2.4: End Behavior

I. End Behavior

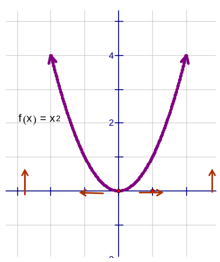
A. How does a function behave as x approaches infinity (on the right) or negative infinity (on the left).

B. We write "the limit as x goes to infinity" as

$$\lim_{x\to\infty}$$

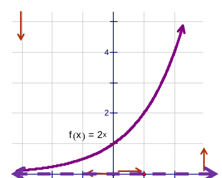
C. Ex 1: Describe the end behavior of $y = x^2$

$\lim_{x\to\infty} x^2 = \infty$ $\lim_{x\to-\infty} x^2 = \infty$



D. Ex 2: Describe the end behavior of $y = 2^x$

$\lim_{x\to\infty} 2^x = \infty$ $\lim_{x\to-\infty} 2^x = \infty$



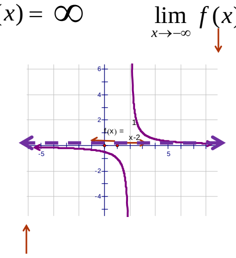
1. If the limit approaches a constant as x goes to + or – infinity, we say it has a horizontal asymptote at y = that constant.
2. Does the function above have a horizontal asymptote?

yes, $y = 0$

E. Ex 3: Examine the end behavior and asymptotes of

$$f(x) = \frac{1}{x-2}$$

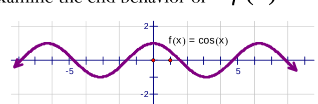
$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = \infty$



Any horizontal asymptotes?

yes, $y = 0$

F. Ex 4: Examine the end behavior of $f(x) = \cos(x)$



Conclusion:

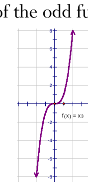
$\lim_{x\to\infty} f(x) =$ Does Not Exist (DNE) $\lim_{x\to-\infty} =$ Does Not Exist (DNE)

G. Ex 5: Determine the end behavior of the odd function

$g(x) = x^3$

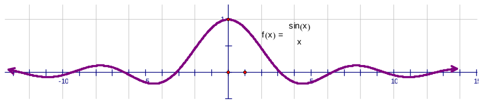
$\lim_{x\to\infty} g(x) = \infty$

$\lim_{x\to-\infty} g(x) = -\infty$



H. Ex 6: Determine the end behavior of the function

$$g(x) = \frac{\sin x}{x}$$



$$\lim_{x \to \infty} g(x) = 0 \qquad \lim_{x \to -\infty} g(x) = 0$$

II. Possible End Behaviors:

There are three possible end behaviors:

1. The values of *f(x)* can increase or decrease without bound (to ∞ or - ∞)
2. The values of *f(x)* can approach some number *L*
3. The values of *f(x)* can follow neither of these patterns (such as oscillating between two values).

CW: LM 1, 2, 5, 6

HW: p. 100-101 #5-7, 11, 15-17

2.5: Parametric Equations

I. What are Parametrics?

A. Normally we define functions in terms of one variable – for example, *y* as a function of *x*.

B. Suppose in a graph, each *(x, y)* depended on a third variable *t*.

C. We would call *t* the parameter and call the equation a parametric equation.

II. Why use parametric equations?

A. Show both vertical and horizontal motion.

B. Easier equations.

C. Easier inverse.

D. Graph non-functions.

E. Graph parts of lines or curves (by restricting *t*).

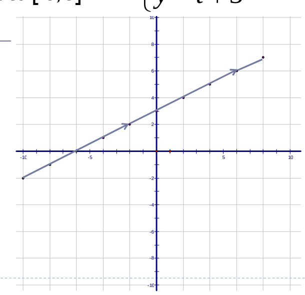
F. Curves have direction.

III. Examples

Ex 1: Graph this parametric equation: $\begin{cases} x = 2t \\ y = t + 3 \end{cases}$

Fill in the table for t values [-5, 5]

t	x	y
−5	−10	−2
−4	−8	−1
−3	−6	0
−2	−4	1
−1	−2	2
0	0	3
1	2	4
2	4	5
3	6	6
4	8	7
5	10	8



EX 2: Inverses

Find the inverse of $y = x^2 - x - 2$

Problems:

- Solving for y when $x = y^2 - y - 2$
- You would have to graph it one half at a time because its not a function.
- Instead, define it as $\begin{cases} x = t \\ y = t^2 - t - 2 \end{cases}$
- Then the inverse is simply (switch x and y) $\begin{cases} x = t^2 - t - 2 \\ y = t \end{cases}$
- Graph them both.

EX 3: Graph a cloud

- Graph $\begin{cases} x = 7\sin t - \sin(7t) \\ y = 7\cos t - \cos(7t) \end{cases}$

Settings: t values: $[-\pi, \pi]$ x and y: [-10,10]

Projectile Motion

- Suppose you toss a ball in the air. Its position (ordered pairs (x, y)) is defined parametrically by this set of equations:

$$\begin{cases} x = v_x t + d_0 \\ y = \frac{-1}{2} g t^2 + v_y t + h_0 \end{cases}$$

- v_x is the horizontal velocity at release
- v_y is the vertical velocity at release
- The point at which you release it is the ordered pair (d_0, h_0)
- g is the gravitational constant (9.8 in m/s^2 or 32 in ft/sec^2)

EX 4: Basketball shot

In a basketball game, a free throw is released from a point 6 feet above the ground and 14 feet from the basket. The vertical velocity at release is 20 ft/sec and horizontal velocity is 14 ft/sec.

(a) Give the equations for the ball's horizontal distance x and vertical distance y at time t:

$$\begin{cases} x(t) = 14t \\ y(t) = -16t^2 + 20t + 6 \end{cases}$$

- (b) Will the ball go directly into the basket? Assume the basket is 10 feet high.

  After 1 second, the ball has traveled 14 feet horizontally, and is at a height of 10 feet, so it will go in

- (c) Look at the info on the graph. What does x, y, and t tell us? Trace it.

  x = horizontal distance

  y = height

  t = time

HW: p. 105-107 #4, 7-10, 12, 15, 20