

1. Determine all solutions to the following equations (exact)

a. $\sin x = -1$

$$x = \frac{3\pi}{2} + 2\pi n$$

b. $\sec x = 2$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2\pi n \text{ or } \frac{5\pi}{3} + 2\pi n$$

c. $\cot x = 0$

$$\tan x = \phi$$

$$\frac{\pi}{2} + \pi n$$

or

$$\frac{3\pi}{2} + 2\pi n, \frac{\pi}{2} + 2\pi n$$

d. $\tan x = -\frac{\sqrt{3}}{3}$

$$x = \frac{5\pi}{6} + \pi n$$

or

$$\frac{5\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$$

2. Determine all solutions to the following equations (approximate)

a. $\cos x = 0.9224$

$$\cos^{-1}(0.9224) \approx 0.397$$

$$Q_I: 0.397 + 2\pi n$$

$$Q_{III}: 2\pi - 0.397 \approx 5.887 + 2\pi n$$

b. $\cot x = -7.770$

$$\tan x = 1/-7.770$$

$$\tan^{-1}(1/7.770)$$

$$Q_I: 0.128$$

$$Q_{II}: \pi - 0.128 \approx 3.014 + 2\pi n$$

$$Q_{IV}: 2\pi - 0.128 \approx 6.155 + 2\pi n$$

3. Determine all solutions to the following over the interval $0 \leq x < 2\pi$ (without calculator)

a. $1 - 3\tan^2 x = 0$

$$1 = 3\tan^2 x$$

$$\frac{1}{3} = \tan^2 x$$

$$\pm \sqrt{\frac{1}{3}} = \tan x$$

$$\pm \frac{\sqrt{3}}{3} = \tan x$$

$$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

b. $\csc^2 x + \csc x - 2 = 0$

$$(\csc x - 1)(\csc x + 2) = 0$$

$$\csc x = 1$$

$$\csc x = -2$$

$$\sin x = 1$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

c. $2\sin x + \csc x = 0$

$$(2\sin x + \frac{1}{\sin x} = 0)\sin x$$

$$2\sin^2 x + 1 = 0$$

$$\sin^2 x = -\frac{1}{2}$$

$$\sin x = \pm \sqrt{-\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

e. $\sin 2x \sin x - \cos x = 0$

$$2\sin x \cos x \sin x - \cos x = 0$$

$$2\sin^2 x \cos x - \cos x = 0$$

$$\cos x (2\sin^2 x - 1) = 0$$

$$\cos x = 0 \quad 2\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

d. $4\sec x \cos x = 1$

$$4\left(\frac{1}{\cos x}\right)(\cos x) = 1$$

$$4 = 1$$

No solution

f. $\sec^2 x + 2 = 3\sec x$

$$\sec^2 x - 3\sec x + 2 = 0$$

$$(\sec x - 1)(\sec x - 2) = 0$$

$$\sec x = 1 \quad \sec x = 2$$

$$\cos x = 1 \quad \cos x = \frac{1}{2}$$

$$x = 0, \frac{\pi}{3}, \frac{5\pi}{3}$$

4. Determine all solutions over the interval $0 \leq x < 2\pi$ (with calculator)

a. $\sin^2 x = 20 - \sin x$

$$\begin{aligned} \sin^2 x + \sin x - 20 &= 0 \\ (\sin x + 5)(\sin x - 4) &= 0 \\ \sin x &= -5 \quad \sin x = 4 \\ \text{No solution} \end{aligned}$$

b. $2 \tan^2 x + 7 \tan x = 15$

$$\begin{aligned} 2 \tan^2 x + 7 \tan x - 15 &= 0 \\ (2 \tan x - 3)(\tan x + 5) &= 0 \\ \tan x &= \frac{3}{2} \quad \tan x = -5 \\ 0.983 \quad 1.768 \\ 4.124 \quad 4.910 \end{aligned}$$

c. $3 \sin^2 x - \cos^2 x = 1$

$$\begin{aligned} 3 \sin^2 x - (1 - \sin^2 x) &= 1 \\ 3 \sin^2 x - 1 + \sin^2 x &= 1 \\ 4 \sin^2 x &= 2 \\ \sin^2 x &= \frac{1}{2} \\ \sin x &= \pm \sqrt{\frac{1}{2}} \\ \sin x &= \pm \frac{\sqrt{2}}{2} \end{aligned}$$

e. $\sec^2 x = 6 \sec x + 2$

$$\begin{aligned} \sec^2 x - 6 \sec x - 2 &= 0 \\ \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)} \\ \sec x &\approx 6.317 \quad \sec x \approx -0.317 \\ \cos x &\approx 0.158 \quad \cos x \approx -3.158 \\ 1.412, 4.871 \end{aligned}$$

d. $\cos 2x = 2 \sin^2 x - 2$

$$\begin{aligned} 1 - 2 \sin^2 x &= 2 \sin^2 x - 2 \\ 3 &= 4 \sin^2 x \\ \frac{3}{4} &= \sin^2 x \\ \pm \sqrt{\frac{3}{4}} &= \sin x \\ \pm \frac{\sqrt{3}}{2} &= \sin x \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

f. $4 \cot^2 x - 4 \cot x = 3$

$$\begin{aligned} 4 \cot^2 x - 4 \cot x - 3 &= 0 \\ (2 \cot x + 1)(2 \cot x - 3) &= 0 \\ \cot x &= -\frac{1}{2} \quad \cot x = \frac{3}{2} \\ \tan x &= -2 \quad \tan x = \frac{2}{3} \\ Q_1: 1.107 \quad Q_2: 2.034 \quad 0.588 \\ Q_3: 5.176 \quad 3.730 \end{aligned}$$

5. On the interval $0 \leq x < 2\pi$, determine where:

a. $\cos^2 x - 2 \cos x < 0$

b. $\sec^2 x - 2 \tan x \geq 4$

c. $\cos 2x + 7 \sin x - 4 > 0$

d. $\sin x \leq \cos 2x$

