9.1 PART I: SECANTS

I. PARALLEL AND PERPENDICULAR LINES

A. Example 1: Write the equation of the line through *(4,3)* that is parallel to $y = \frac{3}{2}x + 1$.

$$y - 3 = \frac{3}{2}(x - 4)$$
$$y = \frac{3}{2}x - 3$$

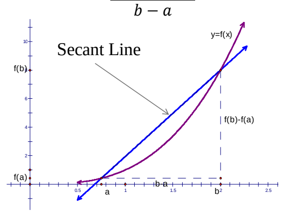
B. Example 2: Write the equation of the line through *(4,3)* that is perpendicular to $y = \frac{3}{2}x + 1$.

$$y - 3 = -\frac{2}{3}(x - 4)$$
$$y = -\frac{2}{3}x + \frac{17}{3}$$

II. AVERAGE RATE OF CHANGE

A. Definition: The Average Rate of Change for a function *f(x)* from *x=a* to *x=b* is:

$$\frac{f(b) - f(a)}{b - a}$$



II. AVERAGE RATE OF CHANGE

B. Example 1: Find the average rate of change on the function $f(x) = x^2 - 4x + 1$ on the interval $[2, 5]$

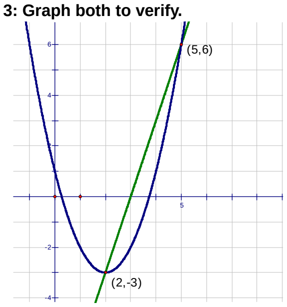
$$r.o.c. = \frac{f(5) - f(2)}{5 - 2} = \frac{6 - (-3)}{3} = 3$$

C. Example 2: Find the equation of the secant line between the points from example 1.

$$y - 6 = 3(x - 5)$$
$$y = 3x - 9$$

II. AVERAGE RATE OF CHANGE

C. Example 3: Graph both to verify.



III. PRACTICE

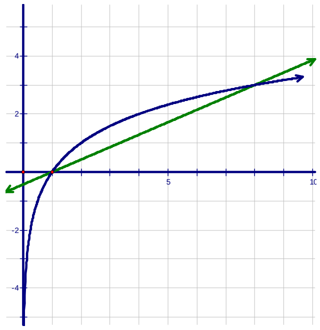
Example 4: Find the average rate of change of $g(x) = \log_2 8$ on $[1, 8]$, as well as the secant line.

$$r.o.c. = \frac{f(8) - f(1)}{8 - 1} = \frac{\log_2 8 - \log_2 1}{7} = \frac{3 - 0}{7} = \frac{3}{7}$$

Secant Line

$$y - 0 = \frac{3}{7}(x - 1)$$
$$y = \frac{3}{7}x - \frac{3}{7}$$

III. PRACTICE



HOMEWORK

9.1 Worksheet 1

9.1 Part II:

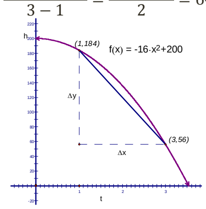
Instantaneous Rates of Change

I. Average vs. Instantaneous Rates of Change

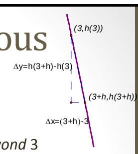
A. Example 1: The position of an object in free fall is given by $h(t) = -16t^2 + 200$, where *h* is measured in feet and *t* is measured in seconds.

a. Find the average rate of speed from *t = 1* to *t = 3*.

$$r.o.c. = \frac{\Delta y}{\Delta x} = \frac{h(3) - h(1)}{3 - 1} = \frac{56 - 184}{2} = 64\frac{ft}{sec}$$



I. Average vs. Instantaneous Rate of Change



b. Find the velocity at *t = 3 (instantaneous).*

To find a slope, we choose a point just beyond 3

$$\frac{\Delta y}{\Delta x} = \frac{h(3+h) - h(3)}{(3+h) - 3}$$

$$\frac{\Delta y}{\Delta x} = \frac{[-16(3+h)^2 + 200] - [-16(3)^2 + 200]}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{56 - 96h - 16h^2 - 56}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{-96h - 16h^2}{h}$$

$$\frac{\Delta y}{\Delta x} = -96 - 16h$$

When *h=0,* $\frac{\Delta y}{\Delta x} = -96\frac{ft}{sec}$

II. Instantaneous Slope

A. Example 2: Find the slope of $f(x) = x^2$ at $x = 4$.

$$\frac{\Delta y}{\Delta x} = \frac{f(4+h) - f(4)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{(4+h)^2 - (4)^2}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{16 + 8h + h^2 - 16}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{8h + h^2}{h}$$

$$\frac{\Delta y}{\Delta x} = 8 + h$$

When *h = 0*

$$\frac{\Delta y}{\Delta x} = 8$$

II. Instantaneous Slope

B. Example 3: Find the slope of $g(x) = x^2 - 4x + 1$ at *x = 3*.

$$\frac{\Delta y}{\Delta x} = \frac{g(3+h) - g(3)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{9 + 6h + h^2 - 12 - 4h + 1 - (-2)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{h^2 + 2h}{h}$$

$$\frac{\Delta y}{\Delta x} = h + 2$$

When *h = 0*

$$\frac{\Delta y}{\Delta x} = 2$$

III. Slope Definition

The slope of the curve *y = f(x)* at *x=a* is:

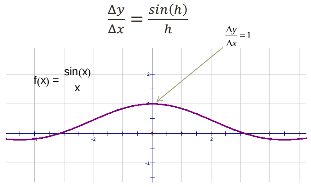
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

IV. Other Functions

A. Example 4: find the slope of $f(x) = sinx$ at *x=0.*

$$\frac{\Delta y}{\Delta x} = \frac{sin(0+h) - sin0}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{sin(h)}{h}$$



$$\frac{\Delta y}{\Delta x} = 1$$

IV. Other Functions

B. Find the slope of $f(x) = e^x$ at $x = 3$

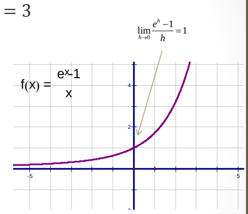
$$m = \frac{\Delta y}{\Delta x} = \frac{f(3+h) - f(3)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{e^{3+h} - e^3}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{e^3 e^h - e^3}{h}$$

$$\frac{\Delta y}{\Delta x} = e^3\left(\frac{e^h - 1}{h}\right)$$

$$\frac{\Delta y}{\Delta x} = e^3$$



Homework

9.1 B worksheet

9.1 Part III:

Tangent and Normal Lines

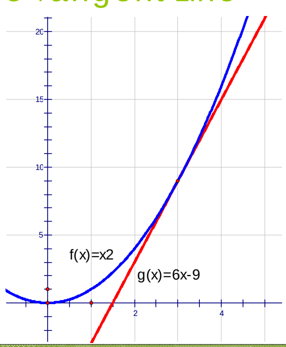
I. The Tangent Line

A. Definition: the tangent line at a point $(a, f(a))$ has the same slope as the instantaneous slope of $f(x)$ at a.

B. Example 1: find the equation for the line tangent to $f(x) = x^2$ at $x = 3$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h}$$
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h} = 6$$
$$(3, f(3)) = (3,9)$$
$$y - 9 = 6(x - 3)$$
$$y = 6x - 9$$

I. The Tangent Line

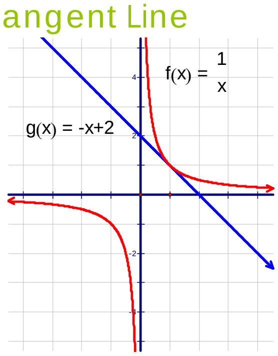


I. The Tangent Line

C. Example 2: Find the slope of the tangent line to the function $f(x) = \frac{1}{x}$ at $x = 1$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h}$$
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h} \frac{(1+h)}{(1+h)}$$
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{1 - (1+h)}{h(1+h)} = -1$$
$$(1, f(1)) = (1,1)$$
$$y - 1 = -1(x - 1)$$
$$y = -x + 2$$

I. The Tangent Line



II. The Normal Line

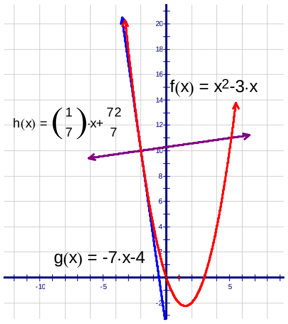
A. Definition: the normal line (think physics) at a point $(a, f(a))$ intersects the tangent line and is perpendicular to it.

B. Example 3: Write the equation for the tangent and normal to the curve $f(x) = x^2 - 3x$ at $x = -2$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{[(-2+h)^2 - 3(-2+h)] - [(-2)^2 - 3(-2)]}{h}$$
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{10 - 7h + h^2 - 10}{h} = -7$$
$$(-2, f(-2)) = (-2,10)$$

Tangent	Normal
$y - 10 = -7(x + 2)$	$y - 10 = \frac{1}{7}(x + 2)$
$y = -7x - 4$	$y = \frac{1}{7}x + \frac{72}{7}$

II. The Normal Line



II. The Tangent Line

C. Find the tangent and normal lines to the function $f(x) = sinx$ at $x = 0$.

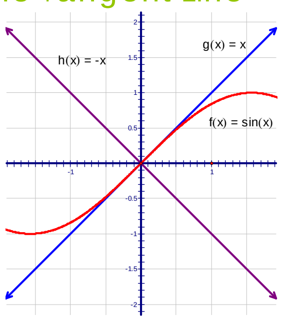
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{sin(0+h) - sin0}{h}$$

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{sinh}{h} = 1$$

$$(0, f(0)) = (0,0)$$

Tangent	Normal
$y - 0 = 1(x - 0)$	$y - 0 = -1(x - 0)$
$y = x$	$y = -x$

II. The Tangent Line



Homework:

9.1 Worksheet #4

9.3 PART I

DERIVATIVE OF A FUNCTION

I. THE DERIVATIVE

A. For a given function $f(x)$,

1. Slope at a point (this is a value)

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

2. Derivative – slope at any point (this is a function)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

B. Notation: these all are used for the derivative:

$$y', f'(x), \frac{dy}{dx}, \frac{df}{dx}, \frac{d}{dx}f(x)$$

II. EXAMPLES

A. Ex 1: Use the definition of the derivative to find $f'(x)$, if $f(x) = x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} (2x + h)$$

$$f'(x) = 2x$$

II. EXAMPLES

B. Ex. 2: Find the slope of $f(x) = x^2$ at $x = 1$, $x = 0$, and $x = -4$.

$$f'(1) = 2(1) = 2$$

$$f'(0) = 2(0) = 0$$

$$f'(-4) = 2(-4) = -8$$

II. EXAMPLES

C. Find the derivative of $f(x) = 7x - 3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{[7(x+h) - 3] - (7x - 3)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{7x + 7h - 3 - 7x + 3}{h}$$

$$f'(x) = \lim_{h \to 0} (7)$$

$$f'(x) = 7$$

II. EXAMPLES

D. Find $f'(x)$ if $f(x) = 2x^2 - 5x + 6$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{[2(x+h)^2 - 5(x+h) + 6] - [2x^2 - 5x + 6\,]}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 6 - 2x^2 + 5x - 6}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h}$$

$$f'(x) = \lim_{h \to 0} (4x + 2h - 5)$$

$$f'(x) = 4x - 5$$

II. EXAMPLES

E. Find $\frac{df}{dx}$ for $f(x) = e^x$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$f'(x) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$
$$f'(x) = \lim_{h\to 0}\frac{e^x e^h-e^x}{h}$$
$$f'(x) = \lim_{h\to 0}\left[e^x\left(\frac{e^h-1}{h}\right)\right]$$

$$f'(x) = (e^x)(1)$$
$$f'(x) = e^x$$

III. HOMEWORK

Worksheet #5

9.3 Part II: The Power Rule

I. The Power Rule

The Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

Notes:

- The derivative of a constant function is 0
- You can use this rule for rational and negative exponents also.

II. Examples

A. Example 1: Find the derivative of each of the following:

1. $y = x^7$

   $y' = 7x^{7-1} = 7x^6$

2. $y = 3x^4$

   $y' = 3(4x^{4-1}) = 12x^3$

3. $y = 4$

   $y' = 0$

4. $y = -4x + 3$

   $y' = -4(1x^{1-1}) + 0 = -4$

II. Examples

5. $f(x) = 3x^2 - 4x + 1$

   $f'(x) = 3(2x^0) - 4(1x^0) = 6x - 4$

6. $f(x) = \sqrt{x}$

   $f(x) = x^{\frac{1}{2}}$

   $f'(x) = \frac{1}{2}x^{\left(\frac{1}{2}-1\right)} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

7. $f(x) = \frac{1}{x}$

   $f(x) = x^{-1}$

   $f(x) = -1x^{-2} = -\frac{1}{x^2}$

II. Examples

8. $f(x) = \frac{1}{x^2}$

   $f(x) = x^{-2}$

   $f'(x) = -2x^{-3} = -\frac{2}{x^3}$

9. $f(x) = 2x^3 + \frac{1}{2x^3}$

   $f(x) = 2x^3 + \frac{1}{2}x^{-3}$

   $f'(x) = 2(3x^2) + \frac{1}{2}(-3x^{-4}) = 6x^2 - \frac{3}{2x^4}$

II. Examples

10. $f(x) = -x^2 + 4$

    $f'(x) = -(2x) + 4 = -2x + 4$

11. $y = x^2(x + 1)$

    $y = x^3 + x^2$

    $y' = 3x^2 + 2x$

II. Examples

B. Given $f(x) = 2x^5 + 3x^4 - 2x^3 + \frac{1}{2}x^2 + x - 253$.

a. $f'(x)$

$$f(x) = 2(5x^4) + 3(4x^3) - 2(3x^2) + \frac{1}{2}(2x^1) + (1x^0) - 0$$
$$f(x) = 10x^4 + 12x^3 - 6x^2 + x + 1$$

b. $f''(x)$

$$f(x) = (40x^3) + 12(3x^2) - 6(2x^1) + (1x^0) + 0$$
$$f(x) = 40x^3 + 36x^2 - 12x + 1$$

III. Homework

9.3 Worksheet #2

9.3 Part III: Applications of Derivatives

I. Horizontal Tangents

A. Reminder: the derivative gives the slope at any point.

B. Ex 1: Find all horizontal tangents for *f(x)* below (When $f'(x) = 0$).

$$f(x) = x^4 - 2x^2 + 2$$
$$f'(x) = 4x^3 - 4x$$

Set derivative to zero

$$0 = 4x^3 - 4x$$
$$0 = 4x(x-1)(x+1)$$
$$x = 0, 1, -1$$

This is where horizontal tangents will occur.

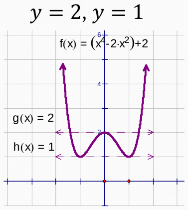
I. Horizontal Tangents

Find *y-values* at those *x-values*

$$f(0) = 2, f(-1) = 1, f(1) = 1$$

Horizontal tangents

$$y = 2, y = 1$$



I. Horizontal Tangents

B. Example 2: Find the x-value for where there is a horizontal tangent for the parabola below:

$$f(x) = ax^2 + bx + c$$

Find derivative

$$f'(x) = 2ax + b$$

Set derivative equal to zero

$$2ax + b = 0$$
$$x = -\frac{b}{2a}$$

Look familiar?

II. Oblique Tangents

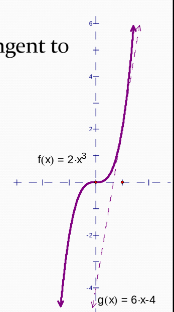
A. Ex 2: Find the equation for the line tangent to $f(x) = 2x^3$ at the point $(-1.-2)$.

Find the slope:

$$f'(x) = 6x^2$$
$$m = f'(-1) = 6(-1)^2 = 6$$

Equation for line:

$$y + 2 = 6(x+1)$$
$$y = 6x + 4$$



III. Increasing/Decreasing

A. Example 3: Where is the function *f(x)* increasing?

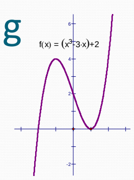
$$f(x) = x^3 - 3x + 2$$

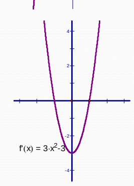
The slope must be positive, so $f'(x) > 0$

$$f'(x) = 3x^2 - 3$$
$$f'(x) = 3(x-1)(x+1)$$

This is positive from:

$$(-\infty, -1) \text{ and } (1, \infty)$$





III. Increasing/Decreasing

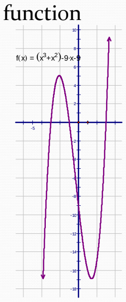
B. Example 4: Sketch the derivative for the function below:

$$f(x) = x^3 + x^2 - 9x - 9$$

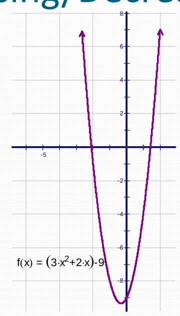
Slope is zero around $x = -2$ and $x = 1.5$

Increasing: $(-\infty, -2) \cup (1.5, \infty)$

Decreasing: $(-2, 1.5)$



III. Increasing/Decreasing



V. Homework

Worksheet #7

9.3 PART IV: FINDING EXTREMA

I. FINDING EXTREMA

A. Maximums and minimums occur at critical points.

B. To find critical points:

1. Find f'(x)
2. Solve the equation *f'(x)=0*.
3. Determine where *f'(x)* is positive and negative
   a. *f'(x)>0 before, f'(x)<0 after → Maximum*
   b. *f'(x)<0 before, f'(x)>0 after → Minimum*

II. EXAMPLE 1:

Find and classify extrema for

$$f(x) = 3x^2 + 12x - 1$$

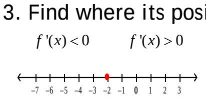
1. Find *f'(x)*

$$f'(x) = 6x + 12$$

2. Set *f'(x)=0*

$$0 = 6x + 12$$
$$x = -2$$
$$f(-2) = -13$$

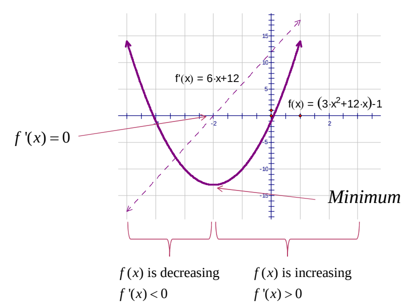
3. Find where its positive and negative



Minimum

$(-2, -13)$

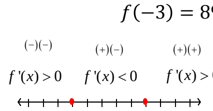
II. EXAMPLE 1



III. EXAMPLE 2

Find and classify extrema for

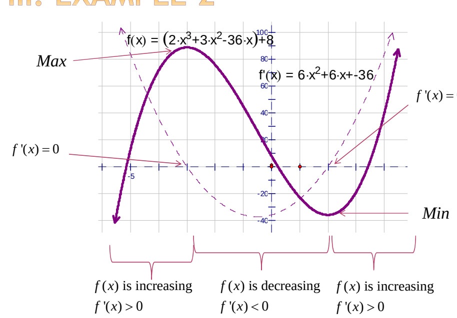
$$f(x) = 2x^3 + 3x^2 - 36x + 8$$
$$f'(x) = 6x^2 + 6x - 36$$
$$0 = 6x^2 + 6x - 36$$
$$0 = x^2 + x - 6$$
$$0 = (x+3)(x-2)$$
$$x = -3, 2$$
$$f(-3) = 89,\ f(2) = -36$$



Maximum: $(-3, 89)$

Minimum: $(2, -36)$

III. EXAMPLE 2

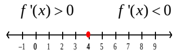


IV. EXAMPLE 3

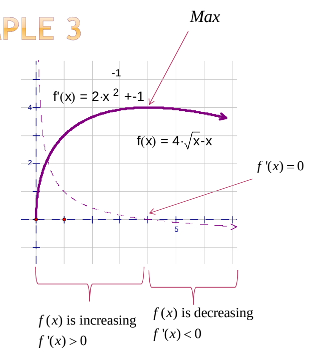
Find and classify extrema for

$$f(x) = 4\sqrt{x} - x = 4x^{\frac{1}{2}} - x$$
$$f'(x) = 2x^{-\frac{1}{2}} - 1 = \frac{2}{\sqrt{x}} - 1$$
$$0 = \frac{2}{\sqrt{x}} - 1$$
$$x = 4$$
$$f(4) = 4$$

Maximum: $(4,4)$



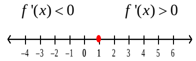
IV. EXAMPLE 3



V. EXAMPLE 4

Find and classify extrema for

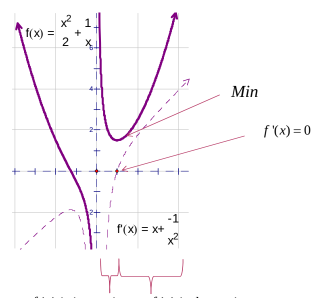
$$f(x) = \frac{x^2}{2} + \frac{1}{x} = \frac{1}{2}x^2 + x^{-1}$$
$$f'(x) = x - x^{-2} = x - \frac{1}{x^2}$$
$$0 = x - \frac{1}{x^2}$$
$$\frac{1}{x^2} = x$$
$$1 = x^3$$
$$1 = x$$
$$f(1) = \frac{3}{2}$$



minimum:

$\left(1, \frac{3}{2}\right)$

V. EXAMPLE 4



VI. HOMEWORK

Max/Min Worksheet

9.4: OPTIMIZATION PROBLEMS

I. PROCESS

How to solve these problems:

1) Read the problem carefully. Distinguish constants from variables.

2) Decide what is being maximized or minimized. This gets the dependent variable.

3) Write the quantity as a function of 1 variable. Use other information in the problems and your knowledge of formulas to help you do this.

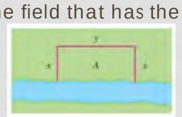
4) Determine an appropriate domain if necessary.

*5) Take the first derivative and find critical points. Test these points (and the end points) to find that you have a max or min.

6) Make sure you answer the question that was asked.

II. EXAMPLE 1

A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?



Maximizing Equation: $A = xy$

Secondary Equation: $2x + y = 2400 \rightarrow y = 2400 - 2x$

II. EXAMPLE 1

In terms of 1 variable:

$$A(x) = x(2400 - 2x)$$
$$A(x) = 2400x - 2x^2$$

Find the derivative

$$A'(x) = 2400 - 4x$$

Find critical points

$$0 = 2400 - 4x$$
$$x = 600ft$$

Calculate Area

$$A(600) = 720{,}000m^2$$

The area is 720,000 square meters

III. EXAMPLE 2

Find two numbers whose sum is 10 for which the sum of their squares is a minimum.

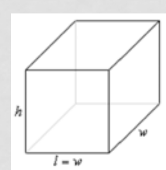
Maximization Equation: $S = x^2 + y^2$

Secondary Equation: $x + y = 10 \rightarrow y = 10 - x$

$$S(x) = x^2 + (10 - x)^2$$
$$S(x) = 2x^2 - 20x + 100$$
$$S'(x) = 4x - 20$$
$$0 = 4x - 20 \longrightarrow x = 5$$
$$solution\text{: } x = 5, y = 5$$

IV. EXAMPLE 3

We want to construct a box with a square base and we have only 10 square meters of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have.



Maximization: $V = lwh = w^2h$

Secondary: $2w^2 + 4wh = 10$

$$4wh = 10 - 2w^2$$
$$h = \frac{10}{4w} - \frac{2w^2}{4w}$$
$$h = \frac{5}{2w} - \frac{w}{2}$$

IV. EXAMPLE 3

$$V(x) = w^2\left(\frac{5}{2w} - \frac{w}{2}\right)$$

$$V(x) = \frac{5w^2}{2w} - \frac{w^3}{2} = \frac{5}{2}w - \frac{1}{2}w^3$$

$$V'(x) = \frac{5}{2} - \frac{3}{2}w^2$$

$$0 = \frac{5}{2} - \frac{3}{2}w^2$$

$$w = \pm\sqrt{\frac{5}{3}} = \pm 1.2910m$$

$$V\left(\sqrt{\frac{5}{3}}\right) \approx 2.1517m^3$$

V. EXAMPLE 4

A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction.

Minimize: $A = 2\pi r^2 + 2\pi rh$ Constraint: $V = \pi r^2 h = 1500$

$$h = \frac{1500}{\pi r^2}$$

V. EXAMPLE 4

$$A(x) = 2\pi r^2 + 2\pi r\left(\frac{1500}{\pi r^2}\right)$$

$$A(x) = 2\pi r^2 + \frac{3000}{r} = 2\pi r^2 + 3000r^{-1}$$

$$A'(x) = 4\pi r - 3000r^{-2} = 4\pi r - \frac{3000}{r^2}$$

$$0 = \frac{4\pi r^3 - 3000}{r^2}$$

$$0 = 4\pi r^3 - 3000$$

$$r = \sqrt[3]{\frac{750}{\pi}} \approx 6.2035cm$$

$$h = \frac{1500}{\pi\left(\sqrt[3]{750/\pi}\right)^2} \approx 12.4070cm$$

$$A\left(\sqrt[3]{\frac{750}{\pi}}\right) \approx 725.3964cm^2$$

VI. HOMEWORK

Maximization Worksheet

9.4: Velocity and Acceleration

I. Position, Velocity, and Acceleration

A. s(t) = the position of a function at time *t*.

B. Average velocity:

$$\bar{v} = \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

C. Instantaneous velocity

$$s'(t) = v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

D. Instantaneous Acceleration

$$a(t) = v'(t) = s''(t)$$

II. Example 1

A dynamite blast launches a heavy rock straight up with an initial velocity of 160 ft / sec.

Its position at time t is given by

$$s(t) = -16t^2 + 160t$$

a. Find the average velocity from *t=1 to* t=4.

$$\frac{\Delta y}{\Delta x} = \frac{s(4) - s(1)}{4 - 1} = \frac{384 - 144}{3} = 80\frac{ft}{s}$$

II. Examples

b. Find the velocity (instantaneous) at *t=1*:

$$v(t) = s'(t) = -32t + 160$$

$$v(1) = -32(1) + 160 = 128\frac{ft}{s}$$

c. How long does it take to reach the highest point?

Set $v(t) = 0$

$$0 = -32t + 160$$

$$t = 5sec$$

II. Examples

D. How high does it go?

$$s(5) = -16(5)^2 + 160(5) = 400ft$$

E. Find an equation for the acceleration at time *t*.

$$s'(t) = v(t) = -32t + 160$$

$$s''(t) = v'(t) = a(t) = -32$$

This is the gravitational constant.

III. Example 2

Given table below:

t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	12.5	26	36.5	44	48.5	50	48.5	44	36.5

Estimate the velocity on the given time intervals given below:

a. [0.5,1.5]

$$\frac{\Delta s}{\Delta t} = \frac{44 - 26}{1.5 - 0.5} = 18\frac{ft}{s}$$

b. [2,3]

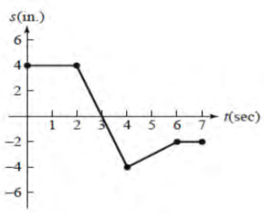
$$\frac{\Delta s}{\Delta t} = \frac{48.5 - 48.5}{3 - 2} = 0\frac{ft}{s}$$

c. [3,4]

$$\frac{\Delta s}{\Delta t} = \frac{36.5 - 48.5}{4 - 3} = 12\frac{ft}{s}$$

IV. Example 3

The graph below shows the position of a particle moving along a horizontal coordinate axis.



IV. Example 3

a. When is the particle moving left?

$(2,4)$

b. When is the particle moving right?

$(4,6)$

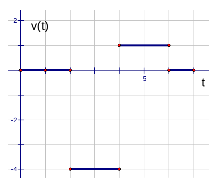
c. When is the particle standing still?

$(0,2), (6,7)$

IV. Example 3

d. Graph the particle's velocity.

t	v(t)
$0 < x < 2$	0
$2 < x < 4$	-4
$4 < x < 6$	1
$6 < x < 7$	0



e. When is the particle moving fastest?

(2,4)

V. Example 4

The position s of a particle moving along a line is given by *s(t)* below for $0 \leq s \leq 6$, where s = *feet* and *t* = *seconds*.

$$s(t) = 2t^3 - 21t^2 + 60t + 3$$

a. Find the displacement during the first 4 seconds.

$$s(4) - s(0) = 35 - 3 = 32\ feet$$

V. Example 4

b. Find the average velocity during the first 4 seconds

$$\frac{s(4) - s(0)}{4 - 0} = \frac{32}{4} = 8\frac{ft}{sec}$$

c. Find the instantaneous velocity at *t=4*.

$$s'(t) = v(t) = 6t^2 - 42t + 60$$

$$v(4) = 6(4)^2 - 42(4) + 60 = -12\frac{ft}{sec}$$

V. Example 4

d. Find the acceleration when *t=4*.

$$s''(t) = v'(t) = a(t) = 12t - 42$$

$$a(4) = 12(4) - 42 = 6\frac{ft}{sec^2}$$

e. Describe when its moving left and right

It changes direction when *v=0*.

$$v(t) = 0 = 6(t-2)(t-5)$$

Right (v(t)>0): $(0,2) \cup (5,6)$

Left (v(t)<0): $(2,5)$

VI. Homework

Worksheet 8