

1. Find the average rate of change in $q(x) = x^2 + 12$ from $x = -1$ to $x = 4$.

$$q(-1) = 13$$

$$q(4) = 28$$

$$\frac{28-13}{4-(-1)} = \frac{15}{5} = 3$$

2. Find the average rate of change in $g(n) = 3n^3 - n^2 + 6$ over the interval $-2 \leq n \leq 2$.

$$g(-2) = -22$$

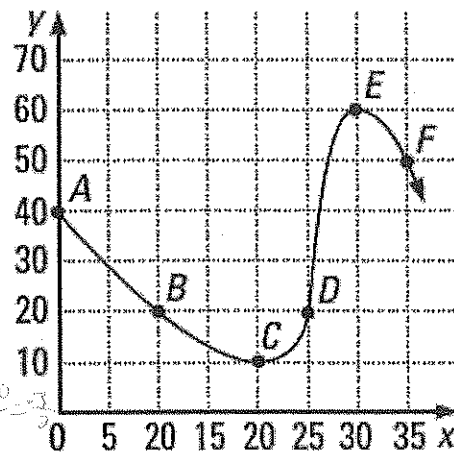
$$g(2) = 26$$

$$\frac{26 - (-22)}{2 - (-2)} = \frac{48}{4} = 12$$

3. Refer to the graph of g at the right. Find the average rate of change in g over each interval.

a. C to E $\frac{60-10}{30-20} = \frac{50}{10} = 5$

b. $0 \leq x \leq 35$ $\frac{50-40}{35-0} = \frac{10}{35} = \frac{2}{7}$



Over what interval does the average rate of change in g have the given value?

c. 0 B to D

d. $-\frac{3}{2}$ A to C

For Questions 4-6, find the derivative of the function at the given value of x .

4. $f(x) = 3x^2 + 4; x = 2$ $f'(x) = 6x$ $f'(2) = 12$

5. $g(x) = -12x + 8; x = .5$ $g'(x) = -12$ $g'(.5) = -12$

6. $h(x) = 22; x = 7$ $h'(x) = 0$ $h(7) = 22$

7. Let $f(x) = -2x^2 + x - 3$ $f'(x) = -4x + 1$

a. Compute $f'(0)$.

$$f'(0) = 1$$

b. Compute $f'(3)$.

$$f'(3) = -11$$

8. The height h in feet of a small rocket t seconds after launch is approximated by $h(t) = 320t - 16t^2$.

$$h'(t) = 320 - 32t$$

- a. Find the instantaneous velocity at time $t = 5$.
 b. Find the instantaneous velocity at time $t = 14$.
 c. Find the instantaneous velocity at time $t = 10$.
 d. At what time does the rocket reach its maximum height?

$$160 \frac{\text{ft}}{\text{s}} = h'(5)$$

$$-128 \frac{\text{ft}}{\text{s}} = h'(14)$$

$$0 \frac{\text{ft}}{\text{s}} = h'(10)$$

$$10 \text{ sec}$$

9. A pebble is dropped from a cliff 60 feet high. The height of the pebble in feet above the ground at time t seconds is given by $h(t) = -16t^2 + 60$.

$$h'(t) = -32t$$

- a. Find the instantaneous velocity of the pebble at time $t = 0.5$ second.
 b. At what time does the pebble hit the ground?
 c. Find the instantaneous velocity of the ball at the moment just before it hits the ground.

$$-16 \frac{\text{ft}}{\text{s}}$$

$$1.936 \text{ sec}$$

$$-61.968 \text{ ft/s} = h'(1.936)$$

$$0 = -16t^2 + 60$$

$$16t^2 = 60$$

$$t^2 = \frac{60}{16}$$

$$t = \pm \sqrt{\frac{60}{16}}$$

10. Refer to the graph of f at the right. Give a value of x for which $f'(x)$ is

a. positive.

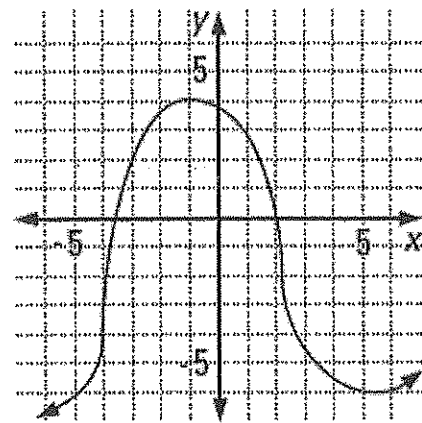
$$(-\infty, -1) \cup (5, \infty)$$

b. negative.

$$(-1, 5)$$

c. zero.

$$\{-1, 5\}$$



For questions 11-14, find the derivative of each function

11. $f(y) = 7y^2$

$$14y$$

12. $g(x) = 7x^2 - 3x$

$$14x - 3$$

13. $p(v) = -4.5v$

$$-4.5$$

14. $q(x) = 94$

$$0$$

15. A particle moves so that the distance s traveled in meters at time t seconds is given by $s(t) = t^2 + 5t - 4$.

- Find the average velocity between 3 and 4 seconds.
- Find the instantaneous velocity of the particle at time $t = 8$.
- What is the initial velocity of the particle (that is, at time $t = 0$ seconds)?

$$s(4) = 32$$

$$s(3) = 20$$

$$\frac{32-20}{4-3} = 12$$

$$s'(t) = 2t + 5$$

$$s'(8) = 16 + 5$$

$$s'(0) = 5$$

16. A particle moves horizontally so that its position in feet to the right of the starting point at time t seconds is given by $f(t) = -t^2 + 5t + 6$.

- At time $t = 8$ seconds, is the particle moving to the right, to the left, or stationary?
- What is the speed of the object at time $t = 8$ seconds?
- What is the acceleration of the object at time $t = 8$ seconds?
- Is the acceleration increasing, decreasing, or staying the same at time $t = 8$ seconds?

$$f'(8) < 0$$

LEFT

$$-11 \frac{\text{ft}}{\text{sec}}$$

$$-2$$

SAME

$$a(t) = f''(t) = -2 \text{ Always}$$

17. A ball is thrown directly upward. Its height h in meters after t seconds is given by the equation $h(t) = -4.9t^2 + 28t + 2$.

- Find the instantaneous velocity at each time.

$$h'(0) = 28$$

i. 0 seconds

$$28 \frac{\text{m}}{\text{s}}$$

$$h'(3) = -1.4$$

ii. 3 seconds

$$-1.4 \frac{\text{m}}{\text{s}}$$

$$a(t) = -9.8$$

- Find the instantaneous acceleration at each time.

i. 0 seconds

$$-9.8$$

ii. 3 seconds

$$-9.8$$

18. A rectangular pen adjacent to a shed is to be enclosed with 40 feet of fencing. What should the dimensions of the pen be in order to maximize the area?

$$2x + y = 40$$

$$y = 40 - 2x$$

$$A = x(40 - 2x)$$

$$A = 40x - 2x^2$$

$$A' = 40 - 4x$$

$$0 = 40 - 4x$$

$$10 \text{ ft} = x$$

$$y = 40 - 2(10) = 20 \text{ ft}$$

10 ft x 20 ft

19. Consider the function f graphed at the right.

a. On what interval(s) is $f'(x)$ positive?

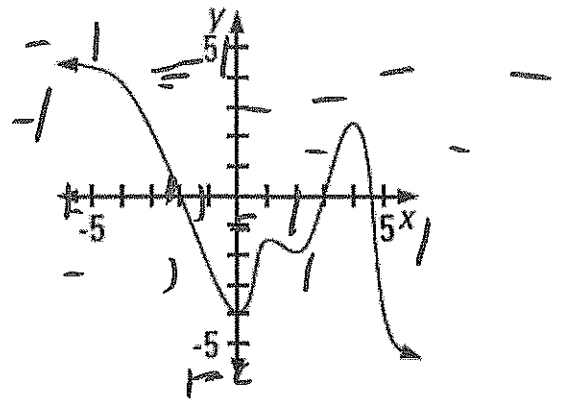
$(0, 1) \cup (3, 4)$

b. On what interval(s) is $f'(x)$ negative?

$(-\infty, 0) \cup (1, 2) \cup (4, \infty)$

c. For what values of x is $f'(x) = 0$?

$\{0, 1, 2, 4\}$



20. The derivative h' of a function h is graphed at the right. Describe the values of x where

a. h is increasing.

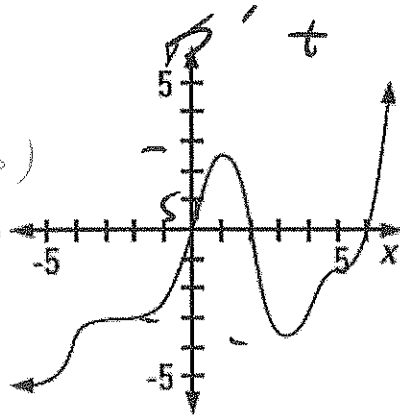
$(0, 2) \cup (6, \infty)$

b. h is decreasing.

$(-\infty, 0) \cup (2, 6)$

c. h has a relative maximum or minimum.

$\{0, 2, 6\}$



21. Suppose g is a function such that $g(x) = -\frac{2}{3}x^3 + 5x^2 - 12x$. Then $g'(x) = -2x^2 + 10x - 12$. Use the first derivative to find each.

a. The interval(s) on which g is increasing.

$(2, 3)$

b. The interval(s) on which g is decreasing.

$(-\infty, 2) \cup (3, \infty)$

c. The points at which g may have a relative maximum or minimum.

$\{2, 3\}$

$$\begin{aligned} -2x^2 + 10x - 12 &= 0 \\ -2(x^2 - 5x + 6) &= 0 \\ -2(x - 2)(x - 3) &= 0 \end{aligned}$$

