Worksheet 6.1

1. Give the domain of each of the following identities

a. $cotx = \frac{cosx}{sinx}$ $\mathbb{R}, x \neq \ldots, -\pi, 0, \pi, 2\pi, \ldots$

b. $sin^2x = 1 - cos^2x$ $\mathbb{R}$

2. For each of the following equations:
a. Try a value for *x* to see if the identity works.
b. Graph on your calculator to verify if it is an identity
c. If it is an identity, identify its domain.

a. $sec^2x - tan^2x = 1$

a) $\sec^2\left(\frac{\pi}{6}\right) - \tan^2\left(\frac{\pi}{6}\right)$
$= \left(\frac{2\sqrt{3}}{3}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^2$
$= \frac{4}{3} - \frac{1}{3} = 1$ ✓

b) YES

c) $\mathbb{R}, x \neq \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

b. $sin\left(\frac{\pi}{2} - x\right) = sin\left(x - \frac{\pi}{2}\right)$

a) $\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$
$\sin\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

b) NO

3. Consider the proposed identity $cos(2x) = 1 - 2sin^2x$

a. Verify the identity for $x = \frac{\pi}{4}$ and $x = \pi$.

$\cos\left(2\left(\frac{\pi}{4}\right)\right) = \cos(\pi/2) = 0$
$1 - 2\left(\sin\left(\frac{\pi}{4}\right)\right)^2 = 1 - 2\left(\frac{\sqrt{2}}{2}\right)^2 = 1 - 2\left(\frac{1}{2}\right) = 0$ ✓

$\cos 2(\pi) = \cos 2\pi = 1$
$1 - 2\sin^2(\pi) = 1 - 2(0) = 1$ ✓

b. Graph both sides to see if they have identical graphs.

YES

Review

4. Find the exact value of each of the following

a. $cos\frac{5\pi}{6}$ $-\frac{\sqrt{3}}{2}$

b. $csc\left(-\frac{\pi}{3}\right)$ $-\frac{2\sqrt{3}}{3}$

c. $tan\frac{5\pi}{3}$ $-\sqrt{3}$

5. Given functions $f(x) = 2x + 1$ and $g(x) = x^2 - 2x$, find a rule and the domain for each of the following.

a. $f + g = x^2 + 1$
$\mathbb{R}$

b. $f - g = -x^2 + 4x + 1$
$\mathbb{R}$

c. $f \cdot g = 2x^3 - 3x^2 - 2x$
$\mathbb{R}$

d. $\frac{f}{g} = \frac{2x+1}{x^2-2x}$
$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

Worksheet 6.2

1. Prove the following identities and give the domain for each.

a. $\cot^2 x + 1 = \csc^2 x$

$\frac{\cos^2 t}{\sin^2 t} + \frac{\sin^2 t}{\sin^2 t} = \frac{\cos^2 t + \sin^2 t}{\sin^2 t}$

$= \frac{1}{\sin^2 t}$

$= \csc^2 t$

b. $\sin x \cot x = \cos x$

$\sin x \cot x = \frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} = \frac{\cancel{\sin x} \cos x}{\cancel{\sin x}}$

$= \cos t$

c. $\cos x \tan x = \sin x$

$\cos x \cdot \frac{\sin x}{\cos x} = \frac{\cancel{\cos x} \sin x}{\cancel{\cos x}}$

$= \sin x$

d. $\tan \cot x = \cos^2 x + \sin^2 x$

$\tan x \cot x = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 1 = \cos^2 x + \sin^2 x$

e. $\csc^2 x \sin x = \dfrac{\sec^2 x - \tan^2 x}{\sin x}$

$\frac{\sec^2 x - \tan^2 x}{\sin x} = \frac{1 + \tan^2 x - \tan^2 x}{\sin x} = \frac{1}{\sin x}$

$= \csc x = \csc x \left(\frac{\sin x}{\sin x}\right) = \csc x \left(\frac{1}{\sin x}\right) \sin x$

$= \csc x \csc x \sin x$

$= \csc^2 x \sin x$

f. $\tan x + \cot x = \sec x \csc x$

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \frac{1}{\cos x \sin x} = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right) = \sec x \csc x$

Review

2. Solve $\left(\dfrac{1}{x} + \dfrac{4}{x^2} = \dfrac{x+4}{5x}\right) 5x^2$

$5x + 20 = x(x+4)$

$5x + 20 = x^2 + 4x$

$0 = x^2 - x - 20$

$0 = (x-5)(x+4)$

$x = 5, -4$

3. Simplify $\dfrac{\dfrac{3}{x} - \dfrac{9}{x^2}}{\dfrac{1}{3x} - 1}$ and state any restrictions on the variable.

$\frac{\frac{3}{x} - \frac{9}{x^2}}{\frac{1}{3x} - 1} \cdot \frac{3x^2}{3x^2} = \frac{9x - 27}{x - 3x^2} = \frac{9(x-3)}{x(x-3)} = \boxed{\frac{9}{x}}$ $x \neq 0, 3$

Simplify each expression:

1. $\tan\theta\cos\theta$

$\frac{\sin x}{\cancel{\cos x}}\cdot\cancel{\cos x} = \sin x$

2. $\tan\left(90^\circ - x\right)$

$\frac{\sin(90-x)}{\cos(90-x)} = \frac{\cos x}{\sin x}$

$= \cot x$

3. $\cos\left(\frac{\pi}{2}-x\right)$

$\sin x$

4. $\left(1+\sin x\right)\left(1-\sin x\right)$

$1+\sin x - \sin x - \sin^2 x$

$1-\sin^2 x$

$\cos^2 x$

5. $\sin^2 x - 1$

$\sin^2 x - (\sin^2 x + \cos^2 x)$

$\sin^2 x - \sin^2 x - \cos^2 x$

$-\cos^2 x$

6. $(\sec x+1)(\sec x-1)$

$\sec^2 x - 1 = 1+\tan^2 x - 1$

$= \tan^2 x$

7. $\tan A\cot A$

$\frac{\cancel{\sin A}}{\cancel{\cos A}}\cdot\frac{\cancel{\cos A}}{\cancel{\sin A}} = 1$

8. $\cot y\sin y$

$\frac{\cos y}{\cancel{\sin y}}\cdot\cancel{\sin y}$

$\cos y$

9. $\cot^2 x - \csc^2 x$

$\cot^2 x - (1+\cot^2 x)$

$\cot^2 x - 1 - \cot^2 x$

-1

10. $\frac{1}{\cos\left(90^\circ-\theta\right)}$

$\frac{1}{\sin\theta} = \csc\theta$

11. $1-\frac{\sin^2\theta}{\tan^2\theta}$

$1-\sin^2\theta \div \frac{\sin^2\theta}{\cos^2\theta}$

$1-\cancel{\sin^2\theta}\cdot\frac{\cos^2\theta}{\cancel{\sin^2\theta}}$

$1-\cos^2\theta = \sin^2\theta$

12. $\frac{1}{\cos^2\theta}-\frac{1}{\cot^2\theta}$

$\sec^2\theta - \tan^2\theta$

$1+\tan^2\theta - \tan^2\theta$

1

13. $\csc^2 x(1-\cos^2 x)$

$\frac{1}{\sin^2 x}(\sin^2 x)$

$\frac{\sin^2 x}{\sin^2 x} = 1$

14. $\cos\theta(\sec\theta-\cos\theta)$

$\cos\theta\left(\frac{1}{\cos\theta}-\cos\theta\right)$

$1-\cos^2\theta$

$\sin^2\theta$

15. $\frac{\sin x\cos x}{1-\cos^2 x}$

$\frac{\cancel{\sin x}\cos x}{\sin^{\cancel{2}} x} = \frac{\cos x}{\sin x}$

$= \cot x$

16. $\dfrac{\tan x+\cot x}{\sec^2 x}$

$\dfrac{\tan x+\cot x}{1+\tan^2 x} = \dfrac{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}{1+\frac{\sin^2 x}{\cos^2 x}}\cdot\dfrac{\sin x\cos^2 x}{\sin x\cos^2 x}$

$= \dfrac{\sin^2 x\cos x+\cos^3 x}{\sin x\cos^2 x+\sin^3 x} = \dfrac{\cos x(\sin^2 x+\cos^2 x)}{\sin x(\cos^2 x+\cos^2 x)} = \dfrac{\cos x}{\sin x}$

$= \cot x$

17. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

$\sin^2 x+\cancel{2\sin x\cos x}+\cos^2 x+\sin^2 x-\cancel{2\sin x\cos x}+\cos^2 x$

$(\sin^2 x+\cos^2 x)+(\sin^2 x+\cos^2 x)$

$1+1$

1

18. $(\sec^2\theta-1)(\csc^2\theta-1)$

$(1+\tan^2\theta-1)(1+\cot^2 x-1)$

$(\tan^2 x)(\cot^2 x)$

1

19. $\dfrac{\tan^2\theta}{\sec\theta+1}+1$

$\dfrac{\sec^2\theta-1}{\sec\theta+1}+1 = \dfrac{(\sec\theta-1)\cancel{(\sec\theta+1)}}{\cancel{\sec\theta+1}}+1$

$= \sec\theta-1+$

$= \sec\theta$

20. $\cos^3 y+\cos y\sin^2 y$

$\cos y(\cos^2 y+\sin^2 y)$

$\cos y$

21. $\dfrac{\sec y+\csc y}{1+\tan y}$

$\dfrac{\frac{1}{\cos y}+\frac{1}{\sin y}}{1+\frac{\sin y}{\cos y}}\cdot\dfrac{\sin y\cos y}{\sin y\cos y} = \dfrac{\sin y+\cos y}{\sin y\cos y+\sin^2 y}$

$= \dfrac{\cancel{\sin y+\cos y}}{\sin y\cancel{(\cos y+\sin y)}} = \dfrac{1}{\sin y} = \csc y$

22. $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{\cos\theta}$

$\dfrac{\cos\theta(\cos\theta)}{(1+\sin\theta)(\cos\theta)}+\dfrac{(1+\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(\cos\theta)}$

$\dfrac{\cos^2\theta+1+2\sin\theta+\sin^2\theta}{(1+\sin\theta)\cos\theta}$

$\dfrac{\cos^2\theta\ 1\ \sin^2\theta+1\ 2\sin\theta}{(1+\sin\theta)\cos\theta}$

$\dfrac{2\ 1\ 2\sin\theta}{(1+\ \theta)\cos\theta}$

$\dfrac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}\cos\theta}$

$\dfrac{2}{\cos\theta} = 2\sec\theta$

23. $\dfrac{\sin^4\theta-\cos^4\theta}{\sin^2\theta-\cos^2\theta}$

$\dfrac{\cancel{(\sin^2\theta-\cos^2\theta)}(\sin^2\theta+\cos^2\theta)}{\cancel{\sin^2\theta-\cos^2\theta}} = 1$

Simplify

1. $\sec^2\alpha - 1$

$1+\tan^2\alpha - 1 = \tan\alpha$

2. $\csc^2 t - \cot^2 t = 1+\cot^2 t - \cot^2 t$

1

3. $\cos t \csc t$

$\cos t \cdot \frac{1}{\sin t} = \frac{\cos t}{\sin t} = \cot t$

4. $\sin\phi \sec\phi$ $\sin\phi \frac{1}{\cos\phi} = \frac{\sin\phi}{\cos\phi} = \tan\phi$

5. $\sin\theta \sec\theta \cot\theta$

$\cancel{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}} = 1$

6. $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x} = \cos x \cdot \frac{1}{\sec x} + \sin x \frac{1}{\csc x}$

$= \cos x \cdot \cos x + \sin x \sin x$ $\cos^2 x + \sin^2 x$

1

7. $\dfrac{\csc x}{\sin x} - \dfrac{\cot x}{\tan x}$

$\frac{1}{\sin x} \div \sin x - \frac{\cos x}{\sin x} \div \frac{\sin x}{\cos x} = \frac{1}{\sin x} \cdot \frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = \csc^2 x - \cot^2 x = 1+\cot^2 x - \cot^2 x = \boxed{1}$

8. $\dfrac{1}{\sin^2 x} - \dfrac{1}{\tan^2 x} = \csc^2 x - \cot^2 x$

$= 1+\cot^2 x - \cot^2 x = 0$

9. $\dfrac{\sec t}{\cos t} - \sec t \cos t$

$\sec t \cdot \frac{1}{\cos t} - 1 = \sec t \cdot \sec t - 1 = \sec^2 t - 1 = \boxed{\tan^2 t}$

10. $1 - \dfrac{\cos^2\alpha}{\cot^2\alpha} = 1 - \cos^2\alpha \div \frac{\cos^2\alpha}{\sin^2\alpha} = 1 - \cos^2\alpha \cdot \frac{\sin^2\alpha}{\cos^2}$

$= 1 \quad \sin^2\alpha$

$\cos^2\alpha$

11. $\dfrac{1+\tan^2\alpha}{\tan^2\alpha}$

$= \frac{1}{\tan^2\alpha} + \frac{\tan^2\alpha}{\tan^2\alpha} = \cot^2\alpha + 1 = \boxed{\csc^2\alpha}$

12. $\dfrac{\sec^2 t - 1}{\sec^2 t}$

$\frac{\sec^2 t}{\sec^2 t} - \frac{1}{\sec^2 t} = 1 - \cos^2 t = \sin^2 t$

13. $\dfrac{\csc^2 x - \cot^2 x}{\sec x}$

$= \frac{1+\cot^2 x - \cot^2 x}{\sec x} = \frac{1}{\sec x} = \cos x$

14. $\cos^2 x^2 + \sin^2 x^2$

$= 1$

15. $\sec^2(\ln x) - \tan^2(\ln x)$

$1+\tan^2(\ln x) - \tan^2(\ln x) = 0$

16. $\dfrac{\sin x}{\csc x} - \sin x \csc x$

$\sin \cdot \frac{1}{\csc x} - \sin x \frac{1}{\cancel{\sin x}} = \sin x \cdot \sin x - 1 = \sin^2 x - 1$

$= \cos$

17. $\dfrac{\sin\theta\cos\theta}{1-\sin^2\theta}$

$= \frac{\sin\theta \cos\theta}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

18. $\tan^2\alpha - \cot^2\alpha + \csc^2\alpha$

$\tan^2 x - \cot^2\alpha + 1 + \cot^2\alpha = \tan^2\alpha + 1$

$= \sec\alpha$

19. $\dfrac{1-\sin^2\alpha}{1-\sin\alpha} - 1$

$\frac{\cancel{(1-\sin\alpha)}(1+\sin\alpha)}{\cancel{1-\sin\alpha}} = 1 - 1 + \sin\alpha = \sin\alpha$

20. $\cos^2 t(\cot^2 t + 1)$

$\cos^2 \cdot \frac{\sin^2 t}{\cos^2 t} + \cos t = \sin^2 t + \cos^2 t$

$= 1$

21. $\sec x - \sin x \tan x$

$\frac{1}{\cos x} - \frac{\sin x \cdot \sin}{\cos} - \frac{1-\sin^2}{\cos} = \frac{\cos^2}{\cos}$

$\cos$

22. $\sin x(\csc x - \sin x)$

$\sin\left(\frac{1}{\sin x} - \sin x\right) = 1 - \sin^2 t$

$= \cos^2 t$

Prove the Stated Identity

1. $\sin^2 x(1+\tan^2 x)=\tan^2 x$

$\sin^2 x(\sec^2 x)$

$\sin^2 x\left(\frac{1}{\cos^2 x}\right)$

$\frac{\sin^2 x}{\cos^2 x}$

$\tan^2 x$

2. $\tan\alpha(\tan\alpha+\cot\alpha)=\sec^2\alpha$

$\tan\alpha\tan\alpha+\tan\alpha\cot\alpha$

$\tan^2\alpha+1$

$\sec^2\alpha$

3. $\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$

$=\cos^2\theta-(1-\cos^2\theta)$

$=\cos^2\theta-1+\cos^2\theta$

$=\boxed{2\cos^2\theta-1}$

4. $\cos^2 x-\sin^2 x=1-2\sin^2 x$

$=1-\sin^2 x-\sin^2 x$

$=\boxed{1-2\sin^2 x}$

5. $\sec^2 x+\csc^2 x=\sec^2 x\csc^2 x$

$\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}=\frac{\sin^2 x+\cos^2 x}{\cos^2 x\sin^2 x}$

$=\frac{1}{\cos^2 x}\cdot\frac{1}{\sin^2 x}$

$=\boxed{\sec^2 x\csc^2 x}$

6. $\tan^2 t-\sin^2 t=\tan^2 t\sin^2 t$

$\frac{\sin^2 t}{\cos^2 t}-\frac{\sin^2 t\cos^2 t}{\cos^2 t}=\frac{\sin^2 t-\sin^2 t\cos^2 t}{\cos^2 t}$

$=\frac{\sin^2 t(1-\cos^2 t)}{\cos^2 t}=\frac{\sin^2 t(\sin^2 t)}{\cos^2 t}$

$=\frac{\sin^2 t}{\cos^2 t}\sin^2 t=\boxed{\tan^2 t\sin^2 t}$

7. $\sec\alpha-\cos\alpha=\sin\alpha\tan\alpha$

$\frac{1}{\cos\alpha}-\frac{\cos^2\alpha}{\cos\alpha}=\frac{1-\cos^2\alpha}{\cos\alpha}$

$=\frac{\sin^2\alpha}{\cos\alpha}$

$=\sin\alpha\cdot\frac{\sin\alpha}{\cos\alpha}$

$=\boxed{\sin\alpha\tan\alpha}$

8. $\csc\theta-\sin\theta=\cos\theta\cot\theta$

$\frac{1}{\sin\theta}-\frac{\sin^2\theta}{\sin\theta}=\frac{1-\sin^2\theta}{\sin\theta}=\frac{\cos^2\theta}{\sin\theta}$

$=\cos\theta\frac{\cos\theta}{\sin\theta}$

$=\cos\theta\cot\theta$

9. $\sec\alpha-\cos\alpha=\sin\alpha\tan\alpha$

~~$\frac{1}{\cos\alpha}-\frac{\cos^2\alpha}{\cos\alpha}=\frac{1-\cos}{}$~~

10. $\sin^4 x+\cos^4 x+2\sin^2 s\cos^2 x=1$

$\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x$

$(\sin^2 x+\cos^2 x)^2=1^2=\boxed{1}$

Verify that each of the following is an identity

1. $(1+\sin\theta)(1-\sin\theta)=\dfrac{1}{\sec^2\theta}$

$(1+\sin\theta)(1-\sin\theta) = 1-\sin^2\theta = \cos^2\theta = \dfrac{1}{\sec^2\theta}$ ✓

2. $\cos^2 x\cot^2 x=\cot^2 x-\cos^2 x$

$\cot^2 x - \cos^2 x = \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\cos^2 x \sin^2 x}{\sin^2 x} = \dfrac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} = \dfrac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}$

$= \cos^2 x\left(\dfrac{\cos^2 x}{\sin^2 x}\right) = \cos^2 x \cot^2 x$ ✓

3. $\tan^4 w+2\tan^2 w+1=\sec^4 w$

$\tan^4 w + 2\tan^2 w + 1 = (\tan^2 w + 1)^2 = (\sec^2 w)^2 = \sec^4 w$ ✓

4. $\sin^2 x(\csc^2 x+\sec^2 x)=\sec^2 x$

$\sin^2 x(\csc^2 x + \sec^2 x) = \sin^2 x\left(\dfrac{1}{\sin^2 x}\right) + \sin^2 x \dfrac{1}{\cos^2 x} = 1 + \dfrac{\sin^2 x}{\cos^2 x}$

$= 1 + \tan^2 x = \sec^2 x$ ✓

5. $\dfrac{\sin x+\cos x}{1-\sin x}=\dfrac{1+\cot x}{\csc x-1}$

$\dfrac{1+\cot x}{\csc x - 1} = \dfrac{1+\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - 1} \cdot \dfrac{\sin x}{\sin x} = \dfrac{\sin x + \cos x}{1 - \sin x}$ ✓

6. $\dfrac{1-\tan x}{1+\tan x}=\dfrac{\cot x-1}{\cot x+1}$

$\dfrac{1-\tan x}{1+\tan x} \cdot \dfrac{\cot x}{\cot x} = \dfrac{\cot x - 1}{\cot x + 1}$ ✓

Prove Each Identity

1. $\cot^2\theta + \cos^2\theta + \sin^2\theta = \csc^2\theta$

$= \cot^2\theta\ (\cos^2\theta + \sin^2\theta)$

$= \cot^2\theta + 1$

$= \csc^2\theta$

✓

2. $\dfrac{\cot\theta - \tan\theta}{\sin\theta\cos\theta} = \csc^2\theta - \sec^2\theta$

$\dfrac{\cot\theta - \tan\theta}{\sin\theta\cos\theta} = \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\sin\theta\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}$

$= \csc^2\theta - \sec^2\theta$

✓

3. $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = \sin\theta\csc\theta$

$\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = \sin\theta\cdot\dfrac{1}{\csc\theta} + \cos\theta\,\dfrac{1}{\sec\theta}$

$= \sin\theta\cdot\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$= \sin\theta\cdot\dfrac{1}{\sin\theta}$

$= \sin\theta\cdot\csc\theta$

✓

4. $\dfrac{1-\sin^2\theta}{1+\cot^2\theta} = \sin^2\theta\cos^2\theta$

$\dfrac{1-\sin^2\theta}{1+\cot^2\theta} = \dfrac{\cos^2\theta}{\csc^2\theta}$

$\cancel{\cos^2\theta\cdot\dfrac{1}{\csc^2\theta}}$

$= \dfrac{1}{\csc^2\theta}\cdot\cos^2\theta$

$= \sin^2\theta\ \cos^2\theta$

✓

1. $\sin 95^\circ \cos 55^\circ + \cos 95^\circ \sin 55^\circ$

$\sin(95+55) = \sin(150) = \boxed{\frac{1}{2}}$

2. $\cos 160^\circ \cos 40^\circ + \sin 160^\circ \sin 40^\circ$

$\cos(160-40) = \cos 120 = \boxed{-\frac{1}{2}}$

3. $\sin(-165^\circ)$

$\sin(-165) = \sin(-120-45) = \sin(-120)\cos(45) - \cos(-120)\sin(45)$
$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

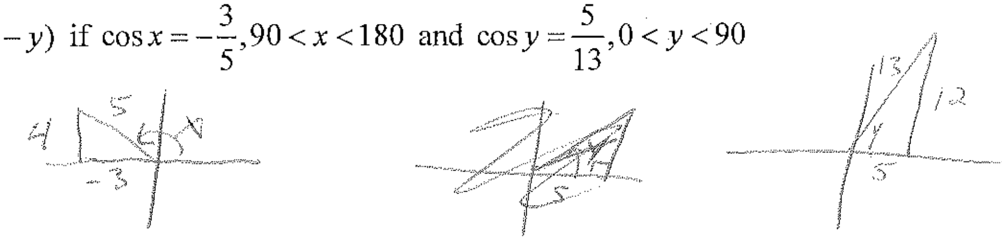
4. $\cos(-75^\circ)$

$\cos(-75) = \cos(-30-45) = \cos(-30)\cos(45) + \sin(-30)\sin(45)$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

5. $\cos(\pi - \theta) = -\cos\theta$

$\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta = (-1)\cos\theta + (0)\sin\theta$
$= -\cos\theta$

6. Find $\cos(x-y)$ if $\cos x = -\frac{3}{5}, 90 < x < 180$ and $\cos y = \frac{5}{13}, 0 < y < 90$



$\cos(x-y) = \cos x \cos y + \sin x \sin y$
$= \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$
$= \frac{-15}{65} + \frac{48}{65}$
$= \boxed{\frac{33}{65}}$

Simplify Each Expression

1. $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$

$\sin(75+15) = \sin 90$
$= \boxed{1}$

2. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) = \cos\left(\frac{6\pi}{12}\right)$
$= \cos\left(\frac{\pi}{2}\right)$
$= \boxed{0}$

3. $\sin 3x \cos 2x - \cos 3x \sin 2x$

$\sin(3x - 2x) = \sin x$

Prove that the given equation is an identity:

4. $\sin(x+\pi) = -\sin x$

$\sin(x+\pi) = \sin x \cos \pi + \cos x \sin \pi$
$= \sin x\,(-1) + \cos x\,(0)$
$= -\sin x$

5. $\cos\left(x + \frac{\pi}{2}\right)$

$\cancel{\cos(x)\sin\left(\frac{\pi}{2}\right) - \sin x \cos\left(\frac{\pi}{2}\right)}$
$\cancel{\cos x\,(1) -}$
$\cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right)$
$\cos x\,(0) - \sin x\,(-1)$
$\sin x$

6. Translate the graph of $y = \sin x$ to the right by π units, and then reflect this curve about the $x-axis$. What graph results?

$y = \sin(-x - \pi) = \sin(-x)\cos\pi - \cos(x)\sin\pi$
$= -\sin x\,(-1) - \cos x\,(0)$
$= \sin x$

Find the exact value of each expression

7. $\cos 75^\circ$

$\cos(30+45)$

$= \cos 30 \cos 45 - \sin 30 \sin 45$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

8. $\cos 105^\circ$

$\cos(60+45)$

$\cos 60 \cos 45 - \sin 60 \sin 45$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

9. $\sin(-15^\circ)$

$\sin(30-45)$

$= \sin 30 \cos 45 - \cos 30 \sin 45$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

10. $\sin\frac{7\pi}{12}$

$\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

Simplify each expression

11. $\sin(30^\circ+\theta)+\sin(30^\circ-\theta)$

$\sin 30 \cos\theta + \cos 30 \sin\theta + \sin 30 \cos\theta - \cos 30 \sin\theta$

$\left(\frac{1}{2}\right)\cos\theta + \frac{\sqrt{3}}{2}\sin\theta + \left(\frac{1}{2}\right)\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$\boxed{\cos\theta}$

12. $\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)$

$\cos\frac{\pi}{3}\cos x - \sin x \sin\frac{\pi}{3} + \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x$

$\left(\frac{1}{2}\right)\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$\boxed{\cos x}$

13. $\cos\left(\frac{3\pi}{2}+x\right)+\cos\left(\frac{3\pi}{2}-x\right)$

$\cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x + \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$(0)\cos x + 0 \cos x$

$\boxed{0}$