

PreCalculus/Trig 3
Solving Trigonometric Equations - Notes

Name: _____
Date: _____

Review from Algebra:

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3, 1$$

$$\left(x = \frac{-1}{x} + 2\right)x$$

$$x^2 = -1 + 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

$$\frac{x+1}{x+2} = \frac{x+3}{2x+4}$$

$$\frac{x+1}{x+2} = \frac{x+3}{2(x+2)}$$

$$x \neq -2$$

$$2(x+1) = x+3$$

$$2x+2 = x+3$$

$$x = 1$$

Example 1 - All possible values.

$$\sin x + \sqrt{2} = -\sin x$$

$$2\sin x = -\sqrt{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4} + 2\pi n$$

$$\frac{7\pi}{4} + 2\pi n$$

Example 2 - All possible values.

$$3\tan^2 x - 1 = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Example 3 - All possible values.

$$\cot x \cos^2 x = 2 \cot x$$

$$\cot x \cos^2 x - 2 \cot x = 0$$

$$\cot x (\cos^2 x - 2) = 0$$

$$\cot x = 0 \quad \cos^2 x = 2$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 x = 2 \quad \cos x = \pm \sqrt{2}$$

$$\emptyset$$

$$\frac{\pi}{2} + n\pi$$

Example 4 - On the interval $[0, 2\pi)$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$$

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Example 5 - On the interval $[0, 2\pi)$

$$2\sin^2 x + 3\cos x - 3 = 0$$

$$2(1 - \cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$0 = 2\cos^2 x - 3\cos x + 1$$

$$0 = (2\cos x - 1)(\cos x - 1)$$

$$\cos x = \frac{1}{2} \quad \cos x = 1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, 0$$

Example 6 - On the interval $[0, 2\pi)$

$$(\cos x + 1)^2 = (\sin x)^2$$

$$\cos^2 x + 2\cos x + 1 = \sin^2 x$$

$$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$$

$$2\cos^2 x + 2\cos x = 0$$

$$2\cos x(\cos x + 1) = 0$$

$$\cos x = 0 \quad \cos x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \pi$$

Example 7 - On the interval $[0, 2\pi)$

$$2\cos 3x - 1 = 0$$

$$\cos 3x = \frac{1}{2}$$

$$3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}$$

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \frac{19\pi}{9}, \frac{23\pi}{9}$$

Example 8 - On the interval $[0, 2\pi)$

$$\sec^2 x - 2\tan x = 4$$

$$1 + \tan^2 x - 2\tan x = 4$$

$$\tan^2 x - 2\tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x = 3 \quad \tan x = -1$$

$$\text{QI: } 1.249$$

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$\text{QIII: } 4.391$$

PreCalculus/Trig 3
Solving Trigonometric Equations - Practice

Name: _____
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1. All possible values.

$$2\cos\theta + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3} + 2\pi n$$

$$\frac{4\pi}{3} + 2\pi n$$

2. All possible values.

$$2\sin^2\theta - \sin\theta = 0$$

$$\sin x (2\sin x - 1) = 0$$

$$\sin x = 0 \quad 2\sin x = \frac{1}{2}$$

~~$$x = 0, \pi$$~~

$$x = 2\pi n$$

$$\pi + 2\pi n$$

$$\frac{\pi}{6} + 2\pi n$$

$$\frac{5\pi}{6} + 2\pi n$$

3. On the interval $[0, 2\pi)$

$$2\sin\theta - 3\csc\theta + 5 = 0$$

$$\left(2\sin x - \frac{3}{\sin x} + 5 = 0\right) \sin x$$

$$2\sin^2 x - 3 + 5\sin x = 0$$

$$2\sin^2 x + 5\sin x - 3 = 0$$

~~$$(2\sin x + 1)(\sin x + 3)$$~~

$$(2\sin x - 1)(\sin x + 3) = 0$$

$$\sin x = \frac{1}{2} \quad \sin x = -3$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$\emptyset$

4. All possible values.

$$\cos\theta = 1 + 2\sec\theta$$

$$\left(\cos\theta \mp 1 + \frac{2}{\cos\theta}\right) \cos\theta$$

$$\cos^2\theta = \cos\theta + 2$$

$$\cos^2\theta - \cos\theta - 2 = 0$$

$$(\cos\theta - 2)(\cos\theta + 1) = 0$$

$$\cos\theta = 2$$

$$\cos\theta = -1$$

$\emptyset$

$$\pi + 2\pi n$$

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5. On the interval $[0, 2\pi)$

$$2\cos^2 \theta + \sin \theta - 1 = 0$$

$$2(1 - \sin \theta) + \sin \theta - 1 = 0$$

$$2 - 2\sin^2 \theta + \sin \theta - 1 = 0$$

$$0 = 2\sin^2 \theta - \sin \theta - 1$$

$$0 = (2\sin \theta + 1)(\sin \theta - 1)$$

$$\sin \theta = -\frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

6. On the interval $[0, 2\pi)$

$$\sin \theta + 2\csc \theta = 3$$

$$\left(\sin \theta + \frac{2}{\sin \theta} = 3 \right) \sin \theta$$

$$\sin^2 \theta + 2 = 3\sin \theta$$

$$\sin^2 \theta - 3\sin \theta + 2 = 0$$

$$(\sin \theta - 2)(\sin \theta - 1) = 0$$

$$\sin \theta = 2$$

$$\emptyset$$

$$\sin \theta = 1$$

$$\theta = \frac{\pi}{2}$$

7. On the interval $[0, 2\pi)$

$$(\cos \theta)^2 = (1 + \sin \theta)^2$$

$$\cos^2 \theta = 1 + 2\sin \theta + \sin^2 \theta$$

$$1 - \sin^2 \theta = 1 + 2\sin \theta + \sin^2 \theta$$

$$0 = 2\sin^2 \theta + 2\sin \theta$$

$$0 = 2\sin \theta (\sin \theta + 1)$$

$$\sin \theta = 0 \quad \sin \theta = -1$$

$$\theta = 0, \pi, \frac{3\pi}{2}$$

8. All possible values.

$$\tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{3} + \pi n$$

$$\frac{2\pi}{3} + \pi n$$