Optimization Homework

1. A ball is thrown straight up in the air from ground level. Its height after t seconds is given by the function $s(t) = -16t^2 + 50t$. When does the ball reach its maximum height? What is its maximum height?

$s'(t) = -32t + 50$

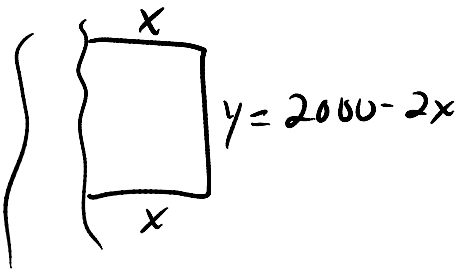
$0 = -32t + 50$

$1.5625 \text{ sec} = t$

$t_{max} = 1.5625$ sec

$S(1.5625) = 39.0625$ FT

2. A farmer has 2,000 feet of fencing to enclose a pasture area. The field will be in the shape of a rectangle and will be placed against a river where there is no fencing needed. What is the largest area field that can be created and what are its dimensions?



$A(x) = x(2000 - 2x)$

$A(x) = 2000x - 2x^2$

$A'(x) = 2000 - 4x$

$0 = 2000 - 4x \rightarrow x = 500$

$x = 500$ FT

$y = 1000$ FT

$A = 500{,}000$ FT^2

3. Two numbers add up to 40. Find the numbers that maximize their product.

$x + y = 40 \rightarrow y = 40 - x$

$xy = P \rightarrow p(x) = x(40 - x)$

$p(x) = 40x - x^2$

$p'(x) = 40 - 2x$

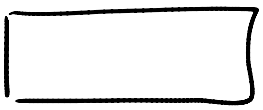
$p'(x) = 0 = 40 - 2x$

$20 = x$

$y = 20$

$x = 20$

4. A rectangle has a perimeter of 71 feet. What length and width should it have so that its area is a maximum? What is its maximum area?



$2w + 2l = 71$

$l = \frac{71 - 2w}{2}$

$A = wl$

$A(w) = w\left(\frac{71 - 2w}{2}\right)$

$A(w) = \frac{71}{2}w - w^2$

$A'(w) = \frac{71}{2} - 2w$

$0 = \frac{71}{2} - 2w$

$w = \frac{71}{4} = 17.75$ FT

$l = 17.75$ FT

$A = 315.0625$ FT^2

5. An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut out to create a box with greatest volume. What is the maximum volume as well?

$V(x) = (30-2x)(16-2x)x$

$V(x) = 4x^3 - 92x^2 + 480x$

$V'(x) = 12x^2 - 184x + 480$

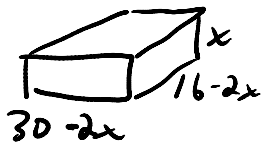
$0 = 12x^2 - 184x + 480$

$0 = 3x^2 - 46x + 120$

$x = \frac{10}{3}$ or 12

$x = 3.\overline{3}$ in

$V(3.\overline{3}) \approx 729.93 \text{ in}^3$



6. A fisheries biologist is stocking fish in a lake. She knows that when there are *n* fish per unit of water, the average weight of each fish will be $W(n) = 500 - 2n$, measured in grams. What is the value of *n* that will maximize the total fish weight after one season? *~~Hint: Total Weight = number of fish x average weight of fish.~~*

$T(n) = n(500-2n)$

$T(n) = 500n - 2n^2$

$T'(n) = 500 - 4n$

$0 = 500 - 4n$

$125 = n$

$T(125) = 31{,}250$ g

MAX $n = 125$ FISH

7. Blood pressure in a patient will drop by an amount $D(x) = 0.025x^2(30 - x)$ where *x* is the amount of drug injected in cm^3. Find the dosage that provides the greatest drop in blood pressure. What is the drop in blood pressure?

$D(x) = 0.075x^2 - 0.0025x^3$

$D'(x) = 0.15x - 0.0075x^2$

$0 = 0.15x - 0.0075x^2$

$0 = 0.15x(1 - 0.05x^2)$

$0 = x$

$0 = 1 - 0.05x^2$

$0.05x^2 = 1$

$x^2 = 20$

$x = \pm 4.47$

$x \approx 4.47$

$D(4.47) \approx 12.76$