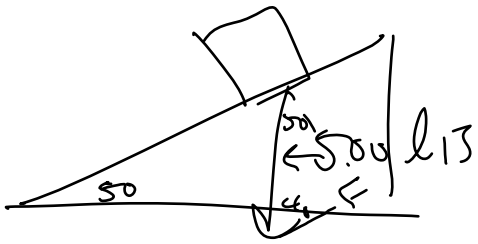
Honors Pre-Calculus

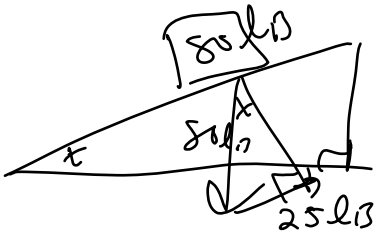
6.1: Vector Word Problems

1. A sled on an inclined plane weighs 500 lb, and the plane makes an angle of 50 degrees with the horizontal. What force, perpendicular to the plane, is exerted on the plane by the sled?



$500 \sin 40 \approx \boxed{321.39 \text{ lb}}$

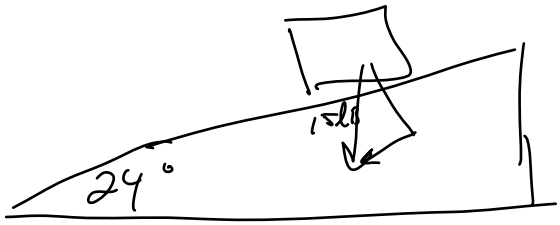
2. If a force of 25 lb is required to move an 80 lb sled up a hill, what angle does the hill make with the horizontal?



$\sin x = \frac{25}{80}$

$x = \sin^{-1}\left(\frac{25}{80}\right) \approx 18.21^\circ$

3. A weight of 15 lb is placed on a smooth plane inclined at an angle of 24 degrees with the horizontal. What force pushing along the plane will just prevent the weight from slipping?

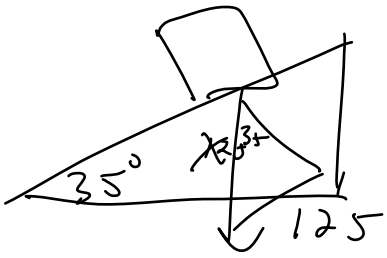


$\sin 24 = \frac{x}{15}$

$15 \sin 24 = x$

$\boxed{6.10 \text{ lb} = x}$

4. What would be the largest weight a person could drag up a slope inclined 35 degrees from the horizontal if that person is able to pull with a force of 125 lb?



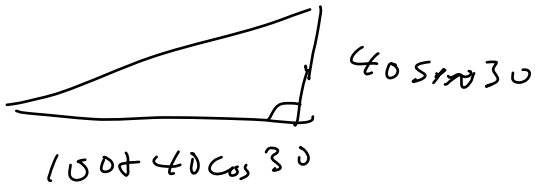
$\sin 35 = \frac{125}{x}$

$x = \frac{125}{\sin 35}$

$\boxed{x \approx 217.93 \text{ lb}}$

1.) 321.39 lbs 2.) 18.21 degrees 3.)6.1 lbs
4.) 217.93 lbs 5.) 136.1 mi, 8.45 degrees 6.) 184.4 degrees, 521.54 mph
7.) 3.92 mph, 0.77 mi, 0.32 hours or 19.35 min 8.) 228.2 degrees, 309 knots

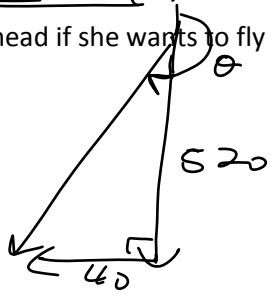
5. A steamer sails 100 miles east and then 40 miles on a heading of 30 degrees North of East. How far is the steamer from its starting point, and at what angle has it traveled from its starting point?



$d \approx 136.12$ mi

$\theta \approx 8.45°$

6. At what bearing and speed should a pilot head if she wants to fly due south at 520 mph when a 40 mph wind is blowing from the east?

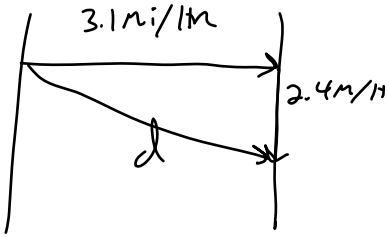


$520^2 + 40^2 = d^2$

$d \approx 521.54$ mph

BEARING ≈ 184.39

7. A river is flowing at 2.4 mph when a girl rows across it. If the girl rows at a still-water speed of 3.1 mph and heads the boat perpendicular to the direction of the current, find the ground speed of the boat. If the river is 1 mile across, how far down the river will she have moved by the time she reaches the far side of the river? How long will it take her to cross the river?



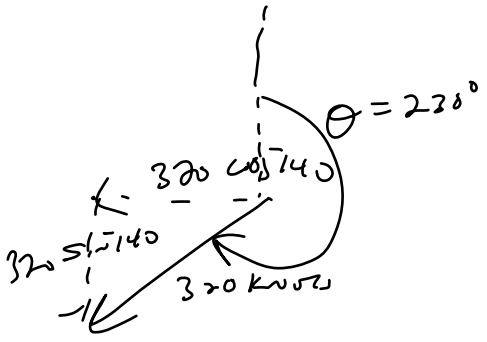
$3.1^2 + 2.4^2 = d^2$

$d \approx 3.9 \frac{mi}{hr}$

$\frac{1}{x} = \frac{3.1}{2.4}$ $\quad x \approx 0.77$ mi

$1 \text{ mi} \cdot \frac{1 \text{ hr}}{3.1 \text{ mi}} \approx 0.32$ hrs

8. An airplane is headed at 230 degrees at 320 knots when a wind of 15 knots is blowing from the south. Find the ground speed of the plane and the wind correction angle.



320 cos 140

320 sin 140 + 15

−245.13

−190.69

$h = 310.57$ knots

$\theta \approx 232.12°$ BEARING

OR

142°

1.) 321.39 lbs 2.) 18.21 degrees 3.)6.1 lbs

4.) 217.93 lbs 5.) 136.1 mi, 8.45 degrees 6.) 184.4 degrees, 521.54 mph

7.) 3.92 mph, 0.77 mi, 0.32 hours or 19.35 min 8.) 228.2 degrees, 309 knots

HCP PRECALCULUS – CIERECH **NAME**_____
Review 6.1-6.3

Answer the following

1) Let $r = 7,\ s = 2,$ **u** $= \langle -1, 4 \rangle$, and **v** $= \langle 5, -2 \rangle$. Find $|su + rv|$.

$|su + rv| = |2\langle -1, 4\rangle + 7\langle 5, -2\rangle| =$

$|\langle 33, -6\rangle| = \sqrt{33^2 + (-6)^2} = \sqrt{1089 + 36} = \sqrt{1125} =$

$15\sqrt{5} \approx 33.541$

2) Find the unit vector in the direction of $\langle 3, 5 \rangle$.

$\frac{1}{|\langle 3,5\rangle|}\langle 3,5\rangle = \frac{1}{\sqrt{34}}\langle 3,5\rangle = \left\langle \frac{3\sqrt{34}}{34}, \frac{5\sqrt{34}}{34} \right\rangle$

3) Let A = (-2, 3) and B = (4, 5).

A. Express the vector $\overrightarrow{AB}$ as a linear combination of **i** $= \langle 1, 0 \rangle$ and **j** $= \langle 0, 1 \rangle$.

$\overrightarrow{AB} = \langle 4+2, 5-3 \rangle = \langle 6, 2 \rangle =$

$6i + 2j$

4) Find **u** $\cdot$ **v** given the angle between **u** and **v** is $\theta = 55°$ and $|u| = 3$ and $|v| = 2$.

$u \cdot v = 3 \cdot 2 \cos 55$

$= 6 \cos 55$

≈ 3.441

5) Let $\mathbf{a} = \langle 6, 2 \rangle$ and $\mathbf{b} = \langle -1, 3 \rangle$. Find $\mathbf{proj_a b}$ $= \frac{b \cdot a}{|a|^2} a$

$= \frac{(-6+6)}{40} \langle 6, 2 \rangle = \boxed{0} \quad \boxed{\langle 0, 0 \rangle}$

6) Eliminate the parameters from the curve C defined by the parametric equations and identify the curve.

$$C = \begin{cases} x = t + 4 \\ y = 5 + t^2 \end{cases}$$

$t = x - 4$

$y = 5 + (x-4)^2 =$

$= 5 + x^2 - 8x + 16$

$y = x^2 - 8x + 21$

parabola opening upward with vertex at (4, 5)

7) Determine the parameterization of the line segment with endpts A = (2, -3) and B = (1,2). $\vec{OP} = \langle x, y \rangle$ $\vec{OA} = \langle 2, -3 \rangle$ $\vec{OB} = \langle 1, 2 \rangle$

$\vec{OP} - \vec{OA} = t(\vec{OB} - \vec{OA})$

$\langle x, y \rangle - \langle 2, -3 \rangle = t(\langle 1, 2 \rangle - \langle 2, -3 \rangle)$

$\langle x-2, y+3 \rangle = t\langle -1, 5 \rangle$

$x - 2 = -t$

$y + 3 = 5t$

$\boxed{\begin{array}{l} x = 2 - t \\ y = 5t - 3 \end{array} \quad 0 \le t \le 1}$

8) Find the angle between $\mathbf{u} = \langle -2, 5 \rangle$ and $\mathbf{v} = \langle 1, -3 \rangle$.

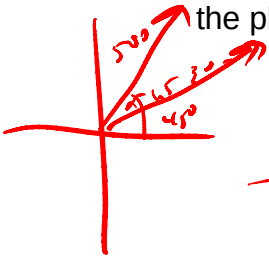
$\theta = \cos^{-1}\left(\frac{-17}{\sqrt{290}}\right)$

$\theta \approx 176.634^\circ$

check

OK

9) An airplane is flying on a bearing of 35^{o} at 550 mph. A wind is blowing with a bearing of 45^{o} at 30 mph. Find the resultant speed and direction of the plane.



$\langle 550\cos 55^\circ + 30\cos 45^\circ,\ 550\sin 55^\circ + 30\sin 45^\circ \rangle$

$\langle 336.68,\ 471.747 \rangle$

≈ 579.568 mph

$\theta = 90 - 54.485$

$\theta \approx 54.485$

35.515° bearing

$a = |u|\cos\theta$

$336.68 = 579.568\cos\theta$

10) A golf ball is hit with an initial velocity of 150 ft/sec at an angle of 29 from the horizontal. Write a parametric equation that represents this situation, and tell me how long the ball is in the air.

$x_1 = (150\cos 29)t$

$y_1 = -16t^2 + (150\sin 29)t$

Window

$T: [0,5]$

$x: [0,750]$

$y: [0,100]$

TABLE: 4.5

$\Delta T: .01$

≈ 4.535 sec.

11) How much work is done in lifting a 45-lb. child 8 feet off the ground? ~~if 100 lbs of force is applied in a direction of $\langle 2,5 \rangle$?~~

a) $45 \to 9.8$

$45 \cdot 8 = 360$ ft-lbs

b) How much work is done moving an object 8 feet up if 100 lb. force is applied in the direction of $\langle 2,5 \rangle$

$\frac{100}{\sqrt{29}}\langle 2,5 \rangle \cdot \langle 0,8 \rangle$

$\frac{100\sqrt{29}}{29}(40) = \frac{4000\sqrt{29}}{29} \approx 742.781$ ft.-lbs

(I think!?!)

12)A Ferris wheel has diameter of 60 feet, it's center is 35 feet off the ground, and it takes 25 seconds to complete one revolution.

at 3 o'clock!

Give the equations for the Ferris wheel.

radians

$x_1 = 30\cos\left(\frac{2\pi t}{25}\right)$

$y_1 = 35 + 30\sin\left(\frac{2\pi t}{25}\right)$

degrees

$x_1 = 30\cos(14.4t)$

$y_1 = 35 + 30\sin(14.4t)$

13.) When Paul throws a ball, he ALWAYS releases it at a height of 5 feet and at an angle of 75 degrees.

Give the equations for the path of the ball if Paul releases the ball with an initial velocity of 70 ft/sec.

rad

$x_1 = 70\left(\cos\frac{5\pi}{12}\right)t$

$y_1 = -16t^2 + 70\left(\sin\frac{5\pi}{12}\right)t + 5$

deg

$x_1 = 70(\cos 75)t$

$y_1 = -16t^2 + (70\sin 75)t + 5$

6.3: Parametric Equations Practice

1. Sid is in a plane flying at an altitude of 3000 meters at a horizontal speed of 300 meters per second, and jumps out of the plane for skydiving.

   a. Write the x- and y-components for Sid's motion as a function of time after he drops out of the plane (before he deploys his parachute). $X(t)=300t$

   $Y(t)=-4.9t^2+3000$

   b. How high off the ground will Sid be if he releases his parachute after 5 seconds?

   $Y(5)=-4.9(5)^2+3000\approx$ **2877.5 m**

   c. How far will he be horizontally from where he jumped the plane?

   $300(5)=$ **1500 m**

2. Justine climbs on the Singapore Flier (the world's largest Ferris Wheel) at the bottom. The wheel takes 37 minutes for one full rotation, has a diameter of 150 meters, and the center is 80 meters above the ground.

   a. Write the parametric equations for Justine as she travels around the wheel.

   $X(t)=75\sin\left(\frac{2\pi}{37}t\right)$ | $Y(t)=-75\cos\left(\frac{2\pi}{37}t\right)+80$

   b. How high will Justine be after 21 minutes?

   $Y(21)\approx$ **148.34 m**

   c. What will the first time be when Justine is 100 meters above the ground?

   $100=-75\cos\left(\frac{2\pi}{37}t\right)+80$ $t\approx$ **10.84 min**

3. Lily and Madelyn are having a catch. They are standing 45 feet apart, and Lily throws the ball with an initial speed of 70 feet per second at an angle of 11 degrees (above the horizontal). When Lily releases the ball, it is 5 feet above the ground.

   a. Write the parametric equations for the motion of the ball.

   $X(t)=(70\cos 11)t$; $Y(t)=-4.9t^2+(70\sin 11)t+5$

   b. What is the horizontal distance of the ball after 0.3 seconds? What is the height?

   $X(0.3)\approx 20.61$ FT $\quad Y(0.3)\approx 8.56$

   c. What is the height of the ball when it reaches Madelyn?

   $70(\cos 11)t=45$

   $t\approx 0.655$

   $Y(0.655)\approx 11.65$ FT

6.3: Parametric Equations and Motion Examples

1. The Independence Day fireworks at Memorial Park are fired at an angle of 82 degrees with the horizontal. The technician firing the shells expects them to explode about 300 feet in the air 4.8 seconds after they are fired.

   a. Find the initial velocity of a shell fired from the ground.

$y(t) = -4.9t^2 + V_0 \sin 82\, t = 300$

$-4.9(4.8)^2 + V_0(\sin 82)(4.8) = 300$

$V_0 \approx 86.87$

   b. Safety barriers will be placed around the launch area to protect spectators. If the barriers are placed 100 yards from the point directly below the explosion of the shells, how far should the barriers be from the point where the fireworks are launched?

$x(t) = 86.87 \cos 82\, t$

$y(t) = -4.9t^2 + 86.87 \sin 82t$

$x(4.8) \approx 86.87 \cos 82 (4.8)$

≈ 58.03

$+300FT$ $358.03FT$

2. Kaci Clark led the Women's Pro Softball League in strikeouts in 1998. Suppose she throws the ball at and angle of 5.2 degrees with the horizontal at a speed of 67 mph. The distance from the pitcher's mound to home plate is 43 feet. If Kaci releases the ball 2.7 feet above ground, how far above the ground is the ball when it crosses home plate? (Hint: 67mph converts to 98.3 ft/s.

   a. Write the parametric equations to model the path of the ball.

$x(t) = 98.3 \cos 5.2\, t$

$y(t) = -4.9t^2 + 98.3 \sin 5.2t + 2.7$

   b. Find the amount of time it will take the softball to travel 43 feet.

$43 = 98.3 \cos 5.2\, t$

$0.44 \text{ SEC} \approx t$

   c. ~~Find the vertical position of the ball at that time.~~

$y(0.44) \approx 5.66 FT$

$\frac{67 mi}{1 Hr} \cdot \frac{1 Hr}{3600 SEC} \cdot \frac{5280 FT}{1 mi} \approx 98.27 \frac{FT}{S}$

HONORS PRECALCULUS

WS: 6.4 Applications of Vectors

1. Find the rectangular coordinates for the polar point $\left(8, 5\pi/4\right)$.

$\left(8\cos\frac{5\pi}{4}, 8\sin\frac{5\pi}{4}\right) = \left(-4\sqrt{2}, -4\sqrt{2}\right)$

2. Find polar coordinates for the rectangular point (-6, -6).

$\sqrt{(-6)^2+(-6)^2} = \sqrt{72} = 6\sqrt{2}$

$\tan^{-1}\left(\frac{-6}{-6}\right) = \frac{\pi}{4}$

$\left(6\sqrt{2}, \frac{5\pi}{4}\right)$ IV QUAD

3. Convert the following equation to polar form. Show work.

$3x+2y=5$

$3r\cos\theta + 2r\sin\theta = 5$

$r = 5/(3\cos\theta + 2\sin\theta)$

4. Convert the following equation to rectangular form. Show work.

$(x-5)^2+(y-6)^2=61$

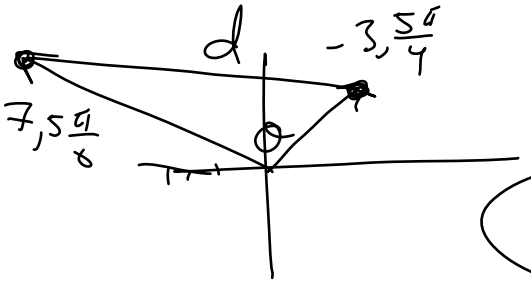
$x^2-10x+25+y^2-12y+36=61$

$x^2+y^2-10x-12y$

$r^2-10r\cos\theta-12r\sin\theta=0$

$r = 10\cos\theta + 12\sin\theta$

5. How would you find the distance between point $\left(-3, \frac{5\pi}{4}\right)$ and $\left(7, \frac{5\pi}{6}\right)$ using the Law of Cosines and a calculator??



$\theta = \frac{5\pi}{6} - \frac{\pi}{4} = \frac{7\pi}{12}$

$d^2 = 7^2+3^2-2(7)(3)\cos\frac{7\pi}{12}$

$d \approx 8.30$

6. Find a polar equation for a limacon with a dimple that is symmetric with respect to the y-axis.

7.. State the domain and range of $r = 3+5\sin\theta$

D: $\mathbb{R}$

$3+5(-1) = -2$

$3+5(1) = 8$

R: $[-2, 8]$

1. Find the rectangular coordinates for the polar point $\left(8, 5\pi/3\right)$.

$8\cos\frac{5\pi}{3},\ 8\sin\frac{5\pi}{3}\Big) = \left(-4,\ -4\sqrt{3}\right)$

2. Find polar coordinates for the rectangular point $\left(-6, -6\sqrt{3}\right)$

$\sqrt{(-6)^2+(-6\sqrt{3})^2} = \sqrt{144} = 12$

$\tan^{-1}\left(\frac{-6\sqrt{3}}{-6}\right) = \pi/3$

$\left(12, \frac{4\pi}{3}\right)$ Quad III

3. Convert the following equation to polar form. Show work.

$2x - 3y = 5$

4. Convert the following equation to rectangular form. Show work.

$(x+6)^2 + (y-5)^2 = 61$

5. How would you find the distance between point $\left(-3, \frac{5\pi}{4}\right)$ and $\left(7, \frac{5\pi}{6}\right)$ using the Law of Cosines and a calculator??

6. Find a polar equation for a limacon with a dimple that is symmetric with respect to the x-axis.

7. State the domain and range of $r = 7 + 10\cos\theta$.

D: $\mathbb{R}$

R: $[-3, 17]$

$7+10(-1) = -3$

$7+10(1) = 17$

HONORS PRECALCULUS

WS: 6.4

I. Convert the polar to rectangular form and describe the graph

1. $r = -2\sin\theta$

$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$x^2 + y^2 + 2y + 1 = 0 + 1$

$x^2 + (y+1)^2 = 1$

CIRCLE, $C = (0, -1)$ $r = 1$

2. $r = -3\cos\theta - 2\sin\theta$

$r^2 = -3r\cos\theta - 2r\sin\theta$

$x^2 + y^2 = -3x - 2y$

$(x^2 + 3x + \frac{9}{4}) + (y^2 + 2y + 1) = 0 + \frac{9}{4} + 1$

$(x + \frac{3}{2})^2 + (y+1)^2 = \frac{13}{4}$

$C = (-\frac{3}{2}, -1)$

$r = \frac{\sqrt{13}}{2}$

3. $r = 3\sec\theta$

$r\cos\theta = 3$

$x = 3$

VERTICAL LINE

II. Convert the rectangular to polar form.

4. $y = -4$

$r\sin\theta = -4$

$r = -4\csc\theta$

5. $2x - 3y = 4$

$2r\cos\theta - 3r\sin\theta = 4$

$r = \frac{4}{2\cos\theta - 3\sin\theta}$

6. $(x-3)^2 + (y+1)^2 = 10$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 10$

$x^2 + y^2 = 6x - 2y$

$r^2 = 6r\cos\theta - 2r\sin\theta$

$r = 6\cos\theta - 2\sin\theta$

HCP PRECALCULUS **NAME**_____

6.4-6.5 Review

1. Give the rectangular coordinates (x, y) for the following point: $(-4, 5\pi/4)$

$\left(-4\cos\frac{5\pi}{4}, -4\sin\frac{5\pi}{4}\right) = \left(-4\left(-\frac{\sqrt{2}}{2}\right), -4\left(-\frac{\sqrt{2}}{2}\right)\right) =$ $(2\sqrt{2}, 2\sqrt{2})$

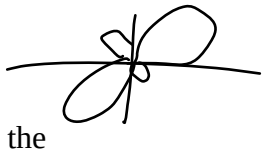
2. Give polar coordinates (r, θ) with $r > 0$ and $0 \le \theta < 2\pi$, for the following point: (8,13)

$\theta = \tan^{-1}\left(\frac{13}{8}\right) \approx 1.019 + n\pi$ $r = \sqrt{233}$ $(\sqrt{233}, 1.019)$

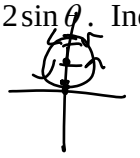
3. Plot the point A with polar coordinates $\left(3, \frac{11\pi}{6}\right)$. Then find two other sets of polar coordinates for this point.

$\left(3, -\frac{\pi}{6}\right)$ $\left(-3, \frac{5\pi}{6}\right)$

4. A. Sketch a graph of $r = 2 + 4\sin 2\theta$ on the following axes from 0 to 2π.



B .Sketch the polar graph of $r = 2\sin\theta$. Include arrows indicating which way the curve is drawn.



5. Convert $2x - 3y = 5$ to polar form.

$2r\cos\theta - 3r\sin\theta = 5$

$r(2\cos\theta - 3\sin\theta) = 5 \Rightarrow r = \frac{5}{2\cos\theta - 3\sin\theta}$

6. Convert $r = 2\sin\theta - 4\cos\theta$ into rectangular form.

$r^2 = 2r\sin\theta - 4r\cos\theta$

$x^2 + y^2 = 2y - 4x$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 0 + 4 + 1$

$(x+2)^2 + (y-1)^2 = 5$

$C = (-2, 1)$ $r = \sqrt{5}$

MORE PRACTICE

1. Find the rectangular coordinates for the polar point $\left(8, \frac{5\pi}{4}\right)$.

$\left(8\cos\frac{5\pi}{4}, 8\sin\frac{5\pi}{4}\right) = \left(8\left(\frac{-\sqrt{2}}{2}\right), 8\left(\frac{-\sqrt{2}}{2}\right)\right) = (-4\sqrt{2}, -4\sqrt{2})$

2. Find polar coordinates for the rectangular point (-6, -6).

$r^2 = (-6)^2 + (-6)^2$ $\quad \theta = \tan^{-1}\left(\frac{-6}{-6}\right)$ $\quad (6\sqrt{2}, \frac{5\pi}{4})$

$r = 6\sqrt{2}$ $\quad \theta = \frac{\pi}{4} + n\pi$ $\quad (-6\sqrt{2}, \frac{\pi}{4})$

3. Convert the following equation to polar form. Show work.

$3x + 2y = 5$

$3r\cos\theta + 2r\sin\theta = 5$

$r(3\cos\theta + 2\sin\theta) = 5$

$r = \frac{5}{3\cos\theta + 2\sin\theta}$

4. Convert the following equation to rectangular form. Show work.

$(x-5)^2 + (y-6)^2 = 61$

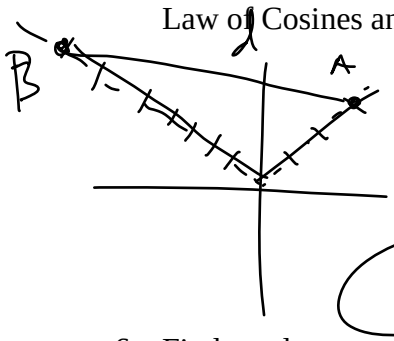
$x^2 - 10x + 25 + y^2 - 12y + 36 = 61$

$x^2 + y^2 = 10x + 12y$

$r^2 = 10r\cos\theta + 12r\sin\theta$

$r = 10\cos\theta + 12\sin\theta$

5. How would you find the distance between point $\left(-3, \frac{5\pi}{4}\right)$ and $\left(7, \frac{5\pi}{6}\right)$ using the Law of Cosines and a calculator?



A $\quad$ B

$\frac{5\pi}{6} - \frac{\pi}{4} = \frac{7\pi}{12}$

$d^2 = (3)^2 + (7)^2 - 2(3)(7)\cos\frac{7\pi}{12}$

$d \approx \sqrt{68.870}$

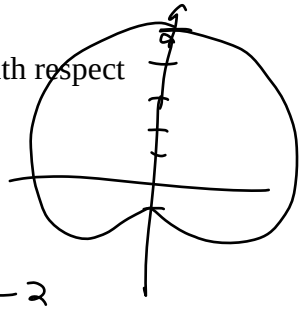
$d \approx 8.299$

6. Find a polar equation for a limaçon with a dimple that is symmetric with respect to the *y*-axis.

$\sin(x)$

$2 > \frac{a}{b} > 1$

$3 + 2\sin\theta$



7.. State the domain and range of $r = 3 + 5\sin\theta$

$3 + 5(-1) = -2$

$3 + 5(1) = 8$

D: $\mathbb{R}$

R: $[-2, 8]$

HONORS PRECALCULUS

WS: 6.6

I. Express each complex number in POLAR form.

1. $-\sqrt{3}+i$

2. $-\sqrt{2}+i\sqrt{2}$

3. $5-5i\sqrt{3}$

II. Express each complex number in RECTANGULAR form.

4. $2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

5. $4cis\frac{5\pi}{6}$

6. $3cis\frac{11\pi}{4}$

III. Find each product or quotient. Then write the result in rectangular form.

7. $\left(4cis\frac{\pi}{3}\right)\left(3cis\frac{5\pi}{3}\right)$

8. $\left(3cis\frac{7\pi}{4}\right)\left(2cis\frac{4\pi}{3}\right)$

9. $\frac{7cis\frac{5\pi}{6}}{14cis\frac{\pi}{4}}$

IV. Find each power. Then write the result in rectangular form.

10. $\left(2-2i\sqrt{3}\right)^5$

11. $(1+i)^8$

12. $\left(\sqrt{2}-i\right)^{10}$

HCP PRECALCULUS – CIERECH
Review CHAPTER 6 **NONCALCULATOR**

1. The unit vector v in the direction of $\langle -8, -15 \rangle$ is $\left\langle -\frac{8}{17}, \frac{-15}{17} \right\rangle$

2. Find the component form of the vector originating from (3,-1) with terminal point (-5, 6). $\langle -8, 7 \rangle$

3. Give the rectangular **coordinates** for each point:

a. $(2\sqrt{2}, \frac{\pi}{4})$ $(2,2)$

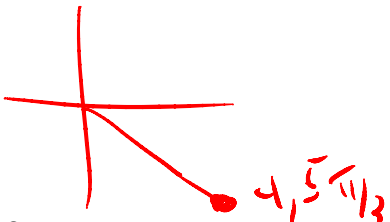
b. $(-2, -30^{o})$ $(-\sqrt{3}, 1)$

4. Give the polar **coordinates**, with $r > 0$ and $0 \le \theta < 2\pi$, for each of the following:

a. $(2,0)$ $(2,0)$

b. $(-1, -\sqrt{3})$ $\left(2, \frac{4\pi}{3}\right)$

5. Plot the point with polar coordinate $(4, \frac{5\pi}{3})$. Then find two other sets of polar coordinates for this point: one set with r < 0 and the other with θ <0.



$\left(-4, \frac{2\pi}{3}\right)$

$\left(4, -\pi/3\right)$

6. Express z_1, z_2, *and* $z_1 z_2$ in polar form if $z_1 = 2 - 2i$ and $z_2 = 1 + \sqrt{3}i$.

$z_1 = 2\sqrt{2}\ \text{cis}\ \frac{7\pi}{4}$

$z_2 = 2\ \text{cis}\ \frac{\pi}{3}$

$z_1 \cdot z_2 = 4\sqrt{2}\ \text{cis}\ \frac{25\pi}{12}$

7. Let $z = 3cis\,120^o$. Find z^2 in polar form and in rectangular form.

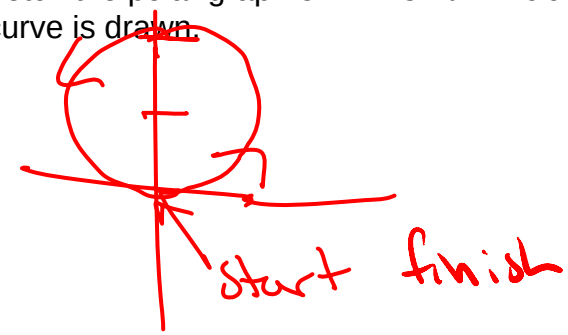
$9\,cis\,240 = 9\left(-\frac{1}{2} + \frac{-\sqrt{3}}{2}i\right) = \boxed{-\frac{9}{2} - \frac{9\sqrt{3}}{2}i}$

8. Find $(1+3i)^3$ using De Moivre's Theorem

$(1+3i)^3 = \left(\sqrt{10}\,cis(1.249)\right)^3 = 10\sqrt{10}\,cis(3.747)$

$= -26 - 18i$

9. Sketch the polar graph of $r = 2\sin\theta$. Include arrows indicating which way the curve is drawn.



θ	r
0	0
$\pi/6$	1
$\pi/4$	$\sqrt{2}$
$\pi/3$	$\sqrt{3}$
$\pi/2$	2

CALCULATOR PART

10. Determine the magnitude of the vector with initial point (-3, 8) and terminal point (5, -2).

$2\sqrt{41}$

11. A vector v has magnitude 5 and direction $\theta = \frac{3\pi}{4}$. Find v.

$\left\langle -\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$

12. A plane is on a bearing of 55^o at a speed of 500 mph. If there is a 35 mph wind at a bearing of 65^o, what s the resultant bearing and speed of the plane? $500\langle\cos 35, \sin 35\rangle + 35\langle\cos 25, \sin 25\rangle =$

$\langle 441.297, 301.507\rangle$ $|v| = 534.503$ mph

bearing of 55.652^o

13. Let $\mathbf{u} = \langle 1,1 \rangle$. Find the vector $\mathbf{v}$ such that $\mathbf{u} \cdot \mathbf{v} = 8$ and $|\mathbf{v}| = \sqrt{32}$.

$V = \langle x, y \rangle$ since $u \cdot v = 8$ $2x^2 - 16x + 32 = 0$

$x + y = 8$ or $y = 8 - x$ $2(x^2 - 8x + 16) = 0$

$2(x-4)^2 = 0$

$\sqrt{x^2+y^2} = \sqrt{32}$ $x = 4$ $\langle 4, 4 \rangle$

$x^2 + (8-x)^2 = 32$ $y = 4$

14. Determine the parameterization of the line segment with end points A = (2, -3) and B = (1, 2).

$\vec{OP} - \vec{OA} = t(\vec{OB} - \vec{OA})$

$\langle x, y \rangle - \langle 2, -3 \rangle = t(\langle 1, 2 \rangle - \langle 2, -3 \rangle)$

$\langle x-2, y+3 \rangle = t\langle -1, 5 \rangle$

$x - 2 = -t$

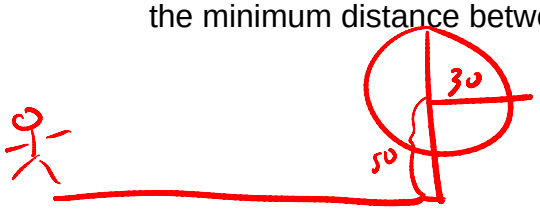
$y + 3 = 5t$

$x = 2 - t$

$y = 5t - 3$ $\quad 0 \le t \le 1$

diameter

15. A Ferris wheel has ~~radius~~ of 60 feet, its center is 50 feet off the ground, and it takes 12 seconds to complete one revolution in the counterclockwise direction. Sally is standing 80 feet to the left of the center of the Ferris wheel. Sally throws a ball the instant Kathy is at the 3 o'clock position. She releases it at a height of 6 feet with an initial velocity of 100 ft/sec. and at an angle of 70 degrees. What is the minimum distance between the ball and Kathy?



KATHY: $x_1 = 30\cos\left(\frac{\pi}{6}t\right)$

$y_1 = 50 + 30\sin\left(\frac{\pi}{6}t\right)$

SALLY: $x_2 = \left(100\cos\frac{7\pi}{18}\right)t - 80$

$y_3 = -16t^2 + \left(100\sin\frac{7\pi}{18}\right)t + 6$

$X_3 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$Y_3 = T$

min. Distance ≈ 60.156
at $t = 2$