

Units, Trigonometry, and Vectors

TOPIC

1

THE GOAL OF PHYSICS IS TO PROVIDE an understanding of the physical world by developing theories based on experiments. A physical theory, usually expressed mathematically, describes how a given physical system works. The theory makes certain predictions about the physical system which can then be checked by observations and experiments. If the predictions turn out to correspond closely to what is actually observed, then the theory stands, although it remains provisional. No theory to date has given a complete description of all physical phenomena, even within a given subdiscipline of physics. Every theory is a work in progress.

The basic laws of physics involve such physical quantities as force, velocity, volume, and acceleration, all of which can be described in terms of more fundamental quantities. In mechanics, it is conventional to use the quantities of **length** (L), **mass** (M), and **time** (T); all other physical quantities can be constructed from these three.

1.1 Standards of Length, Mass, and Time

To communicate the result of a measurement of a certain physical quantity, a *unit* for the quantity must be defined. If our fundamental unit of length is defined to be 1.0 meter, for example, and someone familiar with our system of measurement reports that a wall is 2.0 meters high, we know that the height of the wall is twice the fundamental unit of length. Likewise, if our fundamental unit of mass is defined as 1.0 kilogram and we are told that a person has a mass of 75 kilograms, then that person has a mass 75 times as great as the fundamental unit of mass.

In 1960 an international committee agreed on a standard system of units for the fundamental quantities of science, called **SI** (Système International). Its units of length, mass, and time are the meter, kilogram, and second, respectively.

1.1.1 Length

In 1799 the legal standard of length in France became the meter, defined as one ten-millionth of the distance from the equator to the North Pole. Until 1960, the official length of the meter was the distance between two lines on a specific bar of platinum-iridium alloy stored under controlled conditions. This standard was abandoned for several reasons, the principal one being that measurements of the separation between the lines were not precise enough. In 1960 the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. In October 1983 this definition was abandoned also, and the meter was redefined as the distance traveled by light in vacuum during a time interval of $1/299\,792\,458$ second. This latest definition establishes the speed of light at 299 792 458 meters per second.

1.1.2 Mass

The SI unit of mass, the kilogram, is defined as the mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France (similar to that shown in Fig. 1.1a). As we'll see in Topic 4, mass is a

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Tip 1.1 No Commas in Numbers with Many Digits

In science, numbers with more than three digits are written in groups of three digits separated by spaces rather than commas, so that 10 000 is the same as the common American notation 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

◀ Definition of the meter

◀ Definition of the kilogram



AP Images/Jacques Birnon



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Figure 1.1 (a) International Prototype of the Kilogram, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) A cesium fountain atomic clock. The clock will neither gain nor lose a second in 20 million years.

quantity used to measure the resistance to a change in the motion of an object. It's more difficult to cause a change in the motion of an object with a large mass than an object with a small mass.

1.1.3 Time

Before 1960, the time standard was defined in terms of the average length of a solar day in the year 1900. (A solar day is the time between successive appearances of the Sun at the highest point it reaches in the sky each day.) The basic unit of time, the second, was defined to be $(1/60)(1/60)(1/24) = 1/86\,400$ of the average solar day. In 1967 the second was redefined to take advantage of the high precision attainable with an atomic clock, which uses the characteristic frequency of the light emitted from the cesium-133 atom as its "reference clock." **The second is now defined as 9 192 631 700 times the period of oscillation of radiation from the cesium atom.** The newest type of cesium atomic clock is shown in Figure 1.1b.

1.1.4 Approximate Values for Length, Mass, and Time Intervals

Approximate values of some lengths, masses, and time intervals are presented in Tables 1.1, 1.2, and 1.3, respectively. Note the wide ranges of values. Study these tables to get a feel for a kilogram of mass (this book has a mass of about 2 kilograms), a time interval of 10^{10} seconds (one century is about 3×10^9 seconds), or 2 meters of length (the approximate height of a forward on a basketball team). Appendix A reviews the notation for powers of 10, such as the expression of the number 50 000 in the form 5×10^4 .

Systems of units commonly used in physics are the *Système International*, in which the units of length, mass, and time are the meter (m), kilogram (kg), and second (s); the *cgs*, or *Gaussian*, system, in which the units of length, mass, and time are the centimeter (cm), gram (g), and second; and the U.S. customary system, in which the units of length, mass, and time are the foot (ft), slug, and second. SI units are almost universally accepted in science and industry and will be used throughout the book. Limited use will be made of Gaussian and U.S. customary units.

Table 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Observable Universe	1×10^{26}
Earth to Andromeda	2×10^{22}
Earth to Proxima Centauri	4×10^{16}
One light-year	9×10^{15}
Earth to Sun	2×10^{11}
Earth to Moon	4×10^8
Radius of Earth	6×10^6
World's tallest building	8×10^2
Football field	9×10^1
Housefly	5×10^{-3}
Typical organism cell	1×10^{-5}
Hydrogen atom	1×10^{-10}
Atomic nucleus	1×10^{-14}
Proton diameter	1×10^{-15}

Table 1.2 Approximate Values of Some Masses

	Mass (kg)
Observable Universe	1×10^{52}
Milky Way galaxy	7×10^{41}
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
Shark	1×10^2
Human	7×10^1
Frog	1×10^{-1}
Mosquito	1×10^{-5}
Bacterium	1×10^{-15}
Hydrogen atom	2×10^{-27}
Electron	9×10^{-31}

Table 1.3 Approximate Values of Some Time Intervals

	Time Interval (s)
Age of Universe	5×10^{17}
Age of Earth	1×10^{17}
Age of college student	6×10^8
One year	3×10^7
One day	9×10^4
Heartbeat	8×10^{-1}
Audible sound wave period ^a	1×10^{-3}
Typical radio wave period ^a	1×10^{-6}
Visible light wave period ^a	2×10^{-15}
Nuclear collision	1×10^{-22}

^aA *period* is defined as the time required for one complete vibration.

Some of the most frequently used “metric” (SI and cgs) prefixes representing powers of 10 and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m is 1 kilometer (km). Likewise, 1 kg is equal to 10^3 g, and 1 megavolt (MV) is 10^6 volts (V). It’s a good idea to memorize the more common prefixes early on: femto- to centi-, and kilo- to giga- are used routinely by most physicists.

1.2 The Building Blocks of Matter

A 1-kg (\approx 2-lb) cube of solid gold has a length of about 3.73 cm (\approx 1.5 in.) on a side. If the cube is cut in half, the two resulting pieces retain their chemical identity. But what happens if the pieces of the cube are cut again and again, indefinitely? The Greek philosophers Leucippus and Democritus couldn’t accept the idea that such cutting could go on forever. They speculated that the process ultimately would end when it produced a particle that could no longer be cut. In Greek, *atomos* means “not sliceable.” From this term comes our English word *atom*, once believed to be the smallest particle of matter but since found to be a composite of more elementary particles.

The atom can be naively visualized as a miniature solar system, with a dense, positively charged nucleus occupying the position of the Sun and negatively charged electrons orbiting like planets. This model of the atom, first developed by the great Danish physicist Niels Bohr nearly a century ago, led to the understanding of certain properties of the simpler atoms such as hydrogen but failed to explain many fine details of atomic structure.

Notice the size of a hydrogen atom, listed in Table 1.1, and the size of a proton—the nucleus of a hydrogen atom—one hundred thousand times smaller. If the proton were the size of a ping-pong ball, the electron would be a tiny speck about the size of a bacterium, orbiting the proton a kilometer away! Other atoms are similarly constructed. So there is a surprising amount of empty space in ordinary matter.

After the discovery of the nucleus in the early 1900s, questions arose concerning its structure. Although the structure of the nucleus remains an area of active research even today, by the early 1930s scientists determined that two basic entities—protons and neutrons—occupy the nucleus. The *proton* is nature’s most common carrier of positive charge, equal in magnitude but opposite in sign to the charge on the electron. The number of protons in a nucleus determines what the element is. For instance, a nucleus containing only one proton is the nucleus of an atom of hydrogen, regardless of how many neutrons may be present. Extra neutrons correspond to different isotopes of hydrogen—deuterium and tritium—which react chemically in exactly the same way as hydrogen, but are more massive. An atom having two protons in its nucleus, similarly, is always helium, although again, differing numbers of neutrons are possible.

The existence of *neutrons* was verified conclusively in 1932. A neutron has no charge and has a mass about equal to that of a proton. Except for hydrogen, all atomic nuclei contain neutrons, which, together with the protons, interact through the strong nuclear force. That force opposes the strongly repulsive electrical force of the protons, which otherwise would cause the nucleus to disintegrate.

The division doesn’t stop here; strong evidence collected over many years indicates that protons, neutrons, and a zoo of other exotic particles are composed of six particles called **quarks** (rhymes with “sharks” though some rhyme it with “forks”). These particles have been given the names *up*, *down*, *strange*, *charm*, *bottom*, and *top*. The up, charm, and top quarks each carry a charge equal to $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks each carry a charge equal to $-\frac{1}{3}$ the proton charge. The proton consists of two up quarks and one down quark (see Fig. 1.2), giving the correct charge for the proton, +1. The neutron is composed of two down quarks and one up quark and has a net charge of zero.

Table 1.4 Some Prefixes for Powers of Ten Used with “Metric” (SI and cgs) Units

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{-1}	deci-	d
10^1	deka-	da
10^3	kilo-	k
10^6	mega-	M
10^9	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

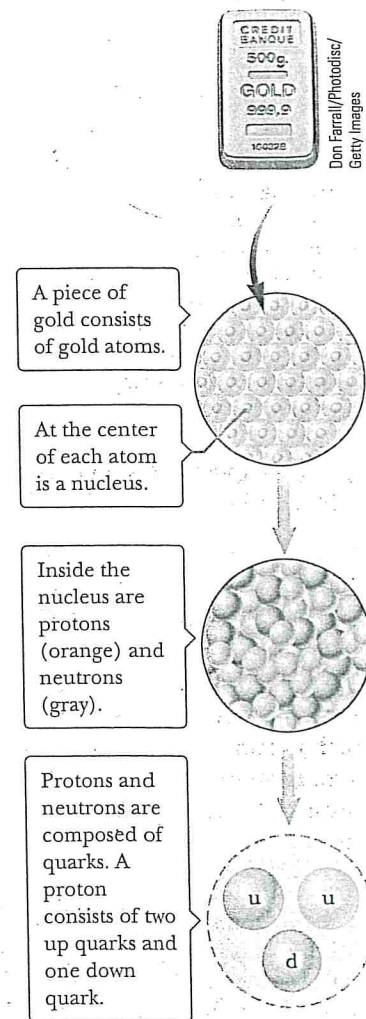


Figure 1.2 Levels of organization in matter.

The up and down quarks are sufficient to describe all normal matter, so the existence of the other four quarks, indirectly observed in high-energy experiments, is something of a mystery. Despite strong indirect evidence, no isolated quark has ever been observed. Consequently, the possible existence of yet more fundamental particles remains purely speculative.

1.3 Dimensional Analysis

In physics the word *dimension* denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are different ways of expressing the dimension of *length*.

The symbols used in this section to specify the dimensions of length, mass, and time are L, M, and T, respectively. Brackets [] will often be used to denote the dimensions of a physical quantity. In this notation, for example, the dimensions of velocity v are written $[v] = L/T$, and the dimensions of area A are $[A] = L^2$. The dimensions of area, volume, velocity, and acceleration are listed in Table 1.5, along with their units in the three common systems. The dimensions of other quantities, such as force and energy, will be described later as they are introduced.

In physics it's often necessary to deal with mathematical expressions that relate different physical quantities. One way to analyze such expressions, called **dimensional analysis**, makes use of the fact that **dimensions can be treated as algebraic quantities**. Adding masses to lengths, for example, makes no sense, so it follows that quantities can be added or subtracted only if they have the same dimensions. If the terms on the opposite sides of an equation have the same dimensions, then that equation may be correct, although correctness can't be guaranteed on the basis of dimensions alone. Nonetheless, dimensional analysis has value as a partial check of an equation and can also be used to develop insight into the relationships between physical quantities.

The procedure can be illustrated by developing some relationships between acceleration, velocity, time, and distance. Distance x has the dimension of length: $[x] = L$. Time t has dimension $[t] = T$. Velocity v has the dimensions length over time: $[v] = L/T$, and acceleration the dimensions length divided by time squared: $[a] = L/T^2$. Notice that velocity and acceleration have similar dimensions, except for an extra dimension of time in the denominator of acceleration. It follows that

$$[v] = \frac{L}{T} = \frac{L}{T^2} T = [a][t]$$

From this it might be guessed that velocity equals acceleration multiplied by time, $v = at$, and that is true for the special case of motion with constant acceleration starting at rest. Noticing that velocity has dimensions of length divided by time and distance has dimensions of length, it's reasonable to guess that

$$[x] = L = L \frac{T}{T} = \frac{L}{T} T = [v][t] = [a][t]^2$$

Here it appears that $x = at^2$ might correctly relate the distance traveled to acceleration and time; however, that equation is not even correct in the case of constant acceleration starting from rest. The correct expression in that case is $x = \frac{1}{2}at^2$.

Table 1.5 Dimensions and Some Units of Area, Volume, Velocity, and Acceleration

System	Area (L^2)	Volume (L^3)	Velocity (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
cgs	cm^2	cm^3	cm/s	cm/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

These examples serve to show the inherent limitations in using dimensional analysis to discover relationships between physical quantities. Nonetheless, such simple procedures can still be of value in developing a preliminary mathematical model for a given physical system. Further, because it's easy to make errors when solving problems, dimensional analysis can be used to check the consistency of the results. When the dimensions in an equation are not consistent, it indicates an error has been made in a prior step.

EXAMPLE 1.1 ANALYSIS OF AN EQUATION

GOAL Check an equation using dimensional analysis.

PROBLEM Show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent velocities, a is acceleration, and t is a time interval.

STRATEGY Analyze each term, finding its dimensions, and then check to see if all the terms agree with each other.

SOLUTION

Find dimensions for v and v_0 .

$$[v] = [v_0] = \frac{\text{L}}{\text{T}}$$

Find the dimensions of at .

$$[at] = [a][t] = \frac{\text{L}}{\text{T}^2} (\text{T}) = \frac{\text{L}}{\text{T}}$$

REMARKS All the terms agree, so the equation is dimensionally correct.

QUESTION 1.1 True or False: An equation that is dimensionally correct is always physically correct, up to a constant of proportionality.

EXERCISE 1.1 Determine whether the equation $x = vt^2$ is dimensionally correct. If not, provide a correct expression, up to an overall constant of proportionality.

ANSWER Incorrect. The expression $x = vt$ is dimensionally correct.

EXAMPLE 1.2 FIND AN EQUATION

GOAL Derive an equation by using dimensional analysis.

PROBLEM Find a relationship between an acceleration of constant magnitude a , speed v , and distance r from the origin for a particle traveling in a circle.

STRATEGY Start with the term having the most dimensionality, a . Find its dimensions, and then rewrite those dimensions in terms of the dimensions of v and r . The dimensions of time will have to be eliminated with v , because that's the only quantity (other than a , itself) in which the dimension of time appears.

SOLUTION

Write down the dimensions of a :

$$[a] = \frac{\text{L}}{\text{T}^2}$$

Solve the dimensions of speed for T:

$$[v] = \frac{\text{L}}{\text{T}} \rightarrow \text{T} = \frac{\text{L}}{[v]}$$

Substitute the expression for T into the equation for $[a]$:

$$[a] = \frac{\text{L}}{\text{T}^2} = \frac{\text{L}}{(\text{L}/[v])^2} = \frac{[v]^2}{\text{L}}$$

Substitute $\text{L} = [r]$, and guess at the equation:

$$[a] = \frac{[v]^2}{[r]} \rightarrow a = \frac{v^2}{r}$$

REMARKS This is the correct equation for the magnitude of the centripetal acceleration—acceleration towards the center of motion—to be discussed in Topic 7. In this case it isn't necessary to introduce a numerical factor. Such a factor is often displayed explicitly as a constant k in front of the right-hand side; for example, $a = kv^2/r$. As it turns out, $k = 1$ gives the correct expression. A good technique sometimes introduced in calculus-based textbooks involves using unknown powers of the dimensions. This problem would then be set up as $[a] = [v]^b[r]^c$. Writing out the dimensions and equating powers of each dimension on both sides of the equation would result in $b = 2$ and $c = -1$.

(Continued)

QUESTION 1.2 True or False: Replacing v by r/t in the final answer also gives a dimensionally correct equation.

EXERCISE 1.2 In physics, energy E carries dimensions of mass times length squared divided by time squared. Use dimensional analysis to derive a relationship for energy in terms of mass m and speed v , up to a constant of proportionality. Set the speed equal to c , the speed of light, and the constant of proportionality equal to 1 to get the most famous equation in physics. (Note, however, that the first relationship is associated with energy of motion and the second with energy of mass. See Topic 26.)

ANSWER $E = kmv^2 \rightarrow E = mc^2$ when $k = 1$ and $v = c$.

1.4 Uncertainty in Measurement and Significant Figures

Physics is a science in which mathematical laws are tested by experiment. No physical quantity can be determined with complete accuracy because our senses are physically limited, even when extended with microscopes, cyclotrons, and other instruments. Consequently, it's important to develop methods of determining the accuracy of measurements.

All measurements have uncertainties associated with them, whether or not they are explicitly stated. The accuracy of a measurement depends on the sensitivity of the apparatus, the skill of the person carrying out the measurement, and the number of times the measurement is repeated. Once the measurements, along with their uncertainties, are known, it's often the case that calculations must be carried out using those measurements. Suppose two such measurements are multiplied. When a calculator is used to obtain this product, there may be eight digits in the calculator window, but often only two or three of those numbers have any significance. The rest have no value because they imply greater accuracy than was actually achieved in the original measurements. In experimental work, determining how many numbers to retain requires the application of statistics and the mathematical propagation of uncertainties. In a textbook it isn't practical to apply those sophisticated tools in the numerous calculations, so instead a simple method, called *significant figures*, is used to indicate the approximate number of digits that should be retained at the end of a calculation. Although that method is not mathematically rigorous, it's easy to apply and works fairly well.

Suppose in a laboratory experiment we measure the area of a rectangular plate with a meter stick. Let's assume the accuracy to which we can measure a particular dimension of the plate is ± 0.1 cm. If the length of the plate is measured to be 16.3 cm, we can only claim it lies somewhere between 16.2 cm and 16.4 cm. In this case, we say the measured value has three significant figures. Likewise, if the plate's width is measured to be 4.5 cm, the actual value lies between 4.4 cm and 4.6 cm. This measured value has only two significant figures. We could write the measured values as 16.3 ± 0.1 cm and 4.5 ± 0.1 cm. In general, a **significant figure is a reliably known digit** (other than a zero used to locate a decimal point). Note that in each case, the final number has some uncertainty associated with it and is therefore not 100% reliable. Despite the uncertainty, that number is retained and considered significant because it does convey some information.

Suppose we would like to find the area of the plate by multiplying the two measured values together. The final value can range between $(16.3 - 0.1 \text{ cm})(4.5 - 0.1 \text{ cm}) = (16.2 \text{ cm})(4.4 \text{ cm}) = 71.28 \text{ cm}^2$ and $(16.3 + 0.1 \text{ cm})(4.5 + 0.1 \text{ cm}) = (16.4 \text{ cm})(4.6 \text{ cm}) = 75.44 \text{ cm}^2$. Claiming to know anything about the hundredths place, or even the tenths place, doesn't make any sense, because it's clear we can't even be certain of the units place, whether it's the 1 in 71, the 5 in 75, or somewhere in between. The tenths and the hundredths places are clearly not significant. We have *some* information about the units place, so that number is significant. Multiplying the numbers at the middle of the uncertainty ranges gives (16.3 cm)

$(4.5 \text{ cm}) = 73.35 \text{ cm}^2$, which is also in the middle of the area's uncertainty range. Because the hundredths and tenths are not significant, we drop them and take the answer to be 73 cm^2 , with an uncertainty of $\pm 2 \text{ cm}^2$. Note that the answer has two significant figures, the same number of figures as the least accurately known quantity being multiplied, the 4.5-cm width.

Calculations as carried out in the preceding paragraph can indicate the proper number of significant figures, but those calculations are time-consuming. Instead, two rules of thumb can be applied. The first, concerning multiplication and division, is as follows: **In multiplying (dividing) two or more quantities, the number of significant figures in the final product (quotient) is the same as the number of significant figures in the least accurate of the factors being combined, where least accurate means having the lowest number of significant figures.**

To get the final number of significant figures, it's usually necessary to do some rounding. If the last digit dropped is less than 5, simply drop the digit. If the last digit dropped is greater than or equal to 5, raise the last retained digit by one.¹

Zeros may or may not be significant figures. Zeros used to position the decimal point in such numbers as 0.03 and 0.007 5 are not considered significant figures. Hence, 0.03 has one significant figure, and 0.007 5 has two.

When zeros are placed after other digits in a whole number, there is a possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous, because we don't know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement.

Using scientific notation to indicate the number of significant figures removes this ambiguity. In this case, we express the mass as $1.5 \times 10^3 \text{ g}$ if there are two significant figures in the measured value, $1.50 \times 10^3 \text{ g}$ if there are three significant figures, and $1.500 \times 10^3 \text{ g}$ if there are four. Likewise, 0.000 15 is expressed in scientific notation as 1.5×10^{-4} if it has two significant figures or as 1.50×10^{-4} if it has three significant figures. The three zeros between the decimal point and the digit 1 in the number 0.000 15 are not counted as significant figures because they only locate the decimal point. Similarly, trailing zeros are not considered significant. However, any zeros written after a decimal point, or between a nonzero number and before a decimal point, are considered significant. For example, 3.00, 30.0, and 300. have three significant figures, whereas 300 has only one. In this book, **most of the numerical examples and end-of-topic problems will yield answers having two or three significant figures.**

For addition and subtraction, it's best to focus on the number of decimal places in the quantities involved rather than on the number of significant figures. **When numbers are added (subtracted), the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum (difference).** For example, if we wish to compute 123 (zero decimal places) $+ 5.35$ (two decimal places), the answer is 128 (zero decimal places) and not 128.35 . If we compute the sum $1.000 1$ (four decimal places) $+ 0.000 3$ (four decimal places) $= 1.000 4$, the result has the correct number of decimal places, namely four. Observe that the rules for multiplying significant figures don't work here because the answer has five significant figures even though one of the terms in the sum, $0.000 3$, has only one significant figure. Likewise, if we perform the subtraction $1.002 - 0.998 = 0.004$, the result has three decimal places because each term in the subtraction has three decimal places.

To show why this rule should hold, we return to the first example in which we added 123 and 5.35, and rewrite these numbers as $123.xxx$ and $5.35x$. Digits written with an x are completely unknown and can be any digit from 0 to 9. Now we

Tip 1.2 Using Calculators

Calculators are designed by engineers to yield as many digits as the memory of the calculator chip permits, so be sure to round the final answer to the correct number of significant figures.

¹Some prefer to round to the nearest even digit when the last dropped digit is 5, which has the advantage of rounding 5 up half the time and down half the time. For example, 1.55 would round to 1.6, but 1.45 would round to 1.4. Because the final significant figure is only one representative of a range of values given by the uncertainty, this very slight refinement will not be used in this text.

line-up $123.xxx$ and $5.35x$ relative to the decimal point and perform the addition, using the rule that an unknown digit added to a known or unknown digit yields an unknown:

$$\begin{array}{r} 123.xxx \\ + 5.35x \\ \hline 128.xxx \end{array}$$

The answer of $128.xxx$ means that we are justified only in keeping the number 128 because everything after the decimal point in the sum is actually unknown. The example shows that the controlling uncertainty is introduced into an addition or subtraction by the term with the smallest number of decimal places.

EXAMPLE 1.3 CARPET CALCULATIONS

GOAL Apply the rules for significant figures.

PROBLEM Several carpet installers make measurements for carpet installation in the different rooms of a restaurant, reporting their measurements with inconsistent accuracy, as compiled in Table 1.6. Compute the areas for (a) the banquet hall, (b) the meeting room, and (c) the dining room, taking into account significant figures. (d) What total area of carpet is required for these rooms?

Table 1.6 Dimensions of Rooms in Example 1.3

	Length (m)	Width (m)
Banquet hall	14.71	7.46
Meeting room	4.822	5.1
Dining room	13.8	9

STRATEGY For the multiplication problems in parts (a)–(c), count the significant figures in each number. The smaller result is the number of significant figures in the answer. Part (d) requires a sum, where the area with the least accurately known decimal place determines the overall number of significant figures in the answer.

SOLUTION

(a) Compute the area of the banquet hall.

Count significant figures:

$$14.71 \text{ m} \rightarrow 4 \text{ significant figures}$$

$$7.46 \text{ m} \rightarrow 3 \text{ significant figures}$$

$$14.71 \text{ m} \times 7.46 \text{ m} = 109.74 \text{ m}^2 \rightarrow 1.10 \times 10^2 \text{ m}^2$$

To find the area, multiply the numbers keeping only three digits:

(b) Compute the area of the meeting room.

Count significant figures:

$$4.822 \text{ m} \rightarrow 4 \text{ significant figures}$$

$$5.1 \text{ m} \rightarrow 2 \text{ significant figures}$$

$$4.822 \text{ m} \times 5.1 \text{ m} = 24.59 \text{ m}^2 \rightarrow 25 \text{ m}^2$$

To find the area, multiply the numbers keeping only two digits:

(c) Compute the area of the dining room.

Count significant figures:

$$13.8 \text{ m} \rightarrow 3 \text{ significant figures}$$

$$9 \text{ m} \rightarrow 1 \text{ significant figure}$$

$$13.8 \text{ m} \times 9 \text{ m} = 124.2 \text{ m}^2 \rightarrow 100 \text{ m}^2$$

To find the area, multiply the numbers keeping only one digit:

(d) Calculate the total area of carpet required, with the proper number of significant figures.

Sum all three answers without regard to significant figures:

$$1.10 \times 10^2 \text{ m}^2 + 25 \text{ m}^2 + 100 \text{ m}^2 = 235 \text{ m}^2$$

The least accurate number is 100 m^2 , with one significant figure in the hundred's decimal place:

$$235 \text{ m}^2 \rightarrow 2 \times 10^2 \text{ m}^2$$

REMARKS Notice that the final answer in part (d) has only one significant figure, in the hundred's place, resulting in an answer that had to be rounded down by a sizable fraction of its total value. That's the consequence of having insufficient information. The value of 9 m, without any further information, represents a true value that could be anywhere in the interval [8.5 m, 9.5 m), all of which round to 9 when only one digit is retained.

QUESTION 1.3 How would the final answer change if the width of the dining room were given as 9.0 m?

EXERCISE 1.3 A ranch has two fenced rectangular areas. Area A has a length of 750 m and width 125 m, and area B has length 400 m and width 150 m. Find (a) area A, (b) area B, and (c) the total area, with attention to the rules of significant figures. Assume trailing zeros are not significant.

ANSWERS (a) $9.4 \times 10^4 \text{ m}^2$ (b) $6 \times 10^4 \text{ m}^2$ (c) $1.5 \times 10^5 \text{ m}^2$

In performing any calculation, especially one involving a number of steps, there will always be slight discrepancies introduced by both the rounding process and the algebraic order in which steps are carried out. For example, consider $2.35 \times 5.89 / 1.57$. This computation can be performed in three different orders. First, we have $2.35 \times 5.89 = 13.842$, which rounds to 13.8, followed by $13.8 / 1.57 = 8.7898$, rounding to 8.79. Second, $5.89 / 1.57 = 3.7516$, which rounds to 3.75, resulting in $2.35 \times 3.75 = 8.8125$, rounding to 8.81. Finally, $2.35 / 1.57 = 1.4968$ rounds to 1.50, and $1.50 \times 5.89 = 8.835$ rounds to 8.84. So three different algebraic orders, following the rules of rounding, lead to answers of 8.79, 8.81, and 8.84, respectively. Such minor discrepancies are to be expected, because the last significant digit is only one representative from a range of possible values, depending on experimental uncertainty. To avoid such discrepancies, some carry one or more extra digits during the calculation, although it isn't conceptually consistent to do so because those extra digits are not significant. As a practical matter, in the worked examples in this text, intermediate reported results will be rounded to the proper number of significant figures, and only those digits will be carried forward. In the problem sets, however, given data will usually be assumed accurate to two or three digits, even when there are trailing zeros. **In solving the problems, the student should be aware that slight differences in rounding practices can result in answers varying from the text in the last significant digit, which is normal and not cause for concern.** The method of significant figures has its limitations in determining accuracy, but it's easy to apply. In experimental work, however, statistics and the mathematical propagation of uncertainty must be used to determine the accuracy of an experimental result.

1.5 Unit Conversions for Physical Quantities

Sometimes it's necessary to convert units from one system to another (see Fig. 1.3). Conversion factors between the SI and U.S. customary systems for units of length are as follows:

$$\begin{aligned} 1 \text{ mi} &= 1609 \text{ m} = 1.609 \text{ km} & 1 \text{ ft} &= 0.3048 \text{ m} = 30.48 \text{ cm} \\ 1 \text{ m} &= 39.37 \text{ in.} = 3.281 \text{ ft} & 1 \text{ in.} &= 0.0254 \text{ m} = 2.54 \text{ cm} \end{aligned}$$

A more extensive list of conversion factors can be found on the front endsheets of this book. In all the given conversion equations, the "1" on the left is assumed to have the same number of significant figures as the quantity given on the right of the equation.

Units can be treated as algebraic quantities that can "cancel" each other. We can make a fraction with the conversion that will cancel the units we don't want, and



Figure 1.3 The speed limit is given in both kilometers per hour and miles per hour on this road sign. How accurate is the conversion?

multiply that fraction by the quantity in question. For example, suppose we want to convert 15.0 in. to centimeters. Because 1 in. = 2.54 cm, we find that

$$15.0 \text{ in.} = 15.0 \text{ in.} \times \left(\frac{2.54 \text{ cm}}{1.00 \text{ in.}} \right) = 38.1 \text{ cm}$$

The next two examples show how to deal with problems involving more than one conversion and with powers.

EXAMPLE 1.4 PULL OVER, BUDDY!

GOAL Convert units using several conversion factors.

PROBLEM If a car is traveling at a speed of 28.0 m/s, is the driver exceeding the speed limit of 55.0 mi/h?

STRATEGY Meters must be converted to miles and seconds to hours, using the conversion factors listed on the front endsheets of the book. Here, three factors will be used.

SOLUTION

Convert meters to miles:

$$28.0 \text{ m/s} = \left(28.0 \frac{\text{m}}{\text{s}} \right) \left(\frac{1.00 \text{ mi}}{1609 \text{ m}} \right) = 1.74 \times 10^{-2} \text{ mi/s}$$

Convert seconds to hours:

$$\begin{aligned} 1.74 \times 10^{-2} \text{ mi/s} &= \left(1.74 \times 10^{-2} \frac{\text{mi}}{\text{s}} \right) \left(60.0 \frac{\text{s}}{\text{min}} \right) \left(60.0 \frac{\text{min}}{\text{h}} \right) \\ &= 62.6 \text{ mi/h} \end{aligned}$$

REMARKS The driver should slow down because he's exceeding the speed limit.

QUESTION 1.4 Repeat the conversion, using the relationship 1.00 m/s = 2.24 mi/h. Why is the answer slightly different?

EXERCISE 1.4 Convert 152 mi/h to m/s.

ANSWER 67.9 m/s

EXAMPLE 1.5 PRESS THE PEDAL TO THE METAL

GOAL Convert a quantity featuring powers of a unit.

PROBLEM The traffic light turns green, and the driver of a high-performance car slams the accelerator to the floor. The accelerometer registers 22.0 m/s². Convert this reading to km/min².

STRATEGY Here we need one factor to convert meters to kilometers and another two factors to convert seconds squared to minutes squared.

SOLUTION

Multiply by the three factors:

$$\frac{22.0 \text{ m}}{1.00 \text{ s}^2} \left(\frac{1.00 \text{ km}}{1.00 \times 10^3 \text{ m}} \right) \left(\frac{60.0 \text{ s}}{1.00 \text{ min}} \right)^2 = 79.2 \frac{\text{km}}{\text{min}^2}$$

REMARKS Notice that in each conversion factor the numerator equals the denominator when units are taken into account. A common error in dealing with squares is to square the units inside the parentheses while forgetting to square the numbers!

QUESTION 1.5 What time conversion factor or factors would be used to further convert the answer to km/h²?

EXERCISE 1.5 Convert 4.50 × 10³ kg/m³ to g/cm³.

ANSWER 4.50 g/cm³

1.6 Estimates and Order-of-Magnitude Calculations

Getting an exact answer to a calculation may often be difficult or impossible, either for mathematical reasons or because limited information is available. In these cases, estimates can yield useful approximate answers that can determine whether a more precise calculation is necessary. Estimates also serve as a partial check if the exact calculations are actually carried out. If a large answer is expected but a small exact answer is obtained, there's an error somewhere.

For many problems, knowing the approximate value of a quantity—within a factor of 10 or so—is sufficient. This approximate value is called an **order-of-magnitude** estimate and requires finding the power of 10 that is closest to the actual value of the quantity. For example, $75 \text{ kg} \sim 10^2 \text{ kg}$, where the symbol \sim means “is on the order of” or “is approximately.” Increasing a quantity by three orders of magnitude means that its value increases by a factor of $10^3 = 1\,000$.

Occasionally the process of making such estimates results in fairly crude answers, but answers ten times or more too large or small are still useful. For example, suppose you're interested in how many people have contracted a certain disease. Any estimates under ten thousand are small compared with Earth's total population, but a million or more would be alarming. So even relatively imprecise information can provide valuable guidance.

In developing these estimates, you can take considerable liberties with the numbers. For example, $\pi \sim 1$, $27 \sim 10$, and $65 \sim 100$. To get a less crude estimate, it's permissible to use slightly more accurate numbers (e.g., $\pi \sim 3$, $27 \sim 30$, $65 \sim 70$). Better accuracy can also be obtained by systematically underestimating as many numbers as you overestimate. Some quantities may be completely unknown, but it's standard to make reasonable guesses, as the examples show.

EXAMPLE 1.6 BRAIN CELLS ESTIMATE

GOAL Develop a simple estimate.

PROBLEM Estimate the number of cells in the human brain.

STRATEGY Estimate the volume of a human brain and divide by the estimated volume of one cell. The brain is located in the upper portion of the head, with a volume that could be approximated by a cube $\ell = 20 \text{ cm}$ on a side. Brain cells, consisting of about 10% neurons and 90% glia, vary greatly in size, with dimensions ranging from a few microns to a meter or so. As a guess, take $d = 10 \text{ microns}$ as a typical dimension and consider a cell to be a cube with each side having that length.

SOLUTION

Estimate the volume of a human brain:

$$V_{\text{brain}} = \ell^3 \approx (0.2 \text{ m})^3 = 8 \times 10^{-3} \text{ m}^3 \approx 1 \times 10^{-2} \text{ m}^3$$

Estimate the volume of a cell:

$$V_{\text{cell}} = d^3 \approx (10 \times 10^{-6} \text{ m})^3 = 1 \times 10^{-15} \text{ m}^3$$

Divide the volume of a brain by the volume of a cell:

$$\text{number of cells} = \frac{V_{\text{brain}}}{V_{\text{cell}}} = \frac{0.01 \text{ m}^3}{1 \times 10^{-15} \text{ m}^3} = 1 \times 10^{13} \text{ cells}$$

REMARKS Notice how little attention was paid to obtaining precise values. Some general information about a problem is required if the estimate is to be within an order of magnitude of the actual value. Here, knowledge of the approximate dimensions of brain cells and the human brain were essential to developing the estimate.

QUESTION 1.6 Would 10^{12} cells also be a reasonable estimate? What about 10^9 cells? Explain.

EXERCISE 1.6 Estimate the total number of cells in the human body.

ANSWER 10^{14} (Answers may vary.)

EXAMPLE 1.7 STACK ONE-DOLLAR BILLS TO THE MOON**GOAL** Estimate the number of stacked objects required to reach a given height.**PROBLEM** How many one-dollar bills, stacked one on top of the other, would reach the Moon?**STRATEGY** The distance to the Moon is about 400 000 km. Guess at the number of dollar bills in a millimeter, and multiply the distance by this number, after converting to consistent units.**SOLUTION**

We estimate that ten stacked bills form a layer of 1 mm.
Convert mm to km:

$$\frac{10 \text{ bills}}{1 \text{ mm}} \left(\frac{10^3 \text{ mm}}{1 \text{ m}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) = \frac{10^7 \text{ bills}}{1 \text{ km}}$$

Multiply this value by the approximate lunar distance:

$$\# \text{ of dollar bills} \sim (4 \times 10^5 \text{ km}) \left(\frac{10^7 \text{ bills}}{1 \text{ km}} \right) = 4 \times 10^{12} \text{ bills}$$

REMARKS That's within an order of magnitude of the U.S. national debt!**QUESTION 1.7** Based on the answer, about how many stacked pennies would reach the Moon?**EXERCISE 1.7** How many pieces of cardboard, typically found at the back of a bound pad of paper, would you have to stack up to match the height of the Washington Monument, about 170 m tall?**ANSWER** $\sim 10^5$ (Answers may vary.)**EXAMPLE 1.8** NUMBER OF GALAXIES IN THE UNIVERSE**GOAL** Estimate a volume and a number density, and combine.**PROBLEM** Given that astronomers can see about 10 billion light-years into space and that there are 14 galaxies in our local group, 2 million light-years from the next local group, estimate the number of galaxies in the observable universe. (Note: One light-year is the distance traveled by light in one year, about 9.5×10^{15} m.) (See Fig. 1.4.)**STRATEGY** From the known information, we can estimate the number of galaxies per unit volume. The local group of 14 galaxies is contained in a sphere a million light-years in radius, with the Andromeda group in a similar sphere, so there are about 10 galaxies within a volume of radius 1 million light-years. Multiply that number density by the volume of the observable universe.

Figure 1.4 In this deep-space photograph, there are few stars—just galaxies without end.

NASA, ESA, S. Beckwith (STScI) and the HUDF Team

SOLUTION

Compute the approximate volume V_{lg} of the local group of galaxies:

$$V_{lg} = \frac{4}{3}\pi r^3 \sim (10^6 \text{ ly})^3 = 10^{18} \text{ ly}^3$$

Estimate the density of galaxies:

$$\begin{aligned} \text{density of galaxies} &= \frac{\# \text{ of galaxies}}{V_{lg}} \\ &\sim \frac{10 \text{ galaxies}}{10^{18} \text{ ly}^3} = 10^{-17} \frac{\text{galaxies}}{\text{ly}^3} \end{aligned}$$

Compute the approximate volume of the observable universe:

$$V_u = \frac{4}{3}\pi r^3 \sim (10^{10} \text{ ly})^3 = 10^{30} \text{ ly}^3$$

Multiply the density of galaxies by V_u :

$$\begin{aligned} \# \text{ of galaxies} &\sim (\text{density of galaxies})V_u \\ &= \left(10^{-17} \frac{\text{galaxies}}{\text{ly}^3} \right) (10^{30} \text{ ly}^3) \\ &= 10^{13} \text{ galaxies} \end{aligned}$$

REMARKS Notice the approximate nature of the computation, which uses $4\pi/3 \sim 1$ on two occasions and $14 \sim 10$ for the number of galaxies in the local group. This is completely justified: Using the actual numbers would be pointless, because the other assumptions in the problem—the size of the observable universe and the idea that the local galaxy density is representative of the density everywhere—are also very rough approximations. Further, there was nothing in the problem that required using volumes of spheres rather than volumes of cubes. Despite all these arbitrary choices, the answer still gives useful information, because it rules out a lot of reasonable possible answers. Before doing the calculation, a guess of a billion galaxies might have seemed plausible.

QUESTION 1.8 About one in ten galaxies in the local group are not dwarf galaxies. Estimate the number of galaxies in the Universe that are not dwarfs.

EXERCISE 1.8 (a) Given that the nearest star is about 4 light-years away, develop an estimate of the density of stars per cubic light-year in our galaxy. (b) Estimate the number of stars in the Milky Way galaxy, given that it's roughly a disk 100 000 light-years across and a thousand light-years thick.

ANSWER (a) 0.02 stars/ly^3 (b) 2×10^{11} stars (Estimates will vary. The actual answer is probably about twice that number.)

1.7 Coordinate Systems

Many aspects of physics deal with locations in space, which require the definition of a coordinate system. A point on a line can be located with one coordinate, a point in a plane with two coordinates, and a point in space with three.

A coordinate system used to specify locations in space consists of the following:

- A fixed reference point O , called the *origin*
- A set of specified axes, or directions, with an appropriate scale and labels on the axes
- Instructions on labeling a point in space relative to the origin and axes

One convenient and commonly used coordinate system is the **Cartesian coordinate system**, sometimes called the **rectangular coordinate system**. Such a system in two dimensions is illustrated in Figure 1.5. An arbitrary point in this system is labeled with the coordinates (x, y) . For example, the point P in the figure has coordinates $(5, 3)$. If we start at the origin O , we can reach P by moving 5 meters horizontally to the right and then 3 meters vertically upward. In the same way, the point Q has coordinates $(-3, 4)$, which corresponds to going 3 meters horizontally to the left of the origin and 4 meters vertically upward from there.

Positive x is usually selected as right of the origin and positive y upward from the origin, but in two dimensions this choice is largely a matter of taste. (In three dimensions, however, there are “right-handed” and “left-handed” coordinates, which lead to minus sign differences in certain operations. These will be addressed as needed.)

Sometimes it's more convenient to locate a point in space by its **plane polar coordinates** (r, θ) , as in Figure 1.6. In this coordinate system, an origin O and a reference line are selected as shown. A point is then specified by the distance r from the origin to the point and by the angle θ between the reference line and a line drawn from the origin to the point. The standard reference line is usually selected to be the positive x -axis of a Cartesian coordinate system. The angle θ is considered positive when measured counterclockwise from the reference line and negative when measured clockwise. For example, if a point is specified by the polar coordinates 3 m and 60° , we locate this point by moving out 3 m from the origin at an angle of 60° above (counterclockwise from) the reference line. A point specified by polar coordinates 3 m and -60° is located 3 m out from the origin and 60° below (clockwise from) the reference line.

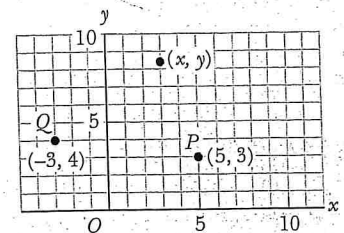


Figure 1.5 Designation of points in a two-dimensional Cartesian coordinate system. Every point is labeled with coordinates (x, y) .

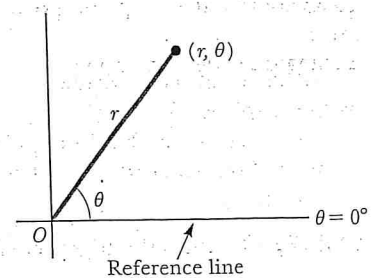


Figure 1.6 The plane polar coordinates of a point are represented by the distance r and the angle θ , where θ is measured counterclockwise from the positive x -axis.

1.8 Trigonometry Review

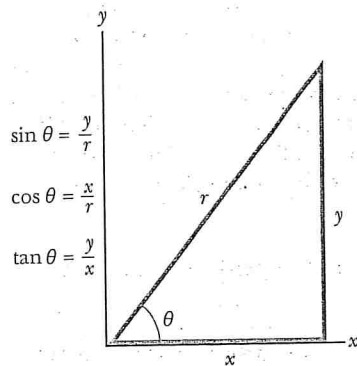


Figure 1.7 Certain trigonometric functions of a right triangle.

Consider the right triangle shown in Figure 1.7, where side y is opposite the angle θ , side x is adjacent to the angle θ , and side r is the hypotenuse of the triangle. The basic trigonometric functions defined by such a triangle are the ratios of the lengths of the sides of the triangle. These relationships are called the sine (sin), cosine (cos), and tangent (tan) functions. In terms of θ , the basic trigonometric functions are as follows:²

$$\begin{aligned}\sin \theta &= \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r} \\ \cos \theta &= \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \\ \tan \theta &= \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x}\end{aligned}\quad [1.1]$$

For example, if the angle θ is equal to 30° , then the ratio of y to r is always 0.50; that is, $\sin 30^\circ = 0.50$. Note that the sine, cosine, and tangent functions are quantities without units because each represents the ratio of two lengths.

Another important relationship, called the **Pythagorean theorem**, exists between the lengths of the sides of a right triangle:

$$r^2 = x^2 + y^2 \quad [1.2]$$

Finally, it will often be necessary to find the values of inverse relationships. For example, suppose you know that the sine of an angle is 0.866, but you need to know the value of the angle itself. The inverse sine function may be expressed as $\sin^{-1}(0.866)$, which is a shorthand way of asking the question “What angle has a sine of 0.866?” Punching a couple of buttons on your calculator reveals that this angle is 60.0° . Try it for yourself and show that $\tan^{-1}(0.400) = 21.8^\circ$. Be sure that your calculator is set for degrees and not radians. In addition, the inverse tangent function can return only values between -90° and $+90^\circ$, so when an angle is in the second or third quadrant, it’s necessary to add 180° to the answer in the calculator window.

The definitions of the trigonometric functions and the inverse trigonometric functions, as well as the Pythagorean theorem, can be applied to *any* right triangle, regardless of whether its sides correspond to x - and y -coordinates.

These results from trigonometry are useful in converting from rectangular coordinates to polar coordinates, or vice versa, as the next example shows.

Tip 1.3 Degrees vs. Radians

When calculating trigonometric functions, make sure your calculator setting—degrees or radians—is consistent with the angular measure you’re using in a given problem.

EXAMPLE 1.9 CARTESIAN AND POLAR COORDINATES

GOAL Understand how to convert from plane rectangular coordinates to plane polar coordinates and vice versa.

PROBLEM (a) The Cartesian coordinates of a point in the xy -plane are $(x, y) = (-3.50 \text{ m}, -2.50 \text{ m})$, as shown in Figure 1.8. Find the polar coordinates of this point. (b) Convert $(r, \theta) = (5.00 \text{ m}, 37.0^\circ)$ to rectangular coordinates.

STRATEGY Apply the trigonometric functions and their inverses, together with the Pythagorean theorem.

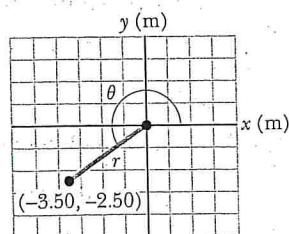


Figure 1.8 (Example 1.9) Converting from Cartesian coordinates to polar coordinates.

²Many people use the mnemonic *SOHCAHTOA* to remember the basic trigonometric formulas: Sine = Opposite/Hypotenuse, Cosine = Adjacent/Hypotenuse, and Tangent = Opposite/Adjacent. (Thanks go to Professor Don Chodrow for pointing this out.)

SOLUTION

(a) Cartesian to polar conversion

Take the square root of both sides of Equation 1.2 to find the radial coordinate:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

Use Equation 1.1 for the tangent function to find the angle with the inverse tangent, adding 180° because the angle is actually in the third quadrant:

$$\tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = \tan^{-1}(0.714) = 35.5^\circ + 180^\circ = 216^\circ$$

(b) Polar to Cartesian conversion

Use the trigonometric definitions, Equation 1.1.

$$x = r \cos \theta = (5.00 \text{ m}) \cos 37.0^\circ = 3.99 \text{ m}$$

$$y = r \sin \theta = (5.00 \text{ m}) \sin 37.0^\circ = 3.01 \text{ m}$$

REMARKS When we take up vectors in two dimensions in Topic 3, we will routinely use a similar process to find the direction and magnitude of a given vector from its components, or, conversely, to find the components from the vector's magnitude and direction.

QUESTION 1.9 Starting with the answers to part (b), work backwards to recover the given radius and angle. Why are there slight differences from the original quantities?

EXERCISE 1.9 (a) Find the polar coordinates corresponding to $(x, y) = (-3.25 \text{ m}, 1.50 \text{ m})$. (b) Find the Cartesian coordinates corresponding to $(r, \theta) = (4.00 \text{ m}, 53.0^\circ)$.

ANSWERS (a) $(r, \theta) = (3.58 \text{ m}, 155^\circ)$ (b) $(x, y) = (2.41 \text{ m}, 3.19 \text{ m})$

EXAMPLE 1.10 HOW HIGH IS THE BUILDING?

GOAL Apply basic results of trigonometry.

PROBLEM A person measures the height of a building by walking out a distance of 46.0 m from its base and shining a flashlight beam towards the top. When the beam is elevated at an angle of 39.0° with respect to the horizontal, as shown in Figure 1.9, the beam just strikes the top of the building. (a) If the flashlight is held at a height of 2.00 m , find the height of the building. (b) Calculate the length of the light beam.

STRATEGY Refer to the right triangle shown in the figure. We know the angle, 39.0° , and the length of the side adjacent to it. Because the height of the building is the side opposite the angle, we can use the tangent function. With the adjacent and opposite sides known, we can then find the hypotenuse with the Pythagorean theorem.

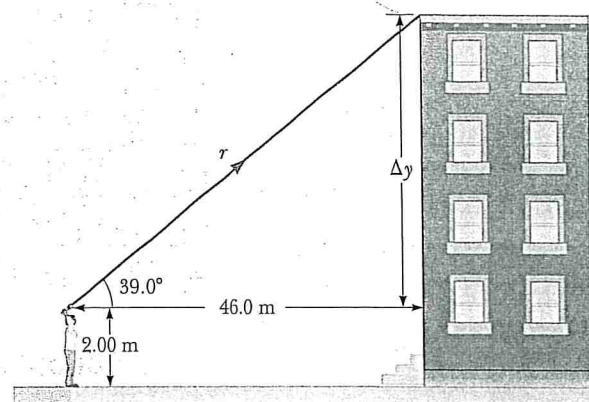


Figure 1.9 (Example 1.10)

SOLUTION

(a) Find the height of the building.

Use the tangent of the given angle:

$$\tan 39.0^\circ = \frac{\Delta y}{46.0 \text{ m}}$$

Solve for the height:

$$\Delta y = (\tan 39.0^\circ)(46.0 \text{ m}) = (0.810)(46.0 \text{ m}) = 37.3 \text{ m}$$

Add 2.00 m to Δy to obtain the height:

$$\text{height} = 39.3 \text{ m}$$

(b) Calculate the length of the light beam.

Use the Pythagorean theorem:

$$r = \sqrt{x^2 + y^2} = \sqrt{(37.3 \text{ m})^2 + (46.0 \text{ m})^2} = 59.2 \text{ m}$$

(Continued)

REMARKS In the next section, right-triangle trigonometry is often used when working with vectors.

QUESTION 1.10 Could the distance traveled by the light beam be found without using the Pythagorean theorem? How?

EXERCISE 1.10 While standing atop a building 50.0 m tall, you spot a friend standing on a street corner. Using a protractor and dangling a plumb bob, you find that the angle between the horizontal and the direction to the spot on the sidewalk where your friend is standing is 25.0° . Your eyes are located 1.75 m above the top of the building. How far away from the foot of the building is your friend?

ANSWER 111 m

1.9 Vectors



Jon Feingersh/Stone/Getty Images

Figure 1.10 A vector such as velocity has a magnitude, shown on the race car's speedometer, and a direction, straight out through the race car's front windshield. The mass of the car is a scalar quantity, as is the volume of gasoline in its fuel tank.

Physical quantities studied in this text fall into two main categories. One type, known as a *scalar quantity*, can be completely described by a single number (with appropriate units) giving its magnitude or size. Some common scalars are mass, temperature, volume, and speed. For example, a car's speed can be completely described by noting the number on its speedometer. A basketball's mass can be specified by measuring a single number with a scale.

The other type of quantity, known as a *vector quantity*, has both a magnitude and a direction. Velocity is a common vector quantity with a magnitude specifying how fast an object is moving and a direction specifying the direction of travel. For example, a car's velocity could be specified by noting it was traveling 60 miles per hour in the direction due north. A car traveling 60 miles per hour in an eastward direction would have a different velocity vector but the same scalar speed. Figure 1.10 illustrates each type of quantity.

In this book, symbols for scalar quantities are shown in italics (e.g., m for mass and T for temperature), and symbols for vector quantities are usually shown in bold with an arrow above the letter (e.g., \vec{v} for velocity). A vector's magnitude is a scalar quantity indicating its length and is shown in italics. For example, the scalar v indicates the magnitude of vector \vec{v} . In general, **a vector quantity is characterized by having both a magnitude and a direction.** By contrast, **a scalar quantity has magnitude, but no direction.** Scalar quantities can be manipulated with the rules of ordinary arithmetic. Vectors can also be added and subtracted from each other, and multiplied, but there are a number of important differences, as will be seen in the following sections.

1.9.1 Equality of Two Vectors

Two vectors \vec{A} and \vec{B} are equal if they have the same magnitude and the same direction. They need not be located at the same point in space. The four vectors in Figure 1.11 are all equal to each other. Moving a vector from one point in space to another doesn't change its magnitude or its direction.

1.9.2 Adding Vectors

When two or more vectors are added, they must all have the same units. For example, it doesn't make sense to add a velocity vector, carrying units of meters per second, to a displacement vector, carrying units of meters. Scalars obey the same rule: It would be similarly meaningless to add temperatures to volumes or masses to time intervals.

Vectors can be added geometrically or algebraically. (The latter is discussed at the end of the next section.) To add vector \vec{B} to vector \vec{A} geometrically, first draw \vec{A} on a piece of graph paper to some scale, such as $1 \text{ cm} = 1 \text{ m}$, so that its

Tip 1.4 Vector Addition vs. Scalar Addition

$\vec{A} + \vec{B} = \vec{C}$ differs significantly from $A + B = C$. The first is a vector sum, which must be handled graphically or with components, whereas the second is a simple arithmetic sum of numbers.

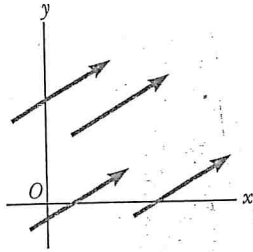


Figure 1.11 These four vectors are equal because they have equal lengths and point in the same direction.

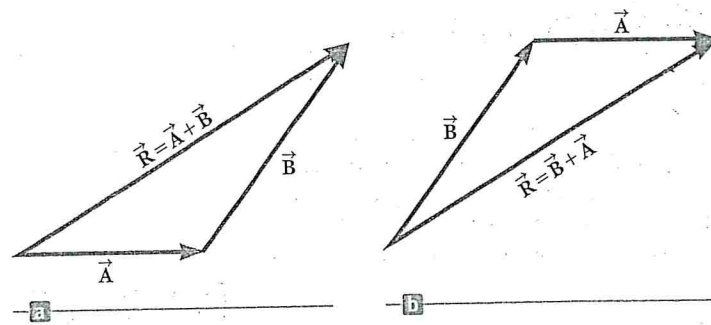


Figure 1.12 (a) When vector \vec{B} is added to vector \vec{A} , the vector sum \vec{R} is the vector that runs from the tail of \vec{A} to the tip of \vec{B} . (b) Here the resultant runs from the tail of \vec{B} to the tip of \vec{A} . These constructions prove that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

direction is specified relative to a coordinate system. Then draw vector \vec{B} to the same scale with the tail of \vec{B} starting at the tip of \vec{A} , as in Figure 1.12a. Vector \vec{B} must be drawn along the direction that makes the proper angle relative to vector \vec{A} . The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is the vector drawn from the tail of \vec{A} to the tip of \vec{B} . This procedure is known as the **triangle method of addition**.

When two vectors are added, their sum is independent of the order of the addition: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. This relationship can be seen from the geometric construction in Figure 1.12b, and is called the **commutative law of addition**.

This same general approach can also be used to add more than two vectors, as is done in Figure 1.13 for four vectors. The resultant vector sum $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$ is the vector drawn from the tail of the first vector to the tip of the last. Again, the order in which the vectors are added is unimportant.

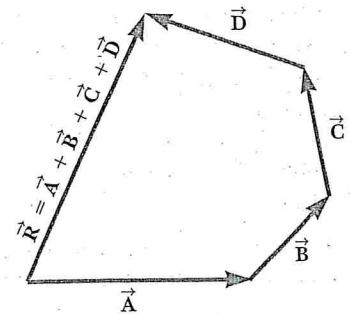


Figure 1.13 A geometric construction for summing four vectors. The resultant vector \vec{R} is the vector that completes the polygon.

1.9.3 Negative of a Vector

The negative of the vector \vec{A} is defined as the vector that gives zero when added to \vec{A} . This means that \vec{A} and $-\vec{A}$ have the same magnitude but opposite directions.

1.9.4 Subtracting Vectors

Vector subtraction makes use of the definition of the negative of a vector. We define the operation $\vec{A} - \vec{B}$ as the vector $-\vec{B}$ added to the vector \vec{A} :

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \quad [1.3]$$

Vector subtraction is really a special case of vector addition. The geometric construction for subtracting two vectors is shown in Figure 1.14.

1.9.5 Multiplying or Dividing a Vector by a Scalar

Multiplying or dividing a vector by a scalar gives a vector. For example, if vector \vec{A} is multiplied by the scalar number 3, the result, written $3\vec{A}$, is a vector with a magnitude three times that of \vec{A} and pointing in the same direction. If we multiply vector \vec{A} by the scalar -3 , the result is $-3\vec{A}$, a vector with a magnitude three times that of \vec{A} and pointing in the opposite direction (because of the negative sign).

Quick Quiz

1.1 The magnitudes of two vectors \vec{A} and \vec{B} are 12 units and 8 units, respectively. What are the largest and smallest possible values for the magnitude of the resultant vector $\vec{R} = \vec{A} + \vec{B}$? (a) 14.4 and 4 (b) 12 and 8 (c) 20 and 4 (d) none of these.

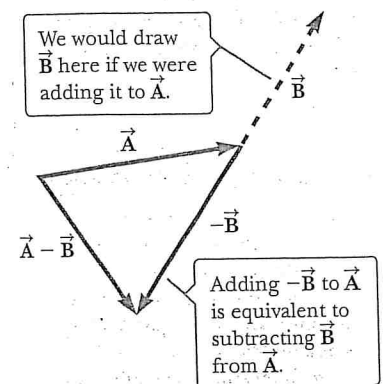


Figure 1.14 This construction shows how to subtract vector \vec{B} from vector \vec{A} . The vector $-\vec{B}$ has the same magnitude as the vector \vec{B} but points in the opposite direction.

EXAMPLE 1.11 TAKING A TRIP

GOAL Find the sum of two vectors by using a graph.

PROBLEM A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as in Figure 1.15. Using a graph, find the magnitude and direction of a single vector that gives the net effect of the car's trip. This vector is called the car's *resultant displacement*.

STRATEGY Draw a graph and represent the displacement vectors as arrows. Graphically locate the vector resulting from the sum of the two displacement vectors. Measure its length and angle with respect to the vertical.

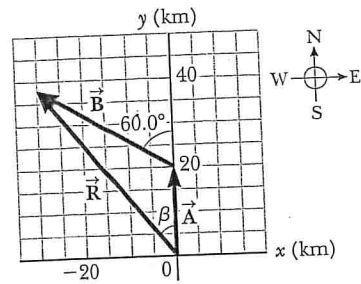


Figure 1.15 (Example 1.11) A graphical method for finding the resultant displacement vector $\vec{R} = \vec{A} + \vec{B}$.

SOLUTION

Let \vec{A} represent the first displacement vector, 20.0 km north, and \vec{B} the second displacement vector, extending west of north. Carefully graph the two vectors, drawing a resultant vector \vec{R} with its base touching the base of \vec{A} and extending to the tip of \vec{B} . Measure the length of this vector, which turns out to be about 48 km. The angle β , measured with a protractor, is about 39° west of north.

REMARKS Notice that ordinary arithmetic doesn't work here: the correct answer of 48 km is not equal to 20.0 km + 35.0 km = 55.0 km!

QUESTION 1.11 Suppose two vectors are added. Under what conditions would the sum of the magnitudes of the vectors equal the magnitude of the resultant vector?

EXERCISE 1.11 Graphically determine the magnitude and direction of the displacement if a man walks 30.0 km 45° north of east and then walks due east 20.0 km.

ANSWER 46 km, 27° north of east

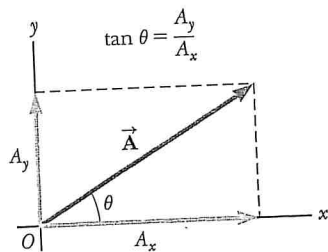


Figure 1.16 Any vector \vec{A} lying in the xy -plane can be represented by its rectangular components A_x and A_y .

Tip 1.5 *x*- and *y*-components

Equation 1.4 for the *x*- and *y*-components of a vector associates cosine with the *x*-component and sine with the *y*-component, as in Figure 1.17a. This association is due *solely* to the fact that we chose to measure the angle θ with respect to the positive *x*-axis. If the angle were measured with respect to the *y*-axis, as in Figure 1.17b, the components would be given by $A_x = A \sin \theta$ and $A_y = A \cos \theta$.

1.10 Components of a Vector

One method of adding vectors makes use of the projections of a vector along the axes of a rectangular coordinate system. These projections are called **components**. Any vector can be completely described by its components.

Consider a vector \vec{A} in a rectangular coordinate system, as shown in Figure 1.16. \vec{A} can be expressed as the sum of two vectors: \vec{A}_x , parallel to the *x*-axis, and \vec{A}_y , parallel to the *y*-axis. Mathematically,

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where \vec{A}_x and \vec{A}_y are the component vectors of \vec{A} . The projection of \vec{A} along the *x*-axis, A_x , is called the *x*-component of \vec{A} , and the projection of \vec{A} along the *y*-axis, A_y , is called the *y*-component of \vec{A} . These components can be either positive or negative numbers with units. From the definitions of sine and cosine, we see that $\cos \theta = A_x/A$ and $\sin \theta = A_y/A$, so the components of \vec{A} are

$$A_x = A \cos \theta \quad [1.4a]$$

$$A_y = A \sin \theta \quad [1.4b]$$

These components form two sides of a right triangle having a hypotenuse with magnitude *A*. It follows that \vec{A} 's magnitude and direction are related to its components through the Pythagorean theorem and the definition of the tangent:

$$A = \sqrt{A_x^2 + A_y^2} \quad [1.5]$$

$$\tan \theta = \frac{A_y}{A_x} \quad [1.6]$$

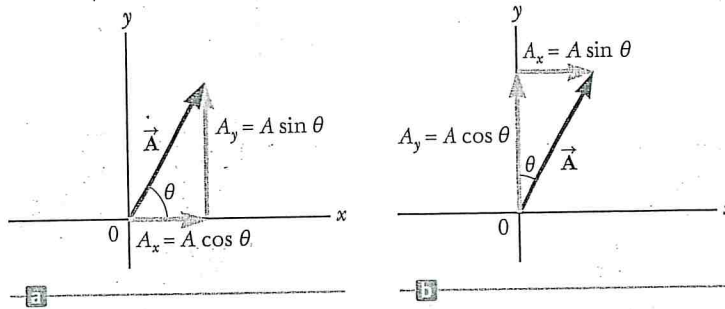


Figure 1.17 The angle θ need not always be defined from the positive x -axis.

To solve for the angle θ , which is measured counterclockwise from the positive x -axis by convention, the inverse tangent can be taken of both sides of Equation 1.6:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

This formula gives the right answer for θ only half the time! The inverse tangent function returns values only from -90° to $+90^\circ$, so the answer in your calculator window will only be correct if the vector happens to lie in the first or fourth quadrant. If it lies in the second or third quadrant, adding 180° to the number in the calculator window will always give the right answer. The angle in Equations 1.4 and 1.6 must be measured from the positive x -axis. Other choices of reference line are possible, but certain adjustments must then be made. (See Tip 1.5 and Fig. 1.17.)

If a coordinate system other than the one shown in Figure 1.16 is chosen, the components of the vector must be modified accordingly. In many applications it's more convenient to express the components of a vector in a coordinate system having axes that are not horizontal and vertical but are still perpendicular to each other. Suppose a vector \vec{B} makes an angle θ' with the x' -axis defined in Figure 1.18. The rectangular components of \vec{B} along the axes of the figure are given by $B_{x'} = B \cos \theta'$ and $B_{y'} = B \sin \theta'$, as in Equations 1.4. The magnitude and direction of \vec{B} are then obtained from expressions equivalent to Equations 1.5 and 1.6.

Quick Quiz

1.2 Figure 1.19 shows two vectors lying in the xy -plane. Determine the signs of the x - and y -components of \vec{A} , \vec{B} , and $\vec{A} + \vec{B}$.

1.3 Which vector has an angle with respect to the positive x -axis that is in the range of the inverse tangent function?

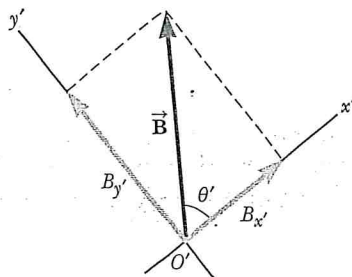


Figure 1.18 The components of vector \vec{B} in a tilted coordinate system.

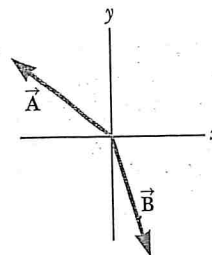


Figure 1.19 (Quick Quizzes 1.2 and 1.3)

Tip 1.6 Inverse Tangents on Calculators: Right Half the Time

The inverse tangent function on calculators returns an angle between -90° and $+90^\circ$. If the vector lies in the second or third quadrant, the angle, as measured from the positive x -axis, will be the angle returned by your calculator plus 180° .

EXAMPLE 1.12 HELP IS ON THE WAY!

GOAL Find vector components, given a magnitude and direction, and vice versa.

PROBLEM (a) Find the horizontal and vertical components of the $d = 1.00 \times 10^2$ m displacement of a superhero who flies from the top of a tall building along the path shown in Figure 1.20a. (b) Suppose instead the superhero leaps in the other direction along a displacement vector \vec{B} to the top of a flagpole where the displacement components are given by $B_x = -25.0$ m and $B_y = 10.0$ m. Find the magnitude and direction of the displacement vector.

STRATEGY (a) The triangle formed by the displacement and its components is shown in Figure 1.20b. Simple trigonometry gives the components relative to the standard xy -coordinate system: $A_x = A \cos \theta$ and $A_y = A \sin \theta$ (Eqs. 1.4). Note that $\theta = -30.0^\circ$, negative because it's measured clockwise from the positive x -axis. (b) Apply Equations 1.5 and 1.6 to find the magnitude and direction of the vector.

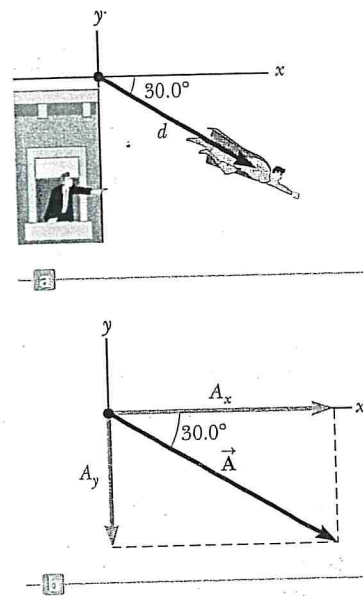


Figure 1.20 (Example 1.12)

SOLUTION

(a) Find the vector components of \vec{A} from its magnitude and direction.

Use Equations 1.4 to find the components of the displacement vector \vec{A} :

(b) Find the magnitude and direction of the displacement vector \vec{B} from its components.

Compute the magnitude of \vec{B} from the Pythagorean theorem:

Calculate the direction of \vec{B} using the inverse tangent, remembering to add 180° to the answer in your calculator window, because the vector lies in the second quadrant:

$$A_x = A \cos \theta = (1.00 \times 10^2 \text{ m}) \cos (-30.0^\circ) = +86.6 \text{ m}$$

$$A_y = A \sin \theta = (1.00 \times 10^2 \text{ m}) \sin (-30.0^\circ) = -50.0 \text{ m}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-25.0 \text{ m})^2 + (10.0 \text{ m})^2} = 26.9 \text{ m}$$

$$\theta = \tan^{-1} \left(\frac{B_y}{B_x} \right) = \tan^{-1} \left(\frac{10.0}{-25.0} \right) = -21.8^\circ$$

$$\theta = 158^\circ$$

REMARKS In part (a), note that $\cos(-\theta) = \cos \theta$; however, $\sin(-\theta) = -\sin \theta$. The negative sign of A_y reflects the fact that displacement in the y -direction is downward.

QUESTION 1.12 What other functions, if any, can be used to find the angle in part (b)?

EXERCISE 1.12 (a) Suppose the superhero had flown 150 m at a 120° angle with respect to the positive x -axis. Find the components of the displacement vector. (b) Suppose instead the superhero had leaped with a displacement having an x -component of 32.5 m and a y -component of 24.3 m. Find the magnitude and direction of the displacement vector.

ANSWERS (a) $A_x = -75$ m, $A_y = 130$ m (b) 40.6 m, 36.8°

1.10.1 Adding Vectors Algebraically

The graphical method of adding vectors is valuable in understanding how vectors can be manipulated, but most of the time vectors are added algebraically in terms of their components. Suppose $\vec{R} = \vec{A} + \vec{B}$. Then the components of the resultant vector \vec{R} are given by

$$R_x = A_x + B_x \quad [1.7a]$$

$$R_y = A_y + B_y \quad [1.7b]$$

So x -components are added only to x -components, and y -components only to y -components. The magnitude and direction of \vec{R} can subsequently be found with Equations 1.5 and 1.6.

Subtracting two vectors works the same way because it's a matter of adding the negative of one vector to another vector. You should make a rough sketch when adding or subtracting vectors, in order to get an approximate geometric solution as a check.

EXAMPLE 1.13 TAKE A HIKE

GOAL Add vectors algebraically and find the resultant vector.

PROBLEM A hiker begins a trip by first walking 25.0 km 45.0° south of east from her base camp. On the second day she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. (a) Determine the components of the hiker's displacements in the first and second days. (b) Determine the components of the hiker's total displacement for the trip. (c) Find the magnitude and direction of the displacement from base camp.

STRATEGY This problem is just an application of vector addition using components, Equations 1.7. We denote the displacement vectors on the first and second days by \vec{A} and \vec{B} , respectively. Using the camp as the origin of the coordinates, we get the vectors shown in Figure 1.21a. After finding x - and y -components for each vector, we add them "componentwise." Finally, we determine the magnitude and direction of the resultant vector \vec{R} , using the Pythagorean theorem and the inverse tangent function.

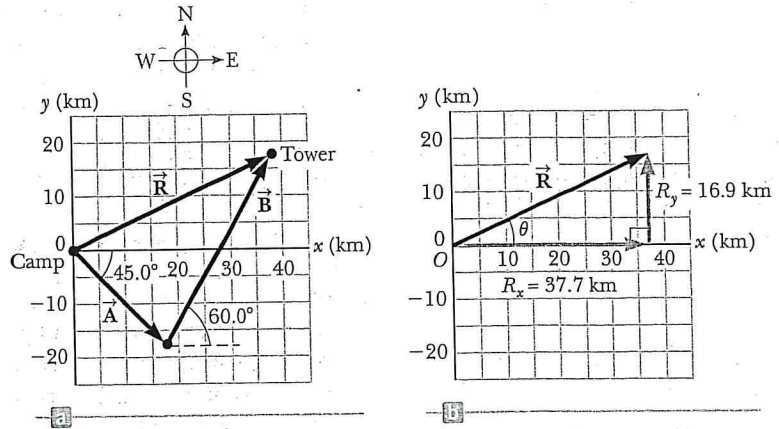


Figure 1.21 (Example 1.13) (a) Hiker's path and the resultant vector. (b) Components of the hiker's total displacement from camp.

SOLUTION

(a) Find the components of \vec{A} .

Use Equations 1.4 to find the components of \vec{A} :

$$A_x = A \cos(-45.0^\circ) = (25.0 \text{ km})(0.707) = 17.7 \text{ km}$$

$$A_y = A \sin(-45.0^\circ) = -(25.0 \text{ km})(0.707) = -17.7 \text{ km}$$

Find the components of \vec{B} :

$$B_x = B \cos 60.0^\circ = (40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B \sin 60.0^\circ = (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$

(b) Find the components of the resultant vector, $\vec{R} = \vec{A} + \vec{B}$.

To find R_x , add the x -components of \vec{A} and \vec{B} :

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = 37.7 \text{ km}$$

To find R_y , add the y -components of \vec{A} and \vec{B} :

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 16.9 \text{ km}$$

(c) Find the magnitude and direction of \vec{R} .

Use the Pythagorean theorem to get the magnitude:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(37.7 \text{ km})^2 + (16.9 \text{ km})^2} = 41.3 \text{ km}$$

Calculate the direction of \vec{R} using the inverse tangent function:

$$\theta = \tan^{-1}\left(\frac{16.9 \text{ km}}{37.7 \text{ km}}\right) = 24.1^\circ$$

REMARKS Figure 1.21b shows a sketch of the components of \vec{R} and their directions in space. The magnitude and direction of the resultant can also be determined from such a sketch.

QUESTION 1.13 A second hiker follows the same path the first day, but then walks 15.0 km east on the second day before turning and reaching the ranger's tower. Is the second hiker's resultant displacement vector the same as the first hiker's, or different?

EXERCISE 1.13 A cruise ship leaving port travels 50.0 km 45.0° north of west and then 70.0 km at a heading 30.0° north of east. Find (a) the components of the ship's displacement vector and (b) the displacement vector's magnitude and direction.

ANSWERS (a) $R_x = 25.3$ km, $R_y = 70.4$ km (b) 74.8 km, 70.2° north of east

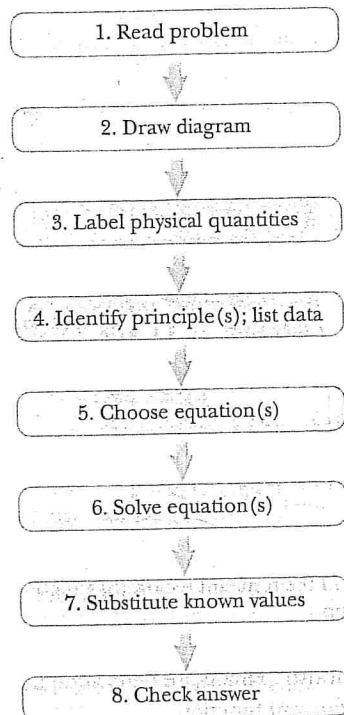


Figure 1.22 A guide to problem solving.

1.11 Problem-Solving Strategy

Most courses in general physics require the student to learn the skills used in solving problems, and examinations usually include problems that test such skills. This brief section presents some useful suggestions to help increase your success in solving problems. An organized approach to problem solving will also enhance your understanding of physical concepts and reduce exam stress. Throughout the book, there will be a number of sections labeled "Problem-Solving Strategy," many of them just a specializing of the list given below (and illustrated in Fig. 1.22).

1.11.1 General Problem-Solving Strategy

Problem

1. Read the problem carefully at least twice. Be sure you understand the nature of the problem before proceeding further.
2. Draw a diagram while rereading the problem.
3. Label all physical quantities in the diagram, using letters that remind you what the quantity is (e.g., m for mass). Choose a coordinate system and label it.

Strategy

4. Identify physical principles, the knowns and unknowns, and list them. Put circles around the unknowns. There must be as many equations as there are unknowns.
5. Equations, the relationships between the labeled physical quantities, should be written down next. Naturally, the selected equations should be consistent with the physical principles identified in the previous step.

Solution

6. Solve the set of equations for the unknown quantities in terms of the known. Do this algebraically, without substituting values until the next step, except where terms are zero.
7. Substitute the known values, together with their units. Obtain a numerical value with units for each unknown.

Check Answer

8. Check your answer. Do the units match? Is the answer reasonable? Does the plus or minus sign make sense? Is your answer consistent with an order of magnitude estimate?

This same procedure, with minor variations, should be followed throughout the course. The first three steps are extremely important, because they get you mentally oriented. Identifying the proper concepts and physical principles assists you in choosing the correct equations. The equations themselves are essential, because when you understand them, you also understand the relationships between the physical quantities. This understanding comes through a lot of daily practice.

Equations are the tools of physics: To solve problems, you have to have them at hand, like a plumber and his wrenches. Know the equations, and understand

what they mean and how to use them. Just as you can't have a conversation without knowing the local language, you can't solve physics problems without knowing and understanding the equations. This understanding grows as you study and apply the concepts and the equations relating them.

Carrying through the algebra for as long as possible (substituting numbers only at the end) is also important, because it helps you think in terms of the physical quantities involved, not merely the numbers that represent them. Many beginning physics students are eager to substitute, but once numbers are substituted it's harder to understand relationships and easier to make mistakes.

The physical layout and organization of your work will make the final product more understandable and easier to follow. Although physics is a challenging discipline, your chances of success are excellent if you maintain a positive attitude and keep trying.

EXAMPLE 1.14 A SHOPPING TRIP

GOAL Illustrate the Problem-Solving Strategy.

PROBLEM A shopper leaves home and drives to a store located 7.00 km away in a direction 30.0° north of east. Leaving the store, the shopper drives 5.00 km in a direction 50.0° west of north to a restaurant. Find the distance and direction from the shopper's home to the restaurant.

STRATEGY We've finished reading the problem (step 1), and have drawn a diagram (step 2) in Figure 1.23 and labeled it (step 3). From the diagram, we recognize that the vector \vec{R} locating the restaurant's position is the sum of vectors \vec{A} and \vec{B} and identify (step 4) the principles involved: the magnitude and direction of a vector. Vectors \vec{A} and \vec{B} are known and vector \vec{R} is unknown.

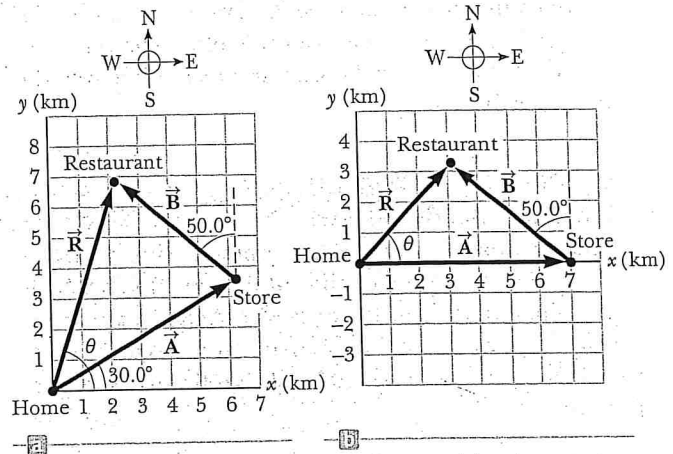


Figure 1.23 (a) (Example 1.14) (b) (Exercise 1.14)

SOLUTION

Express \vec{R} as a vector sum (step 5):

Solve symbolically for the components of \vec{R} (step 6):

Substitute the numbers, with units (step 7). The direction of \vec{B} measured counterclockwise from the positive x -axis is $50.0^\circ + 90.0^\circ = 140.0^\circ$:

Solve for the distance using the Pythagorean theorem:

Solve for the direction:

$$\vec{R} = \vec{A} + \vec{B}$$

$$R_x = A_x + B_x \text{ and } R_y = A_y + B_y$$

$$R_x = (7.00 \text{ km}) \cos(30.0^\circ) + (5.00 \text{ km}) \cos(140.0^\circ) = 2.23 \text{ km}$$

$$R_y = (7.00 \text{ km}) \sin(30.0^\circ) + (5.00 \text{ km}) \sin(140.0^\circ) = 6.71 \text{ km}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.23 \text{ km})^2 + (6.71 \text{ km})^2} = 7.07 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{6.71 \text{ km}}{2.23 \text{ km}}\right) = 71.6^\circ$$

REMARKS In checking (step 8), note that the units match and the answer seems reasonable. A sketch drawn at least approximately to scale can help determine whether or not the distance and angle are reasonable.

QUESTION 1.14 What are the answers if both distances in the problem statement are doubled but the directions are not changed?

EXERCISE 1.14 Find the distance and direction from the shopper's home to the restaurant if the store is located 7.00 km due east from home and the restaurant remains 5.00 km from the store in a direction 50.0° west of north.

ANSWER $R = 4.51 \text{ km}$, $\theta = 45.4^\circ$

SUMMARY

1.1 Standards of Length, Mass, and Time

The physical quantities in the study of mechanics can be expressed in terms of three fundamental quantities: length, mass, and time, which have the SI units meters (m), kilograms (kg), and seconds (s), respectively.

1.2 The Building Blocks of Matter

Matter is made of atoms, which in turn are made up of a relatively small nucleus of protons and neutrons within a cloud of electrons. Protons and neutrons are composed of still smaller particles, called quarks.

1.3 Dimensional Analysis

Dimensional analysis can be used to check equations and to assist in deriving them. When the dimensions on both sides of the equation agree, the equation is often correct up to a numerical factor. When the dimensions don't agree, the equation must be wrong.

1.4 Uncertainty in Measurement and Significant Figures

No physical quantity can be determined with complete accuracy. The concept of significant figures affords a basic method of handling these uncertainties. A significant figure is a reliably known digit, other than a zero used to locate the decimal point. The two rules of significant figures are as follows:

1. When multiplying or dividing using two or more quantities, the result should have the same number of significant figures as the quantity having the fewest significant figures.
2. When quantities are added or subtracted, the number of decimal places in the result should be the same as in the quantity with the fewest decimal places.

Use of scientific notation can avoid ambiguity in significant figures. In rounding, if the last digit dropped is less than 5, simply drop the digit; otherwise, raise the last retained digit by one.

1.5 Unit Conversions for Physical Quantities

Units in physics equations must always be consistent. In solving a physics problem, it's best to start with consistent units, using the table of conversion factors on the front end-sheets as necessary.

Converting units is a matter of multiplying the given quantity by a fraction, with one unit in the numerator and its equivalent in the other units in the denominator, arranged so the unwanted units in the given quantity are canceled out in favor of the desired units.

1.6 Estimates and Order-of-Magnitude Calculations

Sometimes it's useful to find an approximate answer to a question, either because the math is difficult or because information is incomplete. A quick estimate can also be used to check a more detailed calculation. In an order-of-magnitude calculation, each value is replaced by the closest power of ten, which sometimes must be guessed or estimated when the value is unknown. The computation is then carried out. For quick estimates involving known values, each value can first be rounded to one significant figure.

1.7 Coordinate Systems

The Cartesian coordinate system consists of two perpendicular axes, usually called the x -axis and y -axis, with each axis labeled with all numbers from negative infinity to positive infinity. Points are located by specifying the x - and y -values. Polar coordinates consist of a radial coordinate r , which is the distance from the origin, and an angular coordinate θ , which is the angular displacement from the positive x -axis (see Fig. 1.24).

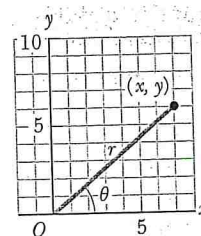


Figure 1.24 A point in the plane can be described with Cartesian coordinates (x, y) or with polar coordinates (r, θ) .

1.8 Trigonometry Review

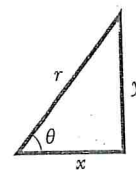
The three most basic trigonometric functions of a right triangle are the sine, cosine, and tangent, defined as follows:

$$\begin{aligned}\sin \theta &= \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{y}{r} \\ \cos \theta &= \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{x}{r} \\ \tan \theta &= \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{y}{x}\end{aligned}\quad [1.1]$$

The Pythagorean theorem is an important relationship between the lengths of the sides of a right triangle:

$$r^2 = x^2 + y^2 \quad [1.2]$$

where r is the hypotenuse of the triangle and x and y are the other two sides (see Fig. 1.25).



1.9 Vectors

A scalar quantity has a magnitude but no direction; a vector quantity has both magnitude and direction. Two vectors \vec{A} and \vec{B} can be added geometrically with the triangle method. The two vectors are drawn to scale on graph paper, with the tail of the second

Figure 1.25 |

vector located at the tip of the first. The **resultant** vector is the vector drawn from the tail of the first vector to the tip of the second.

The negative of a vector \vec{A} is a vector with the same magnitude as \vec{A} , but pointing in the opposite direction. A vector can be multiplied by a scalar, changing its magnitude, and its direction if the scalar is negative.

1.10 Components of a Vector

A vector \vec{A} can be split into two components, one pointing in the x -direction and the other in the y -direction (see Fig. 1.26). These components form two sides of a right triangle having a hypotenuse with magnitude A and are given by

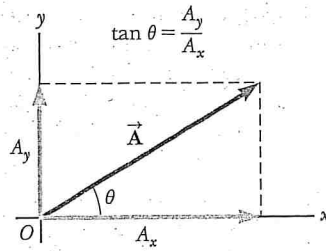


Figure 1.26 A vector can be written in terms of components in the x - and y -directions.

$$A_x = A \cos \theta \quad [1.4a]$$

$$A_y = A \sin \theta \quad [1.4b]$$

The magnitude and direction of \vec{A} are related to its components through the Pythagorean theorem and the definition of the tangent:

$$A = \sqrt{A_x^2 + A_y^2} \quad [1.5]$$

$$\tan \theta = \frac{A_y}{A_x} \quad [1.6]$$

In Equation 1.6, $\theta = \tan^{-1}(A_y/A_x)$ gives the correct vector angle only for vectors with $-90^\circ < \theta < 90^\circ$. If the vector has a negative x -component, 180° must be added to the answer in the calculator window.

If $\vec{R} = \vec{A} + \vec{B}$, then the components of the resultant vector \vec{R} are

$$R_x = A_x + B_x \quad [1.7a]$$

$$R_y = A_y + B_y \quad [1.7b]$$

CONCEPTUAL QUESTIONS

- Estimate the order of magnitude of the length, in meters, of each of the following: (a) a mouse, (b) a pool cue, (c) a basketball court, (d) an elephant, (e) a city block.
- What types of natural phenomena could serve as time standards?
- Find the order of magnitude of your age in seconds.
- An object with a mass of 1 kg weighs approximately 2 lb. Use this information to estimate the mass of the following objects: (a) a baseball, (b) your physics textbook, (c) a pickup truck.
- BIO** (a) Estimate the number of times your heart beats in a month. (b) Estimate the number of human heartbeats in an average lifetime.
- Estimate the number of atoms in 1 cm^3 of a solid. (Note that the diameter of an atom is about 10^{-10} m .)
- BIO** Lacking modern timepieces, early experimenters sometimes measured time intervals with their pulse. Why was this a poor method of measuring time?
- For an angle θ measured from the positive x -axis, the values of $\sin \theta$ and $\cos \theta$ are always (choose one): (a) greater than +1 (b) less than -1 (c) greater than -1 and less than 1 (d) greater than or equal to -1 and less than or equal to 1 (e) less than or equal to -1 or greater than or equal to 1.
- The left side of an equation has dimensions of length and the right side has dimensions of length squared. Can the equation be correct (choose one)? (a) Yes, because both sides involve the dimension of length. (b) No, because the equation is dimensionally inconsistent.
- List some advantages of the metric system of units over most other systems of units.
- BIO** Estimate the time duration of each of the following in the suggested units in parentheses: (a) a heartbeat (seconds), (b) a football game (hours), (c) a summer (months), (d) a movie (hours), (e) the blink of an eye (seconds).
- Suppose two quantities, A and B , have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful. (a) $A + B$ (b) $B - A$ (c) $A - B$ (d) A/B (e) AB
- Answer each question yes or no. Must two quantities have the same dimensions (a) if you are adding them? (b) If you are multiplying them? (c) If you are subtracting them? (d) If you are dividing them? (e) If you are equating them?
- Two different measuring devices are used by students to measure the length of a metal rod. Students using the first device report its length as 0.5 m, while those using the second report 0.502 m. Can both answers be correct (choose one)? (a) Yes, because their values are the same when both are rounded to the same number of significant figures. (b) No, because they report different values.
- If \vec{B} is added to \vec{A} , under what conditions does the resultant vector have a magnitude equal to $A + B$? Under what conditions is the resultant vector equal to zero?
- Under what circumstances would a vector have components that are equal in magnitude?

PROBLEMS

See the Preface for an explanation of the icons used in this problems set.

1.3 Dimensional Analysis

1. The period of a simple pendulum, defined as the time necessary for one complete oscillation, is measured in time units and is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the acceleration due to gravity, in units of length divided by time squared. Show that this equation is dimensionally consistent. (You might want to check the formula using your keys at the end of a string and a stopwatch.)

2. (a) Suppose the displacement of an object is related to time according to the expression $x = Bt^2$. What are the dimensions of B ? (b) A displacement is related to time as $x = A \sin(2\pi ft)$, where A and f are constants. Find the dimensions of A . *Hint:* A trigonometric function appearing in an equation must be dimensionless.
3. **S** A shape that covers an area A and has a uniform height h has a volume $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A , sometimes called the “footprint” of the object, can have any shape and that the height can, in general, be replaced by the average thickness of the object.)
4. **M** Each of the following equations was given by a student during an examination: (a) $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \sqrt{mgh}$ (b) $v = v_0 + at^2$ (c) $ma = v^2$. Do a dimensional analysis of each equation and explain why the equation can't be correct.
5. Newton's law of universal gravitation is represented by

$$F = G\frac{Mm}{r^2}$$

where F is the gravitational force, M and m are masses, and r is a length. Force has the SI units $\text{kg} \cdot \text{m}/\text{s}^2$. What are the SI units of the proportionality constant G ?

6. **Q.C** Kinetic energy KE (Topic 5) has dimensions $\text{kg} \cdot \text{m}^2/\text{s}^2$. It can be written in terms of the momentum p (Topic 6) and mass m as

$$KE = \frac{p^2}{2m}$$

(a) Determine the proper units for momentum using dimensional analysis. (b) Refer to Problem 5. Given the units of force, write a simple equation relating a constant force F exerted on an object, an interval of time t during which the force is applied, and the resulting momentum of the object, p .

1.4 Uncertainty in Measurement and Significant Figures

7. A rectangular airstrip measures 32.30 m by 210 m, with the width measured more accurately than the length. Find the area, taking into account significant figures.

8. Use the rules for significant figures to find the answer to the addition problem $21.4 + 15 + 17.17 + 4.003$.
9. **M** A carpet is to be installed in a room of length 9.72 m and width 5.3 m. Find the area of the room retaining the proper number of significant figures.
10. **Q.C** Use your calculator to determine $(\sqrt{8})^3$ to three significant figures in two ways: (a) Find $\sqrt{8}$ to four significant figures; then cube this number and round to three significant figures. (b) Find $\sqrt{8}$ to three significant figures; then cube this number and round to three significant figures. (c) Which answer is more accurate? Explain.
11. How many significant figures are there in (a) 78.9 ± 0.2 , (b) 3.788×10^9 , (c) 2.46×10^{26} , (d) 0.003 2
12. The speed of light is now defined to be $2.997\,924\,58 \times 10^8$ m/s. Express the speed of light to (a) three significant figures, (b) five significant figures, and (c) seven significant figures.
13. A rectangle has a length of (2.0 ± 0.2) m and a width of (1.5 ± 0.1) m. Calculate (a) the area and (b) the perimeter of the rectangle, and give the uncertainty in each value.
14. The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, and give the uncertainty in each value.
15. The edges of a shoebox are measured to be 11.4 cm, 17.8 cm, and 29 cm. Determine the volume of the box retaining the proper number of significant figures in your answer.
16. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\,2 \times 356.3$; (c) the product $5.620 \times \pi$.

1.5 Unit Conversions for Physical Quantities

17. The Roman cubitus is an ancient unit of measure equivalent to about 0.445 m. Convert the 2.00-m height of a basketball forward to cubiti.
18. A house is advertised as having 1 420 square feet under roof. What is the area of this house in square meters?
19. A fathom is a unit of length, usually reserved for measuring the depth of water. A fathom is approximately 6 ft in length. Take the distance from Earth to the Moon to be 250 000 miles, and use the given approximation to find the distance in fathoms.
20. A small turtle moves at a speed of 186 furlongs per fortnight. Find the speed of the turtle in centimeters per second. Note that 1 furlong = 220 yards and 1 fortnight = 14 days.
21. A firkin is an old British unit of volume equal to 9 gallons. How many cubic meters are there in 6.00 firkins?
22. Find the height or length of these natural wonders in kilometers, meters, and centimeters: (a) The longest cave system in the world is the Mammoth Cave system in Central Kentucky, with a mapped length of 348 miles. (b) In the United States, the waterfall with the greatest single drop is Ribbon Falls in California, which drops 1 612 ft. (c) At 20 320 feet, Mount McKinley in Alaska is America's highest mountain. (d) The deepest canyon in the United States is King's Canyon in California, with a depth of 8 200 ft.

23. A car is traveling at a speed of 38.0 m/s on an interstate highway where the speed limit is 75.0 mi/h. Is the driver exceeding the speed limit? Justify your answer.
24. A certain car has a fuel efficiency of 25.0 miles per gallon (mi/gal). Express this efficiency in kilometers per liter (km/L).
25. The diameter of a sphere is measured to be 5.36 in. Find (a) the radius of the sphere in centimeters, (b) the surface area of the sphere in square centimeters, and (c) the volume of the sphere in cubic centimeters.
26. **AV BIO** Suppose your hair grows at the rate of 1/32 inch per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly atoms are assembled in this protein synthesis.
27. The speed of light is about 3.00×10^8 m/s. Convert this figure to miles per hour.
28. **PI** A house is 50.0 ft long and 26 ft wide and has 8.0-ft-high ceilings. What is the volume of the interior of the house in cubic meters and in cubic centimeters?
29. The amount of water in reservoirs is often measured in acre-ft. One acre-ft is a volume that covers an area of one acre to a depth of one foot. An acre is 43 560 ft². Find the volume in SI units of a reservoir containing 25.0 acre-ft of water.
30. The base of a pyramid covers an area of 13.0 acres (1 acre = 43 560 ft²) and has a height of 481 ft (Fig. P1.30). If the volume of a pyramid is given by the expression $V = bh/3$, where b is the area of the base and h is the height, find the volume of this pyramid in cubic meters.



Figure P1.30

31. A quart container of ice cream is to be made in the form of a cube. What should be the length of a side, in centimeters? (Use the conversion 1 gallon = 3.786 liter.)

1.6 Estimates and Order-of-Magnitude Calculations

Note: In developing answers to the problems in this section, you should state your important assumptions, including the numerical values assigned to parameters used in the solution.

32. Estimate the number of steps you would have to take to walk a distance equal to the circumference of the Earth.
33. **V BIO** Estimate the number of breaths taken by a human being during an average lifetime.
34. **BIO** Estimate the number of people in the world who are suffering from the common cold on any given day. (Answers

may vary. Remember that a person suffers from a cold for about a week.)

35. The habitable part of Earth's surface has been estimated to cover 60 trillion square meters. Estimate the percent of this area occupied by humans if Earth's current population stood packed together as people do in a crowded elevator.
36. **BIO** Treat a cell in a human as a sphere of radius 1.0 μm . (a) Determine the volume of a cell. (b) Estimate the volume of your body. (c) Estimate the number of cells in your body.
37. An automobile tire is rated to last for 50 000 miles. Estimate the number of revolutions the tire will make in its lifetime.
38. **BIO** A study from the National Institutes of Health states that the human body contains trillions of microorganisms that make up 1% to 3% of the body's mass. Use this information to estimate the average mass of the body's approximately 100 trillion microorganisms.

1.7 Coordinate Systems

39. **PI** A point is located in a polar coordinate system by the coordinates $r = 2.5$ m and $\theta = 35^\circ$. Find the x - and y -coordinates of this point, assuming that the two coordinate systems have the same origin.
40. A certain corner of a room is selected as the origin of a rectangular coordinate system. If a fly is crawling on an adjacent wall at a point having coordinates (2.0, 1.0), where the units are meters, what is the distance of the fly from the corner of the room?
41. Express the location of the fly in Problem 40 in polar coordinates.
42. **V** Two points in a rectangular coordinate system have the coordinates (5.0, 3.0) and (-3.0, 4.0), where the units are centimeters. Determine the distance between these points.
43. Two points are given in polar coordinates by $(r, \theta) = (2.00$ m, $50.0^\circ)$ and $(r, \theta) = (5.00$ m, $-50.0^\circ)$, respectively. What is the distance between them?
44. **S** Given points (r_1, θ_1) and (r_2, θ_2) in polar coordinates, obtain a general formula for the distance between them. Simplify it as much as possible using the identity $\cos^2 \theta + \sin^2 \theta = 1$. *Hint:* Write the expressions for the two points in Cartesian coordinates and substitute into the usual distance formula.

1.8 Trigonometry Review

45. **PI** For the triangle shown in Figure P1.45, what are (a) the length of the unknown side, (b) the tangent of θ , and (c) the sine of ϕ ?

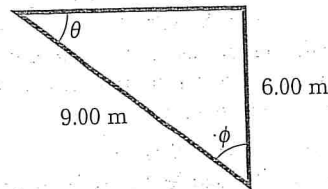


Figure P1.45

46. A ladder 9.00 m long leans against the side of a building. If the ladder is inclined at an angle of 75.0° to the horizontal, what is the horizontal distance from the bottom of the ladder to the building?

47. A high fountain of water is located at the center of a circular pool as shown in Figure P1.47. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation at the bottom of the fountain to be 55.0° . How high is the fountain?

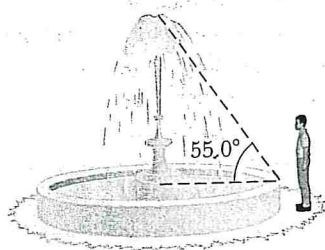


Figure P1.47

48. A right triangle has a hypotenuse of length 3.00 m, and one of its angles is 30.0° . What are the lengths of (a) the side opposite the 30.0° angle and (b) the side adjacent to the 30.0° angle?
49. In Figure P1.49, find (a) the side opposite θ , (b) the side adjacent to ϕ , (c) $\cos \theta$, (d) $\sin \phi$, and (e) $\tan \phi$.

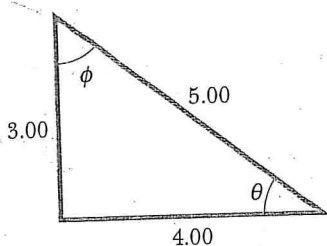


Figure P1.49

50. In a certain right triangle, the two sides that are perpendicular to each other are 5.00 m and 7.00 m long. What is the length of the third side of the triangle?
51. In Problem 50, what is the tangent of the angle for which 5.00 m is the opposite side?
52. A woman measures the angle of elevation of a mountaintop as 12.0° . After walking 1.00 km closer to the mountain on level ground, she finds the angle to be 14.0° . Find the mountain's height, neglecting the height of the woman's eyes above the ground. *Hint:* Distances from the mountain (x and $x - 1$ km) and the mountain's height are unknown. Draw two triangles, one for each of the woman's locations, and equate expressions for the mountain's height. Use that expression to find the first distance x from the mountain and substitute to find the mountain's height.
53. A surveyor measures the distance across a straight river by the following method: starting directly across from a tree on the opposite bank, he walks $x = 1.00 \times 10^2$ m along the riverbank to establish a baseline. Then he sights across to the tree. The angle from his baseline to the tree is $\theta = 35.0^\circ$ (Fig. P1.53). How wide is the river?

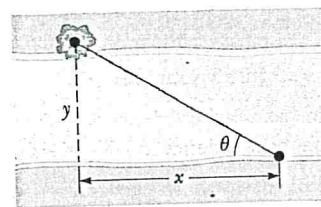


Figure P1.53

1.9 Vectors

54. Vector \vec{A} has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x -axis. Vector \vec{B} also has a magnitude of 8.00 units and is directed along the negative x -axis. Using graphical methods, find (a) the vector sum $\vec{A} + \vec{B}$ and (b) the vector difference $\vec{A} - \vec{B}$.
55. Vector \vec{A} has a magnitude of 29 units and points in the positive y -direction. When vector \vec{B} is added to \vec{A} , the resultant vector $\vec{A} + \vec{B}$ points in the negative y -direction with a magnitude of 14 units. Find the magnitude and direction of \vec{B} .
56. An airplane flies 2.00×10^2 km due west from city A to city B and then 3.00×10^2 km in the direction of 30.0° north of west from city B to city C. (a) In straight-line distance, how far is city C from city A? (b) Relative to city A, in what direction is city C? (c) Why is the answer only approximately correct?
57. Vector \vec{A} is 3.00 units in length and points along the positive x -axis. Vector \vec{B} is 4.00 units in length and points along the negative y -axis. Use graphical methods to find the magnitude and direction of the vectors (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.
58. A force \vec{F}_1 of magnitude 6.00 units acts on an object at the origin in a direction $\theta = 30.0^\circ$ above the positive x -axis (Fig. P1.58). A second force \vec{F}_2 of magnitude 5.00 units acts on the object in the direction of the positive y -axis. Find graphically the magnitude and direction of the resultant force $\vec{F}_1 + \vec{F}_2$.

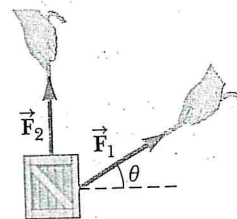


Figure P1.58

59. A roller coaster moves 2.00×10^2 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. Next, it travels 135 ft at an angle of 40.0° below the horizontal. Use graphical techniques to find the roller coaster's displacement from its starting point to the end of this movement.

1.10 Components of a Vector

60. Calculate (a) the x -component and (b) the y -component of the vector with magnitude 24.0 m and direction 56.0° .



61. A vector \vec{A} has components $A_x = -5.00$ m and $A_y = 9.00$ m. Find (a) the magnitude and (b) the direction of the vector.
62. A person walks 25.0° north of east for 3.10 km. How far due north and how far due east would she have to walk to arrive at the same location?
63.  The magnitude of vector \vec{A} is 35.0 units and points in the direction 325° counterclockwise from the positive x -axis. Calculate the x - and y -components of this vector.
64. A figure skater glides along a circular path of radius 5.00 m. If she coasts around one half of the circle, find (a) her distance from the starting location and (b) the length of the path she skated.
65. A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her final position relative to her starting location? (b) What is the length of the path she walked?
66. A quarterback takes the ball from the line of scrimmage, runs backwards for 10.0 yards, and then runs sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a 50.0-yard forward pass straight downfield, perpendicular to the line of scrimmage. How far is the football from its original location?
67. A vector has an x -component of -25.0 units and a y -component of 40.0 units. Find the magnitude and direction of the vector.
68. A map suggests that Atlanta is 730. miles in a direction 5.00° north of east from Dallas. The same map shows that Chicago is 560. miles in a direction 21.0° west of north from Atlanta. Figure P1.68 shows the location of these three cities. Modeling the Earth as flat, use this information to find the displacement from Dallas to Chicago.



Figure P1.68

69.  The eye of a hurricane passes over Grand Bahama Island in a direction 60.0° north of west with a speed of 41.0 km/h. Three hours later the course of the hurricane suddenly shifts due north, and its speed slows to 25.0 km/h. How far from Grand Bahama is the hurricane 4.50 h after it passes over the island?
70. The helicopter view in Figure P1.70 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force a third person would have to exert on the mule to make the net

force equal to zero. The forces are measured in units of newtons (N).

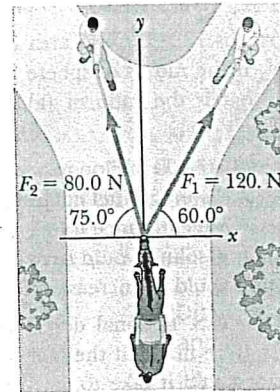


Figure P1.70

71. A commuter airplane starts from an airport and takes the route shown in Figure P1.71. The plane first flies to city A, located 175 km away in a direction 30.0° north of east. Next, it flies for 150. km 20.0° west of north, to city B. Finally, the plane flies 190. km due west, to city C. Find the location of city C relative to the location of the starting point.

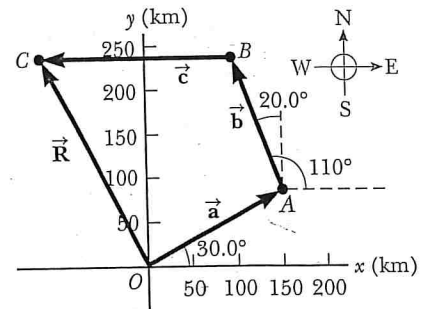







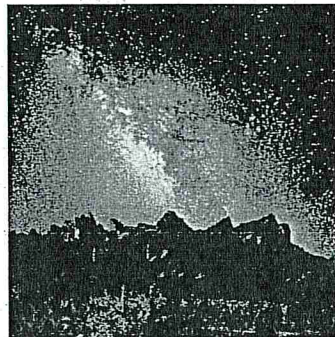
Figure P1.71

Additional Problems

72. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) For a while, federal law mandated that the maximum highway speed would be 55 mi/h. Use the conversion factor from part (a) to find the speed in kilometers per hour. (c) The maximum highway speed has been raised to 65 mi/h in some places. In kilometers per hour, how much of an increase is this over the 55-mi/h limit?
73. The displacement of an object moving under uniform acceleration is some function of time and the acceleration. Suppose we write this displacement as $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can the analysis give the value of k ?
74.  Assume it takes 7.00 minutes to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time interval, in hours, required to fill a 1.00-m^3 volume at the same rate. (1 U.S. gal = 231 in.³)

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75.  One gallon of paint (volume = $3.79 \times 10^{-3} \text{ m}^3$) covers an area of 25.0 m^2 . What is the thickness of the fresh paint on the wall?
76. A sphere of radius r has surface area $A = 4\pi r^2$ and volume $V = (4/3)\pi r^3$. If the radius of sphere 2 is double the radius of sphere 1, what is the ratio of (a) the areas, A_2/A_1 and (b) the volumes, V_2/V_1 ?
77.  Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?
78.  In 2015, the U.S. national debt was about \$18 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming that no interest were charged? (b) A dollar bill is about 15.5 cm long. If 18 trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the planet? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)
79. (a) How many Earths could fit inside the Sun? (b) How many of Earth's Moons could fit inside the Earth?
80.  An average person sneezes about three times per day. Estimate the worldwide number of sneezes happening in a time interval approximately equal to one sneeze.
81. The nearest neutron star (a collapsed star made primarily of neutrons) is about $3.00 \times 10^{18} \text{ m}$ away from Earth. Given that the Milky Way galaxy (Fig. P1.81) is roughly a disk of diameter $\sim 10^{21} \text{ m}$ and thickness $\sim 10^{19} \text{ m}$, estimate the number of neutron stars in the Milky Way to the nearest order of magnitude.



Richard Payne/NASA

Figure P1.81