3.1: Arithmetic Operations on Functions

Pre Calculus II

If f and g are real-valued functions:

$(f+g)(x) = f(x)+g(x)$

$(f-g)(x) = f(x)-g(x)$

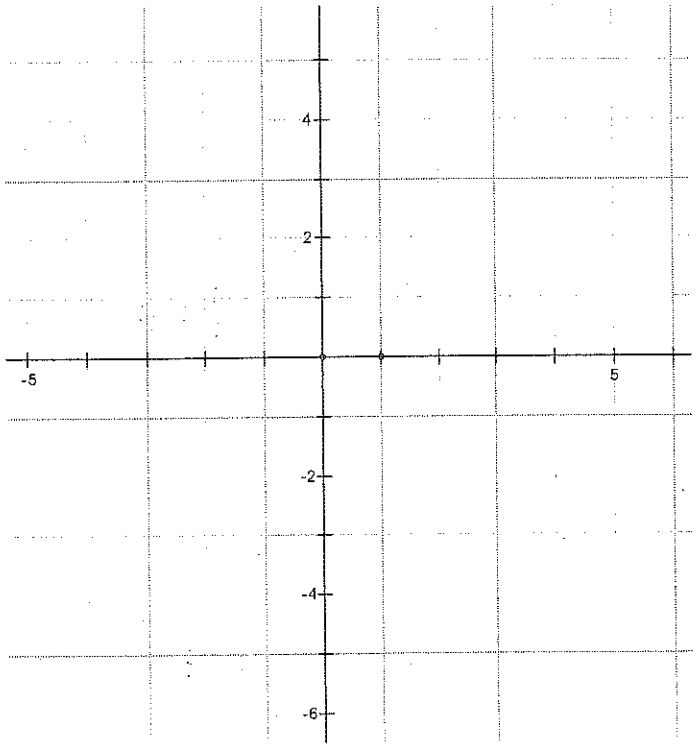
$(f \cdot g)(x) = f(x) \cdot g(x)$ <-- this is multiplication, not composition.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ for $g(x) \neq 0$

EX 1: Use the functions $f(x) = -x^2 + 4$ and $g(x) = \frac{-1}{2}x + 2$

(a) Graph f(x) and g(x)

(b) Use the graphs to graph $(f+g)(x)$
To do this, add the y values of f and g to create the new graph.



(c) Also graph $(f \cdot g)(x)$

(d) Use your graphing calculator to graph $\left(\frac{f}{g}\right)(x)$. Where is it undefined? Why?

EX 2: Given $h(x)=(x-2)^2$ $\quad g(x)=-(x+2)^2$

(a) Algebraically, find a simplified expression for $(h+g)(x)$

(b) Algebraically, find a simplified expression for $(h-g)(x)$

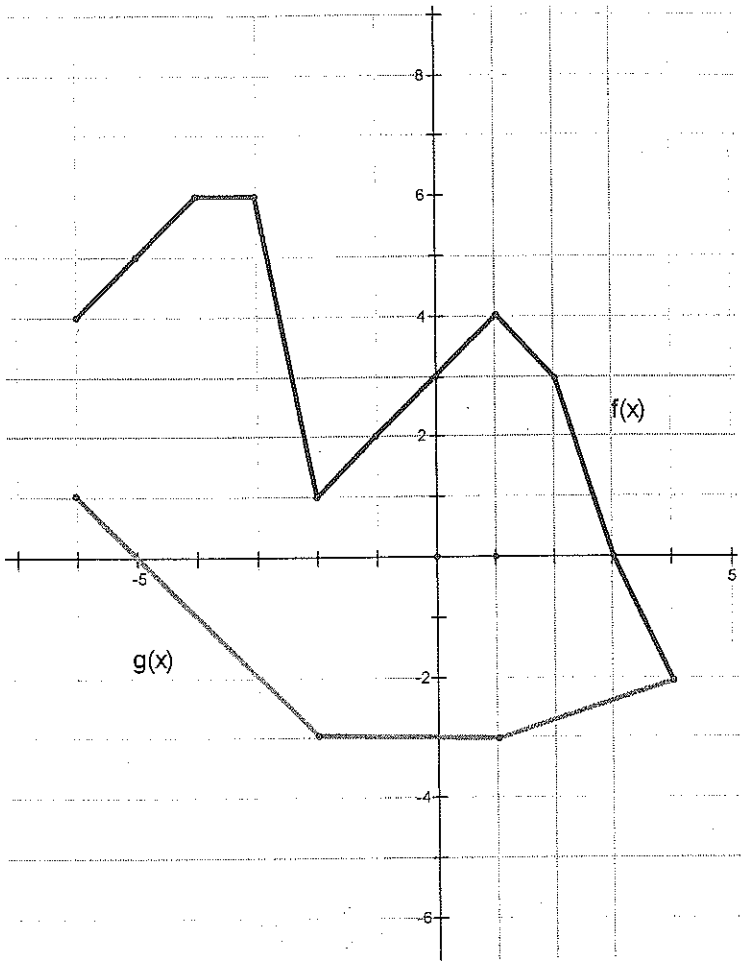
EX 3: Given $f(x)=\frac{1}{2}x-2$ and $g(x)=\frac{-1}{2}x-4$

(a) Algebraically, find (fg)(x) in simplest form (this is multiplication). Then graph on the calculator to confirm.

(b) Algebraically, find $\left(\frac{f}{g}\right)(x)$ in simplest form, then graph to confirm.

EX 4: Use the given graphs of f and g to create a table and graph (f+g)(x) and (f-g)(x)

x	f	g	f+g	f-g
-6				
-5				
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				



HW: p. 151-153 # 5, 7-11, 13, 19-22.

NAME: _____

3.1 WS: Combining Functions Pre Calculus II

Find *f + g, f – g, fg, and f/g* and the domains of each.

1. $f(x) = x^2 - x,\ g(x) = x + 5$

f+g *Domain*_____ *f-g* *Domain*_____

fg *Domain*_____ *f/g* *Domain*_____

2. $f(x) = x^3 + 2x^2,\ g(x) = 3x^2 - 1$

f+g *Domain*_____ *f-g* *Domain*_____

fg *Domain*_____ *f/g* *Domain*_____

3. $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1-x}$

f+g Domain_____ *f-g* Domain_____

fg Domain_____ *f/g* Domain_____

4. $f(x) = \sqrt{9-x^2}$, $g(x) = \sqrt{x^2-1}$

f+g Domain_____ *f-g* Domain_____

fg Domain_____ *f/g* Domain_____

5. $f(x) = \frac{2}{x}$, $g(x) = -\frac{2}{x+4}$

f+g Domain_____ *f-g* Domain_____

fg Domain_____ *f/g* Domain_____

6. $f(x) = \frac{1}{x+1}$, $g(x) = \frac{x}{x+1}$

f+g *Domain*_____ *f-g* *Domain*_____

fg *Domain*_____ *f/g* *Domain*_____

Find the Domain of each section and then f(x).

7. $f(x) = \frac{\sqrt{4-x} + \sqrt{3+x}}{x^2 - 1}$

8. $f(x) = \sqrt{1-x} + \sqrt{x-2}$

NAME: _____

3.2 WS Pre Calculus II Compositions, Inverses of functions

Let $g(x)=x^2$ and $h(x)=2x+1$

1. $g(-2)$

2. $h(3)$

3. $g(h(2))$

4. $h(g(-3))$

5. $(g\circ g)(x)$

6. $(h\circ g)(x)$

For each pair of functions, find $f(g(2))$ and $g(f(2))$

7. $f(x)=3x-2, g(x)=x^2$

8. $f(x)=x^2+1, g(x)=2x-5$

9. $f(x)=x^3, g(x)=x^2-6$

10. $f(x)=x^2+2x+4, g(x)=x-1$

11. $f(x)=5x, g(x)=x+4$

12. $f(x)=x-2, g(x)=x^3$

Let $f(x)=x+3$, $g(x)=x^2$, and $h(x)=2x-3$. Find each composite function.

13. $g(g(x))$

14. $f(f(x))$

15. $f(h(x))$

16. $g(f(x))$

For each pair of functions, find $(h \circ k)(x)$ and $(k \circ h)(x)$

17. $h(x) = x + 7, k(x) = 2x - 3$

18. $h(x) = 5x - 8, k(x) = 3x + 5$

19. $h(x) = x^2 + 2x + 1, k(x) = x - 1$

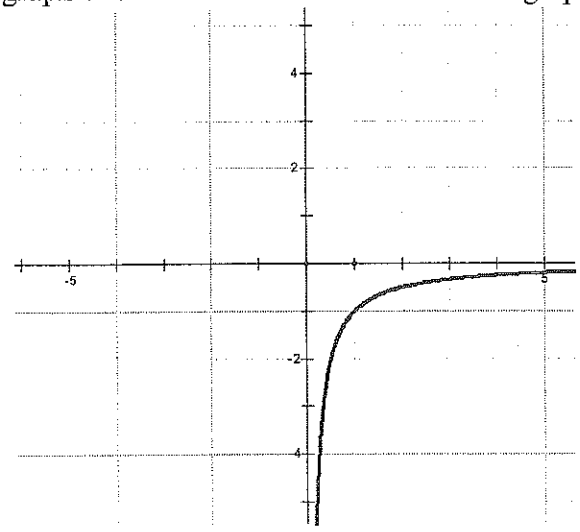
20. $h(x) = x^3, k(x) = x^4$

21. $h(x) = x^2 - x, k(x) = x + 2$

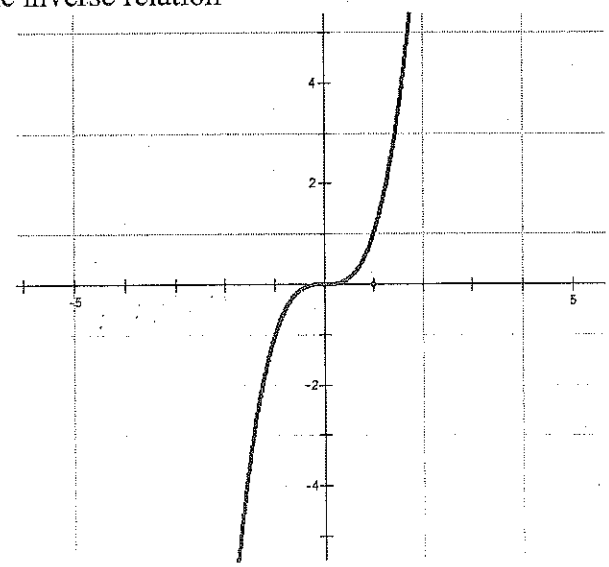
22. $h(x) = 2x + 6, k(x) = \frac{1}{2}x - 3$

The graph of a relation is shown. Sketch the graph of the inverse relation

23.

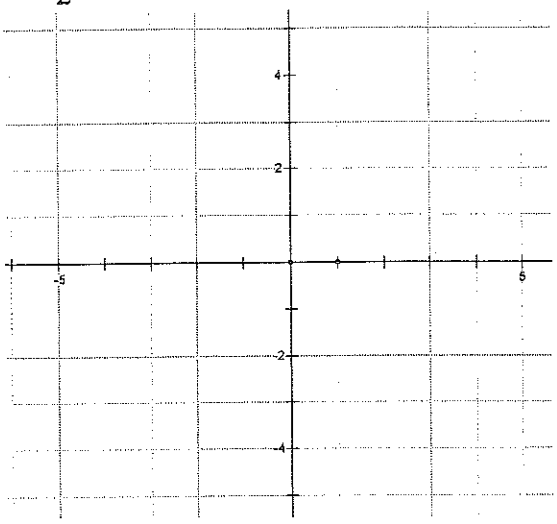


24.

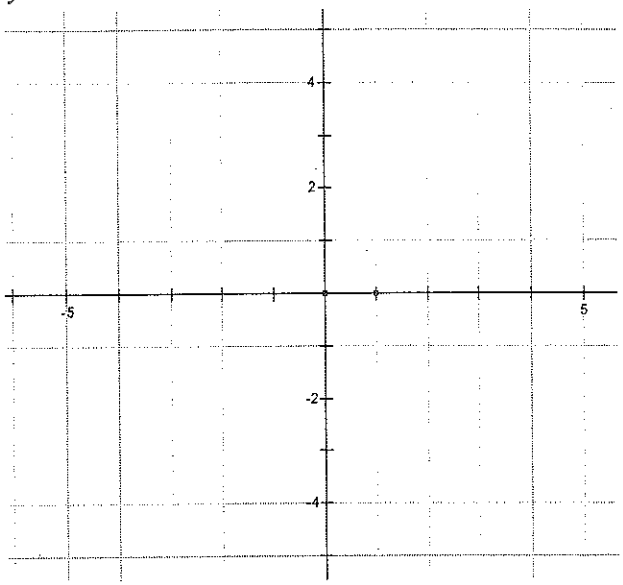


Graph each function and its inverse

25. $y = \frac{1}{2}x + 2$



26. $y = x^3$



For each pair of functions, prove that the functions are inverses of each other

27 $f(x) = x + 7, g(x) = x - 7$

28. $f(x) = 2x + 5, \frac{1}{2}x - \frac{5}{2}$

HW: p. 158-160 # 1-5, 7-19. (for 15 and 16, just use laws of logs)

3.3: The Logic of Equation Solving

Pre Calculus II (CP)

EX 1: Solve $5-x=\sqrt{x+7}$. Justify your steps

Check your answers to make sure both work:

When solving an equation, these steps are non-reversible -- doesn't work in the other direction.

1. Squaring both sides.
2. Multiplying both sides by an expression that may equal zero.

So when you use non-reversible steps in solving an equation, you are limiting the solution to some number of possibilities (often two). You have not finished solving the equation until you substitute each and see if they work.

EX 2: Solve the equation. Are there any non-reversible steps? If so, what are they?

$$\frac{1}{x^2-4x-5}=\frac{1}{3x-15}$$

EX 3: $\sqrt[5]{4n} = 3$ Solve. Note any non-reversible steps.

EX 4: $\log(x^3 + 3x^2 + 5x - 7) = 3\log(x+1)$ Solve. Note any non-reversible steps.

EX 5: $\sqrt{x+14} = x+2$ Solve. Note any non-reversible steps.

HW: p. 165-166 # 1, 4-8, 11-18.

NAME: _____

3.3 WS – Logic of Equation Solving Pre Calculus II (CP)

Solve each equation over the set of real numbers.

1. $x^2 - 6x + 9 = 0$

2. $10y = 4y^2$

3. $(x+2)(x-2) = 3x$

4. $\sqrt{2x+7} = 7$

5. $\sqrt{x+2} - x = 2$

6. $\sqrt[3]{x+1} = -4$

7. $x^{\frac{2}{5}} = 4$

8. $\frac{1}{y^2 - 9} = \frac{1}{y - 3}$

9. $e^{3x-1} = e^{2x}$

10. $\log_{12}(x^2) = \log_{12}(4x\text{-}21)$

Also, p. 167 # 19-23