Pre Calc II 2.7 (Part 1) Exponent Review WS

Simplify the following leaving answers as exact values. No Calculator. Assume all variables represent non-zero real numbers.

Part 1:

1. $a^9 \cdot a^{12}$
2. $(-2x^2y)(-8x^3y^9)$
3. $3x^0$
4. $-2p^0 + 8z^0$

Part 2:

1. $(-2)^{-4}$
2. -2^4
3. $(3x)^{-2}$
4. $3^{-1} + 4^{-1}$

Part 3: Write using only positive exponents.

1. $\frac{4^8}{4^2}$
2. $\frac{7^3}{7^{-5}}$
3. $\frac{s^2}{s^9}$
4. $\frac{r^{-8}}{r^{-3}}$

Part 4:

1. $(3x^2y^5)^2$
2. $\left(\frac{2t^4}{u^7}\right)^4$
3. x^{-4}
4. $\left(\frac{2}{3}\right)^{-3}$

Part 5:

1. $z^{-4} \cdot z^{-11} \cdot z^{5}$

2. $\left(2^{-3}\right)^{2}$

3. $x^{-4} \cdot x^{6} \cdot x^{-2}$

4. $\dfrac{p^{-11}q^{3}}{p^{-5}q^{4}}$

Part 6

1. $100^{\frac{1}{2}}$

2. $-625^{\frac{3}{4}}$

3. $27^{\frac{-2}{3}}$

4. $-125^{\frac{-2}{3}}$

Part 7:

1. $9^{\frac{1}{6}} \cdot 9^{\frac{1}{3}}$

2. $\left(3^{\frac{3}{4}}\right)^{2}$

3. $\left(a^{2}b^{\frac{3}{5}}\right)^{10}$

4. $\sqrt[12]{64^{4}}$

Logistic Growth WS Pre Calc II Show work on loose leaf.

1. The logistic model $P(t)=\dfrac{1500}{1+24e^{-0.75t}}$ models the growth of a population. Use the equation to
   (a) Find the value of k
   (b) Find the carrying capacity
   (c) Find the initial population (let t = 0 and simplify)
   (d) Determine when the population will reach 50% of its carrying capacity

2. Suppose a population is infected with a virus. The number of people infected is modeled by the logistic equation $P(t)=\dfrac{2000}{1+249e^{-0.97t}}$ where p is the population infected (number of people) and t is the number of days since it started spreading.
   (a) How many people are infected initially? (at time t = 0)
   (b) Determine the number of people infected after 1 day, two days and five days.
   (c) Graph the function p(t) and describe its behavior.

3. A population of rabbits is given by the formula $P(t)=\dfrac{1000}{1+122e^{-0.7t}}$

(a) How many rabbits do they start with?

(b) What is the carrying capacity?

(d) How many rabbits will there be after 5 days?

(d) How long will it take until there are 500 rabbits?

4. The number of students infected by measles in a certain school is given by the formula $P(t)=\dfrac{2000}{1+1999e^{-.8t}}$ where t is the number of days after students are first exposed to an infected student.

(a) How many students have measles initially?

(b) How many will have measles after 3 days?

(c) What does the 2000 in the numerator represent?

(d) How long will it be until 175 students are infected?

5. A 2000 gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank.

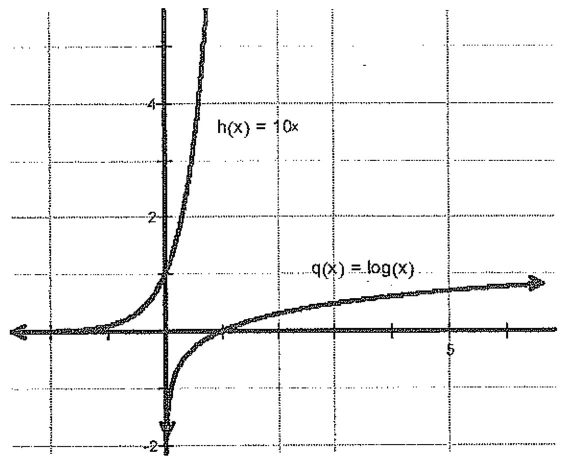
(a) Use t = 0 and the initial value of P = 6 to solve for A. Use k = 0.225. Keep 4 decimal places.

(b) Write your logistic model using the info. found in (a).

(c) How long will it take for the guppy population to reach 100? 125?

2.9: (part 1) Review of Rules of Logarithms Pre Calc II (CP)

EX 1: Given $y = 10^x$, find and graph its inverse: Inverse is *y=log(x)*



DEF: $\mathbf{y = log_{10} x}$ (also written y = log (x)) is the logarithm of x to the base 10.
iff $10^y = x$.

To write in exponential form, make $\log_{10} x = y$ into $10^y = x$.

Often written as a "common log" without the base. log (x) mean base 10.

EX 2: Evaluate $\log_{10} 10{,}000 = y$ (REMEMBER $\log_{10} x = y$ means $10^y = x$)

EX 3: Evaluate $\log_{10} 0.01$ (= y) (First write: 0.01 as a power of 10):

EX 4: Solve for x: $\log x = 3.5$

Logarithms to Bases Other Than 10

Def: If $b > 0$ and $b \neq 1$, then (n) is the log of m to base b written $n = \log_b m$ iff $b^n = m$.

The log of a negative number is not defined.

EX 5: Evaluate $\log_2 64$

EX 6: Evaluate $\log_9 27$

EX 7: Solve for x: $\log_4 32 = x$

EX 8: Solve for x: $\log_x 81 = 4$

EX 9: Solve for x: $\log_3 x = 5$

Think of these properties of logs just like exponents:

For every base b, $\boxed{\log_b 1 = 0}$

For every base b and any real number n, $\boxed{\log_b b^n = n}$

For any base b and for any positive real numbers x, y, $\boxed{\log_b(xy) = \log_b x + \log_b y}$

For any base b and for any positive real numbers x, y, $\boxed{\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y}$

For any base b and for any positive real number x, $\boxed{\log_b(x^n) = n\log_b x}$

EX 10: Find $\log_{12} 1$

EX 11: Solve: $\log_6 36 = y$

EX 12: Simplify: $\log_{15} 5 + \log_{15} 45$

EX 13: Rewrite using rule for quotients and solve: $\log_2\left(\frac{32}{8}\right)$

EX 14: Solve: $\log_3 x = \log_3 15 - \log_3 6$

EX 15: Solve for x: $\log_7 5x = 2\log_7 3 + \log_7 5$

Natural Logs: a log with a base of (e) is a natural log, written ln

$n = \ln m$ is the natural log of m. It is a log with base e.

(ie: $\ln m = \log_e m$ and $m = e^n$)

EX 16: Solve for x: $\ln x = 4$

EX 17: Solve: $\ln e^{-7}$

CHANGE OF BASE RULE: $\log_b a = \dfrac{\log_t a}{\log_t b}$

("log of the number" over "log of the base")

EX 18: Find $\log_4 100$

EX 19: $\log_8 256$

HW: Log WS

2.9 (part 1) Log WS Pre Calc II

Find each logarithm without using a calculator

1. $\log_7 49$
2. $\log_2 16$
3. $\log_2 \frac{1}{8}$
4. $\log_5 \frac{1}{5}$
5. $\log_5 \sqrt{5}$

Solve each equation (Do not use a calculator)

6. $\log_5 x = 2$
7. $\log_6 x = 2$
8. $\log x = 2$
9. $\ln x = 2$
10. $\log_x 121 = 2$
11. $\log_x 64 = 3$
12. $\log_x \left(\frac{1}{2}\right) = -1$
13. $\log_x \sqrt{6} = \frac{1}{2}$

Find each logarithm. (Do not use a calculator)

14. $\log 100$
15. $\log 10{,}000$
16. $\log 0.01$
17. $\log 0.0001$
18. $\log_2 4$
19. $\log_2 32$
20. $\log_2 64$
21. $\log_2 2^{10}$
22. $\log_3 9$
23. $\log_3 27$
24. $\log_3 243$
25. $\log_8 8^8$
26. $\log_5 0.2$
27. $\log_5 \frac{1}{125}$
28. $\log_5 \sqrt[3]{5}$
29. $\log_5 1$
30. $\log_4 64$
31. $\log_4 \frac{-1}{64}$
32. $\log_4 \sqrt[4]{4}$
33. $\log_4 1$
34. $\log_6 36$
35. $\log_6 6$
36. $\log_6 6\sqrt{6}$
37. $\log_6 \sqrt[3]{\frac{1}{6}}$

2.9 (part 2): Solving Exponential Equations Pre Calc II (CP)

Take logs of both sides (or ln) to drop the exponent in front of the log, then solve.

EX 1: Solve $3^x = 57$

EX 2: Solve $3^{2x} = 50$

EX 3: $3e^{2x} = 48$

EX 4: Suppose you invest \$5000 in an account at 4% compounded continuously. How long will it take for the money in the account to reach \$6000 if it is not touched?

EX 5: Suppose the half life of a substance is 400 years. If you start today with 2 grams, how long will it take until 1.78 grams remain?

2.9 (Part 2) WS: Solving Exponential Equations Pre Calc II

1. $10^{-x} = 2$

2. $3^{2x-1} = 5$

3. $2e^{12x} = 17$

4. $4(1 + 10^{5x}) = 9$

5. $2^{3x} = 34$

6. $3^{x/14} = 0.1$

7. $e^{3-5x} = 16$

8. $10^{1-x} = 6^x$

9. A farmer's tractor valued at $50,000 depreciates at an annual rate 10% (yearly). When will its value be $15,000?

10. If John invests $2300 in a savings account with a 6% interest rate compounded annually, how long will it take until John's account has a balance of $4150?

11. How much time is required to double your money if it is invested at 6.25% compounded continuously?

12. Suppose that in any given year, the number of cases of a disease is reduced by 20%. If there are 10,000 cases today, how many years will it take

    a) to reduce the number of cases to 1000?

    b) to eliminate the disease; that is, to reduce the number of cases to less than 1?

PC II

Practice with exponential and logarithmic equations

Name ______________________________ Hour ______

Find the solutions to the exponential equation, correct to 4 decimal places.

1. $10^{-x} = 2$

2. $3^{2x-1} = 5$

3. $2e^{12x} = 17$

4. $4(1 + 10^{5x}) = 9$

5. $2^{3x} = 34$

6. $3^{x/14} = 0.1$

7. $e^{3-5x} = 16$

8. $10^{1-x} = 6^{x}$

9. $x^2 10^x - x10^x = 2(10^x)$

10. $x^2 e^x + xe^x - e^x = 0$

Solve the logarithmic equation for x.

11. $\ln(2 + x) = 1$

12. $\log(x - 4) = 3$

13. $\log_3 (2 - x) = 3$

14. $\log_2 (x^2 - x - 2) = 2$

15. $2\log x = \log 2 + \log (3x - 4)$

16. $\log_5 x + \log_5 (x + 1) = \log_5 20$

17 $\log x + \log (x - 3) = 1$

18. $\ln (x-1) + \ln (x + 2) = 1$

**Solve for x: $\log_2 (\log_3 x) = 4$

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PC II Name:________________________________

Applications of Exponential and Logarithmic Functions

Recall formulas for calculating compounding interest and continuous rates of change:

1. $5000 is invested at 9% per year. Find the time required for the money to double if the interest is compounded a) semiannually b) continuously.

2. The population of the world in 1995 was 5.7 billion, and the estimated relative growth rate is 2% per year. If the population continues to grow at this rate, when will it reach 57 billion?

3. A bacteria culture starts with 10,000 bacteria. The number doubles every 40 minutes.
a) Write a formula for the number of bacteria at time t.
b) Find the number of bacteria after one hour.
c) After how many minutes will there be 50,000 bacteria?