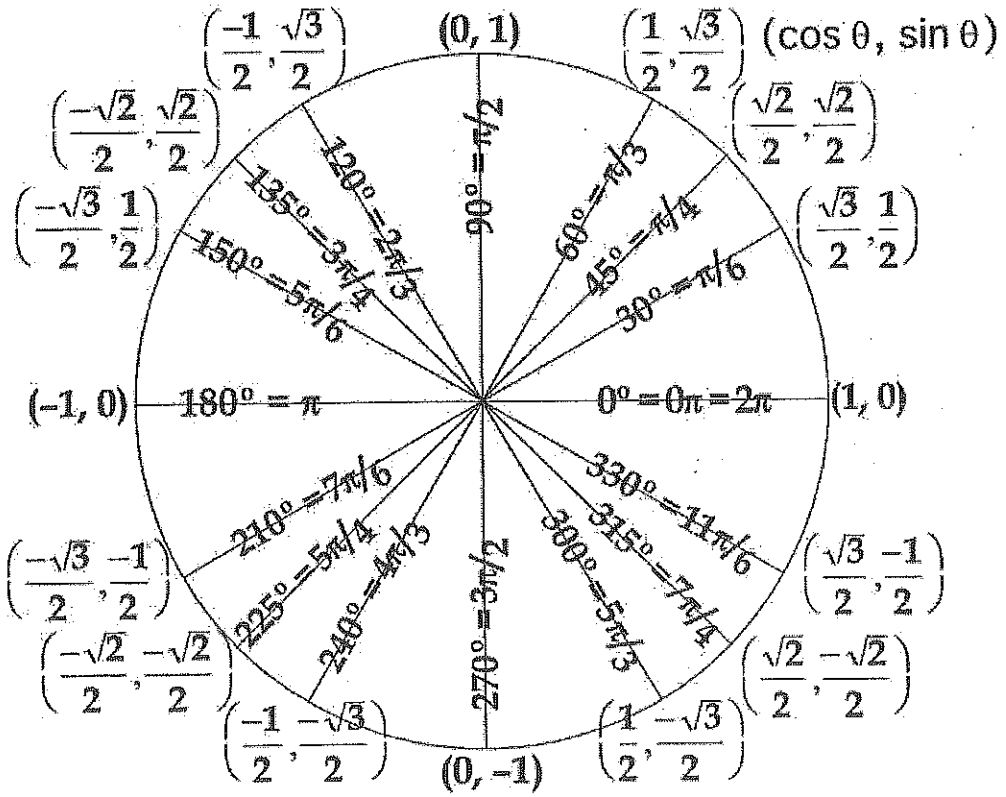
Name: _____ Precalculus II and Discrete Math

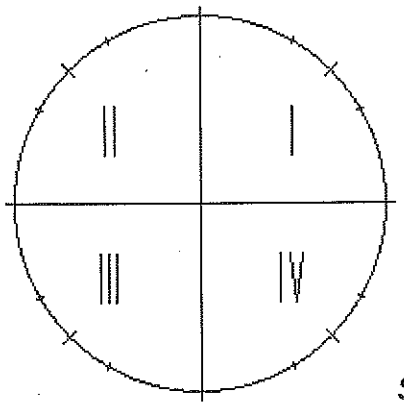
Trig Functions

Upon completion of this packet you will be able to:

- ✓ Define the trigonometric functions of any angle.
- ✓ Apply the definitions of trig functions to solve problems.
- ✓ Know the signs (+/-) of the trigonometric functions in each quadrant.
- ✓ Find a reference angle.
- ✓ Convert angle measures between degrees and radians.
- ✓ Graph the sine and cosine functions.
- ✓ Identify the domain, range, amplitude and period of sine and cosine functions.
- ✓ Describe the maximum/minimum values and end behavior of sine and cosine functions and identify the intervals where the sine and cosine graphs increase/decrease.
- ✓ Graph and describe transformations of the sine and cosine functions.
- ✓ Apply the Law of Sines and Law of Cosines to solve problems.

The Unit Circle: $x^2 + y^2 = r^2$ where r = 1 , $x^2 + y^2 = 1$

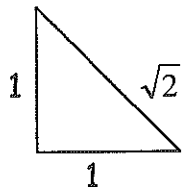




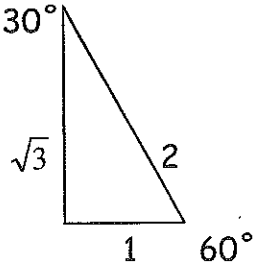
SIMPLIFYING THE UNIT CIRCLE!!!

In each quadrant above, label the trig functions that are positive.

Use reference angles and the relationships of special right triangles to simplify identifying exact values of trig functions.



45° $\left(\frac{\pi}{4}\right)$ triangle or ref. angle



30° $\left(\frac{\pi}{6}\right)$ or 60° $\left(\frac{\pi}{3}\right)$ triangle or ref. angle

Angle Conversions: To convert radians to degrees, multiply by $\left(\frac{180}{\pi}\right)$

To convert degrees to radians, multiply by $\left(\frac{\pi}{180}\right)$

Degrees and Radians

Convert each of the following to radians.

1. 210° $\frac{7\pi}{6}$

2. $-225°$ $-\frac{5\pi}{4}$

3. 450° $\frac{5\pi}{2}$

Convert each of the following to degrees.

4. $\frac{2\pi}{3}$ 120°

5. $\frac{\pi}{12}$ 15°

6. $\frac{7\pi}{18}$ 70°

Complete the following chart:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
degrees	0	30°	45°	60°	90°	180°	270°	360°
sin t	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos t	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tan t	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Ø	0	Ø	0
cot t	Ø	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Ø	0	Ø
sec t	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Ø	-1	Ø	1
csc t	Ø	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	0	Ø	-1	Ø

Determine the quadrant in which the terminal side lies for each angle.

1. $\frac{7\pi}{12}$ II
2. $-\frac{2\pi}{3}$ III
3. 371° I
4. $\frac{14\pi}{5}$ II
5. -156° III
6. 1000° IV
7. 332° IV
8. -240° II

Change each degree measure to radian measure in terms of π.

9. 36° $\frac{\pi}{5}$
10. -250° $-\frac{25\pi}{18}$
11. -145° $-\frac{29\pi}{36}$
12. 6° $\frac{\pi}{30}$
13. 870° $\frac{29\pi}{6}$
14. 18° $\frac{\pi}{10}$
15. -820° $-\frac{41\pi}{18}$
16. 345° $\frac{23\pi}{24}$

Change each radian measure to degree measure.

17. -1 $-\frac{180°}{\pi} \approx -57.296°$
18. 4π 720°
19. -2.56 -146.677°
20. 12.85 736.25°
21. $\frac{3\pi}{16}$ $\frac{135°}{4}$ 33.75°
22. $-\frac{7\pi}{9}$ -140°
23. $\frac{13\pi}{30}$ 78°
24. $-\frac{17\pi}{3}$ -1020°

Find one positive angle and one negative angle that are coterminal with each angle.

25. $70°$ $430°$ $-290°$ 26. $-\frac{2\pi}{5}$ $\frac{8\pi}{5}$ $-\frac{12\pi}{5}$ 27. $-300°$ $60°$ $-660°$ 28. $\frac{3\pi}{4}$ $\frac{11\pi}{4}$ $-\frac{9\pi}{4}$

Find the reference angle for each angle with the given measure.

29. $-20°$ $20°$ 30. $160°$ $20°$ 31. $-545°$ $5°$ 32. $300°$ $60°$

Determine the exact value of each of the following:

1. $\sin\frac{\pi}{6}$ $\frac{1}{2}$ 2. $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$ 3. $\tan\frac{\pi}{6}$ $\frac{\sqrt{3}}{3}$ 4. $\cos\frac{\pi}{2}$ 0

5. $\tan\frac{3\pi}{2}$ $\emptyset$ 6. $\cos\frac{5\pi}{6}$ $-\frac{\sqrt{3}}{2}$ 7. $\sin 0$ 0 8. $\tan\pi$ 0

9. $\sin\frac{7\pi}{4}$ $-\frac{\sqrt{2}}{2}$ 10. $\sin\left(-\frac{\pi}{6}\right)$ $-\frac{1}{2}$ 11. $\tan\left(-\frac{\pi}{3}\right)$ $-\sqrt{3}$ 12. $\sin 3\pi$ 0

13. $\cos\left(-\frac{3\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$ 14. $\cos\frac{5\pi}{2}$ 0 15. $\tan\left(-\frac{7\pi}{4}\right)$ 1 16. $\sin\frac{8\pi}{3}$ $\frac{\sqrt{3}}{2}$

17. $\cos\frac{19\pi}{6}$ $-\frac{\sqrt{3}}{2}$ 18. $\tan 4\pi$ 0 19. $\sin\frac{25\pi}{6}$ $\frac{1}{2}$ 20. $\cos\left(-\frac{7\pi}{3}\right)$ $\frac{1}{2}$

Find all the values of θ for $0 \le \theta \le 2\pi$ that make each of the following statements true.

21. $\cos\theta = \frac{\sqrt{2}}{2}$ $\frac{\pi}{4}, \frac{7\pi}{4}$ 22. $\sin\theta = 0$ $0, 2\pi$ 23. $\tan\theta = 1$ $\frac{\pi}{4}, \frac{5\pi}{4}$ 24. $\cos\theta = -\frac{1}{2}$ $\frac{2\pi}{3}, \frac{4\pi}{3}$

25. $\sin\theta = \frac{\sqrt{3}}{2}$ $\frac{\pi}{3}, \frac{2\pi}{3}$ 26. $\tan\theta = -\frac{\sqrt{3}}{3}$ $\frac{5\pi}{6}, \frac{11\pi}{6}$ 27. $\cos\theta = -1$ π 28. $\sin\theta = -\frac{\sqrt{2}}{2}$ $\frac{5\pi}{4}, \frac{7\pi}{4}$

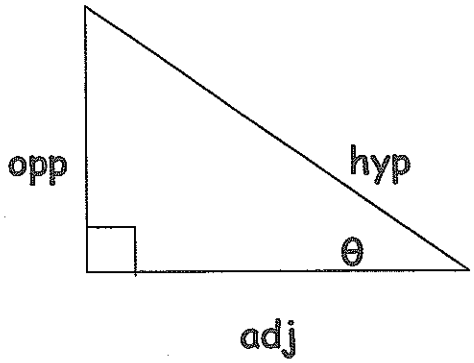
29. $\tan\theta = -\sqrt{3}$ $\frac{2\pi}{3}, \frac{5\pi}{3}$ 30. $\cos\theta = 0$ $\frac{\pi}{2}, \frac{3\pi}{2}$ 31. $\sin\theta = -\frac{1}{2}$ $\frac{7\pi}{6}, \frac{11\pi}{6}$ 32. $\tan\theta = 0$ $0, \pi$

Right Triangle Trigonometry (Reference Angles)

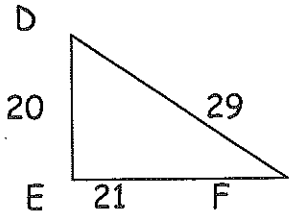
$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\quad \csc\theta = \frac{hyp}{opp} = \frac{r}{y}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\quad \sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ $\quad \cot\theta = \frac{adj}{opp} = \frac{x}{y}$

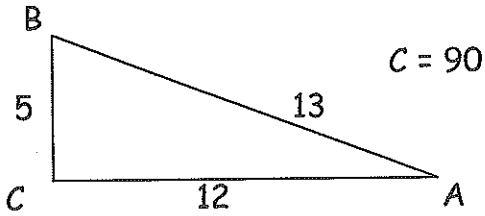


Use the right triangles provided to find each value.



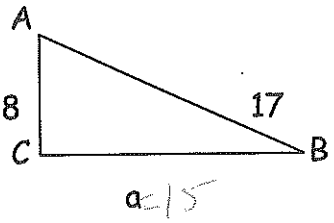
E = 90°

1. sin D 21/29
2. cos D 20/29
3. tan F 20/21
4. cos F 21/29



C = 90°

5. cos B 5/13
6. tan B 12/5
7. sin A 5/13
8. sec A 13/12

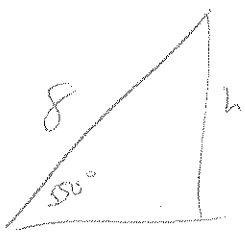


C = 90°

9. a 15
10. m∠A. 61.93°
11. m ∠B. 28.07°

12. A string which runs form the top of a tent to a stake in the ground makes an angle of 50° with the ground. If the string is 8 ft. long, how high is the top of the tent?

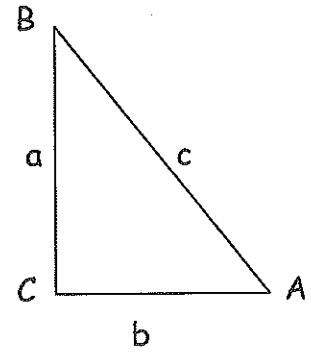
$\sin 50 = \frac{h}{8}$



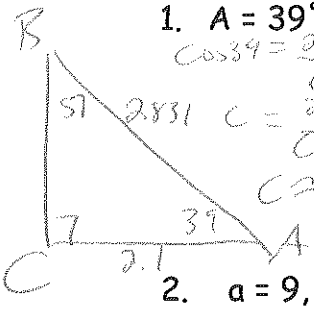
$8 \sin 50 = h$

$6.13 \text{ ft} \approx h$

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.



1. A = 39°, b = 2.1



$\cos 39 = \frac{2.1}{c}$

$c = \frac{2.1}{\cos 39}$

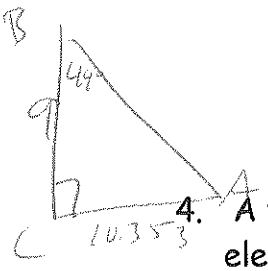
$c \approx 2.831$

$C \approx 2.831$

$B = 51°$

$a \approx 1.898$

2. a = 9, B = 49°

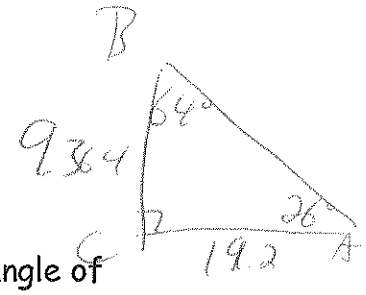


$B \approx 10.353$

$A \approx 41°$

$C \approx 13.718$

3. B = 64° b = 19.2

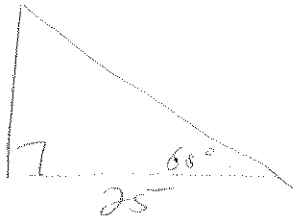


$A = 26°$

$a \approx 9.364$

$c \approx 21.362$

4. A tree casts a shadow 25 ft. long when the measurement of the angle of elevation of the sun is 68°. Find the height of the tree to the nearest foot.

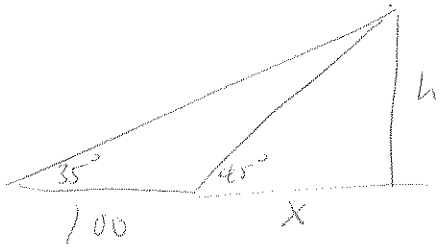


$\tan 68 = \frac{h}{25}$

$25 \tan 68 = h$

$h \approx 61.877 \text{ ft}$

5. The angle of elevation to the top of a building is 35°. 100 feet closer, the angle of elevation is 45°. Find the height of the building to the nearest foot.



$\tan 45 = \frac{h}{x}$

$\tan 35 = \frac{h}{100+x}$

$(100+x)\tan 35 = h$

$\tan 45 = x$

$(100+x)\tan 35 = \tan 45\, x$

$100 + x = \frac{\tan 45}{\tan 35} x$

$100 + x = 1.428x$

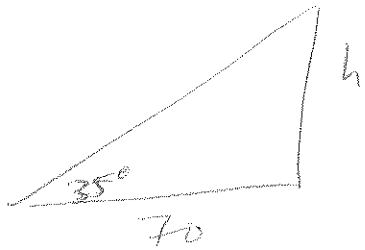
$100 = 0.428x$

$233.584 = x$

$\tan 45 = \frac{h}{233.584}$

$233.584 \text{ ft} \approx h$

6. Attempting to scale a vertical cliff, Jeanie wants to learn its height. From a certain point 70 feet from the base she knows that the angle of elevation is 35°. Find its height.



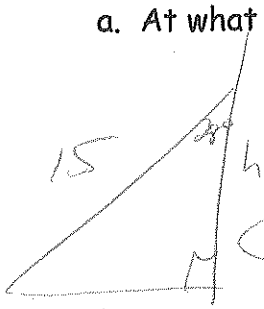
$\tan 35 = \frac{h}{70}$

$70 \tan 35 = h$

$49.015 \text{ ft} \approx h$

7. A 15 ft ladder is placed against a building so that it makes an angle of 28° with the building.

   a. At what height, to the nearest foot, does the ladder touch the building?



$\cos 28 = \frac{h}{15}$

$15 \cos 28 = h$

$13.24 \text{ Ft} \approx h$

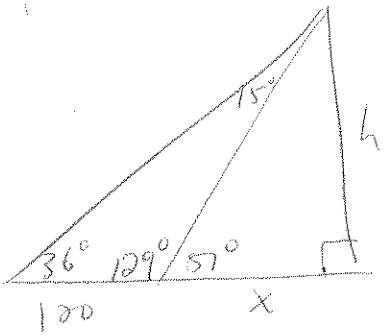
   b. How far from the base of the building is the foot of the ladder?

$\sin 28 = \frac{x}{15}$

$15 \sin 28 = x$

$7.042 \text{ Ft} \approx x$

8. John views the top of a water tower at an angle of elevation of 36°. He walks 120 meters in a straight line toward the tower. Then he sights the top of the tower at an angle of elevation of 51°. How far is John from the base of the tower?



$\tan 51 = \frac{h}{x}$

$x \tan 51 = h$

$\tan 36 = \frac{h}{120+x}$

$(120+x)\tan 36 = h$

$x \tan 51 = (120+x)\tan 36$

$\frac{\tan 51}{\tan 36} x = 120 + x$

$1.69964\, x = 120 + x$

$0.70x \approx 120$

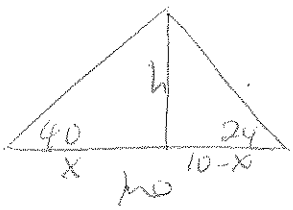
$x \approx 171.504$

$\tan 57 = \frac{h}{171.504}$

$211.79 \text{ Ft} \approx h$

211.79 Ft

9. Two cities are 10 miles apart. An object is seen hovering in the sky above the line joining the two cities. The angles of elevation to the object are 24° and 40°. What is the height of the object about the ground to the nearest tenth of a mile?



$\tan 40 = \frac{h}{x}$

$x \tan 40 = h$

$\tan 24 = \frac{h}{10-x}$

$(10-x)\tan 24 = h$

$x \tan 40 = (10-x)\tan 24$

$\frac{\tan 40}{\tan 24} x = 10 - x$

$1.88x \approx 10 - x$

$2.88x \approx 10$

$x \approx 3.47$

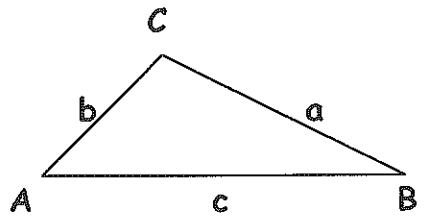
$\tan 40 = \frac{h}{3.47}$

$2.9 \text{ mi} \approx h$

Law of Sines

In any ΔABC with sides a, b and c opposite angles A, B and C;

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Use the Law of Sines to solve each triangle.

1. a = 13.7, A = 25° and B = 78°

$\frac{\sin 25}{13.7} = \frac{\sin 78}{b} = \frac{\sin}{c}$ 77 $C = 77°$

$b \approx 31.71°$ $c \approx 31.59°$

2. b = 795.1, c = 775.6, and B = 51°

$\frac{\sin A}{a} = \frac{\sin 51}{795.1} = \frac{\sin C}{775.6}$

$C \approx 49.30°$ $A \approx 79.70°$ $a \approx 1006.63$

~~130.70°~~

3. A = 38°, B = 63° and c = 15

$\frac{\sin 38}{a} = \frac{\sin 63}{b} = \frac{\sin 79}{15}$

$a \approx 9.41$ $C \approx 79°$

$b \approx 13.62$

4. b = 41, A = 33°, B = 29°

$\frac{\sin 33}{a} = \frac{\sin 29}{41} = \frac{\sin 118}{c}$

$a \approx 46.06$ $c \approx 74.67$ $C = 118°$

5. b = 15.2, A = 12°, C = 57°

$\frac{\sin 12}{a} = \frac{\sin 111}{15.2} = \frac{\sin 57}{c}$

$a \approx 3.39$ $c \approx 13.65$ $B = 111°$

6. a = 32, c = 20, and A = 112°

$\frac{\sin 112}{32} = \frac{\sin B}{b} = \frac{\sin C}{20}$

$C \approx 35.41°$ $B \approx$ ~~32.59~~ $32.59°$ $b \approx 18.59$

or

$C \approx$ ~~144.59°~~ $B =$ ~~[illegible]~~

7. a = 12.4, b = 16.2, A = 47°

Find m $\angle B$.

$\frac{\sin 47}{12.4} = \frac{\sin B}{16.2}$

$B \approx 72.84$ or $107.16°$

8. a = 2.33, B = 58°, C = 74°

Find b.

$\frac{\sin 48}{2.33} = \frac{\sin 58}{b} = \frac{\sin 74}{c}$

$b \approx 2.66$

9. A = 62°, B = 48°, c = 18

Find a.

$\frac{\sin 62}{a} = \frac{\sin 48}{b} = \frac{\sin 100}{18}$

$a \approx 16.14$

10. A = 70°, a = 10, b = 7

Find m $\angle B$.

$\frac{\sin 70}{10} = \frac{\sin B}{7}$

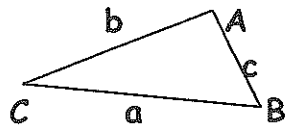
$B \approx 41.13°$ or ~~138.87°~~

Solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

Law of Cosines

In any ΔABC with sides a, b and c opposite angles A, B and C;

$$c^2 = a^2 + b^2 - 2ab\cos C$$



1. a = 1.5, b = 2.3, c = 1.9

~~A ≈ 51.4°~~
A ≈ 40.46°
B ≈ 84.26°
C ≈ 55.28°

2. b = 40, c = 45, A = 51°

a ≈ 36.87
B ≈ 57.47°
C ≈ 71.53°

3. A = 52°, b = 120, c = 160

a ≈ 127.90
B ≈ 47.67°
C ≈ 80.33°

4. a = 15, b = 18, c = 17

A ≈ 50.66
B ≈ 68.13
C ≈ 61.22

5. Find m∠A. a = 3, b = 7 and c = 8

A ≈ 21.79°

6. Find m ∠C. a = 18, b = 10, c = 21

C ≈ 92.71°

7. Find m ∠B. a = 5, b = 13, c = 12

B ≈ 90°

8. Find c. a = 10, b = 7, C = 38°

c ≈ 6.22

9. Find b. a = 17, B = 124°, c = 14

~~b ≈ 28.03~~ b ≈ 27.41°

10. Find a. A = 80°, b = 4, c = 5

a ≈ 5.84

Trigonometric Graphs: Sine and Cosine Functions

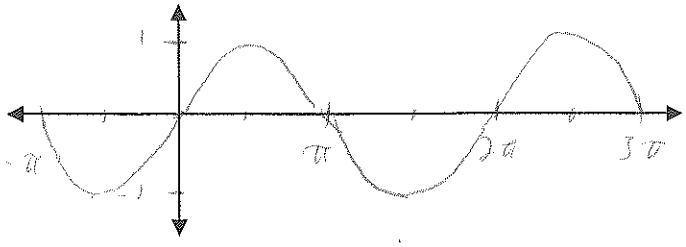
$y = a \sin b(x-c) - d$ and $y = a \cos b(x-c) - d$

a = amplitude b = period

c = phase shift d = vertical shift

1. The "Classic" Sine Function $y = \sin x$

a. Graph the function to the right and indicate following:



b. Domain: $\mathbb{R}$ $(-\infty, \infty)$

c. Range: $[-1, 1]$

d. Period: 2π

e. Amplitude: 1

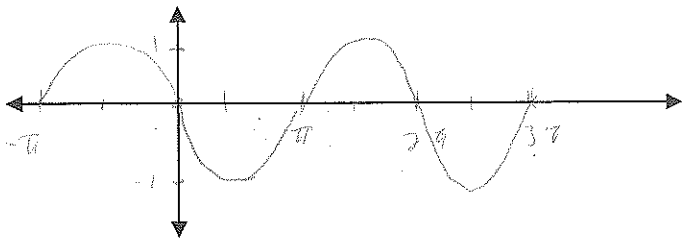
f. Maximum: 1 minimum: -1

g. Increasing: $(0, \frac{\pi}{2})$ $(\frac{3\pi}{2}, 2\pi)$ Decreasing: $(\frac{\pi}{2}, \frac{3\pi}{2})$

h. Special properties ODD

2. $y = -\sin x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-1, 1]$

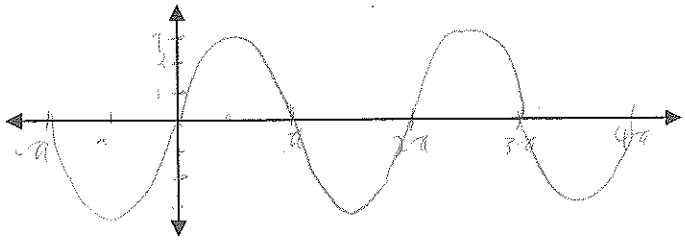
d. Period: 2π

e. Amplitude: 1

f. Describe how this graphed is transformed. (How is it different from the "classic sine function"?) REFLECTED OVER THE X-AXIS

3. $y = 3\sin x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-3, 3]$

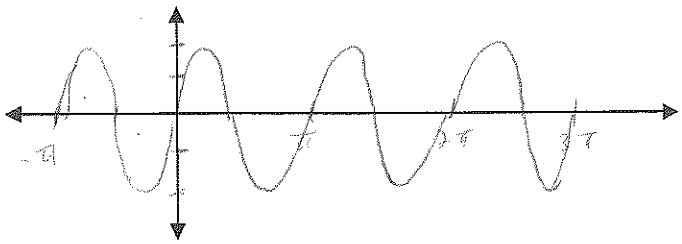
d. Period: 2π

e. Amplitude: 3

f. Describe how this graphed is transformed. VERTICAL STRETCH OF 3

4. $y = 2 \sin 2x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-2, 2]$

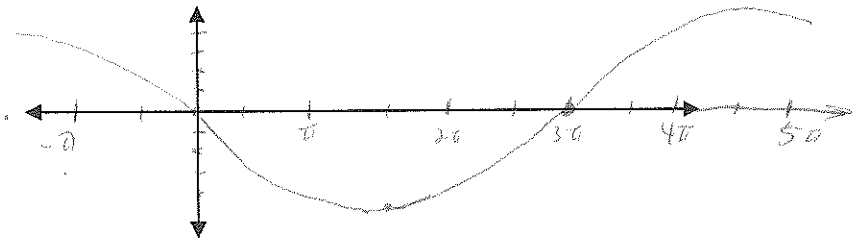
d. Period: π

e. Amplitude: 2

f. Describe how this graphed is transformed. STR VERTICALLY BY 2 / SHRUNK HORIZ BY 1/2

5. $y = -4 \sin (1/3)x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-4, 4]$

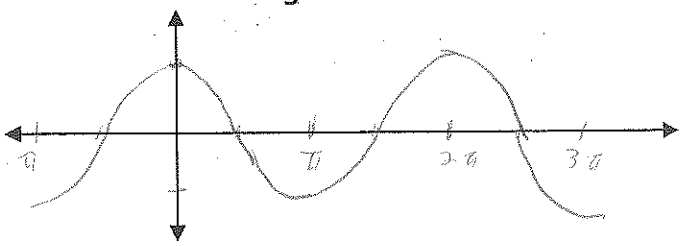
d. Period: 6π

e. Amplitude: 4

f. Describe how this graphed is transformed. REFL OVER X-AXIS, STRETCHED VERT BY 4, STR HORIZ BY 3

6. The "Classic" COSine Function $y = \cos x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-1, 1]$

d. Period: 2π

e. Amplitude: 1

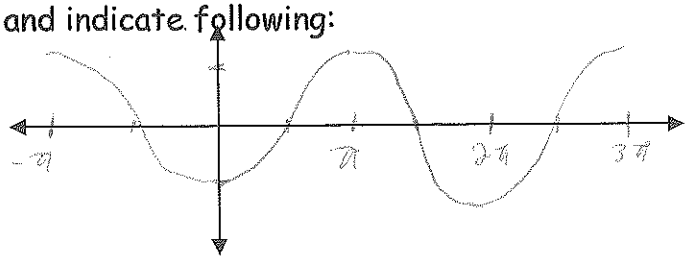
f. Maximum: 1 minimum: -1

g. Increasing: $(\pi, 2\pi)$ Decreasing: $(0, \pi)$

h. Special proper ties EVEN

7. $y = -\cos x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-1, 1]$

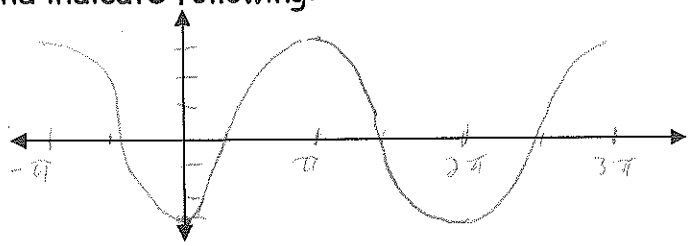
d. Period: 2π

e. Amplitude: 1

f. Describe how this graphed is transformed. (How is it different from the "classic cosine function"?) REFLECTED OVER THE X-AXIS

8. $y = -3 \cos x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-3, 3]$

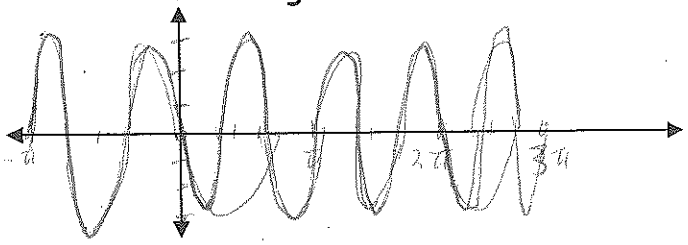
d. Period: 2π

e. Amplitude: 3

f. Describe how this graphed is transformed. REFL OVER THE X-AXIS STRETCHED BY 3 VERTICALLY

9. $y = -3 \cos 3x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-3, 3]$

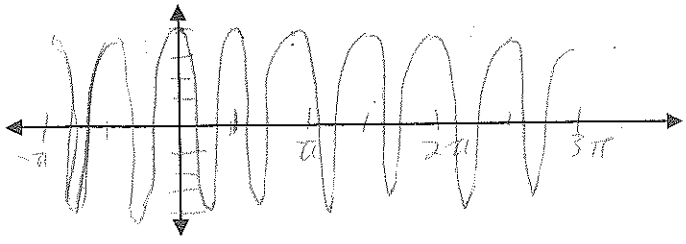
d. Period: $\frac{2\pi}{3}$

e. Amplitude: 3

f. Describe how this graphed is transformed. REFL OVER X-AXIS, STRETCHED VERT BY 3 SHRUNK HORIZ BY 3

10. $y = \frac{1}{2} \cos 4x$

a. Graph the function to the right and indicate following:



b. Domain: $(-\infty, \infty)$

c. Range: $[-\frac{1}{2}, \frac{1}{2}]$

d. Period: $\pi/2$

e. Amplitude: $1/2$

f. Describe how this graphed is transformed. SHRUNK VERT BY $1/2$ SHR HORIZ BY 4