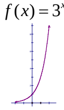
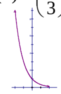
2.7: Analyzing Exponential Functions

I. Compare Exponential Curves

	$f(x)=3^x$	$g(x)=\left(\frac{1}{3}\right)^x$
		
• Domain	$(-\infty,\infty)$	$(-\infty,\infty)$
• Range	$(0,\infty)$	$(0,\infty)$
• Increasing	$(-\infty,\infty)$	$\{\ \}$
• Decreasing	$\{\ \}$	$(-\infty,\infty)$
• Right end behavior	$\lim_{x\to\infty}=\infty$	$\lim_{x\to\infty}=0$
• Left end behavior	$\lim_{x\to-\infty}=0$	$\lim_{x\to-\infty}=\infty$
• Max or Min	*none*	*none*

II. Properties

- <u>Definition:</u> $f(x) = ab^x$ is an **exponential function**
  (where $a\neq 0$, $b>0$, $b\neq 1$)
- <u>**a**</u> is the initial amount and the y-intercept
- <u>**b**</u> is the **growth factor**
- $b>1$ → growth
- $0<b<1$ → decay.
- <u>**x**</u> = the time period.

- EX 1: Do the following represent exponential growth or decay?
  (a) $y=(.75)^x$ (b) $y=3^x$ (c) $y=2\left(\frac{3}{5}\right)^x$ (d) $y=.4(1.04)^x$
  decay *growth* *decay* *growth*

III. Euler's Constant

- Find $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \approx 2.71828...$
- <u>Continuously</u> compounded interest/growth:
  $A = Pe^{rt}$
- <u>Annual growth</u>: $A = P(1+r)^t$

IV. Examples

EX 1: Suppose that $8,000 is invested at 4% for 7 years compounded continuously. Find its value.

$$A = 8000\cdot e^{(0.04)(7)} \approx \boxed{\$10585.04}$$

EX 2: Suppose that $8000 is invested at 4% for 7 year compounded annually. Find its value.

$$A = 8000(1+0.04)^7 \approx \boxed{\$10527.45}$$

Ex. 3: Finding Principal

Suppose you wanted to invest money so that after 10 years, you would have $10,000 in the account. If the interest rate is 4.2% compounded continuously. How much do you need to invest?

$$10,000 = Pe^{(0.042)(10)}$$

$$P = \frac{10,000}{e^{(0.042)(10)}} \approx \boxed{\$6750.47}$$

Homework

p.121-123 # 1-3, 5, 7-9, 10a, 11, 15-18.

2.8 (part 1): Logistic Growth

Pre Calculus II (CP)

Why logistic growth?

- Exponential growth assumes it goes on forever.
- Realistic for a small period of time, but the growth rate usually decreases over time.

Examples of Logistic Growth

- population of a town
- growth / population of animals in a forest
- selling Homecoming tickets
- infecting a population
- Each of these has a number that is called the carrying capacity or maximum population (M).

Equation for Logistic Growth

- Then if growth is logistic,

$$P = \frac{M}{1 + Ae^{-kt}}$$

P = current population
M = maximum population (carrying capacity)
t = time

EXAMPLE

Round Valley park can support no more than 100 grizzly bears. There are 10 bears in the park now. We model the logistic equation with k = 0.1

(a) Find a logistic model

(b) When will the population reach 50 bears?

(c) Graph. Use the window x [0, 75] y [0,110]

(d) Find $\lim_{t \to \infty} P(t)$