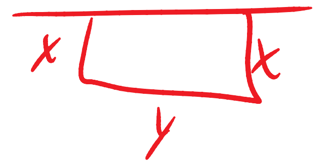
Optimization Problems Notes Pre Calc II

EX 1: A yard is to be enclosed with one side along a building, so it only requires 3 sides of fencing. What is the largest area you can enclose with 1200 feet of fencing?



$$\begin{aligned}&Maximization:\\ &A = xy\\ &Secondary:\\ &2x + y = 1200\\ &\rightarrow y = 1200 - 2x\end{aligned}$$

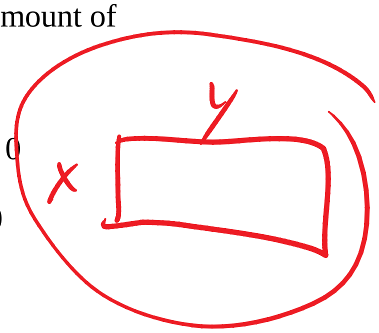
$$\begin{aligned}A(x) &= x(1200 - 2x)\\ A(x) &= 1200x - 2x^2\\ A'(x) &= 1200 - 4x\end{aligned}$$

$$\begin{aligned}1200 - 4x &= 0\\ x &= 300\, ft\\ y &= 1200 - 2(300) = 600\, ft\end{aligned}$$

$$Area = 300\, ft \cdot 600\, ft$$

$$\boxed{Area = 180{,}000\, ft^2}$$

EX 2: Suppose you want to enclose a 5000 square foot yard. What is the least amount of fencing you can use?



$$\begin{aligned}&Minimization:\\ &P = 2x + 2y\\ &Secondary:\\ &5000 = xy\\ &\rightarrow y = \frac{5000}{x}\end{aligned}$$

$$\begin{aligned}P(x) &= 2x + 2\left(\frac{5000}{x}\right)\\ P(x) &= 2x + \frac{10{,}000}{x}\\ P(x) &= 2x + 10{,}000x^{-1}\\ P'(x) &= 2 - 10{,}000x^{-2}\\ P'(x) &= 2 - \frac{10{,}000}{x^2}\end{aligned}$$

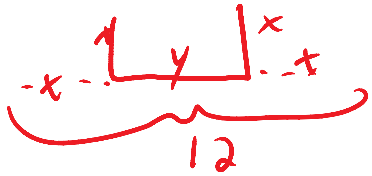
$$\begin{aligned}2 - \frac{10{,}000}{x^2} &= 0\\ 2x^2 &= 10{,}000\\ x^2 &= 5{,}000\\ x &\approx 70.71yd\\ y &\approx \frac{5{,}000}{70.71} \approx 70.71\\ P &\approx 2(70.71) + 2(70.71)\end{aligned}$$

$$\boxed{P \approx 282.84yd}$$

EX 3: A long rectangular sheet of metal 12 inches wide is to be made into a rain gutter by turning up two sides so they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

$A = xy \quad 2x + y = 12$

$y = 12 - 2x$



$A = x(12 - 2x)$

$A = 12x - 2x^2$

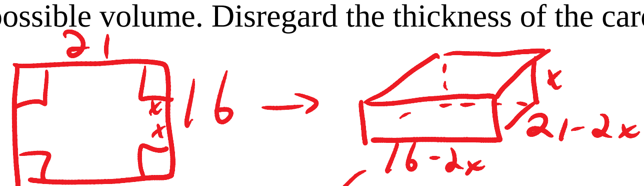
$A'(x) = 12 - 4x$

$0 = 12 - 4x$

$3 = x \quad y = 6$

$A(3) = 18$

EX 4: An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16" wide and 21" long by cutting a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume. Disregard the thickness of the cardboard.



$V(x) = x(16-2x)(21-2x)$

$V(x) = 4x^3 - 74x^2 + 336x$

$V'(x) = (12x^2 - 148x + 336)$

$0 = 12x^2 - 148x + 336$

$0 = 3x^2 - 37x + 84$

$0 = (3x - 28)(x - 3)$

$x = 3, \cancel{9\tfrac{1}{3}}$

$x = 3''$ $\quad V(3) = 450$

EX 5: Find the dimensions that will make a can that holds 1000 cubic centimeters (1 liter) and uses the least amount of material.

$A = 2\pi r h + 2\pi r^2$

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

$A(r) = 2\pi r\left(\frac{1000}{\pi r^2}\right) + 2\pi r^2$

$A(r) = \frac{2000}{r} + 2\pi r^2$

$A(r) = 2000 r^{-1} + 2\pi r^2$

$A'(r) = -2000 r^{-2} + 4\pi r$

$r^2\left(0 = \frac{-2000}{r^2} + 4\pi r\right)$

$0 = -2000 + 4\pi r^3$

$r = \sqrt[3]{\frac{2000}{4\pi}}$

EX 6: Find two non-negative numbers who sum is 16 and whose product is as large as possible.

$P = xy$

$x + y = 16$

$y = 16 - x$

$P(x) = x(16 - x)$

$p(x) = 16x - x^2$

$p'(x) = 16 - 2x$

$0 = 16 - 2x$

$8 = x$

$8, 8$