# 4.1: FACTORS OF INTEGERS AND POLYNOMIALS
# 4.2: THE QUOTIENT – REMAINDER THEOREM

## I. INTEGERS AND MONOMIALS

A. Example 1. for all integers *m* and *n*:
- Is 7 a factor of $14n - 21m^3$?
  Yes
- Is $3mn^3$ a factor of $12m^3n^2$?
  No
- Is -255 a multiple of 15?
  Yes
- Is 31 a multiple of 0?
  No

## II. POLYNOMIALS

A. A function in the form $P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, with $a_n \neq 0$ is a polynomial.

Leading Coefficient — Degree — Constant Term

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

Coefficients

## II. POLYNOMIALS

B. Example 2: What is the degree of each of the following polynomials?

1. $f(x) = 4x^3 - 2x^2 + x - 5$

   3

2. $g(x) = x^3 + 2x^4 - 5x + 9$

   4

3. $h(x) = 95 - 3x^2$

   2

## II. POLYNOMAILS

C. Example 3: Show that $g(x) = x^2 + x - 12$ is a multiple of $f(x) = x - 3$.

$$g(x) = x^2 + x - 12 = (x-3)(x+4)$$

expanded form — factored form

D. Example 4: Write $h(x) = 16 - x^4$ as the product of three polynomials.

$$h(x) = 16 - x^4$$
$$h(x) = (4 + x^2)(4 - x^2)$$
$$h(x) = (4 + x^2)(2 + x)(2 - x)$$

## III. QUOTIENT – REMAINDER THEOREM

A. Quotient Remainder Theorem. For an integer *n* is divided by a positive integer *d:*

$$\frac{n}{d} = q + \frac{r}{d}$$

Quotient — Remainder

or

$$n = q \cdot d + r$$

Note: $0 \leq r < d$

## III. QUOTIENT – REMAINDER THEOREM

B. Example 5. Find *q* and *r* that fit the above theorem for the given *n* and *d* values.

1. *n=19, d=4* $19 = 4(4)+3$ *q=4, r=3*
2. *n=-19, d=3* $-19 = -7(3)+2$ *q=-7, r=2*
3. *n=42,d=12* $42 = 3(12)+6$ *q=3, r=6*
4. *n=-72, d=11* $-72 = -7(11)+5$ *q=-7, r=5*
5. $n = 2m^3 + 4m^2 - 7m + 19,\ d = m^2$

$$2m^3 + 4m^2 - 7m + 19 = (2m+4)(m^2) + (-7m+19)$$

$q = 2m + 4,\ r = -7m + 19$

## III. QUOTIENT – REMAINDER THEOREM

C. Example 6. Letter boxes 5 inches wide are placed on a shelf that is 10 feet 3 inches long. How many boxes will fit? How much space will be left over?

10 feet 3 inches = 123 inches

$123 = 24(5) + 3$

*q=24, r=3*

*24 boxes will fit, and there will be 3 inches left over.*

## HOMEWORK

p. 228-229 # 1-4, 6, 13, 14, 18

p. 235-237 # 1, 3-6, 9, 12, 17 a,b

# 4.3: Polynomial Division and the Remainder Theorem

## I. Long Division

A. Example 1: Divide each of the following

1. $9\overline{)4369}$

2. $2x+1\overline{)6x^3-9x^2+8x+1}$

## I. Long Division

3. $3x^2+2x\overline{)6x^6-20x^5-7x^4+9x^3-x^2-2x+7}$

## I. Long Division

4. Divide $2x^4-15x^2-10x+5$ by $x-3$

## I. Long Division

B. Example 2: If *n(x)* which has degree 6 is divided by *d(x)* which has degree 2, what do you know about the degrees of *q(x)* and *r(x)*.

*q(x)* has degree 4 and *d(x)* has degree of 0 or 1

C. Example 3: If *p(x)* is divided by *d(x)* and the quotient has degree 4 while the remainder has degree 2, what do you know about the degrees of *p(x)* and *d(x)*.

*d(x)* has degree of at least 3, and *p(x)* has degree of at least 7

## II. The remainder theorem

A. If a polynomial p(x) of degree is divided by (x-c), then the remainder is the constant p(c).

B. Use the remainder theorem to calculate the remainder for the following division problem.

$$\frac{y^6-7y^4-18y^2+5y-3}{y-3}$$

c=3

$$(3)^6-7(3)^4-18(3)^2+5(3)-3=\boxed{12}$$

## Homework

p. 241-243 # 2, 3, 5-8, 11, 12, 14, 15

# 4.3 PART II: SYNTHETIC DIVISION

## I. EXAMPLES

A. Divide using synthetic division:

1. $\dfrac{3x^5+5x^4-4x^3+7x+3}{x+2}$

2. $\dfrac{2x^3-7x^2+5}{x-3}$

3. Divide $P(x)=3x^3+x^2-8x-5$ by $x-2$

4. $\dfrac{6x^5+x^4+x^3-3x^2+9}{x+\frac{1}{2}}$

5. $\dfrac{4x^4-2x^2+3x+1}{2x+1}$

## HOMEWORK

- Synthetic Division Worksheet

# 4.4: Zeros of Polynomial Functions

## I. Factors, Zeros, etc.

A. If *f(c)=0*, then:
- $(x - c)$ is a factor of *f*
- *x=c* is a solution (zero) of *f*
- *(c,0)* is an *x-intercept* of *f*
- The remainder of $\frac{f(x)}{x-c}$ *is* 0

## I. Factors, Zeros, etc.

B. Consider $f(x) = x^2 - 5x + 6$.

$$f(2) = (2)^2 - 5(2) + 6 = 0$$

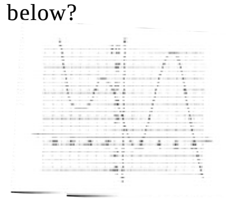
Therefore:
- $(x - 2)$ is a factor of $x^2 - 5x + 6$
- *x=2* is a solution (zero) of *f*
- *(2,0)* is an *x-intercept* of *f*
- The remainder of $\frac{x^2-5x+6}{x-2}$ *is* 0

## II. Number of Zeros

A. Theorem: If *P(x)* is a polynomial of degree *n*, then any horizontal line *y=k* crosses *P(x)* at most *n* times.

B. Example 2: What is the smallest possible degree of *p(x)* graphed below?

5



## II. Number of Zeros

A. A polynomial of degree *n* has at most *n* zeros.

B. Example 1: Find all zeros for

$$p(x) = x^4 - 4x^3 - x^2 + 16x - 12$$

*x=1 and x=2 are zeros*

$$p(x) = (x - 1)(x^3 - 3x^2 - 4x + 12)$$
$$p(x) = (x - 1)(x - 2)(x^2 - x - 6)$$
$$p(x) = (x - 1)(x - 2)(x - 3)(x + 2)$$
$$x = -2,1,2,3$$

## III. Factor Theorem

C. *x-c* is a factor if *P(x)* iff *P(c)=0*

*D. Example 3: Is* $x + 2$ *a factor of* $x^5 - 3x^4 + 8x^2 - 9x + 27$?

$$(-2)^5 - 3(-2)^4 + 8(-2)^2 - 9(-2) + 27 = -3$$

No, since the remainder is not zero.

## Homework

p. 247-249 #2, 3, 5, 6 (some are not real), 8, 9, 13, 15, 18, 19