ s}} = 1.2\text{ m} + 3.2\text{ m}$$
$$= 4.4\text{ m}$$

Using its definition, express and evaluate v_{av}:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}_{1.0\,\text{s to }3.0\,\text{s}}}{\Delta t_{1.0\,\text{s to }3.0\,\text{s}}} = \frac{4.4\,\text{m}}{2.0\,\text{s}} = \boxed{2.2\,\text{m/s}}$$

(*d*) Because the velocity of the particle is *dx*/*dt*, separate the variables and integrate over the interval 1.0 s ≤ *t* ≤ 3.0 s to determine the displacement in this time interval:

$$dx = \left(0.5\ \text{m/s}^3\right)dt$$

so

$$\Delta x_{1.0\,\text{s}\to 3.0\,\text{s}} = \int_{x_0}^{x} dx = \left(0.50\ \text{m/s}^3\right)\int_{1.0\,\text{s}}^{3.0\,\text{s}} t^2\,dt$$

$$= \left(0.50\,\text{m/s}^3\right)\left[\frac{t^3}{3}\right]_{1.0\text{s}}^{3.0\text{s}} = 4.33\ \text{m}$$

$$= \boxed{4.3\ \text{m}}$$

This result is a little smaller than the sum of the displacements found in Part (*b*).

Calculate the average velocity over the 2-s interval from 1.0 s to 3.0 s:

$$\boldsymbol{v}_{\text{av}\,(1.0\text{s}-3.0\text{s})} = \frac{\Delta \boldsymbol{x}_{1.0\text{s}-3.0\text{s}}}{\Delta \boldsymbol{t}_{1.0\text{s}-3.0\text{s}}} = \frac{4.33\ \text{m}}{2.0\ \text{s}} = 2.2\ \text{m/s}$$

Calculate the initial and final velocities of the particle over the same interval:

$$\boldsymbol{v}(1.0\,\text{s}) = \left(0.50\,\text{m/s}^3\right)(1.0\,\text{s})^2 = 0.50\ \text{m/s}$$

$$\boldsymbol{v}(3.0\,\text{s}) = \left(0.50\,\text{m/s}^3\right)(3.0\,\text{s})^2 = 4.5\ \text{m/s}$$

Finally, calculate the average value of the velocities at $t = 1.0$ s and $t = 3.0$ s:

$$\frac{\boldsymbol{v}(1.0\,\text{s}) + \boldsymbol{v}(3.0\,\text{s})}{2} = \frac{0.50\ \text{m/s} + 4.5\ \text{m/s}}{2}$$

$$= 2.5\ \text{m/s}$$

This average of 2.5 m/s is not equal to the average velocity calculated above.

Remarks: The fact that the average velocity was not equal to the average of the velocities at the beginning and the end of the time interval in part (*d*) is a consequence of the acceleration not being constant.

105 •• The velocity of a particle is given by $v_x = (7.0\ \text{m/s}^3)t^2 - 5.0$ m/s, where t is in seconds and v is in meters per second. If the particle is at the origin at $t_0 = 0$, find the position function $x(t)$.

Picture the Problem Because the velocity of the particle varies with the square of the time, the acceleration is not constant. The position of the particle is found by integrating the velocity function.

Express the velocity of a particle as the derivative of its position function:

$$v(t) = \frac{dx(t)}{dt}$$

Separate the variables to obtain:

$$dx(t) = v(t)dt$$

Express the integral of x from $x_0 = 0$ to x and t from $t_0 = 0$ to t:

$$x(t) = \int_{t_0=0}^{x(t)} dx = \int_{t_0=0}^{t} v(t)dt$$

Substitute for $v(t)$ and carry out the integration to obtain:

$$x(t) = \int_{t_0=0}^{t} \left[\left(7.0\,\text{m/s}^3\right)t^2 - (5.0\,\text{m/s})\right]dt$$

$$= \boxed{\left(2.3\,\text{m/s}^3\right)t^3 - (5.0\,\text{m/s})t}$$

106 •• Consider the velocity graph in Figure 2-42. Assuming $x = 0$ at $t = 0$, write correct algebraic expressions for $x(t)$, $v_x(t)$, and $a_x(t)$ with appropriate numerical values inserted for all constants.

Picture the Problem The graph is one of constant negative acceleration. Because $v_x = v(t)$ is a linear function of t, we can make use of the slope-intercept form of the equation of a straight line to find the relationship between these variables. We can then differentiate $v(t)$ to obtain $a(t)$ and integrate $v(t)$ to obtain $x(t)$.

Find the acceleration (the slope of the graph) and the velocity at time 0 (the v-intercept) and use the slope-intercept form of the equation of a straight line to express $v_x(t)$:

$$\boldsymbol{a}_x = \boxed{-10\,\text{m/s}^2}$$

and

$$\boldsymbol{v}_x(\boldsymbol{t}) = \boxed{50\,\text{m/s} + (-10\,\text{m/s}^2)\boldsymbol{t}}$$

Find $x(t)$ by integrating $v(t)$:

$$\boldsymbol{x}(\boldsymbol{t}) = \int \left[\left(-10\,\text{m/s}^2\right)\boldsymbol{t} + 50\,\text{m/s}\right]\boldsymbol{dt}$$

$$= (50\,\text{m/s})\boldsymbol{t} - \left(5.0\,\text{m/s}^2\right)\boldsymbol{t}^2 + \text{C}$$

Using the fact that $x = 0$ when $t = 0$, evaluate C:

$$0 = (50\,\text{m/s})(0) - \left(5.0\,\text{m/s}^2\right)(0)^2 + \text{C}$$

and

$\text{C} = 0$

Substitute to obtain:

$$\boldsymbol{x}(\boldsymbol{t}) = \boxed{(50\,\text{m/s})\boldsymbol{t} - \left(5.0\,\text{m/s}^2\right)\boldsymbol{t}^2}$$

Note that this expression is quadratic in t and that the coefficient of t^2 is negative and equal in magnitude to half the constant acceleration.

Remarks: We can check our result for *x*(*t*) by evaluating it over the 10-s interval shown and comparing this result with the area bounded by this curve and the time axis.

107 ••• Figure 2-43 shows the acceleration of a particle versus time. (*a*) What is the magnitude, in m/s, of the area of the shaded box? (*b*) The particle starts from rest at $t = 0$. Estimate the velocity at $t = 1.0$ s, 2.0 s, and 3.0 s by counting the boxes under the curve. (*c*) Sketch the curve *v* versus *t* from your results for Part (*b*), then estimate how far the particle travels in the interval $t = 0$ to $t = 3.0$ s using your curve.

Picture the Problem During any time interval, the integral of $a(t)$ is the change in velocity and the integral of $v(t)$ is the displacement. The integral of a function equals the "area" between the curve for that function and the independent-variable axis.

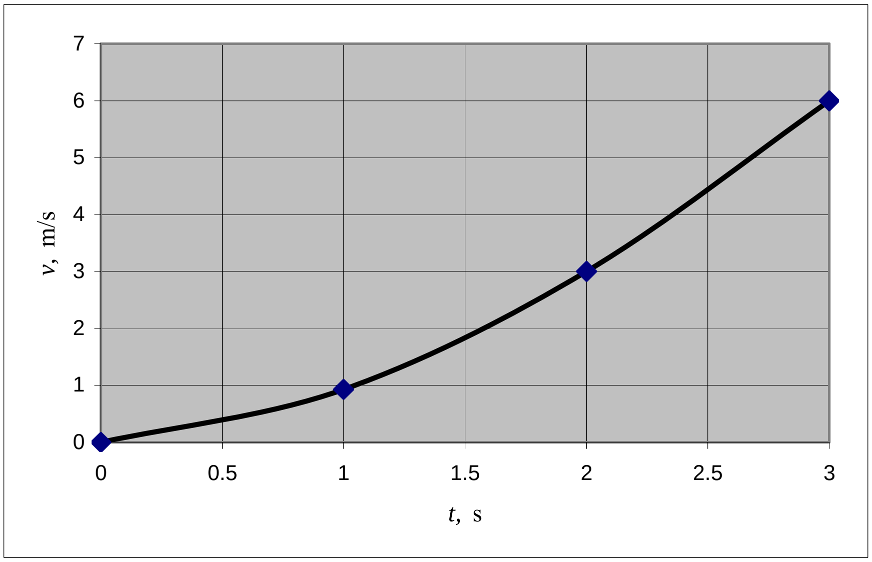
(*a*) Find the area of the shaded grid box in Figure 2-44:

$$Area = \left(0.50\,\frac{\text{m}}{\text{s}^2}\right)(0.50\,\text{s})$$
$$= \boxed{0.25\text{ m/s per box}}$$

(*b*) We start from rest ($v_0 = 0$) at $t = 0$. For the velocities at the other times, count boxes and multiply by the 0.25 m/s per box that we found in Part (*a*):

t	# of boxes	*v*(*t*)
(s)		(m/s)
1	3.7	$\boxed{0.93\text{ m/s}}$
2	12	$\boxed{3.0\text{ m/s}}$
3	24	$\boxed{6.0\text{ m/s}}$

(*c*) The graph of *v* as a function of *t* follows:



$$\boldsymbol{Area} = \boldsymbol{hw} = (1.0\text{ m/s})(0.50\text{ s}) = 0.50\text{ m per box}$$

Count the boxes under the $v(t)$ curve to find the distance traveled:

$$\boldsymbol{x}(3\,\text{s}) = \Delta\boldsymbol{x}(0 \rightarrow 3\,\text{s})$$
$$= (13\,\text{boxes})[(0.50\,\text{m})/\text{box}]$$
$$= \boxed{6.5\,\text{m}}$$

108 ••• Figure 2-44 is a graph of v_x versus t for a particle moving along a straight line. The position of the particle at time $t = 0$ is $x_0 = 5.0$ m.
(*a*) Find x for various times t by counting boxes, and sketch x as a function of t. (*b*) Sketch a graph of the acceleration a as a function of the time t. (*c*) Determine the displacement of the particle between $t = 3.0$ s and 7.0 s.

Picture the Problem The integral of $v(t)$ over a time interval is the displacement (change in position) during that time interval. The integral of a function equals the "area" between the curve for that function and the independent-variable axis. We can estimate this value by counting the number of squares under the curve between given limits and multiplying this count by the "area" of each square. Because acceleration is the slope of a velocity versus time curve, this is a non-constant-acceleration problem. The derivative of a function is equal to the "slope" of the function at that value of the independent variable. We can approximate the slope of any graph by drawing tangent lines and estimating their slopes.

(*a*) To obtain the data for $x(t)$, we must estimate the accumulated area under the $v(t)$ curve at each time interval:

Find the area of a shaded grid box in Figure 2-44:

$$\text{Area} = \boldsymbol{hw} = (1.0\ \text{m/s})(0.50\ \text{s})$$
$$= 0.50\ \text{m per box}$$

We start from rest ($v_o = 0$) at $t_o = 0$. For the position at the other times, count boxes and multiply by the 0.50 m per box that we found above. Remember to add the offset from the origin, $x_o = 5.0$ m, and that boxes below the $v = 0$ line are counted as negative:

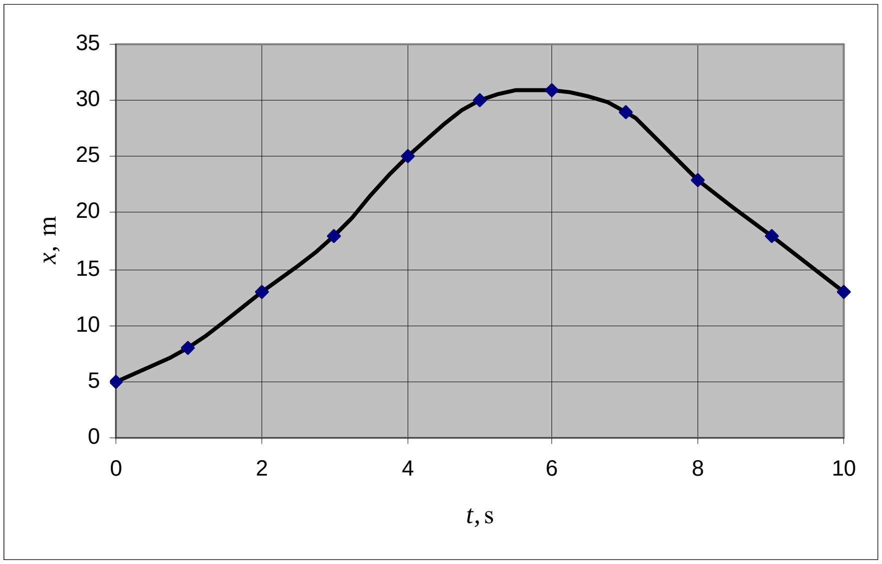
Examples:

$$\boldsymbol{x}(2\ \text{s}) = (15.5\ \text{boxes})\left(\frac{0.50\ \text{m}}{\text{box}}\right) + 5.0\ \text{m}$$
$$= 13\ \text{m}$$

$$\boldsymbol{x}(5\ \text{s}) = (49\ \text{boxes})\left(\frac{0.50\ \text{m}}{\text{box}}\right) + 5.0\ \text{m}$$
$$= 30\ \text{m}$$

$$\boldsymbol{x}(10\ \text{s}) = (51\ \text{boxes})\left(\frac{0.50\ \text{m}}{\text{box}}\right) - (36\ \text{boxes})\left(\frac{0.50\ \text{m}}{\text{box}}\right) + 5.0\ \text{m}$$
$$= 13\ \text{m}$$

A graph of x as a function of t follows:



(b) To obtain the data for $a(t)$, we must estimate the slope ($\Delta v/\Delta t$) of the $v(t)$ curve at several times. A good way to get reasonably reliable readings from the graph is to enlarge it several fold and then estimate the slope of the tangent line at selected points on the graph:

Examples:

$$\boldsymbol{a}(2\text{ s}) = \frac{8.0\text{ m/s} - 4.1\text{ m/s}}{4.9\text{ s}} = 0.8\text{ m/s}^2$$

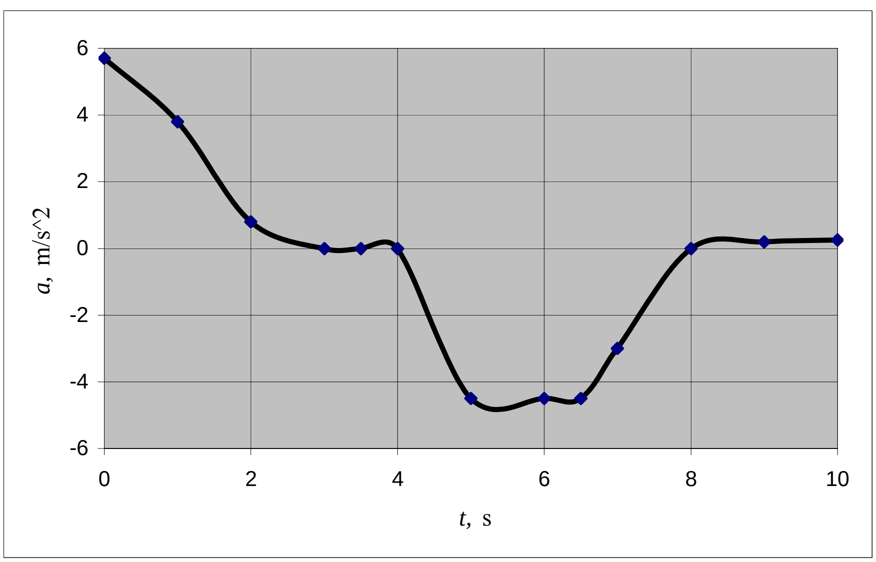
$$\boldsymbol{a}(3\text{ s}) = 0$$

$$\boldsymbol{a}(5\text{ s}) = \frac{-6.0\text{ m/s} - 8.0\text{ m/s}}{7.1\text{ s} - 4.0\text{ s}} = -4.5\text{ m/s}^2$$

$$\boldsymbol{a}(7\text{ s}) = \frac{-6.0\text{ m/s} - 1.4\text{ m/s}}{7.6\text{ s} - 5.0\text{ s}} = -3.0\text{ m/s}^2$$

$$\boldsymbol{a}(8\text{ s}) = 0$$

A graph of a as a function of t follows:



(*c*) The displacement of the particle between *t* = 3.0 s and 7.0 s is given by:

$$\Delta x_{3.0\,\text{s}\rightarrow 7.0\,\text{s}} = x(7.0\,\text{s}) - x(3.0\,\text{s})$$

Use the graph in (*a*) to obtain:

$$x(7.0\,\text{s}) = 29\,\text{m} \text{ and } x(3.0\,\text{s}) = 18\,\text{m}$$

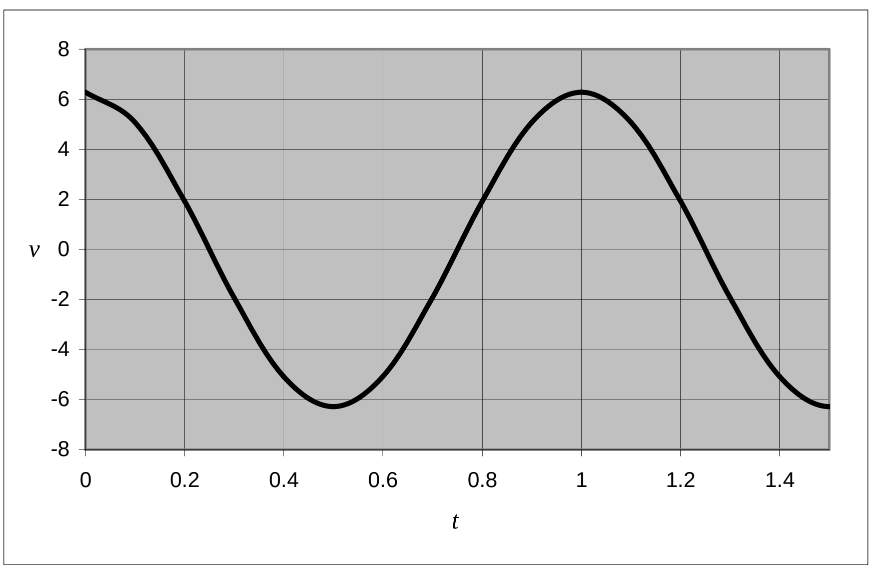
Substitute for Δ*x*(7.0 s) and Δ*x*(3.0 s) to obtain:

$$\Delta x_{3.0\,\text{s}\rightarrow 7.0\,\text{s}} = 29\,\text{m} - 18\,\text{m} = \boxed{11\,\text{m}}$$

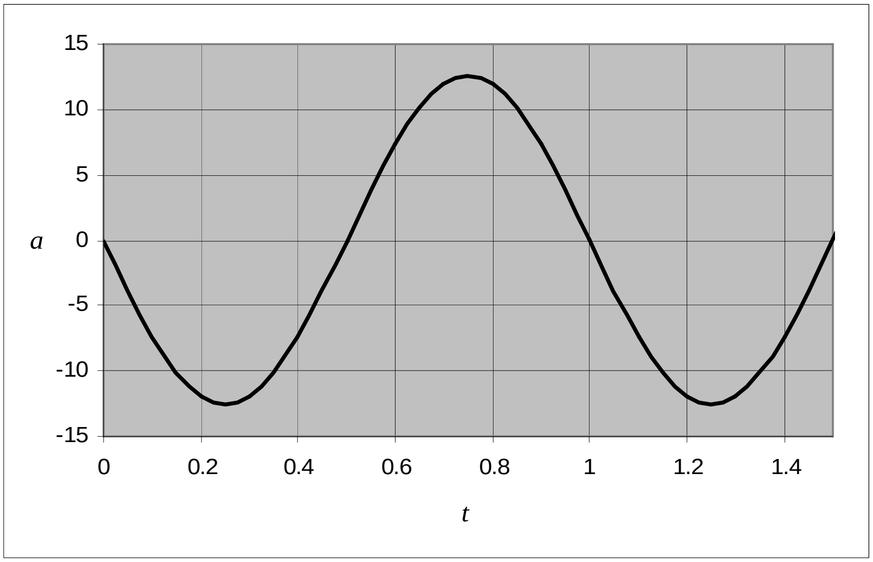
109 ••• **[SSM]** Figure 2-45 shows a plot of *x* versus *t* for a body moving along a straight line. For this motion, sketch graphs (using the same *t* axis) of (*a*) v_x as a function of *t*, and (*b*) a_x as a function of *t*. (*c*) Use your sketches to compare the times when the object is at its largest distance from the origin to the times when its speed is greatest. Explain why they do *not* occur at the same time. (*d*) Use your sketches to compare the time(s) when the object is moving fastest when the time(s) when its acceleration is the largest. Explain why they do *not* occur at the same time.

Picture the Problem Because the position of the body is not described by a parabolic function, the acceleration is not constant.

(*a*) Select a series of points on the graph of *x*(*t*) (e.g., at the extreme values and where the graph crosses the *t* axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating *v* = *dx*/*dt* at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the following graph.



(*b*) Select a series of points on the graph of *v*(*t*) (e.g., at the extreme values and where the graph crosses the *t* axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating *a* = *dv*/*dt* at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the following graph.



(*c*) The points at the greatest distances from the time axis correspond to turn-around points. The velocity of the body is zero at these points.

(*d*) The body is moving fastest as it goes through the origin. At these times the velocity is not changing and hence the acceleration is zero. The maximum acceleration occurs at the maximum distances where the velocity is zero but changing direction rapidly.

110 ••• The acceleration of a certain rocket is given by $a = bt$, where b is a positive constant. (*a*) Find the general position function $x(t)$ if $x_0 = 0$ and $v_x = v_{0x}$ at $t = 0$. (*b*) Find the position and velocity at $t = 5.0$ s if $x_0 = 0$, $v_{0x} = 0$, and $b = 3.0\ \text{m/s}^3$. (*c*) Compute the average velocity of the rocket between $t = 4.5$ s and 5.5 s at $t = 5.0$ s if $x_0 = 0$, $v_{0x} = 0$ and $b = 3.0\ \text{m/s}^3$. Compare this average velocity with the instantaneous velocity at $t = 5.0$ s.

Picture the Problem Because the acceleration of the rocket varies with time, it is not constant and integration of this function is required to determine the rocket's velocity and position as functions of time. The conditions on x and v at $t = 0$ are known as ***initial conditions***.

(*a*) Integrate $a(t)$ to find $v(t)$:

$$v(t) = \int a(t)\,dt = b\int t\,dt = \tfrac{1}{2}bt^2 + \text{C}$$

where C, the constant of integration, can be determined from the initial conditions.

Integrate $v(t)$ to find $x(t)$:

$$x(t) = \int v(t)dt = \int\left[\tfrac{1}{2}bt^2 + \text{C}\right]dt = \tfrac{1}{6}bt^3 + \text{C}t + \text{D}$$

where D is a second constant of integration.

Using the initial conditions, find the constants C and D:

$\boldsymbol{v}_{0x}(0) = 0 \Rightarrow \text{C} = 0$

and

$\boldsymbol{x}_0(0) = 0 \Rightarrow \text{D} = 0$

and so

$x(t) = \boxed{\tfrac{1}{6}bt^3}$

(*b*) Evaluate *v*(5.0 s) and *x*(5.0 s) with C = D = 0 and *b* = 3.0 m/s^3:

$\boldsymbol{v}(5.0\,\text{s}) = \tfrac{1}{2}(3.0\,\text{m/s}^3)(5.0\,\text{s})^2 = \boxed{38\,\text{m/s}}$

and

$\boldsymbol{x}(5.0\,\text{s}) = \tfrac{1}{6}(3.0\,\text{m/s}^3)(5.0\,\text{s}) = \boxed{63\,\text{m}}$

(*c*) The average value of *v*(*t*) over the interval Δ*t* is given by:

$$\bar{v} = \frac{1}{\Delta t}\int_{t_1}^{t_2} v(t)dt$$

Substitute for Δ*t* and *v*(*t*) and evaluate the integral to obtain:

$$\bar{v} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \tfrac{1}{2}bt^2 dt = \frac{b}{6\Delta t}t^3\Big|_{t_1}^{t_2} = \frac{b(t_2^3 - t_1^3)}{6\Delta t}$$

Substitute numerical values and evaluate $\bar{\boldsymbol{v}}$:

$$\bar{v} = \frac{(3.0\,\text{m/s}^3)\left[(5.5\,\text{s})^3 - (4.5\,\text{s})^3\right]}{6(5.5\,\text{s} - 4.5\,\text{s})}$$

$= \boxed{38\,\text{m/s}}$ ... a result in good agreement with the value found in (*b*).

111 ••• **[SSM]** In the time interval from 0.0 s to 10.0 s, the acceleration of a particle traveling in a straight line is given by $a_x = (0.20\,\text{m/s}^3)t$. Let to the right be the +*x* direction. A particle initially has a velocity to the right of 9.5 m/s and is located 5.0 m to the left of the origin. (*a*) Determine the velocity as a function of time during the interval, (*b*) determine the position as a function of time during the interval, (*c*) determine the average velocity between *t* = 0.0 s and10.0 s, and compare it to the average of the instantaneous velocities at the start and ending times. Are these two averages equal? Explain.

Picture the Problem The acceleration is a function of time; therefore it is not constant. The instantaneous velocity can be determined by integration of the acceleration function and the average velocity from the general expression for the average value of a non-linear function.

(*a*) The ***instantaneous velocity*** function *v*(*t*) is the time-integral of the acceleration function:

$$v(t) = \int a_x dt = \int bt\,dt = \frac{b}{2}t^2 + C_1$$

where $b = 0.20\,\text{m/s}^3$

The initial conditions are:

1) $\boldsymbol{x}(0) = -5.0\text{ m}$
and
2) $\boldsymbol{v}(0) = 9.5\text{ m/s}$

Use initial condition 2) to obtain:

$$\boldsymbol{v}(0) = 9.5\text{ m/s} = \boldsymbol{C}_1$$

Substituting in $v(t)$ for b and C_1 yields:

$$\boldsymbol{v}(\boldsymbol{t}) = \boxed{\left(0.10\text{ m/s}^3\right)\boldsymbol{t}^2 + 9.5\text{ m/s}} \quad (1)$$

(b) The ***instantaneous position*** function $x(t)$ is the time-integral of the velocity function:

$$x(t) = \int v(t)dt = \int \left(ct^2 + C_1\right)dt$$

$$= \frac{c}{6}t^3 + C_1 t + C_2$$

where $c = 0.10\text{ m/s}^3$.

Using initial condition 1) yields:

$$\boldsymbol{x}(0) = -5.0\text{ m} = \boldsymbol{C}_2$$

Substituting in $x(t)$ for C_1 and C_2 yields:

$$x(t) = \boxed{\tfrac{1}{6}\left(0.20\text{ m/s}^3\right)t^3 + (9.5\text{ m/s})t - 5.0\text{ m/s}}$$

(c) The average value of $v(t)$ over the interval Δt is given by:

$$\bar{v} = \frac{1}{\Delta t}\int_{t_1}^{t_2} v(t)dt$$

Substitute for $v(t)$ and evaluate the integral to obtain:

$$\bar{v} = \frac{1}{\Delta t}\int_{t_1}^{t_2}\left(\frac{b}{2}t^2 + C_1\right)dt = \frac{1}{\Delta t}\left[\frac{b}{6}t^3 + C_1 t\right]_{t_1}^{t_2} = \frac{1}{\Delta t}\left[\frac{b}{6}t_2^3 + C_1 t_2 - \left(\frac{b}{6}t_1^3 + C_1 t_1\right)\right]$$

Simplifying this expression yields:

$$\bar{v} = \frac{1}{\Delta t}\left[\frac{b}{6}\left(t_2^3 - t_1^3\right) + C_1\left(t_2 - t_1\right)\right]$$

Because $t_1 = 0$:

$$\bar{v} = \frac{1}{\Delta t}\left[\frac{b}{6}t_2^3 + C_1 t_2\right]$$

Substitute numerical values and simplify to obtain:

$$\bar{\boldsymbol{v}} = \frac{1}{10.0\text{ s}}\left[\left(\frac{0.20\text{ m/s}^3}{6}\right)(10.0\text{ s})^3 + (9.5\text{ m/s})(10.0\text{ s})\right] = \boxed{13\text{ m/s}}$$

The average of the initial instantaneous and final instantaneous velocities is given by:

$$\boldsymbol{v}_{\text{av}} = \frac{\boldsymbol{v}(0) + \boldsymbol{v}(10.0\text{ s})}{2} \qquad (2)$$

Using equation (1), evaluate $v(0)$ and $v(10\text{ s})$:

$$\boldsymbol{v}(0) = 9.5\text{ m/s}$$

and

$$\boldsymbol{v}(10\text{ s}) = (0.10\text{ m/s}^3)(10.0\text{ s})^2 + 9.5\text{ m/s} = 19.5\text{ m/s}$$

Substitute in equation (2) to obtain:

$$v_{\text{av}} = \frac{9.5\text{ m/s} + 19.5\text{ m/s}}{2} = \boxed{15\text{ m/s}}$$

v_{av} is not the same as $\bar{\boldsymbol{v}}$ because the velocity does not change linearly with time. The velocity does not change linearly with time because the acceleration is not constant.

112 ••• Consider the motion of a particle that experiences a variable acceleration given by $\boldsymbol{a}_x = \boldsymbol{a}_{0x} + \boldsymbol{b}\boldsymbol{t}$, where a_{0x} and b are constants and $x = x_0$ and $v_x = v_{0x}$ at $t = 0$. (*a*) Find the instantaneous velocity as a function of time. (*b*) Find the position as a function of time. (*c*) Find the average velocity for the time interval with an initial time of zero and arbitrary final time *t*. (*d*) Compare the average of the initial and final velocities to your answer to Part (*c*). Are these two averages equal? Explain.

Determine the Concept Because the acceleration is a function of time, it is not constant. Hence we'll need to integrate the acceleration function to find the velocity of the particle as a function of time and integrate the velocity function to find the position of the particle as a function of time. We can use the general expression for the average value of a non-linear function to find the average velocity over an arbitrary interval of time.

(*a*) From the definition of acceleration we have:

$$\int_{v_{0x}}^{v_x} dv = \int_0^t a\,dt$$

Integrating the left-hand side of the equation and substituting for *a* on the right-hand side yields:

$$v_x - v_{0x} = \int_0^t (a_0 + bt)\,dt$$

Integrate the right-hand side to obtain:

$$v_x - v_{0x} = a_0 t + \tfrac{1}{2}bt^2$$

Solving this equation for v_x yields:

$$v_x = \boxed{v_{0x} + a_0 t + \tfrac{1}{2}bt^2} \qquad (1)$$

(*b*) The position of the particle as a function of time is the integral of the velocity function:

$$\int_{x_0}^{x} dx = \int_0^t v_x(t)\,dt$$

Integrating the left-hand side of the equation and substituting for *a* on the right-hand side yields:

$$x - x_0 = \int_0^t \left(v_{0x} + a_0 t + \frac{b}{2}t^2 \right) dt$$

Integrate the right-hand side to obtain:

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3$$

Solving this equation for *x* yields:

$$x = \boxed{x_{0x} + v_0 t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3}$$

(*c*) The average value of $v(t)$ over an interval Δt is given by:

$$\bar{v}_x = \frac{1}{\Delta t}\int_{t_1}^{t_2} v_x(t)\,dt$$

or, because $t_1 = 0$ and $t_2 = t$,

$$\bar{v}_x = \frac{1}{t}\int_0^t v_x(t)\,dt$$

Substituting for $v_x(t)$ yields:

$$\bar{v}_x = \frac{1}{t}\int_0^t \left(v_{0x} + a_0 t + \tfrac{1}{2}bt^2 \right) dt$$

Carry out the details of the integration to obtain:

$$\bar{v}_x = \frac{1}{t}\left[v_{0x}t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3 \right]_0^t$$

$$= \frac{1}{t}\left[v_{0x}t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3 \right]$$

$$= \boxed{v_{0x} + \tfrac{1}{2}a_0 t + \tfrac{1}{6}bt^2}$$

(*d*) The average of the initial instantaneous and final instantaneous velocities is given by:

$$\boldsymbol{v}_{\text{av},\,\boldsymbol{x}} = \frac{\boldsymbol{v}_{0\boldsymbol{x}} + \boldsymbol{v}_{\boldsymbol{x}}}{2}$$

Using equation (1), substitute to obtain:

$$v_{\text{av},\,x} = \frac{v_{0x} + v_0 + a_0 t + \tfrac{1}{2}bt^2}{2}$$

$$= \frac{2v_{0x} + a_0 t + \tfrac{1}{2}bt^2}{2}$$

$$= \boxed{v_{0x} + \tfrac{1}{2}a_0 t + \tfrac{1}{4}bt^2}$$

$v_{\text{av},\,x}$ is not the same as $\boldsymbol{\bar{v}_x}$ because the acceleration is not constant.

General Problems

113 ••• You are a student in a science class that is using the following apparatus to determine the value of g. Two photogates are used. (Note: You may be familiar with photogates in everyday living. You see them in the doorways of some stores. They are designed to ring a bell when someone interrupts the beam while walking through the door.) One photogate is located at the edge of a 1.00-m high table and the second photogate is located on the floor directly below the first photogate. You are told to drop a marble through these gates, releasing it from rest 0.50 m above the lower gate. The upper gate starts a timer as the ball passes through it. The second photogate stops the timer when the ball passes through its beam. (*a*) Prove that the experimental magnitude of free-fall acceleration is given by $g_{exp} = (2\Delta y)/(\Delta t)^2$, where Δy is the vertical distance between the photogates and Δt is the fall time. (*b*) For your setup, what value of Δt would you expect to measure, assuming g_{exp} is the value (9.81 m/s^2)? (*c*) During the experiment, a slight error is made. Instead of locating the first photogate even with the top of the table, your not-so-careful lab partner locates it 0.50 cm lower than the top of the table. However, she does manage to properly locate the second photogate at a height of 0.50 m above the floor. What value of g_{exp} will you and your partner determine? What percentage difference does this represent from the standard value at sea level?

Picture the Problem The acceleration of the marble is constant. Because the motion is downward, choose a coordinate system with downward as the positive direction and use constant-acceleration equations to describe the motion of the marble.

(*a*) Use a constant-acceleration equation to relate the vertical distance the marble falls to its time of fall and its free-fall acceleration:

$$\Delta \boldsymbol{y} = \boldsymbol{v}_0 \Delta \boldsymbol{t} + \tfrac{1}{2}\boldsymbol{g}_{\text{exp}}(\Delta \boldsymbol{t})^2$$

or, because $v_0 = 0$,

$$\Delta \boldsymbol{y} = \tfrac{1}{2}\boldsymbol{g}_{\text{exp}}(\Delta \boldsymbol{t})^2 \Rightarrow \boldsymbol{g}_{\text{exp}} = \boxed{\frac{2\Delta \boldsymbol{y}}{(\Delta \boldsymbol{t})^2}} \quad (1)$$

(*b*) Solve equation (1) for Δt to obtain:

$$\Delta \boldsymbol{t} = \sqrt{\frac{2\Delta \boldsymbol{y}}{\boldsymbol{g}_{\text{exp}}}}$$

Substitute numerical values and evaluate Δt

$$\Delta \boldsymbol{t} = \sqrt{\frac{2(1.00\text{ m})}{9.81\text{ m/s}^2}} = \boxed{0.452\text{ s}}$$

(*c*) Using a constant-acceleration equation, express the velocity of the marble in terms of its initial velocity, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$ and $a = g$,

$$v_f^2 = 2g\Delta y \Rightarrow v_f = \sqrt{2g\Delta y}$$

Let v_1 be the velocity the ball has reached when it has fallen 0.5 cm, and v_2 be the velocity the ball has reached when it has fallen 0.5 m to obtain.

$$v_1 = \sqrt{2\left(9.81\,\text{m/s}^2\right)\left(0.0050\,\text{m}\right)} = 0.313\,\text{m/s}$$

and

$$v_2 = \sqrt{2\left(9.81\,\text{m/s}^2\right)\left(0.50\,\text{m}\right)} = 3.13\,\text{m/s}$$

Using a constant-acceleration equation, express v_2 in terms of v_1, g and Δt:

$$v_2 = v_1 + g\Delta t \Rightarrow \Delta t = \frac{v_2 - v_1}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta \boldsymbol{t} = \frac{3.13\,\text{m/s} - 0.313\,\text{m/s}}{9.81\,\text{m/s}^2} = 0.2872\,\text{s}$$

The distance the marble falls during this interval is:

$$\Delta y = 1.00\,\text{m} - 0.50\,\text{m} - 0.0050\,\text{m} = 0.50\,\text{m}$$

Use equation (1) to calculate the experimental value of the acceleration due to gravity:

$$g_{\text{exp}} = \frac{2\left(0.50\,\text{m}\right)}{\left(0.2872\,\text{s}\right)^2} = \boxed{12\,\text{m/s}^2}$$

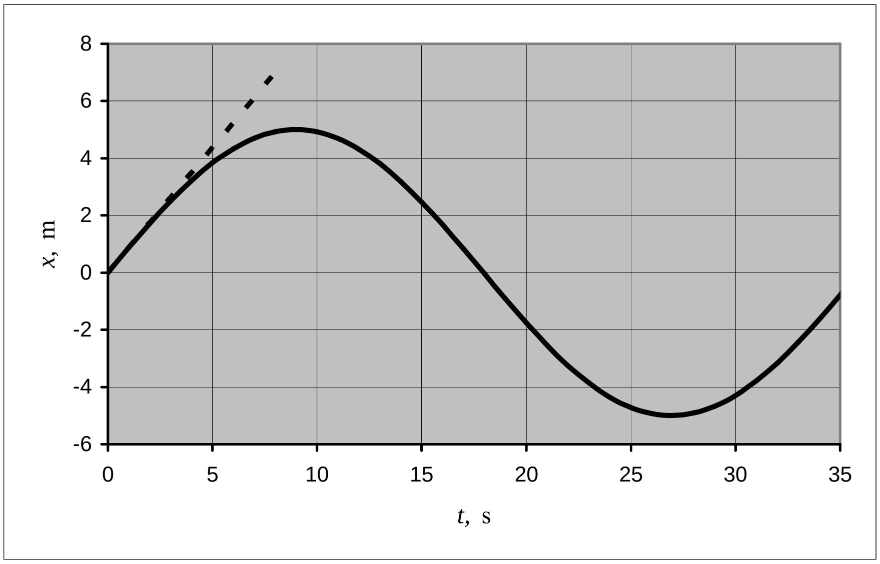
Finally, calculate the percent difference between this experimental result and the value accepted for g at sea level.

$$\%\ \text{difference} = \frac{\left|9.81\,\text{m/s}^2 - 12\,\text{m/s}^2\right|}{9.81\,\text{m/s}^2} = \boxed{22\%}$$

114 ••• The position of a body oscillating on a spring is given by $x = A \sin \omega t$, where A and ω are constants, $A = 5.0$ cm. and $\omega = 0.175\ \text{s}^{-1}$. (*a*) Plot x as a function of t for $0 \le t \le 36$ s. (*b*) Measure the slope of your graph at $t = 0$ to find the velocity at this time. (*c*) Calculate the average velocity for a series of intervals, beginning at $t = 0$ and ending at $t = 6.0$, 3.0, 2.0, 1.0, 0.50, and 0.25 s. (*d*) Compute dx/dt to find the velocity at time $t = 0$. (*e*) Compare your results in Parts (*c*) and (*d*) and explain why your Part (*c*) results approach your Part (*d*) results.

Picture the Problem We can obtain an average velocity, $v_{av} = \Delta x/\Delta t$, over fixed time intervals. The instantaneous velocity, $v = dx/dt$ can only be obtained by differentiation.

(a) The following graph of x versus t was plotted using a spreadsheet program:



(b) Draw a tangent line at the origin and measure its rise and run. Use this ratio to obtain an approximate value for the slope at the origin:

The tangent line appears to, at least approximately, pass through the point (5, 4). Using the origin as the second point,

$$\Delta x = 4 \text{ cm} - 0 = 4 \text{ cm}$$

and

$$\Delta t = 5 \text{ s} - 0 = 5 \text{ s}$$

Therefore, the slope of the tangent line and the velocity of the body as it passes through the origin is approximately:

$$\boldsymbol{v}(0) = \frac{\text{rise}}{\text{run}} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \frac{4\,\text{cm}}{5\,\text{s}} = \boxed{0.8\,\text{cm/s}}$$

(c) Calculate the average velocity for the series of time intervals given by completing the table shown below:

t_0 (s)	t (s)	Δt (s)	x_0 (cm)	x (cm)	Δx (cm)	$v_{av}=\Delta x/\Delta t$ (m/s)
0	6	6	0	4.34	4.34	0.723
0	3	3	0	2.51	2.51	0.835
0	2	2	0	1.71	1.71	0.857
0	1	1	0	0.871	0.871	0.871
0	0.5	0.5	0	0.437	0.437	0.874
0	0.25	0.25	0	0.219	0.219	0.875

(d) Express the time derivative of the position function:

$$\frac{dx}{dt} = A\omega \cos \omega t$$

Substitute numerical values and evaluate $\frac{dx}{dt}$ at $t = 0$:

$$\frac{dx}{dt} = A\omega\cos 0 = A\omega$$
$$= (0.050\text{ m})(0.175\text{ s}^{-1})$$
$$= \boxed{0.88\text{ cm/s}}$$

(*e*) Compare the average velocities from Part (*c*) with the instantaneous velocity from Part (*d*):

As Δt, and thus Δx, becomes small, the value for the average velocity approaches that for the instantaneous velocity obtained in Part (*d*). For $\Delta t = 0.25$ s, they agree to three significant figures.

115 ••• **[SSM]** Consider an object that is attached to a horizontally oscillating piston. The object moves with a velocity given by $v = B\sin(\omega t)$, where B and ω are constants and ω is in s^{-1}. (*a*) Explain why B is equal to the maximum speed v_{max}. (*b*) Determine the acceleration of the object as a function of time. Is the acceleration constant? (*c*) What is the maximum acceleration (magnitude) in terms of ω and v_{max}. (*d*) At $t = 0$, the object's position is known to be x_0. Determine the position as a function of time in terms of in terms of t, ω, x_0 and v_{max}.

Determine the Concept Because the velocity varies nonlinearly with time, the acceleration of the object is not constant. We can find the acceleration of the object by differentiating its velocity with respect to time and its position function by integrating the velocity function.

(*a*) The maximum value of the sine function (as in $v = v_{max}\sin(\omega t)$) is 1. Hence the coefficient B represents the maximum possible speed v_{max}.

(*b*) The acceleration of the object is the derivative of its velocity with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[v_{max}\sin(\omega t)\right]$$
$$= \boxed{\omega v_{max}\cos(\omega t)}$$

Because a varies sinusoidally with time it is *not* constant.

(*c*) Examination of the coefficient of the cosine function in the expression for a leads one to the conclusion that $|\boldsymbol{a}_{max}| = \boxed{\boldsymbol{\omega v}_{max}}.$

(*d*) The position of the object as a function of time is the integral of the velocity function:

$$\int dx = \int v(t)\,dt$$

Integrating the left-hand side of the equation and substituting for v on the right-hand side yields:

$$x = \int v_{\max} \sin(\omega t)dt + C$$

Integrate the right-hand side to obtain:

$$x = \frac{-v_{\max}}{\omega}\cos(\omega t) + C \qquad (1)$$

Use the initial condition $x(0) = x_0$ to obtain:

$$x_0 = \frac{-v_{\max}}{\omega} + C$$

Solving for C yields:

$$C = x_0 + \frac{v_{\max}}{\omega}$$

Substitute for C in equation (1) to obtain:

$$x = \frac{-v_{\max}}{\omega}\cos(\omega t) + x_0 + \frac{v_{\max}}{\omega}$$

Solving this equation for x yields:

$$x = \boxed{x_0 + \frac{v_{\max}}{\omega}[1 - \cos(\omega t)]}$$

116 ••• Suppose the acceleration of a particle is a function of x, where $a_x(x) = (2.0\ \text{s}^{-2})x$. (*a*) If the velocity is zero when $x = 1.0$ m, what is the speed when $x = 3.0$ m? (*b*) How long does it take the particle to travel from $x = 1.0$ m to $x = 3.0$ m?

Picture the Problem Because the acceleration of the particle is a function of its position, it is not constant. Changing the variable of integration in the definition of acceleration will allow us to determine its velocity and position as functions of position.

(*a*) The acceleration of the particle is given by:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Separating variables yields:

$$vdv = adx$$

Substitute for a to obtain:

$$vdv = bxdx$$

where $b = 2.0\ \text{s}^{-2}$

The limits of integration are $v_0 = 0$ and v on the left-hand side and x_0 and x on the right-hand:

$$\int_{v_0=0}^{v} vdv = \int_{x_0}^{x} bxdx$$

Integrating both sides of the equation yields:

$$v^2 - v_0^2 = b\left(x^2 - x_0^2\right)$$

Solve for v to obtain:

$$v = \sqrt{v_0^2 + b\left(x^2 - x_0^2\right)}$$

Now set $v_o = 0$, $x_o = 1.0$ m, $x = 3$ m, and $b = 2.0\ \text{s}^{-2}$ and evaluate the speed:

$$|\boldsymbol{v}(3.0\,\text{m})| = \pm\sqrt{\left(2.0\,\text{s}^{-2}\right)\left[(3.0\,\text{m})^2 - (1.0\,\text{m})^2\right]} = \boxed{4.0\,\text{m/s}}$$

(*b*) From the definition of instantaneous velocity we have:

$$t = \int_0^t dt = \int_{x_0}^{x} \frac{dx}{v(x)}$$

Substitute the expression for $v(x)$ derived in (*a*) to obtain:

$$t = \int_{x_0}^{x} \frac{dx}{\sqrt{b\left(x^2 - x_0^2\right)}}$$

Evaluate the integral using the formula found in standard integral tables to obtain:

$$t = \frac{1}{\sqrt{b}} \int_{x_0}^{x} \frac{dx}{\sqrt{x^2 - x_0^2}} = \frac{1}{\sqrt{b}} \ln\left(\frac{x + \sqrt{x^2 - x_0^2}}{x_0}\right)$$

Substitute numerical values and evaluate t:

$$\boldsymbol{t} = \frac{1}{\sqrt{2.0\,\text{s}^{-2}}} \ln\left(\frac{3.0\,\text{m} + \sqrt{(3.0\,\text{m})^2 - (1.0\,\text{m})^2}}{1.0\,\text{m}}\right) = \boxed{1.2\,\text{s}}$$

117 ••• **[SSM]** A rock falls through water with a continuously decreasing acceleration. Assume that the rock's acceleration as a function of *velocity* has the form $\boldsymbol{a}_y = \boldsymbol{g} - \boldsymbol{b}\boldsymbol{v}_y$ where b is a positive constant. (The $+y$ direction is directly downward.) (*a*) What are the SI units of b? (*b*) Prove mathematically that if the rock enters the water at time $t = 0$, the acceleration will depend exponentially on *time* according to $\boldsymbol{a}_y(\boldsymbol{t}) = \boldsymbol{g}\boldsymbol{e}^{-\boldsymbol{bt}}$. (*c*) What is the terminal speed for the rock in terms of g and b? (See Problem 38 for an explanation of the phenomenon of *terminal speed*.)

Picture the Problem Because the acceleration of the rock is a function of its velocity, it is not constant and we will have to integrate the acceleration function in order to find the velocity function. Choose a coordinate system in which downward is positive and the origin is at the point of release of the rock.

(*a*) All three terms in $\boldsymbol{a}_y = \boldsymbol{g} - \boldsymbol{b}\boldsymbol{v}_y$ must have the same units in order for the equation to be valid. Hence the units of bv_y must be acceleration units. Because the SI units of v_y are m/s, b must have units of $\boxed{\text{s}^{-1}.}$

(*b*) Rewrite $a_y = g - bv_y$ explicitly as a differential equation:

$$\frac{dv_y}{dt} = g - bv_y$$

Separate the variables, v_y on the left, t on the right:

$$\frac{dv_y}{g - bv_y} = dt$$

Integrate the left-hand side of this equation from 0 to v_y and the right-hand side from 0 to t:

$$\int_0^{v_y} \frac{dv_y}{g - bv_y} = \int_0^t dt$$

Integrating this equation yields:

$$-\frac{1}{b}\ln\left(\frac{g - bv_y}{g}\right) = t$$

Solve this expression for v_y to obtain:

$$v_y = \frac{g}{b}\left(1 - e^{-bt}\right) \qquad (1)$$

Differentiate this expression with respect to time to obtain an expression for the acceleration and complete the proof:

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{g}{b}\left(1 - e^{-bt}\right)\right) = \boxed{ge^{-bt}}$$

(*c*) Take the limit, as $t \to \infty$, of both sides of equation (1):

$$\lim_{t\to\infty} v_y = \lim_{t\to\infty}\left[\frac{g}{b}\left(1 - e^{-bt}\right)\right]$$

and

$$v_{\text{t}} = \boxed{\frac{g}{b}}$$

Notice that this result depends only on *b* (inversely so). Thus *b* must include things like the shape and cross-sectional area of the falling object, as well as properties of the liquid such as density and temperature.

118 ••• A small rock sinking through water (see Problem 117) experiences an exponentially decreasing acceleration given by $a_y(t) = ge^{-bt}$, where *b* is a positive constant that depends on the shape and size of the rock and the physical properties of the water. Based upon this, find expressions for the velocity and position of the rock as functions of time. Assume that its initial position and velocity are both zero and that the +*y* direction is directly downward.

Picture the Problem Because the acceleration of the rock is a function of time, it is not constant and we will have to integrate the acceleration function in order to find the velocity of the rock as a function of time. Similarly, we will have to integrate the velocity function in order to find the position of the rock as a function of time. Choose a coordinate system in which downward is positive and the origin at the point of release of the rock.

Separate variables in $a_y(t) = dv_y/dt = ge^{-bt}$ to obtain:

$$dv_y = ge^{-bt}dt$$

Integrating from $t_0 = 0$, $v_{0y} = 0$ to some later time t and velocity v_y yields:

$$v = \int_0^v dv = \int_0^t ge^{-bt}dt = \frac{g}{-b}\left[e^{-bt}\right]_0^t = \frac{g}{b}\left(1-e^{-bt}\right) = \boxed{v_{\mathrm{t}}\left(1-e^{-bt}\right)}$$

where $\boldsymbol{v}_{\mathrm{t}} = \dfrac{\boldsymbol{g}}{\boldsymbol{b}}$

Separate variables in $v = \dfrac{dy}{dt} = v_{\mathrm{t}}\left(1-e^{-bt}\right)$ to obtain:

$$dy = v_{\mathrm{t}}\left(1-e^{-bt}\right)dt$$

Integrate from $t_o = 0$, $y_o = 0$ to some later time t and position y:

$$y = \int_0^y dy = \int_0^t v_{\mathrm{t}}\left(1-e^{-bt}\right)\mathrm{d}tv_{\mathrm{term}}\left[t+\frac{1}{b}e^{-bt}\right]_0^t = \boxed{v_{\mathrm{t}}t - \frac{v_{\mathrm{t}}}{b}\left(1-e^{-bt}\right)}$$

Remarks: This last result is very interesting. It says that throughout its free-fall, the object experiences drag; therefore it has not fallen as far at any given time as it would have if it were falling at the constant velocity, v_t.

119 ••• The acceleration of a skydiver jumping from an airplane is given by $a_y = g - bv_y^2$ where b is a positive constant depending on the skydiver's cross-sectional area and the density of the surrounding atmosphere she is diving through. The $+y$ direction is directly downward. (*a*) If her initial speed is zero when stepping from a hovering helicopter, show that her speed as a function of time is given by $v_y(t) = v_{\mathrm{t}} \tanh(t/T)$ where v_t is the terminal speed (see Problem 38) given by $v_{\mathrm{t}} = \sqrt{g/b}$ and $T = v_{\mathrm{t}}/g$ is a time-scale parameter (*b*) What fraction of the terminal speed is the speed at $t = T$? (*c*) Use a **spreadsheet** program to graph $v_y(t)$ as a function of time, using a terminal speed of 56 m/s (use this value to calculate b and T). Does the resulting curve make sense?

Picture the Problem The skydiver's acceleration is a function of her speed; therefore it is not constant. Expressing her acceleration as the derivative of her speed, separating the variables, and then integrating will give her speed as a function of time.

(a) Rewrite $a_y = g - bv_y^2$ explicitly as a differential equation:

$$\frac{dv_y}{dt} = g - bv_y^2$$

Separate the variables, with v_y on the left, and t on the right:

$$\frac{dv_y}{g - bv_y^2} = dt$$

Eliminate b by using $b = \dfrac{g}{v_t^2}$:

$$\frac{dv_y}{g - \dfrac{g}{v_t^2}v_y^2} = \frac{dv_y}{g\left[1 - \left(\dfrac{v_y}{v_t}\right)^2\right]} = dt$$

or, separating variables,

$$\frac{dv_y}{1 - \left(\dfrac{v_y}{v_t}\right)^2} = gdt$$

Integrate the left-hand side of this equation from 0 to v_y and the right-hand side from 0 to t:

$$\int_0^{v_y} \frac{dv_y}{1 - \left(\dfrac{v_y}{v_t}\right)^2} = g\int_0^t dt = gt$$

The integral on the left-hand side can be found in integral tables:

$$v_t \tanh^{-1}\left(\frac{v_y}{v_t}\right) = gt$$

Solving this equation for v_y yields:

$$v_y = v_t \tanh\left(\frac{g}{v_t}\right)t$$

Because c has units of m^{-1}, and g has units of m/s^2, $(bg)^{-1/2}$ will have units of time. Let's represent this expression with the time-scale factor T:

$$T = (bg)^{-1/2}$$

The skydiver falls with her terminal speed when $a = 0$. Using this definition, relate her terminal speed to the acceleration due to gravity and the constant b in the acceleration equation:

$$0 = g - bv_t^2 \Rightarrow v_t = \sqrt{\frac{g}{b}}$$

Convince yourself that T is also equal to v_t/g and use this relationship to eliminate g and v_t in the solution to the differential equation:

$$v_y(t) = \boxed{v_t \tanh\left(\frac{t}{T}\right)}$$

(*b*) The ratio of the speed at any time to the terminal speed is given by:

$$\frac{v_y(t)}{v_t} = \frac{v_t \tanh\left(\frac{t}{T}\right)}{v_t} = \tanh\left(\frac{t}{T}\right)$$

Evaluate this ratio for $t = T$ to obtain:

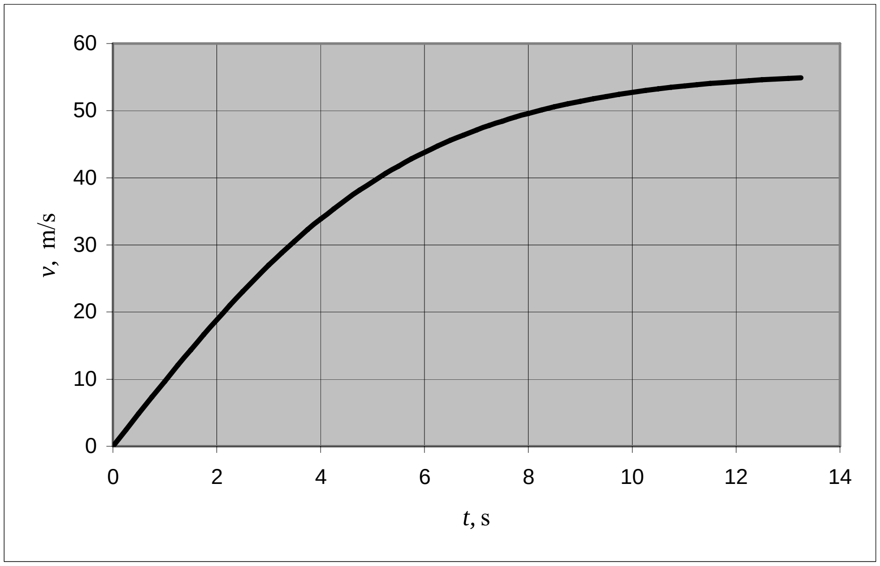
$$\frac{v_y(T)}{v_t} = \tanh\left(\frac{T}{T}\right) = \tanh(1) = \boxed{0.762}$$

(*c*) The following table was generated using a spreadsheet and the equation we derived in Part (*a*) for $v(t)$. The cell formulas and their algebraic forms are:

Cell	Content/Formula	Algebraic Form
D1	56	v_T
D2	5.71	T
B7	B6 + 0.25	$t + 0.25$
C7	B1*TANH(B7/B2)	$v_T \tanh\left(\frac{t}{T}\right)$

	A	B	C
1	v_T =	56	m/s
2	T=	5.71	s
3			
4			
5		t (s)	v (m/s)
6		0.00	0.00
7		0.25	2.45
8		0.50	4.89
9		0.75	7.32
56		12.50	54.61
57		12.75	54.73
58		13.00	54.83
59		13.25	54.93

A graph of v as a function of t follows:



Note that the speed increases linearly over time (that is, with constant acceleration) for about time T, but then it approaches the terminal speed as the acceleration decreases.

120 ••• Imagine that you are standing at a wishing well, wishing that you knew how deep the surface of the water was. Cleverly, you make your wish. Then, you take a penny from your pocket and drop it into the well. Exactly three seconds after you dropped the penny, you hear the sound it made when it struck the water. If the speed of sound is 343 m/s, how deep is the well? Neglect any effects due to air resistance.

Picture the Problem We know that the sound was heard exactly 3.00 s after the penny was dropped. This total time may be broken up into the time required for the penny to drop from your hand to the water's surface, and the time required for the sound to bounce back up to your ears. The time required for the penny to drop is related to the depth of the well. The use of a constant-acceleration equation in expressing the total fall time will lead to a quadratic equation one of whose roots will be the depth of the well

Express the time required for the sound to reach your ear as the sum of the drop time for the penny and the time for the sound to travel to your ear from the bottom of the well:

$$\Delta \boldsymbol{t}_{\text{tot}} = \Delta \boldsymbol{t}_{\text{drop}} + \Delta \boldsymbol{t}_{\text{sound}} \qquad (1)$$

Use a constant-acceleration equation to express the depth of the well:

$$\Delta \boldsymbol{y}_{\text{well}} = \tfrac{1}{2}\boldsymbol{g}\left(\Delta \boldsymbol{t}_{\text{drop}}\right)^2 \Rightarrow \Delta \boldsymbol{t}_{\text{drop}} = \sqrt{\frac{2\Delta \boldsymbol{y}_{\text{well}}}{\boldsymbol{g}}}$$

From the definition of average speed, the time Δt_{sound} required for the sound to travel from the bottom of the well to your ear is given by:

$$\Delta t_{\text{sound}} = \frac{\Delta y_{\text{well}}}{v_{\text{sound}}}$$

Substituting for Δt_{sound} and Δt_{drop} in equation (1) yields:

$$\Delta \boldsymbol{t}_{\text{tot}} = \sqrt{\frac{2\Delta \boldsymbol{y}_{\text{well}}}{\boldsymbol{g}}} + \frac{\Delta \boldsymbol{y}_{\text{well}}}{\boldsymbol{v}_{\text{sound}}}$$

Isolate the radical term and square both sides of the equation to obtain:

$$\left(\Delta \boldsymbol{t}_{\text{tot}} - \frac{\Delta \boldsymbol{y}_{\text{well}}}{\boldsymbol{v}_{\text{sound}}}\right)^2 = \frac{2\Delta \boldsymbol{y}_{\text{well}}}{\boldsymbol{g}}$$

Expanding the left side of this equation yields:

$$(\Delta t_{\text{tot}})^2 - 2\Delta t_{\text{tot}}\frac{\Delta y_{\text{well}}}{v_{\text{sound}}} + \left(\frac{\Delta y_{\text{well}}}{v_{\text{sound}}}\right)^2 = \frac{2\Delta y_{\text{well}}}{g}$$

Rewrite this equation explicitly as a quadratic equation in Δy_{well}:

$$\left(\frac{1}{\boldsymbol{v}_{\text{sound}}}\right)^2 (\Delta \boldsymbol{y}_{\text{well}})^2 - 2\left(\frac{1}{\boldsymbol{g}} + \frac{\Delta \boldsymbol{t}_{\text{tot}}}{\boldsymbol{v}_{\text{sound}}}\right)\Delta \boldsymbol{y}_{\text{well}} + (\Delta \boldsymbol{t}_{\text{tot}})^2 = 0$$

Simplify further to obtain:

$$(\Delta \boldsymbol{y}_{\text{well}})^2 - 2\left(\frac{\boldsymbol{v}_{\text{sound}}^2}{\boldsymbol{g}} + \boldsymbol{v}_{\text{sound}}\Delta \boldsymbol{t}_{\text{tot}}\right)\Delta \boldsymbol{y}_{\text{well}} + (\boldsymbol{v}_{\text{sound}}\Delta \boldsymbol{t}_{\text{tot}})^2 = 0$$

Substituting numerical values yields:

$$(\Delta \boldsymbol{y}_{\text{well}})^2 - 2\left(\frac{(343\text{ m/s})^2}{9.81\text{ m/s}^2} + (343\text{ m/s})(3.00\text{ s})\right)\Delta \boldsymbol{y}_{\text{well}} + [(343\text{ m/s})(3.00\text{ s})]^2 = 0$$

or

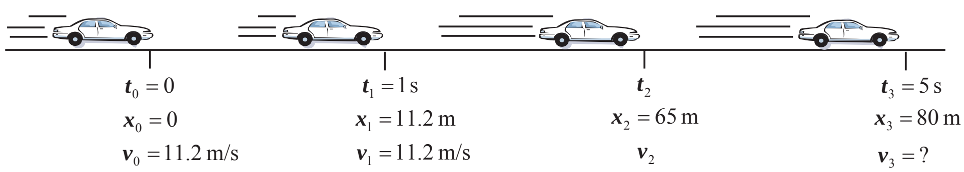
$$(\Delta \boldsymbol{y}_{\text{well}})^2 - (2.604\times10^4\text{ m})\Delta \boldsymbol{y}_{\text{well}} + 1.059\times10^6\text{ m}^2 = 0$$

Use the quadratic formula or your graphing calculator to solve for the smaller root of this equation (the larger root is approximately 26 km and is too large to be physically meaningful):

$$\Delta \boldsymbol{y}_{\text{well}} = 40.73\text{ m} = \boxed{41\text{ m}}$$

121 ••• You are driving a car at the speed limit of 25-mi/h speed limit when you observe the light at the intersection 65 m in front of you turn yellow. You know that at that particular intersection the light remains yellow for exactly 5.0 s before turning red. After you think for 1.0 s, you then accelerate the car at a constant rate. You somehow manage to pass your 4.5-m-long car completely through the 15.0-m-wide intersection just as the light turns red, thus narrowly avoiding a ticket for being in an intersection when the light is red. Immediately after passing through the intersection, you take your foot off the accelerator, relieved. However, down the road, you are pulled over for speeding. You assume that you were ticketed for the speed of your car as it exited the intersection. Determine this speed and decide whether you should fight this ticket in court. Explain.

Picture the Problem First, we should find the total distance covered by the car between the time that the light turned yellow (and you began your acceleration) and the time that the back end of the car left the intersection. This distance is Δx_{tot} = 65.0 m + 15.0 m + 4.5 m = 84.5 m. Part of this distance, Δx_{c} is covered at constant speed, as you think about trying to make it through the intersection in time – this can be subtracted from the full distance Δx_{tot} above to yield Δx_{acc}, the displacement during the constant acceleration. With this information, we then can utilize a constant-acceleration equation to obtain an expression for your velocity as you exited the intersection. The following pictorial representation will help organize the information given in this problem.



Using a constant-acceleration equation relate the distance over which you accelerated to your initial velocity v_1, your acceleration a, and the time during which you accelerated a_{acc}:

$$\Delta x_{\text{acc}} = \Delta x_{\text{tot}} - \Delta x_{\text{c}} = v_1 \Delta t_{\text{acc}} + \tfrac{1}{2} a (\Delta t_{\text{acc}})^2$$

Relate the final velocity v_3 of the car to its velocity v_1 when it begins to accelerate, its acceleration, and the time during which it accelerates:

$$\boldsymbol{v}_3 = \boldsymbol{v}_1 + \boldsymbol{a}\Delta \boldsymbol{t}_{\text{acc}} \Rightarrow \boldsymbol{a} = \frac{\boldsymbol{v}_3 - \boldsymbol{v}_1}{\Delta \boldsymbol{t}_{\text{acc}}}$$

Substitute for a in the expression for Δx_{acc} to obtain:

$$\Delta \boldsymbol{x}_{\text{tot}} - \Delta \boldsymbol{x}_{\text{c}} = \boldsymbol{v}_1 \Delta \boldsymbol{t}_{\text{acc}} + \tfrac{1}{2}\left(\frac{\boldsymbol{v}_3 - \boldsymbol{v}_1}{\Delta \boldsymbol{t}_{\text{acc}}}\right)(\Delta \boldsymbol{t}_{\text{acc}})^2 = \boldsymbol{v}_1 \Delta \boldsymbol{t}_{\text{acc}} + \tfrac{1}{2}(\boldsymbol{v}_3 - \boldsymbol{v}_1)(\Delta \boldsymbol{t}_{\text{acc}})$$

Solving for v_3 yields:

$$\boldsymbol{v}_3 = \frac{2(\Delta \boldsymbol{x}_{\text{tot}} - \Delta \boldsymbol{x}_{\text{c}})}{\Delta \boldsymbol{t}_{\text{acc}}} - \boldsymbol{v}_1$$

Substitute numerical values and evaluate v_3:

$$\boldsymbol{v}_3 = \frac{2(84.5\text{ m} - 11.2\text{ m})}{4.0\text{ s}} - 11.2\text{ m/s} = \boxed{25\text{ m/s}}$$

Because 25 m/s is approximately 57 mi/h, you were approximately 32 mi/h over the speed limit! You would be foolish to contest your ticket.

122 ••• For a spherical celestial object of radius R, the acceleration due to gravity g at a distance x from the *center* of the object is $\boldsymbol{g} = \boldsymbol{g}_0 \boldsymbol{R}^2 / \boldsymbol{x}^2$, where g_0 is the acceleration due to gravity at the object's surface and $x > R$. For the moon, $g_0 = 1.63\text{ m/s}^2$. If a rock is released from rest at a height of $4R$ above the lunar surface, with what speed does the rock impact the moon? *Hint: Its acceleration is a function of position and increases as the object falls. So do not use constant acceleration free-fall equations, but go back to basics.*

Picture the Problem Let the origin be at the center of the moon and the $+x$ direction be radially outward. Because the acceleration of the rock is a function of its distance from the center of the moon, we'll need to change the variables of integration in the definition of acceleration to v and x in order to relate the rock's acceleration to its speed. Separating variables and integrating will yield an expression for the speed of the rock as a function of its distance from the center of the moon.

The acceleration of the object is given by:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Separating variables yields:

$$vdv = adx$$

Because $a = -g$:

$$vdv = -\frac{g_0 R^2}{x^2}dx$$

Expressing the integral of v from 0 to v and x from $4R$ to R yields:

$$\int_0^v vdv = \int_{4R}^{R} -\frac{g_0 R^2}{x^2}dx = -g_0 R^2 \int_{4R}^{R} \frac{dx}{x^2}$$

Carry out the integration to obtain:

$$\frac{v^2}{2} = g_0 R^2 \left[\frac{1}{x}\bigg|_{4R}^{R} \right] = \frac{3}{4} g_0 R$$

Solve for v to obtain an expression for the speed of the object upon impact:

$$v = \sqrt{\frac{3}{2} g_0 R}$$

Substitute numerical values and evaluate v:

$$\boldsymbol{v} = \sqrt{\frac{3}{2}\left(1.63\ \text{m/s}^2\right)\left(1.738\times10^6\ \text{m}\right)} = \boxed{2.06\ \text{km/s}}$$