6.1: GRAPHS OF IDENTITIES

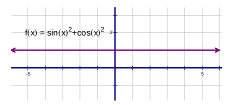
I. Verifying Trigonometric Identities

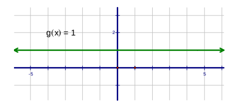
A. Example 1: consider $sin^2x + cos^2x = 1$.

1. Find the domain of the identity

All real numbers

2. Graph to verify that the identity is correct





I. Verifying Trigonometric Identities

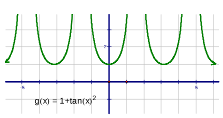
A. Example 1: consider $1 + tan^2x = sec^2x$.

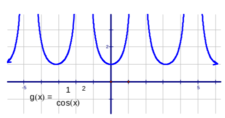
1. Find the domain of the identity

All real numbers, except where $cosx = 0$

$\mathbb{R}, x \neq \cdots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

2. Graph to verify that the identity is correct





I. Verifying Trigonometric Identities

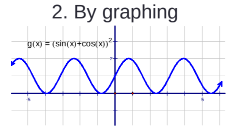
C. Example 3: Decide if $(sinx + cosx)^2 = 1$ is a valid identity.

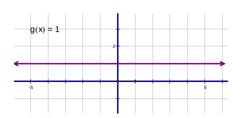
1. By trying a test point:

$$\left(sin\left(\frac{\pi}{4}\right) + cos\left(\frac{\pi}{4}\right)\right)^2 = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)^2$$

$$= \left(\frac{2\sqrt{2}}{2}\right)^2 = \left(\sqrt{2}\right)^2 = 2 \neq 1$$

2. By graphing





I. Verifying Trigonometric Identities

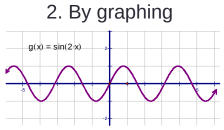
C. Example 3: Decide if $\sin 2x = 2sinxcosx$ is a valid identity.

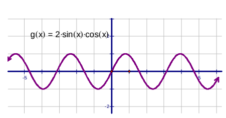
1. By trying a test point:

$$sin\left(2\left(\frac{\pi}{3}\right)\right) = sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$2sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

2. By graphing





II. Homework

6.1 Worksheet

6.2: Proving Identities

I. Proving Identities

A. Prove the following identities and state their domains.

1. $1 + tan^2x = sec^2x$

$$1 + tan^2x = 1 + \frac{sin^2x}{cos^2x}$$

$$1 + tan^2x = \frac{cos^2x}{cos^2x} + \frac{sin^2x}{cos^2x}$$

$$1 + tan^2x = \frac{cos^2x + sin^2x}{cos^2x}$$

$$1 + tan^2x = \frac{1}{cos^2x}$$

$$1 + tan^2x = sec^2x$$

Domain: $\mathbb{R}, x \neq \left\{\dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots\right\}$

I. Proving Identities

2. $tanxcosx = sinx$

$$tanxcosx = \frac{sinx}{cosx} \cdot \frac{cosx}{1}$$

$$tanxcosx = \frac{sinxcosx}{cosx}$$

$$tanxcosx = sinx$$

Domain: $\mathbb{R}, x \neq \left\{\dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots\right\}$

I. Proving Identities

3. $cscxsecxcotx = csc^2x$

$$cscxsecxcotx = \frac{1}{sinx} \cdot \frac{1}{cosx} \cdot \frac{cosx}{sinx}$$

$$cscxsecxcotx = \frac{cosx}{cosx \cdot sin^2x}$$

$$cscxsecxcotx = \frac{1}{sin^2x}$$

$$cscxsecxcotx = csc^2x$$

Domain: $\mathbb{R}, x \neq \left\{\dots, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \dots\right\}$

I. Proving Identities

4. $(sinx + cosx)^2 = 1 + 2sinxcosx$

$$(sinx + cosx)^2 = sin^2x + 2sinxcosx + cos^2x$$

$$(sinx + cosx)^2 = (sin^2x + cos^2x) + 2sinxcosx$$

$$(sinx + cosx)^2 = 1 + 2sinxcosx$$

Domain: $\mathbb{R}$

II. Homework

6.2 Worksheet

6.4-6.5: SUM AND DIFFERENCE FORMULAS

I. COSINE OF A SUM OR DIFFERENCE

A. Cosine of a Sum Formula

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta$$

B. Cosine of a Difference Formula

$$\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta$$

C. Combined

$$\cos(\alpha\pm\beta)=\cos\alpha\cdot\cos\beta\mp\sin\alpha\cdot\sin\beta$$

I. COSINE OF A SUM OR DIFFERENCE

D. Example 1: Evaluate $\cos\frac{\pi}{12}$

First, note $\frac{\pi}{12}=\frac{4\pi}{12}-\frac{3\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cdot\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\cdot\sin\frac{\pi}{4}$$
$$=\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$$
$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

I. COSINE OF A SUM OR DIFFERENCE

E. Verify the following identity:

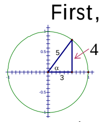
$$\cos\left(\frac{\pi}{2}-x\right)=\sin x$$

$$\cos\left(\frac{\pi}{2}-x\right)=\cos\frac{\pi}{2}\cdot\cos x+\sin\frac{\pi}{2}\sin x$$
$$=(0)\cos x+(1)\sin x$$
$$=\sin x$$

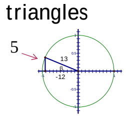
I. COSINE OF A SUM OR DIFFERENCE

F. Evaluate $\cos(\alpha+\beta)$ given that $\cos\alpha=\frac{3}{5}$ $\left(0<\alpha<\frac{\pi}{2}\right)$, and $\cos\beta=-\frac{12}{13}\left(\frac{\pi}{2}<\beta<\pi\right)$.

First, reference triangles



$\sin\alpha=\frac{4}{5}$



$\sin\beta=\frac{5}{13}$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta$$
$$=\left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)-\left(\frac{4}{5}\right)\left(\frac{5}{13}\right)=\frac{-56}{65}$$

II. SINE OF A SUM OR DIFFERENCE

A. Formula:

$$\sin(\alpha\pm\beta)=\sin\alpha\cdot\cos\beta\pm\cos\alpha\cdot\sin\beta$$

B. Example 4: evaluate $\sin 75°$

$$\sin 75°=\sin(45°+30°)$$
$$\sin 75°=\sin 45°\cos 30°+\cos 45°\sin 30°$$
$$\sin 75°=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$\sin 75°=\frac{\sqrt{6}+\sqrt{2}}{4}$$

II. SINE OF A SUM OR DIFFERENCE

C. Evaluate

$$sin(5°)cos(-50°) + cos(5°)sin(-50°)$$
$$= sin(5° + -50°)$$
$$= sin(-45°)$$
$$= -\frac{\sqrt{2}}{2}$$

III. TANGENT OF A SUM OR DIFFERENCE

A. Formula:

$$tan(\alpha \pm \beta) = \frac{tan\alpha \pm tan\beta}{1 \mp tan\alpha \cdot tan\beta}$$

Or

$$tan(\alpha \pm \beta) = \frac{sin(\alpha \pm \beta)}{cos(\alpha \pm \beta)}$$

B. Example 5: Simplify $tan(\pi - \theta)$

$$tan(\pi - \theta) = \frac{tan\pi - tan\theta}{1 + tan\pi \cdot tan\theta}$$
$$tan(\pi - \theta) = \frac{0 - tan\theta}{1 - (0)tan\theta} = -tan\theta$$

IV. HOMEWORK

WS Page 10 in packet (#1-6)

6.6: DOUBLE ANGLE FORMULAS

I. Double angle formulas

A. Derivation:

$$\sin 2\theta = \sin(\theta + \theta)$$

$$\sin 2\theta = \sin\theta \cdot \cos\theta + \cos\theta \cdot \sin\theta$$

$$\sin 2\theta = \sin\theta \cdot \cos\theta + \sin\theta \cdot \cos\theta$$

$$\sin 2\theta = 2\sin\theta \cdot \cos\theta$$

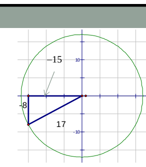
I. Double Angle Formulas

$$\sin 2\theta = 2\sin\theta \cdot \cos\theta$$

$$\cos 2\theta = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

II. Double angle formulas



B. Example 1: Given $\sin\theta = -\frac{8}{17}$ and $\cos\theta < 0$, evaluate each of the following

$\cos\theta = -\frac{15}{17}$

$\tan\theta = \frac{8}{15}$

1. $\sin 2\theta$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{8}{17}\right)\left(-\frac{15}{17}\right) = \frac{240}{289}$$

2. $\cos 2\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{15}{17}\right)^2 - \left(-\frac{8}{17}\right)^2 = \frac{161}{289}$$

3. $\tan 2\theta$

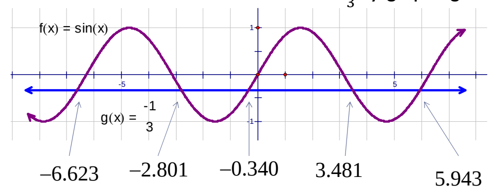
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(\frac{8}{15}\right)}{1 - \left(\frac{8}{15}\right)^2} = \frac{240}{161}$$

6-7: Solving Trig Equations and Inequalities

Part I

I. Solving Equations

A. Ex 1:Find all solutions to $sinx = -\frac{1}{3}$ by graphing.



$$Q_{III}: 3.4801 + 2\pi n$$
$$Q_{IV}: 5.943 + 2\pi n$$

I. Solving Equations

B. Ex 2: Find all solutions to the following equations

1. $cosx = \frac{1}{2}$

$$Q_I: \frac{\pi}{3} + 2\pi n$$
$$Q_{IV}: \frac{5\pi}{3} + 2\pi n$$

2. $cosx = \frac{1}{4}$

$$Q_I: 1.318 + 2\pi n$$
$$Q_{IV}: 4.965 + 2\pi n$$

I. Solving Equations

3. $tanx = 0.573$

$$Q_I: 0.520 + 2\pi n$$
$$Q_{III}: 3.662 + 2\pi n$$

Or

$$0.520 + \pi n$$

4. $secx = -2$

$$cosx = -\frac{1}{2}$$
$$Q_2: \frac{2\pi}{3} + 2\pi n$$
$$Q_{III}: \frac{4\pi}{3} + 2\pi n$$

I. Solving Equations

B. Example 3: Solve $4sin^2x - 1 = 0$

I. Over the interval $0 \le x < 2\pi$

$$4sin^2x - 1 = 0$$
$$sin^2x = \frac{1}{4}$$
$$sinx = \pm\frac{1}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

II. Over the set of all real numbers

$$x = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$$

II. Homework:

Trigonometric Equations Worksheet #1-3

6-7: Solving Trig Equations and Inequalities

Part II

I. Solving Equations With Identities

A. Example 1: Find the solution set for each equation (all real solutions)

1. $cos^2x + sinx + 1 = 0$

$$cos^2x + sinx + 1 = 0$$
$$1 - sin^2x + sinx + 1 = 0$$
$$0 = sin^2x - sinx - 2$$
$$0 = (sinx - 2)(sinx + 1)$$
$$sinx = 2 \qquad sinx = -1$$
$$sinx = \frac{3\pi}{2} + 2\pi n$$

I. Solving Equations With Identities

2. $cotx + csc^2x = 1$

$$cotx + 1 + cot^2x = 1$$
$$cot^2x + cotx = 0$$
$$cotx(cotx + 1) = 0$$
$$cotx = 0 \qquad cotx = -1$$
$$tanx = \emptyset \qquad tanx = -1$$
$$x = \frac{\pi}{2} + 2\pi n, \frac{3\pi}{4} + 2\pi n, \frac{3\pi}{2} + 2\pi n, \frac{7\pi}{4} + 2\pi n$$

I. Solving Equations With Identities

3. $sinx \cdot cosx = \frac{\sqrt{3}}{4}$

$$2(sinx \cdot cosx) = 2\left(\frac{\sqrt{3}}{4}\right)$$
$$2sinx \cdot cosx = \frac{\sqrt{3}}{2}$$
$$sin2x = \frac{\sqrt{3}}{2}$$
$$2x = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n$$
$$x = \frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n$$

I. Solving Equations With Identities

4. $\sqrt{3}cotx + 1 = csc^2x$

$$\sqrt{3}cotx + 1 = 1 + cot^2x$$
$$0 = cot^2x - \sqrt{3}cotx$$
$$0 = cotx(cotx - \sqrt{3})$$
$$cotx = 0 \qquad cotx = \sqrt{3}$$
$$tanx = \emptyset \qquad tanx = \frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{2} + n\pi, \frac{\pi}{6} + n\pi$$

II. Solving Inequalities

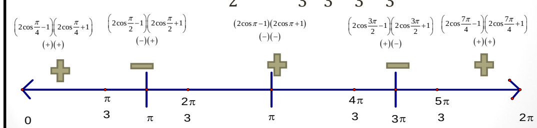
B. Example 2: Solve the following inequalities over the interval $0 \le x < 2\pi$

1. $4cos^2x - 1 \ge 0$

$$(2cosx - 1)(2cosx + 1) \ge 0$$

Critical points:

$$cosx = \pm\frac{1}{2} \to x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$



$$\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$$

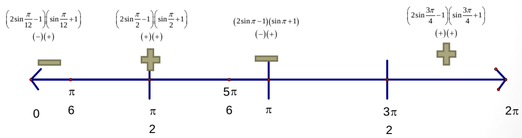
II. Solving Inequalities

2. $2sin^2x + sinx - 1 \geq 0$

$$(2sinx - 1)(sinx + 1) \geq 0$$

Critical points:

$$sinx = \frac{1}{2}, -1 \rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$



$$\left[\frac{\pi}{6}, \frac{5\pi}{6}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$$

III. Homework

Trigonometric Equations Worksheet #1-3