2.1: IDENTIFYING FUNCTIONS

I. Interval Notation:

A. Bounded Intervals

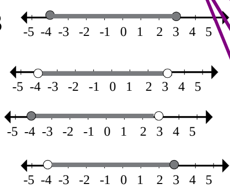
OPEN CLOSED HALF-OPEN

$-4 \le x \le 3$ $[-4,3]$

$-4 < x < 3$ $(-4,3)$

$-4 \le x < 3$ $[-4,3)$

$-4 < x \le 3$ $(-4,3]$



B. Unbounded Intervals

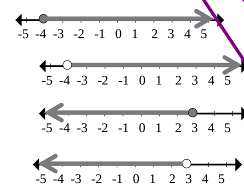
OPEN CLOSED

$x \ge -4$ $[-4,\infty)$

$x > -4$ $(-4,\infty)$

$x \le 3$ $(-\infty,3]$

$x < 3$ $(-\infty,3)$



C. Example 1: Write using interval notation.

1. -1 < x ≤ 3 $(-1,3]$
2. x ≥ 2 $[2,\infty)$
3. x < 0 $(-\infty,0)$

D. Example 2: Write as a compound inequality.

1. (-∞,-5) $x < -5$
2. [-2,∞) $-2 \le x$
3. [-3,0] $-3 \le x \le 0$

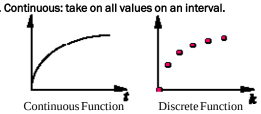
II. Functions

A. A *function f* from a set *A* to *B* is a correspondence in which each element in *A* corresponds to exactly one element of *B*.

1. The domain consists of all input values for the function.
2. The range of the function, and consists of all of the output values.

B. Types of functions

1. Discrete: take on certain values on an interval, but not those in between.
2. Continuous: take on all values on an interval.



B. Example 3: Suppose you need to order sweatshirts for your baseball team, and there is a sale in which you can buy a maximum of twenty at a price of $30.

1. The equation to find the cost is below:

$f(x) = 30x$ *(f(x) notation)*

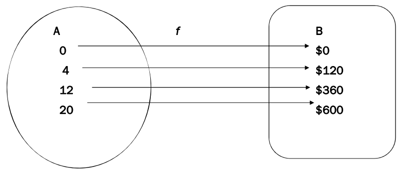
$f : x \to 30x$ *(mapping notation)*

2. This is called the *rule* for this function.
3. For a function *f*, the function values $y = f(x)$ depend on the values of *x* (as the price *depends* on how many sweatshirts you buy)

4. *y* is called the *dependent variable*, and *x* is called the *independent variable*.

5. A function whose *independent* and *dependent* variables have only real number values is called a *real function*.

6. We can display this function in other ways as well, for example, consider this arrow diagram below:



7. The minimum *x-value* of the function is 0 and the maximum is 20, therefore the *domain* is:

$$\{x : 0 \le x \le 20\}$$

$$x \in [0,20]$$

8. The minimum *y-value* of the function is 0, and the maximum is 600, therefore the *range* is:

$$\{y : 0 \le y \le 600\}$$

$$y \in [0,600]$$

III. Example 4:

Find the domain for the real functions below:

$A.\ h : x \to \dfrac{1}{\sqrt{x-5}}$

$x - 5 > 0$

$x > 5$

$\boxed{(5,\infty)}$

$B.\ f(x) = \dfrac{1}{x^2-4}$

$x^2 - 4 \ne 0$

$x^2 \ne 4$

$x \ne \pm 2$

$\boxed{(-\infty,-2)\cup(-2,2)\cup(2,\infty)}$

HOMEWORK

p. 82-83 # 4b-e, 5-8, 12-15.

2.2,2.3(pt1): Finding Maxima and Minima

Pre Calc II

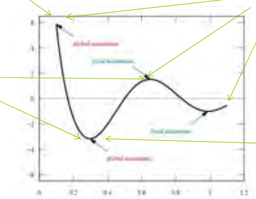
I. Extrema

1. Definitions

A. Local max / min – the highest or lowest value of a function within an interval

B. Absolute max / min – the highest or lowest value over the entire domain of the function.

Note: Max and min are y-values



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2. Quadratics

A. A quadratic has an equation of the form

$$y = ax^2 + bx + c$$

B. Its graph is a parabola.

C. The vertex of a parabola is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ so find x, then sub back in to find y.

D. The vertex is the max or min. of a parabola.

E. EX1: Find the vertex of $y = x^2 + 4x + 6$

$$vertex: \left(\frac{-4}{2(1)}, f\left(\frac{-4}{2(1)}\right)\right) = (-2, f(-2))$$

$$= \boxed{(-2,2)}$$

There is an absolute min of 2, when x=-2

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3. Example 2:

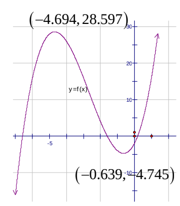
Find relative and absolute extrema for

$$y = x^3 + 8x^2 + 9x - 2$$

Rel Max of 28.597 at x = -4.694

Rel Min of -4.745 at x = -0.639

No Abs Max or Min



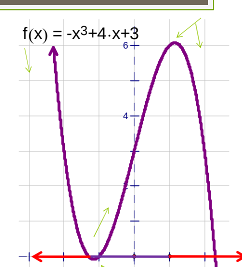
II. Increasing and Decreasing

A. A function is *increasing* on interval *(a,b)* iff as *x* increases, *y* increases.

B. A function is *decreasing* on an interval *(a,b)* iff as *x* increases, *y* decreases.

C. A function is *constant on an interval (a,b)* iff as *x* increases, *y* does not change.

D. Example 3: Calculate the following for the given function



Absolute Max none

Relative Max

$6.08, \text{ when } x = 1.15$

Absolute Min none

Relative Min:

$-0.08, \text{ when } x = -1.15$

Intervals where its increasing $-1.15 < x < 1.15$

Intervals where its decreasing $x < -1.15, 1.15 < x$

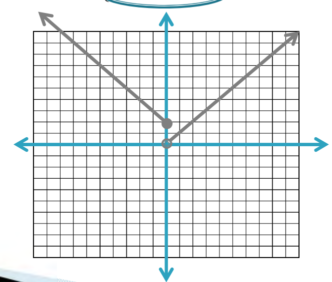
HW: p. 89-90 #1, 3, 6-9, 11, 14

2.2-2.3 Part II: Piecewise Functions

I. Graph the following functions:

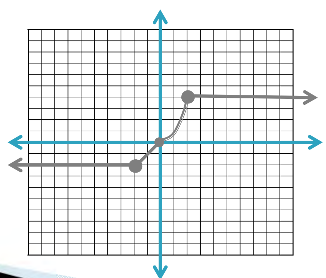
A. Example 1: Graph the following function:

$$f(x)=\begin{cases}-x+2 & x\le 0\\ x & x>0\end{cases}$$



B. Example 2: Graph the following function

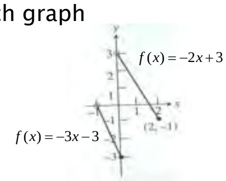
$$g(x)=\begin{cases}-2 & x<-2\\ x & -2\le x\le 0\\ x^2 & 0<x<2\\ 4 & x\ge 2\end{cases}$$



II. Write a function for each graph

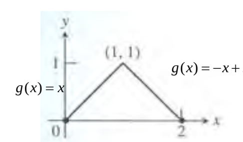
A. Example 3:

$$f(x)=\begin{cases}-3x-3, -1<x\le 0\\ -2x+3, 0<x\le 2\end{cases}$$



B. Example 4:

$$f(x)=\begin{cases}x, 0<x\le 1\\ -x+2, 1<x\le 2\end{cases}$$



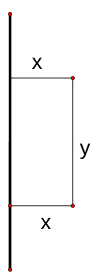
CW / HW: Piecewise Function Worksheet

2.2-2.3 PART III: MAXIMIZATION AND MINIMIZATION

I. Minimization and Maximization Word Problems

A. Steps (you will get a handout with these):

1. Read the problem carefully. Distinguish constants from variables.
2. Decide what is being maximized or minimized. This will be represented by the dependent variable.
3. Write the quantity as a function of 1 variable.
4. Determine an appropriate domain
5. Graph your equation and determine maxima and/or minima. **You may need to adjust window.**
6. Make sure you answer the question that was asked.



B. Example 1: A fence is built along the side of the wall with 100 feet of fence (see picture). Let *x* be the width and *y* be the length. What dimensions will give the maximum area?

$$2x + y = 100 \rightarrow y = 100 - 2x$$

$$Area = x \cdot y$$

$$Area = x \cdot (100 - 2x)$$

$$Area = -2x^2 + 100x$$

$$vertex = \left(\frac{-100}{2(-2)}, f\left(\frac{-100}{2(-2)}\right)\right) = \left(25\ ft, 1250\ ft^2\right)$$

$$\boxed{\text{dimensions} = 25\text{ft by } 50\text{ ft}}$$

C. Example 2: A company has 1000 widgets and they determine that the following equation will determine how many widgets will be sold at a given price *x*.

Number Sold = 1010 – 10x

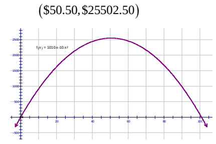
How many widgets should they sell to maximize the profit, and how much money will they make in revenue?

If the price is *x*, the revenue equation will be:

$R(x) = x(1010 - 10x) = 1010x - 10x^2$

The domain that we want is (0,101). Why?

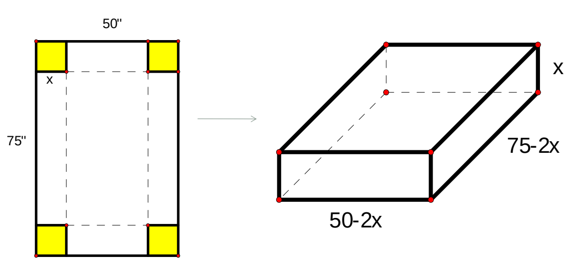
$(\$50.50, \$25502.50)$



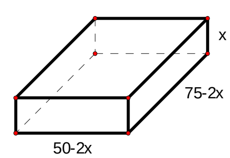
Price for Max = $50.50

Max Revenue = $25,502.50

C. Example 63 A company wants to construct a box. They take a 50" by 75" piece of cardboard, cut out a square of length (*x*) from each corner, and fold the box (with no lid). For what value of *x* will the volume be maximized? What is that maximum volume?



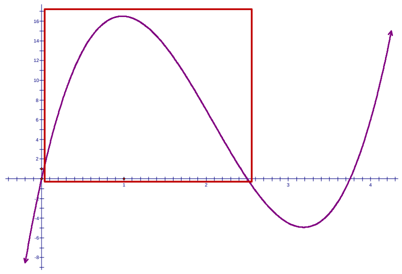
We will now maximize the box



$$V(x) = x(50 - 2x)(75 - 2x)$$

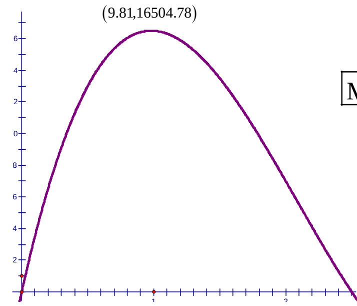
$$V(x) = 4x^3 - 250x^2 + 3750x$$

B. Graph, then estimate the maximum possible volume for the box. What value of *x* gives this volume?



Because of the context, we only want this part:

$$\text{Domain: } 0 < x < 25$$



$\text{Max Vol: } 16505 in^2$

$x \approx 9.81 in$

$\text{Dimesions: } 9.81'' \times 30.38'' \times 55.38''$

CW/HW: optimization problems WS #1