9.1 PART I: SECANTS

I. PARALLEL AND PERPENDICULAR LINES

A. Example 1: Write the equation of the line through *(4,3)* that is parallel to $y = \frac{3}{2}x + 1$.

$$y - 3 = \frac{3}{2}(x - 4)$$
$$y = \frac{3}{2}x - 3$$

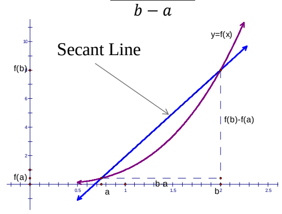
B. Example 2: Write the equation of the line through *(4,3)* that is perpendicular to $y = \frac{3}{2}x + 1$.

$$y - 3 = -\frac{2}{3}(x - 4)$$
$$y = -\frac{2}{3}x + \frac{17}{3}$$

II. AVERAGE RATE OF CHANGE

A. **Definition: The Average Rate of Change for a function *f(x)* from *x=a* to *x=b* is:**

$$\frac{f(b) - f(a)}{b - a}$$



II. AVERAGE RATE OF CHANGE

B. Example 1: Find the average rate of change on the function $f(x) = x^2 - 4x + 1$ on the interval $[2, 5]$

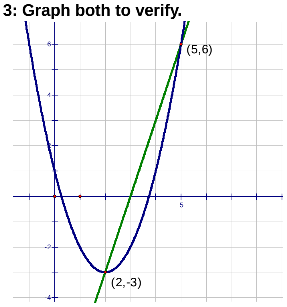
$$r.o.c. = \frac{f(5) - f(2)}{5 - 2} = \frac{6 - (-3)}{3} = 3$$

C. Example 2: Find the equation of the secant line between the points from example 1.

$$y - 6 = 3(x - 5)$$
$$y = 3x - 9$$

II. AVERAGE RATE OF CHANGE

C. Example 3: Graph both to verify.



III. PRACTICE

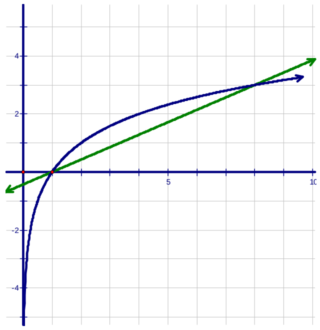
Example 4: Find the average rate of change of $g(x) = \log_2 8$ on $[1, 8]$, as well as the secant line.

$$r.o.c. = \frac{f(8) - f(1)}{8 - 1} = \frac{\log_2 8 - \log_2 1}{7} = \frac{3 - 0}{7} = \frac{3}{7}$$

Secant Line

$$y - 0 = \frac{3}{7}(x - 1)$$
$$y = \frac{3}{7}x - \frac{3}{7}$$

III. PRACTICE



HOMEWORK

9.1 Worksheet 1

9.1 Part II:

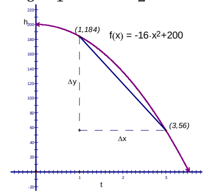
Instantaneous Rates of Change

I. Average vs. Instantaneous Rates of Change

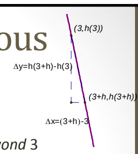
A. Example 1: The position of an object in free fall is given by $h(t) = -16t^2 + 200$, where *h* is measured in feet and *t* is measured in seconds.

a. Find the average rate of speed from *t = 1* to *t = 3*.

$$r.o.c. = \frac{\Delta y}{\Delta x} = \frac{h(3) - h(1)}{3 - 1} = \frac{56 - 184}{2} = 64\frac{ft}{sec}$$



I. Average vs. Instantaneous Rate of Change



b. Find the velocity at *t = 3 (instantaneous).*

To find a slope, we choose a point just beyond 3

$$\frac{\Delta y}{\Delta x} = \frac{h(3+h) - h(3)}{(3+h) - 3}$$

$$\frac{\Delta y}{\Delta x} = \frac{[-16(3+h)^2 + 200] - [-16(3)^2 + 200]}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{56 - 96h - 16h^2 - 56}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{-96h - 16h^2}{h}$$

$$\frac{\Delta y}{\Delta x} = -96 - 16h$$

When *h=0*, $\frac{\Delta y}{\Delta x} = -96\frac{ft}{sec}$

II. Instantaneous Slope

A. Example 2: Find the slope of f$(x) = x^2$ at $x = 4$.

$$\frac{\Delta y}{\Delta x} = \frac{f(4+h) - f(4)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{(4+h)^2 - (4)^2}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{16 + 8h + h^2 - 16}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{8h + h^2}{h}$$

$$\frac{\Delta y}{\Delta x} = 8 + h$$

When *h = 0*

$$\frac{\Delta y}{\Delta x} = 8$$

II. Instantaneous Slope

B. Example 3: Find the slope of g$(x) = x^2 - 4x + 1$ at *x = 3*.

$$\frac{\Delta y}{\Delta x} = \frac{g(3+h) - g(3)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{9 + 6h + h^2 - 12 - 4h + 1 - (-2)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{h^2 + 2h}{h}$$

$$\frac{\Delta y}{\Delta x} = h + 2$$

When *h = 0*

$$\frac{\Delta y}{\Delta x} = 2$$

III. Slope Definition

The slope of the curve *y = f(x)* at *x=a* is:

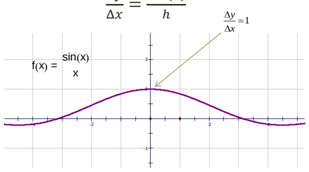
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

IV. Other Functions

A. Example 4: find the slope of $f(x) = sinx$ at $x=0$.

$$\frac{\Delta y}{\Delta x} = \frac{sin(0+h)-sin0}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{sin(h)}{h}$$



$$\frac{\Delta y}{\Delta x} = 1$$

IV. Other Functions

B. Find the slope of $f(x) = e^x$ at $x = 3$

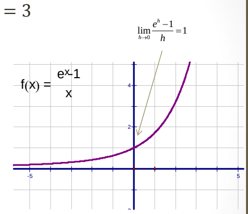
$$m = \frac{\Delta y}{\Delta x} = \frac{f(3+h) - f(3)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{e^{3+h} - e^3}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{e^3 e^h - e^3}{h}$$

$$\frac{\Delta y}{\Delta x} = e^3 \left(\frac{e^h - 1}{h}\right)$$

$$\frac{\Delta y}{\Delta x} = e^3$$



Homework

9.1 B worksheet

9.1 Part III:

Tangent and Normal Lines

I. The Tangent Line

A. Definition: the tangent line at a point $(a, f(a))$ has the same slope as the instantaneous slope of $f(x)$at a.

B. Example 1: find the equation for the line tangent to $f(x) = x^2$ at $x = 3$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h}$$

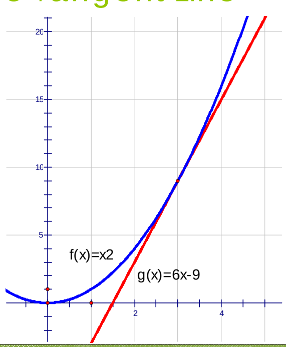
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h} = 6$$

$$(3, f(3)) = (3,9)$$

$$y - 9 = 6(x - 3)$$

$$y = 6x - 9$$

I. The Tangent Line



I. The Tangent Line

C. Example 2: Find the slope of the tangent line to the function $f(x) = \frac{1}{x}$ at $x = 1$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h}$$

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h} \frac{(1+h)}{(1+h)}$$

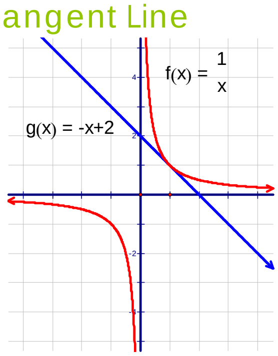
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{1 - (1+h)}{h(1+h)} = -1$$

$$(1, f(1)) = (1,1)$$

$$y - 1 = -1(x - 1)$$

$$y = -x + 2$$

I. The Tangent Line



II. The Normal Line

A. Definition: the normal line (think physics) at a point $(a, f(a))$ intersects the tangent line and is perpendicular to it.

B. Example 3: Write the equation for the tangent and normal to the curve $f(x) = x^2 - 3x$ at $x = -2$.

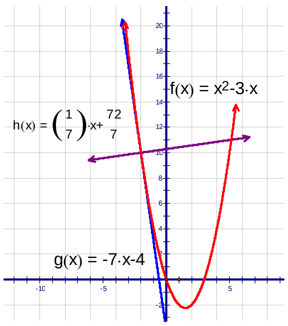
$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{[(-2+h)^2 - 3(-2+h)] - [(-2)^2 - 3(-2)]}{h}$$

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{10 - 7h + h^2 - 10}{h} = -7$$

$$(-2, f(-2)) = (-2,10)$$

Tangent	Normal
$y - 10 = -7(x + 2)$	$y - 10 = \frac{1}{7}(x + 2)$
$y = -7x - 4$	$y = \frac{1}{7}x + \frac{72}{7}$

II. The Normal Line



II. The Tangent Line

C. Find the tangent and normal lines to the function $f(x) = sinx$ at $x = 0$.

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{sin(0+h) - sin0}{h}$$

$$m = \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{sinh}{h} = 1$$

$$(0, f(0)) = (0,0)$$

Tangent

$$y - 0 = 1(x - 0)$$

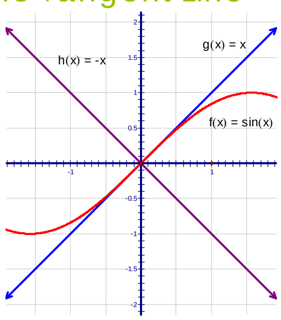
$$y = x$$

Normal

$$y - 0 = -1(x - 0)$$

$$y = -x$$

II. The Tangent Line



Homework:

9.1 Worksheet #4