Pre Calc II 2.7 (Part 1) Exponent Review WS

Simplify the following leaving answers as exact values. No Calculator. Assume all variables represent non-zero real numbers.

Part 1:

1. $a^9 \cdot a^{12}$

a^{9+12}

$\boxed{a^{21}}$

2. $(-2x^2y)(-8x^3y^9)$

$\boxed{16x^5y^{10}}$

3. $3x^0$

3

4. $-2p^0+8z^0$

6

Part 2:

1. $(-2)^{-4}$

$\frac{1}{(-2)^4}$

$\frac{1}{16}$

2. -2^4

-16

3. $(3x)^{-2}$

$\frac{1}{3^2x^2} = \boxed{\frac{1}{9x^2}}$

4. $3^{-1}+4^{-1}$

$\frac{1}{3} + \frac{1}{4}$

$\frac{1\cdot 4}{3\cdot 4} + \frac{1\cdot 3}{4\cdot 3}$

$\frac{4+3}{12} = \frac{7}{12}$

Part 3: Write using only positive exponents.

1. $\frac{4^8}{4^2}$

$4^{8-2} = \boxed{4^6}$

2. $\frac{7^3}{7^{-5}}$

$7^{3-(-5)}$

$\boxed{7^8}$

3. $\frac{s^2}{s^9}$

$s^{2-9} = s^{-7}$

$= \boxed{\frac{1}{s^7}}$

4. $\frac{r^{-8}}{r^{-3}} = r^{-8-(-3)}$

$r^{-5} = \boxed{\frac{1}{r^5}}$

Part 4:

1. $(3x^2y^5)^2$

$3^2 \cdot x^4 \cdot y^{10}$

$9x^4y^{10}$

2. $\left(\frac{2t^4}{u^7}\right)^4$

$\frac{2^4t^{16}}{u^{28}}$

$\boxed{\frac{16t^{16}}{u^{28}}}$

3. x^{-4}

$\boxed{\frac{1}{x^4}}$

4. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

$\frac{3^3}{2^3} = \boxed{\frac{27}{8}}$

2

Part 5:

1. $z^{-4} \cdot z^{-11} \cdot z^{5}$

$z^{-4+-11+5}$

$z^{-10} = \boxed{\frac{1}{z^{10}}}$

2. $\left(2^{-3}\right)^{2}$

$\left(\frac{1}{2^3}\right)^2$

$\frac{1}{2^6}$

$\boxed{\frac{1}{64}}$

3. $x^{-4} \cdot x^{6} \cdot x^{-2}$

$x^{-4+6+-2}$

x^0

$\boxed{1}$

4. $\frac{p^{-11}q^{3}}{p^{-5}q^{4}} = p^{-11-(-5)} q^{3-4}$

$= p^{-6} q^{-1}$

$= \boxed{\frac{1}{p^6 q}}$

Part 6

1. $100^{\frac{1}{2}}$

10

2. $-625^{\frac{3}{4}}$

$-\left((625)^{\frac{1}{4}}\right)^3$

$= 5^3$

$\boxed{-125}$

3. $27^{\frac{-2}{3}}$

$\frac{1}{\left(27^{\frac{1}{3}}\right)^2}$

$\frac{1}{3^2} = \boxed{\frac{1}{9}}$

4. $-125^{\frac{-2}{3}}$

$-\frac{1}{\left(125^{\frac{1}{3}}\right)^2}$

$-\frac{1}{5^2} = \boxed{-\frac{1}{25}}$

Part 7:

1. $9^{\frac{1}{6}} . 9^{\frac{1}{3}}$

$= 9^{\frac{1}{6}+\frac{2}{6}}$

$= 9^{\frac{1}{2}}$

$= \boxed{3}$

2. $\left(3^{\frac{3}{4}}\right)^{2}$

$3^{\frac{6}{4}} = 3^{\frac{3}{2}}$

$= (\sqrt{3})^3$

$= \boxed{3\sqrt{3}}$

3. $\left(a^{2}b^{\frac{3}{5}}\right)^{10}$

$a^{20} b^{\frac{30}{5}}$

$\boxed{a^{20} b^{6}}$

4. $\sqrt[12]{64^{4}}$

$\left((2^6)^4\right)^{\frac{1}{12}}$

$\left(2^{24}\right)^{\frac{1}{12}}$

$2^{\frac{24}{12}}$

$2^2 = \boxed{4}$

Logistic Growth WS   Pre Calc II   Show work on loose leaf.

1. The logistic model $P(t) = \dfrac{1500}{1+24e^{-0.75t}}$ models the growth of a population. Use the equation to

(a) Find the value of k -0.75

(b) Find the carrying capacity 1500

(c) Find the initial population (let t = 0 and simplify)

$$\frac{1500}{1+24e^{-0.75(0)}} = \frac{1500}{1+24} = 60$$

(d) Determine when the population will reach 50% of its carrying capacity

$$750 = \frac{1500}{1+24e^{-0.75t}} \qquad 750(1+24e^{-0.75t}) = 1500 \qquad 1+24e^{-0.75t} = 2 \qquad e^{-0.75t} = \frac{1}{24}$$

$$\ln\frac{1}{24} = -0.75t$$

$$4.24 \approx t$$

2. Suppose a population is infected with a virus. The number of people infected is modeled by the logistic equation $P(t) = \dfrac{2000}{1+249e^{-0.97t}}$ where p is the population infected (number of people) and t is the number of days since it started spreading.

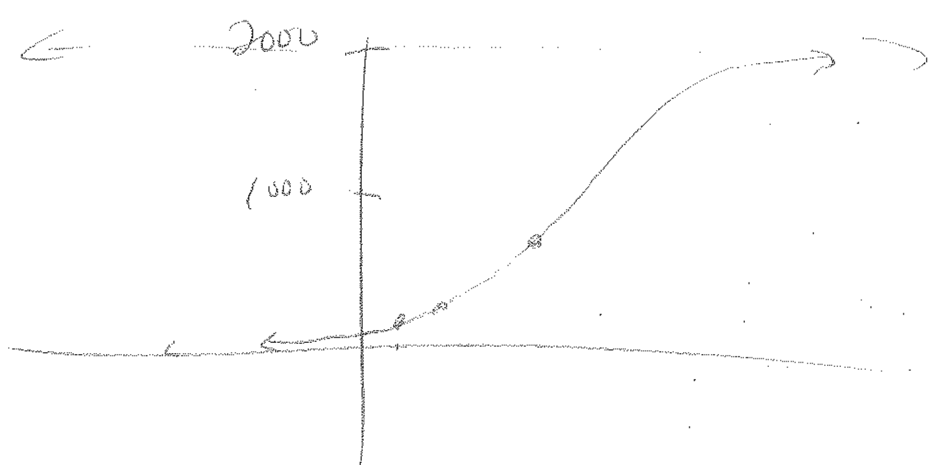
(a) How many people are infected initially? (at time t = 0)

$$\frac{2000}{1+249e^{-0.97(0)}} = \frac{2000}{1+249} = 8$$

(b) Determine the number of people infected after 1 day, two days and five days.

$P(1) = \dfrac{2000}{1+249e^{-0.97(1)}} \approx 21$ PEOPLE $\qquad P(2) \approx 54 \qquad P(5) \approx 678$

(c) Graph the function p(t) and describe its behavior.



3. A population of rabbits is given by the formula $P(t)=\dfrac{1000}{1+122e^{-0.7t}}$

(a) How many rabbits do they start with? $\dfrac{1000}{1+122e^{-0.7(0)}} \approx 8.13 \approx 8$

(b) What is the carrying capacity? 1000 RABBITS

(d) How many rabbits will there be after 5 days? $P(5) \approx 213.49 \approx 213$ RABBITS

(d) How long will it take until there are 500 rabbits?

$500 = \dfrac{1000}{1+122e^{-0.7t}}$ $\quad 1+122e^{-0.7t} = 2$ $\quad 122e^{-0.7t} = 1$ $\quad e^{-0.7t} = \dfrac{1}{122}$ $\quad -0.7t \approx -4.80$ $\quad t \approx 6.86$ DAYS

4. The number of students infected by measles in a certain school is given by the formula $P(t)=\dfrac{2000}{1+1999e^{-.8t}}$ where t is the number of days after students are first exposed to an infected student.

(a) How many students have measles initially? $\dfrac{2000}{1+1999e^{-0.8(0)}} = 1$

(b) How many will have measles after 3 days? $\dfrac{2000}{1+1999e^{-0.8(3)}} \approx 10.97 \approx 11$

(c) What does the 2000 in the numerator represent? CARRYING CAPACITY

(d) How long will it be until 175 students are infected?

$175 = \dfrac{2000}{1+1999e^{-0.8t}}$ $\quad 1+1999e^{-0.8t} = 11.429$ $\quad \approx 6.57$

5. A 2000 gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank.

(a) Use t = 0 and the initial value of P = 6 to solve for A. Use k = 0.225. Keep 4 decimal places.

$P(t) = \dfrac{150}{1+be^{-0.225t}}$ $\quad 6 = \dfrac{150}{1+b(1)}$ $\quad 1+b = \dfrac{150}{6}$ $\quad 1+b = 25$ $\quad b = 24$

(b) Write your logistic model using the info. found in (a).

$P(t) = \dfrac{150}{1+24e^{-0.225t}}$

(c) How long will it take for the guppy population to reach 100? 125?

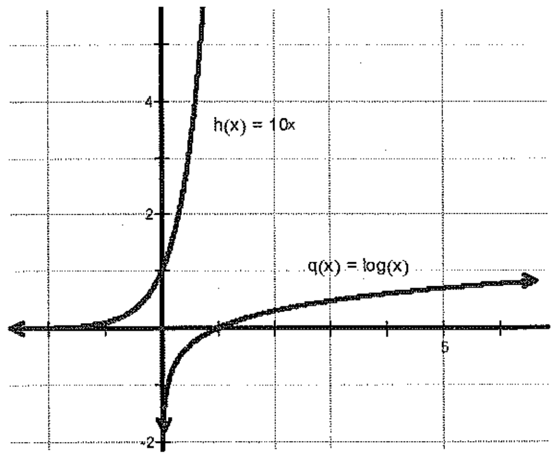
$100 = \dfrac{150}{1+24e^{-0.225t}}$ $\quad 1+24e^{-0.225t} = \dfrac{150}{100}$ $\quad e^{-0.225t} \approx 0.02085$ $\quad t \approx 17.2$

$125 = \dfrac{150}{1+24e^{-0.225t}}$ $\quad t \approx 21.28$

2.9: (part 1) Review of Rules of Logarithms

Pre Calc II (CP)

EX 1: Given $y = 10^x$, find and graph its inverse: Inverse is $y=log(x)$



DEF: $y = \log_{10} x$ (also written $y = \log(x)$) is the logarithm of x to the base 10. iff $10^y = x$.

To write in exponential form, make $\log_{10} x = y$ into $10^y = x$.

Often written as a "common log" without the base. log (x) mean base 10.

EX 2: Evaluate $\log_{10} 10{,}000 = y$ (REMEMBER $\log_{10} x = y$ means $10^y = x$)

$10^y = 10{,}000$

$y = 4$

EX 3: Evaluate $\log_{10} 0.01$ (= y) (First write: 0.01 as a power of 10):

$10^y = 0.01$

$10^y = 10^{-2}$

$y = -2$

EX 4: Solve for x: $\log x = 3.5$

$10^{3.5} \approx 3162.28$

Logarithms to Bases Other Than 10

Def: If $b > 0$ and $b \neq 1$, then (n) is the log of m to base b written $n = \log_b m$ iff $b^n = m$.

The log of a negative number is not defined.

EX 5: Evaluate $\log_2 64$

$2^y = 64$

$2^y = 2^6$

$y = 6$

EX 6: Evaluate $\log_9 27$

$9^y = 27$

$3^{2y} = 3^3$

$2y = 3$

$y = \frac{3}{2}$

EX 7: Solve for x: $\log_4 32 = x$

$4^x = 32$

$2^{2x} = 2^5$

$2x = 5$

$x = \frac{5}{2}$

EX 8: Solve for x: $\log_x 81 = 4$

$x^4 = 81$

$x = \sqrt[4]{81}$

$x = 3$

EX 9: Solve for x: $\log_3 x = 5$

$3^5 = x$

$243 = x$

Think of these properties of logs just like exponents:

For every base b, $\boxed{\log_b 1 = 0}$

For every base b and any real number n, $\boxed{\log_b b^n = n}$

For any base b and for any positive real numbers x, y, $\boxed{\log_b(xy) = \log_b x + \log_b y}$

For any base b and for any positive real numbers x, y, $\boxed{\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y}$

For any base b and for any positive real number x, $\boxed{\log_b(x^n) = n\log_b x}$

EX 10: Find $\log_{12} 1$

$12^y = 1$

$12^y = 12^0$

$y = 0$

EX 11: Solve: $\log_6 36 = y$

$6^y = 36$

$6^y = 6^2$

$y = 2$

EX 12: Simplify: $\log_{15} 5 + \log_{15} 45$

$\log_{15}(5 \cdot 45)$

$\log_{15} 225$

$\log_{15} 15^2$

2

EX 13: Rewrite using rule for quotients and solve: $\log_2\left(\frac{32}{8}\right)$

$\log_2 32 - \log_2 8$

$\log_2 2^5 - \log_2 2^3$

$5 - 3$

2

EX 14: Solve: $\log_3 x = \log_3 15 - \log_3 6$

$\log_3 x = \log_7 \frac{15}{6}$

$x = \frac{15}{6}$

$x = \frac{5}{2}$

EX 15: Solve for x: $\log_7 5x = 2\log_7 3 + \log_7 5$

$\log_7 5x = \log_7 3^2 + \log_7 5$

$\log_7 5x = \log_7 3^2 \cdot 5$

$5x = 3^2 \cdot 5$

$x = 9$

<u>Natural Logs</u>: a log with a base of (e) is a natural log, written ln

$n = \ln m$ is the <u>natural log</u> of m. It is a log with base e.

(ie: $\ln m = \log_e m$ and $m = e^n$)

EX 16: Solve for x: $\ln x = 4$

$e^4 = x$

$\cancel{e}\, x \approx 54.60$

EX 17: Solve: $\ln e^{-7}$

-7

<u>CHANGE OF BASE RULE:</u> $\log_b a = \dfrac{\log_l a}{\log_l b}$

("log of the number" over "log of the base")

EX 18: Find $\log_4 100$

$\dfrac{\log 100}{\log 4}$

3.32

EX 19: $\log_8 256$

$\dfrac{\log 256}{\log 8}$

$2.\overline{6}$

HW: Log WS

2.9 (part 1) Log WS Pre Calc II

Find each logarithm without using a calculator

1. $\log_7 49$ 2

2. $\log_2 16$ 4

3. $\log_2 \frac{1}{8}$ −3

4. $\log_5 \frac{1}{5}$ −1

5. $\log_5 \sqrt{5}$ $\frac{1}{2}$

Solve each equation (Do not use a calculator)

6. $\log_5 x = 2$ $5^2 = 25$

7. $\log_6 x = 2$ $6^2 = 36$

8. $\log x = 2$ $10^2 = 100$

9. $\ln x = 2$ e^2

10. $\log_x 121 = 2$ $x^2 = 121$ $x = 11$

11. $\log_x 64 = 3$ $x^3 = 64$ $x = 4$

12. $\log_x \left(\frac{1}{2}\right) = -1$ $\left(\frac{1}{2}\right)^{-1} = 2$

13. $\log_x \sqrt{6} = \frac{1}{2}$ $x^{\frac{1}{2}} = \sqrt{6}$ $x^{\frac{1}{2}} = 6^{\frac{1}{2}}$ $x = 6$

Find each logarithm. (Do not use a calculator)

14. $\log 100$ 2

15. $\log 10{,}000$ 4

16. $\log 0.01$ −2

17. $\log 0.0001$ −4

18. $\log_2 4$ 2

19. $\log_2 32$ 5

20. $\log_2 64$ 6

21. $\log_2 2^{10}$ 10

22. $\log_3 9$ 2

23. $\log_3 27$ 3

24. $\log_3 243$ 5

25. $\log_8 8^8$ 8

26. $\log_5 0.2$ $\log_5 \frac{1}{5}$ −1

27. $\log_5 \frac{1}{125}$ −3

28. $\log_5 \sqrt[3]{5}$ $\frac{1}{3}$

29. $\log_5 1$ 0

30. $\log_4 64$ 3

31. $\log_4 \frac{1}{64}$ −3

32. $\log_4 \sqrt[4]{4}$ $\frac{1}{4}$

33. $\log_4 1$ 0

34. $\log_6 36$ 2

35. $\log_6 6$ 1

36. $\log_6 6\sqrt{6}$ $\log_6 6 \cdot 6^{\frac{1}{2}}$ $\log_6 6^{\frac{3}{2}}$ $\frac{3}{2}$

37. $\log_6 \sqrt[3]{\frac{1}{6}}$ $\log_6 6^{-\frac{1}{3}}$ $-\frac{1}{3}$

2.9 (part 2): Solving Exponential Equations Pre Calc II (CP)

Take logs of both sides (or ln) to drop the exponent in front of the log, then solve.

EX 1: Solve $3^x = 57$

$\ln 3^x = \ln 57$

$x \ln 3 = \ln 57$

$x = \frac{\ln 57}{\ln 3}$

$x \approx 3.68$

EX 2: Solve $3^{2x} = 50$

$\ln 3^{2x} = \ln 50$

$2x \ln 3 = \ln 50$

$2x = \frac{\ln 50}{\ln 3}$

$x = \frac{1}{2} \frac{\ln 50}{\ln 3}$

$x \approx 1.78$

EX 3: $3e^{2x} = 48$

$e^{2x} = 16$

$2x = \ln 16$

$x = \frac{1}{2} \ln 16$

$x \approx 1.39$

EX 4: Suppose you invest \$5000 in an account at 4% compounded continuously. How long will it take for the money in the account to reach \$6000 if it is not touched?

$6000 = 5000 e^{0.04t}$

$1.2 = e^{0.04t}$

$\ln 1.2 = 0.04t$

$\frac{\ln 1.2}{0.04} = t$

$t \approx 4.56$ yrs

$t \approx 4$ yrs 7 months

EX 5: Suppose the half life of a substance is 400 years. If you start today with 2 grams, how long will it take until 1.78 grams remain?

$1.78 = 2\left(\frac{1}{2}\right)^{t/400}$

$0.89 \approx \left(\frac{1}{2}\right)^{t/400}$

$\log_{\frac{1}{2}} 0.89 = \frac{t}{400}$

$0.168 \approx \frac{t}{400}$

~~$67.25 = t$~~

$t \approx 67.25$ yrs

2.9 (Part 2) WS: Solving Exponential Equations Pre Calc II

1. $10^{-x} = 2$

$-x = \log 2$

$x = -\log 2$

$x \approx -0.301$

2. $3^{2x-1} = 5$

$2x - 1 = \log_3 5$

$2x = \log_3 5 + 1$

$x = \frac{\log_3 5 + 1}{2}$

$x \approx 1.232$

3. $2e^{12x} = 17$

$e^{12x} = \frac{17}{2}$

$12x = \cancel{\log} \frac{17}{2}$

$x = \frac{1}{12} \ln \frac{17}{2}$

$x \approx 0.178$

4. $4(1 + 10^{5x}) = 9$

$1 + 10^{5x} = \frac{9}{4}$

$10^{5x} = \frac{9}{4} - 1$

$10^{5x} = \frac{5}{4}$

$5x = \log (5/4)$

$x = \frac{1}{5} \log \left(\frac{5}{4}\right) \approx 0.0194$

5. $2^{3x} = 34$

$3x = \log_2 34$

$x = \frac{1}{3} \log_2 34$

$x \approx 1.696$

6. $3^{x/14} = 0.1$

$\frac{x}{14} = \log_3 0.1$

$x = 14 \log_3 0.1$

$x \approx -29.343$

7. $e^{3-5x} = 16$

$3 - 5x = \ln 16$

$-5x = \ln 16 - 3$

$x = \frac{\ln 16 - 3}{-5}$

$x \approx 0.0455$

8. $10^{1-x} = 6^x$

$\log 10^{1-x} = \log 6^x$

$(1-x) \log 10 = x \log 6$

$1 - x = x \log 6$

$1 = x + x \log 6$

$1 = x(1 + \log 6)$

$\frac{1}{1 + \log 6} = x$

$x \approx 0.562$

13

PC II

Practice with exponential and logarithmic equations

Name ______________________________ Hour _____

Find the solutions to the exponential equation, correct to 4 decimal places.

1. $10^{-x} = 2$

2. $3^{2x-1} = 5$

3. $2e^{12x} = 17$

4. $4(1 + 10^{5x}) = 9$

5. $2^{3x} = 34$

6. $3^{x/14} = 0.1$

7. $e^{3-5x} = 16$

8. $10^{1-x} = 6^{x}$

9. $x^2 10^x - x10^x = 2(10^x)$

$x^2 10^x - x10^x - 2(10^x) = 0$

$10^x(x^2 - x - 2) = 0$

$10^x(x-2)(x+1) = 0$

$10^x = 0 \quad x-2=0 \quad x+1=0$

$x = 2, -1$

10. $x^2 e^x + xe^x - e^x = 0$

$e^x(x^2+x-1) = 0$

$e^x \neq 0 \quad x^2+x-1=0$

$\frac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$\frac{-1 \pm \sqrt{5}}{2}$

Solve the logarithmic equation for x.

11. $\ln(2 + x) = 1$

$e^1 = 2 + x$

$e - 2 = x$

12. $\log(x - 4) = 3$

$10^3 = x - 4$

$1000 + 4 = x$

$1004 = x$

13. $\log_3 (2 - x) = 3$

$3^3 = 2 - x$

$27 = 2 - x$

$25 = -x$

$-25 = x$

14. $\log_2 (x^2 - x - 2) = 2$

$x^2 - x - 2 = 2^2$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3, -2$

15. $2\log x = \log 2 + \log (3x - 4)$

$\log x^2 = \log[2(3x - 4)]$

$x^2 = 2(3x - 4)$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4, 2$

16. $\log_5 x + \log_5 (x + 1) = \log_5 20$

$\log_5 [x(x+1)] = \log_5 20$

$x(x+1) = 20$

$x^2 + x = 20$

$x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$

$x = -5, 4$

17 $\log x + \log (x - 3) = 1$

$\log[x(x-3)] = 1$

$x(x-3) = 10^1$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = 5, \cancel{-2}$

18. $\ln (x-1) + \ln (x + 2) = 1$

$\ln((x-1)(x+2)) = 1$

$(x-1)(x+2) = e$

$x^2 + x - 2 = e$

$x^2 + x - 2 - e$

$\frac{-1 \pm \sqrt{1 - 4(1)(-2-e)}}{2(1)}$

≈ 1.729

-2.729

**Solve for x: $\log_2 (\log_3 x) = 4$

$\log_3 x = 2^4$

$\log_3 x = 16$

$x = 3^{16}$

$x = 43046721$

15

PC II Name:________________________

Applications of Exponential and Logarithmic Functions

Recall formulas for calculating compounding interest and continuous rates of change:

$A(t)=P(1+\frac{r}{n})^{nt}$ $\quad A(t)=Pe^{rt}$

1. $5000 is invested at 9% per year. Find the time required for the money to double if the interest is compounded a) semiannually b) continuously.

a) $10000=5000(1+\frac{0.09}{2})^{2t}$

$2=(1+\frac{0.09}{2})^{2t}$

$\ln 2=\ln(1+\frac{0.09}{2})^{2t}$

$\ln 2=2t\ln(1.045)$

$\frac{\ln 2}{\ln 1.045}=2t$

$\frac{1}{2}\frac{\ln 2}{\ln 1.045}=t$

$7.87\text{yrs}\approx t$

b) $10000=5000e^{0.09t}$

$2=e^{0.09t}$

$\ln 2=0.09t$

$\frac{\ln 2}{0.09}=t$

$7.70\text{yrs}\approx t$

2. The population of the world in 1995 was 5.7 billion, and the estimated relative growth rate is 2% per year. If the population continues to grow at this rate, when will it reach 57 billion?

$57=5.7e^{0.02t}$

$10=e^{0.02t}$

$\ln 10=0.02t$

$\frac{\ln 10}{0.02}=t$

$t\approx 115.13$

2110.13

During 2110

3. A bacteria culture starts with 10,000 bacteria. The number doubles every 40 minutes.
a) Write a formula for the number of bacteria at time t.
b) Find the number of bacteria after one hour.
c) After how many minutes will there be 50,000 bacteria?

a) $A(t)=10{,}000(2)^{t/40}$

b) $A(60)=10{,}000(2)^{60/40}$

$A(60)\approx 28{,}284$

c) $50{,}000=10{,}000(2)^{t/40}$

$5=2^{t/40}$

$\ln 5=\frac{t}{40}\ln 2$

$40\cdot\frac{\ln 5}{\ln 2}=t$

$t\approx 92.88\text{min}$

1 Hr 33min